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Abstract. In this paper, we consider a one-to-one matching model with two phases; an adolescence phase where individuals meet a number of dates and learn about their aspirations, followed by a matching phase where individuals are matched according to a version of Gale and Shapley's (1962) deferred acceptance (DA) algorithm. Using simulations of this model, we study how the likelihoods of matching and divorce, and also the balancedness and the speed of matching associated with the outcome of the DA algorithm are affected by the size of correlation in the preferences of individuals and by the frequency individuals update their aspirations in the adolescence phase.

Key words: Mate search; one-to-one matching; stability; agent-based simulation.

1 Introduction

In a seminal paper, Gale and Shapley (1962) showed that for every two-sided marriage population where individuals have complete and transitive preferences over potential mates there always exists (between men and women) a one-to-one matching which is stable in the sense that no individual prefers being single to his/her mate and no pair of individuals prefer each other to their mates. They proved this result by proposing an algorithm, namely the deferred acceptance (DA) algorithm, which always produces a stable matching. In this algorithm, which has two symmetric versions with respect to the roles of men and women, individuals in one side of the population, say men, make proposals and individuals in the other side, women, give deferred acceptances or rejections. The algorithm with men proposing stops after any step where no man is rejected or every single man has already proposed to every acceptable women in his preference list.

The DA algorithm, with various extensions for many-to-one matching, has been used in many markets, even before it was proposed by Gale and Shapley (1962).[1] Many of these works have put meaningful restrictions on preferences of individuals who aim to match with each other. For example, in the student placement problem (Balinski and Sönmez, 1999) where a centralized clearinghouse implements a stable matching between colleges and students using some version of the DA algorithm, preferences of colleges over students are usually assumed to commonly derive from some general exam scores of students. On the other hand, in the same problem students are typically allowed to have imperfectly correlated preferences over colleges,

[1] See, for example, Roth (1984, 2002, 2008) for the earliest known inventions as well as many practical uses of this algorithm by the National Resident Matching Program in the United States and by regional medical markets elsewhere. Discussions on similar uses of the algorithm can also be found in Balinski and Sönmez (1999), dealing with the problem of student placement (or centralized school admissions) that is known to exist in several countries including China, Greece, and Turkey, and in Abdulkadiroğlu, Pathak, and Roth (2005) and Pathak and Sönmez (2008) dealing with matching problems in the New York City high schools and Boston public schools, respectively.

reflecting —in addition to the publicly known performance rankings of colleges— students' subjective assessments about the various attributes of colleges, such as academic rigor, research opportunities, job market placements, campus size and location, student body, cost of education, scholarships, accommodations, etc. However, despite the fact that many works in the matching theory assume, like in the case of student placement problem, different degrees of preference correlations for the two sides of the matching population, the impacts of a change in preference correlations on stable matchings is still unexplored. A second gap in the matching literature is that it is lacking an explanation as to how individuals decide on which potential mates in the population they find acceptable as a spouse. Rather, the existing literature implicitly assumes that each individual is born with the knowledge about the set of unacceptable mates and this knowledge remains unchanged during mate search. Obviously, this assumption is too strong and also unrealistic. In practice, an individual can simply decide whether a potential mate is acceptable or not by checking whether the estimated 'value' of this potential mate, i.e., a real number reflecting the aggregated quality of all his/her traits, is above or below his/her own aspiration level, which the individual can learn about during a pre-matching stage where he/she can randomly interact with a number of potential mates.

Motivated by the aforementioned gaps in the stable matching theory, in this paper we will study how the stable matching outcome of the DA algorithm (where men propose and women accept/reject) is affected by the degree of correlation in the preferences of individuals and by the intensity of their learning about their aspirations (and correspondingly about the set of unacceptable mates) during a pre-matching stage. While our research question is, to the best of our knowledge, novel for the literature on stable matchings, a related question has already been studied in a strand of literature on mutual sequential mate search where the focus is only the random formation of 'acceptable' matchings consistent with some demographic patterns. One of the pioneering works in this literature is due to Todd and Miller (1999), from whom we will borrow some part of our model.

Basically, the mate search model of Todd and Miller (1999) involves two phases, an adolescence phase followed by a mating (or matching) phase. In the adolescence phase, individuals randomly meet a number of dates of the opposite gender and after each meeting they mutually exchange information as to whether they have found each other acceptable; i.e., whether each individual finds the mate value of the interacted date above his/her own aspiration level. Using the exchanged information, individuals update their aspirations after each instance of dating, following an adjustment (updating) rule commonly adopted by the whole population. After the adolescence phase is over, individuals enter the mating phase, where they randomly interact with potential mates, and using their aspirations finalized at the end of the adolescence phase they decide on whether to make a proposal to any interacted individual. Individuals mutually proposing to each other become mated and they are removed from the pool of unmated individuals. The mating phase ends after a stage in which the pool of unmated individuals becomes empty or shrinks to a set in which no man and woman are mutually acceptable (and can form a mated couple). Todd and Miller (1999) used the mate search model outlined above to find socially desirable and simple mate search rules (or heuristics) that can be used by individuals in a given matching population to update their aspirations after each dating in the adolescence phase. To compare the performances of several search rules they proposed, they introduced three success measures, namely the mating likelihood measured by the number of mated pairs in the population, the mating balancedness measured by the average mate value of all mated pairs, and the (potential) stability of mating measured by the mean within-pair difference in mate value.

In the past, several works have investigated whether mate search models similar to that of Todd and Miller (1999) can produce demographic patterns consistent with the real data (see, for example, Hills and Todd, 2008; Todd et al. 2005; Simão and Todd, 2003, among others), while some other works have dealt with the robustness of

Todd and Miller's model (1999) or with its improvement. For example, Shiba (2013) and Saglam (2014) studied whether the findings of Todd and Miller (1999) are robust with respect to a change in their assumption that the whole matching population uses the same adjustment rule in the adolescence phase. In another robustness study that is also related to ours, Saglam (2019) relaxed Todd and Miller's (1999) assumption of homogenous (or perfectly correlated) preferences to study the effect of heterogeneity in preferences on the performances of several aspiration adjustment rules. In the same literature, a second study that is closely related to our paper is due to Saglam (2018), who proposed a new adjustment rule, called Take the Weighted Average with the Next Desiring Date (TWAND), and compared its performance to those of the rules proposed by Todd and Miller (1999) for mutual sequential mate search with random acceptable matchings. For his analysis, Saglam (2018) also added the speed of mating to the list of success measures considered by Todd and Miller (1999), and showed that even though his new rule, TWAND, does not perform well with respect to mating stability, it is more balanced than the majority of the adjustment rules of Todd and Miller (1999), and moreover "... in terms of mating likelihood almost as good as, and in terms of mating speed always better than, the most successful —yet also unrealistic— heuristic of Todd and Miller (1999), namely the Mate Value-5 rule, which assumes that individuals in the mating population completely know their own mate values before interacting with any date." (Saglam, 2018b; p. 122)

In this paper, we consider a mate search model integrating various structures in Todd and Miller (1999), Saglam (2019), and Saglam (2018) with the one-to-one matching model of Gale and Shapley (1962). Basically, our model involves two phases, namely the adolescence and matching phases. We borrow the adolescence phase from the mate search model of Todd and Miller (1999); however, instead of their (aspiration) adjustment rules we adopt the adjustment rule of Saglam (2018b), TWAND, because of its aforementioned superior features. On the other hand, we borrow the matching phase in our model from Gale and Shapley (1962). Specifically, we assume that matchings in our model are obtained by the application of the DA algorithm (with men proposing); hence matchings are always stable in our model (with respect to the stability notion of Gale and Shapley (1962) described previously), unlike in the matching model of Todd and Miller (1999), where matchings are obtained through the mutual acceptable proposals of randomly meeting individuals in the matching phase and hence they are not necessarily stable. Also, we allow for, like in Saglam (2019), heterogenous as well as homogenous preferences to study —through some Monte Carlo simulations— the effects of the degree of correlation in preferences and the frequency of aspiration adjustments on the performance of the DA algorithm using four performance measures, three of which we borrow from Todd and Miller (1999) and the remaining one from Saglam (2018b).

The rest of our paper is organized as follows. Section 2 introduces our mate search model, and Section 3 presents our simulation results. Finally, Section 4 concludes.

2 Model

We consider a population involving n men and n women each of whom searches for a mate from the opposite sex. Mate search consists of two phases: an adolescence (dating and learning) phase and a matching phase. In the adolescence phase each individual dates a number of potential mates and learns about his/her own aspiration, a threshold mate value which is equal to the utility of being single. Using the acquired knowledge about his/her aspiration each individual identifies the set of individuals he/she finds acceptable as a spouse, and using his/her knowledge about the mate values of potential mates each individual forms a preference ordering of potential mates. Individuals, given their preference orderings and their identifications about the set of acceptable mates, are then matched in the next phase according to the DA algorithm of Gale and Shapley (1962) where (iteratively) men propose and women reject or accept.

In the described setup, let $v_i(j)$ denote the value (a nonnegative real number) individual i assigns for the mate value of individual j before the adolescence phase begins. This value remains constant during mate search. We say that individual i strictly (weakly) prefers individual j to individual k if $v_i(j) > (\geq)\ v_i(k)$. We assume that no individual knows his/her own mate value; i.e., $v_i(i)$ is unknown to each individual i. (As a matter of fact, each individual will approximate his/her own mate value by his/her aspiration he/she will form in the adolescence phase.) Moreover, the mate values assigned for any individual i by two distinct potential mates j and k may or may not be independent. This last assumption will allow us to study distinct situations where the preferences of men (women) about women (men) are completely correlated, completely uncorrelated, as well as partially correlated. Now, we are ready to present the two phases of our model in more detail.

Adolescence Phase. During this phase, each individual —by randomly dating some potential mates— learns about his/her aspiration, which is assumed to be equal to the utility he/she would derive from being unmatched. Each individual will later use his/her aspiration just before the matching phase to identify the set of acceptable mates, i.e., the set of individuals each of whom he/she will prefer as a spouse to being single.

The adolescence phase involves S consecutive stages of interactions (dating), with S being an integer between 1 and n. Individual i enters the adolescence phase with an initial aspiration, denoted by $a(i,0)$, and then randomly meets a date (a potential mate) of the opposite sex in each stage. More formally, at stage s individual i interacts with the date $d(i,s)$. Comparing the mate value $v_i(d(i,s))$ of this date with his/her aspiration $a(i,s-1)$ formed in the previous stage, individual i decides on whether the date $d(i,s)$ is *acceptable* as a spouse. (In the same way, his/her date independently decides on whether individual i is acceptable.) Next, individual i and his/her date $d(i,s)$ exchange information concerning whether they have found each other acceptable. Taking into account the exchanged information and his/her previous aspiration $a(i, s-1)$, individual i then calculates his/her aspiration $a(i, s)$ for stage s. Individual i makes this calculation with the help of an adjustment (or feedback) rule which is assumed to be commonly used by the whole population. This rule is *Take the Weighted Average with the Next Desiring Date Rule* (TWAND) proposed by Saglam (2018b) for mutual sequential mate search. According to this rule, if individual i learns that his/her date, $d(i, s)$, in stage s of the adolescence phase found him/her acceptable, i.e., $v_{d(x,s)}(i) \geq a(d(i, s), s-1)$, then individual i sets his/her aspiration, $a(i, s)$, in stage s to the weighted average of the mate value of $d(i, s)$, which is $v_i(d(i, s))$, and his/her aspiration $a(i, s-1)$ in the previous stage, using the respective weights of $1/(p+1)$ and $p/(p+1)$ where p denotes the number of stages (before s) in which individual i was found acceptable. On the other hand, if individual i learns that he/she was not found acceptable by his/her date $d(i, s)$ in stage s, i.e., $v_{d(i,s)}(i) < a(d(i, s), s-1)$, then individual i makes no adjustment. More formally, for each individual i and for each stage of dating $s = 1, \ldots, S$, this adjustment satisfies

$$a(i, s) = \begin{cases} \dfrac{1}{p+1}\, v_i(d(i, s)) + \dfrac{p}{p+1}\, a(i, s-1) & \text{if } v_{d(i,s)}(i) \geq a(d(i, s), s-1), \\ a(i, s-1) & \text{otherwise,} \end{cases}$$

where $a(i, 0)$ is given.

Saglam (2018b) observes that in situations where the preferences of men (women) over women (men) are homogenous and all initial aspirations are zero, the aspiration of each individual can be expected to be positive with probability one after the first stage of dating, provided that all mate values are positive. Therefore, as the adolescence phase proceeds, each individual faces the probability of being non-acceptable by some of his/her dates. Because this probability will be higher for individuals with lower mate values, such individuals will have lower levels of aspirations on average.

This means that the lower the mate value of the date, the higher the probability that an individual will be found acceptable. Consequently, the first row of the adjustment rule expressed above will be more frequently called when the date of an individual has a lower mate value, implying that $a(i, s)$ will tend to be lower than $a(i, s-1)$. Given these observations, Saglam (2018) predicts, and also verifies, that under homogenous preferences the average level of adjusted aspirations (calculated for the whole population) will be, after some early stage, always below the average level of mate values and will decrease as the dating and learning proceed. We will check whether this prediction holds under non-homogeneous preferences, as well.

Matching Phase. In this phase, a matching between men and women occur. Formally, a matching is a one-to-one function from the matching population to itself such that for each man and woman pair (m, w) in the matching population it is true that m is the mate of w if and only if w is the mate of m; the mate of m is not a woman if and only if m is single, and the mate of w is not a man if and only if w is single. We will call any two individuals a matched pair if they are the mate of each other.

A matching μ is said to be *acceptable* for an individual if this individual weakly prefers his/her mate at μ to being single. Any two individuals are called a *blocking pair* for a matching μ if they are not matched to one another at μ but strictly prefer one another to their mates at μ. Given these definitions, a matching is called *stable* if it is acceptable for each individual and there exists no blocking pair for it.

A well-known theorem proposed by Gale and Shapley (1962) shows that there always exists a stable matching for any matching population with complete and transitive preferences. They prove this result by showing that a particular proposal and acceptance algorithm, when used by both sides of the marriage population (men and women), always produces a stable matching. This algorithm is known as the deferred acceptance (DA) algorithm which has two versions. In one of them, men make proposals and women give (deferred) acceptances, while in the other version women make proposals and men give (deferred) acceptances.[2] Of these two versions, we will restrict ourselves to the DA algorithm with men proposing (or simply DA-MP algorithm), which can be described as follows. (Clearly, the DA algorithm with women proposing can be obtained by interchanging the roles of men and women in the below steps.)

Step 1: Each man proposes to his most preferred woman in his list of acceptable women (if any). Each woman rejects any unacceptable proposals, and if she has received more than one acceptable proposal, she holds the most preferred one and rejects the others. Then, each rejected man eliminates the woman who rejected him from his list of acceptable women.

Step $k \geq 2$: Any man who was rejected in step $k-1$ makes a proposal to his most preferred woman in his updated list of acceptable women (if any). (If his list is empty, he makes no proposal.) Each woman holds her most preferred acceptable proposal she has received until now and rejects the others. Then, each man rejected in this step eliminates the woman who rejected him from his list of acceptable women. The algorithm terminates when no further proposal is made by any man, and finally each woman is matched to the man (if any) whose proposal she holds.

In any step of the above algorithm if any man (woman) is indifferent between any two women (men), he (she) is allowed to break the tie arbitrarily.

[2] Moreover, as was shown by Gale and Shapley (1962), if all men and women have strict preferences, then the DA algorithm with men proposing always produces men-optimal stable matching; i.e. a stable matching which all men weakly prefer to any other stable matching. Since the focus of our paper will not be on the optimality of stable matchings; we will allow individuals to have indifferences in their preference relations.

3 Results

We will study the effects of correlation in preferences and the frequency of updating aspirations on the matching outcome of the DA-MP algorithm using computer simulations with the help of GAUSS Software Version 3.2.34 (Aptech Systems, 1998). (The program code of the simulations and the resulting data are available from the author upon request.)

For our simulations, we set n, the number of individuals in each sex, to 50 while we vary S, the number of dating stages in the adolescence phase, from 1 to 48 (omitting the values 49 and 50 for geometrically increasing computation time), and for each value of S, we conduct 100 Monte Carlo simulations. In all simulations, we set initial aspirations to zero, i.e., $a(i,0) = 0$ for each individual i. Also, we model the preferences of individuals, and the mate values inducing these preferences, like in Saglam (2019), where the mate values —each individual assigns for the potential mates before the beginning of the adolescence phase— contain in general a common component and a private component. To formalize this, we first pick for each individual i in the matching population a randomly drawn value, denoted by $v^c(i)$, from a uniform distribution of values in $[0, 100]$. We call this particular value the mate value of agent i commonly observed/assigned by all potential mates in the case of completely homogenous (perfectly correlated) preferences. Next, we pick any individual i and any potential mate j from the opposite sex, and set the observation of j about the mate value of i to a randomly drawn value from a uniform distribution of values in $[0, 100]$. We denote this value by $v^p_j(i)$, namely the mate value of individual i privately observed/assigned by individual j in the case of completely heterogenous (perfectly idiosyncratic) preferences. Finally, we pick a real number ω in $[0, 1]$ and define for each individual i and for each potential mate j the value

$$v_j(i) = \omega\, v^c(i) + (1 - \omega)\, v^p_j(i),$$

namely the mate value of individual i privately observed/assigned by individual j

under partially heterogenous preferences associated with the common correlation parameter ω. We should note that the correlation between individuals' preferences is increasing in ω over the interval $[0, 1]$. In our simulations, we vary the correlation parameter inside the set of values $\{0,\ 1/3,\ 2/3,\ 1\}$. Note that $\omega = 0$ and $\omega = 1$ respectively correspond to the cases of uncorrelated (heterogenous) and perfectly correlated (homogenous) preferences, while the other values of ω correspond to some cases of imperfectly correlated preferences.

To analyze how the stable matching outcome of the DA-MP algorithm is affected by a change in the correlation parameter ω or the number of dates S in the adolescence phase, we will consider four performance measures. We borrow these measures from the two-sided mutual search literature that studies the success of various search heuristics. In particular, we borrow the first three of our performance measures from Todd and Miller (1999) and the last one from Saglam (2018b). Before we analyze the success of the DA-MP algorithm according to our performance measures, we will first investigate how the mean aspirations change as individuals interact with a higher number of dates in the adolescence phase.

Our simulations illustrated in Figure 1 show that irrespective of the number of dates, S, the level of correlation in preferences has in general a monotonic, and also negative, effect on the mean aspiration of all individuals. That is, the more correlated the preferences, the lower the mean aspirations. However, the observed effect is negligibly small when the degree of correlation in preferences is not sufficiently high. In Figure 1, we should also observe that the prediction of Saglam (2018b) about the evolution of aspirations can hold in our matching model only if the correlation in preferences is extremely high. In that case, the mean aspiration level rapidly increases during a few initial dating instances and mildly decreases as the number of dating becomes higher. When the correlation in preferences is not sufficiently high ($\omega \leq 2/3$), the frequency of dating, unless it is extremely small, has almost no effect on the mean aspiration level of individuals. As a matter of fact, when ω is $1/3$ or

0, the mean aspiration level rapidly converges to the mean mate value, 50, of the population after a few dating.

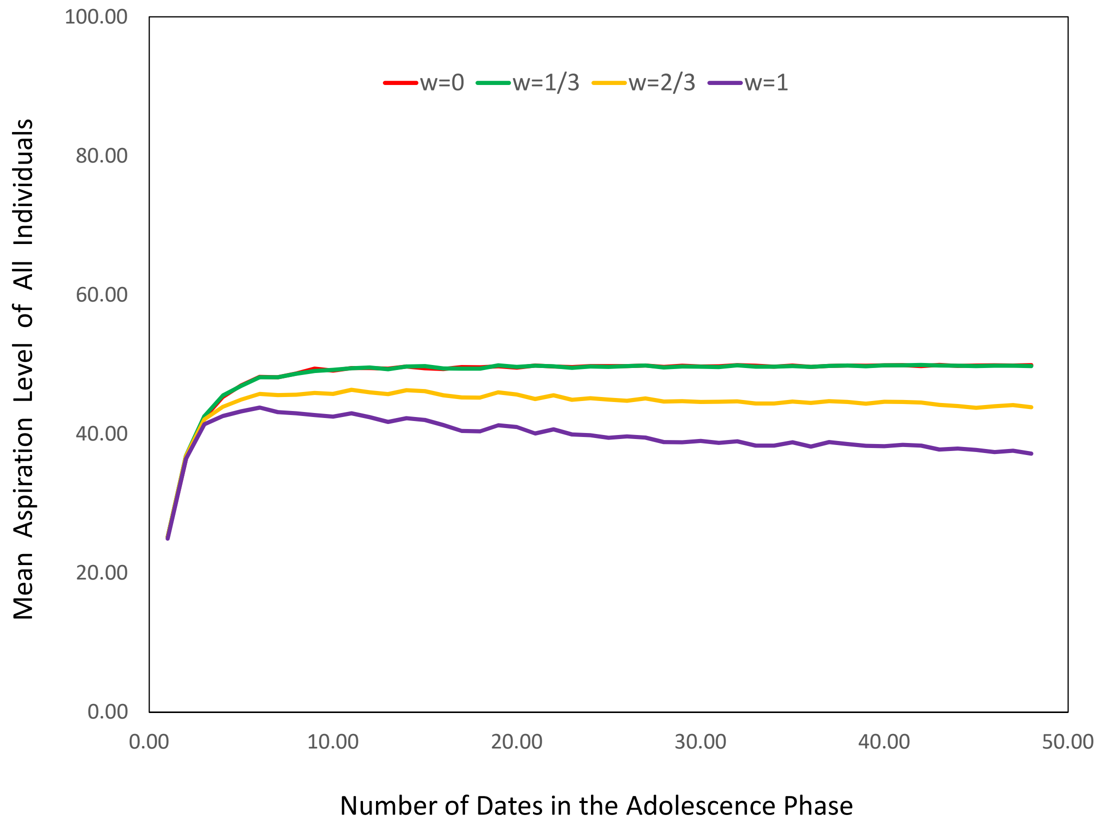

Figure 1.

Below, we will measure the performance of the DA-MP algorithm with respect to our first measure, namely the likelihood of matching, as represented by the percentage of matched in pairs in our population. When the correlation in preferences measured by ω increases, we expect it to produce two effects on the likelihood of matching: a direct effect and an indirect effect. The direct effect works through the increased competition among men over their potential mates due to the nature of the DA-MP algorithm, reducing the likelihood of matching. On the other hand, the indirect effect works through the reduced aspirations of individuals (see Figure 1), expanding the set of acceptable mates for individuals in both sides of the market and increasing thereby their likelihood of matching. Our simulation results, illustrated in Figure 2, show that of these opposite effects the direct effect always dominates the indirect effect. That is, irrespective from the number of dates in the adolescence phase the likelihood of matching is always higher if the preferences of individuals are less correlated (i.e., ω is smaller).

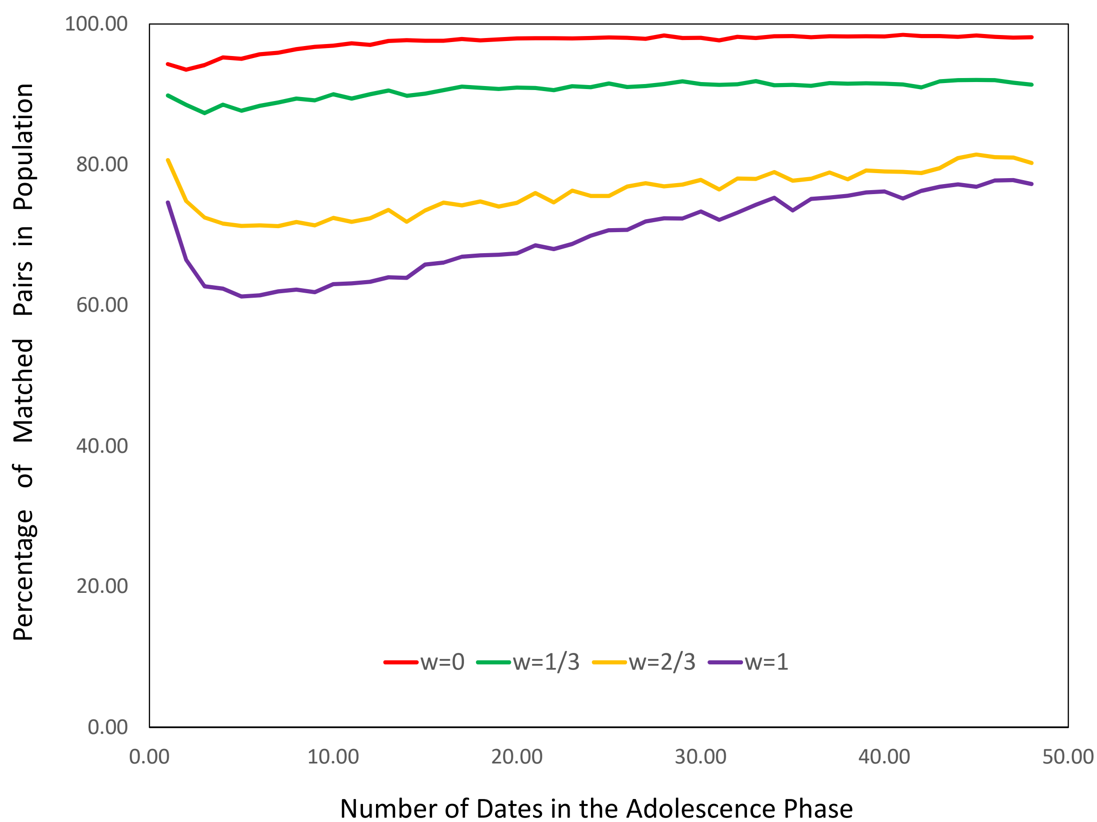

Figure 2.

Figure 2 also shows that when the correlation in individuals' preferences are sufficiently small, the matching likelihood remains fairly stable at a very high level as the number of dates in the adolescence phase increases. On the other hand, when the preferences are sufficiently correlated (i.e., ω is sufficiently high), the matching likelihood initially decreases abruptly with the number of dates over a very small range and mildly increases elsewhere.

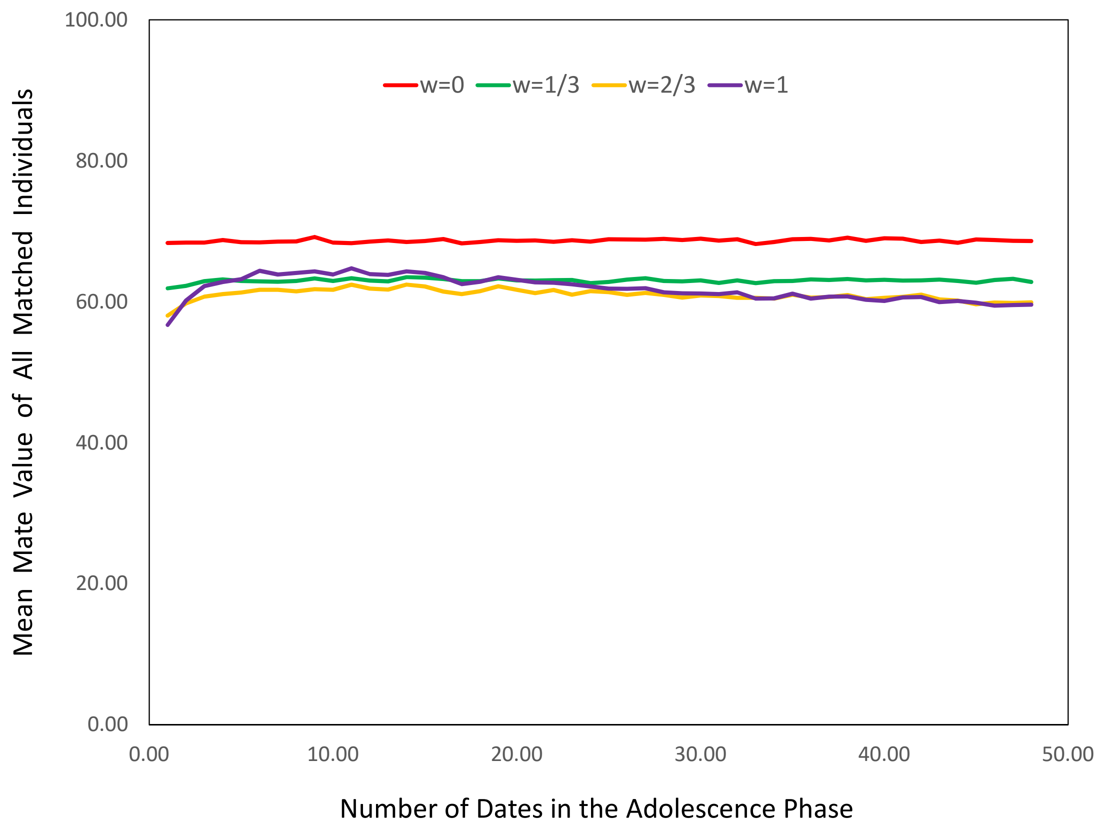

Figure 3.

Our second performance measure is based on the mean mate value of all matched individuals. According to this measure, a matching is considered more successful (more egalitarian or balanced) if the mean mate value of all matched individuals is closer to the mean mate value of the whole population, which is 50 by the assumed distribution of mate values in our simulation. If this measure has a higher (lower) value than 50, then it would indicate that a majority of unmatched individuals have below (above) average mate values. Figure 3 shows that irrespective of the correlation in preferences and the number of dates in the adolescence phase, the mean mate value of all matched individuals is always (significantly) higher than the middle value 50, indicating that the DA-MP algorithm always produces a non-egalitarian, or unbalanced, matching at which the lower segments of the population (in terms of mate values) are more likely to be unmatched than the higher segments. Figure 3 also suggests that the most unbalanced matching is always obtained when there is no correlation in preferences (i.e., $\omega = 0$). On the other hand, when there is some correlation in preferences, the balancedness of matching is found to be affected by both the degree of this correlation and the number of dates interacted in the adolescence phase. As a matter of fact, we observe a positive relationship between the degree of correlation in preferences and the balancedness of the matching outcome only if the number of dates is sufficiently high.

Our third performance measure is the mean within-pair difference in mate value, which can be argued to positively affect the probability of divorce (or the expected divorce rate in the population).[3] Matchings with lower scores with respect to this measure are considered to be more successful as the individuals paired under these matchings are assumed to be less likely to divorce in the future. Our results in Figure 4 illustrate that the number of dates in the adolescence phase has almost negligible impacts on the likelihood of divorce. Moreover, although the degree of correlation in preferences has a remarkable effect on this likelihood irrespective of the number of dates, this effect is not monotonic. As could be predicted, we obtain the worst performance, with the highest within-pair difference in mate values on average, when the preferences are uncorrelated. Surprisingly, the second highest scores arise when the preferences are perfectly correlated. We find that the matching outcome of the DA-MP algorithm becomes more successful in terms of the reduced risk of divorce in cases where preferences are partially correlated than in cases where they are perfectly

[3]Todd and Miller (1999) calls this measure the stability of matching, represented by the inverse of the mean within-pair difference in mate value. We abstain from this calling as we already have a notion of stability, due to Gale and Shapley (1962).

correlated or uncorrelated at all.

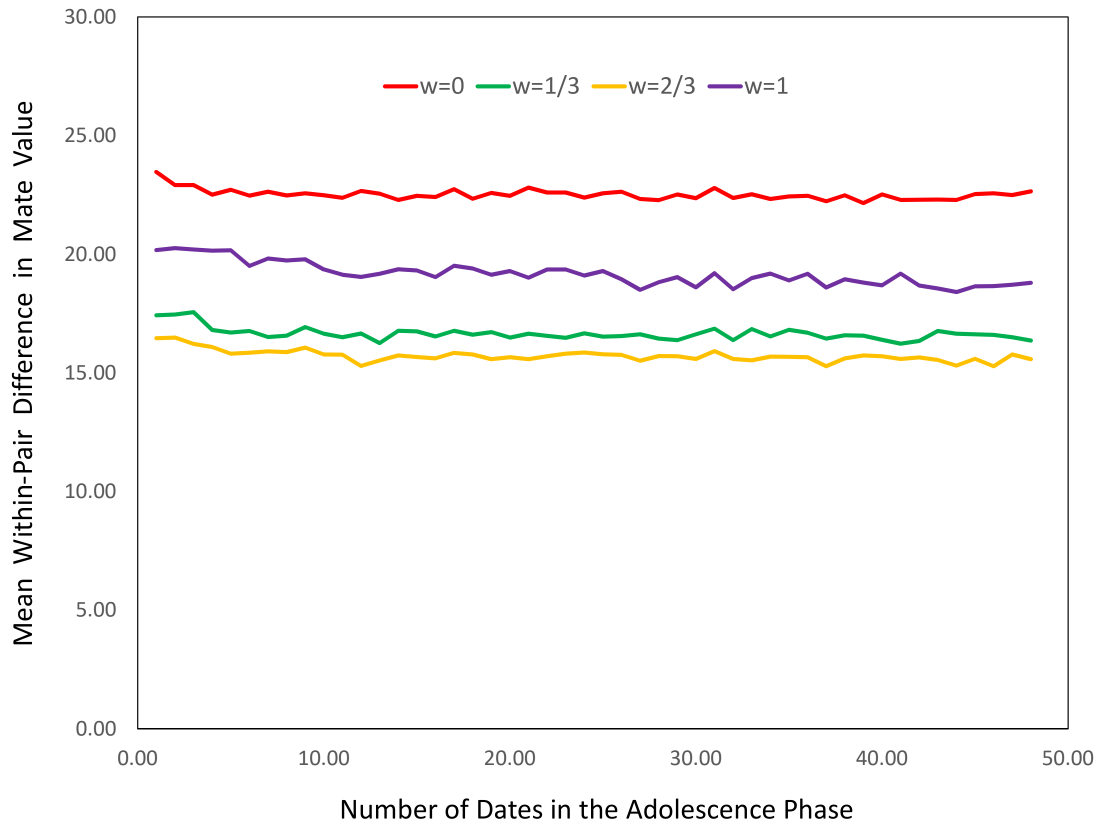

Figure 4.

Our last performance measure is the speed of matching, i.e., the speed of the convergence of the DA-MP algorithm, which is simply represented by the mean number of steps (iterations) in this algorithm. When this number is smaller, we say that the given matching is faster. Figure 5 illustrates that the slowest performance is always obtained when the individuals' preferences are uncorrelated. Moreover, the number of dates, unless it is extremely small, has always small, but also non-predictable, effects on this performance. On the other hand, the best performance is obtained when the preferences are perfectly correlated. In this case, the number of iterations required by the DA-MP algorithm to generate a stable matching becomes very close to the maximal number of potential couples, which is 50 in our simulations.

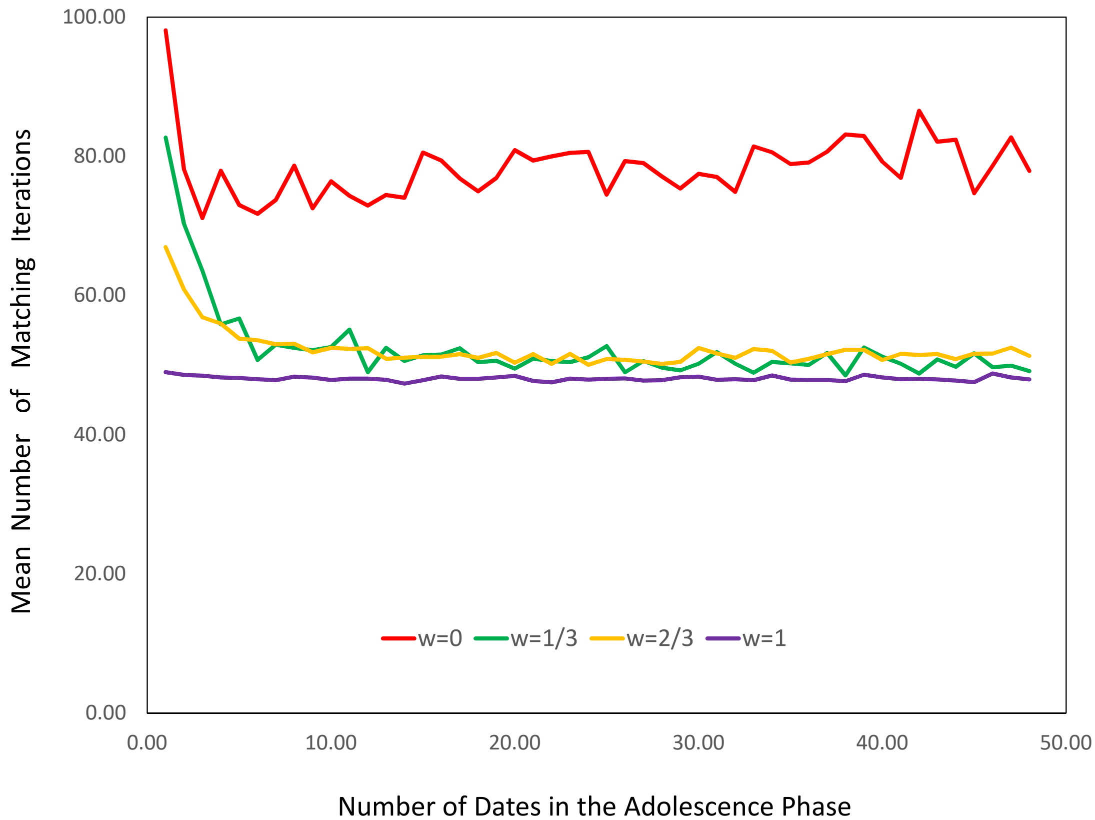

Figure 5.

Figure 5 also shows that the presence of even a small correlation in preferences ($\omega = 1/3$) drastically improves the speed of convergence of the DA-MP algorithm —as compared to the case of no-correlation— unless the number of dates in the adolescence phase is extremely small.

4 Conclusion

In this paper, we have considered a matching model with two phases, an adolescence phase —which we borrow from the mutual sequential mate search model developed by Todd and Miller (1999) and studied by Saglam (2018a,b) among many others—followed by a matching phase where iteratively made proposals by men and (deferred) acceptances by women, according to the DA-MP algorithm proposed by Gale and Shapley (1962), produce one-to-one stable matchings. To evaluate the success of the DA-MP algorithm under several cases of interest, we have considered four performance measures, namely the likelihood of matching, the balancedness of matching, the likelihood of divorce, and the speed of matching; and studied how these measures are affected by the changes in (i) the degree of correlation between the preferences of individuals, and (ii) the frequency individuals date potential mates in the adolescence phase and update their aspirations according to the rule of TWAND proposed by Saglam (2018b).

We have found that irrespective of the number of dates in the adolescence phase the likelihood of finding a mate under a stable matching is always higher if the preferences of individuals are less correlated (or more heterogenous). Moreover, the DA-MP algorithm always produces a non-egalitarian/unbalanced stable matching at which the lower segments of the population (in terms of mate values) are more likely to be unmatched than the higher segments. However, there also exists a positive relationship between the degree of correlation in preferences and the balancedness of matching when the number of dates in the adolescence phase is sufficiently high. In particular, the most unbalanced matching always arises when individuals' preferences are uncorrelated.

As to the likelihood of divorce, we have found that the number of dates in the adolescence phase has negligible effects while the degree of correlation in preferences has considerable, but also non-monotonic, effects. Interestingly, divorce becomes significantly less likely when preferences are partially correlated than when they are perfectly correlated or uncorrelated at all. In particular, the worst case arises when the preferences are uncorrelated. Finally, we have studied the speed of matching in our simulations. To this aim, we have calculated the mean number of iterations in the DA-MP algorithm, and found that this number always attains its highest values (the worst scores) when the preferences of individuals are uncorrelated, while these values are in general not much affected by the number of dates. On the other hand, the fastest stable matchings are generated when the preferences are perfectly correlated.

We should note that in situations where all individuals have strict preferences over potential mates, we can in fact measure, in at least three dimensions, the performance of any stable matching by measuring the performance of the stable matching generated by the DA-MP algorithm, only. The reason is that all performance measures in our study, except for the speed of matching, are calculated using (linear functions of) the mate values of all matched individuals, and these values must be the same for all stable matching outcomes for a given population since a theorem by McVitie and Wilson (1970) shows that under strict preferences the set of unmatched (and therefore the set of matched) men and women must be the same at every stable matching.

Future research may extend our work by comparing the effects of alternative (aspiration) adjustment rules like in Todd and Miller (1999) or Saglam (2018a,b) or by allowing the two sides of the matching population (men and women) to adopt different adjustment rules like in Shiba (2013). Another extension could fruitfully focus on the evolution of the aspiration adjustment rules like in Saglam (2014) and study whether any (aspiration) adjustment rule could arise as a Nash (1950) equilibrium in our model when individuals are allowed to choose any adjustment rule among a finite number of alternatives.

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