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Sasaki, Hiroaki

Graduate School of Economics, Kyoto University

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# North-South Uneven Development and Income Distribution under the Balance of Payments Constraint

Hiroaki Sasaki\*

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## Abstract

This study builds a North-South trade and uneven development model, and investigates the effects of changes in income distribution (the profit share) on economic growth rates of both countries. How a change in the profit share affects both countries' growth rates differs for the short-run equilibrium and the long-run equilibrium. For example, in the short-run equilibrium, an increase in the profit share of the North deteriorates the terms of trade of the South, and then, decreases the growth rate of the South. On the other hand, in the long-run equilibrium, an increase in the profit share of the North either increases or decreases the growth rate of the South through Thirlwall's law.

*Keywords:* North-South trade; Thirlwall's law, uneven development, income distribution

*JEL Classification:* F10; F43; O11; O41

## 1 Introduction

One of the important contributions of the post Keynesian growth theory is the theory of balance of payments constraint growth developed by Thirlwall (1979). This theory states that the growth rate of a country is determined by the equilibrium of trade balance, and is

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\*Graduate School of Economics, Kyoto University. E-mail: sasaki@econ.kyoto-u.ac.jp

named Thirlwall's law after the name of the founder.<sup>1)</sup> Thirlwall's law is given by

$$g_H = \frac{\varepsilon_{EX}}{\varepsilon_{IM}} g_W, \quad (1)$$

where  $g_H$  denotes the growth rate of a home country;  $g_W$ , the growth rate of the rest of the world;  $\varepsilon_{EX}$ , the income elasticity of export demand; and  $\varepsilon_{IM}$ , the income elasticity of import demand. That is, the growth rate of a country is determined by the income elasticity of export demand, the income elasticity of import demand, and the growth rate of the world. Thirlwall's law is derived from the condition that trade balance of a country is in equilibrium: the value of export demand is equal to the value of import demand.<sup>2)</sup>

Dutt (2002) gives an interpretation along North-South trade to Thirlwall's law, and shows that the ratio of the growth rate of the South (a developing country) to the growth rate of the North (a developed country) is equal to the ratio the income elasticity of the Northern import demand to the income elasticity of the Southern import demand.

$$\frac{g_S}{g_N} = \frac{\varepsilon_N}{\varepsilon_S} < 1 \implies g_S < g_N, \quad (2)$$

where  $g_S$  denotes the growth rate of the South;  $g_N$ , the growth rate of the North;  $\varepsilon_N$ , the income elasticity of Northern import demand; and  $\varepsilon_S$ , the income elasticity of Southern import demand. In reality, the income elasticity of Northern import demand is likely to be smaller than that of Southern import demand. Then, the growth rate of the South is less than the growth rate of the North, and hence, the income gap between the two countries will expand through time. Therefore, as long as the two countries are engaged in North-South trade, the income gap between the North and the South will increase.

There are many models that consider development of the North and the South under the North-South trade framework. For example, Findlay (1980) models a situation where the North is a Solow-type economy in which labor and capital are fully employed while the South is a Lewis-type economy in which surplus labor exists and hence, the real wage rate is fixed. Dutt (1996) models a situation where both the North and the South face fixed real wage rates.<sup>3)</sup>

Our model is based on the model of Dutt (2002). Dutt models a situation where the North is a Kalecki-type economy and the South is a Lewis-type economy, and investigates the relationship between the terms of trade and both countries' growth rates in both the

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1) For Thirlwall's law, see also McCombie and Thirlwall (1994), Thirlwall (2012), and Soukiazis and Cerqueira (2012).

2) For further developments of Thirlwall's law, see Blecker (1998) and Nakatani (2012).

3) For the relationship between North-South trade and development of both countries, see also Blecker (1996).

short-run and the long-run equilibria.<sup>4)</sup> In the short run, both countries' capital stocks are assumed to be constant, and the capacity utilization of the North and the terms of trade of the South are endogenous variables. In the long run, both countries' capital stocks evolve by capital investment, and hence, the terms of trade also changes. In the long-run equilibrium, Thirlwall's law holds.

Almost all studies of Thirlwall's law specify ad hoc export and import demand functions, and derive Thirlwall's law by using those functions. In contrast, Dutt (2002) derives Thirlwall's law by specifying production and demand structures of both countries. Hence, Dutt's (2002) model is micro-founded.<sup>5)</sup>

However, Dutt (2002) does not investigate how a change in income distribution affects the growth rates of both countries. A change in labor-management negotiations affects income distribution between workers and capitalists, and the change in income distribution affects economic growth. In two-country models, a change in income distribution of one country can affect the other country through a change in the terms of trade. Therefore, it is important to investigate the effect of a change in income distribution on both countries' growth rates.

For this purpose, we incorporate a Marglin and Bhaduri's (1990) investment function into Dutt's (2002) model, and analyze how changes in income distributions of both countries affect the short-run and long-run equilibrium values. The investment function used in Dutt (2002) is called a Kalecki-type investment function, which is increasing in the capacity utilization rate. In contrast, the Marglin-Bhaduri investment function is increasing in both the capacity utilization rate and the profit share, and widely used in theoretical and empirical analyses of Kaleckian models.<sup>6)</sup> With the Kalecki-type investment function, we usually obtain the result of wage-led growth such that an increase in the profit share (i.e., a decrease in the wage share) decreases the growth rate of the economy. On the other hand, with the Marglin-Bhaduri type investment function, we obtain not only wage-led growth but also profit-led growth such that an increase in the profit share increases the growth rate of the economy. Therefore, we can examine broader possibilities.

The remainder of this paper is organized as follows. Section 2 presents our model.

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4) For a study that models Thirlwall's law under North-South trade, see also Vera (2006). Sasaki (2009) presents a dynamic version of the Ricardian trade model with a continuum of goods that assumes that the North is in full employment while the South faces unemployment. Then, he derives a multi-goods version of Thirlwall's law.

5) For Thirlwall's law, there is a criticism as to why not factor endowments but trade balance equilibrium constrains the long-run growth rate. For this, see Pugno (1998) who presents a model in which Thirlwall's law holds in the long-run equilibrium because of adjustments of some variables.

6) For the basic framework of the Kaleckian model, see Rowthorn (1981). Sasaki (2013) presents a Kaleckian model that captures cyclical fluctuations of the capacity utilization rate, the profit share, and the employment rate.

Section 3 derives the short-run equilibrium, investigates the stability of the short-run equilibrium, and conducts a comparative static analysis of the short-run equilibrium. Section 4 derives the long-run equilibrium, investigate the stability of the long-run equilibrium, and conducts a comparative static analysis of the long-run equilibrium. Section 5 concludes the paper.

## 2 Model

Suppose the world economy that is composed of the North and the South. The North produces the investment-consumption goods, which are used for investment and consumption in both the North and the South. The South produces the investment-consumption goods, which is used for consumption in the North while used for investment and consumption in the South.

The North is a Kalecki-type economy. The principle of effective demand prevails and hence, outputs are determined by effective demand. Capital stocks are not fully utilized. The goods market is in imperfect competition and hence, the price of the Northern goods is determined by mark-up pricing. With mark-up pricing, the profit share, that is, the ratio of total profit income to national income is decided by the mark-up rate. The goods market clears through the adjustment of the capacity utilization rate. The nominal wage rate is assumed to be determined by labor-management negotiations and exogenously given. With the mark-up pricing, the real wage rate that firms in the North face is constant and actual employment is determined by labor demand.

The South is a Lewis-type economy. Say's law prevails and hence, outputs are determined by supply. The goods market of the South is competitive and clears through the adjustment of the price. Capital stocks are fully utilized. In the South, surplus labors exist and hence, the real wage rate is fixed at a certain level. With the fixed real wage rate, actual employment is determined by labor demand.

The Northern goods are produced by employment and capital stock. The production function takes the following Leontief form.

$$Y_N = \min\{E_N/b_N, u_N K_N\}, \quad b_N > 0, \quad (3)$$

where  $Y_N$  denotes the output of the Northern goods;  $E_N$ , employment;  $K_N$ , capital stock;  $b_N$ , the labor input coefficient; and  $u_N$ , the capacity utilization rate.<sup>7)</sup>

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7) Let the potential output be  $Y_N^F$ . Then, the capacity utilization rate is given by  $u_N = Y_N/Y_N^F$ . Suppose that the ratio of capital stock to the potential output  $K_N/Y_N^F$  is constant. Then, the output-capital ratio  $Y_N/K_N$  will be a proxy variable of the capacity utilization rate.

The Southern goods are produced by employment and capital stock. The production function takes the following Leontief form.

$$Y_S = \min\{E_S/b_S, K_S/a_S\}, \quad b_S > 0, a_S > 0, \quad (4)$$

where  $Y_S$  denotes the output of the Southern goods;  $E_S$ , employment;  $K_S$ , capital stock;  $b_S$ , the labor input coefficient; and  $a_S$ , the capital input coefficient.

The price of the Northern goods is determined by unit labor costs multiplied by the mark-up rate.

$$P_N = (1 + z)W_N b_N, \quad 0 < z < 1, \quad (5)$$

where  $P_N$  denotes the price;  $z$ , the mark-up rate; and  $W_N$ , the nominal wage rate. The mark-up rate and the nominal wage rate are exogenously given.

Let the profit share of the North be  $\pi_N$ . Then, by using equation (5), we obtain the following relation.

$$\begin{aligned} \pi_N &= \frac{P_N Y_N - W_N E_N}{P_N Y_N} = 1 - \frac{W_N E_N}{P_N Y_N} \\ &= \frac{z}{1 + z}. \end{aligned} \quad (6)$$

Accordingly, the profit share has a one-to-one relationship with the mark-up rate and is an increasing function of the mark-up rate. This means that the profit share is decided by the mark-up rate. Kalecki himself states that it is the monopoly power that determines the markup rate. Later, many Kaleckians interpret it broadly, and argue that not only monopoly power but also negotiations between workers and firms affect the mark-up rate.

We assume that the real wage rate of the South is constant and exogenously given.

$$\frac{W_S}{P_S} = V_S. \quad (7)$$

With equation (7), the profit share of the South is given by

$$\pi_S = \frac{P_S Y_S - W_S E_S}{P_S Y_S} = 1 - b_S V_S. \quad (8)$$

Since the labor input coefficient and the real wage rate are constant, the profit share is also constant. When  $b_S$  declines by technical progress or  $V_S$  declines for some reason, the profit share of the South increases.

Workers in the North spend all wage income on consumption and therefore do not save.

Capitalists in the North spend a fraction  $s_N$  of profit income on saving and the rest  $1 - s_N$  on consumption. Both workers and capitalists allocate a fraction  $\alpha$  of consumption expenditure to purchase of the Southern goods and the rest  $1 - \alpha$  to purchase of the Northern goods. The fraction  $\alpha$  is assumed to be

$$\alpha = \alpha_0 Y_N^{\varepsilon_N - 1} P^{1 - \mu_N}, \quad P = \frac{P_S}{P_N}, \quad \alpha_0 > 0, \quad \varepsilon_N > 0, \quad \mu_N > 0, \quad (9)$$

where  $\alpha_0$  denotes a positive constant;  $P = P_S/P_N$ , the terms of trade of the South;  $\varepsilon_N$ , the income elasticity of Northern import demand; and  $\mu_N$ , the price elasticity of Northern import demand. According to Dutt (2002), we assume that  $\varepsilon_N < 1$ .

Workers in the South spend all wage income on purchase of the Southern goods. Capitalists in the South spend a fraction  $s_S$  of profit income on saving, a fraction  $\beta$  of the rest  $1 - s_S$  of profit income on purchase of the Northern goods, and the rest  $1 - \beta$  on purchase of the Southern goods. The fraction  $\beta$  is assumed to be

$$\beta = \beta_0 (\pi_S Y_S)^{\varepsilon_S - 1} P^{1 - \mu_S}, \quad \beta_0 > 0, \quad \varepsilon_S > 0, \quad \mu_S > 0, \quad (10)$$

where  $\beta_0$  denotes a positive constant;  $\varepsilon_S$ , the income elasticity of Southern import demand; and  $\mu_S$ , the price elasticity of Southern import demand. According to Dutt (2002), we assume that  $\varepsilon_S > 1$ .

Following Marglin and Bhaduri (1990), we assume that the capital investment function in the North is an increasing function of the capacity utilization rate and the profit share.

$$g_N \equiv \frac{I_N}{K_N} = \gamma_0 + \gamma_1 u_N + \gamma_2 \pi_N, \quad \gamma_0 > 0, \quad \gamma_1 > 0, \quad \gamma_2 > 0, \quad (11)$$

where  $I_N$ , investment; and  $\gamma_i$  ( $i = 0, 1, 2$ ), a positive constant. Dutt (2002) assumes that the investment function is an increasing function of the capacity utilization rate, which corresponds to the case of  $\gamma_2 = 0$  in equation (11).

The value of Northern import from the South is equal to the value of Southern export to the North, which is given by

$$P_S X_S = \alpha (1 - s_N \pi_N) P_N Y_N. \quad (12)$$

From equation (12), the volume of Southern export is given by

$$X_S = \alpha_0 (1 - s_N \pi_N) P^{-\mu_N} Y_N^{\varepsilon_N}. \quad (13)$$

The value of Southern import from the North is equal to the value of Northern export to

the South, which is given by

$$P_N X_N = \beta \pi_S P_S Y_S. \quad (14)$$

From equation (14), the volume of Northern export is given by

$$X_N = \beta_0 \pi_S^{\varepsilon_S} P^{\mu_S} Y_S^{\varepsilon_S}. \quad (15)$$

The excess demand for the Southern goods,  $ED_S$ , is given by

$$ED_S = C_{SS} + I_{SS} + X_S - Y_S, \quad (16)$$

where  $C_{SS}$  denotes Southern consumption demand for the Southern goods; and  $I_{SS}$ , Southern investment demand for the Southern goods. Since  $Y_S = C_{SS} + I_{SS} + M_S$  and  $M_S = X_N/P$  hold, equation (16) can be rewritten as

$$ED_S = X_S - (1/P)X_N. \quad (17)$$

The excess demand for the Northern goods,  $ED_N$ , is given by

$$ED_N = C_{NN} + I_N + X_N - Y_N, \quad (18)$$

where  $C_{NN}$  denotes Northern consumption demand for the Northern goods. Since  $Y_N = C_{NN} + M_N + S_N$  and  $M_N = PX_S$  hold, equation (18) can be rewritten as

$$ED_N = I_N - S_N + X_N - PX_S. \quad (19)$$

### 3 Short-run equilibrium

We define a short run as a situation where both countries' capital stocks  $K_N$  and  $K_S$  are constant. The short-run equilibrium is achieved when  $ED_S = 0$  and  $ED_N = 0$ . From our assumption, the saving of the North is given by

$$S_N = s_N \pi_N Y_N. \quad (20)$$



Therefore, the terms of trade that establishes  $ED_S = 0$  and the capacity utilization rate that establishes  $ED_N = 0$  are given by

$$P^* = \left[ \frac{\alpha_0(1 - s_N\pi_N)}{\beta_0\pi_S^{\varepsilon_S}} (u_N^*K_N)^{\varepsilon_N} \left(\frac{K_S}{a_S}\right)^{-\varepsilon_S} \right]^{\frac{1}{\mu_N + \mu_S - 1}}, \quad (21)$$

$$u_N^* = \frac{\gamma_0 + \gamma_2\pi_N}{s_N\pi_N - \gamma_1}. \quad (22)$$

For the capacity utilization rate to be positive, we need  $s_N\pi_N > \gamma_1$ . This condition means that the response of saving to capacity utilization rate exceeds the response of investment to capacity utilization rate. In the literature of Kaleckian models, this condition is often called the Keynesian stability condition because as will be shown below, it is a condition for the goods market stability. In the following analysis, we assume the Keynesian stability condition.

**Assumption 1.** *The Keynesian stability condition  $s_N\pi_N > \gamma_1$  holds.*

From our assumption, the saving of the South is given by

$$S_S = \frac{s_S\pi_S K_S}{a_S}. \quad (23)$$

Since investment of the South is composed of both the Northern and the Southern goods, we assume that investment of the South is given by

$$I_S = P^\xi S_S, \quad 0 < \xi < 1, \quad (24)$$

where  $\xi$  denotes a parameter that captures the effect of a change in the terms of trade on investment of the South. Substituting equation (23) into equation (24) and dividing the resultant expression by  $K_S$ , we obtain the growth rate of the South.

$$g_S = \frac{s_S\pi_S}{a_S} P^\xi \implies g_S^* = \frac{s_S\pi_S}{a_S} (P^*)^\xi. \quad (25)$$

Therefore, the growth rate of the South is an increasing function of the terms of trade.

Substituting  $u_N^*$  into equation (20), we obtain the growth rate of the North.

$$g_N^* = \frac{s_N(\gamma_0 + \gamma_2\pi_N)\pi_N}{s_N\pi_N - \gamma_1}. \quad (26)$$

We examine the local stability of the short-run equilibrium. Since the excess demand for the Southern goods is adjusted by  $P$  while the excess demand for the Northern goods are

adjusted by  $u_N$ , we assume the following adjustment processes.

$$\dot{P} = \psi \left( X_S - \frac{X_N}{P} \right), \quad \psi > 0, \quad (27)$$

$$\dot{u}_N = \phi \left( \frac{I_N}{K_N} - \frac{S_N}{K_N} + \frac{X_N}{K_N} - \frac{PX_S}{K_N} \right), \quad \phi > 0, \quad (28)$$

where  $\psi$  and  $\phi$  are adjustment parameters. Note that capital stock of the North  $K_N$  is fixed in the short run.

Substituting equations (11), (13), (15), and (20) into equations (27) and (28), we obtain

$$\dot{P} = \psi \left[ \alpha_0(1 - s_N\pi_N)P^{-\mu_N}(u_N K_N)^{\varepsilon_N} - \beta_0\pi_S^{\varepsilon_S} P^{\mu_S-1} \left( \frac{K_S}{a_S} \right)^{\varepsilon_S} \right], \quad (29)$$

$$\dot{u}_N = \phi \left( \gamma_0 + \gamma_1 u_N + \gamma_2 \pi_N - s_N \pi_N u_N - \frac{P \cdot \dot{P}}{\psi K_N} \right). \quad (30)$$

We define the Jacobian matrix corresponding to the above dynamical system as  $\mathbf{J}$ . Each element of  $\mathbf{J}$  is given by

$$J_{11} = \frac{\partial \dot{P}}{\partial P} = -\psi \alpha_0 (1 - s_N \pi_N) (\mu_N + \mu_S - 1) P^{-\mu_N-1} (u_N K_N)^{\varepsilon_N}, \quad (31)$$

$$J_{12} = \frac{\partial \dot{P}}{\partial u_N} = \psi (\varepsilon_N \beta_0 \pi_S^{\varepsilon_S} P^{-\mu_N} u_N^{\varepsilon_N-1} K_N^{\varepsilon_N}) > 0, \quad (32)$$

$$J_{21} = \frac{\partial \dot{u}_N}{\partial P} = -\frac{\phi P}{\psi K_N} J_{11}, \quad (33)$$

$$J_{22} = \frac{\partial \dot{u}_N}{\partial u_N} = \phi \left( \gamma_1 - s_N \pi_N - \frac{P}{\psi K_N} J_{12} \right) < 0. \quad (34)$$

All elements of  $\mathbf{J}$  are evaluated at  $(u_N^*, P^*)$ . With the Keynesian stability condition  $s_N \pi_N > \gamma_1$ , we have  $J_{22} < 0$ .

The necessary and sufficient conditions for the local stability of the short-run equilibrium are that the determinant of  $\mathbf{J}$  is positive and the sum of diagonal elements of  $\mathbf{J}$  are negative, that is,  $\det \mathbf{J} > 0$  and  $\text{tr} \mathbf{J} < 0$ . These are computed as follows:

$$\det \mathbf{J} = \phi J_{11} (\gamma_1 - s_N \pi_N), \quad (35)$$

$$\text{tr} \mathbf{J} = J_{11} + J_{22}. \quad (36)$$

If  $J_{11} < 0$ , both  $\det \mathbf{J} > 0$  and  $\text{tr} \mathbf{J} < 0$  hold because  $J_{22} < 0$  from equation (34) and  $s_N \pi_N > \gamma_1$ . The necessary and sufficient condition for  $J_{11} < 0$  is given by  $\mu_N + \mu_S - 1 > 0$ , which is called the Marshall-Lerner condition. Therefore, we obtain the following proposition.

**Proposition 1.** *The necessary and sufficient condition for the stability of the short-run equilibrium is equivalent to the Marshall-Lerner condition.*

Figure 1 shows the phase diagram for the short run.

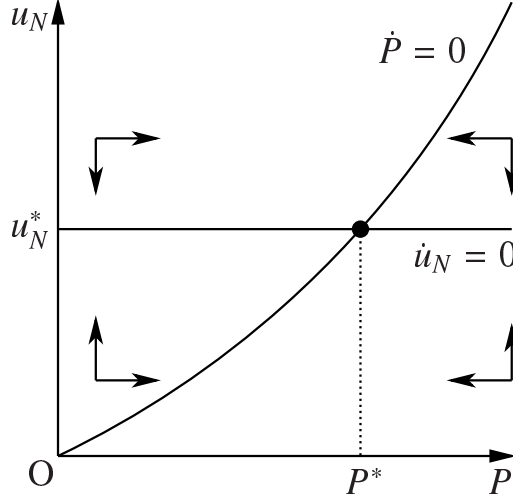


Figure 1: Phase diagram of the capacity utilization rate and the terms of trade in the short run

We investigate the effect of an increase in the profit share of the North on the growth rate of the North.

$$\frac{\partial g_N^*}{\partial \pi_N} = \frac{s_N f(\pi_N)}{(s_N \pi_N - \gamma_1)^2}, \quad (37)$$

where

$$f(\pi_N) = s_N \gamma_2 \left( \pi_N - \frac{\gamma_1}{s_N} \right)^2 - \gamma_0 \gamma_1 - \frac{\gamma_2 \gamma_1^2}{s_N}, \quad (38)$$

$$f(0) = -\gamma_0 \gamma_1 < 0, \quad (39)$$

$$f(1) = s_N \gamma_2 - 2\gamma_1 \gamma_2 - \gamma_0 \gamma_1. \quad (40)$$

With the Keynesian stability condition, we should consider the domain of the profit share  $\pi_N \in (\gamma_1/s_N, 1)$ . The sign of  $\partial g_N^*/\partial \pi_N$  is equal to the sign of  $f(\pi_N)$ . Hence, we should investigate the sign of  $f(\pi_N)$ .

When  $f(1) < 0$ , we always have  $f(\pi_N) < 0$  for  $\pi_N \in (\gamma_1/s_N, 1)$ . Therefore, when  $f(1) < 0$ , the economy exhibits a wage-led growth.

When  $f(1) > 0$ , we define  $\pi_N^c$  as the profit share such that  $f(\pi_N^c) = 0$ . Then, we have

$f(\pi_N) < 0$  for  $\pi_N \in (\gamma_1/s_N, \pi_N^c)$  while  $f(\pi_N) > 0$  for  $\pi_N \in (\pi_N^c, 1)$ . Therefore, the economy exhibits a wage-led growth for  $\pi_N \in (\gamma_1/s_N, \pi_N^c)$  while a profit-led growth for  $\pi_N \in (\pi_N^c, 1)$ .

**Proposition 2.** *When  $s_N\gamma_2 - 2\gamma_1\gamma_2 - \gamma_0\gamma_1 < 0$ , the North exhibits a wage-led growth. When  $s_N\gamma_2 - 2\gamma_1\gamma_2 - \gamma_0\gamma_1 > 0$ , the North exhibits a wage-led growth for  $\pi_N \in (\gamma_1/s_N, \pi_N^c)$  while exhibits a profit-led growth for  $\pi_N \in (\pi_N^c, 1)$ .*

When using the Kalecki-type investment function, that is  $\gamma_2 = 0$  in our model, we always have  $f(\pi_N) < 0$ , that is, only a wage-led growth regime is obtained.

Results for comparative static analysis of the short-run equilibrium are as follows.

First, the effects of an increase in the profit share of the North on the capacity utilization rate, the growth rate of the North, the terms of trade, and the growth rate of the South are given by

$$\pi_N \uparrow \implies u_N^* \downarrow, g_N^* \uparrow \text{ or } \downarrow, P^* \downarrow, g_S^* \downarrow \quad (41)$$

An increase in the profit share of the North deteriorates the terms of trade of the South and hence, decreases the growth rate of the South.

Second, the effects of an increase in the profit share of the South on the capacity utilization rate, the growth rate of the North, the terms of trade, and the growth rate of the South are given by

$$\pi_S \uparrow \implies u_N^* \downarrow, g_N^* \downarrow, P^* \downarrow, g_S^* \uparrow \text{ or } \downarrow \quad (42)$$

An increase in the profit share of the South has two opposite effects on the growth rate of the South. First, an increase in the profit share of the South increases the saving of Southern capitalists, and accordingly, has a positive effect on the growth rate of the South. In contrast, an increase in the profit share of the South deteriorates the terms of trade, and hence, has a negative effect on the growth rate of the South. Depending on which effect dominates, the effect of an increase in the profit share of the South on the growth rate of the South differs.

$$\frac{\partial \log g_S^*}{\partial \log \pi_S} = \frac{\mu_N + \mu_S - 1 - \varepsilon_S}{\mu_N + \mu_S - 1} \gtrless 0. \quad (43)$$

When the Marshall-Lerner condition holds, the denominator of the right-hand side is positive. However, the sign of the numerator is indeterminate. The South is a supply-constrained economy. When an economy is supply-constrained, an increase in the profit share increases the saving and hence, increases the growth rate of the economy if it is under autarky. On the other hand, if it is engaged in international trade, it can be a wage-led growth economy because of the terms of trade effect even if it is a supply-constrained economy.

In summary, for the effects of changes in income distribution, we obtain the following two propositions:

**Proposition 3.** *In the short-run equilibrium, an increase in the profit share of the North either increases or decreases the growth rate of the North while decreases the growth rate of the South.*

**Proposition 4.** *In the short-run equilibrium, an increase in the profit share of the South does not affect the growth rate of the North while either increases or decreases the growth rate of the South.*

## 4 Long-run equilibrium

We define a long run as a situation where the short-run equilibrium always holds and capital accumulation in each country proceeds because of capital investment. That is,  $K_N$  and  $K_S$  evolve in the long run. In this case, the short-run equilibrium value of the terms of trade  $P^*$  also evolves. We define a long-run equilibrium as a situation where  $\dot{P}^* = 0$ .<sup>8)</sup>

We examine the dynamics of the terms of trade. Differentiating  $P^*$  with respect to time, we obtain

$$\frac{\dot{P}^*}{P^*} = \frac{1}{\mu_N + \mu_S - 1} (\varepsilon_N g_N - \varepsilon_S g_S). \quad (44)$$

Equation (44) can be rewritten as

$$\dot{P}^* = \frac{1}{\mu_N + \mu_S - 1} \left[ \varepsilon_N g_N - \frac{\varepsilon_S s_S \pi_S (P^*)^\xi}{a_S} \right] P^*. \quad (45)$$

The long-run equilibrium is defined by  $\dot{P}^* = 0$ , and the long-run equilibrium terms of trade is given by

$$P^{**} = \left[ \frac{\varepsilon_N}{\varepsilon_S} \cdot \frac{a_S}{s_S \pi_S} \cdot \frac{s_N (\gamma_0 + \gamma_2 \pi_N) \pi_N}{s_N \pi_N - \gamma_1} \right]^{\frac{1}{\xi}}. \quad (46)$$

Since the short-run equilibrium growth rate of the North is independent of the terms of trade, the long-run equilibrium growth rate of the North is equal to the short-run growth rate of the North.

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8) In many two-country growth models, as Dutt (2002) also states, the long-run equilibrium is assumed to be a situation where both countries grow at a same rate and the capital stock ratio  $K_N/K_S$  is constant. However, in our and Dutt models, the terms of trade continues to decrease when both countries grow at a same rate, and this situation cannot be the long-run equilibrium.

The long-run equilibrium growth rate of the South is given by

$$g_S^{**} = \frac{\varepsilon_N}{\varepsilon_S} g_N^{**} \implies \frac{g_S^{**}}{g_N^{**}} = \frac{\varepsilon_N}{\varepsilon_S} < 1. \quad (47)$$

This corresponds to Thirlwall's law. That is, if the income elasticity of Southern import demand is larger than the income elasticity of Northern import demand, then the growth rate of the South is smaller than that of the North: the income gap between the two countries will expand through time.

We investigate whether the long-run equilibrium is locally stable. The necessary and sufficient condition for the stability of the long-run equilibrium is given by  $d\dot{P}^*/P^* < 0$  in the neighborhood of the long-run equilibrium. When we actually compute the derivative, we obtain

$$\left. \frac{d\dot{P}^*}{dP^*} \right|_{P=P^{**}} = -\frac{\xi \varepsilon_N g_N}{\mu_N + \mu_S - 1} < 0. \quad (48)$$

Therefore, we obtain the following proposition.

**Proposition 5.** *If the Marshall-Lerner condition is satisfied, then the long-run equilibrium is locally stable.*

Figure 2 shows the phase diagram for the long run.

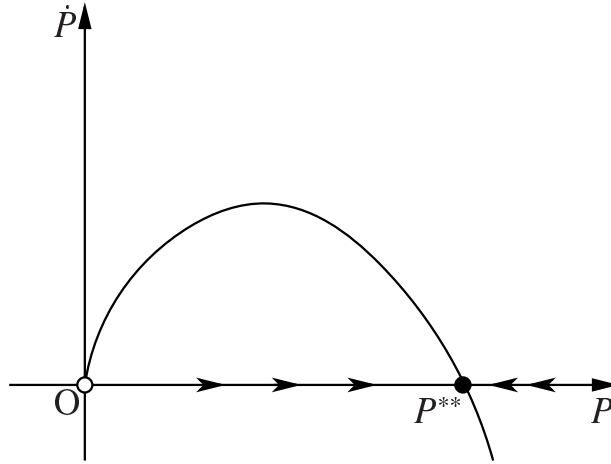


Figure 2: Phase diagram of the terms of trade in the long run

Results of comparative static analysis of the long-run equilibrium are as follows.<sup>9)</sup>

9) In the Appendix, we investigate the effect of an increase in the profit share of each country on the real wage rate of the North in the short-run and the long-run equilibria.

First, the effects of an increase in the profit share of the North on the capacity utilization rate, the growth rate of the North, the terms of trade, and the growth rate of the South are given by

$$\pi_N \uparrow \implies u_N^{**} \downarrow, g_N^{**} \uparrow \text{ or } \downarrow, P^{**} \uparrow \text{ or } \downarrow, g_S^{**} \uparrow \text{ or } \downarrow \quad (49)$$

Since Thirlwall's law holds in the long-run equilibrium, an increase in the profit share of the North has a similar effect on the growth rates of both countries.

Second, the effects of an increase in the profit share of the South on the capacity utilization rate, the growth rate of the North, the terms of trade, and the growth rate of the South are given by

$$\pi_S \uparrow \implies u_N^{**} -, g_N^{**} -, P^{**} \downarrow, g_S^{**} - \quad (50)$$

An increase in the profit share of the South deteriorates the terms of trade but does not affect the growth rate of the South.

**Proposition 6.** *In the long-run equilibrium, an increase in the profit share of the North either increases or decreases the growth rate of the North. When the growth rate of the North increases, the growth rate of the South increases. In contrast, when the growth rate of the North decreases, the growth rate of the South decreases.*

**Proposition 7.** *In the long-run equilibrium, an increase in the profit share of the South does not affect the growth rate of the North and that of the South.*

## 5 Concluding remarks

This study extends the Dutt's (2002) model that describes uneven development between the North and the South under North-South trade, and investigates changes in income distributions on both countries' economic growth rates. In our analysis, to capture both a wage-led growth regime and a profit-led growth regime, we use a Marglin-Bhaduri-type investment function instead of a Kalecki-type investment function that is used in Dutt (2002).

Our analysis shows that how a change in the profit share affects both countries' growth rates differs for the short-run equilibrium and the long-run equilibrium. For example, in the short-run equilibrium, an increase in the profit share of the North deteriorates the terms of trade of the South, and then, decreases the growth rate of the South. On the other hand, in the long-run equilibrium, an increase in the profit share of the North, through Thirlwall's law, either increases or decreases the growth rate of the North. These results suggest that an

income redistribution policy in one country affects the other country's growth rate through international trade.

Income distributions of the North and the South in our model are exogenously given. However, in reality, income distribution is endogenously determined and affected by the economic growth rate. Therefore, to endogenize income distribution is an important issue and will be left for future research,

## Appendix: real wage rate of the North

We investigate the effect of an increase in the profit share of the North on the real wage rate of the North. As in the text, we assume that workers in the North consume both the Northern and the Southern goods. Hence, the real wage rate that workers in the North face  $V_N$  is assumed to be given by

$$\begin{aligned} V_N &= \frac{W_N}{P_S^\alpha P_N^{1-\alpha}} = \frac{1}{P^\alpha(1+z)b_N} \\ &= \frac{1-\pi_N}{P^\alpha b_N}. \end{aligned} \quad (51)$$

The short-run equilibrium value of the real wage rate of the North is given by

$$V_N^* = \frac{1-\pi_N}{b_N} \left[ \frac{\alpha_0(1-s_N\pi_N)}{\beta_0\pi_S^{\varepsilon_S}} (u_N^* K_N)^{\varepsilon_N} \left( \frac{K_S}{a_S} \right)^{-\varepsilon_S} \right]^{\frac{\alpha}{\mu_N+\mu_S-1}}. \quad (52)$$

To investigate the effect of an increase in the profit share of the North on the real wage rate of the North, we take the logarithm of  $V_N^*$  as follows:

$$\log V_N^* = \log(1-\pi_N) - \frac{\alpha_0(u_N^* K_N)^{\varepsilon_N-1} (P^*)^{1-\mu_N}}{\mu_N + \mu_S - 1} [\log(1-s_N\pi_N) + \varepsilon_N \log u_N^*] + \dots \quad (53)$$

Since  $\partial V_N^*/\partial \pi_N = V_N^* \partial \log V_N^*/\partial \pi_N$ , we examine the sign of  $\partial \log V_N^*/\partial \pi_N$ . If  $\alpha$  is constant, then we obtain

$$\frac{\partial \log V_N^*}{\partial \pi_N} = -\frac{1}{1-\pi_N} + \frac{\alpha}{\mu_N + \mu_S - 1} \left( \frac{s_N}{1-s_N\pi_N} - \frac{\varepsilon_N \gamma_2}{\gamma_0 + \gamma_2 \pi_N} + \frac{\varepsilon_N s_N}{s_N \pi_N - \gamma_1} \right) \geq 0. \quad (54)$$

Therefore, an increase in the profit share of the North either increases or decreases the real wage rate of the North in the short-run equilibrium. Actually, let  $\pi_N = 0.3$ ,  $\alpha = 0.5$ ,  $\mu_N = 0.9$ ,  $\mu_S = 2.5$ ,  $s_N = 0.5$ ,  $\varepsilon_N = 0.7$ ,  $\gamma_0 = 0.04$ ,  $\gamma_1 = 0.04$ , and  $\gamma_2 = 0.09$ . Then, we obtain  $u_N^* = 0.61$  and  $g_N^* = 0.09$ . In this case, the economy exhibits a profit-led growth



because a slight increase in  $\pi_N$  increases  $g_N^*$ . We obtain  $\partial \log V_N^*/\partial \pi_N = -0.72 < 0$ . On the other hand, let  $\gamma_1 = 0.09$  and  $\gamma_2 = 0.04$  keeping other parameters same. Then, we obtain  $u_N^* = 0.87$  and  $g_N^* = 0.13$ . In this case, the economy exhibits a wage-led growth because a slight increase in  $\pi_N$  decreases  $g_N^*$ . We obtain  $\partial \log V_N^*/\partial \pi_N = 1.51 > 0$ .

We can easily know that an increase in the profit share of the South increases the real wage rate of the North in the short-run equilibrium.

**Proposition 8.** *Suppose that both countries are located in the short-run equilibrium. Then, an increase in the profit share of the North either increases or decreases the real wage rate of the North. On the other hand, An increase in the profit share of the South increases the real wage rate of the North.*

The long-run equilibrium value of the real wage rate of the North is given by

$$V_N^{**} = \frac{1 - \pi_N}{b_N} \left( \frac{\varepsilon_N}{\varepsilon_S} \cdot \frac{a_S}{s_S \pi_S} \cdot g_N^{**} \right)^{-\frac{\alpha}{\xi}}. \quad (55)$$

To investigate the effect of an increase in the profit share of the North on the real wage rate of the North, we take the logarithm of  $V_N^{**}$  as follows:

$$\log V_N^{**} = \log(1 - \pi_N) - \frac{\alpha_0 (u_N^{**} K_N)^{\varepsilon_N - 1} (P^{**})^{1 - \mu_N}}{\xi} \log g_N^{**} + \dots. \quad (56)$$

Capital stock of the North  $K_N$  continues to increase at the rate of  $g_N^{**} > 0$  at the long-run equilibrium. With  $\varepsilon_N < 1$ , the term  $K_N^{\varepsilon_N - 1}$  approaches zero. Accordingly, the second term of the right-hand side approaches zero. In this case, we obtain

$$\frac{\partial \log V_N^{**}}{\partial \pi_N} = -\frac{1}{1 - \pi_N} < 0. \quad (57)$$

Therefore, an increase in the profit share of the North decreases the real wage rate of the North in the long-run equilibrium.

We can easily know that an increase in the profit share of the South increases the real wage rate of the North in the long-run equilibrium.

**Proposition 9.** *Suppose that both countries are located in the long-run equilibrium. Then, an increase in the profit share of the North decreases the real wage rate of the North. On the other hand, an increase in the profit share of the South increases the real wage rate of the North.*

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