Group size and network formation

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1 Introduction

Interactions among individuals are greatly shaped by their socio-economic attributes. In fact, similarity in attributes is a strong predictor of tie formation. This phenomenon is pervasive and known as homophily.[1] The seminal work by Schelling [1969] further postulates a striking result, namely, segregation arises even in the case in which individuals are happy with a mixed society.

This paper studies processes of integration and segregation using a variation of the symmetric connections model by Jackson and Wolinsky [1996]. In contrast to Jackson and Wolinsky [1996] in which individuals are homogeneous, in this paper individuals are of two types, for instance, defined according to an exogenous dichotomous trait, and exhibit preferences that resemble those in Schelling [1969]. In particular, individuals derive more utility from relations as long as in its neighborhood, their own type is represented above a given fraction. Individuals might have preferences for being surrounded by similar types because is easier to assess their performance in Bagues and Perez-Villadoniga [2013]. In a similar vein Rempel [2017] concludes how people trust more similar others. Interaction costs are higher between different than between similar types. That may reflect disutility of interracial contacts as Battu et al. [2007] and De Martí and Zenou [2017] argue.

Conceptually, this model can be seen as a threshold model of collective action, as Schelling [1969], Schelling [1971] and Granovetter [1978]. In this respect Card et al. [2008] document how in fact whites' actions, specifically, migration patterns, exhibit threshold-like behavior regarding blacks population shares. The magnitude of this threshold is shown to be related to whites attitudes towards interracial contact and also to income levels of blacks and whites.

The main question is which networks emerge in equilibrium when individuals

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1See McPherson et al. [2001] for a comprehensive survey on this topic.

exhibit the aforementioned preferences. As in Jackson and Wolinsky [1996] and Jackson and Rogers [2005] the equilibrium concept is pairwise stability. That is the simplest concept in which individuals have discretion on whether to sever and form relations, overcoming the coordination problems that might arise in Nash equilibrium, in which, dominated strategies, might lead to empty equilibrium networks.[2] Briefly a network is pairwise stable (henceforth, PS) whenever individuals involved in a relation do not want to terminate it and individuals not involved in a relation do not want to establish it. Other papers studying network formation with this focus are De Martí and Zenou [2017], McBride [2006], Johnson and Gilles [2003] and Iijima and Kamada [2017].

Among the previous papers, the closest ones to the current proposal are De Martí and Zenou [2017] and Johnson and Gilles [2003]. De Martí and Zenou [2017] propose an interesting specification in which interactions costs are endogenous and higher between than within communities. Specifically, intercommunity costs are increasing in the relative number of friends of the same type an individual has. Due to the combinatorial complexity that the model entails there are some potentially interesting question that remains to be exploited, as a deeper understanding of the tension between efficiency and stability with an analysis of transfer systems that conciliate both perspectives. In contrast, in the current paper cost are exogenous. By proposing a simpler model in terms of combinatorial complexity the current paper goes further in exploiting the trade-off between efficiency and stability and describing segregation patterns of equilibrium networks. Johnson and Gilles [2003] also consider exogenous costs, that depend on the physical distance between individuals located on the real line. That is the source of heterogeneity. The cost structure in the current paper can be seen as the dichotomous version of the one above. On the benefit side, in the current proposal utility differs with group composition. In contrast to us Johnson and Gilles [2003] do not carry over an analysis of transfers. Further, the relation between segregation and network structure, present in this proposal, is absent in these two aforementioned approaches.

More deeply, the focus of the paper is on the following aspects:

Stability and uniqueness. The analysis is on the conditions under which the completely integrated and segregated networks emerge as equilibria. Completely integrated networks (henceforth, CI) are those in which everyone is connected. Briefly, for CI to be PS the linking costs with individuals of different type should be sufficiently low. For completely segregated network (henceforth, CS) to be PS, the opposite has to hold. Conditions slightly vary as a function of group size, since individuals are concerned with being in a minority group. In contrast to Jackson and Wolinsky [1996], CS naturally emerges as equilibrium due to cost heterogeneity.

[2] See Jackson [2008] for a discussion.

Also in contrast, when CI is PS, is not the unique one. Intuitively, individuals do not always prefer forming new connections, particularly if that leaves their type as a minority in her neighborhood. In this case, there is a full characterization of the class of networks that are PS. These networks exhibit full intraconnection, a situation in which similar types are all connected. Further, regarding links with different types there are three cases: (i) every individual is connected to all individuals of different type, i.e., CI, (ii) each individual of the minority type in the population exactly links to the number of individuals of the majority type that matches the population size of the minority type and (iii) any combination in which some individuals of the minority type connect to all individuals of the majority type and some others connects to the number of individuals of the majority type that matches the population size of the minority group.

Welfare. There is an examination on whether equilibrium networks are also the most socially preferable, in the sense of maximizing the sum of individual utilities. As in Jackson and Wolinsky [1996] there is a tension between efficiency and stability. It is direct to see that CS is not the most socially preferable when PS, since links between the two communities of different generate positive externalities. In particular, a network that bridges the two communities with one link might be socially preferable to CS. There are transfers among individuals that make that bridge network PS. These transfers may reduce inequality among society members.

Segregation and network structures. To assess the segregation that equilibrium networks induce, the focus is on the Spectral Segregation Index by Echenique and Fryer [2007]. Unlike other indexes assessing segregation, as the Dissimilarity or the Isolation Indexes, the Spectral Segregation Index fully relies on individual interactions. The main insight is that there is no a one to one mapping between segregation and welfare. In particular there are networks equally desirable from a welfare perspective, exhibiting different levels of segregation.

2 The model

There is a finite population of $n \geq 3$ individuals. Individuals are of two types, A and B. Let n_t be the number of individuals of type $t = \{A, B\}$ in the population and set $n_A \geq n_B$, w.l.o.g. Let $n = \sum_t n_t$.

Network of relations. Individuals are connected by an undirected network denoted g. Let $g_{ij} = g_{ji} = 1$ if individual i is friend with individual j and $g_{ij} = g_{ji} = 0$ otherwise. Let $t(i)$ denote the type of individual i. The set of neighbors of individual i in network g is $N_i(g) = \{j \neq i | g_{ij} = 1\}$. With some abuse of notation let $N_i(g)$ also denote the cardinality of this set. Analogously, the set (and the cardinality) of neighbors of individual i in network g that are of same type is $N_i^s(g) = \{j \neq$

$i|g_{ij} = 1 \; and \; t(i) = t(j)\}$. Let the fraction of individuals of same type than i in her neighborhood be $p \equiv \dfrac{N_i^s(g)+1}{N_i(g)+1}$.

Preferences. As in Jackson and Wolinsky [1996] individuals derive utility from direct as well as indirect connections. In contrast, individuals derive utility of $0 < \delta < 1$ from each of their connections whenever their own type represents at least one half in their neighborhood and utility $0 < \beta < \delta$ of when this fraction is smaller than one half. Indirect connections follow the same structure with a decay term.[3] Specifically, the value of an indirect connection between i and j decays with its geodesic distance, namely $d(i,j)$.[4] The main reason to use one half as a threshold according to which utility changes is twofold: first, since the concern is whether groups of same type individuals represent a given fraction of the population and pairwise stability involves only one link deviations, the same results on equilibrium networks could be replicated by adapting the conditions regarding to whether one link deviation causes an individual i type to be sufficiently represented in her neighborhood. The problem becomes more a technical than a substantive one. Second, Schelling [1971] uses this fraction in the main arguments. Also, in a reexamination of Schelling proposal, Fagiolo et al. [2007] uses this threshold. In broad terms one half can be understood as a focal point. Regarding cost, only direct links are costly. Establishing links is cheaper between same type individuals than between different type individuals. Specifically, $c_{ij} = c$ if $t(i) = t(j)$ and $c_{ij} = C$ if $t(i) \neq t(j)$ with $C > c$. The utility of individual i in network g is:

$$u_i(g) = \begin{cases} \sum\limits_{j \in g} \delta^{d(i,j)} - \sum\limits_{j \in N_i(g)} c_{ij} & if \quad p \geq 0.5 \\ \sum\limits_{j \in g} \beta^{d(i,j)} - \sum\limits_{j \in N_i(g)} c_{ij} & if \quad p < 0.5 \end{cases}.$$

Equilibrium networks. The equilibrium concept is pairwise stability. Let $g + ij$ denote the network g when the link ij. Analogously $g - ij$ denotes the network g when the link ij has been deleted. Pairwise stability is defined as follows:

Definition. A network g is PS if:

1. For all $ij \in g$, $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$.

2. For all $ij \notin g$, if $u_i(g + ij) > u_i(g)$ then $u_j(g + ij) < u_j(g)$.

Pairwise stability requires that no individual gains from severing an existing link and no pair of individuals that are not connected both gain from forming a direct

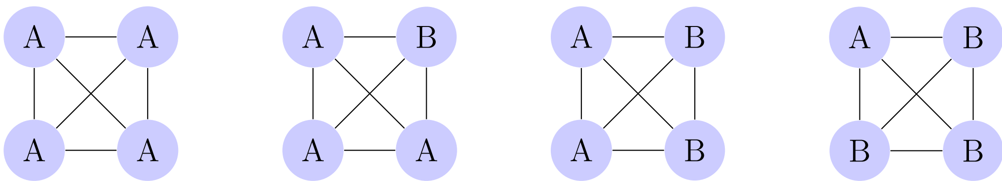
[3] Notice that the neighborhood of individual i is defined as the individuals with whom i has direct connections. There is a brief discussion at the end of the paper on how the model changes when the definition of neighborhood is beyond direct connections.

[4] The geodesic distance between i and j is the minimum number of links needed from i to reach j. If i does not reach j, $d(i,j) = \infty$.

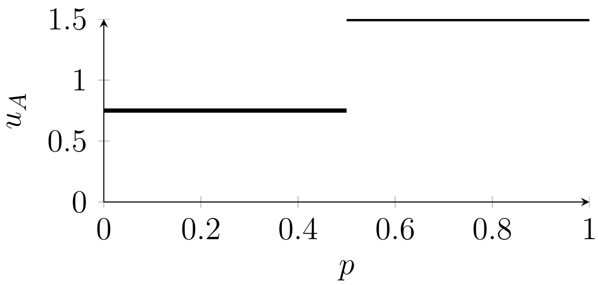
link with each other. Thus, mutual consent is needed to form a link while link severance can be done unilaterally.

Before the results it is worthwhile to briefly analyze the characteristics of this model in relation to Schelling's proposal. While in Schelling' proposal individuals are located on a fixed grid (a line if individuals are defined on one dimension), here the focus is on more general network structures. Moreover in the current proposal mutual consent is needed for a link to be formed while in Schelling's proposal agents choose unilaterally to change their positions to another that is available. With respect to the utility function, in the current proposal, conditional on having the same number of direct and (length of) indirect connections, an individual is better off whenever, in her neighborhood, her type represents at least one half of the population. The next example illustrates it:

Example 1. Consider the following networks. Focus on the individual of type A above on the left, establishing links with three individuals:

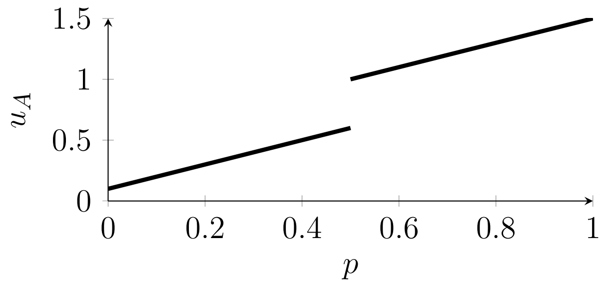


For the sake of exposition, first consider a simpler case in which $c = C$. Recall that $p \equiv \dfrac{N^s_A(g)+1}{N_A(g)+1}$. From left to right $u_A = 3(\delta - c)$ in the first three networks since $p \geq 0.5$, while $u_A = 3(\beta - c)$ in the last one, since $p = 0.25$. Utility of A has the following shape:[5]



Notice that the individual of type A is indifferent among the first three configurations. In particular, there is no strict preference for a segregated neighborhood, i.,e., for p being 1, over a mixed one. That is parallel to Schelling. Observe that in this case CS is never PS. Since any link costs the same, there is not rationale behind that fact that an individual wants to link with some individuals and not with others. Thus, whenever individuals are indifferent between different levels of integration because links are equally costly, in contrast to Schelling, segregation does not emerge. The current model considers $C > c$. Thus utility looks like:

[5]When there is no risk of ambiguity we describe an individual i by her type.



From left to right u_A is $3(\delta-c) > 2(\delta-c)+(\delta-C) > (\delta-c)+2(\delta-C) > 3(\beta-C)$. Was the number of connections fixed, the individual is also happier whenever in her neighborhood their type represent at least one half. Since links are more costly when happen between individual of different types, utility is increasing in the number of links with same type individuals. As in the current model the number of connections matter, a trade-off between the quantity of links and its nature arises. Despite of segregation being strictly preferred when the number of links is fixed, there are costs configurations for which CI is PS. Thus, links quantity plays a crucial role.

3 Results

3.1 Pairwise stable segregated and integrated networks

This section analyses two polar networks. The case in which all individuals are connected to each other, namely, CI, and the case which there are two groups of same type individuals that are completely connected among themselves and completely isolated from the other group, namely, CS. The results are as follows:

Proposition 1. *Let $n_A = n_B$. Then:*

1. *CI is PS iff $C \leq \delta - \delta^2$.*
2. *CS is PS iff $C > \delta + \delta^2 (n_B - 1)$ and $c \leq \delta - \delta^2$.*

Proposition 2. *Let $n_A > n_B$. Then:*

1. *Whenever $n_B = n_A - 1$, CI is PS iff $C \leq (n-1)(\beta - \delta) + \delta - \delta^2$.*
2. *Whenever $n_B < n_A - 1$, CI is PS iff $C \leq min\{\delta - \delta^2, \beta - \beta^2\}$.*
3. *CS is PS iff $C > \delta + \delta^2(n_B - 1)$ and $c \leq \delta - \delta^2$.*

When the cost of maintaining links with same type individuals is sufficiently low while maintaining links with different type individuals is sufficiently costly, CS emerges as equilibrium. In contrast, CI requires that the cost of linking to different type individuals is small enough. The fraction of types A and B in the population slightly alters the conditions under which these networks emerge as equilibria. In particular, under $n_B = n_A - 1$ when types B evaluate whether to sever a link with types A in CI, B types represents exactly one half of its neighborhood, that is why δ and β both enter into play at the margin.[6]

The next result explores uniqueness:

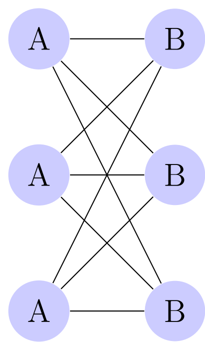
Proposition 3. *Let CS be PS, then it is unique. Let CI be PS. Then, when $n_A - 1 = n_B$ it is unique and when $n_A = n_B$ or $n_A - 1 > n_B$ it is not unique. In the latter case, fully intraconnected networks in which one of the following configurations holds may be also PS:*

1. *Each type B is connected to n_B types A.*

2. *Some types B are connected to all types A and the remaining types B are connected to n_B types A.*

The reasons as to why CI is not unique come from different sources depending on whether $n_A = n_B$ or $n_A - 1 > n_B$. When $n_A = n_B$ and similar types break their type becomes a minority in her neighborhood. Hence they lose a lot from that action. Thus for a wider range of cost of linking similar types, c (in relation to the case in which $n_A - 1 > n_B$) she is willing to maintain similar types links. These wider range for c open the possibility for makes networks to be PS. The next example illustrates it:

Example 2. Let $n_A = n_B$. The bipartite network in which each individual is connected to all individuals of different type and there are no others links is PS iff $C \leq \beta - \beta^3$ and $c > \beta - \beta^2$. Recall that CI is PS under $c < C \leq \delta - \delta^2$. Let $\beta - \beta^3 < \delta - \delta^2$ and $c < C \in (\beta - \beta^2, \beta - \beta^3)$. Hence there are costs structures such that both CI and the bipartite network are PS.

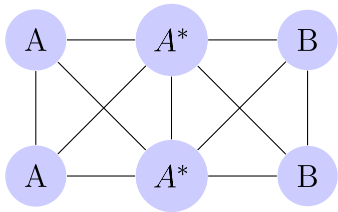


When $n_A - 1 > n_B$, B when individuals of type B break, they are in minority either way. Thus there are not losing a lot for that action. Hence to make the CI network PS, cost of linking with similar types have to be small. That precludes

[6] In Appendix 2: Propositions 8, 9 and 10 are the counterparts of Proposition 1 and 2 with δ_i and β_i.

any non fully intraconnected network from being PS. The next example illustrates networks that are PS, apart from the CI.

Example 3. A PS network, as the one described in point 1 of Proposition 3. For $n_A = 4$ and $n_{A^*} = n_B = 2$, $\delta = 0.9$ and $\beta = 0.1$., $\underline{C} = -3.91$. Thus, for $C \in (0, 0.09)$ that network is PS, together with CI. Whenever $\delta = 0.51$ and $\beta = 0.5$, $\underline{C} = 0.018$. Thus, for $C \in (0.18, 0.249)$ that network is PS, together with CI.



3.2 Welfare

For the purpose of doing a welfare assessment, let the focus being on an utilitarian perspective. Thus, the value of a network g is the sum of individual utilities, formally $v(g) = \sum_i u_i$. A network g is said to be socially preferable to a network g' when the sum of individual utilities is higher for g than for g', that is, when $v(g) > v(g')$. A network g is said to be the most socially preferable when the sum of individuals utilities for g is at least at high as the sum of individual utilities of any other network g', that is, when $v(g) \geq v(g')$, for all possible networks g' that can be formed.

Proposition 4. *Let CI be PS, then it is socially preferable to CS. Further:*

1. *Let $n_A = n_B$ or $n_A - 1 = n_B$. Then CI is the unique most socially preferable.*

2. *Let $n_A - 1 > n_B$ and:*

   (a) *$C \geq 2^{-1}(\delta + \beta) - \delta^2$. Then CI is not the most socially preferable. In particular, all SI yield the same value, are PS the most socially preferable.*

   (b) *$\overline{C} < C < 2^{-1}(\delta + \beta) - \delta^2$. Then the following situations may arise: (i) CI and SI are equally socially preferable in which case they are the most socially preferable, (ii) CI is socially preferable to SI, in which case CI is the most socially preferable and (iii) SI is socially preferable to CI, in which case they are the most socially preferable (and also PS).*

   (c) *$C \leq \overline{C} < 2^{-1}(\delta + \beta) - \delta^2$. Then CI is the unique most socially preferable.*

It is important to notice that there is not tension between stability and efficiency, in the sense that there is always a PS network which is the most socially preferred, either SI or CI.

When $n_A = n_B$ or $n_A - 1 = n_B$ any network that is fully intraconnected is socially preferred to another network, which is, similar to the former in links between different types, with the difference that is not fully intraconnected. Departing from fully intraconnected networks any pair of individuals of different type that are not connected, gain from doing so. Thus CI is the unique most socially preferable. In particular when $n_A - 1 = n_B$ notice that when a type B is connected to $n_A - 1$ types A and evaluates whether to connect to an extra A, the type B passes from majority to minority. For this link to be profitable C has to be below a certain threshold, which is, specifically the one that guarantees that CI is PS (see Proposition 2). That is addressed in point 1.

When $n_A - 1 > n_B$ and types B links to types A beyond $n_A - 1$ more than one individual of type A is involved in the pass from majority to minority. That opens the possibility that other networks are preferred to CI, since types B might be losing a lot from these extra connections. That is addressed in point 2.

The following example illustrates the relation between CI and the network in example 2 for $n_A - 1 > n_B$, point 2. Also the case in which $n_A - 1 = n_B$, point 1.

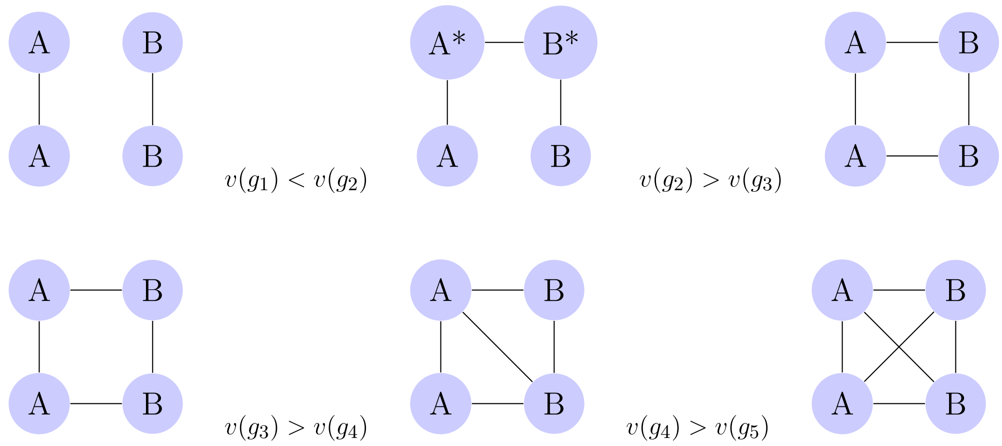
Example 4. Let $n_A = 4$, $n_{A^*} = n_B = 2$, $\delta = 0.9$ and $\beta = 0.1$. The value of CI is $n_A(n_A - 1)(\delta - c) + n_A n_B(\delta - C) + n_B(n_B - 1)(\beta - c) + n_B n_A(\beta - C) = 12(0.9 - c) + 8(0.9 - C) + 2(0.1 - c) + 8(0.1 - C)$. The value of SI in example 3 is $n_A(n_A - 1)(\delta - c) + n_B(n_B - 1)(\delta - c) + 2n_B n_{A^*}(\delta - C) + 2(n_A - n_{A^*})n_B\delta^2 = 12(0.9 - c) + 4(0.9 - C) + 4(0.9)^2 + 2(0.9 - c) + 4(0.9 - C) + 4(0.9)^2$. The difference between the values of SI and CI is $8(0.9)^2 + 2(0.9 - c) - 2(0.1 - c) - 8(0.1 - C) = 7.28 + 8C > 0$. For $C \in (0, 0.09)$, the CI network is PS but not the most socially preferred. Consider now, that $n_A = 3$ and $n_{A^*} = n_B = 2$, so that, $n_A - 1 = n_B$, $\delta = 0.9$. As CI is PS, $C \leq (n - 1)(\beta - \delta) + \delta - \delta^2$ holds (Proposition 2). Notice that in this case $\beta > (4/5)0.9 + 0.81/5 = 0.882$ for the upper bound on C to be strictly positive. Let $\beta = 0.89$. The value of CI is $6(0.9 - c) + 6(0.9 - C) + 2(0.89 - c) + 6(0.89 - C)$. The value of a fully intraconnected network in which, further, each B is connected to $n_A^* = 2$ is $6(0.9 - c) + 4(0.9 - C) + 2(0.9)^2 + 2(0.9 - c) + 4(0.9 - C) + 2(0.9)^2$. The difference between the value of the last network and the value of the CI is $4C - 0.28$. CI is not the most socially preferable when PS if $C > 0.07$. However for CI to be PS, $C \leq 0.05$. Both conditions are incompatible. Thus the alternative network is not socially preferred to CI. The reason is that CI is PS when types B do not suffer a great loss by being in minority, i.e., when β is sufficiently close to δ.

When CS is PS it might not be the most socially preferred, due to the positive externalities that links generate. However, CS is socially preferable to CI due to the high linking costs between different type individuals. The results are summarized below.

Proposition 5. *Let CS be PS, then it is socially preferred to CI. Further, if cost of linking with individuals of different types are not so high, CS is not the most socially preferred. In particular, a network that results from CS by adding a link between* *two individuals of different type yields higher value.*

For sufficiently low costs of interacting with individuals of different type, the positive externalities that this links generates on the unconnected individuals overcome the cost borne by the connected individuals.[7] The next example illustrates it:

Example 5. CS is PS but not the most socially preferable.



In comparing these networks, the common part due to same type links, $4(\delta - c)$, is omitted. Thus, $v(g_1) = 0$, $v(g_2) = 2(\delta - C) + 4\delta^2 + 2\delta^3$. For $C \in (\delta + \delta^2, \delta + 2\delta^2 + \delta^3)$ then $v(g_2) > v(g_1)$. Moreover, $v(g_3) = 4(\delta - C) + 4\delta^2$, $v(g_4) = 6(\delta - C) + 2\delta^2$, $v(g_5) = 8(\delta - C)$. Since $C > \delta$, then $v(g_3) > v(g_4) > v(g_5)$.[8] The network in which there is one link connecting the two communities, that is, the bridge network, is the most socially preferable. The increase in the number of individuals entails however a difficulty when evaluating which networks are the most socially preferable, due to the exponential increase in the possible combination of crossed links to be considered. Since the bridge network yields higher value that CS under the conditions above, there is an examination of the possible transfers among individuals that make it PS.

Thus, let a transfer rule be a function $t : g \rightarrow \mathbb{R}^n$ such that $\sum_i t_i(g) = 0$. It proposes a redistribution of payoffs within a network g. The analysis is on transfer rules that make the bridge network PS when $n_A = n_B \geq 2$. The reason is that first, group size higher than two makes the analysis more general as highlighted below. Second, making both groups equally represented makes the analysis more neat since symmetric. The intuition on what happens whenever groups are of different size is however straightforward. When the group size of individuals of type A is huge, individuals of type B would potentially be willing to pay more in order to preserve the link that connect both communities, since they benefit for a higher number of indirect connections. The result is as follows:

[7] Specifically, for $C \in (\delta + (n_B - 1)\delta^2, \overline{C})$ with $\overline{C} = \delta + (n_A - 1)\delta^2 + (n_B - 1)\delta^2 + (n_B - 1)(n_A - 1)\delta^3$.

[8] g_2 is still socially preferable to g_3 when g_3 consists on the same A having two links, one with each with B.

Proposition 6. *Let $n_A = n_B \geq 2$. Then there exist transfer rules that make the bridge network PS. In any of them A^* and B^* receive a subsidy for their link. Two of these rules are:*

1. *The egalitarian rule, that is, transfers t_i for each i such that $u_i + t_i = \frac{\sum_i u_i}{n}$. Under this rule both A^* and B^* receive the same subsidy, and the remaining individuals all pay the same. That is:*
   (a) $t_{A^*} = t_{B^*} = \frac{(4-n)(n_B-1)\delta^2 + (n-2)(C-\delta) + 2(n_B-1)^2\delta^3}{n} > 0.$
   (b) $t_A = t_B = \frac{(4(n_B-1)-n)\delta^2 + 2(\delta - C) - 2(n_B-1)\delta^3}{n} < 0.$
2. *Under certain conditions on C and c, the transfer rule that leaves A^* and B^* indifferent between forming or not their link. That is $t_{A^*} = t_{B^*} = C - \delta - (n_B - 1)\delta^2 > 0$. Among the several options to pay the overall subsidy of $2t_{A^*}$, the remaining individuals may all pay the same, that is, $t_i = \frac{2(\delta + (n_B-1)\delta^2 - C)}{n-2} < 0$ for every $i \neq A^*, B^*$ or each $i \neq A^*, B^*$ may pay $t_i \in [0, C - \delta - (n_B - 1)\delta^2]$.*

The egalitarian rule compensates A^* and B^* for the positive externalities they generate with their link and the cost they incur when linking, net of the utility of indirect connections of distance two they already derive from that link. That is, due to that crossed link, there are distance three connections that benefit the remaining individuals. In particular, $n_A - 1$ individuals benefit from distance three connections with others $n_B - 1$ individuals and vice versa. There are two out of n individuals responsible for that crossed link and hence compensated. Moreover, A^* and B^* incur in a cost $C - \delta$ each for that link. This amount is paid by the remaining $n - 2$ individuals. The total utility out of distances two is $4(n_B - 1)\delta^2$ and A^* and B^* already get from $(n_B - 1)\delta^2$ each, that should be discounted in the transfer. Notice that when $n_A = n_B = 2$, $t_{A^*} = t_{B^*} = 2^{-1}(C - \delta + \delta^3) > 0$ and $t_A = t_B = 2^{-1}(\delta - \delta^3 - C) < 0$. With respect to the second transfer rule. Consider that one individual chosen uniformly at random in her group subsidizes the individual of her own type for the crossed link. In this case individuals are ex-ante indifferent between the transfer rule in which each pays for sure the same, and a lottery in which each is chosen uniformly at random within her group, that is, with probability $(n_B - 1)^{-1}$, to subsidize her type, given that $n_B = n/2$. The reason as to why the transfer rule according to which types A or B pay exactly the positive externalities they benefit from does not make the bridge network PS is because, those individuals might instead break the link with either A^* or B^* and benefit form distance four with the remaining individuals of different types, specially when the group size of these individuals is high enough. Notice that this does not happen when $n = 4$ because a disconnected individuals gets zero utility.[9]

Regarding the relation between inequality and transfers, it is direct that the distribution of utilities after the egalitarian transfer rule Lorenz dominates any other distribution of utilities resulting according to any other transfer rule, and in particular the one described in point 2 in the above result.[10] Regarding the transfer system describe in point 2 of Proposition 6 the remark is as follows:

Remark. *Consider that transfers leave A^* and B^* indifferent between forming or not their link. The distribution of the utilities when the remaining individuals all pay the same to subsidize that link Lorenz dominates any other distribution of utilities in which not all of the individuals all pay the same.*

First, notice that A^* and B^* get $(n_A - 1)(\delta - c)$ each in any distribution. When all the remaining individuals all pay the same, $(n_A - 1)(\delta - c)$ is the lowest utility. Specifically, the distribution of utilities after transfers looks like $(n_A - 1)(\delta - c), (n_A - 1)(\delta - c), u..., u$ where:

$$u = (n_A - 1)(\delta - c) + \delta^2 + \delta^3(n_A - 1) - \frac{2}{n-2}(C - \delta - \delta^2(n_A - 1)) > (n_A - 1)(\delta - c).$$

The contrary implies that $C > \delta + (n_A - 1)\delta^2 + (n_B - 1)\delta^2 + 2(n_B - 1)(n_A - 1)\delta^2$, which contradicts the upper bound on C when the bridge network is socially preferable to CS (see proposition 5). When not all of the remaining individuals pay the same, two cases may arise. The first one is when the lowest utility, u', of individuals who pay more than in the distribution in which all pay the same, is still higher than $(n_A - 1)(\delta - c)$. The distribution of utilities after transfers is: $(n_A - 1)(\delta - c), (n_A - 1)(\delta - c), u', ..., u, ..., u''$ where $u'' > u > u' > (n_A - 1)(\delta - c)$. When not all individuals pay the same, any partial sum of utilities up to the individuals with utility up to u is strictly smaller in this distribution than the corresponding sum in the distribution in which all pay the same. The same hold for any partial sum including individuals such that $u'' > u$. As the subsidy is constant, the magnitude of the utility loss at the bottom of the distribution always overcomes the utility gain of the partial sum. Both partial sums are equal only when all individuals are considered. The second case is when for (some of) those individuals that pay more, its utility is below $(n_A - 1)(\delta - c)$.[11] The distribution is: $u', ..., (n_A - 1)(\delta - c), (n_A - 1)(\delta - c), u'', ..., u, ..., u'''$ where $u' < (n_A - 1)(\delta - c)$, $(n_A - 1)(\delta - c) < u'' < u$ and $u''' > u$. The argument is parallel than above.

[9] See Appendix 2.

[10] The criterion of Lorenz dominance establishes that, given two distributions $x = (x_1, ..., x_n)$ and $y = (y1, ..., y_n)$ with $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$, x Lorenz dominates y if for each $m = 1...n$, $\sum_{i=1}^m x_i \geq \sum_{i=1}^m y_i$ holds.

[11] That is the case when two individuals pay $C - \delta - \delta^2(n_B - 1)$ each. Then her utility falls below $(n_A - 1)(\delta - c)$ when $C > \delta + \delta^2(n_B - 1) + \delta^3(n_B - 1) + \delta^2$.

3.3 Network structure and segregation

As De Martí and Zenou [2017] point out, one interesting question is how the network structure emerging in equilibrium induces segregation. This paper uses the Spectral Segregation Index by Echenique and Fryer [2007] to measure segregation in the resulting equilibrium networks. This index measures segregation of a group based on the intensity of the interactions only among the members of that group. Hence the main ingredient is the intensity of interactions of every pair of connected group members. In order to recover this ingredient from the model, let d_i be the degree of individual i in an equilibrium network, including herself w.l.o.g. For the computation of the index the assumption is that the intensity of interactions is inversely related to the the degree. In particular, the intensity of interactions of an individual with each of her friends is $1/d_i$. Thus an individual with 5 friends pays $1/6$ of attention to each of them. Let $d_{A\to B}$ the degree distribution of types A when considering only their connections to types B. Let SSI^i, $i = \{A, B\}$ be the Spectral Segregation Index of group i.

Proposition 7. *Consider the class of networks in Proposition* 3*, point 1. Then:*

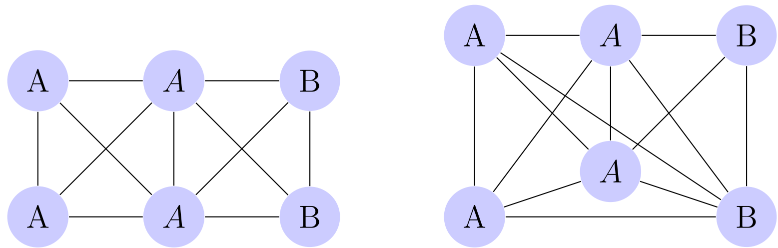
1. *When the same n_B types A are connected to types B, $SSI^A = \frac{n_B}{n} + \frac{n_A - n_B}{n_A}$ and that is the maximum value the index takes. Minimize SSI^A is equivalent to minimize the variance of $d_{A\to B}$.*
2. *$SSI^B = \frac{1}{2}$.*

Consider the class of networks in Proposition 3*, point 2. Then:*

1. *Minimize SSI^A is also equivalent to minimize the variance of $d_{A\to B}$.*
2. *Let $n_A \geq n_B^2$. Then SSI^A is higher for networks in point 1 than for networks of point 2.*
3. *Let $n_A < n_B^2$. Consider the class of networks in Proposition 3, point 2, where each of the $\tilde{n}_B < n_B$ types B connects to all A and the remaining $n_B - \tilde{n}_B$ types each connects to the n_B types A. When each of the $n_B - \tilde{n}_B$ types connects to the same types A in networks in point 2 than in networks in point 1, the index is lower in the former class of networks than in the latter.*
4. *SSI^B is higher for networks in point 1 than for networks in point 2.*

The next example illustrates points 3 and 4.

Example. Let $n_A < n_B^2$ and consider the following two networks:



For the network in the left (prop 3.1), $SSI^A = \frac{n_B}{n} + \frac{n_A - n_B}{n_A} = \frac{2}{6} + \frac{2}{4} = 0.83$ and $SSI^B = 0.5$. For the network on the right (prop 3.2). $SSI^A = 0.73$. Observe that there is one B in this network connected to the same types than its corresponding B in the network on the left. Further $SSI^B = 0.42$. In general, if the remaining types does not behave in the same way in both networks, that result does not hold. For networks in prop 3.1 $\underline{SSI}^A = \frac{n_B^2 - zn_A}{n_A + z + 1} + \frac{n_A - n_B^2 - zn_A}{n_A + z}$ where $z \geq 1$ is the integer such that $zn_A \leq n_B^2$ and $(z+1)n_A \geq n_B^2$ is the minimal value of the index. For networks in prop 3.2, $\overline{SSI}^A = \frac{n_B}{n_A + n_B} + \frac{n_A - n_B}{n_A - 1}$ is the maximal value of the index. Let $n_A = 21$ and $n_B = 19$. Thus $z = 17$. In this case $\underline{SSI}^A = 0.55$ and $\overline{SSI}^A = 0.57$. Let $n_A = 5$ and $n_B = 3$. Thus $z = 2$. In this case $\underline{SSI}^A = 0.73$ and $\overline{SSI}^A = 0.71$.

Finally, in the CI network $SSI^i = n_i/(n_i + n_j)$ where $i, j = \{A, B\}$ and $i \neq j$. In the CS network the index is 1 for A and B.

Notice that here an interesting point arises, which is to notice that there is no a one to one mapping between welfare, when measured as the sum of individual utilities, and segregation. The class of networks in Proposition 3, point 1, all have the same value, however different levels segregation. In a nutshell more segregation does not have to imply less welfare in our model. the driving mechanism behind this result is the trade off between the number of relations at the cost of being in minority. As Echenique et al. [2006] illustrate in their study of the effects of within school segregation in the U.S., blacks that are more segregated have lower test scores, but also are less likely to smoke, a widespread behavior among whites. More segregated Asians also have lower test scores but report to be happier at school. The authors conclude that while it is well documented that segregation across schools exacerbates differences in achievements, segregation within schools does not have an important effect on grades or social behavior.

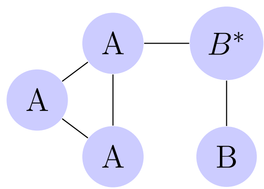
4 Discussion

Redefining neighbors. Up to now the assumption was that individuals only care about distance one connections when evaluating whether their own type is above a given threshold. A more general setting considers individuals consider as neighborhoods also those at higher distances. Let $p_{d(i,j)}$ be proportion own type in individual i neighborhood when the neighborhood is defined as connections up to distance $d(i,j)$. The utility of individual i net of costs is:

$$u_i(g) = \sum_{j \in g} (\mathbf{1}\delta + (1 - \mathbf{1})\beta)^{d(i,j)}$$

where $\mathbf{1}$ is the indicator of whether $p_{d(i,j)} \geq 0.5$. Notice that this utility implies that if when jumping from distance n to distance $n+1$ own types goes from majority to minority (or vice versa), only the utility form indirect connections at at distance between n and $n+1$ are affected. The next example illustrates it:

Example 6. In the following network $u_{B^*} = 2\delta - c - C + 2\delta^2$ using the model in the main body while $u_{B^*} = 2\delta - c - C + 2\beta^2$ using the extended version. The reason is that B types represents one half of the neighborhood when that is defined at distance one only. At distance two B types represent two fifths of the population.



Individuals now care about being in majority also at higher distances, hence different types at higher distances as considered as neighbors and thus the utility goes down. Intuitively, conditions for segregation are easier to meet in this case as the following remark points out.

Remark. *CS is PS iff* $c \leq \delta - \delta^2$ *and* $C \geq min\{\delta + \delta^2(n_B - 1), \delta + \beta^2(n_A - 1)\}$.

When $n_A = n_B$ any individual's type is represented with one half in the distance two case, when establishing a link. Hence that individuals evaluates its utility with δ in both cases. That is the first part within the min operator. However when $n_A > n_B$ and a type B establishes a link, she is in minority when her neighborhood is defined as those individuals up to distance two. That is why she evaluates type A links with β. For type A there is no change since she is in majority anyway. For, at least, the same range of C complete segregation is PS, i.e., individuals become more segregationist. This new specification does not affect the decision of breaking links with same type individuals.

Multiplicity of equilibria. Consider that $n_A - 1 > n_B$. One might think which network to expect as a function of the starting point. The notion of improving paths introduced by Jackson and Watts [2002] might help to make these predictions. That is, starting at a given network which networks we could expect to emerge in equilibrium through a sequence of networks with the following characteristics:

every network differs from the previous one just by adding or deleting a single link. The notion of improving paths captures this feature of the dynamic process. An improving path is sequence of networks that emerge when individuals form or sever links based on the improvement the resulting network offers relative to the current network. In order for a link to be deleted, the individual making the decision must gain with the decision and does not need the consent of the other individual involved. If a network arises from the previous one by adding a link, both individuals involved should benefit from it.[12] Consider as a starting point a fully intraconnected network in which, further, each B is connected to $\tilde{n}_A < n_B$ types A. In this case, Since $c < C \leq min\{\delta - \delta^2, \beta - \beta^2\}$ the first individual in the sequence will never delete a link with same types, and in fact, this is true at any point in time. Thus, in any adjacent network to the starting point any individual must be willing to form a link. In particular consider that B form a link with A. In this case type A gains since she is in majority always and B also gain given the bounds on C. One can consider a sequence of adjacent networks in which types B, one in a row complete all the links with types A, up to the point in which every type B is connected to exactly n_B types A. In this way, the network in proposition 3, point 1, arises, which is PS under some restrictions in c and C. Any PS network is not, by definition, in the improving path of any other network, thus the process stops.

Consider as a starting point a fully intraconnected networks in which, further, each B is connected to $\tilde{n}_A \in (n_B, n_A)$ types A. Since $C \leq min\{\delta - \delta^2, \beta - \beta^2\}$ the first individual in the sequence will never delete same type links. That is true at any point in time. In any adjacent network to the starting point any individual must be willing to form a link with different types. In particular considers that B forms a link with A. A type A gains since she is in majority always. A Type B also gains given the bounds on C. One can consider a sequence of adjacent networks in which types B, one in a row complete all the links with types A, up to the point in which every type B is connected to all types A. The CI network arises as a product of this process, which stops there. Notice than in this case the minority group B is willing to establish relations with members of the majority group. As Cheng and Yamamura [1957] points out group size is a factor affecting assimilation of minority groups. That is members of the minority group are willing to engage in marriage with members of the majority group in other to assimilate to the majority culture.

5 Proofs

Proof of Proposition 1. Let $n_A = n_B$ and focus on type A w.l.o.g.

CI. Notice that $(N_A^s + 1)/(N_A + 1) = n_A/n = 0.5$. If a type A is deciding to

[12]Notice that the notion of improving path inherits the properties of pairwise stability. It is also a myopic concept in the sense that individuals do not evaluate the consequences that their actions may have on others' choices.

break a link with another type A the fraction of own type in her neighborhood can be written as N_A^s/N_A. Notice that $N_A^s/N_A \geq (N_A^s+1)/(N_A+1)$ implies that $N_A^s \geq N_A$ which is a contradiction, since the number of same type friends is strictly smaller than the total number of friends. Hence $N_A^s/N_A < 0.5$. Thus links in CI worth δ while links if type A severs a link with another type A links worth $\beta < \delta$. A type A does not sever a link with another A if:

$$\underbrace{(n-1)\delta - n_B C - (n_A-1)c}_{u_A(g)} \geq \underbrace{(n-2)\beta + \beta^2 - n_B C - (n_A-2)c}_{u_A(g-AA)}$$

or

$$c \leq (\delta-\beta)(n-1) + \beta - \beta^2. \tag{1}$$

When type A evaluates breaking a link with type B, $(N_A^s+1)/N_A > 0.5$. Thus, links worth δ before and after breaking that link. Thus, a A does not sever a link with B if:

$$\underbrace{(n-1)\delta - n_B C - (n_A-1)c}_{u_A(g)} \geq \underbrace{(n-2)\delta + \delta^2 - (n_B-1)C - (n_A-1)c}_{u_A(g-AB)}$$

or

$$C \leq \delta - \delta^2. \tag{2}$$

Notice that RHS of (2) < RHS of (1). The contrary would imply that $\beta^2 - \delta^2 \geq (\delta-\beta)(n-1) + \beta - \delta$, which is a contradiction since the LHS of this equation is negative while its RHS is positive. As $c < C$ only condition (2) matters.

CS. Notice that $(N_A^s+1)/(N_A+1) = 1$. Regardless of whether a type A is deciding to break a link with another type A the fraction of own type in her neighborhood is still 1, so links always worth δ. A type A does not sever a link with another type A if:

$$\underbrace{(n_A-1)(\delta-c)}_{u_A(g)} \geq \underbrace{(n_A-2)(\delta-c) + \delta^2}_{u_A(g-AA)}$$

or

$$c \leq \delta - \delta^2. \tag{3}$$

When type A evaluates forming a link with type B, the new fraction of own types can be written as $(N_A^s+1)/(N_A+2)$, which is at least 0.5. To see so notice that $(N_A^s+1)/(N_A+2) < 0.5$ implies that $N_A^s < 0.5N_A$. That can only hold when $N_A^s = 0$ which in turn implies that $n_A = n_B = 1$ and hence $n = 2$, but by assumption $n > 2$. Thus, links always worth δ. Therefore, a type A does not form a link with a type B if:

$$\underbrace{(n_A-1)(\delta-c)}_{u_A(g)} > \underbrace{(n_A-1)(\delta-c) + \delta + \delta^2(n_B-1) - C}_{u_A(g+AB)}$$

or

$$C > \delta + \delta^2(n^B - 1). \tag{4}$$

Conditions (3) and (4) characterize CS. Finally set $n_B = n/2$.

■

Proof of Proposition 2. Let $n_A > n_B$.

CI. First, consider types A. If a type A breaks a link with another type A the new fraction of own type in her neighborhood can be written as N_A^s/N_A or equivalently $(n_A - 1)/(n - 1)$. Notice that $(n_A - 1)/(n - 1) < 0.5$ implies that $n_A - 1 < n_B$, which is a contradiction since $n_A > n_B$ implies that $n_A - 1 \geq n_B$. Thus, links before and after breaking this worth δ. A type A does not sever a link with another type A if:

$$\underbrace{(n-1)\delta - (n_A - 1)c - n_B C}_{u_A(g)} \geq \underbrace{(n-2)\delta + \delta^2 - (n_A - 2)c - n_B C}_{u_A(g - AA)}$$

or

$$c \leq \delta - \delta^2. \tag{5}$$

When type A evaluates breaking a link with type B, the new fraction of own types can be written as $(N_A^s + 1)/N_A > 0.5$. Links worth δ before and after breaking the link. Thus, A does not sever a link with B if:

$$\underbrace{(n-1)\delta - (n_A - 1)c - n_B C}_{u_A(g)} \geq \underbrace{(n-2)\delta + \delta^2 - (n_A - 1)c - (n_B - 1)C}_{u_A(g - AB)}$$

or

$$C \leq \delta - \delta^2. \tag{6}$$

Since $c < C$, condition (6) is the most restrictive.

Second, consider types B. If a type B breaks a link with another type B the fraction of own type in her neighborhood can be written as $N_B^s/N_B < 0.5$. So links before and after breaking the link worth β. Analogous calculations than those to get (5) lead to:

$$c \leq \beta - \beta^2. \tag{7}$$

If a type B breaks a link with a type A the fraction of own type in her neighborhood can be written as $(N_B^s + 1)/N_B$ or equivalently $n_B/(n - 1)$. Notice also that $n_B/(n - 1) \leq 0.5$. The contrary would imply that $n_B > n_A - 1$ which is a contradiction since $n_A > n_B$ implies that $n_A - 1 \geq n_B$. There are two cases: if $n_A - 1 > n_B$ then $(N_B^s + 1)/N_B < 0.5$. Analogous calculations than those to get (6) lead to:

$$C \leq \beta - \beta^2. \tag{8}$$

If $n_B = n_A - 1$ then $(N_B^s + 1)/N_B = 0.5$. Links before and after worth β and δ, respectively. Thus , a type B does not sever a link with a type A if:

$$\underbrace{(n-1)\beta - n_A C - (n_B - 1)c}_{u_B(g)} \geq \underbrace{(n-2)\delta + \delta^2 - (n_A - 1)C - (n_B - 1)c}_{u_B(g-BA)}$$

or

$$C \leq (n-1)(\beta - \delta) + \delta - \delta^2. \tag{9}$$

.

Under $n_A - 1 > n_B$ conditions are $(5)-(8)$. Since $c < C$, (6) and (8) are the most restrictive. Combining the two yields $C \leq min\{\delta - \delta^2, \beta - \beta^2\}$. Under $n_A - 1 = n_B$ conditions are $(5)-(7)$ and (9). Let $\beta > (n-2)\delta/(n-1) + (n-1)^{-1}\delta^2$ for the LHS of (9) to be strictly positive. Notice that $(n-1)(\beta - \delta) + \delta - \delta^2 < i - i^2$, $i = \delta, \beta$. That is direct for $i = \delta$. For $i = \beta$ the contrary implies that $\beta^2 - \delta^2 \geq (\delta - \beta)(n-1) + \beta - \delta$ which is a contradiction. Hence, condition (9) is the one that matters.

CS. First, consider type A. The analysis is analogous to the one in Proposition 1. A type A does not sever a link with another type A if condition (3) holds and does not form a link with a type B if condition (4) holds. Second, consider types B. The reasoning is the same but condition (4) modifies to:

$$C > \delta + \delta^2(n_A - 1). \tag{10}$$

Condition (3) has to be satisfied for same types being completely connected. Since $n_A > n_B$, RHS of (10) > RHS of (4). Since link formation is mutual consent, no link between A and B is formed if and only if (4) holds. ■

Proof of Proposition 3. Let CS be PS. Thus, $c \leq \delta - \delta^2$ and $C > \delta + \delta^2(n_B - 1)$. The claim is that a network with links between different types cannot be PS. The argument is that, in particular, types A always want to break those links. Notice that, first, whenever a type A is in majority, by severing a link with B, A remains so. The worst case scenario for A is when she loses a lot by breaking that link with B. That happens when: (i) A is breaking the only link she has to types B, (ii) no other types A have links to B and (iii) that B with whom A is breaking, has direct connections to all the remaining $n_B - 1$ types B, that are all directly connected among themselves. Thus A is losing a direct connection that does not become of higher order (i.e., it disappears). Also, A is entirely losing all indirect $n_B - 1$ connections of order 2. The change in utility is $-\delta + C - (n_B - 1)\delta^2$. Severing that link is profitable if $C > \delta + \delta^2(n_B - 1)$. Exactly as prescribed by the conditions for CS to be PS.[13] Second, let A go from minority to majority when severing the link. The worst case scenario for severing a link with B arises in the

[13]The case in which A is in minority and remains so after severing a link, is analogous just changing δ for β. Thus $C > \beta + \beta^2(n_B - 1)$ also holds.

same situation as above. In this case A is entirely losing the utility of one direct connection and $n_B - 1$ connections of order 2. She further gains, $\delta^n - \beta^n$ for unchanged indirect connections, denoted u, of order $n \geq 2$. In short, the change in utility is $-\beta + C - (n_B - 1)\beta^2 + \sum_{j=1}^{u}(\delta^{d(i,j)} - \beta^{d(i,j)})$. Severing a link is profitable whenever $C > \beta + (n_B - 1)\beta^2 - \sum_{j=1}^{u}(\delta^{d(i,j)} - \beta^{d(i,j)})$, which is implied by the conditions for CS to be PS. Thus no network with links between different types is PS. Finally, since $c \leq \delta - \delta^2 < \delta - \delta^n$ non-connected similar types, gain when linking. Thus only CS is PS.

Let CI be PS and $n_A = n_B$. Thus $c < C \leq \delta - \delta^2$. Let $n_A = n_B > 2$. The bipartite network is PS iff $C \leq \beta - \beta^3$ and $c > \beta - \beta^2$.[14] Let $\beta - \beta^3 < \delta - \delta^2$ and $c < C \in (\beta - \beta^2, \beta - \beta^3)$, both the bipartite network and CI are PS. Let CI be PS and $n_A - 1 > n_B$. Thus $c < C \leq min\{\delta - \delta^2, \beta - \beta^2\}$. Any PS network is fully intraconnected. When two individuals of same type form a link they remain in (i) minority and gain $-c + \beta - \beta^n$ each (ii) majority and gain $-c + \delta - \delta^n$ each or (iii) pass from minority to majority and gain $-c + \delta - \beta^n$ each. From each of the existing direct connections the gain is $\delta - \beta$. The same happens for the remaining indirect connections with the appropriate powers of δ and β. Further, this link can only reduce length of indirect connections. In every case links with similar types increase utility. Thus consider fully intraconnected networks. Focus on links between different types. A type A always accepts connections with types B since $-C + \delta - \delta^2 \geq 0$. A network in which each B is connected to $\tilde{n}_A \in (n_B, n_A)$ types A is not PS since $-C + \beta - \beta^2 \geq 0$. Thus, B also gains from an extra link with A. A network in which each B is connected to $\tilde{n}_A < n_B$ types A is not PS since a type B gets $-C + \delta - \delta^2 \geq 0$ from a extra link with A, when by doing so it remains in majority. For a network in which every B connects to exactly $n_{A^*} = n_B$ types A to be PS the conditions are: B does not link to type $A \neq A^*$ if $\underbrace{(n_B - 1)(\delta - c) + n_{A^*}(\delta - C) + (n_A - n_{A^*})\delta^2}_{u_B(g)} >$ $\underbrace{(n_B - 1)(\beta - c) + (n_{A^*} + 1)(\beta - C) + (n_A - n_{A^*} - 1)\beta^2}_{u_B(g+BA)}$ or

$$C > n_{A^*}(\beta - \delta) + (n_A - n_{A^*})(\beta^2 - \delta^2) + \beta - \beta^2 + (n_B - 1)(\beta - \delta). \tag{11}$$

A^* does not break with B and vice versa if

$$C \leq \delta - \delta^2. \tag{12}$$

Notice that the RHS of (11)$< min\{\delta - \delta^2, \beta - \beta^2\}$. Hence under

$$c < C \in (n_A^*(\beta - \delta) + (n_A - n_A^*)(\beta^2 - \delta^2) + \beta - \beta^2, min\{\delta - \delta^2, \beta - \beta^2\}), \tag{13}$$

[14] The bipartite network is one in which each individual of a given type is connected to all individuals of different type and there no other connections.

networks in this class are PS. A network in which some types B are connected to $\tilde{n}_A \in (n_B,n_A)$ types A and other types B are connected to $\tilde{n}_A < n_B$ types A is not PS. By the same reasoning as above, types B connected to $\tilde{n}_A \in (n_B,n_A)$ types A not connected A and B want to form links by the same reasoning as above. Types B connected to $\tilde{n}_A < n_B$ want to form new links with types A up to n_B for sure. Consider the network in which some B are connected all A and others B to exactly n_B types A. In this network a link between A and B is not severed since $C \leq min\{\delta - \delta^2, \beta - \beta^2\}$. Types B connected to n_B types A do not form a link with other A if (11) holds. Thus, (13) guarantees that networks in this class are PS.

Let CI be PS and $n_A - 1 = n_B$. Thus $C \leq (n-1)(\beta - \delta) + \delta - \delta^2$. Notice that $(n-1)(\beta - \delta) + \delta - \delta^2 \leq i - i^2$, $i = \{\delta, \beta\}$. That is direct when $i = \delta$. When $i = \beta$ the contrary would imply that $(n-1)(\beta - \delta) + \delta - \beta \geq \delta^2 - \beta^2$, which is a contradiction. Thus, by the same reasoning as above only the PS networks above can be PS also in this case. However, the networks in which (some) types B connect to n_B types A is not PS. Since $n_A^* = n_A - 1$, (11) becomes:

$$C > (n-1)(\beta - \delta) + \delta - \delta^2. \tag{14}$$

That contradicts the condition for CI to be PS. Thus B wants to form a link with $A \neq A^*$, who accepts. Thus, only CI is PS. ■

Proof of Proposition 4. Let $n_A = n_B$ or $n_A - 1 = n_B$. The proof goes as follows: first, it is shown that any network is less socially preferable than a counterpart network which have the same crossed links as the first and is also fully intraconnected. Second, it is shown that departing from the fully intraconnected network all individuals gain by completing links with different types, hence CI is the unique most socially preferable.[15] Let $n_A = n_B$ and focus on type A w.l.o.g. Let $\tilde{n}_A$ and $\tilde{n}_B$ the number of types A and B to whom a type A is linked, respectively. The ratio of own type in her neighborhood when she is connected to all types A is $n_A/(n_A + \tilde{n}_B) \geq 0.5$, hence links worth δ. The ratio of own type in her neighborhood in any other case is $(\tilde{n}_A + 1)/(\tilde{n}_A + \tilde{n}_B + 1)$. This ratio can be smaller, equal of higher than one half. First, let $(\tilde{n}_A + 1)/(\tilde{n}_A + \tilde{n}_B + 1) < 0.5$. Then links worth β. Let Δu_A be a lower bound for the change in utility when a type A goes from not being connected to all A to be connected to all of them. Thus $\Delta u_A = (\delta - \beta)\tilde{n}_A + (\delta - \beta)\tilde{n}_B + (n_A - 1 - \tilde{n}_A)(\delta - c) - (n_A - 1 - \tilde{n}_A)\beta^2 + (n_B - \tilde{n}_B)(\delta^n - \beta^n)$ where $(\delta - \beta)\tilde{n}_A$ and $(\delta - \beta)\tilde{n}_B$ are the changes in utility of existing connections, $(n_A - 1 - \tilde{n}_A)(\delta - c)$ is the utility of new connections with same type individuals, $(n_A - 1 - \tilde{n}_A)\beta^2$ is the highest lost related to indirect connections with types A that have become direct, since those individuals could have been at distance more than

[15] In contrast to Jackson and Wolinsky [1996], it is not true that individuals always gain from forming links, so their proof does not hold here.

two. Finally, $(n_B - \tilde{n}_B)(\delta^n - \beta^n)$ is the change in utility of indirect connections to types B at distance $n \geq 2$. That is the smallest gain because through connections to A, indirect distances could also be potentially reduced. Since CI is PS, $c \leq \delta - \delta^2 < \delta - \beta^2$. Hence $\Delta u_A > 0$. Second, let $(\tilde{n}_A+1)/(\tilde{n}_A+\tilde{n}_B+1) \geq 0.5$. Then in both cases links worth δ and $\Delta u_A = (n_A - 1 - \tilde{n}_A)(\delta - c) - (n_A - 1 - \tilde{n}_A)\delta^n \geq 0$ with $n \geq 2$, since $c \leq \delta - \delta^2$. There is also some positive value due to indirect distances with individuals of different types potentially reduced. Thus, fully intraconnected networks are socially preferable to their counterparts. Once a network is fully intraconnected two individuals of different types always gain when forming a link with a different type since $C \leq \delta - \delta^2$. Thus, CI is the unique most socially preferable. Let $n_A - 1 = n_B$. The proof for types A is the same as above. For types B, the ratio of own type in her neighborhood when she is connected to everyone of own type is $n_B/(n_B + \tilde{n}_A)$ which can be higher, equal of smaller than one half. Let $n_B/(n_B + \tilde{n}_A) \geq 0.5$, the proof is the same than above, thus fully intraconnected networks are socially preferable to their counterparts. With respect to crossed links, since $C \leq \delta - \delta^2$ types A gain when linking to B. Since $n_B/(n_B + n_A) < 0.5$ for any B, the value of links pass from δ to β when linking to the last A. Hence for B to gain with this link $(n_B - 1)(\beta - \delta) + (n_A - 1)(\beta - \delta) + \beta - C - \delta^2 \geq 0$ has to hold. That is equivalent to $C \leq (n-1)(\beta - \delta) + \delta - \delta^2$, which precisely guarantees that CI is PS. Hence the CI network is the uniquely most socially preferable. Let $n_B/(n_B + \tilde{n}_A) < 0.5$. Notice that $\tilde{n}_B + 1 \leq n_B$. Which implies that $(\tilde{n}_B + 1)/(\tilde{n}_A + \tilde{n}_B + 1) < 0.5$. Hence links before and after, completing all the connections with other types B, worth β. Thus $\Delta u_B = (n_B - 1 - \tilde{n}_B)(\beta - c) - (n_B - 1 - \tilde{n}_B)\beta^n$, with ≥ 2. There is also some positive value due to indirect distances with individuals of different types potentially reduced. Recall that $c \leq C \leq (n-1)(\beta - \delta) + \delta - \delta^2$ for the CI network to be PS. It is direct that $(n-1)(\beta - \delta) + \delta - \delta^2 < \beta - \beta^2$. Thus, $\Delta u_B > 0$. With respect to crossed links individuals of both types gain by the same arguments as above. Hence CI is the unique most socially preferable. Let $n_A - 1 > n_B$. Thus, $C \leq min\{\delta - \delta^2, \beta - \beta^2\}$ for CI to be PS. Thus, any pair of non-connected individuals of same type gain by linking. Focus then on fully intraconnected networks. The value of CI is:

$$n_A(n_A - 1)(\delta - c) + n_A n_B(\delta - C) + n_B(n_B - 1)(\beta - c) + n_B n_A(\beta - C). \quad (15)$$

Consider networks in Proposition 3, point 1. They are fully intraconnected and further each type B is connected to $n_{A^*} = n_B$ types A. First notice that regardless of which types A each B is connected to, all these networks yield the same value. To see that notice that the value of same type links is the same in all these networks. Regarding crossed links, for types B the value is also the same since each B is connected to the same number of types A. For types A notice that any network of this class can be reached by modifying one link at a time on the network in which all B are connected to the same types A. In each one link deviation, only utilities of the two types A involved change by the amount $\delta - \delta^2$ and opposite sign. Thus, w.l.o.g, let each B links to the same types A. The value of this network is:

$$n_A(n_A-1)(\delta-c)+2n_{A^*}n_B(\delta-C)+2(n_A-n_{A^*})n_B\delta^2+n_B(n_B-1)(\delta-c). \quad (16)$$

Using $n_{A^*} = n_B$ and after some algebra, (16) - (15) can be written as:

$$n_B[(n_B-n_A)(\delta-C)+2(n_A-n_B)\delta^2+(n_B-1)(\delta-\beta)+n_B(\delta-C)-n_A(\beta-C)] \quad (17)$$

or equivalently

$$n_B[-n_A(\delta-2\delta^2+\beta-2C)+n_B(2\delta-2\delta^2-2C)+(n_B-1)(\delta-\beta)] \quad (18)$$

Recall that $C \leq min\{\delta-\delta^2, \beta-\beta^2\}$ for CI to be PS. Then $C \geq 2^{-1}(\delta+\beta-2\delta^2)$ suffices for (18) ≥ 0 and thus CI is not the most socially preferable. Recall that the focus is on fully intraconnected networks: since $C \leq min\{\delta-\delta^2, \beta-\beta^2\}$ types A always gain when linking to types B. Each B also gains when linking up to n_B types A. When B connects to more than n_B types A, the claim is that any type B loses more than the gain of types A. The utility of B with n_B links to types A is:

$$(n_B-1)(\delta-c)+n_B(\delta-C)+(n_A-n_B)\delta^2. \quad (19)$$

The utility of B with $e \in [1, n_A-n_B]$ extra links to types A is:

$$(n_B-1)(\beta-c)+(n_B+e)(\beta-C)+(n_A-n_B-e)\beta^2. \quad (20)$$

Thus, (20) – (19) ≥ 0 for:

$$C \leq \frac{(2n_B-1)(\beta-\delta)+(n_A-n_B)(\beta^2-\delta^2)}{e}+\beta-\beta^2. \quad (21)$$

Since $\beta-\delta<0$, the RHS of (21) is the highest whenever $n_B=1$ and $e=n_A-n_B$. In that case C is bounded above by $\beta-\delta^2 < 2^{-1}(\delta+\beta)-\delta^2$ as $n_A \to \infty$. Then $C \geq 2^{-1}(\delta+\beta)-\delta^2$. and (21) are incompatible, meaning that any type B is worse off when connecting to extra types A. The gain of types A with e extra links is:

$$e(\delta-\delta^2-C). \quad (22)$$

The absolute value of the loss of type B is:

$$eC+(2n_B-1)(\delta-\beta)+e(\beta^2-\beta)+(n_A-n_B)(\delta^2-\beta^2). \quad (23)$$

The claim is that (23) − (22) > 0. That is, any type B loses more than the gain of types A. (23) − (22) is:

$$2eC + 2(n_B - 1)(\delta - \beta) + e(\beta^2 + \delta^2 - \beta - \delta) + (n_A - n_B)(\delta^2 - \beta^2). \tag{24}$$

Let $2^{-1}(\delta + \beta) - \delta^2 < 0$. Setting $C = 0$ (as an exteme case) and $e = n_A - n_B$ reduces the value of (24), which becomes:

$$(n_B - 1)(\delta - \beta) + n_B(\delta - \beta) + (n_A - n_B)(2\delta^2 - \beta - \delta). \tag{25}$$

Since $(\delta + \beta) < 2\delta^2$ thus $(25) > 0$.

Let $2^{-1}(\delta + \beta) - \delta^2 > 0$. Rewrite $e = n_A - n_B - x$ with $x \in [0, n_A - n_B - 1]$. Thus (24) becomes:

$$n_B(3\delta - \beta - 2\delta^2 - 2C) + \beta - \delta + n_A(2C + 2\delta^2 - \beta - \delta) + x(\delta + \beta - \beta^2 - \delta^2 - 2C). \tag{26}$$

Since $C \geq 2^{-1}(\delta + \beta) - \delta^2 > 0$ the term accompanying n_A is non-negative. Since, $C \leq min\{\beta - \beta^2, \delta - \delta^2\}$ direct algebra shows that the terms accompanying x and n_B are both positive. Thus (26) is the smallest whenever $x = 0$, $n_B = 1$ and $n_A = 3$.[16] Thus (26) reduces to:

$$4C + 4\delta^2 - 3\beta - \delta, \tag{27}$$

which is positive since $C \geq 2^{-1}(\delta + \beta) - \delta^2 > 4^{-1}(\delta + 3\beta) - \delta^2$. Thus, the networks in proposition 3, point 1, are the most socially preferable. These networks are PS for $C \in (\underline{C}, min\{\delta - \delta^2, \beta - \beta^2\})$, where $\underline{C} = n_B(\beta - \delta) + (n_A - n_B)(\beta^2 - \delta^2) + \beta - \beta^2 + (n_B - 1)(\beta - \delta)$ (see expression (13) in the proof of Proposition 3). It turns out that $\underline{C} < 2^{-1}(\delta + \beta) - \delta^2$. The contrary implies that $(4n_B - 1)(\beta - \delta) + 2(n_A - n_B - 1)(\beta^2 - \delta^2) > 0$, which is a contradiction. As $C > 2^{-1}(\delta + \beta) - \delta^2$. These networks are PS together with CI.

Consider now that $C < 2^{-1}(\delta + \beta) - \delta^2$ and focus on CI, hence $x = 0$. Hence the RHS of (21) at $x = 0$ is $\overline{C} = \beta - \delta^2 + \dfrac{(2n_B - 1)(\beta - \delta)}{n_A - n_B}$. Evaluating (26) at $\overline{C}$ and $x = 0$ yields (after some algebra): $(n - 1)(\beta - \delta) < 0$. Thus every type B gains when linking to all A, who also gain as argued above. Thus, for $C = \overline{C}$, CI is socially preferred to the networks in proposition 3, point 1. That is also the case when $C < \overline{C}$ since (26) is decreasing in C when $x = 0$. To compare CI with any other network that form by adding extra links, notice that in this case $x > 0$. Hence for $C \leq \overline{C}$ the gain that any type B enjoys by connecting to e extra types A is smaller when $e \neq n_A - n_B$ ($x > 0$) than when $e = n_A - n_B$ $(x = 0)$.[17] Since (26) increases with x, CI is the uniquely most socially preferable. Finally, let

[16] Recall that the framework is such that $n_A - 1 > n_B$.

[17] That gain may even be a loss.

$C \in (\overline{C}, 2^{-1}(\delta+\beta)-\delta^2)$. There are values of C, n_A and n_B, δ and β such that CI and SI are equally preferred, CI is socially preferred to SI or vice versa. Let CI be socially preferred to SI. That means that (26) < 0 at $x = 0$. For any $x > 0$ (26) is less negative. Thus CI is the most socially preferred. Let SI be socially preferred to CI, thus (26) > 0 at $x = 0$. For any $x > 0$, (26) is more positive. Thus SI is the most socially preferred. Let SI be and CI and equally preferred, thus,(26) $= 0$ at $x = 0$. For any $x > 0$, (26) is more positive. Thus SI and CI are the most socially preferred. Let CI and SI be equally preferred. That means that (26) $= 0$. For any $x > 0$, (26) become positive. Thus CI and SI are the most socially preferable. Regarding pairwise stability notice that $\overline{C} > \underline{C}$, hence SI are PS

Regarding the comparison between CI and CS, the value of CS is:

$$n_A(n_A-1)(\delta-c)+n_B(n_B-1)(\delta-c). \tag{28}$$

The value of CI when $n_A = n_B$ is:

$$n_A(n_A-1)(\delta-c)+n_B(n_B-1)(\delta-c)+2n_An_B(\delta-C). \tag{29}$$

and when $n_A > n_B$ is:

$$n_A(n_A-1)(\delta-c)+n_B(n_B-1)(\beta-c)+n_An_B(\delta-C)+n_An_B(\beta-C). \tag{30}$$

Since $C < \beta < \delta$, (29) $>$ (28). In comparing (28) and (30), suppose that CS yields higher value than CI. That is, let $(n_B-1)(\delta-c) \geq (n_B-1)(\beta-c)+n_A(\delta-C)+n_A(\beta-C)$. That implies that:

$$n_A(\delta+\beta-2C) \leq (n_B-1)(\delta-\beta) \tag{31}$$

When $n_A - 1 > n_B$ then $C \leq (n-1)(\beta-\delta)+\delta-\delta^2 < i-i^2$, $i = \delta, \beta$, for CI to be PS (see the proof of Proposition 2). Hence the LHS (30)>0 and is the smallest whenver $n_A - 2 = n_B$. That implies that $n_B \leq 0$, which is a contradiction. Thus CS cannot yield higher value than CI. When $n_A - 1 = n_B$ the LHS (30)>0 as well, and the same contradiction is reached.

■

Proof of Proposition 5. Let CS be PS. Thus, $C > \delta+\delta^2(n_B-1)$ and $c \leq \delta-\delta^2$. Denote by g_s the network under segregation and g_{s+AB} the network which differs form the segregated one by adding a link between A and B. Let $\mu = n_A(n_A-1)(\delta-c)+n_B(n_B-1)(\delta-c)$. Then, $v(g_s) = \mu$ and $V(g_{s+AB}) = \mu+2\delta-2C+(n_B-1)\delta^2+(n_A-1)\delta^2+(n_A-1)(\delta^2+(n_B-1)\delta^3)+(n_B-1)(\delta^2+(n_A-1)\delta^3)$. That $v(g_{s+AB}) < v(g_s)$ implies that $C > \delta+(n_B-1)\delta^2+(n_A-1)\delta^2+(n_B-1)(n_A-1)\delta^3$. Hence for $C \in (\delta+(n_B-1)\delta^2, \delta+(n_B-1)\delta^2+(n_A-1)\delta^2+(n_B-1)(n_A-1)\delta^3)$, this network with and extra link is socially preferable to CS.

Denote by g_I the network under complete integration. Then $V(g_I) = n_A(n_A - 1)(\delta - c) + n_B(n_B - 1)(\delta - c) + 2n_An_B(\delta - C)$ when $n_A = n_B$ and $V(g_I) = n_A(n_A - 1)(\delta - c) + n_B(n_B - 1)(\beta - c) + n_An_B(\delta - C) + n_An_B(\beta - C)$ when $n_A > n_B$. Since $C > \delta$, $v(g_I) < v(g_s)$. Hence CS is socially preferable to CI.

■

Proof of Proposition 6. Let $n_A = n_B > 2$. Consider the bridge network with A^* and B^* being the individuals that bridge the two communities. Denote by A and B the remaining individuals of either type. Notice that $u_{A^*} = u_{B^*} = (n_A - 1)(\delta - c) + \delta - C + (n_B - 1)\delta^2$ and $u_A = u_B = (n_A - 1)(\delta - c) + \delta^2 + (n_B - 1)\delta^3$.[18] The system of transfers that sustains the bridge network as an equilibrium has to satisfy the following conditions:

(a) A link between A and B does not form whenever $u_A + t_A \geq u_{A+AB}$ and/or $u_B + t_B \geq u_{B+BA}$. For A the expression becomes $(n_A-1)(\delta-c)+\delta^2+(n_B-1)\delta^3+t_A \geq (n_A-1)(\delta-c)+\delta-C+(n_B-1)\delta^2$. Thus $t_A \geq \delta-C+(n_B-1)(\delta^2-\delta^3)-\delta^2$. Analogously, $t_B \geq \delta - C + (n_A - 1)(\delta^2 - \delta^3) - \delta^2$.

(b) A link between A^* and B does not form whenever $u_{A^*} + t_{A^*} \geq u_{A^*+A^*B}$ and/or $u_B + t_B \geq u_{B+BA^*}$. That leads to $t_{A^*} \geq \delta - \delta^2 - C$ and/or $t_B \geq \delta - C + (n_A - 1)(\delta^2 - \delta^3) - \delta^2$.

(c) Analogously, a link between B^* and A does not form whenever $t_{B^*} \geq \delta - \delta^2 - C$ and/or $t_A \geq \delta - C + (n_B - 1)(\delta^2 - \delta^3) - \delta^2$.

(d) A^* and A do not sever their link whenever $u_{A^*} + t_{A^*} \geq u_{A-A^*A}$ and $u_A + t_A \geq u_{A-AA^*}$. Thus, $t_{A^*} \geq c - \delta + \delta^2$ and $t_A \geq c - \delta - (n_B - 1)(\delta^3 - \delta^4) + \delta^3$.

(e) Analogously, B^* and B do not sever their link whenever $t_{B^*} \geq c - \delta + \delta^2$ and $t_B \geq c - \delta - (n_A - 1)(\delta^3 - \delta^4) + \delta^3$.

(f) A^* and B^* does not sever their link whenever $u_{A^*} + t_{A^*} \geq u_{A^*-A^*B^*}$ and $u_{B^*} + t_{B^*} \geq u_{B^*-B^*A^*}$. Thus, $t_{A^*} \geq C - \delta - (n_B - 1)\delta^2$ and $t_{B^*} \geq C - \delta - (n_A - 1)\delta^2$

(g) A and A (resp. B and B) do not sever their link if $t_A \geq c - \delta + \delta^2$ (resp. $t_B \geq c - \delta + \delta^2$).

Recall that CS is PS but the bridge network is socially preferable to it. Hence $c \leq \delta - \delta^2$ and $C \in (\delta+(n_B-1)\delta^2, \delta+(n_B-1)\delta^2+(n_B-1)\delta^2+(n_B-1)(n_B-1)\delta^3)$. By (f), $t_{A^*}, t_{B^*} > 0$. Also, (f) implies the conditions for t_{A^*} and t_{B^*} in (b) and (c) and in (d) and (e).

With respect to t_A and t_B conditions (d), (e) and (g) have to be satisfied. Regarding (a), at least one. The comparison thus concerns $c-\delta-(n_A-1)(\delta^3-\delta^4)+\delta^3$,

[18] For the sake of simplicity the argument in the utility is omitted.

$c-\delta+\delta^2$, and $\delta-C+(n_B-1)(\delta^2-\delta^3)-\delta^2$, the lower bounds for t_A and t_B in (d), (e), (g) and (a). Given the conditions on c and C for CS to be PS, these bounds are negative. Notice that $c-\delta+\delta^2 > c-\delta-(n_A-1)(\delta^3-\delta^4)+\delta^3$. The contrary implies that $\delta^2 < \delta^3-(n_A-1)(\delta^3-\delta^4)$, which is a contradiction since $\delta^2 > \delta^3$. Then (g) implies (d) and (e).

Regarding (g) and (a), the comparison is between $c-\delta+\delta^2$ and $\delta-C+(n_B-1)(\delta^2-\delta^3)-\delta^2$. Let $c-\delta+\delta^2 \geq \delta-C+(n_A-1)(\delta^2-\delta^3)-\delta^2$. That is, let (g) imply (a). Hence transfers are of the from $t_{A^*}, t_{B^*} \geq C-\delta-(n_B-1)\delta^2 > 0$ and $t_A, t_B \geq c-\delta+\delta^2 < 0$. The inequality above implies that:

$$C+c \geq 2(\delta-\delta^2)+(n_B-1)(\delta^2-\delta^3). \tag{32}$$

Notice that whenever $\delta-\delta^2-c \geq C-\delta-(n_B-1)\delta^2$ every individual could pay $2(n-2)^{-1}(C-\delta-(n_B-1)\delta^2)$. That would be the transfer system such that A^* and B^* each is compensated for the loss due to their link, and everyone else pays the same amount.The previous inequality implies that:

$$C+c \leq 2\delta+(n_B-2)\delta^2. \tag{33}$$

First notice, that the RHS (31) < RHS (32). The contrary implies that $-\delta^2 > \delta^3$, which is a contradiction. Thus, both conditions are compatible. Further they have to be also compatible with the fact that $C \in (\delta+(n_B-1)\delta^2, \delta+(n_B-1)\delta^2+(n_B-1)\delta^2+(n_B-1)(n_B-1)\delta^3)$ by PS of CS.

Notice that (32) establishes an upper bound on C, that is $C \leq 2\delta+(n_B-2)\delta^2-c$. This upper bound has to be higher than $\delta+(n_B-1)\delta^2$, the lower bound on C for CS to be PS. Since $2\delta+(n_B-2)\delta^2-c > \delta+(n_B-1)\delta^2$ for $c < \delta-\delta^2$. There are C compatible with these requirements. Analogously notice that (31) establishes a lower bound on C, that is, $C \geq 2(\delta-\delta^2)+(n_B-1)(\delta^2-\delta^3)-c$. This lower bound has to be smaller than $\delta+(n_B-1)\delta^2+(n_B-1)\delta^2+(n_B-1)(n_B-1)\delta^3$, the upper bound on C for CS to be PS. Thus, $2(\delta-\delta^2)+(n_B-1)(\delta^2-\delta^3)-c < \delta+2(n_B-1)\delta^2+(n_B-1)^2\delta^3$ or $c > \delta-2\delta^2-\delta^3-\delta^2(n_B-1)-\delta^3(n_B-1)^2$. For $\delta \in (0.41, 1)$ the RHS of this inequality is negative, so it holds for any c. Summing up the transfer system in which A^* and B^* are exactly compensating for establishing the link, and the remaining agents all pay the same, makes the bridge networks PS. Notice that any other payment of the (n-2) individuals such that each pays $t_i \in [0, C-\delta-\delta^2(n_B-1)]$ also works.

The egalitarian transfer system makes any network PS, since societal and individual incentives are aligned, see Jackson [2008] pp 174-175. The computations are omitted and come from solving the equation $u_i(g)+t_i^e(g) = n^{-1}\sum_{i=1}^{n} u_i(g)$ for every i, where g is the bridge network. Thus, every individual has to get the same utility, which is the value of the network, divided by the number of individuals in society. That is achieved through transfers.

■

Proof of Proposition 7. SSI^A is the largest eigenvalue of the matrix $\mathbf{A}$ which describes interactions only among types A, see Echenique and Fryer [2007]. Let $a_{ij} \in [0,1]$ be a typical entry of $\mathbf{A}$ describing the intensity of the relation between i and j. By theorem 8.1.22 in Horn and Johnson [1990]:

$$\min_{1 \leq j \leq n} \sum_i a_{ij} \leq SSI^A \leq \max_{1 \leq j \leq n} \sum_i a_{ij}.$$

Since $a_{ij} = 1/d_i$, all columns of $\mathbf{A}$ sum up to the same value, $\sum_{i=1}^{n_A}(n_A + n_B^i)^{-1}$, where n_B^i is the number of types B a type A is connected to and n_A enters due to full intraconnection.[19] Thus SSI^A equals that sum. Due to full intraconnection n_A is fixed. Thus the only aspect that alters the index is how links to types B distribute. The SSI^A reacts to this distribution as follows: consider A' and A with A' having more links to B than A. Denote the number of links to B by $n_{\tilde{B}}$ and $n_{\hat{B}}$, for A' and A, respectively. Thus, $n_{\tilde{B}} > n_{\hat{B}}$. By changing one link from A' to A the index always decreases except when A has just one link less than A', in which case remains constant. Due that one link change only entries of A' and A change in any column of $\mathbf{A}$. For A', her entry increases from $1/(n_A + n_{\tilde{B}})$ to $1/(n_A + n_{\tilde{B}} - 1)$, that is, by $1/(n_A + n_{\tilde{B}})(n_A + n_{\tilde{B}} - 1)$. For A, her entry decreases from $1/(n_A + n_{\hat{B}})$ to $1/(n_A + n_{\hat{B}} + 1)$, that is, by $1/(n_A + n_{\hat{B}})(n_A + n_{\hat{B}} + 1)$. If $n_{\tilde{B}} - 1 \geq n_{\hat{B}} + 1$, then the decrement overcomes the increment. Thus, overall the index decreases. Let $n_{\tilde{B}} - 1 < n_{\hat{B}} + 1$ and notice that this only happens whenever $n_{\tilde{B}} - n_{\hat{B}} = 1$. The decrement and the increment are equal and so is the index. Thus whenever a redistribution in the above sense is possible, it reduces the index. As a consequence the case in which types A accumulate the highest number of links to B each, makes the index maximal. The minimum number of types A accumulating n_B links each is n_B. Hence, $SSI^A = \sum_{i=1}^{n_B}(n_A + n_B)^{-1} + \sum_{i=n_B+1}^{n_A}(n_A)^{-1} = n_B/n + (n_A - n_B)/n_A$.

The result above implies that to minimize the index links have to be as evenly distributed as possible across types A. Otherwise there is always a one link transfers that reduces the index. The way of achieving that slight varies depending on the relation between the number of links to be distributed, which is n_B^2, and the number of the n_A types A receiving these links. Let $n_A = n_B^2$, then the index is minimal when each A has exactly one link. The variance of $d_{A \to B}$ is zero and thus minimal. Let $n_A > n_B^2$, then the index is minimal when n_B^2 types A have one link each and $n_A - n_B^2$ have no links. Notice that any other distribution can be achieved starting from the one above, by transferring links, one in a row from A to A' when A has at most the same number of links than A'. At each step A has one extra link and A'

[19]The formula does not imply that all possible n_B^i are compatible at the same time. Due to the considered class of networks types B connects to n_B types A each. What is true is that, regardless of which types A are involved, the sum across columns is the same.

has one link less. Any transfer in this fashion increases the index. It also increases the variance of $d_{A\to B}$. To see that, let $m \equiv n_B^2/n_A$ denote the mean $d_{A\to B}$, which is constant. Let the transfer be from an individual with $\hat{n}_B$ links to an individual with $\tilde{n}_B \geq \hat{n}_B$ links. The variance of $d_{A\to B}$ changes by $\alpha = (\tilde{n}_B+1-m)^2-(\tilde{n}_B-m)^2$ and $\beta = (\hat{n}_B-1-m)^2-(\hat{n}_B-m)^2$. Specifically $\alpha = 1+2\tilde{n}_B-2m$ and $\beta = 1-2\hat{n}_B+2m$ and $\alpha+\beta = 2+2(\tilde{n}_B-\hat{n}_B) \geq 0$. Thus, in any other distribution the variance is higher. Let $n_A < n_B^2$. Notice that there always exists an integer $z \geq 1$ such that $zn_A \leq n_B^2$ and $(z+1)n_A \geq n_B^2$. Let the n_A types have z links each and $n_B^2 - zn_A \leq n_A$ types A have in addition one extra link each. The total number of links is $(n_B^2 - zn_A)(z+1) + (n_A - (n_B^2 - zn_A))z = n_B^2$. Any other redistribution starting form this one arises as above. Thus the index and the variance are positively related and they are minimal in the proposed distribution.

Consider the class of networks in Proposition 3, point 2. Links reduce the intensity of interactions (by reducing a_{ij} in the columns of $\mathbf{A}$). Thus the maximal value of the index for types A obtains when just one type B is connected to all A and the remaining $n_B - 1$ types has each n_B connections to types A. Only notice that each A has $n_A + 1$ links to start with, due to the type B connected to all A. Again the only thing that matters for the index is how links to B distribute and the result above applies. Thus whenever n_B types A have $n_B - 1$ links each, the index is maximal. Thus, the maximal value of the index is $\frac{n_B}{n_A+1+n_B-1} + \frac{n_A-n_B}{n_A+1} = \frac{n_B}{n_A+n_B} + \frac{n_A-n_B}{n_A+1}$. Focus now on the minimum value of the index for networks in proposition 3, point 1. Let $n_B^2 = n_A$. The minimum value of the index achieves when each A has just one link to types B. In this case the minimum value of the index is $\frac{n_A}{n_A+1} = \frac{n_B}{n_A+1} + \frac{n_A-n_B}{n_A+1}$. This minimum value of the index is higher that the maximum value above. Let $n_B^2 < n_A$. For networks in proposition 3, point 1, the minimum value of the index achieves when n_B^2 types A have one link to types B each and $n_A - n_B^2$ types A have no links. In this case the index is $\frac{n_B^2}{n_A+1} + \frac{n_A-n_B^2}{n_A} > \frac{n_B^2}{n_A+1} + \frac{n_A-n_B^2}{n_A+1} \geq \frac{n_B}{n_A+n_B} + \frac{n_A-n_B}{n_A+1}$. Thus, when $n_B^2 \leq n_A$ the minimum value of the index for networks in proposition 3, point 1 is higher than the maximum value of the index for networks in proposition 3, point 2.

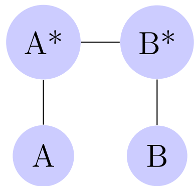
Consider now networks in proposition 3, point2. Let $\tilde{n}_B < n_B$ types B connects to all types A and the remaining $n_B - \tilde{n}_B$ connects to n_B types A each. Consider networks in proposition1, point 1, such that $n_B - \tilde{n}_B$ types B connects in the same way than above to n_B types A and the remaining $\tilde{n}_B$ types B connects each to n_B types A in either way. Regardless of the distribution of links of the latter $\tilde{n}_B$ types, their counterparts in networks in proposition 3, point 2, have more links, in fact they are connected to all A. Thus the value of the index have to be lower in this case.

Finally, regarding types B, for networks in proposition 3, point 1 $SSI^B = n_B/2n_B = 0.5$ since each B is connected to all others of same type and also to n_B types A. For networks in proposition 3, point 2, the maximum value of the index is $\dfrac{n_B-1}{2n_B}+\dfrac{1}{n_A+n_B}<0.5$ when just one B connects to all A and the remaining B connects to n_B types A.

■

6 Appendix 1. Transfers when $n_A = n_B = 2$.

Consider network g_2 in example 2, where $u_{A*} = u_{B*} = \delta - c + \delta - C + \delta^2$ and $u_A = u_B = \delta - c + \delta^2 + \delta^3$.



(1) A and B do not form a link if $t_A \geq \delta - \delta^3 - C$ and/or $t_B \geq \delta - \delta^3 - C$.
(2) A^* and B do not form a link if $t_{A^*} \geq \delta + \delta^2 - C$ and/or $t_B \geq \delta - \delta^3 - C$.
(3) A and B^* do not form a link if $t_{B^*} \geq \delta + \delta^2 - C$ and/or $t_A \geq \delta - \delta^3 - C$.
(4) A and A^* do not delete their link if $t_A \geq c - \delta - \delta^2 - \delta^3$ and $t_{A^*} \geq c - \delta$.
(5) B and B^* do not delete their link if $t_B \geq c - \delta - \delta^2 - \delta^3$ and $t_{B^*} \geq c - \delta$.
(6) A^* and B^* do not delete their link if $t_{A^*} \geq C - \delta - \delta^2$ and $t_{B^*} \geq C - \delta - \delta^2$.

Since CS is PS, $C > \delta + \delta^2$. By (6), $t_{A^*} > 0$ and $t_B > 0$. Notice that (6) implies, on the one hand, (2) and (3) and, on the other hand, (4) and (5), for t_{A^*} and t_{B^*}, respectively. From (4) and (5), $t_A \geq c - \delta - \delta^2 - \delta^3$ and $t_B \geq c - \delta - \delta^2 - \delta^3$ have to hold. For t_A and t_B in (1) and (2), notice that both $c - \delta - \delta^2 - \delta^3 \geq \delta - \delta^3 - C$ or $c - \delta - \delta^2 - \delta^3 < \delta - \delta^3 - C$ may hold. Let $c - \delta - \delta^2 - \delta^3 \geq \delta - \delta^3 - C$ hold. That implies that $C + c \geq 2\delta + \delta^2$. Thus, (4) and (5) imply (1) and (2) for A and B. Transfers are such that: $t_{A^*} \geq C - \delta - \delta^2$, $t_{B^*} \geq C - \delta - \delta^2$, $t_A \geq c - \delta - \delta^2 - \delta^3$ and $t_B \geq c - \delta - \delta^2 - \delta^3$. Now notice that $\delta + \delta^2 + \delta^3 - c \geq C - \delta - \delta^2$. The contrary would imply that $C + c > 2\delta + 2\delta^2 + \delta^3$ which is a contradiction since $C < \delta + 2\delta^2 + \delta^3$ and $c \leq \delta - \delta^2$. Hence there are transfer rules that make the bridge network PS. The egalitarian transfer rule makes any network PS. Another rule is the one under which A^* and B^* receive $C - \delta - \delta^2$. That is, they are exactly compensated for the loss they experience when linking, with $t_i = C - \delta - \delta^2$, $i = A, B$. Finally, let $t_A = t_B = c - \delta - \delta^2 - \delta^3$. Since $u_A = u_B = \delta - c + \delta^2 + \delta^3$ this transfer leaves types A and B with zero utility, all of it being transferred to A^* and B^*.

7 Appendix 2. Stability and efficiency whenever δ_i and β_i differ across individuals.

This appendix reexamines equilibrium networks when:

$$u_i(g) = \begin{cases} \sum_{j\in g} \delta_i^{d(i,j)} - \sum_{j\in N_i(g)} c_{ij} & if \quad p \geq 0.5 \\ \sum_{j\in g} \beta_i^{d(i,j)} - \sum_{j\in N_i(g)} c_{ij} & if \quad p < 0.5. \end{cases}$$

Proposition 8. *Let $n_A = n_B$. Then:*

1. *CI is PS iff*
$$C \leq \min_i \ \delta_i - \delta_i^2.$$

2. *CS is PS iff*
$$c \leq \min_i \ \delta_i - \delta_i^2$$
and for no pair of individuals such that $t(i) \neq t(j)$, it holds that
$$C < \delta_i + \delta_i^2(n_B - 1) \quad and \quad C < \delta_j + \delta_j^2(n_A - 1).$$

Proof of Proposition 8. CI. Conditions (1)-(2) in the proof of proposition 1 are individual specific. For CI to be PS, the restrictions below have to hold:

$$c \leq \min_i \ (\delta_i - \beta_i)(n-1) + \beta_i - \beta_i^2. \tag{34}$$

$$C \leq \min_i \ \delta_i - \delta_i^2. \tag{35}$$

In this case RHS (34) < RHS (33). To see that, consider different pairs (β_i, δ_i), (β_j, δ_j),...,(β_k, δ_k). Consider $i \equiv arg\min_m \ \delta_m - \delta_m^2$ and $j \equiv arg\min_m \ (n-1)(\beta_m - \delta_m) + \delta_m - \delta_m^2$, $m = 1, 2..., n$. Let $j = i$. By the proof of Proposition 1, RHS (34) < RHS (33). Let $j \neq i$. In this case the RHS (34) is the smallest for the pair i. In particular the RHS (34) is smaller for i than for j, otherwise it would not be the minimum. Moreover, the RHS of (33) at $j >$ RHS (34) at $j >$ RHS (34) at i. Thus the RHS (34) has to be smaller than the RHS of (33).

CS. Conditions (3)-(4) in the proof of Proposition 1 are individual specific. Thus, no individual breaks a link with a similar type if:

$$c \leq \min_i \ \delta_i - \delta_i^2. \tag{36}$$

Further there cannot be two individuals of different type, such that $C < \delta_i + \delta_i^2(n_B - 1)$ and $C < \delta_j + \delta_j^2(n_A - 1)$, $i \neq j$, since in this case they link.

■

Proposition 9. *Let $n_A - 1 > n_B$. Then:*

1. *CI is PS iff*

$$C \leq min\{\min_{i:t(i)=A} \delta_i - \delta_i^2, \min_{i:t(i)=B} \beta_i - \beta_i^2\}.$$

2. *CS is PS under the same conditions than in Proposition 8.*

Proof of Proposition 9. CI. Conditions (5)-(8) in the proof of Proposition 2 are individual specific. For CS to be PS, the restrictions below have to hold:

$$c \leq \min_{i:t(i)=A} \delta_i - \delta_i^2. \quad (37)$$

$$C \leq \min_{i:t(i)=A} \delta_i - \delta_i^2. \quad (38)$$

$$c \leq \min_{i:t(i)=B} \beta_i - \beta_i^2. \quad (39)$$

$$C \leq \min_{i:t(i)=B} \beta_i - \beta_i^2. \quad (40)$$

Since c<C, (37) and (39) guarantee that CS is PS. Thus:

$$C \leq min\{\min_{i:t(i)=A} \delta_i - \delta_i^2, \min_{i:t(i)=B} \beta_i - \beta_i^2\} \quad (41)$$

CS. The proof is analogous than the one of Proposition 8.

■

Proposition 10. *Let $n_A - 1 = n_B$. Then:*

1. *CI is PS iff*

$$C \leq min\{\min_{i:t(i)=A} \delta_i - \delta_i^2, \min_{i:t(i)=B} (n-1)(\beta_i - \delta_i) + \delta_i - \delta_i^2\}.$$

2. *CS is PS under the same conditions than in Proposition 8.*

Proof of Proposition 10. CI. Conditions (5)-(7) and (9) in the proof of Proposition 2 are individual specific. The restrictions below have to hold:

$$c \leq \min_{i:t(i)=A} \delta_i - \delta_i^2. \quad (42)$$

$$C \leq \min_{i:t(i)=A} \delta_i - \delta_i^2. \quad (43)$$

$$c \leq \min_{i:t(i)=B} \beta_i - \beta_i^2. \quad (44)$$

$$C \leq \min_{i:t(i)=B} (n-1)(\beta_i - \delta_i) + \delta_i - \delta_i^2. \quad (45)$$

For types A (42) has to hold. Since c<C (42) implies (41). For types B, (43) and (44) have to hold. It turns out that RHS (44) $<$ RHS (43). Consider the pairs (β_i, δ_i), (β_j, δ_j),...,(β_k, δ_k). Let $i \equiv arg\min_m \ \beta_m - \beta_m^2$ and $j \equiv arg\min_m \ (n-1)(\beta_m - \delta_m) + \delta_m - \delta_m^2$, $m = 1, 2..., n_B$. Let $j = i$. By the proof of Proposition 2, RHS (44) $<$ RHS (43). Let $j \neq i$. The RHS (44) is the smallest for the pair j. In particular the RHS (44) is smaller for j than for i, otherwise j would not minimize that expression. Moreover, the RHS of (43) at $i >$ RHS (44) at $i >$ RHS (44) at j. Thus RHS (44) $<$ RHS (43) has to hold. Thus, compare (42) and (44). When among types B, $\beta_i = 0.5 \ \forall \ i$, $\delta_i = \{0.51, 0.52\}$ and $n = 11$, RHS (44) is 0.15 and 0.049 respectively. When among types A, $\delta_i = \{0.51, 0.52\}$ RHS (42) $>$ RHS (44), thus (44) is the most restrictive. Further, when among types A, $\delta_i \leq 0.05$, the RHS (42) ≤ 0.048, and hence the most restrictive.

CS. See the proof of Proposition 8.

■

Proposition 11. *Let CI be PS. Whenever $n_A = n_B$ or $n_A - 1 = n_B$ CI is socially preferable to CS. Whenever $n_A - 1 > n_B$ either CI is socially preferable to CS or vice versa. Let CS be PS. Then either CI is socially preferable to CS or vice versa.*

explain that

Proof of Proposition 11. First, let CI be PS. The value of CS and CI are the analogous to the ones in expressions, (27) and (28) respectively. When $n_A = n_B$ the value of CI further incorporates (apart form the value of same type links which is the same in CI and CS) the value of links with different types, $n_B \sum_{i=1}^{n_A} (\delta_i - C) + n_A \sum_{i=1}^{n_B} (\delta_i - C)$. By Proposition 8, for CI to be PS it has to be that:

$$C \leq \min_i \ \delta_i - \delta_i^2. \tag{46}$$

The claim is that (45) implies that $C < \delta_i$ for each i. Thus CI is socially preferable to CS. To see that, first consider that the RHS (45) minimizes at $\delta_k > 0.5$. Hence $C \leq \delta_k - \delta_k^2$. Notice that $\delta_i \in [1-\delta_k, \delta_k]$ for each individual i, otherwise δ_k would not minimize (45). In particular $\delta_i = 1-\delta_k$ is the minimum value that δ_i might take. Let $C > 1-\delta_k$. That implies that $C > 1-\delta_k+(1-\delta_k)^2 = \delta_k-\delta_k^2$, which is a contradiction. Second, consider that $\delta_k < 0.5$. In this case that $\delta_i \in [\delta_k, 1-\delta_k]$ for each individual i, otherwise δ_k would not minimize (45). It holds that $C \leq \delta_k - \delta_k^2 < \delta_k$. Thus C is smaller than any other $\delta_i \in [\delta_k, 1-\delta_k]$.[20] When $n_A > n_B$, focus on the analogs of (27) and (29). The value of CS is:

[20] The case in which $\delta_k = 0.5$ requires that any other $\delta_i = 0.5$ otehwise 0.5 would not minimize (45), hence it is the case in which $\delta_i = \delta$ for all i as in the main body.

$$(n_A - 1)\sum_{i=1}^{n_A}(\delta_i - c) + (n_B - 1)\sum_{i=1}^{n_B}(\delta_i - c). \tag{47}$$

The value of CI is:

$$(n_A - 1)\sum_{i=1}^{n_A}(\delta_i - c) + (n_B - 1)\sum_{i=1}^{n_B}(\beta_i - c) + n_B\sum_{i=1}^{n_A}(\delta_i - C) + n_A\sum_{i=1}^{n_B}(\beta_i - c). \tag{48}$$

Let (46) > (47), that is let:

$$(n_B - 1)\sum_{i=1}^{n_B}(\delta_i - c) \geq (n_B - 1)\sum_{i=1}^{n_B}(\beta_i - c) + n_B\sum_{i=1}^{n_A}(\delta_i - C) + n_A\sum_{i=1}^{n_B}(\beta_i - c). \tag{49}$$

or

$$(n_B - 1)\sum_{i=1}^{n_B}(\delta_i - \beta_i) \geq n_B\sum_{i=1}^{n_A}(\delta_i - C) + n_A\sum_{i=1}^{n_B}(\beta_i - c). \tag{50}$$

The claim is that (49) cannot hold. There are two cases. First, Let $n_A - 1 = n_B$, then CS is PS iff $C \leq min\{\min_{i:t(i)=A}\ \delta_i - \delta_i^2, \min_{i:t(i)=B}\ (n-1)(\beta_i - \delta_i) + \delta_i - \delta_i^2\}$. (see proposition 10). The minimum of these two values has to be positive, otherwise CI is not PS. Thus, for every pair (β_i, δ_i) for types B, $\beta_i \geq \frac{(n-2)\delta_i}{n-1} + \frac{\delta_i^2}{n-1}$ and in particular for the pair (β_i, δ_i) that minimizes $(n-1)(\beta_i - \delta_i) + \delta_i - \delta_i^2$. Let $\beta_i = \frac{(n-2)\delta_i}{n-1} + \frac{\delta_i^2}{n-1}$ for every individual i of type B. In this case the LHS (49) is as high as possible and the RHS (49) is as small as possible, everything else fixed. The claim is that even in that case:

$$(n_B - 1)\sum_{i=1}^{n_B}(\delta_i - \beta_i) \geq n_A\sum_{i=1}^{n_B}(\beta_i - c) \tag{51}$$

cannot hold, and hence (49) cannot hold. In particular it has to hold that:

$$\sum_{i=1}^{n_B}(\delta_i - \beta_i) < \sum_{i=1}^{n_B}(\beta_i - c). \tag{52}$$

Suppose in contrast that it holds that:

$$\sum_{i=1}^{n_B}(\delta_i - 2\beta_i + c) \geq 0. \tag{53}$$

Notice that for every i, $\delta_i - 2\beta_i = \delta_i - 2\left[\frac{(n-2)\delta_i}{n-1} + \frac{\delta_i^2}{n-1}\right] = \frac{(3-n)\delta_i}{n-1} - \frac{2\delta_i^2}{n-1} < 0$ and hence, $c > (n-1)^{-1}((n-3)\delta_i + 2\delta_i^2) > 0$, for some summands in (52) to be non-negative. Notice that $(n-1)^{-1}((n-3)\delta_i + 2\delta_i^2)$ increases with n. Thus it takes the minimum value at $n = 3$. In this case $c > \delta_i^2$. Recall that $c < C \leq (n-1)(\beta_i - \delta_i) + \delta_i - \delta_i^2$ has to hold for every pair, since in particular costs are smaller than $\min_{i:t(i)=B}\ (n-1)(\beta_i - \delta_i) + \delta_i - \delta_i^2\}$. Recall also that $\beta_i = (n-1)^{-1}((n-2)\delta_i + \delta_i^2$. Both inequalities on c are compatible whenever $\delta_i^2 < (n-1)(\beta_i - \delta_i) + \delta_i - \delta_i^2$ for the considered β_i. However that implies that $2\delta_i^2 < 2\delta_i^2 - \delta_i$, which is a contradiction. Hence every summand in (52) is negative. As a consequence, was $n_B - 1 = n_A$ (50) could not hold, so it cannot hold either for $n_B - 1 < n_A$. That implies that the $RHS(49) > LHS(49)$, which contradicts the assumption that CS is socially preferable to CI. Second, let $n_A - 1 > n_B$, then CI is PS iff $C \leq min\{\min_{i:t(i)=A}\ \delta_i - \delta_i^2, \min_{i:t(i)=B}\ \beta_i - \beta_i^2\}$, (see proposition 9). Let $n_A = 11$ with 3 individuals defined by $(\delta_1, \beta_1) = (0.002, 0.0001)$, 3 individuals defined by $(\delta_2, \beta_2) = (0.09, 0.0002)$ and 5 individuals defined by $(\delta_3, \beta_3) = (0.18, 0.0003)$. Let $n_B = 9$ with 3 individuals defined by $(\delta_1, \beta_1) = (0.3, 0.01)$, 3 individuals defined by $(\delta_2, \beta_2) = (0.4, 0.02)$ and 3 individuals defined by $(\delta_3, \beta_3) = (0.6, 0.2)$. In this case, for types B, $\beta_i - \beta_i^2$ takes the minimum value of 0.0099 at $\beta_1 = 0.01$ and for types A, $\delta_i - \delta_i^2$ takes the minimum value of 0.001996 at $\delta_1 = 0.002$. Hence set $c = 0.0017 < C = 0.0018 < 0.001996$. In this case the value of CI, according to (47) is 34.78. The value of CS according to (46) is 42.63. Hence CS is socially preferable to CI. Now, for types B, change parameters to $(\delta_1, \beta_1) = (0.3, 0.29)$, 3 individuals defined by $(\delta_2, \beta_2) = (0.4, 0.39)$ and 3 individuals defined by $(\delta_3, \beta_3) = (0.6, 0.4)$. In this case, for types B, $\beta_i - \beta_i^2$ takes the minimum value of 0.2059 at $\beta_1 = 0.29$. The minimum of types A is the same. Thus, let $c = 0.0017 < C = 0.0018 < 0.001996$. In this case the value of CI, according to (47) is 83.23. The value of CS according to (46) is 42.63. Hence CI is socially preferable to CS.

Second, let CS be PS. Thus conditions in Proposition 8 are satisfied. For $n_A = n_B$ the values of CS and CI differ by $n_B \sum_{i=1}^{n_A}(\delta_i - C) + n_A \sum_{i=1}^{n_B}(\delta_i - C)$. Let $\delta_i = 0.8$ and $\delta_i = 0.5$ for A and B, resp.. Let $n_A = n_B = 11$. For $C \in (0.5 + 10(0.5^2), 0.8 + 10(0.8^2)) = (3, 7.2)$ and $c \leq 0.8 - 0.8^2$ CS is PS. Since $C > \delta_i\ \forall\ i$, CI is socially preferable to CS. Let $\delta_i = 0.8$ and $\delta_i = 0.1$ for A and B, resp. Let $n_A = n_B = 2$. For $C \in (0.1 + 0.1^2, 0.8 + 0.8^2) = (0.11, 1.44)$ and $c \leq 0.1 - 0.1^2$ CS is PS. Let $C > 1 > \delta_i$, $\forall\ i$, then CI is socially preferable to CS. Let $C = 0.2$, then $n_B \sum_{i=1}^{n_A}(\delta_i - C) = 2(0.8 - 0.2 + 0.8 - 0.2) = 2.4$ and $n_A \sum_{i=1}^{n_B}(\delta_i - C) = 2(0.11 - 0.2 + 0.11 - 0.2) = -0.36$. Thus CI is socially preferable to CS.

Let $n_A > n_B$. The focus is on expression (49). Let individuals of type A be defined by $\delta_i = 0.8$ and $\beta_i \in (0, 1)$ (possibly) different across individuals. Let types

B be defined by $(\delta_i, \beta_i) = (0.1, 0.09)$ Let $n_B = 2$ and $n_A = 3$. In this case for $C \in (0.11, 2.08)$ and $c = 0.05 < 0.1 - 0.1^2 = 0.09$ CS is PS. For $C = 0.2$ (49) does not hold. Specifically its LHS takes value $2(0.1 - 0.09) = 0.02$ while its RHS takes value $6(0.8 - 0.2) + 6(0.09 - 0.05) = 3.84$. When $n_B = 2$ and $n_A > 3$ the same holds. Thus CI is socially preferable to CS. On the contrary, whenever, everything else equal, $C = 1.2$ the RHS (49) is negative. Thus CS is socially preferable to CI.

■

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