# Market Efficiency and Volatility Persistence of Cryptocurrency during Pre- and Post-Crash Periods of Bitcoin: Evidence based on Fractional Integration

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## Abstract

This paper investigates both market efficiency and volatility persistence in 12 cryptocurrencies during pre-crash and post-crash periods. We were motivated by the erroneous belief of some authors that driving currency, Bitcoin is inefficient. By considering robust fractional integration methods in linear and nonlinear set up, we found that markets of Bitcoin and most altcoins considered in our samples can be dubbed as efficient, and these are highly volatile particularly in the post-crash sample that we are now. These volatilities will then persist for shorter period than in the pre-crash period. Our work therefore renders important information to cryptocurrency market participants and portfolio managers.

**Keyword:** Bitcoin; Cryptocurrency; Market efficiency; Fractional integration; Virtual currency
**JEL Classification:** C22

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## 1. Introduction

As a result of 2007/2018 crash in global financial market, individuals and traders have lost interests in investing in economic policy driven investments such as stocks, foreign exchange rates, oil and gold. Pricing of these assets are driven by global markets, of which are influenced by US economic and political activities, thus these are traditional market system. As a result, global market is moving to a non-traditional monetary system, independent of government policy and politics. Cryptocurrency, the digital currency has become another investment source accepted in the global market (see Weber, 2016). This market is popularized with the introduction of the first digital coin, the Bitcoin in 2009 priced as low as $5 per coin, and since then, well over 2000s cryptocurrency types are being traded 24/7 on the internet.[1] For the first time, having attained steady and astronomical increase in prices till late 2017 to around $19000 per coin, Bitcoin crashed and cause prices of other alternative coins to fall (see Cointelegraph, 2018). Other alternative coins (altcoins) include Dash, Doge, Ethereum, Litecoin, Monero, Ripple, Stellar, Vertcoin, etc, introduced due to rising popularity of Bitcoin. Litecoin for example was introduced to conserve the computing power for mining coin, Dash was introduced to render faster processing and improved privacy protection (Ciaian and Rajcaniova, 2018). All other cryptocurrency types have reasons of being in existence. Yet, none of the other cryptocurrency types has overtaken Bitcoin's price. While the process to accept or reject cryptocurrency by economists and regulators is ongoing, press and various financial blogs keep circulating the awareness. Cryptocurrency is known differently for its huge volatility, whereas there are very few papers considering this. The predictability of the price that is efficiency of the market could also be of interests to portfolio managers and traders.

[1] Bitcoin was introduced by Satoshi Nakamoto in 2009 in a white paper published online. Cryptocurrency generally is a new solution to some internet enthusiasts since the introduction of internet: as a form of digital cash that is "peer-to-peer" and open sourced-based (Beer and Weber, 2015). The offer by Nakamoto allows a platform to create more private currency and "buy and sell" takes place without recourse to central banks and commercial banks.

Meanwhile, various academic papers from economic-finance angles have concentrated on influence of other assets such as gold and stocks on cryptocurrency (see Barber et al., 2012; Glasser et al., 2014; Dyhrberg, 2016; Corbet et al., 2018; Bouri et al., 2018).

The level of market efficiency is useful in the evaluation of the investment environment, in the description of the financial market and in the knowing how to develop the market further. The efficiency/inefficiency of cryptocurrency market will render useful information to market players. Efficiency market posits that nothing but own past information predicts the future dynamics of market price, that is other influence such as domestic and macroeconomic policy do not influence price in this context. The standard definition of efficient market in Fama (1965) says: “In an efficient market, at any point in time, the actual price of a security will be a good estimate of its intrinsic value”. Fama (1970) then developed the Efficiency Market Hypothesis (EMH) which states that prices of assets already contain past information and in the event of new information, there is a quick adjustment to the price such that the security is valued correctly. That is are returns of cryptocurrency predictable? Returns are expected to be be unpredictable for EMH to hold, while in an inefficient market, returns are predictable (see Lim and Brooks, 2011). In an inefficient market, returns are predictable and it is possible for investors to make abnormal returns. The random walk hypothesis implies market efficiency since market returns are unpredictable in such a time process. Thus, as investors try to beat the market more, the market becomes more efficient. A form of market efficiency defined in Fama (1970) is the weak form efficiency where current asset prices are expected to reflect all information in the market transactional data and no technical data analysis could help in realizing abnormal returns from such dataset.

Efficiency and volatility are inseparable since efficiency is a function of market returns while volatility is a function of variation from such returns and persistence is the time it takes to fizzle out. These variations are proxied as absolute or squared returns. Investigating efficiency and volatility persistence in cryptocurrency markets will therefore interest readers. In an efficient market, short period of time is expected for the effect of price shocks/volatility to fizzle out.

In the present paper, we investigate market efficiency and volatility persistence in some highly priced and capitalized cryptocurrencies based on daily data from 7 August 2015 to 28 November 2018. We considered samples up to late 2017 crash and samples after the crash in order to remove influence of structural breaks in the market returns. We used fractional integration techniques on the returns to test for the hypothesis of market efficiency while we squared returns are used as proxy for volatility where long memory evidence in squared returns imply the extent of volatility persistence. Our approach of fractional integration estimation is robust as it allows for both nonlinearity and possible smooth breaks in the returns and squared returns of cryptocurrency series, noting that Perron (2006) unit root-break test different break dates when one is not too sure of specification of constant and intercept in the testing regression.

Several prominent papers have examined the market inefficiency of the cryptocurrency, with specific focus on the most valuable of all, bitcoin. Some studies (Urquhart, 2016; Bariviera, 2017; Nadarajah and Chu, 2017; among others) found informational efficiency in the bitcoin return series, which was dependent on the sample period considered. The informational efficiency was observed in periods covering recent years. However, the various studies confirmed that the returns on cryptocurrencies are often times characterized by some salient features, which include unit roots, autocorrelation, non-linearity and long range dependence (see also Balcilar et al., 2017; Bouri et al., 2018 and Yaya, et al., 2018; among others). As consistent with returns on asset prices, returns on cryptocurrencies also exhibit large volatility and strong persistence (see Cheah et al., 2018; Yaya, et al., 2018; among others). In contrast to the stance of the proponents of the characteristics of market efficiency of cryptocurrencies, some other studies, (Cheah et al., 2018; among others) find non-homogeneous informational inefficiency, as well as cointegrating relationships among the markets investigated. On the adoption of cryptocurrencies as alternative hedging options (see Dyhrberg, 2016; Bouri et al., 2017a), others (see Bouri et al., 2017b) differ from this stance, thus suggesting cryptocurrencies as just diversifier devices rather than hedge instruments.

Methodologically, market efficiency and/or inefficiency of cryptocurrencies have been recently studied using a battery of market efficiency test such as Runs test, Ljung-Box, Automatic Variance Ratio test, Bartels test, BDS and R/S Hurst (see Urquhart, 2016; Nadarajah and Chu, 2017; among others); Hurst exponent (Bariviera, 2017), fractionally cointegrated VAR framework - FCVAR (Cheah et al., 2018); Multifractal Detrended Cross-correlation Analysis – MF-DCCA (Zhang et al., 2018); Detrended Fluctuation Analysis - DFA (Bariviera, 2017; Tiwari et al., 2018); among others.

Our approach of market efficiency and volatility persistence is different from that of previous authors on efficiency and volatility persistence in Bitcoin and alternative cryptocurrency since it is series-based. Those literature considered Bitcoin or other cryptocurrency in relationship with other asset prices, trying to find possible cointegration relationships, while others use different approaches. The EMH has rightly defined market efficiency as random walk evidence that is the unpredictability of returns. Then, how does the volatility persist?

The rest of the paper is structured as follows: Section 2 presents the time series analysis approach used in this paper. As mentioned in the introduction, this is the fractional integration methodology in the linear and nonlinear case for returns and squared returns of cryptocurrency series. In Section 3, we present the data, some pre-tests and empirical results. Section 4 renders the concluding remarks.

## 2. Time Series analysis approach

Fractional integration framework is used throughout to investigate both market efficiency and volatility persistence in this paper. Recall that the unit integration by Dickey and Fuller (1979) (the Augmented Dickey-Fuller) test is based on three regression models of no constant, constant only and linear trend with constant. In the same spirit, Robinson (1994) set up a testing framework for the three testing regression models. The approach uses the Lagrange Multiplier (LM) test, with the model,

$$y_t = \alpha + \beta t + x_t; \qquad t = 1, 2, ...., \tag{1}$$

with,

$$(1-L)^d x_t = u_t; \quad t = 0, \pm 1, \pm 2, .... \tag{2}$$

where the linear model in (1) applies to the three standard cases of (i) no constant (i.e. $\alpha = \beta = 0$); an intercept ($\alpha$ unknown and $\beta = 0$ a priori); and a linear time trend (i.e. $\alpha$ and $\beta$ unknown). The regressor $x_t$ is the time series under investigation, to be fractionally differenced with exponent *d*. Robinson (1994) tests the null hypothesis,

$$H_0 : d = d_0 \tag{3}$$

where $d_0$ is any real value. The time series $y_t$ is the resulting covariance stationary process obtained after the integration process has been carried out on the series $x_t$. Since this estimation approach is parametric, it is therefore imperative to specify a particular functional form for the $I(0)$ error term, $u_t$. Though, there are different functional forms, say Autoregressive [AR(1)] or seasonal ARMA (Autoregressive Moving Average) as the dataset may permit. Our datasets are daily frequency, thus seasonal AR is not expected. We therefore consider only the white noise (uncorrelated error) case. Equations (1) and (2) can be combined as one equation as

$$y_t^* = \alpha_0 1_t^* + \beta_0 t_t^* + u_t, \quad t = 1, 2, ... \tag{4}$$

where $y_t^* = (1-L)^{d_0} y_t$, $1_t^* = (1-L)^{d_0} 1_t$; $t_t^* = (1-L)^{d_0} t_t$. Since $u_t$ is I(0), then it is straightforward to estimate $\alpha_0$ and $\beta_0$ in (4) by ordinary least squares (OLS) methods. The LM statistics for Robinson (1994) test has the functional form given by:

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a}, \tag{5}$$

where $T$ is the sample size, and

$$\hat{a} = \frac{-2\pi}{T} \sum_j^* \psi(\lambda_j) g_u(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g_u(\lambda_j; \hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{T} \left( \sum_j^* \psi(\lambda_j) \psi(\lambda_j)' - \sum_j^* \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \left( \sum_j^* \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \sum_j^* \hat{\varepsilon}(\lambda_j) \psi(\lambda_j)' \right);$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g_u(\lambda_j; \hat{\tau}),$$

where $I(\lambda_j)$ is the periodogram of $\hat{u}_t$, and $\hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau)$, with $T^*$ as a suitable subset of the $R^q$ Euclidean space, and $\lambda_j = 2\pi j/T$, with the summations in * above equations are bounded for all frequency in the spectrum.

To test for the hypothesis of market efficiency, we expect to find randomness in the return of cryptocurrency. Thus, fractional integrated parameters $d$ are expected to be insignificantly different from 0. Significance of this parameter then implies autocorrelations of lagged observations to the current time series, hence the returns seem to be predictable, this implying market inefficiency. The squared log-returns act as proxy to volatility persistence whenever series-based approach to volatility persistence is considered.[2] The transformed series

[2] Model-based volatility persistence uses the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model of Bollerslev (1986).

is expected to contain long memory, a case where observations distant in time lag correlate with current time observation.

For robustness, we investigate possible structural break in the data sample by means of a Fourier smooth function which induces nonlinearity. This function allows for nonlinear smooth break in the time series of interest, particularly when the form of the break, and the break date are unknown. Fourier function is first considered for unit root case in Becker, Enders and Lee (2006) and Enders and Lee (2012a,b). Also, we were motivated by authors such as Dolado, Gonzalo and Mayoral (2002) who proposed the first Dickey-Fuller fractional integration model. Our proposed test is an extension of the linear model by Robinson (1994) to the nonlinear case which uses flexible Fourier form (FFF) to mimic nonlinearities in the process (see Gil-Alana and Yaya, 2018). Technically, by extending (4) to the nonlinear case proposed in Gil-Alana and Yaya (2018), we write,

$$y_t^* = \alpha_0 1_t^* + \beta_0 t_t^* + \lambda_1 \sin_{1,t}^* + \gamma_1 \cos_{1,t}^* + x_t, \quad t = 1, 2, \ldots \tag{6}$$

where $\sin_{1,t}^* = (1-L)^{d_0} \sin\left(\frac{2\pi t}{T}\right)$ and $\cos_{1,t}^* = (1-L)^{d_0} \sin\left(\frac{2\pi t}{T}\right)$. Significance of at least one of the Fourier parameters $\lambda_1$ and $\gamma_1$ implies nonlinearity in the time series.

## 3. Data and Empirical Result

The dataset are the daily prices of cryptocurrency from 7 August 2015 and 28 November 2018. Based on high price and market capitalizations, we have only included 12 cryptocurrencies in our analysis.[3] The cryptos are: Bitcoin, Dash, Digibyte, Doge, Ethereum, Litecoin, Maidsafecoin, Monero, Nem, Ripple, Stellar and Vertcoin. We obtained log-transformed prices for returns and squared returns. Plot of these are given in Figure 1. The price series in each plot is observed to depart from its original trends, which may be an indication of the presence of

[3] For dataset downloads, visit https://coinmetrics.io/data-downloads/.

structural breaks. Therefore, in addition to the full sample period considered, the series was further sub-divided into different two sample periods – the pre-crash and post-crash periods. The adopted point for the sub-division was the peak point with respect to the bitcoin prices, being the most valuable cryptocurrency (Yaya et al., 2018). Consequently, the chosen break date was 17th December, 2017.[4] Thus, each plot contains the actual, returns and squared returns series of each cryptocurrency. Cryptocurrency returns are characterized by volatility clustering and notable jumps at different time periods, particularly in the post-crash periods. This is quite noticeable in the log-returns for Bitcoin, Dash, Digibyte, Doge, Litecoin, Maidsafecoin, Monero, Ripple, Stellar and Vertcoin.

**INSERT FIGURE 1 ABOUT HERE**

Descriptive statistics, presented in Table 1 gives the statistical distribution of the cryptocurrencies as it gives more reliable results than the graphical approach in Figure 1. We consider the actual price, log-returns and squared returns of the cryptocurrency series under three different sample periods: the full sample, the pre-crash sample and the post-crash sample periods with sample sizes 1210, 864 and 346, respectively. The results are presented in three different panes, each displaying the statistical properties of the cryptocurrency prices and the log-returns (see Table 1). Pane A and Pane B show the summary statistics for the cryptocurrency prices and the log-return series under the three sample periods. Bitcoin is the highest cryptocurrency and is averagely priced at 3596.6 USD, 1495 USD and 8194 USD in the full sample, pre-crash and post-crash samples, respectively. On the other hand; Doge, the least valuable cryptocurrency among the considered cryptocurrencies, is averagely priced at 0.001 USD, 0.002 USD and 0.005 USD in the full, pre-crash and post-crash sample periods, respectively. Regardless of the cryptocurrency considered, prices seemed to be more volatile

[4] Note that it took some altcoins about 30-40 days to react to react to price crash induced from Bitcoin market while some altcoins reacted within the first week of price crash.

in post-crash period than in the pre-crash sample period, since the observed standard deviation values are higher in the former than in the latter sample period. This may not be completely disconnected from the speculation induced by the cryptocurrency crash of December 17, 2017. Also, the cryptocurrency prices are not normally distributed, given the statistically significant of Jarque-Bera statistics, which formally combines the skewness and kurtosis.

On the statistical distribution of the log-returns of cryptocurrency (see Pane B in Table 1), Nem had the highest log-returns of 0.0023 and 0.0043 in the full sample and the pre-crash sample periods, respectively, while Stellar had the highest log-returns of -0.0008 in the post-crash sample period. The least log-returns in the full, pre-crash and post-crash sample periods were observed for Maidsafecoin (0.0006, 0.0016) and Vertcoin (-0.0042), respectively. The log-returns were found to be highly volatile, with the standard deviations that were twice the means in most cases, and also not normally distributed.

**INSERT TABLE 1 ABOUT HERE**

Since market efficiency is a function of returns (price changes), volatility comes into play (high, medium or low) and the function of time the volatility will face out is the volatility persistence, which is an autocorrelation issue in the absolute or squared return or both as proxies. Starting with the case of market efficiency, investigated for the whole sample period (Table 2a), pre-crash sample (Table 2b) and post-crash sample (Table 2c), we found evidences to support market efficiency of some cryptocurrencies. The null hypothesis of random walk for market efficiency is unrejected in the case of Bitcoin, Dash, Digibyte and Ethereum markets in the full sample, pre-crash and after crash samples (see Bartos, 2015). During the pre-crash sample, evidence to support market inefficiency is found in Nem and Stellar since random walk is evident, while the linear time trend specification points to non-rejection of market efficiency in Stellar. In the post-crash sample, market becomes more inefficient, as we observe rejection of random walk in returns in Litecoin, Maidsafecoin, Monero and Vertcoin in the three test regression specifications.

**INSERT TABLE 2a ABOUT HERE**

**INSERT TABLE 2b ABOUT HERE**

**INSERT TABLE 2c ABOUT HERE**

The results presented in Tables 2a-2c are based on the assumption of linearity in log-returns of cryptocurrencies. We then check for robustness by means of nonlinear fractional integration using flexible Fourier function described in the methodology part.[5] Noting that Fourier function allows one to model remaining structural breaks in returns as smooth breaks rather than instantaneous breaks. In Tables 3a-3c, it is interesting to find that nonlinearity is found in the full return sample, as expected, at least in form of a break during the crash for some cryptocurrency. The coefficient of cosine part of the nonlinear function, $\gamma$ is significant in Bitcoin, Dash, Ethereum, Litecoin, Monero, Nem, Ripple and Vertcoin implying the relevance of nonlinear fractional integration results here. Now, looking at the estimated $d$ parameter for returns, we find, in addition to four cases of market inefficiency (Doge, Maidsafecoin, Ripple and Stellar), Vertcoin indicating rejection of null hypothesis of random walk of market hypothesis. Similarly, in the case of pre-crash and post-crash samples.

**INSERT TABLE 3a ABOUT HERE**

**INSERT TABLE 3b ABOUT HERE**

**INSERT TABLE 3c ABOUT HERE**

Next, we consider issue of volatility persistence in cryptocurrency analyzing fractional integration parameter in the squared returns series. Though we found volatility in the post-crash sample to be higher than that of the pre-crash sample. This is actually the expectation of

[5] The nonlinear fractional integration framework considers only the case of intercept, since time trend coefficients are insignificant in the linear cases.

many researchers since they have found that lesser volatility during bear periods compared to bull periods (Gomez et al., 2004; Gonzalez et al, 2005). The results of volatility persistence for the linear case are presented in Tables 4a, 4b and 4c for full sample, pre-crash and post-crash samples. We only find evidence of no persistence of volatility in the case of Ethereum across the three sampled periods, while the remaining 11 cryptocurrencies indicate evidence of significant volatility persistence. In the full sample, Vertcoin has the highest volatility persistence, and similarly in the pre-crash sample. Next to this is Stellar. Lowest significant volatility is found Maidsafecoin. Results of volatility persistence observed during post-crash periods are not consistent with that of pre-crash since highest persistence of volatility is found in Doge, across the three testing regression model, while low volatility persistence is found in Dash, Nem, Ripple and Maidsafecoin. In this subsample, volatility persistence is lower than that in the pre-crash sample, as the observed volatility takes shorter time than in the pre-crash period to fizzle out.

**INSERT TABLE 4a ABOUT HERE**

**INSERT TABLE 4b ABOUT HERE**

**INSERT TABLE 4c ABOUT HERE**

As we did earlier by introducing nonlinear smooth break function to capture possible structural breaks in the time series at hand, we, similarly apply this to the squared returns with the results presented in Tables 5a-5c. We first observe some nonlinearities, in squared returns of Bitcoin, Dash, Doge, Litecoin (in the full sample), in Bitcoin and Litecoin (in pre-crash sample) and in Bitcoin, Dash, Ethereum, Litecoin, Maidsafecoin, Monero, Nem, Ripple and Stellar (in post-crash sample). By comparing the d estimates in the linear and nonlinear cases, we still find only Ethereum with no-significant volatility persistence for the three testing regression specifications. In the pre-crash sample, Digibyte is found to be insignificant due to loss power and increase in the degree of freedom of the test statistic. The results computed for the case of post-crash showed more nonlinearities, with nonlinearity found for squared return of Bitcoin and other eight (8) altcoins. Meanwhile, significant d values are found in Digibyte, Doge, Stellar and Vertcoin.

**INSERT TABLE 5a ABOUT HERE**

**INSERT TABLE 5b ABOUT HERE**

**INSERT TABLE 5c ABOUT HERE**

## 4. Conclusion

We have considered in this paper efficiency, volatility and its persistence in 12 cryptocurrency markets, with data samples from 7 August 2015 and 28 November 2018. The considered cryptocurrency are the Bitcoin, Dash, Digibyte, Doge, Ethereum, Litecoin, Maidsafecoin, Monero, Nem, Ripple, Stellar and Vertcoin. The findings obtained about the level of market efficiency of cryptocurrency indicate evidence of random walk in returns of most cryptocurrencies including Bitcoin, which is contrary to what is published by some authors who found inefficiency in Bitcoin. Thus future Bitcoin values are unpredictable. Our approach of investigating market efficiency is novel and robustness is checked based on nonlinearity. Apart from that, we follow closely the definition of EMH by Fama (1970) on returns unpredictability or being a random walk process to imply market efficiency. Though volatility is found to be higher during the post-crash period but this will persist for shorter period compared to pre-crash period.

As a result of this unpredictability of returns in Bitcoin particularly, traders cannot boast of making abnormal profits in the cryptocurrency markets as revealed in the findings. Our approach of analysis in this paper follows Gil-Alana et al. (2018) on efficiency and volatility persistence of Baltic stock markets, and we therefore welcome criticism. This work will therefore interest market participants and portfolio managers in a number of ways.

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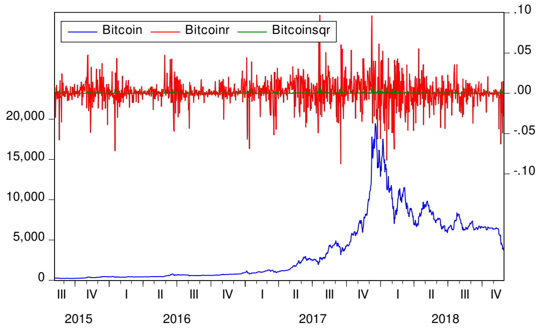
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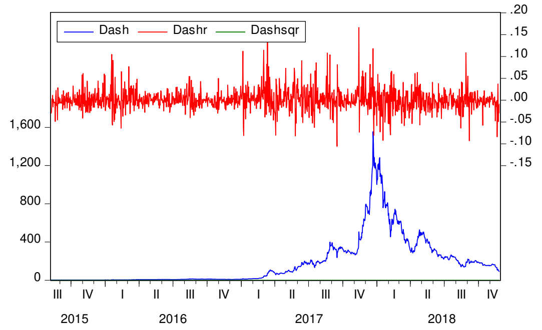
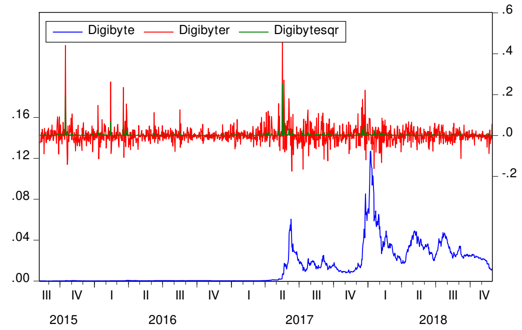
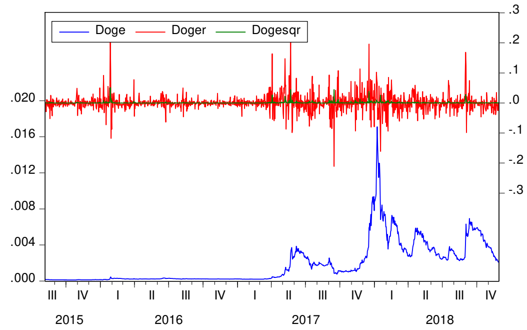
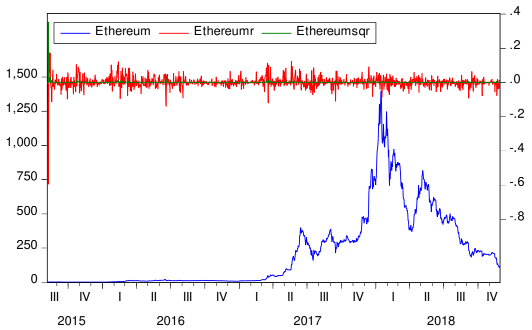
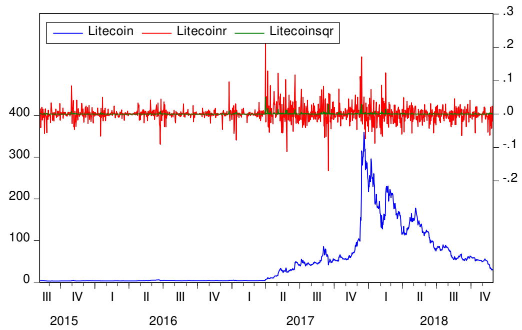
2015 2016 2017 2018

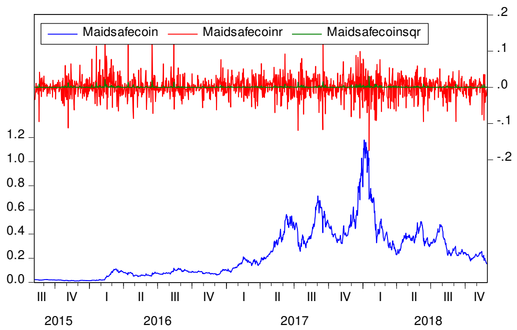

Dash
Dashr
Dashsqr
1,600
1,200
800
400
0
.20
.15
.10
.05
.00
-.05
-.10
-.15
III IV I II III IV I II III IV I II III IV
2015 2016 2017 2018


Digibyte
Digibyter
Digibytesqr
.16
.12
.08
.04
.00
.6
.4
.2
.0
-.2
III IV I II III IV I II III IV I II III IV
2015 2016 2017 2018


Doge
Doger
Dogesqr
.020
.016
.012
.008
.004
.000
.3
.2
.1
.0
-.1
-.2
-.3
III IV I II III IV I II III IV I II III IV
2015 2016 2017 2018


Ethereum
Ethereumr
Ethereumsqr
1,500
1,250
1,000
750
500
250
0
.4
.2
.0
-.2
-.4
-.6
-.8
III IV I II III IV I II III IV I II III IV
2015 2016 2017 2018


Litecoin
Litecoinr
Litecoinsqr
400
300
200
100
0
.3
.2
.1
.0
-.1
-.2
III IV I II III IV I II III IV I II III IV
2015 2016 2017 2018


Maidsafecoin
Maidsafecoinr
Maidsafecoinsqr
1.2
1.0
0.8
0.6
0.4
0.2
0.0
.2
.1
.0
-.1
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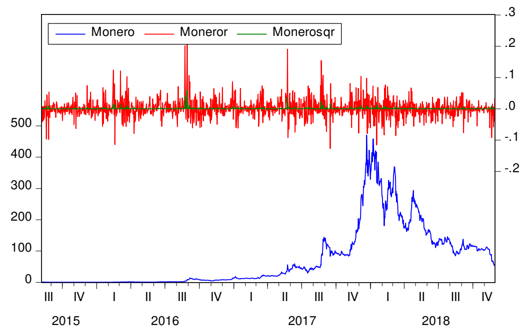
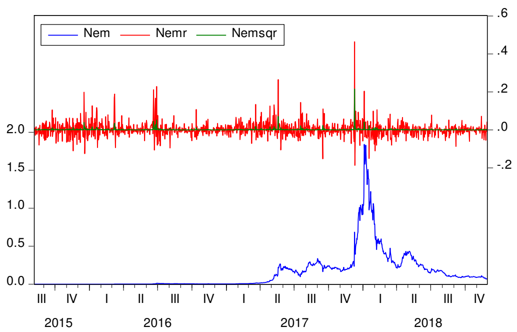
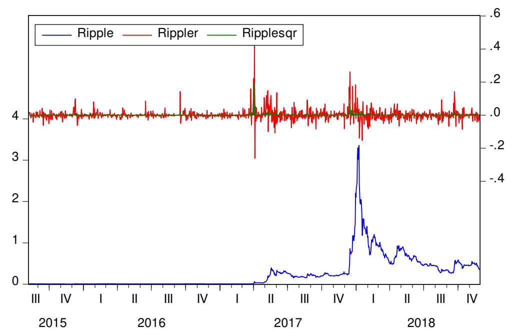
III IV I II III IV I II III IV I II III IV
2015 2016 2017 2018


Monero
Moneror
Monerosqr
500
400
300
200
100
0
.3
.2
.1
.0
-.1
-.2
III IV I II III IV I II III IV I II III IV
2015 2016 2017 2018


Nem
Nemr
Nemsqr
2.0
1.5
1.0
0.5
0.0
.6
.4
.2
.0
-.2
III IV I II III IV I II III IV I II III IV
2015 2016 2017 2018


Ripple
Rippler
Ripplesqr
4
3
2
1
0
.6
.4
.2
.0
-.2
-.4
III IV I II III IV I II III IV I II III IV
2015 2016 2017 2018

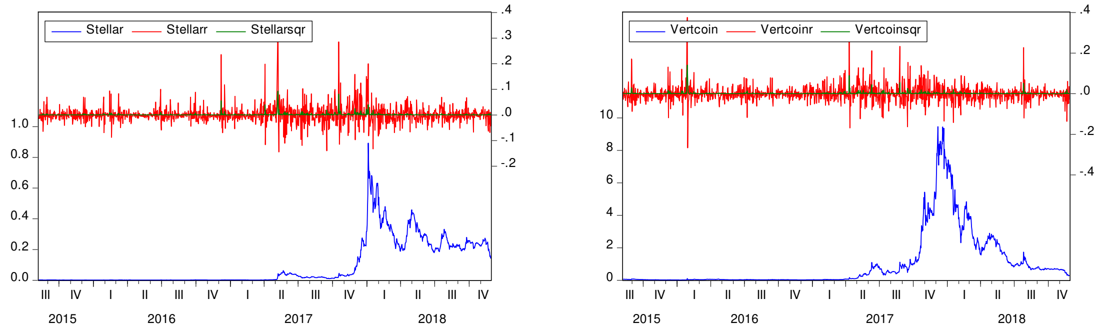



**Figure 1: Plots of Returns, Absolute and Squared returns**

## Table 1: Descriptive Statistics

| | Bitcoin | Dash | Digibyte | Doge | Ethereum | Litecoin | Maidsafecoin | Monero | Nem | Ripple | Stellar | Vertcoin |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Pane A: Prices** | | | | | | | | | | | | |
| ***Full Sample*** | | | | | | | | | | | | |
| Mean | 3596.646 | 177.316 | 0.013 | 0.002 | 213.463 | 47.745 | 0.234 | 73.391 | 0.143 | 0.256 | 0.093 | 0.996 |
| Maximum | 19475.8 | 1555.59 | 0.127 | 0.017 | 1397.48 | 359.13 | 1.18 | 470.29 | 1.840 | 3.360 | 0.892 | 9.460 |
| Minimum | 210.07 | 2.08 | 0.000 | 0.000 | 0.432 | 2.64 | 0.012 | 0.369 | 0.000 | 0.004 | 0.001 | 0.018 |
| Std. Dev. | 3961.201 | 250.49 | 0.018 | 0.002 | 278.344 | 65.205 | 0.209 | 98.401 | 0.245 | 0.408 | 0.145 | 1.74 |
| Jarque-Bera | 403.8** | 2328.5** | 3042.7** | 2329.2** | 626.3** | 1302.3** | 634.8** | 741.3** | 12569.1** | 12851.3** | 962.2** | 3372.4** |
| ***Pre-Crash Sample*** | | | | | | | | | | | | |
| Mean | 1715.545 | 87.842 | 0.004 | 0.001 | 85.793 | 18.379 | 0.172 | 27.167 | 0.061 | 0.068 | 0.011 | 0.52 |
| Maximum | 19475.8 | 1014.51 | 0.06 | 0.006 | 700.59 | 315.36 | 0.716 | 328.06 | 0.684 | 0.862 | 0.235 | 9.46 |
| Minimum | 210.07 | 2.08 | 0.000 | 0.000 | 0.432 | 2.64 | 0.012 | 0.369 | 0.000 | 0.004 | 0.001 | 0.018 |
| Std. Dev. | 2582.042 | 156.871 | 0.008 | 0.001 | 136.454 | 32.481 | 0.173 | 48.94 | 0.105 | 0.108 | 0.023 | 1.39 |
| Jarque-Bera | 9832.8** | 3663.8** | 4426.6** | 1071.1** | 520.6** | 42763.3** | 187.4** | 6163.8** | 1298.5** | 2076.9** | 38921.2** | 12245.9** |
| ***Post-Crash Sample*** | | | | | | | | | | | | |
| Mean | 8293.962 | 400.74 | 0.035 | 0.005 | 532.267 | 121.076 | 0.389 | 188.818 | 0.347 | 0.725 | 0.298 | 2.183 |
| Maximum | 19118.3 | 1555.59 | 0.127 | 0.017 | 1397.48 | 359.13 | 1.18 | 470.29 | 1.84 | 3.36 | 0.892 | 9.44 |
| Minimum | 3765.95 | 88.68 | 0.011 | 0.002 | 107.91 | 29.23 | 0.153 | 53.12 | 0.068 | 0.263 | 0.144 | 0.28 |
| Std. Dev. | 2705.482 | 297.007 | 0.019 | 0.002 | 286.66 | 68.613 | 0.21 | 96.06 | 0.353 | 0.493 | 0.117 | 1.95 |
| Jarque-Bera | 260.2** | 186.4** | 1002.9** | 642.8** | 24** | 47.4** | 384.3** | 50.3** | 485.6** | 1622.3** | 374.3** | 211.9** |
| **Pane B: Log-Returns** | | | | | | | | | | | | |
| ***Full Sample*** | | | | | | | | | | | | |
| Mean | 0.0009 | 0.0012 | 0.0019 | 0.0009 | 0.0013 | 0.0007 | 0.0006 | 0.0016 | 0.0023 | 0.0014 | 0.0015 | 0.0005 |
| Maximum | 0.0971 | 0.1664 | 0.5004 | 0.2263 | 0.174 | 0.2252 | 0.1476 | 0.2465 | 0.464 | 0.4391 | 0.3058 | 0.3758 |
| Minimum | -0.0878 | -0.1057 | -0.1745 | -0.2107 | -0.5943 | -0.1698 | -0.1746 | -0.1267 | -0.1871 | -0.2613 | -0.1448 | -0.2667 |
| Std. Dev. | 0.0171 | 0.0257 | 0.045 | 0.03 | 0.0341 | 0.025 | 0.0298 | 0.0309 | 0.0396 | 0.0331 | 0.0364 | 0.0429 |
| Jarque-Bera | 1149.8** | 1487.3** | 28667.9** | 6904.8** | 312176.0** | 9166.6** | 520.1** | 2875.3** | 22151.8** | 71054.0** | 12543.5** | 8385.1** |
| ***Pre-Crash Sample*** | | | | | | | | | | | | |
| Mean | 0.0021 | 0.0029 | 0.0031 | 0.0018 | 0.0028 | 0.0022 | 0.0016 | 0.0031 | 0.0043 | 0.0023 | 0.0023 | 0.0024 |
| Maximum | 0.097 | 0.166 | 0.5 | 0.226 | 0.174 | 0.225 | 0.148 | 0.247 | 0.464 | 0.439 | 0.306 | 0.376 |
| Minimum | -0.088 | -0.106 | -0.175 | -0.211 | -0.594 | -0.17 | -0.119 | -0.127 | -0.187 | -0.261 | -0.145 | -0.267 |
| Std. Dev. | 0.016 | 0.025 | 0.048 | 0.029 | 0.037 | 0.025 | 0.029 | 0.032 | 0.042 | 0.033 | 0.038 | 0.046 |
| Jarque-Bera | 1507.9** | 1562.2** | 23058.9** | 9735.4** | 218287.7** | 10698.8** | 392.6** | 2833.2** | 17099.9** | 86303.6** | 10592.1** | 5817.4** |
| ***Post-Crash Sample*** | | | | | | | | | | | | |
| Mean | -0.002 | -0.0032 | -0.0014 | -0.0013 | -0.0024 | -0.0029 | -0.0019 | -0.0023 | -0.0028 | -0.0009 | -0.0008 | -0.0042 |
| Maximum | 0.061 | 0.118 | 0.222 | 0.169 | 0.064 | 0.124 | 0.099 | 0.098 | 0.204 | 0.184 | 0.2 | 0.228 |
| Minimum | -0.083 | -0.093 | -0.162 | -0.161 | -0.092 | -0.088 | -0.175 | -0.115 | -0.15 | -0.153 | -0.133 | -0.142 |
| Std. Dev. | 0.019 | 0.026 | 0.037 | 0.033 | 0.024 | 0.024 | 0.032 | 0.028 | 0.032 | 0.033 | 0.032 | 0.035 |
| Jarque-Bera | 74.1** | 118.1** | 343.6** | 257.1** | 43.4** | 111** | 150.8** | 37.9** | 577** | 892** | 712.3** | 74.4** |

**Table 2a: Estimates of d for the Log-returns in the whole sample**

| | No regressors | An intercept | A linear time trend |
|---|---|---|---|
| Bitcoin | 0.0280 (0.0221) | 0.0281 (0.0221) | 0.0252 (0.0222) |
| Dash | 0.0050 (0.0202) | 0.0050 (0.0202) | -9.78E-05 (0.0204) |
| Digibyte | 0.0370 (0.0220) | 0.0370 (0.0220) | 0.0359 (0.0221) |
| Doge | **0.0635 (0.0227)** | **0.0635 (0.0228)** | **0.0634 (0.0227)** |
| Ethereum | 0.0435 (0.0227) | 0.0437 (0.0227) | 0.0407 (0.0229) |
| Litecoin | 0.0257 (0.0214) | 0.0258 (0.0214) | 0.0251 (0.0214) |
| Maidsafecoin | **-0.0430 (0.0209)** | **-0.0431 (0.0209)** | **-0.0489 (0.0213)** |
| Monero | -0.0008 (0.0210) | -0.0008 (0.0211) | -0.0043 (0.0213) |
| Nem | -0.0231 (0.0196) | -0.0216 (0.0197) | -0.0332 (0.0202) |
| Ripple | **0.0563 (0.0209)** | **0.0564 (0.0209)** | **0.0563 (0.0209)** |
| Stellar | **0.0622 (0.0227)** | **0.0622 (0.0227)** | **0.0619 (0.0228)** |
| Vertcoin | -0.0368 (0.0217) | -0.0370 (0.0217) | -0.0373 (0.0217) |

Note, significant parameter *d* are in bold.

**Table 2b: Estimates of d for the Log-returns in the Pre-Crash sample**

| | No regressors | An intercept | A linear time trend |
|---|---|---|---|
| Bitcoin | 0.0240 (0.0266) | 0.0241 (0.0266) | 0.0004 (0.0279) |
| Dash | -0.0149 (0.0249) | -0.0146 (0.0249) | -0.0318 (0.0259) |
| Digibyte | 0.0330 (0.0267) | 0.0331 (0.0267) | 0.0319 (0.0268) |
| Doge | 0.0492 (0.0278) | 0.0493 (0.0278) | 0.0439 (0.0279) |
| Ethereum | 0.0341 (0.0280) | 0.0342 (0.0280) | 0.0316 (0.0281) |
| Litecoin | 0.0381 (0.0260) | 0.0381 (0.0260) | 0.0152 (0.0273) |
| Maidsafecoin | -0.0449 (0.0257) | -0.0449 (0.0257) | -0.0452 (0.0257) |
| Monero | -0.0004 (0.0260) | -0.0004 (0.0260) | -0.0078 (0.0264) |
| Nem | **-0.0534 (0.0243)** | **-0.0534 (0.0243)** | **-0.0542 (0.0243)** |
| Ripple | 0.0468 (0.0244) | 0.0468 (0.0244) | 0.0373 (0.0250) |
| Stellar | **0.0641 (0.0274)** | **0.0642 (0.0274)** | 0.0530 (0.0281) |
| Vertcoin | -0.0392 (0.0267) | -0.0392 (0.0267) | -0.0763 (0.0294) |

Note, significant parameter *d* are in bold.

**Table 2c: Estimates of d for the Log-returns in the Post-Crash sample**

| | No regressors | An intercept | A linear time trend |
|---|---|---|---|
| Bitcoin | -0.0094 (0.0430) | -0.0094 (0.0432) | -0.0106 (0.0434) |
| Dash | -0.0693 (0.0433) | -0.0693 (0.0434) | -0.0700 (0.0435) |
| Digibyte | -0.0033 (0.0428) | -0.0034 (0.0428) | -0.0064 (0.0429) |
| Doge | 0.0813 (0.0448) | 0.0813 (0.0448) | 0.0811 (0.0447) |
| Ethereum | 0.0179 (0.0420) | 0.0180 (0.0420) | 0.0050 (0.0429) |
| Litecoin | **-0.0860 (0.0432)** | **-0.0860 (0.0433)** | **-0.0860 (0.0433)** |
| Maidsafecoin | **-0.0935 (0.0405)** | **-0.0933 (0.0405)** | **-0.0934 (0.0406)** |
| Monero | **-0.0991 (0.0430)** | **-0.0992 (0.0431)** | **-0.0995 (0.0430)** |
| Nem | -0.0424 (0.0420) | -0.0427 (0.0421) | -0.0430 (0.0421) |
| Ripple | 0.0657 (0.0438) | 0.0658 (0.0438) | 0.0661 (0.0437) |
| Stellar | 0.0369 (0.0435) | 0.0369 (0.0435) | 0.0341 (0.0434) |
| Vertcoin | **-0.1226 (0.0411)** | **-0.1229 (0.0413)** | **-0.1241 (0.0415)** |

Note, significant parameter *d* are in bold.

**Table 3a: Estimates of d for Log-returns for nonlinear Fourier function case in whole sample**

| | $d$ | $intercept$ , $\alpha$ | $\lambda$ | $\gamma$ |
|---|---|---|---|---|
| Bitcoin | 0.0151 (0.0228) | -9.13E-06 (0.0005) | 5.58E-05 (0.0008) | **-0.0018 (0.0007)** |
| Dash | -0.0217 (0.0216) | 3.00E-05 (0.0006) | 1.10E-03 (0.0009) | **-0.0033 (0.0009)** |
| Digibyte | 0.0333 (0.0222) | 3.12E-05 (0.0016) | -3.64E-04 (0.0022) | -0.0027 (0.0022) |
| Doge | **0.0598 (0.0230)** | -4.58E-05 (0.0013) | -4.21E-04 (0.0017) | -0.0021 (0.0016) |
| Ethereum | 0.0329 (0.0234) | -7.01E-05 (0.0012) | 1.20E-03 (0.0017) | **-0.0036 (0.0016)** |
| Litecoin | 0.0056 (0.0225) | -1.79E-05 (0.0007) | -3.60E-04 (0.0010) | **-0.0032 (0.0010)** |
| Maidsafecoin | **-0.0537 (0.0216)** | 4.59E-05 (0.0006) | 1.36E-03 (0.0009) | -0.0015 (0.0009) |
| Monero | -0.0196 (0.0221) | 8.74E-06 (0.0008) | 1.42E-03 (0.0011) | **-0.0032 (0.0011)** |
| Nem | -0.0386 (0.0204) | 6.85E-06 (0.0009) | 2.40E-03 (0.0013) | **-0.0029 (0.0013)** |
| Ripple | **0.0477 (0.0214)** | -8.02E-06 (0.0013) | -6.80E-04 (0.0017) | **-0.0034 (0.0017)** |
| Stellar | **0.0542 (0.0232)** | -6.84E-06 (0.0015) | -1.79E-03 (0.0020) | -0.0030 (0.0019) |
| Vertcoin | **-0.0645 (0.0232)** | 2.80E-05 (0.0008) | -3.31E-04 (0.0012) | **-0.0045 (0.0013)** |

In bold, significant estimates. Standard errors of intercept and slope as well as that of Fourier function parameters are given in parentheses.

**Table 3b: Estimates of d for Log-returns for nonlinear Fourier function case in Pre-crash sample**

| | $d$ | $intercept$ , $\alpha$ | $\lambda$ | $\gamma$ |
|---|---|---|---|---|
| Bitcoin | 0.0072 (0.0279) | 4.53E-06 (0.0006) | -1.13E-03 (0.0008) | 0.0013 (0.0008) |
| Dash | -0.0252 (0.0258) | 2.90E-05 (0.0007) | -1.85E-03 (0.0011) | 0.0000 (0.0011) |
| Digibyte | 0.0281 (0.0271) | 7.48E-05 (0.0019) | -2.78E-03 (0.0026) | 0.0016 (0.0026) |
| Doge | 0.0456 (0.0281) | 5.71E-05 (0.0013) | -1.30E-03 (0.0017) | 0.0010 (0.0017) |
| Ethereum | 0.0336 (0.0281) | -7.40E-05 (0.0016) | -7.28E-04 (0.0021) | -0.0004 (0.0021) |
| Litecoin | 0.0227 (0.0273) | -3.74E-06 (0.0010) | -2.46E-03 (0.0014) | 0.0013 (0.0013) |
| Maidsafecoin | -0.0456 (0.0257) | 4.27E-05 (0.0007) | 2.86E-06 (0.0011) | -0.0006 (0.0011) |
| Monero | -0.0025 (0.0263) | -6.00E-06 (0.0011) | -5.17E-04 (0.0015) | -0.0010 (0.0015) |
| Nem | **-0.0538 (0.0244)** | -6.84E-06 (0.0010) | -4.25E-04 (0.0015) | 0.0002 (0.0016) |
| Ripple | 0.0373 (0.0252) | 3.20E-05 (0.0014) | -3.27E-03 (0.0019) | 0.0003 (0.0019) |
| Stellar | **0.0581 (0.0280)** | 1.07E-04 (0.0019) | -2.45E-03 (0.0024) | 0.0017 (0.0024) |
| Vertcoin | **-0.0650 (0.0288)** | 2.00E-05 (0.0010) | **-3.96E-03 (0.0016)** | 0.0015 (0.0016) |

In bold, significant estimates. Standard errors of intercept and slope as well as that of Fourier function parameters are given in parentheses.

**Table 3c: Estimates of d for Log-returns for nonlinear Fourier function case in Post-crash sample**

| | $d$ | $intercept$ , $\alpha$ | $\lambda$ | $\gamma$ |
|---|---|---|---|---|
| Bitcoin | -0.0294 (0.0447) | 2.15E-05 (0.0009) | -7.71E-04 (0.0013) | -0.0023 (0.0013) |
| Dash | -0.0768 (0.0439) | -1.47E-05 (0.0009) | -1.00E-03 (0.0014) | -0.0013 (0.0015) |
| Digibyte | -0.0070 (0.0432) | -4.94E-05 (0.0019) | -2.56E-04 (0.0027) | -0.0017 (0.0027) |
| Doge | 0.0766 (0.0452) | -1.21E-04 (0.0027) | -2.61E-03 (0.0034) | -0.0015 (0.0033) |
| Ethereum | 0.0148 (0.0424) | -1.00E-04 (0.0014) | 1.03E-03 (0.0020) | -0.0005 (0.0019) |
| Litecoin | **-0.0884 (0.0434)** | -1.98E-05 (0.0008) | 1.22E-04 (0.0013) | -0.0011 (0.0013) |
| Maidsafecoin | **-0.1025 (0.0412)** | -1.00E-04 (0.0010) | -3.04E-04 (0.0015) | -0.0021 (0.0016) |
| Monero | **-0.1003 (0.0431)** | 1.23E-05 (0.0009) | -5.87E-04 (0.0014) | -0.0006 (0.0015) |
| Nem | -0.0448 (0.0422) | -2.00E-04 (0.0013) | -1.12E-03 (0.0020) | 0.0005 (0.0021) |
| Ripple | 0.0620 (0.0441) | -1.36E-05 (0.0025) | -1.87E-03 (0.0032) | 0.0020 (0.0032) |
| Stellar | 0.0366 (0.0435) | -3.66E-05 (0.0021) | -8.41E-04 (0.0028) | 0.0002 (0.0028) |
| Vertcoin | **-0.1301 (0.0417)** | 3.74E-05 (0.0009) | -8.71E-04 (0.0015) | -0.0017 (0.0016) |

In bold, significant estimates. Standard errors of intercept and slope as well as that of Fourier function parameters are given in parentheses.

**Table 4a: Estimates of d for the squared returns in the whole sample**

| | No regressors | An intercept | A linear time trend |
|---|---|---|---|
| Bitcoin | **0.1683 (0.0208)** | **0.1683 (0.0207)** | **0.1683 (0.0207)** |
| Dash | **0.1253 (0.0213)** | **0.1254 (0.0213)** | **0.1220 (0.0215)** |
| Digibyte | **0.0684 (0.0208)** | **0.0684 (0.0208)** | **0.0676 (0.0208)** |
| Doge | **0.1718 (0.0221)** | **0.1719 (0.0221)** | **0.1684 (0.0224)** |
| Ethereum | 0.0324 (0.0214) | 0.0328 (0.0215) | 0.0275 (0.0217) |
| Litecoin | **0.1304 (0.0211)** | **0.1304 (0.0211)** | **0.1256 (0.0215)** |
| Maidsafecoin | **0.1103 (0.0214)** | **0.1104 (0.0214)** | **0.1102 (0.0214)** |
| Monero | **0.1191 (0.0221)** | **0.1191 (0.0221)** | **0.1190 (0.0221)** |
| Nem | **0.1200 (0.0238)** | **0.1200 (0.0238)** | **0.1195 (0.0238)** |
| Ripple | **0.2264 (0.0243)** | **0.2264 (0.0243)** | **0.2257 (0.0244)** |
| Stellar | **0.3132 (0.0263)** | **0.3131 (0.0262)** | **0.3126 (0.0263)** |
| Vertcoin | **0.4183 (0.0306)** | **0.4183 (0.0306)** | **0.4182 (0.0306)** |

Note, significant parameter *d* are in bold.

**Table 4b: Estimates of d for the squared returns in the Pre-Crash sample**

| | No regressors | An intercept | A linear time trend |
|---|---|---|---|
| Bitcoin | **0.1977 (0.0274)** | **0.1976 (0.0274)** | **0.1801 (0.0286)** |
| Dash | **0.1361 (0.0260)** | **0.1361 (0.0260)** | **0.1218 (0.0270)** |
| Digibyte | **0.0617 (0.0250)** | **0.0617 (0.0250)** | **0.0608 (0.0251)** |
| Doge | **0.1541 (0.0273)** | **0.1541 (0.0273)** | **0.1409 (0.0280)** |
| Ethereum | 0.0306 (0.0254) | 0.0311 (0.0256) | 0.0264 (0.0257) |
| Litecoin | **0.1367 (0.0256)** | **0.1367 (0.0256)** | **0.1117 (0.0272)** |
| Maidsafecoin | **0.1175 (0.0272)** | **0.1176 (0.0272)** | **0.1174 (0.0271)** |
| Monero | **0.1190 (0.0269)** | **0.1190 (0.0269)** | **0.1170 (0.0270)** |
| Nem | **0.1212 (0.0288)** | **0.1212 (0.0288)** | **0.1198 (0.0287)** |
| Ripple | **0.2412 (0.0293)** | **0.2412 (0.0293)** | **0.2374 (0.0296)** |
| Stellar | **0.3364 (0.0320)** | **0.3365 (0.0320)** | **0.3311 (0.0326)** |
| Vertcoin | **0.4502 (0.0370)** | **0.4502 (0.0370)** | **0.4501 (0.0371)** |

Note, significant parameter *d* are in bold.

**Table 4c: Estimates of d for the squared returns in the Post-Crash sample**

| | No regressors | An intercept | A linear time trend |
|---|---|---|---|
| Bitcoin | **0.1234 (0.0332)** | **0.1246 (0.0334)** | 0.0373 (0.0375) |
| Dash | **0.0836 (0.0376)** | **0.0844 (0.0378)** | 0.0436 (0.0401) |
| Digibyte | **0.1537 (0.0403)** | **0.1560 (0.0407)** | **0.0938 (0.0441)** |
| Doge | **0.3795 (0.0543)** | **0.3815 (0.0543)** | **0.3677 (0.0552)** |
| Ethereum | 0.0607 (0.0392) | 0.0608 (0.0392) | 0.0326 (0.0416) |
| Litecoin | **0.0866 (0.0351)** | **0.0867 (0.0351)** | 0.0202 (0.0394) |
| Maidsafecoin | **0.0905 (0.0363)** | **0.0906 (0.0364)** | 0.0185 (0.0405) |
| Monero | **0.1107 (0.0344)** | **0.1119 (0.0346)** | 0.0374 (0.0386) |
| Nem | **0.0878 (0.0357)** | **0.0878 (0.0357)** | 0.0044 (0.0401) |
| Ripple | **0.0897 (0.0367)** | **0.0905 (0.0369)** | 0.0545 (0.0376) |
| Stellar | **0.1557 (0.0375)** | **0.1565 (0.0376)** | **0.1170 (0.0401)** |
| Vertcoin | **0.1300 (0.0462)** | **0.1302 (0.0462)** | **0.1229 (0.0471)** |

Note, significant parameter *d* are in bold.

**Table 5a: Estimates of d for Squared returns for nonlinear Fourier function case in whole sample**

| | $d$ | $intercept$, $\alpha$ | $\lambda$ | $\gamma$ |
|---|---|---|---|---|
| Bitcoin | **0.1395 (0.0224)** | 2.60E-06 (0.0001) | **-2.22E-04 (0.0001)** | -0.0001 (0.0001) |
| Dash | **0.1059 (0.0225)** | -1.73E-06 (0.0001) | **-2.92E-04 (0.0001)** | -0.0002 (0.0001) |
| Digibyte | **0.0646 (0.0210)** | 6.10E-06 (0.0005) | -5.39E-04 (0.0006) | -0.0006 (0.0006) |
| Doge | **0.1581 (0.0231)** | -7.25E-06 (0.0003) | **-6.70E-04 (0.0003)** | -0.0003 (0.0003) |
| Ethereum | 0.0310 (0.0218) | 5.03E-05 (0.0004) | 1.00E-04 (0.0005) | 0.0008 (0.0005) |
| Litecoin | **0.1008 (0.0230)** | 5.66E-06 (0.0001) | **-4.94E-04 (0.0002)** | **-0.0004 (0.0002)** |
| Maidsafecoin | **0.1064 (0.0216)** | -1.01E-05 (0.0001) | -1.90E-04 (0.0001) | 0.0000 (0.0001) |
| Monero | **0.1136 (0.0225)** | 2.65E-06 (0.0002) | -5.63E-06 (0.0002) | -0.0003 (0.0002) |
| Nem | **0.1189 (0.0238)** | -1.08E-05 (0.0005) | -2.83E-04 (0.0006) | -0.0002 (0.0005) |
| Ripple | **0.2211 (0.0248)** | 2.53E-05 (0.0008) | -7.21E-04 (0.0009) | -0.0008 (0.0008) |
| Stellar | **0.3065 (0.0269)** | 6.70E-05 (0.0011) | -9.38E-04 (0.0011) | -0.0010 (0.0010) |
| Vertcoin | **0.4180 (0.0306)** | 8.65E-05 (0.0028) | -3.94E-04 (0.0025) | -0.0003 (0.0021) |

In bold, significant estimates. Standard errors of intercept and slope as well as that of Fourier function parameters are given in parentheses.

**Table 5b: Estimates of d for Squared returns for nonlinear Fourier function case in Pre-crash sample**

| | $d$ | $intercept$, $\alpha$ | $\lambda$ | $\gamma$ |
|---|---|---|---|---|
| Bitcoin | **0.1786 (0.0290)** | 2.12E-06 (0.0001) | -1.00E-04 (0.0001) | **0.0002 (0.0001)** |
| Dash | **0.1191 (0.0274)** | -8.13E-06 (0.0001) | -3.16E-04 (0.0002) | 0.0002 (0.0002) |
| Digibyte | 0.0487 (0.0258) | -7.14E-06 (0.0005) | -8.98E-04 (0.0007) | 0.0012 (0.0007) |
| Doge | **0.1375 (0.0285)** | 6.35E-06 (0.0003) | -4.18E-04 (0.0003) | 0.0006 (0.0003) |
| Ethereum | 0.0263 (0.0261) | 5.20E-05 (0.0005) | 9.69E-05 (0.0007) | 0.0012 (0.0007) |
| Litecoin | **0.1095 (0.0277)** | 1.06E-05 (0.0002) | **-5.45E-04 (0.0002)** | 0.0004 (0.0002) |
| Maidsafecoin | **0.1120 (0.0276)** | -1.30E-05 (0.0001) | 1.25E-04 (0.0002) | 0.0001 (0.0002) |
| Monero | **0.1179 (0.0270)** | 2.42E-06 (0.0002) | -6.30E-05 (0.0003) | -0.0001 (0.0003) |
| Nem | **0.1160 (0.0291)** | 3.16E-05 (0.0006) | 3.25E-04 (0.0007) | 0.0009 (0.0007) |
| Ripple | **0.2363 (0.0298)** | 3.21E-05 (0.0011) | -1.15E-03 (0.0012) | 0.0003 (0.0011) |
| Stellar | **0.3321 (0.0326)** | -0.0002 (0.0016) | -1.12E-03 (0.0015) | 0.0006 (0.0013) |
| Vertcoin | **0.4496 (0.0371)** | -0.0005 (0.0039) | -3.26E-04 (0.0034) | 0.0008 (0.0029) |

In bold, significant estimates. Standard errors of intercept and slope as well as that of Fourier function parameters are given in parentheses.

**Table 5c: Estimates of d for Squared returns for nonlinear Fourier function case in Post-crash sample**

| | $d$ | *intercept*, $\alpha$ | $\lambda$ | $\gamma$ |
|---|---|---|---|---|
| Bitcoin | 0.0268 (0.0370) | 3.39E-06 (0.0000) | **2.60E-04 (0.0001)** | **0.0002 (0.0001)** |
| Dash | 0.0506 (0.0401) | 8.42E-06 (0.0001) | 1.89E-04 (0.0001) | **0.0003 (0.0001)** |
| Digibyte | **0.1269 (0.0429)** | 1.00E-04 (0.0004) | 5.51E-04 (0.0004) | 0.0008 (0.0004) |
| Doge | **0.3736 (0.0551)** | 4.20E-04 (0.0010) | -1.45E-04 (0.0009) | 0.0007 (0.0008) |
| Ethereum | 0.0201 (0.0423) | -2.69E-06 (0.0001) | 1.36E-04 (0.0001) | **0.0002 (0.0001)** |
| Litecoin | 0.0079 (0.0398) | -5.32E-06 (0.0001) | **3.23E-04 (0.0001)** | **0.0003 (0.0001)** |
| Maidsafecoin | 0.0399 (0.0396) | -1.21E-05 (0.0001) | **4.45E-04 (0.0002)** | **0.0004 (0.0002)** |
| Monero | 0.0444 (0.0381) | 6.51E-06 (0.0001) | **3.87E-04 (0.0001)** | **0.0004 (0.0001)** |
| Nem | 0.0255 (0.0395) | -2.30E-05 (0.0002) | **6.32E-04 (0.0002)** | **0.0006 (0.0002)** |
| Ripple | 0.0458 (0.0390) | 1.34E-05 (0.0002) | 2.03E-04 (0.0003) | **0.0009 (0.0003)** |
| Stellar | **0.1286 (0.0395)** | 4.49E-05 (0.0003) | 4.44E-04 (0.0004) | **0.0007 (0.0004)** |
| Vertcoin | **0.1270 (0.0465)** | 1.90E-05 (0.0004) | 1.74E-04 (0.0005) | 0.0002 (0.0004) |

In bold, significant estimates. Standard errors of intercept and slope as well as that of Fourier function parameters are given in parentheses.