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Banks Risk Taking and Creditors Bargaining Power *

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Abstract

We analyze the influence of unsecured debt (subdebt) on risk-shifting in banks whose assets are risky debt claims. We assume that the stockholders and subdebt-holders jointly decide on risk-shifting. We show that replacing part of the stock with subdebt: (1) leads to fewer risk-shifting events, but can lead to higher levels of risk, depending on the relative bargaining power, (2) does not change the level of risk-shifting when side payments are possible, and (3) may yield the surprising result that risk-shifting increases with tighter regulatory control.

Keywords: Risk-taking, asset risk, financial institutions, stress test, leverage, bargaining.
JEL Classification: G21, G28, G32, G38

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1 Introduction

Excessive risk-taking by banks may lead to costly bailouts and spillovers to the rest of the financial system and the real economy (Allen and Gale, 2000; Leitner, 2005; Gai and Kapadia, 2010; Gofman, 2017), prompting governments to supervise banks and curb their risk-taking. However, as the size and complexity of financial institutions increases (Varotto and Zhao, 2018), the regulators' ability to control banks' asset risk using traditional supervisory techniques, such as minimum capital requirements and supervisory review, erodes (Basel Committee on Banking Supervision, 1999; Morgan and Stiroh, 2001; Berger, Davies, and Flannery, 2000; DeYoung, Flannery, Lang, and Sorescu, 2001).

In response, banks are encouraged to issue, in addition to their stock, unsecured debt (hereafter: subdebt), which is subordinated to their deposits. The subdebt serves as a buffer against declines in asset value and consequently protects depositors from costly failures (Calomiris, 1999; Evanoff and Wall, 2000). Moreover, subdebtholders are considered sophisticated creditors that increase banks' transparency (Hart and Zingales, 2011). Therefore, they may be able to affect banks' behavior ("direct discipline") in a way that is believed to be aligned with the deposit insurer's incentive (Flannery, 2001; Vashishtha, Chen, Goldstein, and Huang, 2018). In addition, investors can indirectly discipline banks by incorporating risk premia into subdebt yield spread, as it provides a signal of the banks' risk to the regulator and other market participants who can take prompt corrective actions (Gorton and Santomero, 1990; Dewatripont and Tirole, 1993).

Following the 2007–2009 financial crisis, the use of subdebt as a monitoring tool has been questioned due to mixed evidence on its effectiveness. On the one hand, the empirical literature finds that subdebt reduced banks' risk-taking, both during the financial crisis and the period following it (Danisewicz, McGowan, Onali, and Schaeck, 2018; Nguyen, 2013; John, Mehran, and Qian, 2010; Belkhir, 2013). On the other hand, many financial institutions with subdebt as part of their capital structure defaulted or were bailed out using taxpayers' money (Calomiris and Herring, 2013).[1]

The theoretical literature on the effect of subdebt mainly assumes either that stockholders determine the level of asset risk under some restrictions from regulators (Niu, 2008; Chen

[1]The pre-crisis literature on the informativeness of subdebt spreads about the issuing banks' financial condition presented mixed results as well (Flannery and Sorescu, 1996; Evanoff and Wall, 2001; Hancock and Kwast, 2001; Morgan and Stiroh, 2001; Sironi, 2003; Balasubramnian and Cyree, 2011; Krishnan, Ritchken, and Thomson, 2005; Avery, Belton, and Goldberg, 1988; Gorton and Santomero, 1990).

and Hasan, 2011; Hilscher and Raviv, 2014; Blum, 2002)), or that subdebtholders determine the level of asset risk (Gorton and Santomero, 1990). However, in light of the new Basel III accord (Basel Committee on Banking Supervision, 2010), which pushes banks to disclose an unprecedented amount of information by participation in periodic stress tests (Goldstein and Sapra, 2014; Goldstein and Leitner, 2018), the effect of joint control, where both the subdebtholders and the stockholders affect the choice of the level of asset risk, should be considered (including the possibility of side payments between the two stakeholders). In addition, the theoretical literature ignores the fact that an increasing portion of banks' assets are in the form of risky debt claims, as shown by Hanson, Shleifer, Stein, and Vishny (2015) and discussed and analyzed recently by Nagel and Purnanandam (2015) and Gornall and Strebulaev (2018).

Motivated by this gap in the literature and the inconsistent assessments of the ability of subdebt to mitigate risk-taking during the 2007–2009 financial crisis, our goal is to enhance the understanding of the effect of subdebt on banks' risk-taking and stability. We consider the effectiveness of subdebt in mitigating risk-taking, which depends on transparency, the bargaining power of subdebtholders, and the interaction of these elements with capital adequacy and with the supervisory processes. To this end, we begin by modeling the fair value of a bank's different liabilities using a framework in which the bank's assets are risky debt claims with a limited upside (Nagel and Purnanandam, 2015; Gornall and Strebulaev, 2018; Dermine and Lajeri, 2001; Peleg Lazar and Raviv, 2017). Next, we apply a game-theoretic approach, to the strategic bargaining interaction between the bank's claimholders and its borrowers, to find the equilibrium level of asset risk. The bank's risk-taking depends on (1) the position of each of the claimholders who affect the level of asset risk, (2) the bargaining power of the subdebtholders and their ability to observe the level of asset risk, and (3) the corrective measures that are taken by the regulator when information on risk-taking is disclosed.

Consistent with the corporate finance literature, we assume that the asset value of the bank's borrower follows a geometric Brownian motion (Merton, 1974). Due to the legal limited liability of the stockholders, the borrower's stock value is a convex function of its asset value and its value is replicated by a call option on the borrower's asset (Merton, 1977). Thus, the value of the borrower's stock always increases with asset risk. We deviate from traditional banking models in assuming that the bank's assets are risky debt claims whose value is contingent on the borrowers' asset value and its risk is determined by the borrower's asset risk. This means that the value of the bank's assets is limited from above by the face value of the borrower's loan. Consequently, the values of the bank's stock is also limited from above and cannot exceed the difference between the face value of the borrower's loan and the total face value of the bank's debt (deposits and subdebt). Finally, the subdebtholders' payoff is capped from above by the face value of their debt (Black and Cox, 1976).

We assume a regulator conducts periodic audits, at which time the bank's asset risk is set (to a level we name initial asset risk) in accordance with the regulator's policy (Ronn and Verma, 1986; Marcus and Shaked, 1984), but that between regulatory audits, risk-shifting might occur.[2] By contrast, since banks are efficient at monitoring and limiting the risk of their borrowers (Datta, Iskandar-Datta, and Patel, 1999; Ahn and Choi, 2009), the borrower cannot increase its level of asset risk above the initial level, at any time, without the bank's consent. In addition, consistent with the literature, we assume that depositors are incapable of monitoring the bank's risk as doing so is difficult and costly (Morgan, 2002; Caprio and Levine, 2002; Flannery, Kwan, and Nimalendran, 2013). Taken together, our assumptions suggest that the level of asset risk between regulatory audits is determined in a bargaining process between the bank's stockholders and subdebtholders, with the outcome depending on their relative power.

To explore the effect of subdebtholder's bargaining power on risk-taking we consider several cases. We first analyze the benchmark case in which the subdebtholders cannot affect the level of asset risk, possibly due to unobservable asset risk. In this case, the equilibrium level of asset risk is the result of the strategic interaction between the stockholders of the bank and the stockholders of the borrower. This case is in line with criticism made during the 2007–2009 financial crisis that subdebtholders are limited in their ability to enforce market discipline (Flannery, Kwan, and Nimalendran, 2013; Calomiris and Herring, 2013). We show that in equilibrium, risk-shifting occurs only when the borrower is in financial distress and its asset value is below a threshold equal to the discounted geometric mean of the face value of the borrower's debt and the face value of the bank's total debt. The stockholders' relatively low appetite for risk-taking is due to the limited upside of the bank's stock.

Next, we analyze the scenario where the subdebtholders choose the level of asset risk.

[2]Several papers suggest that since the regulator conducts on-site examinations of banks, it is better than other claimants at uncovering negative information; however, the additional information that is revealed, becomes stale within a few months (Berger and Davies, 1998; Dahl, Hanweck, and O'Keefe, 1995; Flannery and Houston, 1999; Cole and Gunther, 1998; DeYoung, Flannery, Lang, and Sorescu, 1998), suggesting that risk-shifting is possible between audit events.

We prove analytically that risk-shifting occurs only when the bank is in financial distress and the value of its assets is below the discounted geometric mean of the face value of the bank's deposits and the face value of its total debt. Therefore, in equilibrium, risk-shifting occurs for a smaller range of the borrower's asset value than when the equityholders are in control, and when it occurs, the level of asset risk is lower. Thus, even in this extreme case, we find that risk-shifting is not avoided completely. This result is similar to the analysis in Gorton and Santomero (1990) who show that subdebtholders are motivated to shift risk when asset value is below the threshold discussed above. However, Gorton and Santomero (1990) assume that the value of the subdebt increases with asset risk, whereas we show, using a closed-form solution, that the value of subdebt is bell-shaped with respect to asset risk and, as a result, there is an interior solution that maximizes the value of subdebt. This means that the increase in risk we predict in the event of financial distress is more moderate than that predicted by Gorton and Santomero (1990).

Following these two extreme cases where only one of the claimholders chooses the equilibrium level of asset risk, we explore the (more realistic) case where risk is determined jointly by the two claimholders as the outcome of a bargaining process. In this case, subdebtholders observe the chosen level of asset risk; i.e., they have the ability to identify a shift in asset risk taken by the stockholders. If, following an increase in risk by stockholders, subdebtholders are not better off relative to their state with the initial level of asset risk, the value of the subdebt decreases while the value of equity increases. These price changes signal a risk-shifting event triggering an on-site audit (or stress-test event), which leads the regulator to take corrective actions and to restore the bank's asset risk to its initial level. We solve this case by applying the concept of an asymmetric Nash bargaining solution (Nash, 1950; Kalai, 1977) and find that the range of asset values for which risk-shifting occurs is identical to the range found when the level of asset risk is chosen to maximize the value of subdebt. In addition, when risk-shifting occurs in equilibrium, the level of asset risk is between the level that maximizes the value of the stock and the level that maximizes the value of the subdebt, and this level of asset risk decreases with the subdebtholders' bargaining power.

We extend our model to address a concern raised in previous literature, by allowing side payments between the stockholders and the subdebtholders.[3] We solve this case in two

[3]Furlong and Keeley (1987) show that equityholders can compensate uninsured debtholders for increased risk in the form of higher promised interest rates. Calomiris (1999) and Chen and Hasan (2011) discuss the need to regulate the design of subordinated debt, including its maturity and maximum allowable yield, in order to assure that subdebtholders are motivated to control stockholders' risk-taking.

steps. First, the stockholders and the subdebtholders jointly choose the level of asset risk that maximizes the sum of their payoffs. Next, the side payment is chosen as the unique Nash solution to the bargaining problem of dividing the joint payoff between the two claimholders. We find that risk-shifting takes place for a larger range of the borrower's asset values than when the subdebtholders determine the level of asset risk, but for a smaller range than when the stockholders do. In addition, when risk-shifting occurs in equilibrium, the level of asset risk is between the level that maximizes the value of the subdebt and the level that maximizes the value of the stock.

The above threshold for risk-shifting and the equilibrium level of asset risk are identical to those of a bank funded by just stock and deposits, with no subdebt. Therefore, this last case is used as a benchmark that enables us to study the effect of replacing stock with subdebt. We show that a liability structure with subdebt leads to fewer risk-shifting events than a liability structure with only stock and deposits. However, when the bargaining power of stockholders is relatively high, replacing stock with subdebt can lead to a higher level of asset risk in equilibrium.

Since the 2007–2009 financial crisis, the issue of bank transparency has been heavily debated. The Basel III international regulatory framework (Basel Committee on Banking Supervision, 2010) calls for financial institutions to increase transparency by conducting periodic stress tests that involve an unprecedented amount of disclosure. We contribute to this debate by showing that when the subdebtholders' bargaining power is low, suggesting low transparency, more restrictive regulatory corrective measures in the form of a lower asset risk prescribed at the time of an audit can motivate claimholders to agree on a higher level of asset risk in the bargaining process. Thus, the efficiency of subdebt as a disciplinary tool declines with the enforcement of traditional regulatory tools, such as capital adequacy and on-site supervision, suggesting that the two measures can be substitutes rather than complements. This analysis is related to Chen and Hasan (2011) who show that more frequent audits lead to less risk-shifting, while we show that more restrictive measures taken at those audits lead to more risk-shifting between audits.

Our paper is also related to two recent strands of the literature. The first studies the effect of the issuance of "bail in" instruments on bank behavior and risk taking. "Bail in" instruments are financial instruments that aim to prevent the costly bail outs of financial institutions using taxpayers money. Instead "bail in" instruments either write down a financial institution's debt or convert it into common stock at time of financial distress (Chen, Glasserman, Nouri, and Pelger, 2017; Pelger, 2012; Glasserman and Nouri, 2012; Hilscher and Raviv, 2014; Martynova and Perotti, 2018). The second strand of the literature studies the incentive schemes and capital structures of financial institutions that lead to deviation from the optimal social level of risk, and suggest methods to realign financial institutions' behavior (Eufinger and Gill, 2016; Wong, 2018; Albuquerque, Cabral, and Guedes, 2018).

The rest of the paper is organized as follow. Section 2 describes the liability structures of the bank and its borrower and expresses the values of their different claims. Section 3 discusses the sensitivity of these valuations to the level of the borrower's asset risk. Section 4 analyzes the extent to which risk-shifting occurs under different scenarios. Section 5 presents a numerical example of results, and Section 6 concludes.

2 Liability Structure and Valuations

In this section we describe the liability structures of the bank and the borrower and express the value of their different claims, which are used in Section 3 to find the preferred level of asset risk of each of the claimholders. These levels of asset risk are used to analyze equilibrium risk-shifting in Section 4. In addition, in this section we define the cost of deposit insurance. For convenience, all the notations are summarized in Appendix A.

2.1 The borrower's liability structure

A corporation is funded by stock with market value S_C and by a single loan with face value F_C and market value B_C. The loan is a zero-coupon loan maturing at time T and the bank is its sole creditor. The value of the corporation's assets, V_C, under the risk-neutral measure follows a geometric Brownian motion according to the following equation:

$$dV_{C,t} = rV_{C,t}dt + \sigma V_{C,t}dW,$$

where r is the instantaneous risk-free rate of return, σ is the instantaneous volatility of the corporation's assets, and dW is a standard Wiener process under the risk-neutral probability measure.

A default event occurs at debt maturity, T, if the corporation's asset value, $V_{C,T}$, is lower than its face value of debt. If default occurs, the creditor takes over the corporation and realizes the residual assets of the corporation, $V_{C,T}$. Otherwise, debt is fully paid and the creditor, the bank, receives the total face value of debt, F_C. Therefore, the corporation's payoff at debt maturity is equal to $B_{C,T} = \min[V_{C,T}, F_C]$. This expression can be rearranged and expressed as $B_{C,T} = F_C - \max[F_C - V_{C,T}, 0]$. As discussed in Merton (1974), this payoff is equivalent to the payoff of a risk-free debt with a face value of F^C and a short position in a European put option. Therefore, the present value of the corporation's debt is given by

$$B_{C,t} = F_C \cdot e^{-r(T-t)} - Put_t(V_{C,t}, F_C, \sigma, T-t, r),$$

where $Put_t(V_{C,t}, F_C, \sigma, T-t, r)$ is the value at time t of a European put option on the corporation's asset value and where the option's strike price is equal to the face value of debt F_C, the level of asset risk is σ, and the time to maturity is $(T-t)$. Under the above-described geometric Brownian motion the value of the option can be found using the Black and Scholes (1973) equation.

Since the stock is the residual claim, its payoff at debt maturity is $S_{C,T} = \max[V_{C,T} - F_C, 0]$. This payoff can be replicated by a European call option on the value of the corporation's assets, with a strike price equal to its face value of debt (Galai and Masulis, 1976). Therefore the value of stock at time t is

$$S_{C,t} = Call_t(V_{C,t}, F_C, \sigma, T-t, r), \tag{1}$$

where $Call_t(V_{C,t}, F_C, \sigma, T-t, r)$ is the value of a European call option according to the Black and Scholes (1973) equation.

2.2 The bank's liability structure

We consider a bank funded by stock with a market value of S_B, deposits with a total face value of F_{Dep} and a market value of B_{Dep}, and zero-coupon subdebt with a face value of F_{Sub} and a market value of B_{Sub}. Deposits are insured by the government or a government agency. We follow the classic papers Marcus and Shaked (1984) and Ronn and Verma (1986) in defining deposits as debt claims that mature at the time of the regulatory audit, T. This is also consistent with a newer finding of Berger, Davies, and Flannery (2000) who show that supervisory assessments following an on-site inspection (audit) are more accurate than the market in predicting changes in bank performance. Following a similar logic, we assume that the subdebt matures at time T as well. Since the bank's asset is the loan that funds the activity of the corporation, described in Section 2.1, we can express the payoff of the bank's assets at maturity as

$$V_{B,T} = B_{C,T} = F_C - \max[F_C - V_{C,T}, 0]. \tag{2}$$

Thus, the asset value of the bank prior to maturity is

$$V_{B,t} = B_{C,t} = F_C \cdot e^{-r(T-t)} - Put_t(V_{C,t}, F_C, \sigma, T-t, r).$$

The depositors are the senior claimholders, and therefore the bank pays at time T the minimum between the value of the bank's assets and the face value of the bank's deposits, F_{Dep}, which can be expressed as

$$\begin{aligned} B_{Dep,T} &= \min\left[V_{B,T}, F_{Dep}\right] = F_{Dep} - \max\left[F_{Dep} - V_{B,T}, 0\right] \\ &= F_{Dep} - \max\left[F_{Dep} - F_C + \max\left[F_C - V_{C,T}, 0\right], 0\right]. \end{aligned}$$

As the bank funds the borrower's loan using both stock and debt, the face value of the borrower's debt, F_C, must be higher than the face value of the bank's own debt, $(F_{Dep}+F_{Sub})$, and, therefore, also higher than F_{Dep}. Under this assumption, we can express the depositor's payoff at maturity as $B_{Dep,T} = F_{Dep} - \max[F_{Dep} - V_{C,T}, 0]$. This payoff can be replicated by a long position in a risk-free debt with a face value of F_{Dep} and a short position in a European put option on the borrower's assets, with a strike price equal to the face value of the bank's deposits. Therefore, the value of the bank's deposits at any time t prior to debt maturity can be expressed as

$$B_{Dep,t} = F_{Dep} \cdot e^{-r(T-t)} - Put_t(V_{C,t}, F_{Dep}, \sigma, T-t, r).$$

The subdebtholders receive at debt maturity a face value of F_{Dep} if the value of the borrower's assets is above the bank's total face value of debt. Otherwise, the payoff is the maximum between zero and the difference between the value of the bank's assets and the face value of the deposits. This payoff can be rearranged and expressed as

$$B_{Sub,T} = \max\left[V_{C,T} - F_{Dep}, 0\right] - \max\left[V_{C,T} - (F_{Dep} + F_{Sub}), 0\right],$$

which is equivalent to a long position in a European call option with a strike price equal to the face value of deposits, F_{Dep}, and a short position in a European call option with a strike price equal to the face value of the bank's total debt, $(F_{Dep} + F_{Sub})$. Therefore, the value of the subdebt prior to debt maturity is

$$B_{Sub,t} = Call_t(V_{C,t}, F_{Dep}, \sigma, T-t, r) - Call_t(V_{C,t}, F_{Dep} + F_{Sub}, \sigma, T-t, r). \quad (3)$$

Since the bank's stockholders are the residual claimholders, their payoff at maturity is $S_{B,T} = \max\left[V_{B,T} - (F_{Dep} + F_{Sub}), 0\right]$. If the bank is solvent at maturity, the stockholders receive a payoff of $F_C - (F_{Dep} + F_{Sub})$, which is the maximum payoff that the bank's stockholders can receive. This differs from the basic structural approach in which the stockholders' payoff is unbounded. When we expand this payoff it can be expressed as

$$S_{B,T} = \max[V_{C,T} - (F_{Dep} + F_{Sub}), 0] - \max[V_{C,T} - F_C, 0].$$

This payoff can be replicated by a long position in a European call option, with a strike price equal to the face value of the bank's total debt, $(F_{Dep} + F_{Sub})$, and a short position in a European call option, with a strike price equal to the face value of the corporation's debt, F_C. Therefore, the value of the bank's stock prior to debt maturity is

$$S_{B,t} = Call_t(V_{C,t}, F_{Dep} + F_{Sub}, \sigma, T-t, r) - Call_t(V_{C,t}, F_C, \sigma, T-t, r). \quad (4)$$

The value of the bank's assets and the payoff to each of the bank's claimholders at debt maturity is described in Figure 1.

2.3 The cost of deposit insurance

A regulator conducts periodic audits to assess the bank's asset risk, which is then used to estimate the bank's deposit insurance premium; i.e., at the time of the audit the bank pays a fair insurance premium reflecting its capital structure and risk. Since the premium is not adjusted between audits to reflect changes in the quality of the bank's asset, it does not affect the bank's risk-taking motivation between audits.[4]

If at debt maturity the value of the bank's asset is below the face value of its deposits,

[4]The literature on deposit insurance shows that regulators lack the resources, information, or incentive to correctly assess bank risk and to charge the deposit insurance premium accordingly (Allen, Carletti, Goldstein, and Leonello, 2015; Anginer, Demirguc-Kunt, and Zhu, 2014; Chan, Greenbaum, and Thakor, 1992; Freixas and Rochet, 1998).

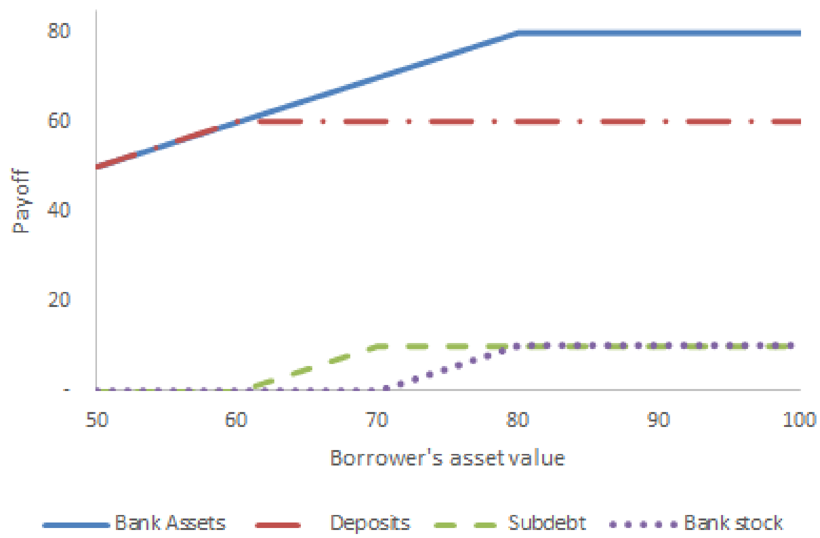


Figure 1: The bank's asset value and the payoff of the depositors, subdebtholders, and stockholders at debt maturity. The face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and the face value of its subdebt is $F_{Sub} = 10$.

the insurer compensates the depositors with the difference between the two. Thus, the cost of deposit insurance equals the maximum between zero and the difference between the face value of the secured deposits and the value of the bank's asset: $DI_T = \max[F_{Dep} - V_{B,T}, 0]$. Replacing $V_{B,T}$ above by Equation 2 we find that $DI_T = \max[F_{Dep} - V_{C,T}, 0]$. As discussed in Merton (1977) and Crouhy and Galai (1991), this payoff is equivalent to a long put option on the corporation's asset with a strike price equal to the face value of the bank's deposits. Following convention, we normalize and use the value of deposit insurance per dollar (DIPD) of insured deposits, which is defined as

$$DIPD_t = \frac{Put_t(V_{C,t}, F_{Dep}, \sigma, T-t, r)}{F_{Dep}}. \tag{5}$$

The value of deposit insurance increases with the borrower's asset risk, and decreases with its asset value.

3 Preferences regarding the Level of Asset Risk

In this section, we build on the pricing equations for the different securities developed in Section 2, and find the different claimholders' preferences for asset risk. Throughout the paper we assume that the sole objective of each claimholder is to maximize the market value of its claim. We define σ_0 as the borrower's asset risk at the time of the last regulatory audit. In what follows we study the preferences of the different claimholders at some time $t \in (0, T)$ after the loan contract is set but before the time of the next audit, which is also the time of debt maturity.

3.1 Preferences of the borrower's stockholders

In line with classic agency theory (Jensen and Meckling, 1976), the borrower's stockholders are always better off increasing the level of asset risk (under the assumption that their goal is to maximize the market value of their own holdings).

Claim 1. *The value of the stock of the bank's borrower increases with the level of asset risk.*

Proof. Recall that the market value of the corporation's stock is equal to the value of a European call option (Eq. 1): $S_{C,t} = Call_t(V_{C,t}, F_C, \sigma, T - t, r)$. It is well known that the value of a call option strictly increases with the level of option risk σ (see, e.g., Jensen and Meckling, 1976). □

3.2 Preferences of the bank's stockholders

The payoff of the stock is similar to that of a subordinated debt when the value of assets is uncapped and therefore the asset value threshold for risk-shifting is identical to the one developed by Gorton and Santomero (1990) for the case of subdebt of a corporation with uncapped assets. Specifically, the characterization of the bank's stockholders' preferences regarding the corporation's asset risk crucially depends on the threshold (Appendix B)

$$V^*_{S_B} \equiv e^{-r(T-t)} \sqrt{(F_{Dep} + F_{Sub}) \cdot F_C}, \tag{6}$$

which is a function of the geometric mean of the face value of the borrower's debt and the face value of the bank's total debt. In situations where the borrower's asset value is above this threshold, the value of the bank's stock decreases with risk.

By contrast, in setups where the borrower's asset value is below the threshold, the relationship between the market value of the bank's stock and the borrower's asset risk is hump-shaped (unimodal), and the value of the bank's stock reaches its maximum when the borrower's asset risk is (as proven in Appendix B)

$$\sigma_{S_B}^{max} \equiv \arg\max_{\sigma} S_B = \sqrt{\frac{1}{T-t} \ln\left(\frac{(F_{Dep} + F_{sub}) \cdot F_C}{(V_{C,t})^2}\right) - 2r}. \quad (7)$$

The level of asset risk preferred by the bank's stockholders, $\sigma_{S_B}^{max}$, is an increasing function of the borrower's leverage (F_C to $V_{C,t}$) and the bank's leverage defined as the face value of the bank's debt to the value of its asset ($F_{Dep} + F_{sub}$ to $V_{C,t}$). Thus, the level of asset risk preferred by the bank's stockholders depends on the financial risk both of the bank and of its borrower.

We find that $\sigma_{S_B}^{max} > \sigma_0$ if and only if the corporation's asset value is below a second threshold defined as (Appendix B):

$$V_{S_B}^{**} \equiv e^{-\left(r+\frac{\sigma_0^2}{2}\right)(T-t)} \sqrt{(F_{Dep} + F_{Sub}) \cdot F_C}. \quad (8)$$

Note that $V_{S_B}^{*}$ is higher than $V_{S_B}^{**}$ for any positive initial risk $\sigma_0 > 0$. Further note that both $V_{S_B}^{*}$ and $V_{S_B}^{**}$ depend on the geometric mean of the face values of the bank's total debt and the corporation's debt. Since the face value of the bank's debt is lower than that of the corporation's debt, both thresholds are lower than the face value of the corporation's debt. Thus, when the value of the borrower's assets crosses these thresholds the borrower is already in financial distress. The following result summarizes the above analysis of the value of the bank's stock as a function of its borrower's asset risk.

Proposition 1. *The value of the bank's stock is (1) decreasing with the value of its borrower's asset risk if $V_{S_B}^{*} < V_{C,t}$, and (2) hump-shaped (unimodal) in the asset risk if $V_{S_B}^{*} > V_{C,t}$, and, in this case, its maximum is obtained for the level of asset risk $\sigma_{S_B}^{max}$. Moreover, the level of asset risk that maximizes the value of the bank's stock is higher than the initial level of asset risk (i.e., $\sigma_{S_B}^{max} > \sigma_0$) if and only if $V_{C,t} < V_{S_B}^{**}$.*

3.3 Preferences of the subdebtholders

The payoff of the subdebt has a similar structure to the payoff of the bank's stock. Both payoffs are zero if the value of the asset is below some lower threshold and both payoffs are upward sloping above this threshold till some upper threshold is reached where the payoff is capped. However, while the value of the stock is capped when the value of the borrower's assets is above the face value of its debt, F_C, the subdebt is capped when the value of the borrower's assets is above the total face value of the bank's debt, $F_{Sub} + F_{Dep}$, which is actually the level at which the payoff of the stockholders changes from zero to positive. The value of the borrower's assets above which the payoff of the subdebt becomes positive is equal to the face value of the bank's deposits, F_{Dep}. Therefore, we can simply replace the threshold in Equation (6) to obtain the threshold that determines the sensitivity of the subdebt to asset risk

$$V^*_{B_{sub}} \equiv e^{-r(T-t)}\sqrt{F_{Dep} \cdot (F_{Dep} + F_{sub})}. \tag{9}$$

This threshold is a function of the geometric mean of the face value of deposits and of the bank's total debt. Observe that (1) the threshold $V^*_{B_{sub}}$ does not depend on the borrower's level of debt, and (2) the subdebtholders' threshold for risk-shifting is strictly lower than the stockholders' threshold for risk-shifting (i.e., $V^*_{B_{sub}} < V^*_{S_B}$). In situations where the corporation's asset value is above this threshold, the market value of the subdebt decreases with the level of asset risk. In realizations where the value of the borrower's assets is below the threshold, the relationship between the market value of the subdebt claim and the level of asset risk is hump-shaped, and the maximum value of the subdebt claim is reached at the level of asset risk defined as

$$\sigma^{max}_{B_{sub}} \equiv \arg\max_{\sigma} B_{Sub,t}(\sigma) = \sqrt{\frac{1}{T-t}\ln\left(\frac{(F_{Dep} + F_{sub}) \cdot F_{Dep}}{(V_{C,t})^2}\right) - 2r}. \tag{10}$$

It is immediate that $\sigma^{max}_{B_{sub}} < \sigma^{max}_{S_B}$ (since $F_{Dep} < F_{Dep} + F_{Sub} < F_C$), which implies that subdebtholders always prefer a lower level of asset risk compared to the bank's shareholders. Finally, the value of subdebt is maximized at a positive level of asset risk, $\sigma^{max}_{B_{Sub}} > \sigma_0$, if and only if the corporation's asset value is below a second threshold defined as

$$V^{**}_{B_{sub}} \equiv e^{-\left(r+\frac{\sigma_0^2}{2}\right)(T-t)}\sqrt{(F_{Dep} + F_{Sub}) \cdot F_{Dep}}. \tag{11}$$

Note that $V^*_{B_{sub}}$ is above $V^{**}_{B_{sub}}$ for each positive level of initial risk $\sigma_0 > 0$. In addition, both $V^*_{B_{sub}}$ and $V^{**}_{B_{sub}}$ depend on the geometric mean of the face value of the bank's total debt and the face value of its deposits; i.e., both thresholds are below the face value of bank debt.

Thus, when the value of the borrower's assets crosses these thresholds the bank is already in financial distress.

The following result summarizes the above analysis of the value of the bank's stock as a function of the level of asset risk.

Proposition 2. *The market value of the subdebt claim is (1) decreasing in the borrower's level of asset risk if* $V^*_{B_{Sub}} < V_{C,t}$*, and (2) hump-shaped (unimodal) in the level of asset risk if* $V^*_{B_{Sub}} > V_{C,t}$*, and, in this case, its maximum is obtained for the level of asset risk* $\sigma^{max}_{B_{Sub}}$*. Moreover, the level of asset risk that maximizes the value of the subdebt is higher than the initial level of risk (i.e.,* $\sigma^{max}_{B_{Sub}} > \sigma_0$*) if and only if* $V_{C,t} < V^{**}_{B_{Sub}}$*.*

It should be noted that our analysis of the asset threshold for risk shifting for each of the banks liabilities is a generalization of the specific solution of Gorton and Santomero (1990) for the case of subdebt of a non-financial firm, and that our analysis of the level of risk that maximizes the value of a liability is a generalization of the specific case of a bank having only deposit and stock, which is analyzed in Peleg Lazar and Raviv (2017).

3.4 Illustration of the preferences of the various claimholders

The risk preferences of the bank's subdebtholders and stockholders and the preferences' dependence on asset value are illustrated in this section and demonstrated in Figure 2. Each panel illustrates for a different asset value, the value of both the bank's stock and the bank's subdebt as a function of the level of the asset risk of the borrower and the level of asset risk that maximizes the value of each claim. The figure also illustrates that the stockholders prefer a higher level of asset risk compared to the subdebtholders, as discussed above. The face value of the corporation's loan is 80, the face value of the subdebt is 10, and the face value of the deposits is 60. The risk-free rate is 1% and time to maturity is one year.

Panel 2a illustrates a high asset value where $V_{C,t} = 100$, which is above the threshold $V^*_{S_B} = 74.1$ (Eq. 6). In this case, the value of both the stock and the subdebt is decreasing in the level of asset risk; i.e., the value of both claims is maximized with zero risk. Panel 2b is an example of an intermediate asset value where $V_{C,t} = 70$, which is below $V^*_{S_B} = 74.1$ but still above $V^*_{B_{Sub}} = 64.2$ (Eq. 9). In this case, the value of the stock is hump-shaped with respect to the level of asset risk and is maximized with a level of asset risk of 33.7%. However, the value of the subdebt is decreasing in the level of asset risk and therefore is maximized with zero risk. Panel 2c illustrates a low asset value of $V_{C,t} = 62$, which is below

$V^*_{B_{Sub}} = 64.2$. In this case, the value of both the stock and the subdebt is hump-shaped with respect to the level of asset risk but the value of the stock is maximized at a higher level $\sigma^{max}_{S_B} = 59.7\%$ than that of the subdebt $\sigma^{max}_{B_{Sub}} = 26.2\%$. In addition, the value of deposit insurance per dollar of insured deposits (Eq. 5) with asset risk $\sigma^{max}_{B_{Sub}} = 26.2\%$ is $DIPD_t = 8.5\%$, whereas with the higher asset risk of $\sigma^{max}_{S_B} = 59.7\%$ the deposit insurance would increase to $DIPD_t = 21.6\%$.

4 Analysis of Risk-Shifting

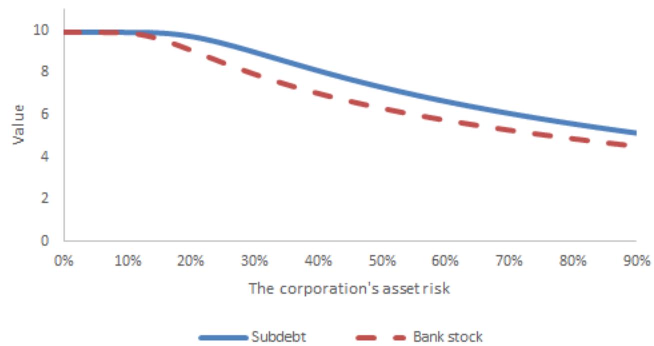
In this section we build on the expressions developed in Section 3 to to analyze the extent to which risk shifting occurs under different scenarios regarding how the control over the bank is divided between the shareholders and the subdebtholders.

4.1 Framework of analysis

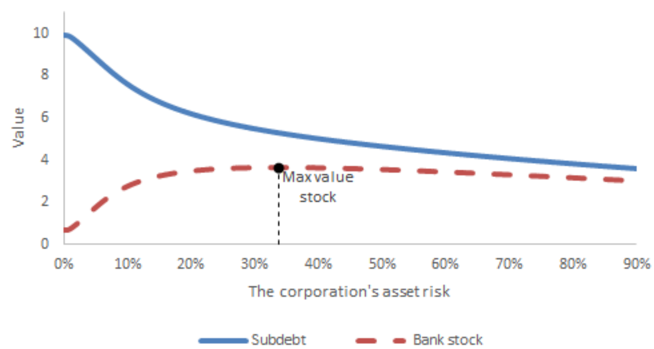
We assume a regulator conducts periodic audits to assess and align the bank's level of asset risk with the regulatory policy.[5] Accordingly, the bank sets the face value of the borrower's loan, F_C, to account for the borrower's initial level of asset risk, σ_0. Thus, at the time of an audit, the bank's liabilities, stock, subdebt, and secured deposits are fairly priced according to the borrower's level of asset risk and leverage. Although at the time of the audit the level of asset risk is set to the initial level, at all times between the periodic audits the bank can shift its level of asset risk by allowing the borrower to change its level of asset risk. In our model, we focus on such points in time and inquire into the conditions under which risk-shifting can occur.

Risk-shifting can occur if the borrower's claimholders (stockholders and creditors) agree on a specific level of risk; i.e, they are all better off with risk-shifting. Since the creditor is the bank, the bank's own claimholders must reach an agreement on the borrower's level of asset risk. The bank's control rights remain an open issue, specifically the ability of subdebtholders to affect the chosen level of asset risk. This ability is a function of the information available to the subdebtholders, specifically the ability to observe the borrower's level of asset risk

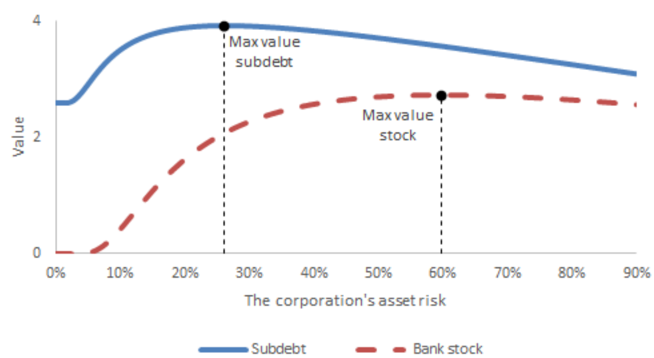
[5]The question of what is the optimal level of asset risk that should be set at the time of a regulatory audit as well as the relation between the optimal level and the actual level that the regulator chooses is beyond the scope of our analysis. However, it is clear that at the time of an audit the regulator can force some level of risk, which we call the initial level of risk.



(a) The borrower's asset value: $V_C = 100$



(b) The borrower's asset value: $V_C = 70$



(c) The borrower's asset value: $V_C = 62$

Figure 2: The value of the bank's stock and the value of its subordinated debt as a function of the borrower's level of asset risk. The face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and the face value of its subordinated debt is $F_{Sub} = 10$. These values for the bank yield a leverage (in book values) of 87.5%. In addition, the time to maturity is one year and the risk free-rate is $r = 1\%$.

and the corrective measures that the regulator can take upon receiving such information. Therefore, in our analysis we cover a wide spectrum of possible states and study the effect of the subdebtholders' relative bargaining power over risk-shifting. The chosen cases either mark the upper and lower boundaries for risk-shifting or refer to intermediate cases that are a result of different claimholders' bargaining power.

First, we consider the extreme case where the bank's stockholders hold all control rights and therefore can choose any level of asset risk accepted by the bank's borrower. Second, we examine the other extreme case, where the subdebtholders determine the level of asset risk while the stockholders have no say. Although this extreme case is highly improbable in reality, it serves as an informative benchmark, marking the upper bound for the creditor's rights. Next we move to an intermediate bargaining framework where any change in th level of asset risk requires the joint consent of the bank's stockholders and subdebtholders, and their joint decision is the result of a bargaining process. Under this framework, risk-shifting can occur only if both the bank's stockholders and subdebtholders are better off with respect to the initial level of asset risk set at the previous audit. Finally, we allow the stockholders and the subdebtholders to exchange side payments, while jointly deciding the level of asset risk. All results presented in this section are summarized in Table 1.

Claim 1 in Section 3.1 implies that the borrower's stockholders always prefer the highest level of asset risk allowed by the bank and would never agree to decrease risk. Therefore, in the rest of our analysis we focus on the bank's controller's choice of the maximum allowed level of asset risk of the borrower, which must be weakly higher than the initial level of risk in order for the borrower's stockholders to agree. We assume that this maximum allowed level of risk is feasible given the technological limitations governing the borrower's assets, and that the borrower's shareholders always shift the level of asset risk to it.

4.2 Asset risk is determined by the stockholders

In this subsection we assume that the bank's stockholders hold all control rights, while the subdebtholders are unable to restrict the stockholders' risk-taking. This extreme case is consistent with a scenario in which subdebtholders cannot observe the level of asset risk of the borrower. As a result, the value of the subdebt need not decline in a risk-shifting event and the regulator would not receive a signal to take corrective measures and to restore the asset risk to its initial level. This scenario is in line with the view that subdebtholders did not restrict excessive risk-taking during the 2007–2009 financial crisis because they did not have the ability to do so and not due to lack of motivation (Gorton and Santomero, 1990; Calomiris and Herring, 2013).

Proposition 1 in Section 3.2 characterized the value of the bank's stock for different asset values. An immediate corollary is that risk-shifting occurs in the above scenario if and only if the value of the borrower's asset is below the threshold, $V^{**}_{S_B}$.

Corollary 3. *When the bank's stockholders hold the bank's control rights, risk-shifting occurs if and only if* $V_{C,t} < V^{**}_{S_B}$. *When there is risk-shifting, the level of asset risk increases to* $\sigma^{max}_{S_B}$.

This case highlights that even when the subdebtholders are unable to restrict the level of asset risk, risk-shifting is limited to states where the bank's borrower is in financial distress and the level of asset risk is limited by $\sigma^{max}_{S_B}$, and is not unbounded as suggested by Gorton and Santomero (1990).[6]

4.3 Asset risk is determined by the subdebtholders

In this subsection we assume that the subdebtholders possess the bank's control rights. This is an extreme case of the view that market discipline can be used to restrict bank risk and in particular that the subdebtholders possess "direct influence" on banks' risk-taking (as defined in Flannery, 2001).

Proposition 2 in Section 3.3 characterized the payoff to the bank's stockholders for different asset values. An immediate corollary is that risk-shifting occurs in this second scenario if and only if the value of the corporation's asset is below the threshold, $V^{**}_{B_{Sub}}$

Corollary 4. *When the subdebtholders control the bank's decisions, risk-shifting occurs if and only if* $V_{C,t} < V^{**}_{B_{Sub}}$. *When there is risk-shifting, the level of asset risk increases to* $\sigma^{max}_{B_{Sub}}$.

Observe that when the bank is controlled by subdebtholders, risk-shifting is less pronounced than in the case where the bank is controlled by the shareholders in the following two ways: (1) when the corporation's asset value is between $V^{**}_{B_{sub}}$ and $V^{**}_{S_B}$, risk-shifting occurs only when the bank is controlled by the stockholders and does not occur when it is controlled by the subdebtholders (consistent with the example in Figure 2b) and (2) when the corporation's asset value is below $V^{**}_{B_{sub}}$, there is risk shifting in both cases, but the risk-shifting

[6]Since the bank's stockhoders' payoff is limited from above, their utility from the level of asset risk is concave. This would not be the case if the bank's asset value was unbounded, in which case the equilibrium level of asset risk would be unbounded as well.

Table 1: Summary of Risk-shifting Analysis

<table>
<tr><td rowspan="2">Control & bargaining framework</td><td colspan="4">Corporation's asset value (V_C)</td></tr>
<tr><td>$V_C > V^{**}_{S_B}$</td><td>$V_C \in (V^{**}_{S_B+B_{Sub}}, V^{**}_{S_B})$</td><td>$V_C \in (V^{**}_{B_{Sub}}, V^{**}_{S_B+B_{Sub}})$</td><td>$V_C < V^{**}_{B_{Sub}}$</td></tr>
<tr><td>Stockholders control the bank</td><td rowspan="4">No risk-shifting</td><td colspan="3">Risk-shifting to $\sigma^{max}_{S_B}$</td></tr>
<tr><td>Joint control with side payments</td><td rowspan="3">No risk-shifting</td><td colspan="2">Risk-shifting to $\sigma^{max}_{S_B+B_{sub}} \in (\sigma^{max}_{B_{Sub}}, \sigma^{max}_{S_B})$</td></tr>
<tr><td>Joint control without side payments</td><td rowspan="2">No risk-shifting</td><td>Risk-shifting to $\sigma^{max}_{\alpha} \in (\sigma^{max}_{B_{Sub}}, \sigma^{max}_{S_B})$</td></tr>
<tr><td>Subdebtholders control the bank</td><td>Risk-shifting to $\sigma^{max}_{B_{Sub}}$</td></tr>
</table>

is less pronounced when the bank is controlled by the subdebtholders, i.e., $\sigma^{max}_{B_{sub}} < \sigma^{max}_{S_B}$ (consistent with the example in Figure 2c). A comparison of the two cases is presented in Table 1.

4.4 Asset Risk is determined in a joint decision

The previous two cases are extreme ones where the bank is solely controlled by either the stockholders or by the subdebtholders. The analysis in this section refers to the rest of the spectrum, where any change in the level of asset risk requires the joint consent of both stockholders and subdebtholders. Specifically, we assume that allowing the borrower to increase its level of asset risk requires the joint agreement of the bank's stockholders and subdebtholders, and when a disagreement arises, the level of asset risk remains at its initial level, σ_0. This assumption reflects the ability of the subdebtholders to observe the level of asset risk. In the case where the subdebtholders are not better off increasing the level of asset risk the price of the subdebt decreases and the yield spread over the risk-free rate increases. On the other hand, the value of the bank's stock increases. When such information is revealed through the market value of the two securities, the regulator instructs the bank to lower the level of asset risk to its initial level, i.e., the level that was set at the last audit and used to price the bank's deposit insurance premium.

A necessary condition for the described bargaining process is that subdebtholders can observe the level of asset risk. Thus, a regime shift from the case where only the stockholders determine the level of risk to the cases with bargaining can be a result of a change in investor sophistication or an increase in asset transparency, such as the one brought about by the disclosure rules that followed Basel III (Vashishtha, Chen, Goldstein, and Huang, 2018).

4.4.1 Analysis of the bargaining game

As is common in the game-theoretic literature, we model this strategic interaction as a bargaining situation (see, e.g., Osborne and Rubinstein (1990), Chapter 7, for a textbook introduction). Specifically, we assume that an exogenous parameter $\alpha \in [0,1]$ describes the bargaining power of stockholders relative to subdebtholders. The case of $\alpha = 1$ ($\alpha = 0$) corresponds to a state in which stockholders (subdebtholders) hold all the bargaining power; i.e., the stockholders (subdebtholders) present a "take-it-or-leave-it" offer regarding the maximal allowed level of asset risk, and the subdebtholders (stockholders) can either accept or reject this offer, without an opportunity to present a counteroffer. The case where α=0.5 corresponds to a symmetric state, where both claimholders possess the same bargaining power.

The solution concept we apply to capture the joint decision of stockholders and subdebtholders is the asymmetric Nash bargaining solution (Nash, 1950; Kalai, 1977), according to which the maximal level of asset risk that is chosen at the end of the bargaining process is

$$\sigma_{\alpha}^{max} = \arg\max_{\substack{\sigma \geq \sigma_0 \\ S_{B,t}(\sigma) \geq S_{B,t}(\sigma_0) \\ B_{Sub,t}(\sigma) \geq B_{Sub,t}(\sigma_0)}} \left(\left(S_{B,t}\left(\sigma\right) - S_{B,t}\left(\sigma_0\right) \right)^{\alpha} \cdot \left(B_{Sub,t}\left(\sigma\right) - B_{Sub,t}\left(\sigma_0\right) \right)^{1-\alpha} \right). \tag{12}$$

4.4.2 Solution of the bargaining game

The following proposition characterizes the maximal level of asset risk σ_{α}^{max} jointly chosen by stockholders and subdebtholders. Specifically, it shows that (1) the condition for risk-shifting is the same as in the case where the bank is controlled by subdebtholders, and (2) if there is risk-shifting, the level of asset risk is increasing in α, and is between the levels of risk found in the two previous cases (a comparison of the solution of this case with those of the two previous cases is presented in Table 1).

Proposition 5. *When the bank is jointly controlled by subdebtholders and stockholders, risk-shifting occurs if and only if* $V_{C,t} < V_{B_{Sub}}^{**}$. *When there is risk-shifting, the level of asset risk*

σ_α^{max} is (1) increasing in α, and (2) between the level of asset risk decided upon in the case of sole control by the subdebtholders and the level decided upon in the case of sole control by the stockholders (i.e., $\sigma_{B_{Sub}}^{max} \leq \sigma_\alpha^{max} \leq \sigma_{S_B}^{max}$).

Proof. We begin by showing that risk-shifting occurs if and only if $V_{C,t} < V_{B_{Sub}}^{**}$. Proposition 2 immediately implies that there exists a higher level of asset risk $\sigma > \sigma_0$ that induces a higher value for the subdebt relative to the initial level of risk, if and only if $V_{C,t} < V_{B_{Sub}}^{**}$. Moreover, by Proposition 1 and the inequality $V_{B_{Sub}}^{**} < V_{S_B}^{**}$, if the subdebtholders achieve a higher value (relative to its value given the initial risk), so do the stockholders. Thus, due to Eq. 12, there exists a higher level of risk $\sigma > \sigma_0$ that induces a higher value for both the stock and the subdebt (relative to the value induced for them by the initial risk) if and only if $V_{C,t} < V_{B_{Sub}}^{**}$.

Next, we focus on the case where there is risk-shifting (i.e., $V_{C,t} < V_{B_{Sub}}^{**}$). Observe that due to Propositions 1 and 2, (1) both the expressions $(S_{B,t}(\sigma) - S_{B,t}(\sigma_0))$ and $(B_{Sub,t}(\sigma) - B_{Sub,t}(\sigma_0))$ are increasing in σ for low levels of asset risk satisfying $\sigma < \sigma_{B_{Sub}}^{max}$, (2) $(S_{B,t}(\sigma) - S_{B,t}(\sigma_0))$ is increasing in σ, while $(B_{Sub,t}(\sigma) - B_{Sub,t}(\sigma_0))$ is decreasing in σ for intermediate levels of asset risk satisfying $\sigma_{B_{Sub}}^{max} < \sigma < \sigma_{S_B}^{max}$, and (3) both expressions are decreasing in σ for high levels of asset risk satisfying $\sigma > \sigma_{S_B}^{max}$. These observations and the definition of σ_α^{max} in Eq. 12 imply that σ_α^{max} is increasing in α, and that $\sigma_{B_{Sub}}^{max} \leq \sigma_\alpha^{max} \leq \sigma_{S_B}^{max}$. □

4.4.3 Illustration

We demonstrate the motivation of the bank's subdebtholders and stockholders in Figure 3, using the parameters discussed in Section 3.4. When the borrower's asset value is low, $V_C = 62$, the value of the subdebt is higher with any level of asset risk between $\sigma_0 = 10\%$ and $\overline{\sigma_0} = 64\%$ then it is with the initial level of risk (Figure 3b). Therefore, when stockholders suggest an increase in asset risk to a level within this range, the subdebtholders are better off as well and the price of the two securities increases. Since the regulator cannot distinguish between an increase in the bank's asset value and an increase in the bank's asset risk, as the price of the two securities behaves similarly in these two scenarios, the regulator does not receive any price signal and therefore will not take any corrective actions. For example, when $\alpha = 0.8$ the equilibrium level of risk is $\sigma_{0.8}^{max}(10\%) = 40.6\%$, which is between the risk preferred by the stockholders, 59.7%, and the risk preferred by the subdebtholders, 26.2%. The value of deposit insurance per dollar of insured deposits (Eq. 5) with asset risk $\sigma_{0.8}^{max}(10\%) = 40.6\%$ is $DIPD_t = 14.2\%$, which is higher than its value using the risk preferred by subdebtholders and lower than its value using the risk preferred by stockholders (Section 3.4).

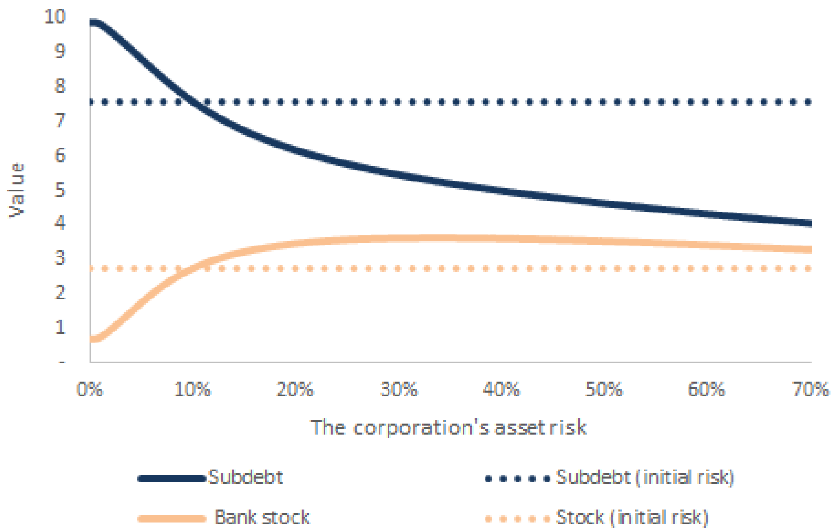
4.4.4 Impact of initial risk level

In the previous section we interpreted σ_0 in Equation 12 as the initial level of the borrower's asset risk set in the contract between the borrower and the bank. We assumed that if the bank's claimholders cannot agree on a higher level of risk then risk remains unchanged at its initial level. However, we can also interpret σ_0 as the level of asset risk that is set by the regulator when it learns from the subdebtholders that the bank took part in risk-shifting. A higher σ_0 is consistent with regulator forbearance while a lower one is consistent with a regulator that is intolerant of risk-shifting. The extreme case where $\sigma_0 = 0\%$ is consistent with the regulator closing any bank that she finds has engaged in risk-shifting. Under this interpretation, we find that more severe regulatory corrective measures may have the opposite effect to the one intended and actually increase equilibrium risk. This result is even more striking if we interpret σ_0 as the level of asset risk that the claimholders believe the regulator will set; i.e., if claimholders believe that the regulator is more intolerant of risk-shifting, then the equilibrium level of asset risk will increase when the bank is jointly controlled and side payments are not possible.

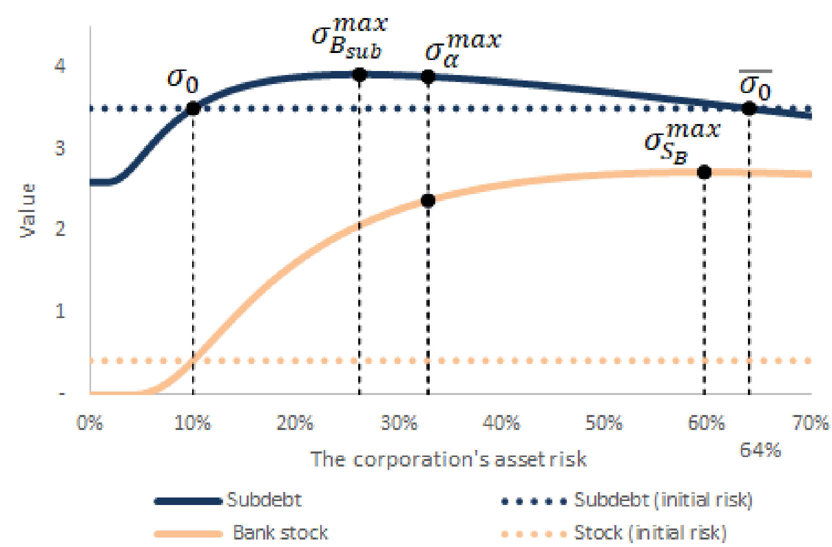
In what follows we characterize conditions under which we get surprising comparative statics with respect to the initial level of asset risk, namely, that reducing the initial risk level, which is set during an audit period, increases the equilibrium level off asset risk induced by the bargaining process.

The sufficient conditions for this counterintuitive result are that: (1) subdebtholders are willing to increase asset risk above its initial level, i.e., $V_{C,t} < V^{**}_{B_{Sub}}$, (2) but they do not agree to increase asset risk to the level that maximizes the value of the bank's stock, i.e., $B_{sub,t}(\sigma^{max}_{S_B}) < B_{sub,t}(\sigma_0)$, and (3) the stockholders' bargaining power α is sufficiently high. Formally:

Proposition 6. *Assume that the bank is jointly controlled by the subdebtholders and the stockholders and that $V_{C,t} < V^{**}_{B_{Sub}}$. Further assume that $B_{sub,t}(\sigma^{max}_{S_B}) < B_{sub,t}(\sigma_0)$. Then for each $\sigma' < \sigma_0$, there exists $\overline{\alpha} < 1$ such that the equilibrium level of risk-shifting is higher when the initial level of asset risk decreases from σ_0 to σ', i.e., $\sigma^{max}_{\alpha}(\sigma_0) < \sigma^{max}_{\alpha}(\sigma')$, for*



(a) The corporation's asset value: $V_C = 70$



(b) The corporation's asset value: $V_C = 62$

Figure 3: The value of the bank's stock and the value of its subordinated debt as a function of the level of the asset risk of the borrower. The dashed lines represent the value of the bank's subdebt and the value of its stock with an initial level of asset risk of $\sigma_0 = 10\%$. The face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and the face value of its subordinated debt is $F_{Sub} = 10$. These face values yield for the bank a leverage of 87.5%. In addition, the time to maturity is one year and the risk-free rate is $r = 1\%$ and $\alpha = 0.8$.

each $\alpha > \overline{\alpha}$.

Proof. The assumption that $V_{C,t} < V^{**}_{B_{Sub}}$ implies by Proposition 5 that there is risk-shifting, that the payoff function of both the stockholders and the subdebtholders is hump-shaped, and that the levels of asset risk that maximize the values of the stock and the subdebt satisfy $\sigma_0 < \sigma^{max}_{B_{Sub}} < \sigma^{max}_{S_B}$. Let $\overline{\sigma_0} > \sigma_0$ (resp., $\overline{\sigma'} > \sigma'$) be the level of asset risk that induces the same payoff for the subdebtholders as the one induced by the initial risk level, i.e., $B_{sub,t}(\overline{\sigma_0}) = B_{sub,t}(\sigma_0)$ (resp., $B_{sub,t}\left(\overline{\sigma'}\right) = B_{sub,t}(\sigma'))$. The inequality $B_{sub,t}(\sigma^{max}_{S_B}) < B_{sub,t}(\sigma_0)$ and the definition of the Nash bargaining solution imply that $\sigma^{max}_{\alpha}(\sigma_0) < \overline{\sigma_0} < \sigma^{max}_{S_B}$ for each bargaining power α. Next, observe that $\sigma^{max}_{\alpha}(\sigma')$ converges to $min(\overline{\sigma'}, \sigma^{max}_{S_B}) > \overline{\sigma_0}$ as α converges to 1. This implies that there exists $\overline{\alpha} < 1$ such that $\sigma^{max}_{\alpha}(\sigma_0) < \overline{\sigma_0} < \sigma^{max}_{\alpha}(\sigma')$ for each $\alpha > \overline{\alpha}$. □

In the example in Section 4.4.3 we show that when $\sigma' = 10\%$, the equilibrium level of asset risk is $\sigma^{max}_{0.8}(10\%) = 40.6\%$. If the level of asset risk imposed by the regulator when risk-shifting is observed through a change in the price of the securities increases to $\sigma_0 = 15\%$, the equilibrium level of asset risk decreases to $\sigma^{max}_{0.8}(15\%) = 35.3\%$, as illustrated in Figure 4.

This result can also be seen in Figure 5, which presents the equilibrium level of asset risk when the initial risk is 10% (in blue) versus when the initial risk is 20% (in red) and for $\alpha = 0.8$ (dotted) and $\alpha = 0.2$ (dashed). In contrast to the case of complete control by one of the claimholders, the initial level of asset risk has a reverse effect on the equilibrium level of asset risk.

4.5 Joint control with side payments

Finally, we consider an extension where the stockholders and subdebtholders jointly control the bank and the stockholders' relative bargaining power is $\alpha \in (0, 1)$, but now we introduce the option of side payments paid by the stockholders to the subdebtholders or vice versa. Side payments allow the paying side to directly influence the receiving side's risk preferences by altering their payoff function.

Payments from stockholders to subdebtholders can be implemented by increasing the interest rate of the subdebt claim above the rate that the subdebtholders would request

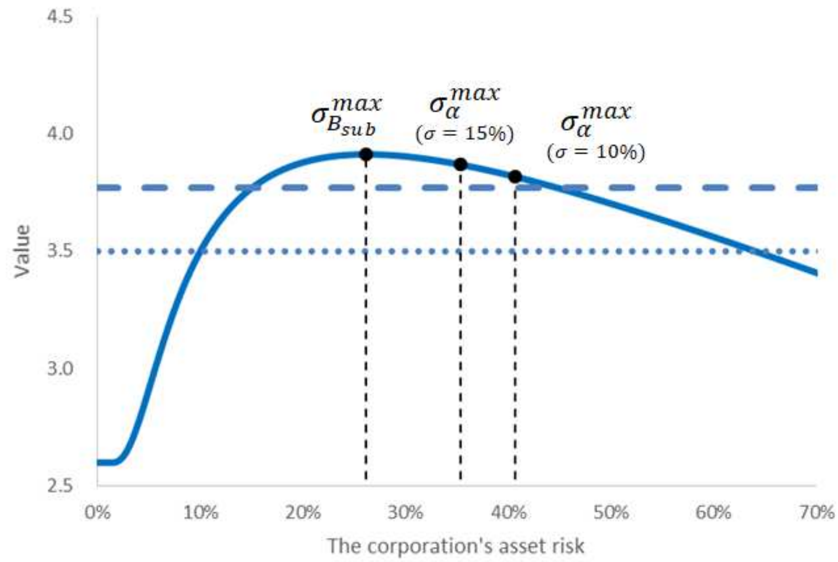


Figure 4: The value of the bank's subdebt versus the borrower's level of asset risk for different initial levels of asset risk. The face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and the face value of its subordinated debt is $F_{Sub} = 10$. In addition, the time to maturity is one year, the risk free-rate is $r = 1\%$, and $\alpha = 0.8$. The dotted and dashed lines represent the value of subdebt with levels of asset risk of 10% and 15%, respectively.

without the side payments. Several papers expressed concerns about this possibility and suggested that regulators cap the interest payments on subdebt (Furlong and Keeley, 1987; Calomiris, 1999; Chen and Hasan, 2011). Payments from subdebtholders to stockholders can be implemented through deviations or threat of deviation from the absolute priority rule (Weiss, 1990).

Now the two sides have to jointly decide on both (1) how much to allow the borrower to increase the level of asset risk (if at all) and (2) how much to require the stockholders to pay the subdebtholders in a side payment of X_α dollars (which might be either positive or negative). As is standard in the bargaining literature, we find the solution to each one of these decisions separately in a two-step solution as follows: the two sides jointly choose (1) the level of asset risk that maximizes the sum of their claims, and (2) the side payment that is the unique Nash solution to the bargaining problem of dividing the joint value of the claims between the two parties. Therefore, an analysis of this case requires first a characterization of how the total value of both the subdebt and the stock depends on the level of risk, which is done in Section 4.5.1, and then an application of this analysis to the solution of the bargaining situation with side payments, which is done in Section 4.5.2.

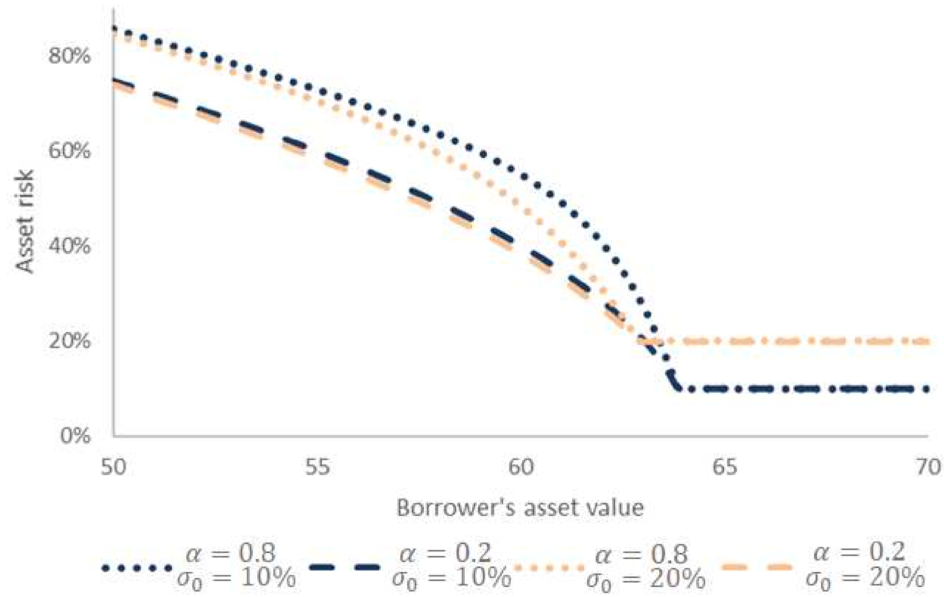


Figure 5: The borrower's level of asset risk for different initial levels of asset risk and alphas. The initial risk, σ_0, is 10% for the darker lines and 20% for the lighter lines. The face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and the face value of its subordinated debt is $F_{Sub} = 10$. In addition, the time to maturity is one year and the risk-free rate is $r = 1\%$.

4.5.1 Analysis of the sum of claims: Stock and subdebt

The joint payoff of stockholders and subdebtholders is the sum of payoffs from Eq. 3 and Eq. 4:

$$S_{B,t} + B_{Sub,t} = Call_t(V_{C,t}, F_{Dep}, \sigma, T-t, r) - Call_t(V_{C,t}, F_C, \sigma, T-t, r). \tag{13}$$

Since the sum of payoffs is a portfolio of two call options, analogous arguments to those presented in Propositions 1 and 2 show that the characterization of the sum of the value of the the two claims (i.e., the bank's stock and its subdebt) crucially depends on the threshold (see also the analysis in Appendix B):

$$V^*_{S_B+B_{sub}} \equiv e^{-r(T-t)}\sqrt{F_{Dep} \cdot F_C}, \tag{14}$$

which is a function of the geometric mean of the face value of the bank's deposits and of the borrower's loan. Observe that (1) the threshold $V^*_{S_B+B_{sub}}$ does not depend on the size of the subdebt or on the relative bargaining power α, and (2) this threshold is strictly

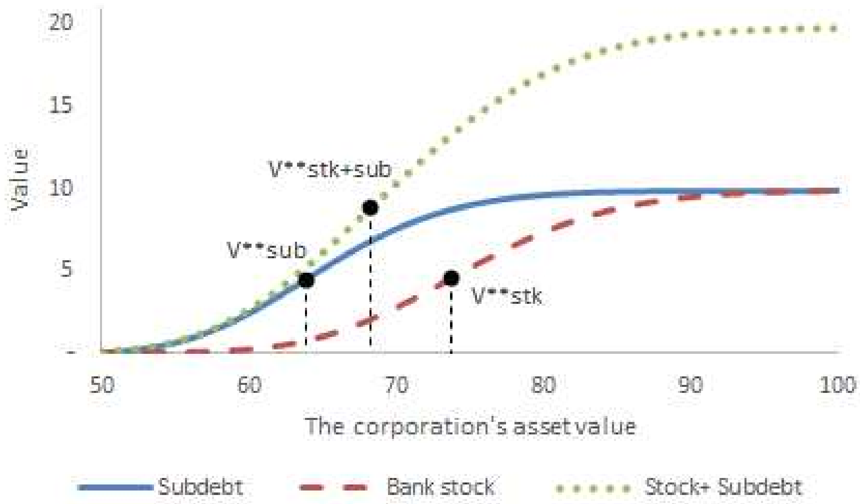


Figure 6: The value of the bank's stock and subdebt and the sum of both. The face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and the face value of its subordinated debt is $F_{sub} = 10$. The initial level of the borrower's asset risk is $\sigma_0 = 10\%$. The risk free-rate is $r = 1\%$ and the time to maturity is one year.

between the thresholds of risk-shifting for the subdebtholders and for the stockholders, i.e., $V^*_{B_{sub}} < V^*_{S_B+B_{sub}} < V^*_{S_B}$, as demonstrated in Figure 6.

In situations where the borrower's asset value is above this threshold, the total value of the stock and the subdebt decreases with the level of asset risk. By contrast, in situations where the value of the borrower's assets is below the threshold, the relationship between the total value of the stock and the subdebt and the level of asset risk is hump-shaped, and the maximum value of the stock and the subdebt is achieved with the risk level of (by analogous arguments to Propositions 1 and 2):

$$\sigma^{max}_{S_B+B_{sub}} \equiv \arg\max_{\sigma}(S_B + B_{sub})(\sigma) \equiv \sqrt{\frac{1}{T-t}\ln\left(\frac{F_C \cdot F_{Dep}}{(V_{C,t})^2}\right) - 2r}. \tag{15}$$

It is immediate that $\sigma^{max}_{B_{sub}} < \sigma^{max}_{S_B+B_{sub}} < \sigma^{max}_{S_B}$ (since $F_{Dep} < F_{Dep}+F_{sub} < F_C$), which implies that the level of asset risk that maximizes the total value of the two claims is between the level that maximizes the value of the subdebt and the level that maximizes the value of the stock.

Finally, analogous arguments to those used to prove Propositions 1 and 2 show that $\sigma^{max}_{S_B+B_{sub}} > \sigma_0$ if and only if the borrower's asset value is below a second threshold defined as

$$V^{**}_{S_B+B_{sub}} \equiv e^{-\left(r+\frac{\sigma_0^2}{2}\right)(T-t)}\sqrt{F_C \cdot F_{Dep}}. \tag{16}$$

Note that (1) $V^{**}_{S_B+B_{sub}}$ is above $V^{*}_{S_B+B_{sub}}$ whenever assets are risky, i.e., $\sigma \neq 0$, (2) $V^{**}_{S_B+B_{sub}}$ is between the thresholds for risk-shifting for the subdebt and for the stock, i.e., $V^{**}_{B_{Sub}} < V^{**}_{S_B+B_{sub}} < V^{**}_{S_B}$, and, (3) this threshold does not depend on the relative bargaining power α, either.

The following proposition summarizes the above analysis of the value of the bank's stock as a function of the asset's risk.

Proposition 7. *The total value of the bank's stock and subdebt $S_{B,t}+B_{Sub,t}$ is (1) decreasing in the level of asset risk if $V_{C,t} > V^{*}_{S_B+B_{sub}}$, (2) hump-shaped (unimodal) in the level of asset risk if $V_{C,t} > V^{*}_{S_B+B_{sub}}$, and, in this case, its maximum is obtained for a level of asset risk $\sigma^{max}_{S_B+B_{sub}}$. Moreover, the level of asset risk that maximizes the sum of the payoffs is higher than the initial level of risk (i.e., $\sigma^{max}_{S_B+B_{sub}} > \sigma_0$) if and only if $V_{C,t} < V^{**}_{S_B+B_{sub}}$.*

It is important to note that the problem faced by the two claimholders in this case is the same as the one faced by stockholders in a bank with no subdebt. In both cases, the party in control can increase its payoff by extracting value only from the depositors. This means that if side payments are possible, the depositors and the deposit insurer will not benefit from regulation requiring banks to issue subdebt to replace stock; in fact, the state of the depositors and the deposit insurer will not change at all.

4.5.2 Bargaining solution with side payments

Proposition 7 implies the following characterization of risk-shifting in the Nash bargaining solution in the case of joint control of the bank with side payments.

Corollary 8. *When the bank is jointly controlled by the subdebtholders and the stockholders (with a relative bargaining power of α for the stockholders) and side payments are feasible, risk-shifting occurs if and only if $V_{C,t} < V^{**}_{S_B+B_{sub}}$. When there is risk-shifting, the level of risk-shifting (that is independent of α) is $\sigma^{max}_{S_B+B_{sub}} \in (\sigma^{max}_{B_{sub}}, \sigma^{max}_{S_B})$.*

Observe that the introduction of side payments increases the range of market values of the asset for which risk-shifting occurs. Specifically, without side payments risk-shifting occurs if and only if $V_{C,t} < V^{**}_{B_{Sub}}$, with side payments risk-shifting also occurs in the interval $V^{**}_{B_{Sub}} < V_{C,t} < V^{**}_{S_B+B_{sub}}$. A comparison of the different cases is presented in Table 1.

Next we characterize the amount paid as part of the bargaining solution.

Proposition 9. *When the bank is jointly controlled by the subdebtholders and the stockholders (with a relative bargaining power of α for the stockholders) and side payments are feasible, then side payments are used if and only if there is risk-shifting (i.e., if $V_{C,t} < V^{**}_{S_B+B_{sub}}$). In this case, the (possibly negative) amount x_α that the stockholders pay to the subdebtholders is*

$$x_\alpha = \arg\max_{x \in R} \left(\left(S_{B,t}\left(\sigma^{max}_{S_B+B_{sub}}\right) - S_{B,t}\left(\sigma_0\right) - x_\alpha \right)^\alpha \cdot \left(B_{Sub,t}\left(\sigma^{max}_{S_B+B_{sub}}\right) - B_{Sub,t}\left(\sigma_0\right) + x_\alpha \right)^{1-\alpha} \right).$$

Observe that the side payment x_α is decreasing in α; i.e., lower the stockholders' bargaining power α, the higher their side payment to the subdebtholders.

5 A Numerical Analysis: Changes in Capital Structure and in Regulatory Policy

The equilibrium level of asset risk for different control rights and different bargaining rules is analyzed in Section 4. However, the analysis of each case is conducted separately. The aim of this section is to summarize and to compare the results of all cases by presenting a numerical analysis. The analysis is focused on the effect of changes in the bank's capital structure as well as changes in the regulator's forbearance policy and their effects on the equilibrium level of asset risk and on the cost of default.

The analysis is conducted using our base-case parameters, discussed in Section 3.4. The face value of the corporation's loan is 80, the face value of the subdebt is 10, and the face value of the deposits is 60. These face values yield for the bank a leverage of 87.5%. In addition, the time to maturity is one year and the risk-free rate is 1%. We consider an initial level of asset volatility of 10%, similar to the level of risk of investment grade bonds (Huang and Huang, 2012).

When the bank's assets are a risky debt claim, as in our framework of analysis, the highest asset value for which risk-shifting occurs is the discounted geometric average of the face value of the borrower's debt and the total face value of the bank's debt, which is equal to 73.7. However, risk-shifting at this point occurs only if the bank's stockholders possess the control rights, as presented in Table 2. In fact, when the stockholders control the bank, risk-shifting is possible for any asset value below 73.7. For example, if the corporation's asset value is 70, risk-shifting will occur only if stockholders control the bank, in which case the equilibrium level of asset risk is 33.7% and the value of deposit insurance per dollar of insured deposits is 7.3%.

If the bank is jointly controlled and side payments are possible, risk-shifting can occur for any asset value below the discounted geometric average of the face value of the borrower's debt and the face value of the bank's deposits, which is equal to 68.3. For example, if the borrower's asset value decreases to 65, risk-shifting takes place either when stockholders are in control, in which case the equilibrium level of asset risk is 51.2%, which corresponds to a cost of deposit insurance per dollar (DIPD) of 16.6%, or when the bank is jointly controlled and side payments are possible, in which case subdebtholders are able to restrict the increase in the equilibrium level of asset risk to 32.8%, which corresponds to a DIPD cost of 9.3%. This case is important since, as shown in the previous section, a capital structure with subdebt where side payments are possible yields identical results to the case where subdebt is swapped with stock. Thus, if the subdebtholder cannot affect the level of risk as in the first case, a capital structure with stock only is superior. However, as can be seen from the table, side payments clearly decrease market monitoring, since if side payments were restricted, subdebtholders would not agree to an increase in risk at all. As discussed above, while the equilibrium level of asset risk is unaffected by stockholders' relative bargaining power, the payment the stockholders must make to the subdebtholders to prevent them from approaching the regulator decreases with the stockholders' relative bargaining power.

The last threshold that effects risk-shifting is the discounted geometric average between the face value of the deposits and the total face value of the bank's debt. At this level risk-shifting can occur if the subdebtholders have a complete control over the chosen level of asset risk or if there is bargaining between the bank's claimholders. For example, if the borrower's asset value further decreases to 62, risk-shifting can take place in each of the cases of different control rights, but its degree increases with the stockholders' relative bargaining power. The equilibrium level of asset risk is highest when stockholders have full control, 59.7%, in which case DIPD receives its highest value of 8.5%. The lowest level of asset risk is observed when subdebtholders have full control, 26.2% (similarly the cost of DIPD is the lowest at 26.2%). If claimholders have joint control and side payments are not possible the equilibrium level of asset risk is between 26.2% and 59.7%, depending on the relative

Table 2: Numerical Analysis: Equilibrium level of asset risk and the cost of deposit insurance

<table>
<tr><th rowspan="2">Control and bargaining framework</th><th colspan="4">Corporation's asset value (V_C)</th></tr>
<tr><th>$V_C > 73.7$</th><th>$V_C = 70$</th><th>$V_C = 65$</th><th>$V_C = 62$</th></tr>
<tr><td>Stockholders control</td><td rowspan="4">No risk-shifting</td><td>Risk-shifting to 33.7%
DIPD=7.3%</td><td>Risk-shifting to 51.2%
DIPD=16.6%</td><td>Risk-shifting to 59.7%
DIPD=21.6%</td></tr>
<tr><td>Joint control with side payments
$\alpha = 0.2/\alpha = 0.5/\alpha = 0.8$</td><td rowspan="3">No risk-shifting</td><td>Risk-shifting to 32.8%
Side payment: 1.62/1.28/0.95
DIPD=9.3%</td><td>Risk-shifting to 45.0%
Side payment: 1.72/0.98/0.23
DIPD=15.9%</td></tr>
<tr><td>Joint control without side payments
$\alpha = 0.2/\alpha = 0.5/\alpha = 0.8$</td><td rowspan="2">No risk-shifting</td><td>Risk-shifting to
28.5%/32.9%/40.6%
DIPD: 9.4%/11.2%/14.2%</td></tr>
<tr><td>Subdebtholders control</td><td>Risk-shifting to 26.2%
DIPD=8.5%</td></tr>
</table>

The numerical analysis refers to the case where the face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and the face value of its subordinated debt is $F_{sub} = 10$. The initial level of asset risk is $\sigma_0 = 10\%$. The risk-free rate is $r = 1\%$ and the time to maturity is one year. The table presents the equilibrium level of asset risk as well as the cost of deposit insurance, in terms of the percentage of the face value of deposits, for different levels of asset value and subdebtholder's bargaining power.

bargaining power. In the case where claimholders have joint control and side payments are possible, or if the bank has no subdebt and is funded by stock and deposits, the equilibrium level of asset risk is 45% and the cost of DIPD is 15.9%.

A similar picture emerges in Panel (a) of Figure 7, which presents the equilibrium level of the borrower's asset risk. The equilibrium level of asset risk is at its highest level when the bank is controlled by the stockholders and at its lowest when it is controlled by the subdebtholders. The equilibrium level of asset risk is between these two levels when the bank is jointly controlled and risk-shifting occurs only for asset values for which subdebtholders would increase risk if they controlled the bank. The introduction of side payments may increase or decrease the equilibrium asset risk, depending on the stockholders' relative bargaining power. A mirror imeage of Panel (a) emerges from Panel (b) of Figure 7 which presents the bank's cost of deposit insurance per dollar of insured deposits. As can be seen, a higher level of equilibrium risk translates to a higher cost of deposit insurance.

The effect of replacing stock with subdebt depends on the relative power of the subdebtholders and the stockholders. If subdebtholders do not have the ability to affect the level of asset risk, the issuance of subdebt will increase the equilibrium level of asset risk due to the increased leverage. If subdebtholders have some bargaining power, the introduction of subdebt will decrease the equilibrium level of asset risk for asset values in the range between $V^{**}_{B_{Sub}}$ and $V^{**}_{S_B+B_{sub}}$, since with subdebt there will be no risk-shifting in this range at all. However, the effect of the issuance of subdebt on the equilibrium level of asset risk for asset values below $V^{**}_{B_{Sub}}$ depends on the relative bargaining power of the stockholders, which is captured in our model by the parameter α. When stockholders' bargaining power is low enough, the introduction of subdebt will decrease the level of asset risk in equilibrium for all asset values. By contrast, when stockholders have relatively high bargaining power, the level of asset risk decreases following the issuance of subdebt only for asset values above some threshold (which is below $V^{**}_{B_{Sub}}$), but it increases following the issuance of subdebt for asset values below this threshold. The threshold decreases with the stockholders' bargaining power α.

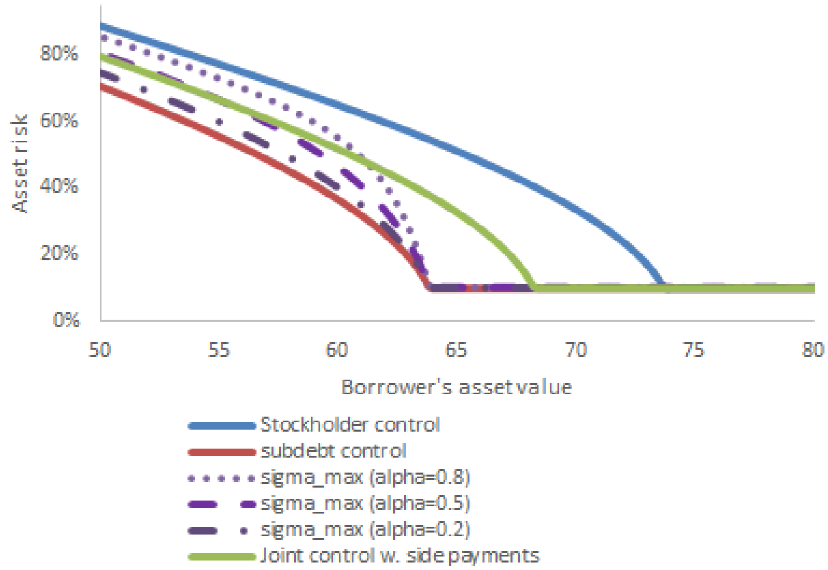
In our example, using the base-case parameters, we find that when α is smaller than 0.45, the introduction of subdebt to replace stock always leads to a decrease in the equilibrium level of asset risk relative to a bank with no subdebt. By contrast, when α is above 0.45, the equilibrium level of asset risk with subdebt is higher than in a bank with no subdebt for low enough asset values. For example, when $\alpha = 0.8$, the introduction of subdebt leads to a lower equilibrium level of asset risk for asset values above 61.1, but it leads to a higher such level for asset values below 61.1.

Figure 8 presents the effect of the amount of subdebt on the equilibrium level of asset risk. When subdebtholders have any level of bargaining power, as the subdebt that replaces the stock increases and consequently the stock decreases, the equilibrium level of asset risk increases. This is shown in Panel (a) for the case where stockholders control the bank and in Panel (b) for the case where subdebtholders control the bank.

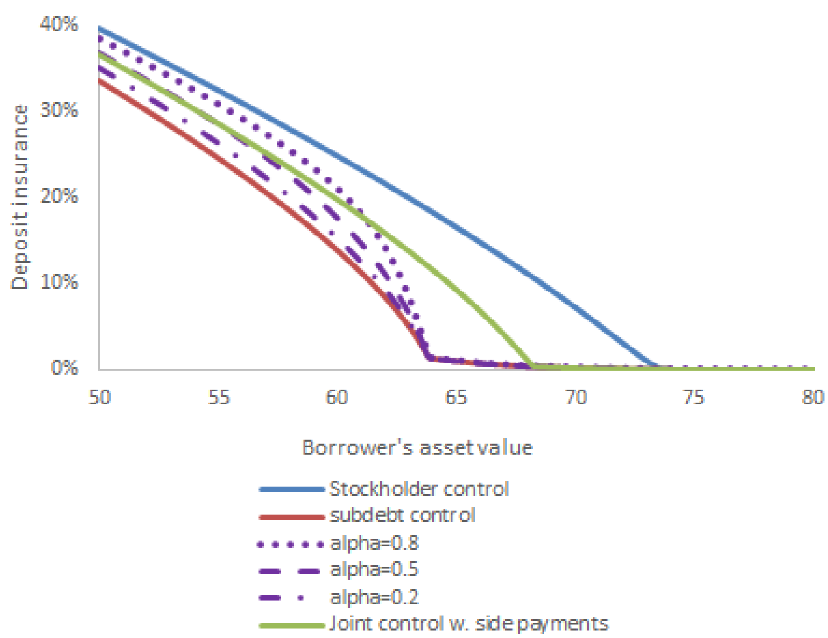
The equilibrium level of asset risk is not affected by a bank's leverage ratio if side payments are possible and consequently the issuance of subdebt has no effect on the equilibrium level of asset risk.

6 Conclusion

The accepted belief among policymakers and regulators before the 2007–2009 financial crisis was that uninsured subordinated creditors can effectively reduce financial institutions' excessive risk-taking. However, after the 2007–2009 financial crisis the effectiveness of subdebt as a monitoring tool was questioned. On the one hand, many financial institutions with

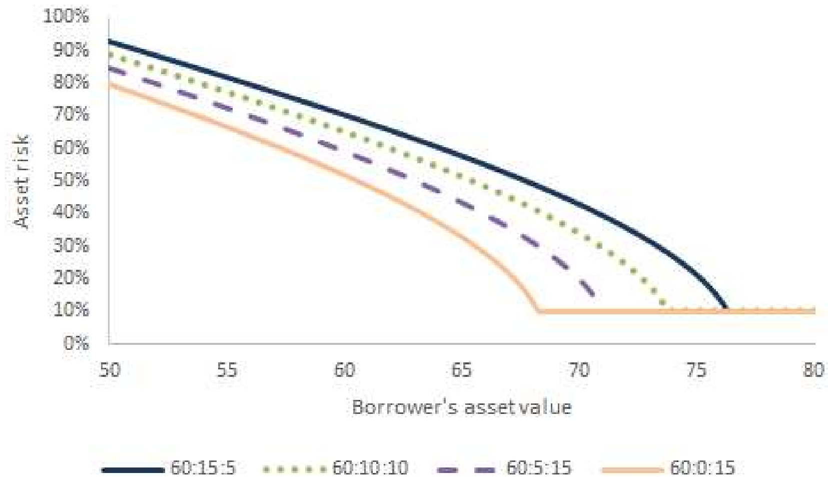


(a) Asset risk

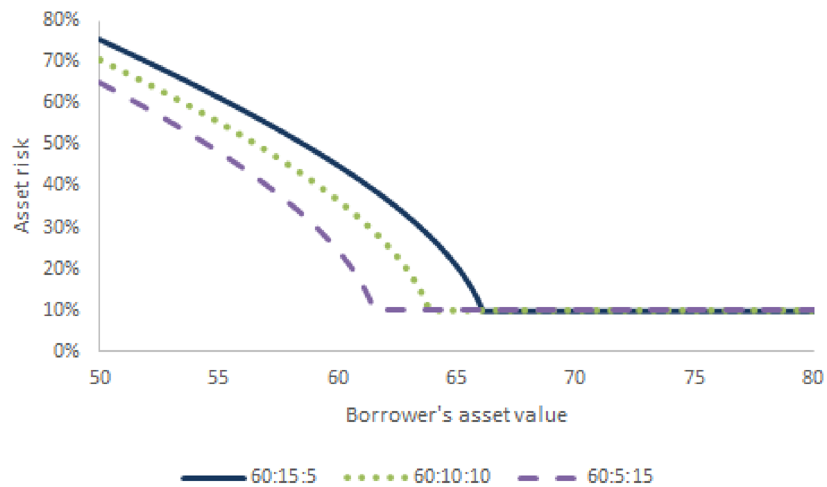


(b) Value of deposit insurance per dollar of insured deposits

Figure 7: The equilibrium level of the borrower's asset risk and the value of insured deposits per dollar of insured deposits. The figure depicts the equilibrium level of asset risk for all the cases discussed in Section 4. The face value of the borrower's debt is $F_C = 80$. The face value of the bank's deposits is $F_{Dep} = 60$ and the face value of its subordinated debt is $F_{sub} = 10$. The initial level of risk is $\sigma_0 = 10\%$. In addition, the time to maturity is one year and the risk-free rate is $r = 1\%$.



(a) Asset risk chosen by stockholders



(b) Asset risk chosen by subdebtholders

Figure 8: The equilibrium level of the borrower's asset risk for different amounts of subdebt. The top panel of the figure depicts the equilibrium level of asset risk chosen by the stockholders while the bottom panel depicts the equilibrium level of asset risk chosen by the subdebtholders. The different lines correspond to different face values of subdebt, F_{sub}, ranging from 15 to zero, i.e., a bank funded only by stock and deposits. The face value of the borrower's debt is $F_C = 80$ and the face value of the bank's deposits is $F_{Dep} = 60$. The initial level of asset risk is $\sigma_0 = 10\%$. In addition, the time to maturity is one year and the risk-free rate is $r = 1\%$.

subdebt as part of their capital structure were bailed out using taxpayers' money. On the other hand, the empirical literature found that subdebt can reduce bank's risk-taking. Motivated by these mixed conceptions and the revision to banking regulation (Basel III) that followed the crisis, we enhance the understanding of subdebt's effect on bank's risk-taking by analyzing the interaction between the bank's different claimholders. The analysis of how the equilibrium level of asset risk varies with the bargaining power of the subsdebtholders allows us to reconcile different theories on the ineffectiveness of subdebt as a disciplinary tool and also to see the connection between these theories and the empirical research on subdebt.

We find that when the stockholder fully controls the bank's decision on the level of its borrower's asset risk, risk-shifting takes place when the value of the borrower's assets is below the discounted geometric average of the face value of the debt of the borrower and the face value of the total debt of the bank. Thus, risk-shifting occurs only if the bank's borrower is in financial distress and in such a case the equilibrium level of asset risk has an interior solution and is not unlimited as predicted by Gorton and Santomero (1990). This case is in line with the claim that subdebtholders cannot affect a bank's risk-taking as they are uninformed about the bank's level of asset risk. In the other extreme case, where the subdebtholders control the bank's level of asset risk, risk-shifting occurs only when the borrower's asset value is below the discounted geometric mean of the face value of the bank's total debt and the face value of its deposits. Thus, risk-taking is limited to fewer states of the borrower's asset value and, when it does occur, the levels of asset risk are lower than when the stockholders control the level of risk. This case rests on the assumption that subdebtholders are informed about the bank's level of asset risk, but risk-shifting can still occur when they are better off with such a shift.

We further analyze the (more realistic) case where the stockholders and subdebtholders jointly control the bank. In this case the equilibrium level of asset risk is between the levels in the case where the subdebtholders control asset risk and the case where the stockholders do. Still, the asset values for which risk-shifting takes place are the same as when the subdebtholders determine the level of asset risk. We show that risk-shifting decreases as the bargaining power of the subdebtholders increases. In addition, we show that replacing stock with subdebt leads to fewer asset values for which risk-shifting can take place, but when the bargaining power of the stockholders is relatively high, replacing stock with subdebt can lead to a higher level of risk in equilibrium. When side payments between the claimholders are possible, the range of states for which risk-shifting occurs increases, but to a lower level than when the stockholders control asset risk, and the equilibrium level of asset risk can be below the one that is achieved in a process with bargaining and no side payments. We also show that risk-shifting behavior in this case is identical to what would be observed if subdebt was replaced by additional stock.

Following the 2007–2009 financial crisis, the issue of bank transparency was debated. The international regulatory framework Basel III calls for financial institutions to increase transparency by conducting stress tests involving an unprecedented amount of disclosure. On the one hand, transparency can prevent excessive risk-shifting by banks. On the other hand, it is often argued that transparency has significant disadvantages in banking, given the role of banks in liquidity provision and risk-sharing. We contribute to this debate by showing, counterintuitively, that as the level of asset risk that is determined in a regulatory audit event decreases, the equilibrium level of asset risk increases (when the bargaining power of the stock holders is sufficiently high). As a result, a more restrictive corrective measure imposed by the regulator can motivate claimholders to agree on a higher level of asset risk in the bargaining process. Thus, the efficiency of subdebt as a disciplinary tool declines with the enforcement of traditional regulatory tools such as on-site supervision.

Appendix A: Notation

The bank's borrower (the corporation)	
S_C	Value of stock of the borrower.
F_C	Face value of debt of the borrower.
B_C	Value of the borrower's debt.
T	Time in which all debt matures (time till next audit).
V_C	Value of the borrower's assets.
μ	Instantaneous expected return on the borrower's assets.
σ	Instantaneous volatility of the borrower's assets (asset risk).
dW	Standard Wiener process.
The bank	
V_B	Value of bank's assets.
S_B	Value of bank's stock.
F_{Dep}	Face value of deposits.
B_{Dep}	Value of deposits.
F_{Sub}	Face value of subdebt.
B_{Sub}	Market value of subordinated debt.
DI	Value of deposit insurance.
$V_{S_B}^{*}$	Threshold below which the bank's stockholders prefer risk higher than zero.
$V_{S_B}^{**}$	Threshold below which the bank's stockholders prefer risk higher than the initial risk.
$\sigma_{S_B}^{max}$	Stockholders' preferred asset risk when $V_C < V_{S_B}^{*}$.
$V_{B_{Sub}}^{*}$	Threshold below which the subdebtholders prefer risk higher than zero.
$V_{B_{Sub}}^{**}$	Threshold below which the subdebtholders prefer risk higher than the initial risk.
$\sigma_{B_{Sub}}^{max}$	Subdebtholders' preferred asset risk when $V_{B_{Sub}}^{*} < V_{C,t}$.
α	Bargaining power of the stockholders relative to the subdebtholders.
σ_{α}^{max}	Preferred asset risk when stockholders and subdebtholders have joint control.
X_{α}	Side payments.
$V_{S_B+B_{sub}}^{*}$	Threshold below which the bank's claimholders prefer risk higher than zero.
$V_{S_B+B_{sub}}^{**}$	Threshold below which the bank's claimholders prefer risk higher than the initial risk.
$\sigma_{S_B+B_{sub}}^{max}$	Preferred risk with side payments.

Appendix B: Proofs

The payoff of both the bank's stockholders (Eq. 4) and its subdebtholders (Eq. 3), as well as the sum of their payoffs (Eq. 13), are all equivalent to a portfolio of two call options on the value of the borrower's assets where each payoff is defined by options with a different strike price. All these payoffs can be represented generally, using the strike prices F_1 and F_2, as

$$P_t = Call_t(V_{C,t}, F_1, \sigma, T-t, r) - Call_t(V_{C,t}, F_2, \sigma, T-t, r).$$

To find the level of the borrower's asset risk that maximizes the value of any of these payoffs we calculate the derivative of the value of equity with respect to asset risk. Since it is well known that

$$\frac{\partial call_t}{\partial \sigma} = \frac{\sqrt{T}}{\sqrt{2\pi}} \cdot V^C \cdot e^{-\frac{1}{2}\cdot(d_1(F_i))^2},$$

where $d_1 = \frac{1}{\sigma\sqrt{T-t}} \cdot \left[\ln\left(\frac{V_{C,t}}{F_i}\right) + \left(r + \frac{1}{2}\sigma^2\right) \cdot (T-t)\right]$, we get

$$\frac{\partial P_t}{\partial \sigma} = \frac{\sqrt{T}}{\sqrt{2\pi}} \cdot V^C \cdot e^{-\frac{1}{2}\cdot(d_1(F_1))^2} - \frac{\sqrt{T}}{\sqrt{2\pi}} \cdot V^C \cdot e^{-\frac{1}{2}\cdot(d_1(F_2))^2}.$$

After rearranging it can be shown that the derivative is equal to

$$\frac{\partial P_t}{\partial \sigma} = \frac{\sqrt{T-t}}{\sqrt{2\pi}} \cdot V_{C,t} \cdot e^{-\frac{1}{2\cdot T\cdot\sigma^2}} \left[e^a - e^b\right],$$

where a and b are defined as

$$a = -2 \cdot \ln V_{C,t} \cdot \ln F_1 + (\ln(F_1))^2 - 2 \cdot \ln(F_1) \cdot \left(r + \frac{\sigma^2}{2}\right) \cdot (T-t)$$

$$b = -2 \cdot \ln V_{C,t} \cdot \ln F_2 + (\ln(F_2))^2 - 2 \cdot \ln(F_2) \cdot \left(r + \frac{\sigma^2}{2}\right) \cdot (T-t).$$

The payoff is maximized with respect to the level of asset risk in cases where the first derivative equals zero. This happens when either $V_{C,t} = 0$ or $a = b$. Since the first option is of no interest economically we focus on the second option. We find that $a = b$ when

$$V_{C,t} = e^{-\left(r+\frac{1}{2}\sigma^2\right)\cdot(T-t)} \cdot \sqrt{F_1 \cdot F_2}. \tag{B.1}$$

Based on Equation B.1, we define V^{**} as the borrower's asset value for which the claim's value is maximized given a level of asset risk σ. This threshold is identical to the one found in Black and Cox (1976) and Gorton and Santomero (1990) for subdebtholders. We also note that the derivative changes its sign from positive to negative above the threshold, V^{**}; i.e., the bank's claimholder would like to increase risk below that level and to decrease it above that level of assets.

Alternatively, fixing the level of assets, we find that $a = b$ when

$$\sigma_{max} = \sqrt{\frac{1}{T-t} \ln \left(\frac{F_1 \cdot F_2}{(V_{C,t})^2} \right) - 2r}. \tag{B.2}$$

However, for both Equations B.1 and B.2 to hold, that is, for an internal solution to exist, the corporation's asset value must be below V^* defined as $V^* \equiv e^{-r \cdot (T-t)} \sqrt{F_1 \cdot F_2}$.

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