Timing of RD Decisions and Output Subsidies in a Mixed Duopoly with Spillovers

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Timing of R&D Decisions and Output Subsidies in a Mixed Duopoly with Spillovers

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Abstract

This study considers a mixed duopoly with research spillovers and examines the interplay between firms' R&D decisions and government's output subsidies. We investigate and compare the timing of the game between ex-ante R&D and ex-post R&D decisions where the R&D decisions are chosen before the output subsidy is determined in the former case while the order is reversed in the latter case. We show that the equilibrium outcomes can be opposite between the two cases because both public and private firms have different objectives in choosing R&D investments, but the spillovers rate is a key factor that determines their incentives. In particular, we show that the output subsidy is smaller (larger) and the welfare is larger (smaller) under the ex-ante R&D decisions for a higher (lower) degree of spillovers rate. Finally, privatization increases the welfare in both cases only when spillovers rate is weak.

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Keywords: Mixed duopoly; Research spillovers, Ex-ante R&D; Ex-post R&D, Output subsidy.

1. Introduction

As innovations have intensified the market competition among the firms, policy makers have emphasized the importance of R&D activities and thus have enacted various policies to encourage them. Among the effective policy alternatives in the real world, governments have been continuously increasing its intervention of R&D initiatives through public institutions and organizations, aiming facilitation of innovations development. Accordingly, investigations of the welfare consequences of R&D activities with their associated policies are contemporary and practical.

There have been considerable empirical and theoretical works on the R&D incentives, and policy implications on innovation and competition are presently gaining importance in the literature. Further, a significant number of researches concluded that the presence of R&D spillovers is critical to assess the welfare effect of governmental intervention.[1] In particular, recent government policy is spreading

[1] See, for example, d'Aspremont and Jacquemin (1988), Katsoulacos and Ulph (1988), Kamien et al. (1992), Jaffe et al. (1993), Delbono and Denicolo (1993), Poyago-Theotoky (1995, 1998, 1999), Lee (1998), Beath, et al. (1998) and Keller (2002) for early discussion.

to mixed markets where the public firm competes with the private firms under the different ownership structures.

For instance, Zikos (2007) analyzed the policy mix of output and R&D subsidies in a mixed duopoly, and showed that the first-best can be obtained under full nationalization. Gil Molto, et al. (2011) studied the role of R&D subsidies and showed that privatization decrease welfare for any degree of spillovers. Kesavayuth and Zikos (2013) and Lee et al. (2017) compared the different effects between output and R&D subsidies as a policy instrument in mixed oligopolies.[2] In particular, Lee et al. (2017) showed that social welfare is higher under the output subsidy than that under the R&D subsidy regardless of the degree of privatization. Haruna and Goel (2017) emphasized that government might have to choose to subsidize output rather than R&D in the case that R&D subsidy is relatively more complicated than output subsidy. Even though valuable conclusions are spread out among different papers, there are still controversies on the effectiveness of R&D policy.

Moreover, all mentioned studies did not consider the strategic behaviors of the firms in the time-inconsistent setting where the government cannot commit its policy rule. Firms might choose different R&D investment decisions under the expectation of the introduction of subsidy policy. In fact, firm's decisions on R&D investment have different characteristics depending on its contractual scopes and time horizons under long-term or short-term contracts. The long-term contract is mostly irreversible and indivisible even though the government policy is changed. It is widely accepted to perceive the implementation of long-term contract as a profitable mechanism that helps to avoid the risks that involves high sunk investments. These investments mainly presents in a form of infrastructure investments (or capital intensive assets) which facilitate generation of innovations. On the other hand, short-term contract investment is relatively reversible, divisible and flexible according to the government policy. Mostly these investments are characterized by comparably lower time-spending and consequently counted as relatively low risk expenditures. For instance, divisible R&D expenditures dedicated to knowledge adjustment by offering trainings or flexible design by supporting the upgraded equipment and expert procedures.[3]

The purpose of this analysis is to incorporate different timings of R&D decisions made by the firms into the model along with implementing output subsidy policy by government and to observe firms competition in quantity production.[4] This approach complements with shedding some light to research works on the mixed oligopoly by analyzing the effect of government interventions and the timing of

[2] Recent development on R&D policy in mixed markets includes Lee and Tomaru (2017), Lee and Muminov (2017), Lee, et al. (2017), Haruna and Goel (2017) and Kesavayuth, et al (2018).

[3] Regarding different characteristics between divisible and indivisible R&D investments and time-consistency problem in an environmental policy, see, Petrakis and Xepapadeas (1999), Dijkstra (2002), Poyago-Theotoky and Teerasuwannajak (2002), Moner-Colonques and Rubio (2015), Leal et al. (2018) and Garci, et al. (2018).

[4] Recent analysis of the timing of privatization policies in mixed oligopolies includes Xu, et al. (2017), Lee et al. (2018), Lian et al. (2018), Xu and Lee (2018) and Escrihuela-Villar and Gutiérrez-Hita (2018).

R&D decisions with research spillovers. In the following analysis, we investigate the timing of two different firm's decisions on R&D investments toward the output subsidies, ex-ante R&D and ex-post R&D decisions. In the former case of an R&D-then-subsidy order, we consider the long-term investments which are sunk and irreversible at the time of government subsidization and thus the R&D decisions are indivisibly chosen before the output subsidy is determined. Thus, firms might preemptively select their R&D levels by expecting that the regulator sets the output subsidy after they have determined their R&D investments. In the latter case of a subsidy-then-R&D order, we consider the short-term investments which are flexibly adjustable at the time of government subsidization and thus the R&D decisions are divisibly chosen after the output subsidy is announced. Thus, firms might select their R&D levels after observing the output subsidy that the regulater sets with the commitment power.

In the presence of R&D spillovers, we consider a mixed duopoly and analyze the interplay between the strategic choice of R&D decisions toward the government's output subsidy policy. This paper compares the following two scenarios: (i) Ex-post R&D case – the output subsidy is determined **before** the two firms engage in cost reduction; (ii) Ex-ante R&D case – the output subsidy is determined **after** the two firms engage in cost reduction and highlights its policy implications in output subsidy policy. Under the heterogeneous objective functions between public and private firms in a mixed market, we show that the equilibrium outcomes can be opposite between the two cases. The results from the comparison are summarized as follows: the social welfare in scenario (i) is higher than that in scenario (ii) if and only if the degree of technological spillovers is weak. In addition to the comparison, this paper also considers whether privatization of the public firm improves social welfare or not. The social welfare under the mixed duopoly in scenario (i) is higher than that with privatization in scenario (i) for almost all the degrees of technological spillovers. Also, the social welfare under the mixed duopoly in scenario (ii) is higher than that with privatization in scenario (ii) if and only if the degree of technological spillovers is not weak. This finding is in contrast to Gil Molto, et al. (2011) who examined R&D subsidy and showed that privatization decrease welfare for any degree of spillovers.

The reminder of the paper is structured as follows. In section 2, we present a Cournot duopoly model with government's output subsidy, in which R&D and output competition between public and private firms occurs. In sections 3 and 4, we analyze firms' ex-ante and ex-post R&D decisions, respectively, depending on the timing of government's output subsidy policy. In section 5, we provide main findings and compare the outcomes in the two different timings. Finally, we conclude our analysis in section 6.

2. The Model

We consider a duopoly market where firms 0 and 1 produce homogeneous goods. Let the inverse demand function be $P(Q) = a - Q$ where P is the market price, $Q(= q_0 + q_1)$ is the market output, and q_0 and q_1 are the outputs of firm i $(= 0,1)$, respectively. The consumer surplus is denoted as $CS = Q^2/2$.

As assumed in a standard model of the cost function with R&D spillovers, we consider the following cost of output production and R&D investment, respectively:[5]

$$C(q_i, x_i) = \big(c - x_i - dx_j\big)q_i + q_i^2 \quad \text{and} \quad \Gamma(x_i) = x_i^2 \quad for \quad i \neq j = 0,1. \tag{1}$$

where $a > c > 0$ and x_i denotes the amount of R&D investment for firm i. The initial cost c is reduced by each firm's R&D investment, x_i, and rival's R&D investment, dx_j, where $d \in [0,1]$ denotes the R&D spillovers rate. It implies that R&D investment can reduce a firm's own cost by x_i and the rival firm's cost by dx_i per unit of output, depending on the R&D spillovers rate. Note that the R&D investment exhibits decreasing returns to scale where the firm has to spend x_i^2 to implement cost-reducing R&D, x_i.

We also assume that each firm receives an output subsidy, where s denotes the per-unit subsidy rate to output. Then, the profit function of the firm is as follows:

$$\pi_i = \big(a - q_0 - q_1\big)q_i - \big(c - x_i - dx_j\big)q_i - q_i^2 - x_i^2 + s\, q_i, \quad i \neq j = 0,1. \tag{2}$$

We define social welfare as the sum of consumer surplus and firms' profit minus total subsidy, which is given by:

$$W = CS + \pi_0 + \pi_1 - s(q_0 + q_1). \tag{3}$$

Note that the subsidies are financed from taxpayers in a lump-sum manner, so that they do not directly influence welfare.[6]

Finally, we consider a mixed duopoly where a private firm is owned by private investors who are seeking for profit-maximization while a public firm is owned by the welfare-maximizing government.

In the following sections, we shall consider two alternative cases of R&D decisions in the presence of R&D spillovers, each featuring a three-stage game between the government and firms. In particular, we examine the properties of different equilibria between ex-ante and ex-post R&D decisions. In the

[5] As analyzed in Gil-Moltó, et al. (2011), Kesavayuth and Zikos (2013) and Haruna and Goel (2017), we assume a quadratic production cost function, which is standard in mixed market literature, to rule out the uninteresting case of a public monopoly. Then, the production cost shows that a firm's R&D investment shifts its marginal cost function downwards, $\partial C/\partial q_i = c - x_i - dx_j + 2q_i$, but does not alter its slope.

[6] Note that there might be a welfare loss when the output subsidy requires public finance, which has distorting effects on the welfare through a tax on labor income. Recent analysis of excess burden of taxation in mixed markets is provided in Matsumura and Tomaru (2013, 2015) and Xu and Lee (2018).

former case, firms select their R&D levels simultaneously and independently, and then the government sets the output subsidy. In the latter case, the government sets the output subsidy then the firms, taking the subsidy rate as given, choose R&D investment simultaneously and independently. Finally, in the third stage, the public and the private firms compete in quantities in a Cournot fashion. We solve the subgame perfect Nash equilibrium of these games by backward induction.

3. Ex-post R&D decisions: a subsidy-then-R&D case

In this game, the government chooses the level of output subsidy before the firms undertake R&D investment. Then, each firm decides its R&D investment in the second stage, taking the output subsidy rate as given.

In the third stage, public firm chooses q_0 to maximize the social welfare, *W*, while the private firm chooses q_1 to maximize its own profit, π_1, simultaneously. The first-order conditions provide the following equilibrium output level of each firm and the total output, respectively:

$$q_0 = \frac{3(a-c) + (4-d)x_0 + (4d-1)x_1 - s}{11}. \tag{4}$$

$$q_1 = \frac{2(a-c) + (3d-1)x_0 + (3-d)x_1 + 3s}{11}. \tag{5}$$

$$Q = \frac{5(a-c) + (3+2d)x_0 + (2+3d)x_1 + 2s}{11}. \tag{6}$$

It shows that public firm's output decreases but private firm's output increases as the output subsidy increases, i.e., $\frac{\partial q_0}{\partial s} < 0$, $\frac{\partial q_1}{\partial s} > 0$. Thus, the output subsidy works for the firm's output decisions differently. However, the total output increases as the output subsidy or R&D investment increases.

In the second stage, both firms choose R&D investments to maximize their objective functions. Using the first-order conditions, we obtain the following reaction functions:

$$x_0(x_1) = \frac{(a-c)(31+28d) - (3-9d)s - (14-d(87-14d))x_1}{197 + 14d(2-3d)}. \tag{7}$$

$$x_1(x_0) = \frac{2(3-d)(2(a-c) + 3s - (1-3d)x_0)}{103 + 2(6-d)d}. \tag{8}$$

It shows that the R&D decisions are initially strategic substitutes and become strategic complements as the degree of the spillovers increases. That is, $\frac{\partial x_0}{\partial x_1} \gtrless 0$ if $d \gtrless 0.165$ and $\frac{\partial x_1}{\partial x_0} \gtrless 0$ if $d \gtrless 0.333$.

Solving the reaction functions provides the following equilibrium R&D investments as a function of the output subsidy *s*:

$$x_0^B = \frac{11(a-c)(25+2(18-d)d)+3(d(77-2(10-d)d)-17)s}{11(167+2(25-d)(1-d)d)}. \quad (9)$$

$$x_1^B = \frac{22(a-c)(9-d^2)+6(3-d)\big(18+d(2-3d)\big)s}{11(167+2(25-d)(1-d)d)}. \quad (10)$$

We employ superscript B to denote the equilibrium under the ex-post R&D decisions. It shows that the private firm's R&D investment is increasing in output subsidy while the public firm's R&D is dependent upon the degree of spillovers effect, i.e., $\frac{\partial x_1^B}{\partial s} > 0$ and $\frac{\partial x_0^B}{\partial s} \gtrless 0$ if $d \gtrless 0.235$. That is, if d is small (large), the public firm's R&D is decreasing (increasing) in output subsidy. Thus, the public firm increases its R&D investment when the spillovers rate is high where both firms can enjoy the R&D performances. The total R&D, $X^B = x_0^B + x_1^B$, is increasing in output subsidy.

Then, we have the following equilibrium outputs of the second stage:

$$q_0^B = \frac{11(a-c)(53+d(31-18d))-(215-d(163-d(11+20d-6d^2)))s}{11(167+2(25-d)(1-d)d)}. \quad (11)$$

$$q_1^B = \frac{11(a-c)(3+d)+3(18+(2-3d)d)s}{167+2(25-d)(1-d)d}. \quad (12)$$

It shows that output subsidy induces the private firm to enlarge its output while it restricts the output of public firm, i.e., $\frac{\partial q_0^B}{\partial s} < 0 < \frac{\partial q_1^B}{\partial s}$, irrespective of the spillovers rate. However, the decrease in the public firm's output will be outweighed by the increase in the private firm's output. Thus, the total industry output increases with output subsidy.

In the first stage, the government chooses the optimal output subsidy under the ex-post R&D decisions:

$$s^B = \frac{11(a-c)(6159+d(10255+2d(419-3d(773-d(15+2d(50-9d))))))}{E}. \quad (13)$$

where $E = 250025 + 136138d - 168501d^2 - 21604d^3 + 34566d^4 - 7496d^5 + 368d^6 + 240d^7 - 36d^8 > 0$.

Note that $s^B > 0$ and $\frac{\partial s^B}{\partial d} > 0$ for any $d \in [0,1]$. Thus, the optimal output subsidy is always positive and increases as d increases.

By substituting equation (13) into equations (9), (10), (11) and (12), we obtain the following equilibrium R&D investment and output levels produced by public and private firms, respectively:

$$x_0^B = \frac{2(a-c)(17774+d(34511+d(1624-d(10991-d(1463+d(233-6d(23-3d)))))))}{E}. \quad (14)$$

$$x_1^B = \frac{2(a-c)(3-d)(6483+d(5539-2d(772+d(655-d(77+42d-9d^2)))))}{E}. \quad (15)$$

$$q_0^B = \frac{(a-c)(71420 + d(61043 - d(42667 + 2d(8253 - d(5171 - 735d + 81d^2)))))}{E}. \quad (16)$$

$$q_1^B = \frac{11(a-c)(6483 + d(5539 - 2d(772 + d(655 - d(77 + 42d - 9d^2)))))}{E}. \quad (17)$$

The public firm's R&D investment and both firms' outputs increase as the spillovers increases, i.e., $\frac{\partial x_0^B}{\partial d} > 0$ and $\frac{\partial q_i^B}{\partial d} > 0$. But, we have $\frac{\partial x_1^B}{\partial d} \gtreqless 0$ if $d \gtreqless 0.079$. That is, if d is large (small), the private firm increases (decreases) R&D investment. Thus, the private firm increases its R&D investment when the spillovers rate is high where both firms can enjoy the R&D performances under the optimal subsidy. However, the total R&D increases as spillovers increase, i.e., $\frac{\partial(x_0^B + x_1^B)}{\partial d} > 0$.

Lemma 1. Under the ex-post R&D decisions, we have $x_0^B \gtreqless x_1^B$ and $q_0^B \lesseqgtr q_1^B$ if $d \gtreqless 0.067$.

It states that the public firm's R&D investment (output production) is higher (lower) than that of the private firm unless the spillovers rate is sufficiently low. Note that when $d = 0$, public firm's R&D investment is always lower than that of the private firm, which is reverse for output, i.e., $x_0^B < x_1^B$ and $q_0^B > q_1^B$, respectively. Thus, the public (private) firm is more (less) aggressive in R&D investment but less (more) aggressive in output with the spillovers rate is high while the results are reversed when the spillovers rate is low.

The resulting profits of the public and private firms are respectively:

$$\pi_0^B = \frac{\begin{array}{c}(a-c)^2(8675789676 + d(16004329315 - d(2709604131 + \\ d(15464622157 + d(499710091 - 2d(2895192378 - \\ d(127469999 + d(464202703 - 2d(44721677 + \\ d(6824434 - d(3333877 - 3d(198820 - 9d(2555 - 4d(46 - 3d)))))))))))))\end{array}}{E^2}. \quad (18)$$

$$\pi_1^B = \frac{2(a-c)^2(103 + 2(6-d)d)(6483 + d(5539 - 2d(772 + d(655 - d(77 + 42d - 9d^2)))))^2}{E^2}. \quad (19)$$

Then, we have that $\frac{\partial \pi_i^B}{\partial d} > 0$ and $\pi_0^B \lesseqgtr \pi_1^B$ if $d \gtreqless 0.067$. Thus, both firms' profits increase as the spillovers increase, but the profit of the private firm is larger than that of the public firm unless the spillovers rate is sufficiently low.

Finally, we obtain the social welfare under the ex-post R&D decisions:

$$W^B = \frac{(a-c)^2(142787 + d(121054 - d(58211 + 2d(14088 - d(5741 - 24d(26 - 3d))))))}{2E}. \quad (20)$$

Then, social welfare increases as *d* increases.

4. Ex-ante R&D decisions: an R&D-then-subsidy case

In this game, the government chooses the level of output subsidy after both public and private firms undertake R&D investment simultaneously. Then, private firm can strategically decide its R&D investment in the first stage, before the government decides the level of output subsidy. Note that the government and public firm have the same objective function.

The last stage in output choice is the same as in the previous ex-post case, where the equilibrium outputs of the firms are derived in (4) and (5).

In the second stage, the government chooses the welfare maximizing output subsidy rate, taking the firms' investment levels in R&D as given. The first order condition of this problem yields the following optimal output subsidy:

$$s^A = \frac{2(a-c) - (1-3d)x_0 + (3-d)x_1}{8}. \tag{21}$$

We also employ superscript *A* to denote the equilibrium under the ex-ante R&D decisions. It shows that output subsidy is increasing in the private firm's R&D investment, i.e., $\frac{\partial s^A}{\partial x_1} > 0$, while it depends on the public firm's R&D investment, i.e., $\frac{\partial s^A}{\partial x_0} \gtreqless 0$ if $d \gtreqless 0.333$. That is, if d is small (large), the output subsidy is decreasing (increasing) in the public firm's R&D. Thus, the private firm has a higher incentive to increase R&D while the public firm's decision depends on the spillovers rate.

In the first stage, both firms choose their R&D investment levels, taking into account how the government is going to respond in the second stage. The first-order conditions give the following reaction functions:

$$x_0(x_1) = \frac{2(a-c)(1+d) - (1-(6-d)d)x_1}{13 + (2-3d)d}. \tag{22}$$

$$x_1(x_0) = \frac{(3-d)(2(a-c) - (1-3d)x_0)}{23 + (6-d)d}. \tag{23}$$

It shows that the R&D decisions are strategic substitutes or strategic complements where the slope of reaction function of each firm is negative or positive, depending on the spillovers rate. That is, $\frac{\partial x_0^A}{\partial x_1^A} \gtreqless 0$ if $d \gtreqless 0.172$ and $\frac{\partial x_1^A}{\partial x_0^A} \gtreqless 0$ if $d \gtreqless 0.333$.

Solving the reaction functions yields the following equilibrium R&D investments:

$$x_0^A = \frac{(a-c)(5 + (12-d)d)}{(1+d)(37 - (18-d)d)}. \tag{24}$$

$$x_1^A = \frac{(a-c)(9-d^2)}{(1+d)(37 - (18-d)d)}. \tag{25}$$

The public firm's R&D investment increases as the spillovers increases, i.e., $\frac{\partial x_0^A}{\partial d} > 0$, while the private firm's R&D investment depends on the spillovers rate, i.e., $\frac{\partial x_1^A}{\partial d} \gtrless 0$ if $d \gtrless 0.892$. However, the total R&D increases as spillovers increase, i.e., $\frac{\partial (x_0^A + x_1^A)}{\partial d} > 0$.

Lemma 2. Under the ex-ante R&D decisions, we have $x_0^A \gtrless x_1^A$ if $d \gtrless 0.333$.

It states that the public firm's R&D investment is higher (lower) than that of the private firm if the R&D spillovers rate is high (low). For example, in the absence of the spillovers, i.e., $d = 0$, private firm's R&D investment is always higher than that of the public firm, i.e., $x_0^A < x_1^A$. That is, the private firm with a lower spillovers rate has a higher incentive to invest R&D strategically. However, the presence of the spillovers can significantly change this result. Thus, public firm is more (less) aggressive in R&D investment with a higher (lower) spillovers rate, which induces a larger amount of total R&D to the society.

By substituting (24) and (25) into (21), we obtain the following optimal output subsidy rate:

$$s^A = \frac{4(a-c)(3+d)}{(1+d)(37-(18-d)d)}. \tag{26}$$

Note that $s^A > 0$ for $d \in [0,1]$ and $\frac{\partial s^A}{\partial d} \gtrless 0$ if $d \gtrless 0.193$. Thus, the optimal output subsidy is always positive and increases (decreases) for a higher (lower) degree of spillovers rate.

From (4) and (5), the equilibrium outputs of both firms are obtained:

$$q_0^A = \frac{2(a-c)(5+3(2-d)d)}{(1+d)(37-(18-d)d)}. \tag{27}$$

$$q_1^A = \frac{4(a-c)(3+d)}{(1+d)(37-(18-d)d)}. \tag{28}$$

Lemma 3. Under the ex-ante R&D decisions, we have $q_0^A \gtrless q_1^A$ if $d \gtrless 0.333$.

It states that the public firm becomes more (less) aggressive in output production for a higher (lower) spillovers rate. Note that the threshold of R&D spillovers rate is the same with that of R&D investment in Lemma 1, but private firm's output is always higher than that of the public firm, i.e., $q_0^A < q_1^A$ if $d = 0$. Therefore, the public (private) firm invests more (less) R&D and produces more (less) output when the spillovers rate is high while the results are reversed when the spillovers rate is low.

The resulting profits of the firms are as follows:

$$\pi_0^A = \frac{(a-c)^2(3+5d)(65-d(7+d(33-7d)))}{(1+d)^2(37-(18-d)d)^2}. \tag{29}$$

$$\pi_1^A = \frac{(a-c)^2(3+d)^2(23+(6-d)d)}{(1+d)^2(37-(18-d)d)^2}. \tag{30}$$

Then, we have that $\frac{\partial \pi_0^A}{\partial d} \gtreqless 0$ if $d \gtreqless 0.701$ and $\frac{\partial \pi_1^A}{\partial d} \gtreqless 0$ if $d \gtreqless 0.052$. Thus, private (public) firms' profits increase (decrease) unless the spillovers rate is sufficiently low (high).

Finally, we obtain the social welfare under the ex-ante R&D decisions:

$$W^A = \frac{4(a-c)^2(95+d(47-d(67-13d)))}{(1+d)(37-(18-d)d)^2}. \tag{31}$$

Then, social welfare increases as *d* increases.[7]

5. Comparisons and Discussions

5-1. Comparisons

We provide comparisons between the ex-ante and ex-post R&D decisions, and explain how the strategic R&D choices of the private firm affect the output subsidies, equilibrium outcomes and social welfare.

Proposition 1. $s^B \lesseqgtr s^A$ if $d \lesseqgtr 0.155$.

It states that spillovers rate is a key factor that determined the size of output subsidy. Under the ex-ante R&D decisions, output subsidy is always higher than that of the ex-post R&D decision with lower spillovers. Thus, the government should encourage the R&D activities in a mixed duopoly under the ex-ante R&D decisions where the government cannot commit its output subsidy policy.

Then, we have the following comparisons with equilibrium outcomes in mixed duopolies:

Lemma 4. In a mixed duopoly, we have:

1. $x_0^B \gtreqless x_0^A$ if $d \lesseqgtr 0.164$; $x_1^B < x_1^A$ and $X^B < X^A$ for any $d \in [0,1]$
2. $q_0^B \gtreqless q_0^A$ if $d \lesseqgtr 0.193$; $q_1^B \lesseqgtr q_1^A$ if $d \lesseqgtr 0.319$; $\mathrm{Q}^B \lesseqgtr Q^A$ if $d \lesseqgtr 0.460$
3. $\pi_0^B \lesseqgtr \pi_0^A$ if $d \lesseqgtr 0.103$; $\pi_1^B \lesseqgtr \pi_1^A$ if $d \lesseqgtr 0.113$

[7] It represent that public firm has an incentive to full share its R&D performances with its rival firm in order to improve social welfare when it can endogeneize the spillovers rate. For the discussion on the endogeneous sharing of R&D performances, see Katsoulacos and Ulph (1988), Jaffe et al. (1993), Poyago-Theotoky (1999), Keller (2002) and Lee and Muminov (2017).

Due to the different subsidy rate, the spillovers rate will affect the equilibrium outcomes in each case differently. Lemma 4 represents that the output subsidy under the ex-ante R&D decisions is higher (lower) than that under the ex-post R&D decisions when the spillovers rate is low (high). And, higher output subsidy increases private firm's output but decreases public firm's output. The total output is still higher under the ex-ante R&D decisions when the spillovers rate is low. On the other hand, with a higher output subsidy, private firm's R&D investment increases but the public firm's R&D decreases. The total R&D is always higher under the ex-ante R&D decisions irrespective of the spillovers rate.

Proposition 2. $W^B \underset{>}{\overset{<}{=}} W^A$ if $d \underset{<}{\overset{>}{=}} 0.219$.

It should be emphasized under the ex-ante R&D decisions, social welfare is lower (higher) than that of the ex-post R&D decisions when the spillovers is small (large).

5-2. Discussion on Privatization

5-2-1. Ex-post R&D decisions: a subsidy-then-R&D case

When the government privatizes the public firm, both private firms undertake R&D investment after the government chooses the level of output subsidy. In the last output choice stage, private firms choose q_i to maximize their own profit, π_i, simultaneously.
Then the resulting equilibrium outcomes in a private duopoly are given as:

$$s^{BP} = \frac{(a-c)(217+2d(77-19d))}{788-4d(61-17d)}. \tag{32}$$

$$x_i^{BP} = \frac{15(a-c)(4-d)}{394-2d(61-17d)} \tag{33}$$

$$q_i^{BP} = \frac{225(a-c)}{788-4d(61-17d)} \tag{34}$$

We employ superscript *BP* to denote the equilibrium under the ex-post R&D decisions in a private duopoly model.

Proposition 3. Under the ex-post R&D decisions, $s^B \underset{<}{\overset{>}{=}} s^{BP}$ if $d \underset{<}{\overset{>}{=}} 0.151$.

This implies that the spillovers rate critically affects the optimal output subsidy under the ex-post R&D decisions when the government implements privatization policy. This result is contrast to the ex-ante R&D decisions where the privatization policy reduces the output subsidy rate. In particular, when the spillovers rate is not so small, the optimal output subsidy increases under the ex-post R&D

decisions when the government implements privatization policy. Furthermore, $s^B < s^{BP}$ in the absence of spillovers, i.e, $d = 0$.
Finally, the resulting social welfare is as follows:

$$W^{BP} = \frac{225(a-c)^2}{788 - 4d(61 - 17d)}. \tag{35}$$

Proposition 4. Under the ex-post R&D decisions, $W^B < W^{BP}$ if $0.008 < d < 0.059$, while $W^B > W^{BP}$ otherwise.

Thus, privatization policy decreases social welfare when the spillovers rate is strong under the ex-post R&D decisions. This finding is in contrast to Gil Molto, et al. (2011) who examined R&D subsidy and showed that privatization decrease welfare for any degree of spillovers.

5-2-2. Ex-ante R&D decisions: an R&D-then-subsidy case

When the government privatizes the public firm, both private firms strategically undertake R&D investment before the government chooses the level of output subsidy. In the last stage, both firms choose their outputs to maximize their profits in (2), respectively and simultaneously.
The equilibrium outcomes of ex-ante R&D decisions are given as:

$$s^{AP} = \frac{12(a-c)}{41 - (6-d)d} \tag{36}$$

$$x_i^{AP} = \frac{(a-c)(7-d)}{41 - (6-d)d}, \tag{37}$$

$$q_i^{AP} = \frac{12(a-c)}{41 - (6-d)d} \tag{38}$$

We employ superscript *AP* to denote the equilibrium under the ex-ante R&D decisions in a private duopoly model.

Proposition 5. Under the ex-ante R&D decisions, $s^A > s^{AP}$ for any $d \in [0,1]$.

This implies that irrespective of the spillovers rate, the optimal output subsidy decreases under the ex-ante R&D decisions when the government implements privatization policy. This is because the R&D decisions are sunk and thus the output subsidy is not effective on the R&D decisions but effective on output decisions after privatization. Then, the privatization induces lower total R&D investments and lower output subsidy under ex-ante R&D decision.
Finally, we obtain the resulting social welfare in a private duopoly:

$$W^{AP} = \frac{2(a-c)^2(239+(14-d)d)}{(41-(6-d)d)^2}. \tag{39}$$

Proposition 6. Under the ex-ante R&D decisions, $W^A \gtreqless W^{AP}$ if $d \gtreqless 0.238$.

Thus, privatization policy decreases social welfare for higher spillovers rate under the ex-ante R&D decisions. This result is also contrast to the ex-post R&D decisions where the privatization policy can increase the welfare even if the spillovers rate is not so weak. Nevertheless, under the ex-ante R&D decisions, social welfare in a private duopoly is always higher than that of mixed duopoly in the absence of spillovers, i.e., $W^A < W^{AP}$ for $d = 0$. This implies that "Privatization Neutrality Theorem" does not hold in the ex-ante R&D decision case.[8]

6. Conclusion

We considered a mixed duopoly and analyzed the interplay between the strategic choice of R&D decisions toward the government's output subsidy policy in the presence of R&D spillovers. In the case that the regulator announces an output subsidy, we examined the properties of different equilibria between ex-ante and ex-post R&D decisions. We also compared an R&D-then-subsidy case and a subsidy-then-R&D case and highlighted its policy implications in output subsidy policy.

Our findings show that the optimal output subsidy decreases irrespective of the spillovers rate under the ex-ante R&D decisions when the government implements privatization policy. In a contrast, the spillovers rate critically affects the optimal output subsidy under the ex-post R&D decisions when the government implements privatization policy. In particular, the optimal output subsidy increases for lower spillovers rate under the ex-post R&D decisions in a private oligopoly. We also found that privatization policy decreases (increase) social welfare for higher (lower) spillovers rate under the ex-ante R&D decisions. However, social welfare decreases when the spillovers rate is not so small under the ex-post R&D decisions when the government implements privatization policy. Finally, another interesting finding is that the social welfare is always larger (smaller) in the ex-ante R&D decisions case than in the ex-post one for higher (lower) degree of spillover rate in a mixed oligopoly models.

There still remain many limitations mainly because of its model-specific assumptions. The analysis of different market structure, such as Cournot, Bertrand, and Stackelberg competition with differentiated products should be examined for the robustness of the findings. Also, understanding the effects of endogenous entry and foreign penetration can be promising topics for future research.

[8] In the literature of mixed oligopolies, the government can achieve the same welfare consequences under the optimal output subsidy irrespective of privatization policy, which is called as "Privatization Neutrality Theorem." Recent analysis is provided in Matsumura and Okumura (2013, 2017), Lee and Tomaru (2017) and Lee, et al. (2017).

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