Shareholder Activism Externalities

Li, Zhan

University of Winnipeg

7 August 2017

Shareholder Activism Externalities

Zhan Li*

First Version: August, 2017

This Version: January, 2019

Abstract

Shareholder activism increases the non-target firm's outside option and reduces its CEO's outside option, which leads to higher firm profit and lower CEO compensation. Due to this positive externality, the activist's intervention is inefficiently low. Several extensions further generate a number of novel insights: The liquidity of the CEO talent market exacerbates the externality; common ownership alleviates the externality but exacerbates the free-rider problem, ultimately reducing market efficiency; regulating activists' interventions decreases market efficiency when similar firms compete for different CEO talents.

Keywords: Shareholder activism, externality, common ownership.

JEL Classification: G34, J33, D86

*Faculty of Business and Economics, University of Winnipeg, Winnipeg, MB, R3B 2E9, Canada. Email: z.li@uwinnipeg.ca. This paper is partly based on my Ph.D. thesis. I am extremely grateful for the guidance and support from my thesis advisor Hector Chade and my dissertation committee member Natalia Kovrijnykh and Alejandro Manelli. I also thank Galina Vereshchagina for her generous help.

1 Introduction

The rise of shareholder activism since 1980s has been seen as a major force in corporate governance. Activists initiate changes in target firms by tearing down takeover defense, ousting CEOs, changing board structure, and challenging executive compensation plans. Recent empirical evidence shows that the impact of shareholder activism reaches beyond target firms, as industry peers make similar improvements and changes, generating significant financial and real effects (Zhu, 2013; Aslan and Kumar, 2016; Bourveau and Schoenfeld, 2017; Gantchev et al., Forthcoming; Feng et al., 2018).

This paper takes a new look at the positive externality of shareholder activism. While existing literature emphasizes the product market competition and the threat of intervention as the main channels for externalities, I draw attention to the factor market competition, namely, the competition for CEO talent. I show that shareholder activism generates positive externalities because both firms and CEOs' outside options are endogenously determined and interlinked in the market equilibrium. I derive an activist's optimal level of intervention and discuss its implications on market efficiency. I further study how the liquidity of the CEO talent market, common ownership (when an activist holds ownership stakes in multiple firms), and regulation affect the efficiency of shareholder activism.

Specifically, I consider one activist, two firms, and three CEOs in a three-period model. Firms are the same ex ante and CEOs have different talents. In the first period, the activist launches a costly activism campaign to reduce the target firm's governance deficiencies such as removing poison pills, declassifying the board, increasing board independence, and adopting confidential shareholder voting. In the second period, firms of different governance deficiencies match with CEOs of different talents. Equilibrium matching patterns, outside options, and payoffs are determined in this period. In the third period, each CEO produces a stochastic cash flow, and the marginal product of CEO talent increases with firm size. The cash flow can only be observed by the CEO, who reports it to the firm. The firm monitors the CEO by auditing her report, and the effectiveness of monitoring decreases with the firm's governance deficiencies.[1]

In equilibrium, the target firm—with governance deficiencies reduced by the activist—matches with a CEO of lower talent. Intuitively, lower governance deficiencies lead to more effective monitoring and lower CEO compensation, which reduces the firm's competitiveness for CEO talent. However, despite matching with a less talented CEO, the target firm earns a higher profit than the non-target firm because the gain from more effective monitoring and lower CEO compensation outweighs the loss from matching with a less talented CEO.

An important insight from the model is that shareholder activism generates a positive externality to the non-target firm through the market for CEO talent. Intuitively, the non-target firm's outside option is determined by its profit from deviating to match with the less talented CEO at the target firm, and the non-target CEO's outside option is determined by her wage from deviating to match with the target firm. Shareholder activism increases the target firm's profit and lowers its CEO's wage, which increases the non-target firm's deviation profit and reduces its CEO's deviation wage. Thus, shareholder activism leads to higher firm profit and lower CEO compensation at the non-target firm. Due to this positive externality, the activist sets inefficiently high governance deficiencies at the target firm.

I proceed to discuss how the liquidity of the CEO talent market affects the shareholder activism externality. When the CEO talent market becomes more liquid—due to more transferable CEO talent or smaller search frictions—the externality becomes more severe. Intuitively, when the CEO talent market is more liquid, the non-target firm and its CEO's outside options are more linked to the target firm. This increased linkage increases the inefficiency of shareholder activism.

[1] For example, monitoring is less effective when the lack of board independence, a form of governance deficiencies, is higher.

The empirical literature shows that CEO talents have become more transferable and outside hires are more pervasive over the last several decades (e.g., Murphy and Zabojnik 2004; Custódio et al. 2013), thus, the shareholder activism externality plays an increasingly important role in determining market efficiency.

Next, I discuss shareholder activism under common ownership (when an activist cross-holds multiple firms). It is not clear whether common ownership increases or decreases the efficiency of shareholder activism: Common ownership internalizes the positive externality by holding both the target and non-target firm; however, it also reduces the activist's stakes at the target firm, which exacerbates free-riding from non-activist shareholders (Grossman and Hart, 1980; Shleifer and Vishny, 1986).

To understand this tradeoff, I compare a cross-holding activist with a blockholder activist who optimally allocates her stakes in the target and non-target firm. The comparison generates two findings. First, the blockholder activist optimally concentrates all her stakes in the target firm. Thus, the need to internalize the externality in shareholder activism is unlikely to account for the rise of common ownership over the last several decades. Second, the cross-holding activist, who allocates shares in the non-target firm, sets higher governance deficiencies than the blockholder activist. Thus, common ownership exacerbates the inefficiency of shareholder activism.

Finally, I examine whether regulation can improve the efficiency of shareholder activism. I consider a regulator mandating a universal governance deficiency ceiling on two firms of different sizes, with two activists launching activism campaigns in the two firms. The effectiveness of the regulation depends on the structure of the CEO talent market: If the CEO talent market is competitive on the demand side (i.e., firms have similar sizes), the regulation results in less efficient shareholder activism.

To understand the intuition, consider two firms: Firm 1 and firm 2 with firm 1 as the larger firm. Firm 1 matches with a high talent CEO, and firm 2 matches with a low talent CEO. When firm 1's size is similar to firm 2, firm 1's size advantage is not enough to attract the high talent CEO, thus, activist 1 optimally sets higher governance deficiencies at firm 1 than activist 2 at firm 2, whose governance deficiencies are still inefficiently high due to the positive externality. However, a universal governance deficiency ceiling, which tries to remedy the inefficiency by reducing governance deficiencies at firm 2, will be too low for firm 1. Under such a regulation, overregulation occurs and market efficiency decreases.

This paper is related to the literature on shareholder activism externalities. The existing literature discusses two channels. The first is the product market channel: Activism generates negative effects on rival firms through product market competition (Aslan and Kumar, 2016). The second is the threat channel: The possibility of activism disciplines managers and improves performance at non-target firms (Zhu, 2013; Gantchev et al., Forthcoming; Feng et al., 2018; Bourveau and Schoenfeld, 2017). This paper identifies a novel channel of externalities that work through interlinked firm and CEO outside options. To my best knowledge, this is the first paper to model the effect of the CEO talent market on the externality and efficiency of shareholder activism.

Acharya and Volpin (2010) and Dicks (2012) analyze corporate governance externalities which also arise from firms' competition for CEO talent. Their papers focus on the inefficient contracting between firms and CEOs, while this paper focuses on activists' inefficient choices of governance characteristics, which themselves are determinants of the contracting environment studied in Acharya and Volpin (2010) and Dicks (2012). Thus, this paper is complementary to Acharya and Volpin (2010) and Dicks (2012): Shareholder activism can improve the contracting environment by decreasing governance deficiencies, however, activism has its own externalities and inefficienicies.

2 The Model

2.1 Model Setup

I consider a market of one activist, two firms, and three CEOs. A CEO can only work for one firm, a firm can only hire one CEO, and the activist can only launch an activism campaign at one firm. CEO $j \in \{1,2,3\}$'s talent is t_j with $t_1 > t_2 > t_3$. Governance deficiencies, such as the presence of poison pills or the lack of director independence, are present at both firms. Firms are the same ex ante: firm $i \in \{1,2\}$'s governance deficiencies are $\bar{e}$ and firm i's size is s. Each firm has 1 share outstanding. For simplicity, I assume the activist fully owns the target firm. CEOs' talents $\{t_1,t_2,t_3\}$ and firms' governance deficiencies $\bar{e}$ are common knowledge. CEOs and the activist are all risk-neutral.

The model has three periods. In period 1, the activist launches a costly activism campaign at the target firm to reduce its governance deficiencies. For example, the activist incurs a costly proxy fight to declassify the board. In period 2, firms compete with each other to match with the high talent CEO. Equilibrium matching pattern and equilibrium payoffs are determined in this period. In period 3, each firm contracts with its CEO to determine how the CEO will be monitored and compensated. Figure 1 plots the timing of the model.

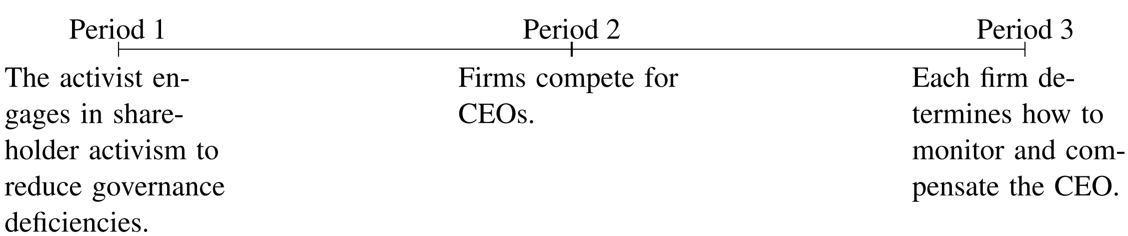


Figure 1: **Timing of the Model.**

Note that the activist's ownership at the target firm is exogenously given, thus, this paper does not speak about how the activist's accumulation of stakes is affected by stock market liquidity or strategic interactions between the activist and non-activist shareholders, both of which are extensively studied in the existing literature on shareholder activism.[2] Furthermore, by assuming the activist fully owns the target firm, the externality is *not* driven by the free-riding from non-activist shareholders, who receive the benefits of activism without sharing the costs (Grossman and Hart, 1980; Shleifer and Vishny, 1986).

2.2 Solution

2.2.1 Optimal Contract in Period 3

I focus on a single firm in period 3 and solve the optimal contract between the firm and its CEO. The period-3 problem is a simplified costly state verification model à la Border and Sobel (1987). For simplicity, I strip away subscripts from notations.

The firm's cash flow in period 3 is stochastic: it equals st, the product of firm size and CEO talent, with probability p and 0 otherwise. The distribution is common knowledge, but the realized cash flow $o \in \{st, 0\}$ is only observable to the CEO, who submits a report $r \in \{st, 0\}$ to the firm. Depending on the report r, the firm monitors the CEO by auditing her report with probability g_r and pays an auditing cost of sg_r. The effectiveness of auditing decreases in governance deficiencies.[3] Conditional on auditing, the firm discovers the realized cash flow with probability $\frac{1}{e}$, where e measures the firm's governance deficiencies. Apparently, the unconditional probability of discovering the realized cash flow is $\frac{g_r}{e}$.

The contract between the firm and the CEO specifies the probability of auditing g_r and CEO compensation $\{w_r, w_{or}\}$, where w_r is contingent on report r if there is no auditing or there is audit-

[2] For example, Grossman and Hart (1980); Shleifer and Vishny (1986); Maug (1998); Kahn and Winton (1998); Aghion et al. (2004); Faure-Grimaud and Gromb (2004); Admati and Pfleiderer (2009); Edmans (2009); Edmans and Manso (2010).

[3] For example, as the lack of board independence (a form of governance deficiencies) increases, directors become more tied to the CEO and more reluctant to oppose the CEO. Thus, board monitoring becomes less effective.

ing but the firm fails to discover the realized cash flow, and w_{or} is contingent on report r and realized output o if there is auditing and the firm discovers the realized cash flow. I denote the optimal contract by $C = \{g_r, w_r, w_{or}\}$, which can also be written as $C = \{g_{st}, g_0, w_{st}, w_0, w_{st,st}, w_{st,0}, w_{00}, w_{0,st}\}$ for $o, r \in \{st, 0\}$. The timing of the period-3 contract is shown in Figure 2.

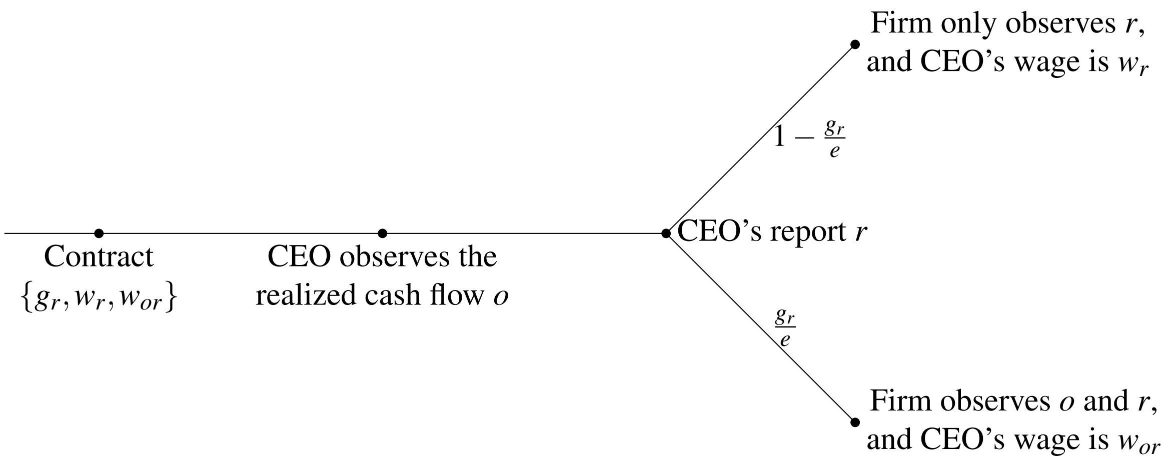


Figure 2: **Timing of Period 3.** Note that o refers to the realized cash flow and r refers to the reported cash flow.

By revelation principle, I look for the truth-telling equilibrium. When the realized cash flow is st, if the CEO reports truthfully, her wage is w_{st} with probability $1-\frac{g_{st}}{e}$ and $w_{st,st}$ with probability $\frac{g_{st}}{e}$; if she falsifies the report, her payoff is $w_0 + st - 0$ with probability $1-\frac{g_0}{e}$ and $w_{st,0}$ with probability $\frac{g_0}{e}$, where $st-0$ is the difference between the realized and reported cash flow. Incentive compatibility requires that the CEO prefers reporting the realized cash flow st truthfully:

$$\left(1-\frac{g_{st}}{e}\right) w_{st} + \frac{g_{st}}{e} w_{st,st} \geq \left(1-\frac{g_0}{e}\right)(w_0 + st - 0) + \frac{g_0}{e} w_{st,0}$$

Similarly, when the realized cash flow is 0, incentive compatibility requires

$$\left(1-\frac{g_0}{e}\right)w_0+\frac{g_0}{e}w_{00}\geq\left(1-\frac{g_{st}}{e}\right)(w_{st}+0-st)+\frac{g_{st}}{e}w_{0,st}$$

The firm signs contract C with the CEO to maximize the expected total cash flow pst net of expected CEO compensation $p\left(\left(1-\frac{g_{st}}{e}\right)w_{st}+\frac{g_{st}}{e}w_{st,st}\right)+(1-p)\left(\left(1-\frac{g_0}{e}\right)w_0+\frac{g_0}{e}w_{00}\right)$ and expected auditing cost $psg_{st}+(1-p)sg_0$, subject to a set of constraints including the incentive compatibility conditions:

$$\max_{C}\ pst-p((1-\frac{g_{st}}{e})w_{st}+\frac{g_{st}}{e}w_{st,st})-(1-p)((1-\frac{g_0}{e})w_0+\frac{g_0}{e}w_{00})-psg_{st}-(1-p)sg_0 \tag{1}$$

$$st.\ p((1-\frac{g_{st}}{e})w_{st}+\frac{g_{st}}{e}w_{st,st})+(1-p)((1-\frac{g_0}{e})w_0+\frac{g_0}{e}w_{00})\geq v \tag{PK}$$

$$(1-\frac{g_{st}}{e})w_{st}+\frac{g_{st}}{e}w_{st,st}\ \geq(1-\frac{g_0}{e})(w_0+st-0)+\frac{g_0}{e}w_{st,0} \tag{$ICst$}$$

$$(1-\frac{g_0}{e})w_0+\frac{g_0}{e}w_{00}\geq(1-\frac{g_{st}}{e})(w_{st}+0-st)+\frac{g_{st}}{e}w_{0,st} \tag{$IC0$}$$

$$w_0,w_{00},w_{0,st},w_{st},w_{st,st},w_{st,0}\geq 0 \tag{LL}$$

$$w_{st},w_{st,st}\leq st, w_0,w_{00}\leq 0 \tag{FC}$$

$$g_0,g_{st}\in[0,1]$$

where PK is the promise-keeping constraint, and v is the promised utility, which satisfies $v<pst$ in equilibrium (See Section 2.2.2). ICst and IC0 are the CEO's incentive compatibility constraints when realized cash flow is st and 0. LL is the limited liability constraint, and FC is the feasibility constraint. I solve the optimal contract under the assumption of $e<\frac{pt}{1-p}$. If $e\geq\frac{pt}{1-p}$, the firm never audits the CEO's report because auditing is too costly.

Lemma 1. *The expected CEO compensation equals the promised utility v, and the probability of* *auditing is given by* $g_{st} = 0$ *and* $g_0 = e\left(1 - \frac{v}{pst}\right)$.

Lemma 1 shows that g_0, the probability of auditing when the reported cash flow is 0, increases in governance deficiencies e. Intuitively, higher governance deficiencies reduce the effectiveness of auditing, thus, the firm compensates less effective auditing with a higher probability of auditing. Lemma 1 also shows that the firm only audits the CEO when she reports the low cash flow. Intuitively, the CEO can only benefit from falsely reporting the low cash flow when the realized cash flow is high. Upon receiving a report of high cash flow, the firm knows the report must be true and thus decides not to audit it.

Hereafter, I drop g_{st} from the discussion and call g_0 the probability of monitoring or monitoring intensity, which decreases in CEO compensation v. Intuitively, CEO compensation and monitoring are *substitutes* in incentivizing the CEO to report the realized cash flow truthfully. As CEO compensation increases, the firm sets a weaker monitoring intensity.

Substituting the optimal contract in Lemma 1 to the firm's objective function (1) yields the expected profit function:

$$\pi(s,e,t,v) = \left(1 - \frac{(1-p)e}{pt}\right)(pst - v) \tag{2}$$

Equation (2) shows that firm profit increases in CEO talent and decreases in CEO compensation. The firm prefers higher CEO talent and lower CEO compensation. However, a more talented CEO always demands a higher wage in equilibrium. Thus, higher CEO talent is also more costly to acquire. Section 2.2.2 discusses this tradeoff faced by firms in the two-sided matching market.

2.2.2 Matching Equilibrium in Period 2

In period 2, the activist has reduced the target firm's governance deficiencies from $\bar{e}$, while the non-target firm's governance deficiencies stay at $\bar{e}$. I ask how firms of different governance deficiencies match with CEOs of different talents and what are their equilibrium payoffs. Hereafter, I call the non-target firm *firm 1* and the target firm *firm 2*. Apparently, firm 1's governance deficiencies e_1 and firm 2's governance deficiencies e_2 satisfy $\bar{e} = e_1 > e_2$.

The equilibrium matching pattern needs to satisfy the feasibility and stability requirements. Feasibility requires that firm profit and CEO compensation belong to the utility possibility set, whose Pareto frontier is defined by equation (2). Stability requires that no firm or CEO prefers to match with each other or stay unmatched by breaking their current match. To satisfy the stability requirement, it is necessary to maintain the following assumption throughout this paper:

Assumption 1. $\bar{e} < \frac{pt_j}{1-p}, j \in \{1,2,3\}$

With the upper bound on governance deficiencies imposed by Assumption 1, equation (2) shows that firms' profits are always positive, thus both firms will prefer matching with a CEO to staying unmatched. Clearly, Assumption 1 implies $e_i < \frac{pt_3}{1-p}, i \in \{1,2\}$.

Lemma 2. *In equilibrium, firm 1 matches with CEO 1, firm 2 matches with CEO 2, and CEO 3 is unmatched.*

A firm with lower governance deficiencies matches with a less talented CEO, thus, there is positive assortative matching between governance deficiencies and CEO talent in equilibrium. If governance deficiencies refer to the lack of board independence, Lemma 2 implies that a firm of higher board independence matches with a CEO of lower talent. Intuitively, lower governance deficiencies lead to more effective monitoring and lower CEO compensation, which reduces the firm's competitiveness for CEO talent.

Next, I solve equilibrium payoffs, which are generally not uniquely determined for matching models. In particular, equilibrium payoffs can be parameterized by the least talented CEO (CEO 3)'s compensation v_3 in this model. Lemma 2 shows that CEO 3 is unmatched in equilibrium, thus, I assume CEO 3's wage equals her autarky payoff, which is normalized to be 0.

Furthermore, the matching model is discrete, thus, there are match-specific rents left for bargaining between firms and their CEOs. Firm i's profit and CEO i's wage are solved by the following Nash bargaining game:

$$\begin{aligned} &\max_{\pi_i, v_i} (\pi_i - \pi_{i,o})^{1-\phi_i}(v_i - v_{i,o})^{\phi_i} \\ &s.t.\ \pi_i = \left(1 - \frac{(1-p)e_i}{pt_i}\right)(pst_i - v_i) \end{aligned} \tag{3}$$

where $\pi_{i,o}$ denotes firm i's outside option, $v_{1,o}$ denotes CEO i's outside option, and ϕ_i denotes CEO i's bargaining power.

Proposition 1. *In the matching equilibrium,*

i. CEO 1's outside option $v_{1,o}(e_2)$ strictly increases in firm 2's governance deficiencies e_2, and firm 1's outside option $\pi_{1,o}(e_2)$ strictly decreases in e_2.

ii. Firm 1's profit $\pi_1 = \phi_1(1 - \frac{(1-p)e_1}{pt_1})(pst_1 - v_{1,o}(e_2)) + (1-\phi_1)\pi_{1,o}(e_2)$ strictly decreases in both e_1 and e_2.

Part i. shows that higher governance deficiencies at firm 2 reduce firm 1's outside option and increase CEO 1's outside option. Intuitively, firm 1's outside option $\pi_{1,o}$ is its profit from deviating to match with CEO 2, and CEO 1's outside option $v_{1,o}$ is her wage from deviating to match with firm 2. As firm 2's governance deficiencies e_2 increases, firm 2's profit becomes lower, its CEO's compensation becomes higher, and monitoring becomes weaker. Thus, firm 1's outside option deteriorates because CEO 2 is more expensive for firm 1 to match with, and CEO 1's outside option improves because firm 2 is willing to pay more and monitor less to match with CEO 1. Part ii. shows that higher governance deficiencies at firm 2 reduce firm 1's profit, which can be understood from the Nash bargaining game (3) and Part i. Part ii. also shows that the externality is more related to firm 1's outside option when CEO 1's bargaining power is weaker.

Appendix 4 solves firm 1 and CEO 1's outside options. CEO 1 and CEO 2's compensation can be explicitly solved from the Nash bargaining problem (3)

$$v_1(v_2) = (1-\phi_1)\frac{ps(t_1-t_2)+\left(1-\frac{(1-p)e_2}{pt_2}\right)v_2}{1-\frac{(1-p)e_2}{pt_1}} + \phi_1\frac{ps(t_1-t_2)+\left(1-\frac{(1-p)e_1}{pt_2}\right)v_2}{1-\frac{(1-p)e_1}{pt_1}} \tag{4}$$

$$v_2 = \phi_2\frac{ps(t_2-t_3)}{1-\frac{(1-p)e_2}{pt_2}} \tag{5}$$

By equation (4), it is easy to verify that $v_1 < pst_1$ and $v_2 < pst_2$.[4] Thus, the assumption of $v < pst$ in Section 2.2.1 is satisfied in equilibrium. Equation (4) also shows that CEO 1 gains her bargaining power from her marginal talent over CEO 2: If $t_1 = t_2$, she would have the same wage as CEO 2.

Proposition 1 also shows that higher governance deficiencies at firm 1 reduce its profit. However, firm 1 is the firm matching with the more talented CEO. Thus, higher governance deficiencies increase a firm's competitiveness for CEO talent but decrease its profit for a given match. The following corollary shows that firm 1 earns a lower equilibrium profit than firm 2. Thus, the cost of higher governance deficiencies outweighs the benefit of higher CEO talent in equilibrium.

Corollary 1. *In matching equilibrium, $\pi_1 < \pi_2$ and $v_1 > v_2$. That is, the firm with strictly higher governance deficiencies earns strictly lower profit, and the CEO with strictly higher talent earns a strictly higher wage.*

[4] From equation (4), v_1 is strictly increasing in v_2. By $v_2 = \phi_1\frac{ps(t_2-t_3)}{1-\frac{(1-p)e_2}{pt_2}} < pt_2$, CEO 1's wage $v_1 < v_1(v_2 = pst_2) = pst_1$.

To see why, assume CEO 2's wage is weakly higher than CEO 1, then firm 2 can form a new match with CEO 1 and pay her a wage of $v_2 + \varepsilon$. For a sufficiently small ε, both firm 2 and CEO 1 will be better off because firm 2 benefits from a strictly higher CEO talent while paying a marginally higher wage and CEO 1 earns a strictly higher wage. Firm 2 and CEO 1 will break their current match and rematch with each other, which violates the stability requirement of the matching equilibrium. The same argument can be made to show $\pi_1 < \pi_2$.

2.2.3 Shareholder Activism in Period 1

The activist launches an activism campaign in period 1, and firm 2 is the target firm.[5] An activism campaign is costly to the activist, who needs to pay fees on disclosure, compliance, proxy advisors, governance consultants, and public relation professionals.[6] Specifically, the activist incurs a cost of $\frac{k}{2}(\bar{e} - e_2)^2$ to reduce the target firm's governance deficiencies from $\bar{e}$ to e_2. I assume the activism cost is sufficiently large, $k > \frac{1-p}{\bar{e}}s$, such that an interior solution exists on $[0, \bar{e}]$.[7] The payoff to the activist, who has full ownership of firm 2, is firm 2's period-2 profit net of period-1 activism costs. The activist solves the following problem:

$$\max_{e_2} \pi(s, e_2, t_2, v_2(e_2)) - \frac{k}{2}(\bar{e} - e_2)^2 \tag{6}$$

where $\pi(s, e_2, t_2, v_2)$ is the target firm's period-2 profit, given by equation (2).

To examine the efficiency of the activist's solution e_2^*, I compare it to efficient governance deficiencies e_2^o. I consider a social planner who maximizes total firm profits net of the shareholder activism cost, while respecting the matching equilibrium in period 2 and the optimal contract in

[5]The profit from targeting either firm is the same to the activist because firms are the same ex ante.

[6]Gantchev (2013) finds that an activism campaign ending in a proxy fight has average costs of \$10.71 million, which reduce shareholder returns by more than two-thirds.

[7]If $k \leq \frac{1-p}{\bar{e}}$, the activist always sets $e_2 = 0$.

period 3. Efficient governance deficiencies e_2^o is solved by

$$\max_{e_2} \pi(s,t_1,\bar{e},v_1(e_2)) + \pi(s,t_2,e_2,v_2(e_2)) - \frac{k}{2}(\bar{e}-e_2)^2 \tag{7}$$

where $\pi(s,t_1,\bar{e},v_1(e_2))$ and $\pi(s,t_2,e_2,v_2(e_2))$ are given by equation (2). The activist ignores the positive externality of her activism campaign, and solving the activist and the social planner's problem yields the following proposition.

Proposition 2. *The activist sets inefficiently high governance deficiencies. Formally, $e_2^* > e_2^o$.*

Two comments are in place. First, the cost function $\frac{k}{2}(\bar{e}-e_2)^2$ only accounts for the variable cost of shareholder activism. Apparently, conditional on launching the activism campaign, the presence of a fixed cost will not change the optimal choice of the activist or the social planner. However, the decision to launch the activism campaign depends on the fixed cost. More formally, let $\underline{\pi} = \max\limits_{e_2} \pi(s,e_2,t_2,v_2) - \dfrac{k}{2}(\bar{e}-e_2)^2$ and $\underline{\pi}^o = \max_{e_2} \pi(s,t_1,\bar{e},v_1(e_2)) + \pi(s,t_2,e_2,v_2) - \frac{k}{2}(\bar{e}-e_2)^2$. It is easy to prove that $\underline{\pi} < \underline{\pi}^o$. If shareholder activism incurs a fixed cost of d and the fixed cost is mildly large ($\underline{\pi} < d < \underline{\pi}^o$), then the activist will not launch the activism campaign while the social planner will. This suggests that the activist engages in too little activism in the presence of a fixed cost. Second, by assuming that the activist has full ownership stake in the target firm, this section does not speak about the free-rider problem: non-activist shareholders benefit from the activism without sharing the cost. If the activist has fractional ownership in the target firm, the inefficiency will be driven by both the positive externality and free-riding, which will be discussed in Section 3.2.

Note that equation (4) and (5) show that both CEOs' wages increase in firm 2's governance deficiencies. It is also clear that both firms' profits and monitoring intensities decrease in firm 2's governance deficiencies. Proposition 2 shows that social planner sets lower governance deficiencies than the activist. Thus, both CEOs' wages are strictly lower, and firms' monitoring intensities and profits are strictly higher under the social planner than under the activist.

3 Extensions

3.1 The Liquidity of the CEO Talent Market

The main model shows that outside options, determined by payoffs from matching with other partners in the market, are important channels to generate externalities. In this section, I consider how market frictions affect outside options and thus the shareholder activism externality. The friction I consider is the illiquidity of the CEO talent market: If a match breaks down, a CEO matches with a new firm and a firm matches with a new CEO with probability λ. Thus, the parameter λ is a measure of liquidity in the CEO talent market. Conceptually, labor market illiquidity can be understood as a result of search frictions (the CEO is only successful in locating a new firm with probability λ) or the limited transferability of CEO talent (the CEO's skill is only transferable to a new firm with probability λ).[8]

I first determine the equilibrium matching pattern. Unlike the main model, there may exist multiple equilibria when CEO talent is not perfectly transferable. For example, if $\lambda = 0$, any equilibrium matching pattern is possible because a matched partner is strictly preferred to any unmatched partner. In the section, I select the equilibrium matching pattern which can survive any liquidity parameter λ.

[8]Note that most theoretical papers in the firm-CEO matching literature implicitly assume that CEO talent is perfectly transferable. However, CEO talent has firm or industry-specific components, which limit its transferability. Custódio et al. (2013) measure the generality of CEO talent using a general ability index (GAI) and find different levels of generalities among different CEOs: GAI has a mean of zero with a standard deviation of 1, the minimum GAI is -1.504, and the maximum GAI is 7.23.

Lemma 3. *There always exists an equilibrium matching pattern such that firm 1 matches with CEO 1, firm 2 matches with CEO 2, and CEO 3 is unmatched.*

Next, I discuss equilibrium payoffs under the specific equilibrium matching pattern shown in Lemma 3. Like the main model, firms and CEOs bargain over the matching surplus, and outside options are endogenously determined. With probability λ, CEO 1's outside option is $v_{1,o}$, which is CEO 1's wage from deviating to match with firm 2, and with probability $1-\lambda$, CEO 1's outside option is 0. Thus, CEO 1's expected outside option is $\lambda v_{1,o}$. Similarly, firm 1's outside option is $\lambda \pi_{1,o}$. Firm i and CEO i's payoffs are given by the following Nash bargaining game:

$$\max_{\pi_i, v_i} (\pi_i - \lambda \pi_{i,o})^{1-\phi_i} (v_i - \lambda v_{i,o})^{\phi_i} \tag{8}$$
$$s.t.\ \pi_i = \left(1 - \frac{(1-p)e_i}{pt_i}\right)(pst_i - v_i)$$

which solves firm 1's profit $\pi_1 = (1-\phi_1)(1-\frac{(1-p)e_1}{pt_1})(pst_1 - \lambda v_{1,o}(e_2)) + \phi_1 \lambda \pi_{1,o}(e_2)$.

Similar to the main model, I use e_2^* to denote the activist's optimal choice of governance deficiencies and e_2^o to denote the social planner's efficient choice of governance deficiencies. The externality can be measured by the difference between the private solution and the socially-efficient solution $|e_2^* - e_2^o|$.

Proposition 3. *The shareholder activism externality is more severe when CEO talent market is more liquid. Formally,* $|e_2^* - e_2^o|$ *strictly increases in* λ.

Intuitively, when the CEO talent market is more liquid (either because search frictions are smaller or CEO talents are more transferable), CEO 1 and firm 1's outside options are more tied to firm 2's governance deficiencies e_2. For example, when $\lambda = 0$, the shareholder activism externality disappears because firm 1's profit is independent of e_2. The empirical literature shows that CEO talents have become more transferable and outside hires are more pervasive over the last several decades (e.g., Murphy and Zabojnik 2004; Custódio et al. 2013), thus, shareholder activism externalities play an increasingly important role in determining market efficiency.

3.2 Common Ownership

This section asks whether common ownership—when an activist has ownership in multiple firms—can improve market efficiency. On the one hand, when an activist cross-holds multiple firms, she maximizes portfolio firms' value instead of individual firms' value, thus, common ownership has the potential to internalize the positive externality within cross-held firms (Hansen and Lott, 1996; Rubin, 2006). On the other hand, by spreading ownership into multiple firms, common ownership reduces the activist's stake at the target firm, which exacerbates the free-rider problem (Grossman and Hart, 1980; Shleifer and Vishny, 1986).

To understand this tradeoff, I compare an activist (blockholder activist, hereafter) who optimally allocates her ownership in the target firm with an activist (cross-holding activist, hereafter) who passively cross-holds both the target and non-target firm. I examine whether shareholder activism is more efficient under the cross-holding activist than under the blockholder activist.

Note that theoretically, the cross-holding activist can launch activism campaigns in both firms. However, activists generally have small stakes in the target firm and the fixed cost of shareholder activism is likely to be large.[9] I thus make the following assumption:

Assumption 2. *The fixed cost of shareholder activism is sufficiently large such that the cross-holding activist will only engage in shareholder activism in one firm.*

The blockholder activist has a total of $\alpha < 1$ shares. She allocates $x \in [0, \alpha]$ shares to the non-

[9] For example, Brav et al. (2008) show that activist hedge funds' median ownership in target firms is 6.30%, using data from 2001 to 2006.

target firm to capture the positive externality from her activism campaign at the target firm. She solves the following problem:

$$\max_{e_2, x \in [0,\alpha]} x\pi(s, t_1, \bar{e}, v_1(e_2)) + (\alpha - x)\pi(s, t_2, e_2, v_2(e_2)) - \frac{k}{2}(\bar{e} - e_2)^2 \tag{9}$$

Lemma 4. *The blockholder activist optimally sets $x = 0$. That is, she concentrates all her stakes in the target firm.*

The explanation is as follows. Corollary 1 shows that the target firm has a strictly higher profit than the non-target firm. Thus, moving one share to the target firm from the non-target firm strictly increases the activist's payoff, and the activist optimally concentrates all her shares in the target firm.

Lemma 4 implies that instead of internalizing the externality across portfolio firms, considerations such as decreasing product market competition and increasing implicit collusion through common ownership are more likely to account for the rise of common ownership over recent years.[10]

Appendix 4 shows that the optimal governance deficiencies $e_2^*(\alpha) = e_2^* + (1-\alpha)\frac{1-p}{k}s > e_2^*$, where e_2^* is the activist's optimal choice of governance deficiencies with 100% ownership stakes in the target firm; Appendix 4 further shows that

$$e_2^b(\alpha) = e_2^o + \underbrace{\frac{1}{k}\left(1 - \frac{(1-p)\bar{e}}{pt_1}\right)\frac{\partial v_1(e_2)}{\partial e_2}}_{\text{positive externality}} + \underbrace{(1-\alpha)\frac{1-p}{k}s}_{\text{free-riding}} \tag{10}$$

where e_2^o is the social planner's efficient choice of governance deficiencies. Thus, with fractional ownership, the inefficiency of shareholder activism is caused by the activist's failure to internalize

[10]For recent work on the rise and anti-competitive effects of common ownership, see Azar (2017); He and Huang (2017); Azar et al. (2018).

the positive externality and free-riding from non-activist shareholders. Furthermore, equation (10) shows that the positive externality does not depend on the activist's position α, thus, assuming full ownership stakes in the main model yields a general result on shareholder activism externality.

Next, I compare the blockholder activist's solution with that of the cross-holding activist, who has a total of α shares and passively allocates $y \in (0, \alpha)$ shares in the non-target firm. The cross-holding activist solves

$$\max_{e_2} y\pi(s, t_1, \bar{e}, v_1(e_2)) + (\alpha - y)\pi(s, t_2, e_2, v_2(e_2)) - \frac{k}{2}(\bar{e} - e_2)^2 \tag{11}$$

where $y \in (0, \alpha)$ are the shares passively allocated in the non-target firm. I use e_2^c to denote the cross-holding activist's optimal choice of governance deficiencies.

Proposition 4. *The cross-holding activist sets higher governance deficiencies at the target firm if more shares are allocated in the non-target firm. Formally,* $\frac{\partial e_2^c}{\partial y} > 0$.

Allocating more shares in the non-target firm enables the activist to better internalize the positive externality, however, it also exacerbates the free-riding problem from non-activist shareholders. Proposition 4 implies that the cost of free-riding dominates.

Note that the blockholder activist is an activist with zero shares allocated in the non-target firm. Thus, Proposition 4 implies that the blockholder activist sets lower governance deficiencies than the cross-holding activist. Simple comparative statics also shows that the cross-holding activist earns a strictly lower profit than the blockholder activist. Thus, in the presence of a fixed cost, the cross-holding activist is less likely to engage in shareholder activism than the blockholder activist. The discussion is summarized in the following corollary.

Corollary 2. *Compared with concentrated ownership, shareholder activism under common ownership is less efficient:*

i. The cross-holding activist launches fewer shareholder activism campaigns.

ii. The cross-holding activist sets higher governance deficiencies.

From 2004 to 2016, index fund assets grew nearly fivefold from $554 billion to $2.6 trillion and index funds' share of long-term mutual fund assets more than doubled from 9.0 percent to 19.3 percent (Investment Company Institute, 2017). If index funds—who hold up to thousands of firms—displace large blockholders, common ownership increases at the expense of concentrated ownership. Corollary 2 implies that the increased popularity of index funds can reduce the efficiency of shareholder activism.[11]

3.3 Regulation

Can regulation improve the efficiency of shareholder activism? Proposition 2 implies that a regulator can fully restore market efficiency by imposing the efficient level of governance deficiencies e_2^o on the target firm. However, in a market with thousands of firms, regulating each individual firm is highly impractical because of the regulator's limited access to firm-level information, enforcement costs, or legal challenges mounted by corporate lobbyists. Thus, I consider a more realistic approach: The regulator sets a universal governance deficiency ceiling for all firms. For example, the Sarbanes-Oxley Act mandates that public firms have fully independent audit, compensation and governance committees, which essentially sets an upper bound on the lack of board independence for all US-listed public firms. I ask whether policies like this can help internalize the externality of shareholder activism and improve market efficiency.

To this end, I modify the main model by considering two activists, who simultaneously launch

[11] Index funds generally don't directly engage in shareholder activism. However, Brav et al. (2008) and Appel et al. (2016) show that passive investors play a key role in influencing firms' governance choices through their large voting blocs—passive ownership is associated with less support for management proposals and more support for shareholder-initiated governance proposals.

activism campaigns in period 1. Two firms have different sizes, denoted by s_1 and s_2 for firm 1 and 2. Without loss of generality, I assume $s_1 \geq s_2$. Each activist can only target one firm and each firm can only be targeted by one activist. I further assume $t_2 = t_3$, which implies CEO 2's wage $v_2 = 0$ (by equation (5)). This assumption greatly simplifies my analysis without changing the model's qualitative predictions. The rest of the setup is the same as the main model.

3.3.1 Matching in Period 2

I start from period 2 because period-3 contract is the same as the main model. I consider two candidate equilibrium matching patterns: firm 1 matches with CEO 1 (thus, firm 2 matches with CEO 2) and firm 1 matches with CEO 2 (thus, firm 2 matches with CEO 1). Generally, it is challenging to solve the equilibrium matching pattern when firms differ in both governance deficiencies and size because matching is multi-dimensional. However, it is possible to characterize the equilibrium matching pattern with only two firms.

Lemma 5. *In the matching equilibrium, CEO 3 always stays unmatched; firm 1 matches with CEO 1 and firm 2 matches with CEO 2*

i. when firm 1's size $s_1 \geq \frac{t_1}{t_1 - t_2} s_2$

ii. if and only if $e_1 \geq -\frac{p t_1 (s_1 - s_2)}{(1-p) s_2} + \frac{s_1}{s_2} e_2$ *when firm 1's size* $s_1 \in [s_2, \frac{t_1}{t_1 - t_2} s_2)$

When firm 1's size is sufficiently large, it matches with the more talented CEO irrespective of its governance deficiencies; when firm 1's size is smaller than $\frac{t_1}{t_1 - t_2} s_2$ (but still larger than s_2), it matches with the more talented CEO if and only if its governance deficiencies are sufficiently large. When both firms' sizes are the same, Lemma 5 reduces to Lemma 2: The firm with higher governance deficiencies matches with the more talented CEO.

Intuitively, the equilibrium matching pattern is determined by firms' competitiveness for CEO talent. A firm with higher governance deficiencies is more competitive because it offers higher

CEO compensation, and a firm with a larger size is more competitive because a CEO is more productive in a larger firm. Thus, when firm 1 is sufficiently large, it outcompetes firm 2 for CEO talent and matches with the high talent CEO; when firm 1'size is smaller, firm 2 with sufficiently high governance deficiencies can outcompete firm 1 and match with the high talent CEO.

Figure 3 plots matching patterns under different governance deficiencies, with 1-1 matching referring to the matching pattern of "firm 1 matches with CEO 1" and 1-2 matching referring to the matching pattern of "firm 1 matches with CEO 2". Figure 3b shows the case of $s_1 \in \left(s_2, \frac{t_1}{t_1-t_2}s_2\right)$, where the line connecting $\bar{e_1} = \frac{t_1 s_2 + s_1 t_2 - t_1 s_1}{s_2 t_2}\frac{pt_2}{1-p} < \frac{pt_2}{1-p}$ and $\underline{e_2} = \frac{pt_1(s_1-s_2)}{(1-p)s_1} < \frac{pt_2}{1-p}$ determines the boundary of two different matching patterns.[12] Note that both $\bar{e_1}$ and $\underline{e_2}$ decrease in s_1, thus, the 1-2 region decreases as s_1 increases, that is, the matching pattern of "firm 1 matches with CEO 2" becomes less likely as firm 1's size becomes larger.[13] When $s_1 \geq \frac{t_1}{t_1-t_2}s_2$, 1-2 region disappears, and "firm 1 matches with CEO 1" is the unique matching pattern, shown by Figure 3a; when s_1 equals s_2, 1-2 regions becomes the largest, shown by Figure 3c.

Next, I solve the equilibrium payoffs. Similar to the main model, I solve the Nash bargaining game between firm 1 and CEO 1. Equation (4) implies that, with $v_2 = 0$, CEO 1's wage is given by

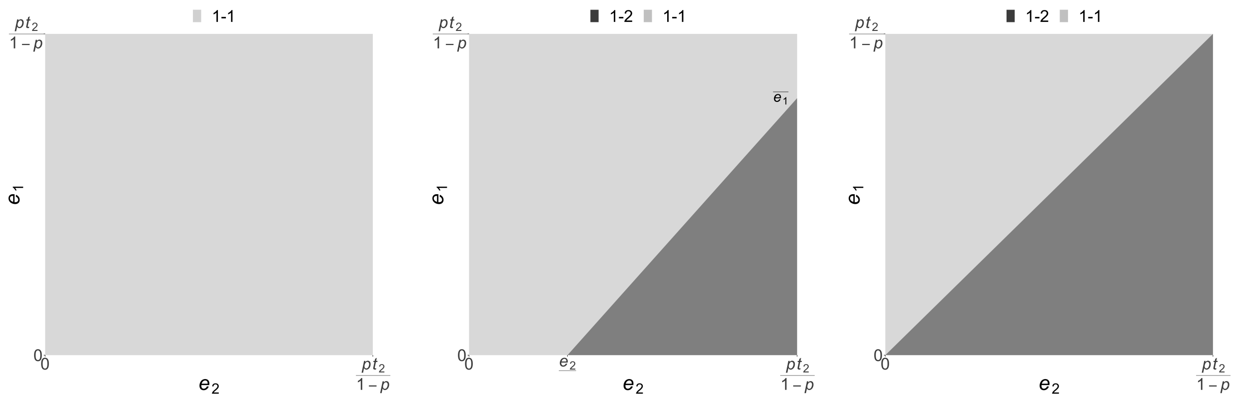
$$v_1 = (1-\phi_1)\frac{ps_2(t_1-t_2)}{1-\frac{(1-p)e_2}{pt_1}} + \phi_1\frac{ps_1(t_1-t_2)}{1-\frac{(1-p)e_1}{pt_1}} \tag{12}$$

if CEO 1 matches with firm 1, and

$$v_1 = \phi_1\frac{ps_2(t_1-t_2)}{1-\frac{(1-p)e_2}{pt_1}} + (1-\phi_1)\frac{ps_1(t_1-t_2)}{1-\frac{(1-p)e_1}{pt_1}} \tag{13}$$

[12] To prove $\underline{e_1} < \frac{pt_2}{1-p}$, it suffices to prove $\frac{t_1 s_2 + s_1 t_2 - t_1 s_1}{s_2 t_2} < 1$, which is equivalent to $(s_1-s_2)(t_1-t_2) > 0$. And to prove $\underline{e_2} = \frac{pt_1(s_1-s_2)}{(1-p)s_1} < \frac{pt_2}{1-p}$, it suffices to prove $\frac{pt_1(s_1-s_2)}{ps_1 t_2} < 1$, which is equivalent to $s_1 < \frac{ps_2 t_1}{pt_1 - pt_2} = \frac{t_1}{t_1-t_2}s_2$.

[13] $\bar{e_1} = \frac{t_1 s_2 + s_1 t_2 - t_1 s_1}{s_2 t_2}\frac{pt_2}{1-p}$ decreases in s_1 because $t_2 < t_1$, and $\underline{e_2} = \frac{pt_1(s_1-s_2)}{(1-p)s_1} = \frac{pt_1}{1-p}(1-\frac{s_2}{s_1})$ apparently decreases in s_1.



(a) $s_1 \geq \frac{pt_1 s_2 - v_2}{pt_1 - pt_2}$ (b) $s_2 < s_1 < \frac{t_1}{t_1 - t_2} s_2$ (c) $s_1 = s_2$

Figure 3: **Period-2 Equilibrium Matching Patterns**. This figure plots different matching patterns under governance deficiencies $(e_1, e_2) \in [0, \frac{pt_2}{1-p}] \times [0, \frac{pt_2}{1-p}]$. "1-1" refers to the matching pattern of "firm 1 matches with CEO 1" and "1-2" refers to the matching pattern of "firm 1 matches with CEO 2". For Figure 3b, $\bar{e}_1 = \frac{t_1 s_2 + s_1 t_2 - t_1 s_1}{s_2 t_2} \frac{pt_2}{1-p} < \frac{pt_2}{1-p}$, and $\underline{e_2} = \frac{pt_1(s_1 - s_2)}{(1-p)s_1} < \frac{pt_2}{1-p}$.

if CEO 1 matches with firm 2. Firms' profits depend on the equilibrium matching pattern: $\pi_i = \left(1 - \frac{(1-p)e_i}{pt_i}\right)(ps_i t_i - v_i), i \in \{1,2\}$ if firm 1 matches with CEO 1 and $\pi_i = \left(1 - \frac{(1-p)e_i}{pt_i}\right)(ps_i t_{-i} - v_{-i}), i \in \{1,2\}, -i \in \{1,2\} \backslash i$ if firm 1 matches with CEO 2.

3.3.2 Nash Equilibrium in Period 1

Denote activist $i \in \{1,2\}$'s governance deficiency choice in period 1 by e_i^*. I look for (e_1^*, e_2^*) which satisfies: (1) Nash equilibrium in period 1, that is, activist i can not earn a higher payoff by deviating from e_i^*, given the other activist $-i$'s choice e_{-i}^*. (2) matching equilibrium in period 2, that is, given governance deficiencies (e_1^*, e_2^*) in period 1, matching is stable.

Despite the simplicity of the model, solving Nash equilibrium is non-trivial because activists' governance deficiency choices in period 1 affect the period-2 matching, which is complicated by multidimensionality (firms differ in both governance deficiencies and size). Thus, I consider two special cases: Firm 1 is sufficiently large, and firm 1 is sufficiently small (but still larger than firm

2). The lemma below shows that the equilibrium matching patterns are the same under the two special cases, which greatly simplifies the policy analysis below.

Lemma 6. *When firm 1's size is either sufficiently larger than firm 2 or similar to firm 2, firm 1 matches with CEO 1 and firm 2 matches with CEO 2. Formally, there exists $\bar{s_1} > \underline{s_1}$ such that for $s_1 \in [s_2, \underline{s_1}] \cup [\bar{s_1}, +\infty)$, firm 1 matches with CEO 1 and firm 2 matches with CEO 2.*

Next, to examine the efficiency of the competitive market solution (e_1^*, e_2^*), I solve the efficient levels of governance deficiencies by considering a social planner who sets both firms' governance deficiencies to maximize total firm profits net of activism costs.

$$\max_{e_1,e_2} \pi(s_1,t_1,e_1,v_1(e_2)) + \pi(t_2,s_2,e_2,v_2) - \frac{k}{2}(\bar{e}-e_1)^2 - \frac{k}{2}(\bar{e}-e_2)^2 \tag{14}$$

I denote the social planner's solution by (e_1^o, e_2^o). The social planner's solution (e_1^o, e_2^o) and the competitive market solution (e_1^*, e_2^*) satisfy

Lemma 7. *Both activists set inefficiently high governance deficiencies. Formally, $e_1^o < e_1^*$ and $e_2^o < e_2^*$.*

Intuitively, activist 2 sets inefficiently high governance deficiencies because she ignores the positive externality from her activism campaign at firm 2. Furthermore, due to the strategic complementarity between the two activists, activist 1 sets inefficiently high governance deficiencies if activist 2 sets inefficiently high governance deficiencies.

Next, I consider a regulator who chooses a governance deficiency ceiling e_r, and both activists must set the level of governance deficiencies: $e_1, e_2 < e_r$. The proposition below shows that the structure of the CEO talent market is important for the effectiveness of policy interventions.

Proposition 5. *When firm 1's size is sufficiently larger than firm 2, the regulator sets the governance deficiency ceiling at $e_r = e_2^o$, and the market restores to full efficiency. When firm 1's size is similar to firm 2, the regulator should do nothing.*

Intuitively, the inefficiency of shareholder activism can be remedied by reducing governance deficiencies at firm 2, whose governance deficiencies are inefficiently high due to the shareholder activism externality. When firm 1's size is similar to firm 2, firm 1's size advantage is not enough to win the talent competition. Thus, activist 1 optimally sets higher governance deficiencies at firm 1 than firm 2 to match with the high talent CEO. In this case, a governance deficiency ceiling, which further reduces firm 2's governance deficiencies to remedy shareholder activism inefficiency, will be too low for firm 1 such that overregulation occurs and market efficiency further decreases. The regulator should do nothing.

When firm 1 is sufficiently larger than firm 2, activist 1 optimally sets lower governance deficiencies at firm 1 than firm 2 to match with the high talent CEO because firm 1's size advantage is enough to win the talent competition. In this case, a governance deficiency ceiling set at a level lower than firm 2's governance deficiencies but still higher than firm 1's governance deficiencies can internalize the shareholder activism externality at firm 2 and avoid over-regulating firm 1.

4 Conclusion

This paper examines the positive externality of shareholder activism. Shareholder activism strengthens the non-target firm's outside option and weakens its CEO's outside option, both of which are linked to the target firm through the market for CEO talent. Shareholder activism leads to lower CEO compensation, higher firm profit, and more effective monitoring at the non-target firm. Due to this positive externality, the activist's intervention is inefficiently low.

I proceed to discuss how the inefficiency of shareholder activism is affected by the liquidity of the CEO talent market, common ownership, and regulation. When the CEO talent market is more liquid—either because of more transferable CEO talent or lower search frictions—the shareholder activism externality is more severe. I next ask whether common ownership—when an activist holds ownership stakes in multiple firms—can affect the efficiency of shareholder activism. Common ownership internalizes the positive externality by holding multiple firms; however, it also reduces the activist's stakes at the target firm, which exacerbates free-riding from non-activist shareholders. I show that the cost of free-riding dominates and common ownership results in less efficient shareholder activism. I then ask whether a regulator mandating a universal governance deficiency ceiling can improve market efficiency. I show that the effect of the regulation depends on the distribution of firm size: If firms have similar sizes, the regulation results in less efficient shareholder activism and the regulator should do nothing; if one firm is sufficiently larger than the other, the regulation leads to more efficient shareholder activism.

Appendix A. Proofs

Proof of Lemma 1

First, note that ICst and IC0 become stronger if $w_{st,0}$ and $w_{0,st}$ become smaller, thus, the firm sets $w_{st,0} = w_{0,st} = 0$. LL and FC imply $w_0 = w_{00} = 0$. The left hand side of IC0 is zero. The right hand side of IC0 is (weakly) negative because $w_{0,st} = 0$ and $w_{st} - st \leq 0$ (implied by FC), thus, IC0 is satisfied and the CEO will not falsify a report of st when the realized cash flow is 0. This implies a report of cash flow st must be true and the firm will not audit the report. Thus, $g_{st} = 0$. Substituting $w_0 = w_{00} = 0$, $w_{st,0} = w_{0,st} = 0$, and $g_{st} = 0$ to program (1) yields

$$\pi(s,e,t,v) = \underset{g_0, w_{st}}{Max}\ pst - pw_{st} - (1-p)\, sg_0$$

$$t.t.\ pw_{st} \geq v \qquad (PC)$$

$$w_{st} \geq \left(1 - \frac{g_0}{e}\right) st \qquad (ICst)$$

ICst must be binding, if not, the firm can increase its profit by reducing g_0. Thus, $g_0 = e\left(1 - \frac{w_{st}}{st}\right)$. Substituting $w_{st} = \left(1 - \frac{g_0}{e}\right) st$ to the above maximization problem yields

$$\pi(s,e,t,v) = \underset{g_0}{Max}\ pst\frac{g_0}{e} - (1-p)\, sg_0$$

$$t.t.\ pst\left(1 - \frac{g_0}{e}\right) \geq v \qquad (PC)$$

PC must be binding, if not, the firm can increase its profit by increasing g_0, thus $g_0 = e\left(1 - \frac{v}{pst}\right)$. This implies $w_{st} = \frac{v}{p}$ and the expected CEO compensation $pw_{st} = v$. Note that substituting $g_0 = e\left(1 - \frac{v}{pst}\right)$ to the firm's profit function yields $\pi(s,e,t,v) = \left(1 - \frac{(1-p)e}{pt}\right)(pst - v)$.

Proof of Lemma 2

First, I prove that CEO 3 is unmatched in equilibrium. Without loss of generality, assume firm $i \in \{1,2\}$ matches with CEO 3, then there must exist an unmatched CEO $j \in \{1,2\}$ who receives an autarky payoff of 0. Assume CEO 3's wage is $v_3 \geq 0$, then firm i's profit is $\pi(s, e_i, t_3, v_3)$. Because $t_j > t_3, j \in \{1,2\}$, thus $\pi(s, e_i, t_j, v_3) > \pi(s, e_i, t_3, v_3)$. Thus, both firm i and CEO j can be strictly better off by matching with each other, and CEO 3 is unmatched in equilibrium.

Because firms are only different in governance deficiencies and CEOs are only different in talents, the matching model is one-dimensional. The firm's profit function (2) exhibits non-transferability: a one dollar decrease in CEO compensation does not lead to a one dollar increase in firm profit. In this case, the equilibrium matching pattern cannot be pinned down by the cross partial derivative with respect to governance deficiencies and talents (Legros and Newman, 2007). Instead, I draw the results on matching with nontransferable utilities from Legros and Newman (2007) and Chade et al. (2017). I use equation (9) of Chade et al. (2017), which is a more tractable version of Legros and Newman (2007)'s "generalized increasing difference" condition, to prove Lemma 2. Note that the matching pattern shown in Lemma 2 is positive assortative: a firm of higher governance deficiencies matches with a CEO of higher talent. Equation (9) of Chade et al. (2017) implies that PAM is the equilibrium matching pattern if and only if $\pi_{et}(s,e,t,v) \geq \frac{\pi_t(e,t,v)}{\pi_v(e,t,v)} \pi_{ev}(s,e,t,v)$, where $\pi_{et}(s,e,t,v)$ is the cross-partial of the firm's profit function (2) on e and t, and $\pi_t(s,e,t,v)$ is the first order derivative of the firm's profit function on CEO talent t. π_{ev} and π_v are similarly defined. Note that $\pi_v(s,e,t,v) < 0$, thus $\pi_{et}(s,e,t,v) \geq \frac{\pi_t(e,t,v)}{\pi_v(e,t,v)} \pi_{ev}(s,e,t,v)$ can be written as

$$\pi_{ev}(s,e,t,v)\,\pi_t(s,e,t,v) - \pi_{et}(s,e,t,v)\,\pi_v(s,e,t,v) \geq 0$$

Simple algebra yields the sufficient and necessary condition $(1-p)(pst-v) \geq 0$, which holds because $1 \geq p$ and $pst \geq v$.

Proof of Proposition 1

To prove part i., I first solve CEO 2's wage. CEO 2's outside option $v_{2,o} = 0$ because there does not exist a firm smaller than firm 2. Firm 2's outside option $\pi_{2,o} = \pi(s, e_2, t_3, 0)$, which is firm 2' profit from deviating to match with CEO 3. Solving firm 2 and CEO 2's Nash bargaining game (3) yields

$$v_2(e_2) = \phi_1 \frac{ps(t_2 - t_3)}{1 - \frac{(1-p)e_2}{pt_2}} \tag{15}$$

which strictly increases in e_2. To solve firm 1 and CEO 1's outside option, note that $\pi_{1,o}(e_2)$ is firm 1's profit from deviating to match with CEO 2, thus,

$$\pi_{1,o}(e_2) = \pi(s, e_1, t_2, v_2) = \left(1 - \frac{(1-p)e_1}{pt_2}\right)(pst_2 - v_2(e_2)) \tag{16}$$

and $v_{1,o}(e_2)$ is CEO 1's wage from deviating to match with firm 2, thus $v_{1,o}$ is solved by $\pi(s, e_2, t_1, v_{1,o}) = \pi(s, e_2, t_2, v_2)$, which yields

$$v_{1,o}(e_2) = pst_1 - \frac{1 - \frac{(1-p)e_2}{pt_2}}{1 - \frac{(1-p)e_2}{pt_1}}(pst_2 - v_2(e_2)) \tag{17}$$

And because $v_2(e_2)$ strictly increases in e_2, then $\pi_{1,o}(e_2)$ strictly decreases in e_2. Substituting $v_2(e_2)$ into $v_{1,o}(e_2)$ and after some algebra, $\frac{\partial v_{1,o}(e_2)}{\partial e_2} = (1-p)s \frac{1 - \frac{pt_2 - \phi_1 p(t_2 - t_3)}{pt_1}}{\left(1 - \frac{(1-p)e_2}{pt_1}\right)^2} > 0$. That is, $v_{1,o}(e_2)$ strictly increases in e_2.

To prove Part ii., note that π_1 can be solved from the bargaining problem (3). And Part i. implies that π_1 strictly decreases in e_2. Furthermore, solving bargaining problem (3) also yields

CEO 1's compensation $v_1 = (1-\phi_1)v_{1,o} + \phi_1(pst_1 - \frac{\pi_{1,o}}{(1-\frac{(1-p)e_1}{pt_1})})$. Substituting equation (16) and (17) into v_1 yields equation (4).

Proof of Corollary 1

I start with CEO compensation. Note that CEO 1's wage $v_1 \geq v_{1,o}$, where $v_{1,o} = pst_1 - \frac{1-\frac{(1-p)e_2}{pt_2}}{1-\frac{(1-p)e_2}{pt_1}}(pst_2 - v_2)$ is her outside option. Thus, $v_1 - v_2 \geq v_{1,o} - v_2 = (t_1 - t_2)\frac{ps - \frac{(1-p)e_2}{pt_1 t_2}v_2}{1-\frac{(1-p)e_2}{pt_1}} > 0$, which holds because $t_1 > t_2$, $pst_2 > v_2$ and $1 - \frac{(1-p)e_2}{pt_1} > 0$ by Assumption 1. For firm profits, the following holds:

$$
\begin{aligned}
\pi_1 - \pi_2 &= \left(1 - \frac{(1-p)e_1}{pt_1}\right)(pst_1 - v_1) - \pi_2 \\
&\leq \left(1 - \frac{(1-p)e_1}{pt_1}\right)(pst_1 - v_{1,o}) - \pi_2 \\
&= \left(1 - \frac{(1-p)e_1}{pt_1}\right)\left(pst_1 - pst_1 + \frac{\pi_2}{1 - \frac{(1-p)e_2}{pt_1}}\right) - \pi_2 \\
&= \left(\frac{1 - \frac{(1-p)e_1}{pt_1}}{1 - \frac{(1-p)e_2}{pt_1}} - 1\right)\pi_2 < 0
\end{aligned}
$$

The first inequality holds because $v_1 \geq v_{1,o}$, and the second inequality holds because $e_1 > e_2$.

Proof of Proposition 2

Substituting $v_2(e_2) = \phi_2 \frac{ps(t_2 - t_3)}{1-\frac{(1-p)e_2}{pt_2}}$ into the activist 2's objective function $\left(1 - \frac{(1-p)e_2}{pt_2}\right)(pst_2 - v_2(e_2)) - \frac{k}{2}(\bar{e} - e_2)^2$, it is straightforward to show the objective function is strictly concave in e_2. First order condition yields the activist's optimal solution $e_2^* = \bar{e} - \frac{(1-p)s}{k}$. Under the assumption of $k > \frac{1-p}{\bar{e}}s$, $e_2^* \in [0, \bar{e}]$.

The social planner's problem (7) can be written as

$$\max_{e_2} \left(1-\frac{(1-p)\bar{e}}{pt_1}\right)(pst_1 - v_1(e_2)) + \left(1-\frac{(1-p)e_2}{pt_2}\right)(pst_2 - v_2(e_2)) - \frac{k}{2}(\bar{e}-e_2)^2 \tag{18}$$

with $v_1(e_2)$ given by equation (4). The second order condition for (18) is

$$SOC = -\left(1-\frac{(1-p)e_1}{pt_1}\right)\frac{\partial^2 v_1(e_2)}{\partial e_2^2} - k$$

By equation (4), after some algebra, $\frac{\partial^2 v_1(e_2)}{\partial e_2^2} > 0$. Thus, $SOC < 0$. The social planner's objective function (18) is strictly concave. First order condition yields the social planner's optimal choice $e_2^o = e_2^* - \frac{1}{k}\left(1-\frac{(1-p)\bar{e}}{pt_1}\right)\frac{\partial v_1(e_2)}{\partial e_2} < e_2^*$.

Proof of Lemma 3

The Lemma can be proved more formally by showing that both CEOs and firms will not deviate to match with other partners in the market. However, this lemma is essentially a corollary of Lemma 2. Lemma 2 shows that firm 1 will not match with CEO 2 in equilibrium when CEO talents are perfectly transferable. This result is stronger when CEO 2's talent is only partially transferable to firm 1. The same logic applies to firm 2 and CEO 2.

Proof of Proposition 3

I first solve CEO 2's wage. CEO 2's outside option $v_{2,o} = 0$ because there does not exist a firm smaller than firm 2. Firm 2's outside option $\pi_{2,o} = \pi(s, e_2, t_3, 0)$, which is firm 2' profit from deviating to match with CEO 3. Solving firm 2 and CEO 2's Nash bargaining game (8) yields $v_2(e_2,\lambda) = \phi_2 \frac{ps(t_2-\lambda t_3)-(1-\lambda)s(1-p)e_2}{1-\frac{(1-p)e_2}{pt_2}}$.

The activist's problem is $\max_{e_2} \pi(s,e_2,t_2,v_2(e_2,\lambda)) - \frac{k}{2}(\bar{e}-e_2)^2$. It is straightforward to verify that the objective function is strictly concave, thus, the first order condition yields the activist's optimal choice of governance deficiencies $e_2^* = \bar{e} - \frac{1-\phi_2(1-\lambda)}{k}s(1-p)$. The social planner's problem (7) can be written as

$$\max_{e_2} \left(1-\frac{(1-p)\bar{e}}{pt_1}\right)(pst_1 - v_1(e_2,\lambda)) + \left(1-\frac{(1-p)e_2}{pt_2}\right)(pst_2 - v_2(e_2,\lambda)) - \frac{k}{2}(\bar{e}-e_2)^2 \quad (19)$$

with $v_1(e_2) = (1-\phi_1)\lambda v_{1,o} + \phi_1(pst_1 - \frac{\lambda \pi_{1,o}}{(1-\frac{(1-p)e_1}{pt_1})})$. Similar to the proof of Proposition 1, the outside options are given by

$$\pi_{1,o}(e_2,\lambda) = \left(1-\frac{(1-p)e_1}{pt_2}\right)(pst_2 - v_2(e_2,\lambda))$$
$$v_{1,o}(e_2,\lambda) = pst_1 - \frac{1-\frac{(1-p)e_2}{pt_2}}{1-\frac{(1-p)e_2}{pt_1}}(pst_2 - v_2(e_2,\lambda))$$

It is straightforward to verify that the objective function (19) is strictly concave (after some tedious algebra). First order condition yields the social planner's optimal choice e_2^o and $e_2^* - e_2^o = \frac{1}{k}\left(1-\frac{(1-p)\bar{e}}{pt_1}\right)\frac{\partial v_1(e_2,\lambda)}{\partial e_2}$. Note that $\frac{\partial^2 v_1(e_2,\lambda)}{\partial \lambda \partial e_2} = (1-\phi_1)\left(\frac{\partial v_{1,o}}{\partial e_2} + \lambda \frac{\partial^2 v_{1,o}}{\partial e_2 \partial \lambda}\right) - \phi_1 \frac{\left(\frac{\partial \pi_{1,o}}{\partial e_2} + \lambda \frac{\partial^2 \pi_{1,o}}{\partial \lambda \partial e_2}\right)}{1-\frac{(1-p)e_1}{pt_1}}$. It is easy to verify that $\frac{\partial v_{1,o}}{\partial e_2} > 0$, $\frac{\partial^2 v_{1,o}}{\partial \lambda \partial e_2} > 0$, $\frac{\partial \pi_{1,o}}{\partial e_2} < 0$, and $\frac{\partial^2 \pi_{1,o}}{\partial \lambda \partial e_2} < 0$. Thus,

$$\frac{\partial(e_2^* - e_2^o)}{\partial \lambda} = \frac{1}{k}\left(1-\frac{(1-p)\bar{e}}{pt_1}\right)\frac{\partial^2 v_1(e_2,\lambda)}{\partial \lambda \partial e_2} > 0$$

That is, shareholder activism externality is more severe when the CEO talent market is more liquid.

Proof of Lemma 4

Denote $\pi_b(x) = \max_{e_2} x\pi(s,t_1,\bar{e},v_1(e_2)) + (\alpha - x)\pi(s,t_2,e_2,v_2(e_2)) - \frac{k}{2}(\bar{e}-e_2)^2$. By the envelope theorem, $\frac{\partial \pi_b(x)}{\partial x} = \pi(s,t_1,\bar{e},v_1(e_2^*)) - \pi(s,t_2,e_2^*,v_2(e_2^*)) < 0$, where "$<$" follows from Corollary 1. The blockholder activist optimally chooses $x = 0$, that is, she concentrates all her stakes in the target firm.

With $x = 0$, the blockholder activist's optimal choice is $e_2^b = argmax_{e_2}\ \alpha\pi(s,t_2,e_2,v_2(e_2)) - \frac{k}{2}(\bar{e}-e_2)^2$. Similar to the proof of Proposition 2, $\alpha\pi(s,t_2,e_2,v_2(e_2)) - \frac{k}{2}(\bar{e}-e_2)^2$ is strictly concave in e_2, thus, the first order condition yields the optimal solution $e_2^b(\alpha) = \bar{e} - \alpha\frac{1-p}{k}s$. And the proof of Proposition 2 shows that $e_2^* = \bar{e} - \frac{1-p}{k}s$, thus, $e_2^*(\alpha) = e_2^* + (1-\alpha)\frac{1-p}{k}s > e_2^*$. Using $e_2^o = e_2^* - \frac{1}{k}\left(1 - \frac{(1-p)\bar{e}}{pt_1}\right)\frac{\partial v_1(e_2)}{\partial e_2}$ from the proof of Proposition 2, I can show that

$$e_2^b(\alpha) = e_2^o + \underbrace{\frac{1}{k}\left(1 - \frac{(1-p)\bar{e}}{pt_1}\right)\frac{\partial v_1(e_2)}{\partial e_2}}_{\text{positive externality}} + \underbrace{(1-\alpha)\frac{1-p}{k}s}_{\text{free-riding}}$$

Proof of Proposition 4

The result can be proved using simple comparative statics. Denote

$$f(y,e_2) = y\pi(s,t_1,\bar{e},v_1(e_2)) + (\alpha - y)\pi(s,t_2,e_2,v_2(e_2)) - \frac{k}{2}(\bar{e}-e_2)^2$$

Then $\frac{\partial^2 f(y,e_2)}{\partial y \partial e_2} = \left(1 - \frac{(1-p)\bar{e}}{pt_1}\right)\left(-\frac{\partial v_1(e_2)}{\partial e_2}\right) + (1-p)s$. Note that

$$
\begin{aligned}
\frac{\partial v_1(e_1,e_2)}{\partial e_2} &= (1-\phi_1)\frac{ps(t_1-t_2)+\phi_2 ps(t_2-t_3)}{(1-\frac{(1-p)e_2}{pt_1})^2}\frac{(1-p)}{pt_1}+\phi_1\frac{\left(1-\frac{(1-p)e_1}{pt_2}\right)\phi_2}{1-\frac{(1-p)e_1}{pt_1}}\frac{ps(t_2-t_3)}{\left(1-\frac{(1-p)e_2}{pt_2}\right)^2}\frac{(1-p)}{pt_2}\\
&< (1-\phi_1)\frac{ps(t_1-t_2)+ps(t_2-t_3)}{(1-\frac{(1-p)e_2}{pt_1})^2}\frac{(1-p)}{pt_1}+\phi_1\frac{1-\frac{(1-p)e_1}{pt_2}}{1-\frac{(1-p)e_1}{pt_1}}\frac{ps(t_2-t_3)}{\left(1-\frac{(1-p)e_2}{pt_2}\right)^2}\frac{(1-p)}{pt_2}
\end{aligned}\tag{20}
$$

$$
\begin{aligned}
&< (1-\phi_1)\frac{ps(t_1-t_3)}{(1-\frac{(1-p)\bar{e}}{pt_1})^2}\frac{(1-p)}{pt_1}+\phi_1\frac{1-\frac{(1-p)e_1}{pt_2}}{1-\frac{(1-p)e_1}{pt_1}}\frac{ps(t_2-t_3)}{\left(1-\frac{(1-p)\bar{e}}{pt_2}\right)^2}\frac{(1-p)}{pt_2}\\
&< (1-\phi_1)\frac{ps(t_1-t_3)}{(1-\frac{(1-p)\bar{e}}{pt_1})^2}\frac{(1-p)}{pt_1}+\phi_1\frac{ps(t_2-t_3)}{\left(1-\frac{(1-p)\bar{e}}{pt_2}\right)^2}\frac{(1-p)}{pt_2}
\end{aligned}
$$

where the first inequality is obtained by setting $\phi_2 = 1$, the second inequality is from $e_2 < \bar{e}$, and the third inequality is from $\frac{1-\frac{(1-p)e_1}{pt_2}}{1-\frac{(1-p)e_1}{pt_1}} < 1$. Thus,

$$
\begin{aligned}
\frac{\left(1-\frac{(1-p)\bar{e}}{pt_1}\right)\frac{\partial v_1(e_1,e_2)}{\partial e_2}}{(1-p)s} &< (1-\phi_1)\frac{p(t_1-t_3)}{(1-\frac{(1-p)\bar{e}}{pt_1})}\frac{1}{pt_1}+\phi_1\frac{p(t_2-t_3)}{\left(1-\frac{(1-p)\bar{e}}{pt_2}\right)}\frac{1}{pt_2}\\
&= (1-\phi_1)\frac{p(t_1-t_3)}{pt_1-(1-p)\bar{e}}+\phi_1\frac{p(t_2-t_3)}{pt_2-(1-p)\bar{e}}\\
&< (1-\phi_1)+\phi_1 = 1
\end{aligned}
$$

where the first inequality is implied by (20) and the second inequality is by Assumption 1. Thus, $\frac{\partial^2 f(y,e_2)}{\partial y \partial e_2} = \left(1-\frac{(1-p)\bar{e}}{pt_1}\right)\left(-\frac{\partial v_1(e_2)}{\partial e_2}\right)+(1-p)s > 0$, and the cross-holding activist's optimal choice of governance deficiencies e_2^c strictly increases in y, her stakes in the non-target firm.

Proof of Lemma 5

The matching is multidimensional because firms differ in both governance deficiencies and size. Thus, existing results on one-dimensional matching (e.g., Becker, 1973; Legros and Newman, 2007) cannot be applied here. However, there are only two firms and two CEOs in the model, thus, there are only two possible matching patterns, and I can use the stability requirement to find out the equilibrium matching patterns.

Consider that both firms submit wage offers to CEO 1. Denote v_1^i as firm i's highest wage offer for CEO 1. Firm i prefers to match with CEO 1 if and only if $\pi(s_i,e_i,t_1,v_1^i) \geq \pi(s_i,e_i,t_2,v_2)$, where $\pi(s_i,e_i,t_2,v_2)$ is firm i's profit by matching with CEO 2. Firm i's maximum wage offer for CEO 1 is thus solved by $\pi(s_i,e_i,t_1,v_1^i) = \pi(s_i,e_i,t_2,v_2)$, which yields $v_1^i = \frac{ps_i(t_1-t_2)}{1-e_i\frac{1-p}{pt_1}}$ by noting that $v_2 = 0$ under the assumption of $t_2 = t_3$. Firm 1 matches with CEO 1 if and only if it outbids firm 2, that is, $v_1^1 \geq v_1^2$, which yields $e_1 \geq -\frac{pt_1(s_1-s_2)}{(1-p)s_2} + \frac{s_1}{s_2}e_2$. When $s_1 \geq \frac{t_1}{t_1-t_2}s_2$,

$$
\begin{aligned}
-\frac{pt_1(s_1-s_2)}{(1-p)s_2} + \frac{s_1}{s_2}e_2 &\leq -\frac{pt_1(s_1-s_2)}{(1-p)s_2} + \frac{s_1}{s_2}\frac{pt_2}{1-p} \\
&= \frac{p}{(1-p)s_2}(s_1t_2 - t_1s_1 + t_1s_2) \leq 0
\end{aligned}
$$

where the first inequality follows from $e_2 < \frac{pt_2}{1-p}$ (Assumption 1). Thus, when $s_1 \geq \frac{t_1}{t_1-t_2}s_2$, $e_1 \geq -\frac{pt_1(s_1-s_2)}{(1-p)s_2} + \frac{s_1}{s_2}e_2$ always holds ,and firm 1 always matches with CEO 1.

Proof of Lemma 6

I prove a stronger version of Lemma 6: For $s_1 = s_2$ or $s_1 \geq \frac{t_1}{t_1-t_2}s_2$, firm 1 matches with CEO 1 and firm 2 matches with CEO 2 in period 2, and the unique Nash equilibrium in period 1 is given by

$$e_1^* = \bar{e} - \frac{(1-p)}{k}\left(s_1 - (1-\phi_1)\frac{ps_2\,(t_1-t_2)}{pt_1-(1-p)e_2^*}\right) \tag{21}$$

$$e_2^* = \bar{e} - \frac{1-p}{k}s_2 \tag{22}$$

To prove (e_1^*, e_2^*) is the unique Nash equilibrium, I show that (1) given (e_1^*, e_2^*), matching is stable, (2) activist i's optimal choice of governance deficiencies is e_i^*, given the other activist's choice e_{-i}^*.

I first prove the matching is stable. When $s_1 \geq \frac{t_1}{t_1-t_2}s_2$, Lemma 5 shows that "firm 1 matches with CEO 1" is the unique equilibrium matching pattern. Thus, period-2 matching is stable for any given (e_1^*, e_2^*). When $s_1 = s_2 = s$, clearly, $e_1^* > e_2^*$, and Lemma 2 implies that period-2 matching is stable.

Next, I prove that (e_1^*, e_2^*) is the Nash equilibrium in period 1. When $s_1 \geq \frac{t_1}{t_1-t_2}s_2$, "firm 1 matches with CEO 1 and firm 2 matches with CEO 2" is the unique equilibrium matching pattern by Lemma 5. Given matching with CEO 2, activist 2 solves $\max_{e_2}\left(1-\frac{(1-p)e_2}{pt_2}\right)(ps_2t_2 - v_2) - \frac{k}{2}(\bar{e}-e_2)^2$, which yields $e_2^* = \bar{e} - \frac{1-p}{k}s_2$; given matching with CEO 1, activist 1 solves $\max_{e_1}\left(1-\frac{(1-p)e_1}{pt_1}\right)(ps_1t_1 - v_1(e_1, e_2^*)) - \frac{k}{2}(\bar{e}-e_1)^2$ which yields

$$e_1^* = \bar{e} - \frac{(1-p)}{k}\left(s_1 - (1-\phi_1)\frac{ps_2\,(t_1-t_2)}{pt_1-(1-p)e_2^*}\right)$$

Thus, (e_1^*, e_2^*) is the Nash equilibrium when $s_1 \geq \bar{s_1}$. When $s_1 = s_2 = s$, I first prove the existence of a pure Nash equilibrium by using the Debreu-Glicksberg-Fan Theorem. First, activists' strategy set $[0, \bar{e}]$ is compact and convex. Second, when $e_i \leq e_{-i}$, firm i matches with CEO 2, and $\pi(e_i, e_{-i}) = \left(1-\frac{(1-p)e_i}{pt_2}\right)(pst_2 - v_2) - \frac{k}{2}(\bar{e}-e_i)^2$, which is concave in e_i; when $e_i > e_{-i}$, firm i matches with

CEO 1, and $\pi(e_i,e_{-i}) = \left(1-\frac{(1-p)e_i}{pt_1}\right)(pst_1 - v_1(e_{-i})) - \frac{k}{2}(\bar{e}-e_i)^2$, which is also concave in e_i. Third, I prove that $\pi(e_i,e_{-i})$ is continuous in e_{-i}, which is clearly true for $e_{-i} < e_i$ and $e_{-i} > e_i$ for a given e_i, thus, I only need to prove $\pi(e_i,e_{-i})$ is continuous at $e_{-i} = e_i$. The left limit

$$\begin{aligned}
\lim_{e_{-i}\to e_i^-} \pi(e_i,e_{-i}) &= \left(1-\frac{(1-p)e_i}{pt_1}\right)\left(pst_1 - \lim_{e_{-i}\to e_i^-} v_1(e_{-i})\right) - \frac{k}{2}(\bar{e}-e_i)^2 \\
&= \left(1-\frac{(1-p)e_i}{pt_1}\right)\left(pst_1 - \frac{ps(t_1-t_2)+\left(1-\frac{(1-p)e_i}{pt_2}\right)v_2}{1-\frac{(1-p)e_i}{pt_1}}\right) - \frac{k}{2}(\bar{e}-e_i)^2 \\
&= \left(1-\frac{(1-p)e_i}{pt_1}\right)(pst_2 - v_2) - \frac{k}{2}(\bar{e}-e_i)^2 = \lim_{e_{-i}\to e_i^+} \pi(e_i,e_{-i})
\end{aligned}$$

Thus, $\pi(e_i,e_{-i})$ is continuous at $e_{-i} = e_i$. $\pi(e_i,e_{-i})$ is continuous in e_{-i} on $[0,\bar{e}]$. Similarly, I can prove that $\pi(e_i,e_{-i})$ is continuous in e_i on $[0,\bar{e}]$. Thus, $\pi(e_i,e_{-i})$ is continuous on $[0,\bar{e}]\times[0,\bar{e}]$. According to the Debreu-Glicksberg-Fan theorem, there exists a pure strategy Nash Equilibrium. Next, it is easy to show that any $e_i \neq e_1^*$ and $e_i \neq e_2^*$ cannot be optimal for activist i whether firm i matches with CEO 1 or CEO 2. Thus, (e_1^*, e_2^*) is the unique Nash equilibrium for $s_1 = s_2 = s$. By continuity, (e_1^*, e_2^*) is the unique Nash equilibrium if s_1 is sufficiently close to s_2.

Proof of Lemma 7

The social planner's objective function (14) is concave (see the proof of Proposition 2). The first order condition yields (e_1^o, e_2^o).

$$e_1^o = \bar{e} - \frac{(1-p)}{k}\left(s_1 - (1-\phi_1)\frac{ps_2(t_1-t_2)}{pt_1-(1-p)e_2^o}\right) \tag{23}$$

$$e_2^o = \bar{e} - \frac{1-p}{k}s_2 - \frac{1}{k}\left(1-e_1^o\frac{1-p}{pt_1}\right)\frac{\partial v_1(e_2^o)}{\partial e_2^o} \tag{24}$$

Comparing equation (22) and equation (24) yields $e_2^o = e_2^* - \left(1 - e_1^o \frac{1-p}{pt_1}\right) \frac{\partial v_1(e_2^o)}{\partial e_2^o} < e_2^*$. By equation (21), equation (23), and $e_2^* > e_2^o$, I can prove $e_1^o < e_1^*$.

Proof of Proposition 5

Denote the universal governance deficiency ceiling by e_r. The regulator maximizes total firm profits net of activism costs, subject to activists' incentive constraints:

$$\max_{e_1,e_2,e_r} \pi(s_1,e_1,t_1,v_1(e_2)) + \pi(s_2,e_2,t_2,v_2) - \frac{k}{2}(\bar{e}-e_1)^2 - \frac{k}{2}(\bar{e}-e_2)^2$$

$$s.t.\ e_1 = \min\{e_r, e_1^*\}$$

$$e_2 = \min\{e_r, e_2^*\}$$

First, equation (23) shows that e_1^o is strictly decreasing in s_1. Equation (12) shows that $\frac{\partial v_1(e_2^o)}{\partial e_2^o}$ is independent of s_1, thus, e_2^o is independent of s_1 by equation (24). Therefore, when firm 1's size s_1 is sufficiently larger than firm 2's size s_2, $e_1^o < e_2^o$. Let the regulator set $e_r = e_2^o$. Under such a regulation, activist 2 sets her optimal choice of governance deficiencies at $e_2^o = e_r$ because her objective function is quadratic in e_2. Comparing equation (21) and (23) shows that if activist 2 sets governance deficiencies at the efficient level, activist 1 optimally sets governance deficiencies at the efficient level e_1^o, which satisfies the regulatory constraint because $e_1^o < e_2^o = e_r$.

When $s_1 = s_2$. Note first that $e_1^* > e_2^*$ when $s_1 = s_2$ (see the proof of Lemma 6). I consider two cases when e_r is binding. First, if $e_2^* < e_r < e_1^*$, then activist 2 sets firm 2's governance deficiencies at e_2^*; activist 1 sets firm 1's governance deficiencies at e_r because her payoff is quadratic in e_1 and e_1^* is her optimal choice without regulation. Activist 2 earns the same payoff and activist 1 is strictly worse off under the regulation. The regulator should do nothing. Second, the regulator sets $e_r \leq e_2^*$. Activist 2's payoff function is quadratic and maximized at e_2^* without regulation, thus she sets $e_1 = e_r$ under the regulation. By equation (21), activist 1's payoff is maximized at

$$\bar{e} - \frac{(1-p)}{k}\left(s_1 - \frac{1}{2}\frac{ps_2\,(t_1 - t_2)}{pt_1 - (1-p)\,e_r}\right) > e_2^* \geq e_r$$

where the first inequality can be proved similar to the proof of $e_1^* > e_2^*$ (see the proof of Lemma 6). Thus, activist 1 sets $e_1 = e_r$ under the regulation because her payoff is quadratic in e_1. Because both activists choose e_r, they will earn the same payoff. Thus, maximizing total activists' payoffs is equivalent to maximizing activist 2's payoff

$$\max_{e_r} \left(1 - \frac{(1-p)e_r}{pt_2}\right)(ps_2t_2 - v_2) - \frac{k}{2}(\bar{e} - e_r)^2$$

and the regulator sets $e_r = e_2^*$. Compared with the unregulated market outcome, activist 2 earns the same payoff, and activist 1 earns a strictly lower payoff. By the continuity argument, when firm 1 and firm 2's sizes are sufficiently similar, the regulator should do nothing instead of setting a universal governance deficiency ceiling for the two activists.

References

Acharya, V., and P. Volpin. 2010. Corporate governance externalities. *Review of Finance* 14:1–33.

Admati, A. R., and P. Pfleiderer. 2009. The "Wall Street Walk" and shareholder activism: Exit as a form of voice. *The Review of Financial Studies* 22:2645–2685.

Aghion, P., P. Bolton, and J. Tirole. 2004. Exit options in corporate finance: Liquidity versus incentives. *Review of Finance* 8:327–353.

Appel, I. R., T. A. Gormley, and D. B. Keim. 2016. Passive investors, not passive owners. *Journal of Financial Economics* 121:111–141.

Aslan, H., and P. Kumar. 2016. The product market effects of hedge fund activism. *Journal of Financial Economics* 119:226–248.

Azar, J. 2017. Portfolio diversification, market power, and the theory of the firm. Working paper, University of Navarra.

Azar, J., M. C. Schmalz, and I. Tecu. 2018. Anticompetitive effects of common ownership. *The Journal of Finance* .

Becker, G. S. 1973. A theory of marriage: Part I. *Journal of Political economy* 81:813–846.

Border, K. C., and J. Sobel. 1987. Samurai accountant: A theory of auditing and plunder. *The Review of economic studies* 54:525–540.

Bourveau, T., and J. Schoenfeld. 2017. Shareholder activism and voluntary disclosure. *Review of Accounting Studies* 22:1307–1339.

Brav, A., W. Jiang, F. Partnoy, and R. Thomas. 2008. Hedge fund activism, corporate governance, and firm performance. *The Journal of Finance* 63:1729–1775.

Chade, H., J. Eeckhout, and L. Smith. 2017. Sorting through search and matching models in economics. *Journal of Economic Literature* 55:493–544.

Custódio, C., M. A. Ferreira, and P. Matos. 2013. Generalists versus specialists: Lifetime work experience and chief executive officer pay. *Journal of Financial Economics* 108:471–492.

Dicks, D. L. 2012. Executive compensation and the role for corporate governance regulation. *Review of Financial Studies* 25:1971–2004.

Edmans, A. 2009. Blockholder trading, market efficiency, and managerial myopia. *The Journal of Finance* 64:2481–2513.

Edmans, A., and G. Manso. 2010. Governance through trading and intervention: A theory of multiple blockholders. *The Review of Financial Studies* 24:2395–2428.

Faure-Grimaud, A., and D. Gromb. 2004. Public trading and private incentives. *Review of financial Studies* 17:985–1014.

Feng, F., Q. Xu, and H. Zhu. 2018. Caught in the Crossfire: How the Threat of Hedge Fund Activism Affects Creditors. Working paper, University of Notre Dame.

Gantchev, N. 2013. The costs of shareholder activism: Evidence from a sequential decision model. *Journal of Financial Economics* 107:610–631.

Gantchev, N., O. Gredil, and C. Jotikasthira. Forthcoming. Governance under the gun: Spillover effects of hedge fund activism. *Review of Finance* .

Grossman, S. J., and O. D. Hart. 1980. Takeover bids, the free-rider problem, and the theory of the corporation. *The Bell Journal of Economics* pp. 42–64.

Hansen, R. G., and J. R. Lott. 1996. Externalities and corporate objectives in a world with diversified shareholder/consumers. *Journal of Financial and Quantitative Analysis* 31:43–68.

He, J. J., and J. Huang. 2017. Product market competition in a world of cross-ownership: Evidence from institutional blockholdings. *The Review of Financial Studies* 30:2674–2718.

Investment Company Institute. 2017. 2017 Investment Company Fact Book. Tech. rep., Investment Company Institute, Washington, DC.

Kahn, C., and A. Winton. 1998. Ownership structure, speculation, and shareholder intervention. *The Journal of Finance* 53:99–129.

Legros, P., and A. F. Newman. 2007. Beauty is a beast, frog is a prince: assortative matching with nontransferabilities. *Econometrica* 75:1073–1102.

Maug, E. 1998. Large shareholders as monitors: is there a trade-off between liquidity and control? *The journal of finance* 53:65–98.

Murphy, K., and J. Zabojnik. 2004. CEO pay and appointments: A market-based explanation for recent trends. *The American Economic Review* 94:192–196.

Rubin, A. 2006. Diversification and corporate decisions. *Corporate Ownership & Control* 3:209–212.

Shleifer, A., and R. W. Vishny. 1986. Large shareholders and corporate control. *Journal of political economy* 94:461–488.

Zhu, H. 2013. The preventive effect of hedge fund activism. Working paper, University of Oklahoma.