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Farsightedness in Games: Stabilizing Cooperation in International Conflict

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Abstract

We show that a cooperative outcome—one that is at least next-best for the players—is not a Nash equilibrium (NE) in 19 of the 57 2 × 2 strict ordinal conflict games (33%), including Prisoners' Dilemma and Chicken. Auspiciously, in 16 of these games (84%), cooperative outcomes are nonmyopic equilibria (NMEs) when the players make farsighted calculations, based on backward induction; in the other three games, credible threats induce cooperation. More generally, in all finite normal-form games, if players' preferences are strict, farsighted calculations stabilize at least one Pareto-optimal NME. We illustrate the choice of NMEs that are not NEs by two cases in international relations: (i) no first use of nuclear weapons, chosen by the protagonists in the 1962 Cuban missile crisis and since adopted by some nuclear powers; and (ii) the 2015 agreement between Iran, and a coalition of the United States and other countries, that has been abrogated by the United States but has forestalled Iran's possible development of nuclear weapons.

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1. Introduction

The standard solution concept in noncooperative game theory is that of Nash equilibrium (NE). However, what might be considered a "cooperative outcome" in a significant number of games is not an NE.[1] The best-known examples of such games are Prisoners' Dilemma and Chicken.

To justify cooperation in such games, the usual approach is to posit repeated play of a game. According to the folk theorem of noncooperative game theory, all Pareto-optimal outcomes become NEs if (i) the repetition is infinite or has no definite end and (ii) the players are sufficiently patient. But most real-life games are not played, *de novo*, again and again; moreover, the resulting plethora of NEs in repeated play has little predictive power.

In this paper, we take a different approach: Starting from any outcome, players make farsighted calculations of where play will terminate after a finite series of moves and countermoves *within* a game. In almost all 2 × 2 strict ordinal *normal-form games*—representable by a payoff matrix in which the players are assumed to make independent strategy choices—cooperative outcomes that are not NEs stabilize as *nonmyopic equilibria* (NMEs), an alternative equilibrium concept we will define and illustrate with Prisoners' Dilemma, Chicken, and other games.

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[1] It is the strategies associated with an outcome, not the outcome itself, that define an NE. Because a unique pair of strategies is associated with each outcome in the 2 × 2 strict ordinal games that we analyze, for brevity we identify NEs in these games by the outcomes rather than the strategies that yield them.

NMEs are based on rules wherein play commences at an outcome (or *initial state*)—rather than with the choice of strategies—and players can move or countermove from that state according to rules that we will specify. If players would not move from an initial state, anticipating all possible moves and countermoves in a game of complete information, then that state is an NME. (A state may also be an NME if players would move to it from another state—not just stay at it if they start there—which we discuss later.) In Prisoners' Dilemma and Chicken, the cooperative outcome in each game is an NME when play commences at it.

In a few games, however, cooperative outcomes that are not NEs are also not NMEs if play starts there. Fortunately, cooperation in them can be induced in these games by one player's credible threat of a Pareto-inferior outcome if its opponent does not comply with the threat.

Thereby we show that in all 2×2 strict ordinal games that have cooperative outcomes that are not NEs, either (i) the nonmyopic stability of a cooperative outcome or (ii) one player's credible threat of a worse outcome (for both players) stabilizes cooperation in these games. After enumerating these games in section 2, we spell out the rules of play and rationality rules for calculating NMEs and determining credible threats in them in section 3.

In particular, we distinguish the 2×2 games that have cooperative outcomes, at least one of which is an NE, from games with no cooperative outcomes. In some games, the NMEs are "boomerang NMEs," whereby players have an incentive to move back and forth between two diagonally opposite NMEs. In Prisoners' Dilemma, the

noncooperative outcome, which is not Pareto-optimal, and the cooperative outcome, which is, are NMEs. In all other 2 × 2 games, the NMEs are Pareto-optimal.

In section 4, we extend the analysis to all finite games (2-person and *n*-person) in normal form, wherein we define a cooperative outcome to be a maximin outcome. Like the 2 × 2 games, cooperative outcomes in these games may not be NMEs. But these games always contain at least one Pareto-optimal NME.

In section 5, we suggest that two real-world conflicts in international relations can be modeled by games in which cooperation, while not an NE, was achieved. The players in the first conflict, the 1962 Cuban missile crisis, were the United States and the Soviet Union, both nuclear powers. The crisis has been modeled by different 2 × 2 games, whose cooperative outcomes are not NMEs; we do not describe these models here, because they have been developed in detail elsewhere. But we argue that they illustrate why, since 1962, some nuclear states have agreed to no first use of nuclear weapons, even though they might benefit, in an immediate or myopic sense, from their first use.

The second conflict, which we analyze in more detail, is the game played between Iran and a coalition that includes the United States and other countries. It culminated in a 2015 agreement whereby Iran submitted to rigorous inspections of its nuclear facilities— to prevent the enrichment of uranium for possible use in weapons—in return for the gradual lifting of economic sanctions that had been imposed on it. Although this agreement has been abrogated by the Trump administration, Iran has continued to abide by it, suggesting that it is still nonmyopically stable.

In section 6, we discuss the normative implications of justifying cooperation, if it is not myopically stable, with NMEs or credible threats. This more expansive view for

ameliorating conflict may give policy makers a greater incentive to consider the long-term consequences of their choices, even when an agreement in the short term is unstable.

2. 2 × 2 Games with Cooperative Outcomes That Are Not NEs

There are 78 2 × 2 strict ordinal games in normal (strategic) form that are distinct in the sense that no interchange of players, strategies, or a combination of players and strategies can transform one of these games into another (Rapoport and Guyer, 1966; Rapoport, Gordon, and Guyer, 1976). Let (x, y) be the ordinal payoffs to the players, where x is the payoff to the row player and y is the payoff to the column. Assume the following ranking: 4 = best, 3 = next best, 2 = next worst, and 1 = worst. Of the 78 games, 57 are *conflict games* (73%), in which there is no mutually best (4,4) outcome and which we focus on next.

In 44 of the 57 conflict games (77%), there are either one or two *cooperative outcomes* (x, y) that are at least next best for both players—(3,3), (3,4), or (4,3). It turns out that the cooperative outcomes in 25 of the 44 conflict games are NEs, which break down as follows:[2]

(i) 22 games have *one* cooperative outcome that is also an NE (games 1-4, 7-9, 12-17, 20-21, 23-24, 38-41, 53);

(ii) 3 games have *two* cooperative outcomes, both of which are NEs (games 51, 54-55).

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[2] For reference purposes, we show the numbers of these games that are given in the classification scheme of Brams (1994, 2011); for other classification schemes, see Bruns (2015) and references therein.

We do not consider these games further, because cooperation does not need to be stabilized in them. In fact, all the cooperative outcomes in these games are both NEs and NMEs.

In the remaining 44 – 25 = 19 games, which constitute 33% of the 57 conflict games, exactly one cooperative outcome is not an NE in pure strategies:[3]

   (iii) 4 games have *two* cooperative outcomes, one of which is not an NE (games 33-34, 36-37);

   (iv) 15 games have *one* cooperative outcome, which is not an NE (games 22, 27-32, 35, 46-50, 56-57).

In section 3, we show that in 16 of the 19 games (84%), the non-NE cooperative outcome can be stabilized as an NME, which we will illustrate with Prisoners' Dilemma and Chicken (these games happen to be the only symmetric games of the 19).[4] In the 3 games in which this is not possible, credible threats can induce the choice of a cooperative outcome.

To summarize, exactly 1/3 of the 57 conflict games have cooperative outcomes that are not NEs. Except for 3 of these 19 games, all the cooperative outcomes are NMEs

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[3] Because ordinal games do not permit the calculation of mixed strategies, we do not consider them in our analysis. But even if one could attribute utilities to the players' rankings, the interpretation of mixed strategies is problematic; moreover, the players' expected payoffs from mixed-strategy NEs are Pareto-inferior to at least one pure-strategy outcome in a game.

[4] A 2 × 2 ordinal game is *symmetric* if the payoffs can be arranged so that the players have the same ranking of outcomes along the main diagonal, and their off-diagonal rankings are mirror images of each other on each side of the diagonal.

when play starts there, which stabilizes cooperation when the players are farsighted in calculating whether or not it is rational for them to depart from cooperation. Even in the 3 games in which farsightedness is not sufficient to deter a player from moving from a cooperative outcome, a credible threat by one player, which we describe in section 3 as well as define NMEs, renders it rational for both players choose the non-NE cooperative outcome.

3. Nonmyopic Equilibria (NMEs) and Credible Threats in the 19 Games with a Cooperative Outcome That Is Not an NE

In Figure 1, we depict the 2×2 games of Prisoners' Dilemma (game 32) and Chicken (game 57). The NEs in Prisoners' Dilemma and Chicken, from which neither player would depart because it would do immediately worse if it did, are underscored; the NMEs are shown in boldface.

Figure 1 about here

In addition, the NMEs from every state are given in brackets below each outcome. Thus, for example, the NME from the cooperative outcome in each game, (3,3), is [3,3]—that is, if (3,3) is the initial state, the players would not depart from this outcome when they calculate the consequences of possible moves and countermoves from (3,3) by each player.

To describe how NMEs are calculated from each state, we begin by specifying four *rules of play*:[5]

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[5] Brams (1994, 2011) gives a full account of the *theory of moves* (*TOM*), in which the concepts of TOM are developed in depth, whereas in this section we define NMEs, credible threats, and a few ancillary concepts

1. Play starts at an outcome, called the *initial state,* which is at the intersection of the row and column of a 2 × 2 payoff matrix.

2. Either player can unilaterally switch its strategy, and thereby change the initial state into a new state, in the same row or column as the initial state. Call the player that switches, who may be either R or C, player 1.

3. Player 2 can respond by unilaterally switching its strategy, thereby moving the game to a new state.

4. The alternating responses continue until the player (player 1 or player 2) whose turn it is to move next chooses not to switch its strategy. When this happens, the game terminates in a *final state,* which is the *outcome* of the game.

Note that the sequence of moves and countermoves is strictly alternating: First, say, R moves, then C moves, and so on, until one player stops, at which point the state reached is final and, therefore, the outcome of the game.[6]

The use of the word "state" is meant to convey the temporary nature of an outcome before players decide to stop switching strategies. We assume that no payoffs accrue to players from being in a state unless it is the final state and, therefore, becomes the outcome (which could be the initial state if the players choose not to move from it).

Rule l differs radically from the corresponding rule of play in standard game theory, in which players simultaneously choose strategies in a normal-form (matrix) game, which determines its outcome. Instead of starting with strategy choices, we

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that are used later. In section 4, we give new results on games with cooperative outcomes that are not NEs, and in section 5 we define boomerang NMEs, illustrate NMEs that are not maximin, and prove one theorem about the existence of at least one Pareto-optimal NME in all normal-form games.

[6] Rules that allow for backtracking are analyzed in Willson (1998) and applied in Zeager, Ericcson, and Williams (2013).

assume that players are already in some state at the start of play (the status quo) and receive payoffs from this state only if they choose to stay. Based on these payoffs, they decide, individually, whether or not to change this state in order to do better, which may be physical moves or a thought experiment that anticipates future choices.

In summary, play of a game starts in a state, at which players accrue payoffs only if they remain in that state so that it becomes the outcome of the game. If they do not remain, they still know what payoffs they would have accrued had they stayed; hence, they can make a rational calculation of the advantages of staying versus moving. They move precisely because they calculate that they can do better by switching states, anticipating a better outcome when the move-countermove process comes to rest.

Rules l-4 say nothing about what causes a game to end but only when: Termination occurs when a "player whose turn it is to move next chooses not to switch its strategy" (rule 4). But when is it rational not to continue moving, or not to move in the first place from the initial state?

To answer this question, we posit a rule of *rational termination*. It prohibits a player from moving from an initial state unless doing so leads to a better (not just the same) final state, based on the following rule:

5. A player will not move from an initial state if this move

  (i) leads to a less preferred final state (i.e., outcome); or

  (ii) returns play to the initial state (i.e., makes the initial state the outcome) or to any other previously visited state.

We discuss and illustrate shortly how rational players, starting from some initial state, determine by backward induction what the outcome will be.

Condition (i) of rule 5, which precludes moves that result in an inferior state, needs no defense. But condition (ii), which precludes moves that will cause players to cycle back to the initial state, is worth some elaboration. It says that if it is rational for play of

a game to cycle back—in either a clockwise or counterclockwise direction—to the initial state after player 1 moves, player 1 will not move in the first place. After all, what is the point of initiating the move-countermove process if play simply returns to "square one," given that the players receive no payoffs along the way (i.e., before an outcome is reached)?

Not only is there no gain from cycling but, in fact, there may be a loss because of so-called transaction costs—including the psychic energy spent—that players suffer by virtue of making moves that, ultimately, do not change the situation. Therefore, it seems sensible to assume that player 1 will not trigger a move-countermove process if it only returns the players to the initial state, making it the outcome.
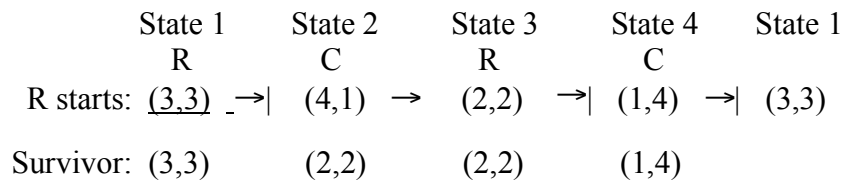
We call rule 5 a *rationality rule,* because it provides the basis for players to determine whether they do better by moving from a state or remaining in it. But another rationality rule is needed to ensure that both players take into account each other's calculations before deciding to move from the initial state. We call this rule the *two-sidedness rule*, which assumes that the players have complete information about each others' preferences that is common knowledge, and act according to the preceding five rules:

6. Each player takes into account the consequences of the other player's rational choices, as well as its own, in deciding whether to move from the initial state or subsequently, based on backward induction (to be defined and illustrated shortly). If it is rational for one player to move and the other player not to move from the initial state, then the player who moves takes *precedence*: Its move overrides the player who stays, so the outcome is that induced by the player who moves.

Because players have complete information, they can look ahead and anticipate the consequences of their moves. We next demonstrate, using backward induction, that if

(3,3) is the initial state in Prisoners' Dilemma, the players would not move from this state, making it the NME from this state.

Assume R moves first from (3,3), moving play to (4,1), whence play continues cycling counterclockwise back to (3,3), progressing from (4,1) to (2,2) to (1,4) to (3,3). The player (R or C) who makes the next move, shown below each state, alternates:[7]

	State 1	State 2	State 3	State 4	State 1
	R	C	R	C	
R starts:	(3,3) →|	(4,1) →	(2,2) →|	(1,4) →|	(3,3)
Survivor:	(3,3)	(2,2)	(2,2)	(1,4)	

The *survivor* is determined by working backwards, after a putative cycle has been completed, which is calculated in the following manner. Assume that the players' alternating moves have taken them counterclockwise from (3,3) eventually to (1,4), at which point C must decide whether to stop at (1,4) or complete the cycle and return to (3,3).

Clearly, C prefers (1,4) to (3,3), so (1,4) is listed as the survivor below (1,4): Because C would *not* move the process back to (3,3) should it reach (1,4), the players know that if the move-countermove process reaches this state, the outcome will be (1,4). We indicate that it is not rational for C to move on from (1,4) by the vertical line blocking the arrow emanating from (1,4), which we refer to as *blockage*: A player will always stop at a blocked state, wherever it is in the progression.

Would R at the prior state, (2,2), move to (1,4)? Because R prefers (2,2) to the survivor at (1,4)—namely, (1,4)—the answer is no. Once again there is blockage, and

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[7] Effectively, this is a game tree, or game in extensive form, showing a sequence of alternating choices of the players, except that instead of branching from top to bottom, as is the usual representation, the choices of the players go sideways, from left to right. More conventional game trees that illustrate TOM calculations are given in Taylor and Pacelli (2008).

(2,2) becomes the new survivor when R must choose between stopping at (2,2) and moving to (1,4).

At the state prior to (2,2), (4,1), C would prefer staying at (2,2) rather than moving to (4,1), so (2,2) again is the survivor if the process reaches (4,1). However, at the initial state, (3,3), because R prefers (3,3) to (2,2), (3,3) becomes the survivor at the initial state, and there is again blockage. But in this case we call the blockage *stoppage*, because it occurs for the *first* time from the initial state, (3,3); we underscore (3,3) to indicate that it is the last surviving state.

The fact that (3,3) is the survivor at the initial state (3,3) means that it is rational for R not to move from (3,3). That is, after working *backward* from C's choice of completing or not completing the cycle at (1,4), the players can reverse the process and, looking *forward,* determine that it is rational for R not to move from (3,3). Likewise, it is also rational for C not to move from (3,3) because of the symmetry of Prisoners' Dilemma.

An analogous argument shows that it is not rational for either player to move from (2,2), making this outcome an NME from itself, as shown in Figure 1. Applying backward induction from (4,1) and (1,4) indicates that C and R, respectively—each receiving only a payoff of 1—would move to (2,2), where play would stop. But, in fact, we amend this calculation of NMEs from (4,1) and (1,4) according to the following convention:

> **Two-sidedness convention (TSC):** If one player (say, C), by moving, can induce a better state for itself than by staying—but R by moving can induce a state Pareto-superior to C's induced state—then R will move, even if it otherwise would prefer to say, to effect a better outcome.

To illustrate this convention, observe that from (4,1), C can induce a better state for itself by moving to (2,2). But because R, by moving first to (3,3), can induce a state

Pareto-superior to (2,2)—that is, one better for both players—it behooves R *not* to stay at (4,1) but instead to move to (3,3). Moreover, it is also in C's interest to defer its move to (2,2) to enable R to implement (3,3).

Although we could make TSC a new rule (i.e., rule 7), it seems better to call it a "convention" because it clarifies a circumstance when rule 6 (i.e., the two-sidedness rule) is operative—that is, when a move by a player takes precedence. In our example, although C at (4,1) immediately benefits by moving to (2,2) rather than staying at (4,1), TSC says that it nevertheless is rational for C to stay in order to allow R to move first to (3,3).

Although TSC is applicable to Prisoners' Dilemma, it does not apply to Chicken. Whereas in Prisoners' Dilemma a beneficial move by a player from an initial state may lead to a Pareto-inferior outcome (in Prisoners' Dilemma, to (2,2) from (4,1) or (1,4)), the NMEs from the four initial states in Chicken are all Pareto-optimal, as shown in Figure 1.

But Chicken introduces a new wrinkle into the calculation of NMEs in games, for which we list two NMEs, separated by a slash, from states (4,2), (2,4), and (1,1). In the case in which (1,1) is the initial state, for example, [2,4]/[4,2] indicates that if R moves first, the NME is [2,4], whereas if C moves first the NME is [4,2]. This renders (1,1) an *indeterminate state*, because either of these two NMEs could occur, depending on which player moves first from (1,1).

If one player can dictate the order of moves from an indeterminate state, we say that it has *order power*. Thus, if (1,1) is the initial state and R has order power, it would be rational for it to force C to move first, because R prefers that the NME be (4,2) rather than (2,4).

By contrast, if (4,2) or (2,4) is the initial state, the player with order power would prefer to move first rather than second. For example, from (4,2), the NME is (3,3) if R moves first and (2,4) if C moves first; because R prefers (3,3) to (2,4), it would choose to move first if it possessed order power.

In 16 of the 19 games (84%) in which a cooperative outcome is not an NE, including Prisoners Dilemma and Chicken, this outcome is an NME when the cooperative outcome is the initial state. We list these games in Figure 2, wherein below every cooperative outcome—(3,3) or (3,4) in the upper left of each game—this outcome appears in brackets, indicating that it is the NME from this state. Observe that these cooperative outcomes are also NMEs from other states in several of the 19 games.

Other outcomes may be NMEs from different states. In particular, in four games (33, 34, 36, and 37) there is a second cooperative outcome, (4,3), that is both an NE, and an NME from itself, as shown in Figure 2.

Figure 2 about here

There are three games, shown in Figure 1, in which there is a cooperative outcome, (3,3), that is not an NE or an NME from itself. Instead, starting at (3,3), the NME is (2,4) in all these games (22, 49, and 57).

Fortunately, a threat by one player (T) in these games can induce (3,3) if the threat is *credible*—that is, the threatened player ($\overline{T}$) believes that T will carry out its threat, which will produce a worse outcome for both players than some other outcome in the game. We emphasize that T's exercise of a threat does not depend on backward induction from an initial state but instead on the threat of choosing one of its strategies, which we assume is communicated by T to $\overline{T}$.

Threats come in two varieties, compellent and deterrent, which is a distinction made by Schelling (1964) and formalized by Brams (1994, 2011):

Compellent threat: A compellent threat is a threat by T to stay at a particular strategy to induce $\overline{T}$ to choose its, as well as T's, best outcome associated with that strategy.

Deterrent threat: A deterrent threat is a threat by T to switch to another strategy

to induce $\overline{T}$ to choose an outcome associated with T's initial strategy that is better for both players than the threatened state.

Whereas a deterrent threat deters an opponent from moving to a state, a compellent threat forces an opponent to move to a desired state.

Games 49 and 56 illustrate R's compellent threat of choosing its first strategy, and refusing to move from it, thereby presenting C with a choice between (1,2) and (3,3) in game 49, and between (1,1) and (3,3) in game 56. Obviously, C in each game would prefer (3,3), associated with its first strategy, when R sticks with its first strategy, which "compels" C also to choose its first strategy. Thereby R can induce (3,3) in each game, which is Pareto-superior to (1,2) in game 49, and (1,1) in game 56, if it has a credible threat of choosing its first strategy. We call (1,2) in game 49 and (1,1) in game 56 the *breakdown outcomes*, and (3,3) the *threat outcome* in each game.

Game 22 in Figure 1 illustrates R's deterrent threat of choosing its second strategy, thereby presenting C with a choice between (4,2) and (1,2), C's two worst outcomes. Clearly, C would prefer (1,2). But this outcome is worse for both players than (3,3), associated both players' first strategies. Thus, it is rational for C to choose its first strategy when R agrees to choose its first strategy if R's threat of choosing its second strategy is credible.

R's credible compellent or deterrent threats in games 22, 49, and 56 induce the choice of the cooperative outcome, (3,3), in all three games, which is not an NME when (3,3) is the initial state. It is true that (3,3) is an NME from other states in games 49 and 56, but in game 22 this is not the case—the NE of (2,4) is the NME from every state in this game. Thus, a compellent or deterrent threat by R in these games, if credible, provides an alternative mechanism for inducing the cooperative outcome.

In summary, in 16 of the 19 games in which there is a cooperative outcome that is not an NE, it is an NME from itself as well as from other outcomes in several of these

games. For the three games in which the cooperative outcome is not an NME from itself, either a compellent or deterrent threat can induce the choice of this outcome if one player has a credible threat (for details, see Brams, 1994, 2011). These results underscore how either farsighted thinking or credible threats can induce a cooperative outcome in all 2 × 2 games in which cooperation is not a NE.

4. Extensions of the Analysis

We have said nothing about the 57 – 19 = 38 2 × 2 conflict games (67%) in which every cooperative outcome is an NE (26 games) or there is no cooperative outcome (12 games). In the former 26 games, the cooperative outcomes are all NMEs as well as NEs, so they are both myopically and nonmyopically stable. In the 3 games (51, 54, and 55) with two NEs/NMEs, which one will be chosen depends on where play commences (i.e., the initial state) unless threat power—or some other kind of power (e.g., order or moving power) that creates an asymmetry in the players' capabilities (Brams, 1994, 2001)—decides this question.

We would expect cooperation to prevail in these games, because no other outcome in them gives both players at least a next-best outcome (i.e., 3). However, if there are two cooperative outcomes (there cannot be three cooperative outcomes in a 2 × 2 game), there may be a conflict about which one will be chosen. This conflict would seem most severe in the 3 games (51, 54, and 55, which are not shown) in which both cooperative outcomes— (3,4) and (4,3)—are NEs as well as NMEs.

In the 12 conflict games (21%) that contain no cooperative outcome, the only possible NMEs are (2,3), (3,2), (2,4) or (4,2), because an outcome that is worst for one or both players (i.e., 1) can never be an NME (or an NE). Eight of these games have a

unique NME: In 4 games (10, 11, 25, 26) it is (2,3) or (3,2), and in 4 games (5, 6, 18, 19, 22) it is (2,4) or (4,2). The remaining 4 games (42, 43, 44, 45) have two NMEs, neither of which is an NE that might give it some greater claim to being chosen.[8]

The picture that emerges from this brief overview of the 57 2 × 2 conflict games is that the vast majority (45, or 79%) have cooperative outcomes and, except in game 22 (see Figure 1), they are always NMEs. This is not to say that there may not be conflict— in particular, about which of two NMEs will be chosen. But it is probably the 12 games without cooperative outcomes, wherein an NME is always a next-worst outcome (i.e., 2) for one player, that make cooperation most tenuous.

To return to our earlier analysis in section 2 of the 19 games in which there is a unique NME that is not an NE, it is pleasing that cooperation can be stabilized in 16 of these games when play commences at them. But when play starts elsewhere, other NMEs may be chosen, including (2,2) in Prisoners' Dilemma (game 32), which is Pareto-inferior to (3,3) and the unique NE in this game, or (2,4) or (4,2) in Chicken (game 57), which are the two pure-strategy NEs in this game.[9]

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[8] The lack of pure-strategy NEs in these 4 games means that their solution in standard game theory can only be in mixed strategies, provided the players associate cardinal utilities with the ranks of the four outcomes. In that case, they can calculate how to randomize, according to some probability distribution, their choices between their two pure (i.e., single) strategies. There are other 2 × 2 conflict games that do not have pure-strategy Nash equilibria, but these 4 games differ in having two NMEs, neither of which is a cooperative outcome.

[9] Chicken has a third NE in mixed strategies, but it is not defined in games with in which preferences are ordinal (see note 8).

Even if play starts at a cooperative NME, however, one player may have an incentive to move from it to another NME, from which the other player would, in turn, have an incentive to return to the original NME. Both outcomes are NMEs, because the players would move to them from some other outcome—making them *attractors*—even though at least one player would not stay at them.

This was true in the case of the cooperative (3,3) outcome in games 49 and 56 in Figure 1. Starting at (3,3) in these games the players would move to the diagonally opposite outcome, (2,4), whence they would return to (3,3), making both of these outcomes *boomerang NMEs*: There would be a bouncing back and forth between them.

 "Ping-pong NMEs" could serve equally well to describe this back-and-forth movement. In fact, the two NEs in Chicken are boomerang NMEs. At (2,4), R benefits by moving first from (2,4) to (1,1), whence C would move to (4,2) and play would terminate; at (4,2), C benefits by moving first from (4,2) to (1,1), whence R would move to (2,4) and play would terminate.

To be sure, if C has order power in the former case and R does in the latter, these players would move first to (3,3), where play would terminate. Besides Chicken (game 57) and games 49 and 56 in Figure 1, there are five other games in which the two outcomes along a diagonal are boomerang NMEs as well as being NEs: games 51, 52, 53, 54, 55 (not shown).

Although at least one cooperative outcome is an NME in all these games, once play reaches it, the players may not be motivated to stay. Enduring international rivalries, such as that between Israel and several Arab countries over the past 70 years and between France and Germany in past centuries, reflect the unsteadiness of even

NMEs, when—after a war has been fought and there is a lull in hostilities—war resumes

some time later. In these conflicts, while there may be a temporary reprieve from

conflict, it is not steadfast, because a rival who was defeated will often have an incentive

to resume the conflict if it believes it can do better in the next encounter.

What about 2-person conflict games larger than 2 × 2? In fact, they exhibit some

of the same features that we have observed in the 2 × 2 games. Consider the following 2

× 3 game, in which the two players rank the six outcomes from best (6) to worst (1):

(6,1)	(4,4)	(1,6)
(5,2)	(3,3)	**(2,5)**

Define a *cooperative outcome* to be one that maximizes the minimum payoff to

the players, which is (4,4) in this game. Like (3,3) in game 22 (Figure 1), (4,4) is not an

NME, because C would move to (1,6), whence R would move to (2,5), from which C

would not move, because subsequent rational moves would return play to (4,4), where C

would do worse. By similar reasoning, it is not difficult to show that starting from any

outcome other than (4,4), the players would stay at or move to (2,5), making it the unique

NME in the 2 × 3 game (it is also the unique NE and so is shown in boldface and also

underscored).[10]

This game illustrates that a maximin outcome like (4,4)—or (3,3) in game 22—

which one might expect to be nonmyopically stable, will not always be so. Instead, a

more one-sided outcome like (2,5)—or (2,4) in game 22—may be the only NME. Like

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[10] In Brams (1994, pp. 11-17), rules of play for larger games are illustrated by a 3 × 3 game.

(2,4) in game 22, (2,5) in the 2 × 3 game is associated with the dominant strategy of one player (C).

Other outcomes one might not expect to be NMEs include (2,2) in Prisoners' Dilemma (game 32). This outcome is Pareto-inferior to the cooperative outcome, (3,3), which is also an NME—not only from itself but also from (4,1) and (1,4)—whereas (2,2) is an NME only if play starts at this state. In fact, (2,2) in Prisoners Dilemma is the only Pareto-inferior NME in all 57 of the 2 × 2 conflict games, suggesting that Pareto-inferior NMEs in larger games are likely to be rare. Fortunately, in both 2 × 2 and larger games, it is never the case that all NMEs are Pareto-inferior if the preferences of players are strict, as we prove next.

Theorem. *In every finite normal-form game (2-person or n-person), at least one NME is Pareto-optimal*.

Proof. Assume the contrary—that all NMEs are Pareto-dominated. Consider the NME from some outcome, and assume it is Pareto-dominated by some Pareto-optimal outcome X (there must be at least one X).

Consider all backward-induction paths from X back to itself. X will displace all outcomes, including NMEs, that it Pareto-dominates along the path back to itself.

Similarly, other Pareto-dominated NMEs from Xs that dominate them cannot, as assumed, be NMEs. Instead, these Xs must be NMEs from themselves or go into other Xs. Because there is at least one X, there must be at least one Pareto-optimal NME. Q.E.D.

To illustrate Theorem, recall that (2,2) is a Pareto-dominated NME from itself in Prisoners' Dilemma (game 32). But starting at (3,3), which Pareto-dominates (2,2), (3,3) displaces (2,2) along the path back to itself, rendering (3,3) the NME from itself.

As an illustration of Theorem in an *n*-person game, consider the following $2 \times 2 \times 2$ game:

(3,7,3)	**(5,6,5)**		(4,5,4)	(7,3,6)
(2,4,1)	(8,1,2)		(1,8,7)	(6,2,8)

In this game, in which the third player chooses the left or right matrix and receives the third payoff in every state, (5,6,5), shown in boldface, is the unique Pareto-optimal NME from all states, but it is not an NE—the Pareto-nonoptimal (4,5,4), which is underscored, is the unique NE.

To show that (5,6,5) is the unique NME, we note that the NMEs in the left and right matrices are (5,6,5) and (4,5,4), respectively, if we treat each matrix as a distinct game. Because the third player prefers the NME in the left matrix to the one in the right matrix, the NME in the 3-person game will be (5,6,5), wherever play starts, when the third player chooses the left matrix. Thus, like Prisoners' Dilemma, this is another example of a game in which the unique NME Pareto-dominates the unique NE, illustrating the advantage to the players of thinking ahead rather than myopically.[11]

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[11] All our examples assume that the preferences of the players are strict. If they are not, more than one NME may survive the backward induction process because of ties. But ties have no effect on Theorem—at

5. No First Use of Nuclear Weapons and the 2015 Iran Agreement
on Nuclear Weapons

No First Use of Nuclear Weapons

The United States was the first country to develop, and the only country ever to use, nuclear weapons in war. The two atomic bombs dropped on Hiroshima and Nagasaki in August 1945 brought World War II to a close when Japan surrendered after the second bomb was dropped.

Since the Soviet Union (now Russia) developed nuclear weapons in 1949, seven other countries are now known to possess such weapons (China, France, India, Israel, North Korea, Pakistan, and the United Kingdom). Each has threatened their use if attacked, but three countries (China, India, and Israel) have gone further by declaring that they will not be the first to introduce them into a conflict (https://en.wikipedia.org/wiki/No_first_use).

With the exception of China, whose declaration is unqualified, India and Israel have indicated minor qualifications in their declarations. Taken at face value, however, they have pledged no first use (NFU) of nuclear weapons. All the other nuclear powers have said they would use their weapons only defensively—in retaliation against a nuclear attack—and some have said they would never use such weapons against an attack by a country that did not possess nuclear weapons.

The most serious threat of a nuclear attack occurred during the Cuban missile crisis in October 1962. This confrontation between the Soviet Union and the United

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least one NME must be Pareto-optimal in the sense of there being no other outcome that is weakly preferred to it by the players.

States is sometimes modeled as a game of Chicken (game 57 in Figures 1 and 2), though Brams (1994, 2011) argues that a different game (game 30 in Figure 2) is a more accurate representation of choices in the crisis.

In both Chicken and game 30, the (3,3) cooperative outcome is not an NE, but it is an NME from itself. In fact, in game 30, it is an NME wherever play starts, whereas in game 57 it is not an NME from (1,1).

It is an NME from (3,3) in game 30, and also from (4,2) and (2,4) if the player receiving a payoff of 4 has order power and can move to (3,3) before the player receiving a payoff of 2 moves to (1,1). This is plausible, because moving to (1,1) would likely curtail all future moves if it produced a nuclear winter, from which neither side could recover.

Whether game 30 or Chicken offers the more realistic model of the Cuban missile crisis—or any future nuclear confrontation—(3,3) in both games is an NME from itself. Therefore, if neither player during a confrontation initiates a strike against its foe, cooperation is in the long-term interest of both players, taking into account rational moves and countermoves away from it.[12] Thereby both games offer at least a partial explanation of why antagonisms between nuclear powers, including India and Pakistan in

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[12] Other game-theoretic models of the Cuban missile crisis describe the sequential choices of players and the imperfect information they had at different junctures; see, for example, Dixit et al. (2014). Most of these models assume an extensive-form game, which underlies player calculations of nonmyopic equilibria in the normal-form representations of the crisis. The normal form has the advantage of being not only economical but also facilitating the comparison of different assumptions about player preferences (e.g., in comparing, for example, Chicken and game 30).

recent decades and the United States and North Korea today, have not escalated to the nuclear level.

The 2015 Iran Agreement on Nuclear Weapons

In 2012, exactly fifty years after the Cuban missile crisis, several countries and the International Atomic Energy Agency feared that Iran might be attempting to develop a nuclear capability that could be used for military purposes. Israel, in particular, believed that Iran was enriching uranium in order to develop nuclear weapons that could be used against it. It had suspected such surreptitious activities earlier, but in 2012 it claimed they posed an imminent threat to its existence.

Iran denied that developing nuclear weapons was its intention, despite the discovery of previously hidden nuclear-production facilities. It said that it desired to enrich uranium only as an alternative energy source to be used for civilian purposes.

Israeli Prime Minister Benjamin Netanyahu threatened to attack Iran and destroy its nuclear capability unless there was proof, based on the rigorous inspection of its suspected nuclear facilities, that Iran was not developing nuclear weapons. (A number of Israeli leaders opposed such an attack, arguing that at best it might delay but not stop Iran's acquisition of nuclear weapons.) Israel and Iran were at an impasse, with Iran denying international inspectors access to the facilities in question.

Because of its refusal, Iran suffered ever more severe economic sanctions imposed by the United States, the European Union, and other countries. But a carrot was held out, with the sanctioners offering to relax or lift the sanctions if Iran agreed to allow inspections and credibly commit to halting any efforts that could lead to the production of

nuclear weapons. However, a number of countries, including China and Russia, opposed the use of sanctions.

The most immediate danger of armed conflict arose from Israel's threat to attack Iran's nuclear-production facilities. More specifically, Israel's position was that, failing an agreement, it would attack Iran's facilities before a point of no return—called a "zone of immunity" by Israeli Defense Minister Ehud Barak—was reached. The point that would set off an attack would be the time just before these facilities became sufficiently hardened (they were inside a mountain) to be effectively impregnable.

Whether the United States would actively participate in such an attack, or covertly facilitate it, was unclear. On March 8, 2012, President Barack Obama said the United States "will always have Israel's back," which signaled that he was supportive of Israel's concern but did not spell out exactly what the United States would do to aid Israel.

Worth noting is that in the 1970s, Israel began producing, but never publicly acknowledged possessing, nuclear weapons, though it is now presumed to have about eighty nuclear warheads (Arms Control Association, 2016). It did say, however, that it would not be the first party to introduce them into a conflict.

In claiming that Iran's acquisition of nuclear weapons threatened its existence, Israel implied that it would use every means short of nuclear weapons to arrest Iran's development of them if economic sanctions or covert actions failed. The latter had included assassinations and cyberwarfare, which had disrupted Iran's enrichment of uranium.

Unlike the superpowers during the Cold War, Israel was unwilling to rely on its own nuclear deterrent—that is, the threat of MAD ("mutual assured destruction")—

perhaps in part because it feared that terrorists could gain control of Iranian nuclear weapons and act "crazily," without concern for what Israel's response might be. Also, Israel's small physical size made its survival an issue, even after retaliation from an attack, whereas Iran's ability to absorb a retaliatory strike was greater, possibly giving it an incentive to preempt with nuclear weapons.

We present in Figure 3 a game (game 27 in Figure 2) to model the conflict between Iran and Israel.[13] In this game, Iran chooses between developing (D) or not developing ($\overline{\text{D}}$) nuclear weapons, and Israel chooses between attacking (A) or not attacking ($\overline{\text{A}}$) Iran's nuclear facilities.

Figure 3 about here

We assume Israel's ranking to be $\overline{\text{D}}\,\overline{\text{A}} > \text{DA} > \overline{\text{D}}\,\text{A} > \text{D}\,\overline{\text{A}}$. As justification, there is little doubt that Israel would most prefer a cooperative solution ($\overline{\text{D}}\,\overline{\text{A}}$), in which Iran does not develop nuclear weapons and so no attack is required, and would least

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[13] In a situation wherein there is incomplete information about the preferences of players or player types, it would be appropriate to model this uncertainty in a game. Jelnov, Tauman, and Zeckhauser (2018) propose such a model in which it is uncertain whether a country, which may or may not allow inspections, possesses nuclear weapons; its opponent, which wishes to deter their use, has only imperfect intelligence about its opponent's possession. In the case of Iran, it was clear that it did not yet possess nuclear weapons; the question was whether it would possess them in the future and whether Israel, thinking that it would, would attack to prevent their development. We answer this question by showing that even though game 27 possesses no pure-strategy NE, the NME of this game is the outcome we would expect farsighted players, looking ahead, to reach. Thereby it offers an explanation for the peaceful resolution of the Iran-Israel conflict, at least in 2015.

prefer that Iran develop nuclear weapons without Israel's making an effort to stop their production $(D\overline{A})$. Between attacking weapons that are being developed (DA) and mistakenly attacking weapons that are not being developed $(\overline{D}A)$, we assume that Israel would prefer the former: An attack on weapons being developed would certainly create a major crisis, but it would be seen by Israel as a measure that was essential to its survival.[14]

As for Iran, we assume that its most preferred outcome is to develop nuclear weapons without being attacked $(D\overline{A})$, and its least preferred outcome is not to develop nuclear weapons and be attacked anyway $(\overline{D}A)$. In between, we assume that Iran prefers the cooperative outcome $(\overline{D}\,\overline{A})$ to the noncooperative outcome (DA), which could lead to a major conflict and even war after the attack.

D is a dominant strategy for Iran, and the unique NE in game 27 is the noncooperative outcome (DA). Unfortunately for both countries, this outcome, (2,3), is Pareto-inferior to (3,4), but the strategies that yield (3,4), $\overline{D}\,\overline{A}$, are not an NE.

In July 2015, P5+1—the five permanent members of the U.N. Security Council plus Germany and the European Union—reached an agreement with Iran for robust inspections of its nuclear facilities that would prevent the significant enrichment of uranium, which could produce nuclear weapons, for fifteen years, as well as several other measures to inhibit Iran's development of nuclear weapons. Although Israel opposed this

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[14] But there is the delicate question of whether Israel's detection of uranium enrichment or actual weaponization, or something in between, would constitute a *casus belli* for Israel to attack Iran's nuclear facilities.

agreement, the agreement defused a volatile situation that could have led to an Israeli attack, perhaps implicitly if not explicitly supported by the United States.

Why didn't Israel attack earlier, as it continually threatened to do from 2012 to 2015? We think that it probably had intelligence that Iran was not approaching the zone of immunity and, as well, that Iran did not have the capability, or even the intention, of producing nuclear weapons. In that case, Israel would prefer $\overline{A}$ to A.

Iran, we presume, knew that Israel, as well as the United States, could closely track its progress in its nuclear program. While not knowing exactly what these countries knew about its activities, it could predict that Israel, with a high probability, would choose $\overline{A}$. Furthermore, because the Joint Comprehensive Plan of Action (JCPOA) provided for the gradual lifting of sanctions if Iran verifiably abided by its commitment not to develop nuclear weapons, there would be benefits in its choice of $\overline{D}$.

In summary, we have suggested that the conflict between Iran and Israel over Iran's possible development of nuclear weapons can plausibly be represented by game 27. The cooperative outcome in this game is not an NE, but it is an NME wherever play starts.

Although it appears that Iran attempted to enrich its supply of uranium before Israel's threat of attacking its nuclear facilities became imminent, as the conflict escalated, it became in Iran's interest to choose not to develop nuclear weapons for two reasons: (i) its fear of an attack on its production facilities; and (ii) the continued tightening of economic sanctions, which is not in our game but which was certainly a significant factor in inducing Iran to reach an settlement. When agreement was finally achieved in 2015 after long and arduous negotiations that are detailed in Parsi (2017), it

became no longer in Israel's interest to attack or threaten to attack Iran. True, both before and after the 2016 presidential election, Donald Trump disparaged the agreement and in May 2018 officially withdrew the United States from it, but as of this writing it is generally acknowledged that Iran continues to abide by it.

6. Conclusions

We have shown that in 16 of 19 2 × 2 strict ordinal conflict games with a cooperative outcome that is not a Nash equilibrium (NE), this outcome is a nonmyopic equilibrium (NME) when it is an initial state. In the three games in which this is not the case, either a compellent or deterrent threat by one player, if credible, can induce this outcome. Moreover, this outcome is an NME from other states in several games and, when it is not, can generally be induced by credible threats from other states.

All 2 × 2 conflict games have at least one Pareto-optimal NME—including Prisoners' Dilemma, which does not have a Pareto-optimal NE. We proved that all larger two-person and *n*-person normal-form games in which preferences are strict also have Pareto-optimal NMEs, though they may not be maximin.

The NMEs in several 2 × 2 games are boomerang NMEs, suggesting that a cooperative NME may be displaced by another NME that, while Pareto-optimal, may not be cooperative. However, this also means that a noncooperative NME like (2,4) in games 49 and 56 (Figure 1) can be transformed into a cooperative NME—(3,3) in these games—if play does not commence at (3,3).

We discussed two examples in international relations in which the cooperative outcome in games that model international conflicts were not NEs but NMEs from themselves. To model no first use, we suggested Chicken and game 30, both of which

have been used to model the Cuban missile crisis, as models of confrontation between nuclear powers in which the cooperative outcome is not an NE but is an NME. Moreover, it may be reinforced by compellent or deterrent threats.

Similarly, in the Iran-Israel conflict over the former's possible development of nuclear weapons, the agreement reached in 2015 was a cooperative outcome that we modeled by game 27. Before the agreement was reached, however, there was temptation on both sides to escalate the conflict. But Israel's threat of attack as well as economic sanctions made it in the long-run interest of both sides to defuse the confrontation, which nevertheless required difficult negotiations over many months before a compromise was hammered out. It continues to hold up, despite President Donald Trump's abrogation, and in 2018, withdrawal from the agreement, suggesting its long-term stability.

NMEs seem well to model the achievement of cooperation in international conflicts that NEs do not support. Farsighted thinking generally facilitates players' escape from Pareto-dominated NEs, enabling them to reach Pareto-optimal NMEs that stabilize cooperative outcomes.

Figure 1

Five Games with One Cooperative Outcome (Upper Left) That Are Not NEs

Two Symmetric Games in Which (3,3) is an NME from Itself

Prisoners' Dilemma (Game 32)   Chicken (Game 57)

<div align="center">

		C	
		(3,3) [3,3]	(1,4) [3,3]
R		(4,1) [3,3]	**(2,2)** [2,2]

		C	
		(3,3) [3,3]	**(2,4)** [3,3]/[4,2]
R		**(4,2)** [3,3]/[2,4]	(1,1) [4,2]/[2,4]

</div>

Three Games in Which (3,3) Is Not an NME from Itself but Inducible by Threats

Game 22   Game 49   Game 56

<div align="center">

		C	
		(3,3)^Rd [2,4]	**(2,4)** [2,4]
R		(4,1) [2,4]	(1,2) [2,4]

		C	
		(3,3)^Rc [2,4]	(1,2) [2,4]/[3,3]
R		(4,1) [2,4]	**(2,4)** [3,3]

		C	
		(3,3)^Rc [2,4]	(1,1) [2,4]/[3,3]
R		**(4,2)** [4,2]	**(2,4)** [3,3]

</div>

Key:

Nash equilibria (NEs) underscored

Nonmyopic equilibria (NMEs) in boldface

NMEs from each initial state in brackets

Compellent (c) and deterrent (d) threat outcomes inducible by row (R)

Figure 2

**16 Games with at Least One Cooperative Outcome (Upper Left) That Is
Not an NE but Is an NME from Itself**

4 Games with Two Cooperative Outcomes

33			34			36			37	
(3,4)	(1,2)		**(3,4)**	(2,2)		**(3,4)**	(2,1)		**(3,4)**	(2,2)
[3,4]	[4,3]		[3,4]	[4,3]		[3,4]	[4,3]		[3,4]	[4,3]
(4,3)	(2,1)		**(4,3)**	(1,1)		**(4,3)**	(1,2)		**(4,3)**	(1,1)
[4,3]	[4,3]		[4,3]	[4,3]		[4,3]	[4,3]		[4,3]	[4,3]

12 Games with One Cooperative Outcome

27			28			29			30	
(3,4)	(1,2)		**(3,4)**	(1,3)		**(3,4)**	(2,3)		**(3,3)**	(1,4)
[3,4]	[3,4]		[3,4]	[3,4]		[3,4]	[3,4]		[3,3]	[3,3]
(4,1)	(2,3)		(4,1)	(2,2)		(4,1)	(1,2)		(2,2)	(4,1)
[3,4]	[3,4]		[3,4]	[3,4]		[3,4]	[3,4]		[3,3]	[3,3]

31			Prisoners' Dilemma (32)			35			46	
(3,4)	(2,2)		**(3,3)**	(1,4)		**(3,4)**	(1,3)		**(3,4)**	(2,1)
[3,4]	[3,4]		[3,3]	[3,3]		[3,4]	[4,2]		[3,4]	[4,2]
(4,1)	(1,3)		(4,1)	**(2,2)**		**(4,2)**	(2,1)		**(4,2)**	(1,3)
[3,4]	[3,4]		[3,3]	[2,2]		[4,2]	[4,2]		[4,2]	[3,4]

47			48			50			Chicken (57)	
(3,3)	(2,1)		**(3,4)**	(1,1)		**(3,4)**	(2,3)		**(3,3)**	**(2,4)**
[3,3]	[4,2]		[3,4]	[3,4]		[3,4]	[4,2]/[3,4]		[3,3]	[3,3]/[4,2]
(4,2)	(1,4)		**(4,2)**	(2,3)		**(4,2)**	(1,1)		**(4,2)**	(1,1)
[4,2]	[3,3]		[4,2]	[3,4]		[3,4]	[3,4]/[4,2]		[3,3]/[2,4]	[2,4]/[4,2]

Key:

Nash equilibria (NEs) underscored

Nonmyopic equilibria (NMEs) in boldface

NMEs from each initial state in brackets

Figure 3

Iran-Israel Conflict (Game 27)

Israel

	Don't Attack: $\overline{A}$	Attack: A
Don't Develop: $\overline{D}$	Peaceful settlement **(3,4)** [3,4]	Attack unwarranted (1,2) [3,4]
Develop: D	Attack justified but not carried out (4,1) [3,4]	Attack justified and carried out (2,3) [3,4]

(Iran on left side)

Key:

Nash equilibrium (NE) underscored

Nonmyopic equilibrium (NME) in boldface

NMEs from each initial state in brackets

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