The division of labor and the extent of the market

Zhou, Haiwen

22 January 2019

The Division of Labor and the Extent of the Market[∃]

Haiwen Zhou

Abstract: A firm's degree of specialization is modeled as the number of different goods it produces. When a firm chooses its degree of specialization, it faces a tradeoff between the fixed cost and the marginal cost of production. A firm's degree of specialization is shown to increase with the extent of the market. Meanwhile, the real wage rate, as a measure of the extent of the market, is endogenously determined in the model and is shown to increase with the division of labor.

Keywords: Division of labor, Extent of the market, Specialization, Increasing returns to scale

JEL Classification Number: A10

1. Introduction

Why is the labor productivity in some countries very high while in other countries very low? For Adam Smith [17], productivity depends on the division of labor. The division of labor depends on the extent of the market, which in turn depends on the division of labor. Thus, there is a mutual dependence between the division of labor and the extent of the market. To Smith, the extent of the market is proportional to the wealth and the population of a country.[1] A country's wealth is related to a country's real wage rate. Thus the extent of the market is determined by the real wage rate and population. When firms can enter and exit an industry freely, the real wage rate will be determined by the zero profit condition and will be endogenously determined. Thus, the extent of the market is ultimately related to the total population.

Surprisingly no formal research concerning the mutual dependence between the division of labor and the extent of the market has been conducted. Understanding the mutual dependence between the division of labor and the extent of the market in a formal model is important for many reasons. First, a formal model will be very useful in understanding the mechanism of the mutual dependence between the division of labor and the extent of the market. Second, a formal model will be helpful in empirical research on this issue. A formal model will point out which variables may be relevant for empirically testing the theory. Finally, a formal model will be helpful in understanding other issues related to the division of labor. For example, how is the division of labor related to international trade? Studying how the division of labor changes with

[∃] The author thanks an anonymous referee and Antonio Ciccone for their very valuable and constructive suggestions. I thank Deborah Minehart, Robert Schwab, and Daniel Vincent for their valuable guidance and advice. I also thank Mingchao Chen, David Selover, and Xiaokai Yang for their helpful advice.

[1] See Smith [17], Volume I, p.23.

the opening of international trade within the framework of the mutual dependence between the division of labor and the extent of the market will provide us with a consistent view of both issues.

In this paper, I study the mutual dependence between the division of labor and the extent of the market in a formal general equilibrium model. When a firm chooses its degree of specialization, a tradeoff between the fixed cost and the marginal cost of production is involved. There are an infinite number of production technologies. A more specialized technology is modeled as a technology with a relatively higher fixed cost of production and a lower marginal cost of production. Thus it is suitable for larger scale production. A less specialized technology is modeled as a technology with a lower fixed cost and a higher marginal cost of production and is thus appropriate for smaller scale production.

With this innovative way of modeling production technologies, I show that a firm's degree of specialization increases with the extent of the market. The reason is that when the population is larger, a more specialized technology will be adopted. The real wage rate as a measure of the extent of the market is endogenously determined. As a more specialized technology is adopted, the average cost of production falls. As each firm earns a profit of zero in equilibrium, the real wage rate increases with the division of labor. Thus, the mechanism for the mutual dependence between the division of labor and the extent of the market is established. I also show that the opening of international trade increases welfare as a more specialized technology is adopted and the average production cost decreases.

One issue in formalizing Smith's idea, as Young [22] points out, is that the division of labor is related to increasing returns to scale. Increasing returns to scale may lead to the monopolization of an industry. However, we do not observe many monopolized industries in reality. To reconcile theory and the empirical evidence, Stigler [18] argues that the different functions performed by a firm have different cost structures. When one function enjoys decreasing costs, another function may be experiencing increasing costs. Thus increasing returns to scale in one function is not enough to lead to the monopolization of an industry. Stigler did not provide a formal model to support his argument. In this paper, as firms engage in Cournot competition, even with the presence of increasing returns to scale in production, the existence of multiple firms producing the same product is feasible and the industry will not be monopolized.

Thus, this paper provides another avenue through which increasing returns to scale may coexist with competition.

The literature on specialization can be roughly grouped into two large categories: partial equilibrium studies and general equilibrium studies.[2] For partial equilibrium studies, see Baumgardner [2], Kim [9], and Locay [11]. These models are very helpful in increasing our understanding of the division of labor. However, having shown that the degree of specialization increases with the extent of the market, they are not able to show how the extent of the market would be affected by the division of labor. The mutual dependence between the division of labor and the extent of the market calls for a general equilibrium framework to tackle this issue. For general equilibrium studies related to specialization in the literature, see Romer [13], Ciccone [4], Ciccone and Matsuyama [5], and Weitzman [19]. None of the above papers studies the range of goods produced by firms. Yet the range of goods produced by firms is clearly an important dimension of specialization. In this paper, the number of different goods produced by each firm is a key element of the model. Yang and Borland [20] study the number of goods produced by a consumer-producer. In their model, an individual may produce more than one product at the beginning. Each producer will eventually become a monopolist because of the advantages accumulated from learning by doing in production. This is different from this model as there are always multiple firms producing the same product.

In this model, with the fixed cost of production, the production function displays increasing returns to scale.[3] This is similar to the setup in Krugman's [10] seminal paper. However, there are some important differences between Krugman [10] and this model. First, Krugman [10] did not study a firm's choice of the degree of specialization. In this model, in contrast, a firm's choice of the number of different goods to produce is crucial. Second, the total number of goods available for production is fixed in this model. In Krugman [10], it is assumed that the economy is able to produce any of a large number of goods. Third, there is only one monopolistic firm producing each good in Krugman's [10] model. In this model, there are multiple firms producing the same good.

[2] See Yang and Ng [21] for an excellent survey of the literature on specialization.
[3] In Rosen [15], the increasing returns to utilization of human capital are emphasized. Indivisibilities imply fixed elements of human capital investment that are independent of subsequent utilization, thus *ex ante* identical individuals have the incentives to specialize in one skill.

The paper is organized as follows. First, I study a representative consumer's choices. Each consumer inelastically supplies one unit of labor. A consumer chooses her quantity of consumption of each good to maximize utility. Second, I study a representative firm's choices. A firm chooses the number of goods and the quantity of each good to produce in order to maximize profits. Then, the conditions for the labor market and the goods markets to clear and for the existence of free market entry and exit are imposed. These conditions and the conditions for a representative consumer's utility maximization and a representative firm's profit maximization define the equilibrium. By analyzing the equilibrium conditions, the mutual dependence between the division of labor and the extent of market is established. Finally, I conduct some comparative static studies concerning various relationships, such as the relationship between the per capita consumption and the extent of the market.

2. The Model

The model is set up as follows. It is assumed that there are L identical consumers. Consistent with Smith [17] and Young [22], L is related to the extent of the market.[4] Each consumer supplies one unit of labor.[5] Let N denote the total number of different goods produced in the economy. N is a very large number and is exogenously given. Let c_i denote a representative consumer's amount of consumption of good i, and $i \in \{1,...N\}$. The utility from consuming c_i is $u(c_i)$. It is assumed that $u' > 0$ and $u'' < 0$. A consumer's total utility from consuming the goods is denoted by U and it takes the following form,

$$U = \sum_{i=1}^{N} u(c_i). \tag{1}$$

In equation (1), it is assumed that the utilities are additively separable when a consumer consumes different kinds of goods. Also, goods enter into the utility function in a symmetric

[4] To Young [22], the extent of the market is determined by buying power, i.e., the capacity to absorb a large annual output of goods (See Young [22], p.533). Young's point of view is consistent with Smith's view as a larger and richer country has a higher buying power.

[5] In Rosen [14], workers differ in their distribution of skills. Division of labor thus comes about as a result of comparative advantage. In this model, all workers have the same abilities. This is consistent with Smith [17] (Volume I, pp.19-20), as he states "The difference of natural talents in different men is, in reality, much less than we are aware of: and the very different genius which appears to distinguish men of different professions, when grown up to maturity, is not upon many occasions so much the cause, as the effect of the division of labor. The difference between the most dissimilar characters, between a philosopher and a common street porter, for example, seems to arise not so much from nature, as from habit, custom, and education."

way, as in Dixit and Stiglitz [6]. For horizontally differentiated products, another way of modeling the substitutability among goods is that each consumer has her most preferred product, as in the tradition of Hotelling [8] and Salop [16]. Baumgardner [2] and Kim [9] adopt Salop's approach in their study on the division of labor.

First, I study a representative consumer's choices. A representative consumer chooses the amount of consumption of each good to maximize her utility, subject to the constraint that she cannot spend more than her income. It is assumed that a consumer has only wage income. Each consumer inelastically supplies one unit of labor. Let w denote the nominal wage rate and p_i denote the price of commodity i, then a consumer's budget constraint is

$$\sum_{i=1}^{N} p_i c_i \leq w . \tag{2}$$

Let λ denote the Lagrange multiplier and the shadow price of income associated with (2). The following first order conditions are needed for a consumer's utility maximization,

$$u'(c_i) = \lambda p_i \text{ for all } i . \tag{3}$$

Define the elasticity of demand as $\varepsilon_i \equiv -u'/u''c_i$. Since $u'>0$ and $u''<0$, ε_i is positive.

Second, I study a representative firm's choices. When there are multiple firms producing the same product, the firms are assumed to engage in Cournot competition. A firm takes the nominal wage rate as given and chooses the total number of different goods to produce. Let n denote the total number of different goods a representative firm produces, and $n \leq N$. For these n different goods a firm chooses to produce, a firm will also choose the amount of production of each good. Let x_i denote the amount of product i produced by the firm.

In this paper, n represents a firm's degree of specialization. As n decreases, the degree of specialization increases. To simplify the analysis, the integer constraint on n is ignored here. To produce n different products, a firm incurs a total fixed cost of $f(n)$ units of labor. So each product's fixed cost of production is $f(n)/n$ units of labor. Thus, the fixed cost of production varies with the number of different goods produced but not with the quantity of each good produced. It is assumed that $f'>0$ and $f''<0$. The motivation for the assumption that $f'>0$ is that total fixed cost of production will increase when more goods are produced. Also, the property that the fixed cost of production for each good increases as fewer goods are produced (or $d(f/n)/dn<0$) is desirable. This requirement motivates the assumption that $f''<0$.

In addition to the fixed cost of production, a firm also incurs a marginal cost of $\beta(n)$ units of labor to produce each unit of a good. Here the marginal cost is constant with respect to x, but the marginal cost with respect to n is not constant. It is assumed that $\beta' > 0$. The motivation for this assumption is that the marginal cost of production for each unit of output decreases as the degree of specialization increases. One illustration for the assumptions on costs is that one production technology may need a lot of machines, and the fixed cost of production from buying the machines is high. This technology is suitable for large scale production as the marginal cost of production is low. Another production technology does not employ any machines, and the fixed cost is small. However, the marginal cost of production is high.

In summary, to produce x units of each of the n different goods in a firm's production set, the total units of labor needed will be $f(n) + \beta(n)nx$. As a firm becomes more specialized, the fixed cost of producing each good goes up, but the marginal cost of production goes down. This is the basic tradeoff faced by a firm in choosing the degree of specialization.[6] This tradeoff has not been rigorously explored in the literature and this innovation is a main contribution of this paper.[7]

This paper focuses on a symmetric equilibrium. In a symmetric equilibrium, the number of firms producing each good is the same, and the same amount of each good will be produced. Each firm will have the same degree of specialization, and each consumer will have the same consumption bundle. Because of the symmetry in this model, $p_i = p$, $n_i = n$, $x_i = x$, $c_i = c$, and $\varepsilon_i = \varepsilon$. The subscript of a variable is sometimes dropped since no confusion will arise from doing so.

When a firm produces the same amount of n different goods in its production set, its total revenue is npx. A firm's cost of production is $fw + n\beta xw$, thus a firm's profit is

---

[6] To Adam Smith, the benefit of the division of labor comes from three sources. First, the workers' dexterity increases as specialization increases. Second, time on switching from one type of work to another is reduced. Third, a great number of machines are adopted. My specification of production technology is consistent with the third benefit. One technology uses many machines (high fixed cost) and the marginal production cost is small, while another technology does not use any machine and involves a high marginal cost.

[7] Young [22] (p. 530) illustrates intuitively that some technologies are more specialized and suitable for larger scale production, while some other methods are less specialized and suitable for handling smaller production needs. To him, it would be wasteful to make a hammer to drive a single nail. It would be wasteful to furnish a factory with an elaborate set of specially constructed equipment to produce a small level of output.

$$\pi = \sum_{i=1}^{n} p_i x_i - fw - \sum_{i=1}^{n} \beta x_i w = npx - fw - n\beta xw. \tag{4}$$

A firm will choose its degree of specialization optimally. Taking the first order condition with respect to n leads to

$$px - f'w - (\beta' n + \beta)xw = 0. \tag{5}$$

The following second order condition is necessary for a firm's profit maximization and is assumed to be satisfied,

$$f'' + (2\beta' + \beta'' n)x > 0. \tag{6}$$

A firm will also choose its quantities of production optimally. Taking the first order condition with respect to x in (4) leads to

$$p_i + x_i \frac{\partial p_i}{\partial x_i} - \beta w = 0. \tag{7}$$

3. Equilibrium

In this section, conditions for the labor and goods market to clear and the free entry and exit condition are imposed, and an equilibrium is established.

First, consider the goods market for product i. Each consumer demands c_i units of good i, and the total demand for this good is Lc_i. Suppose that there are m firms producing this good. Thus m is a measure of market structure. As each firm supplies x_i units of good i, the total supply of this good will be mx_i. For the demand and supply of this good to clear, the following condition (8) is needed,

$$Lc_i = mx_i. \tag{8}$$

Second, consider the labor market. Because each firm produces n goods and there are m firms producing each good, the total number of firms in the economy is mN/n. Each firm needs $f + n\beta x$ units of labor, and the total demand for labor will be $mN(f + \beta nx)/n$. The total supply of labor is L. For the labor market to clear, the following condition (9) is needed,

$$\frac{mN}{n}(f + \beta nx) = L. \tag{9}$$

Finally, free entry and exit ensure that each firm will obtain a profit of zero in equilibrium. From (4), the zero profit condition is

$$npx - fw - \beta nxw = 0. \tag{10}$$

It is clear that $w^r \equiv w/p$ is the real wage rate. The following proposition illustrates the relationship between the degree of specialization and the real wage rate.

Proposition 1 *The real wage rate is positively related to the degree of specialization.*

Proof: From (10), it can be shown that

$$x = \frac{fw^r}{n(1-\beta w^r)}. \tag{11}$$

From (5) and (11), w^r can be expressed as a function of n,

$$w^r = \frac{f - nf'}{\beta f + \beta' nf - n\beta f'}. \tag{12}$$

Differentiating (12) with respect to n leads to

$$\frac{dw^r}{dn} = -\left(\frac{nf''}{f - nf'} + \frac{2\beta' f + \beta'' fn - \beta f'' n}{\beta f + \beta' nf - \beta f' n}\right) w^r.$$

For $dw^r / dn < 0$, the following inequality (13) is needed,

$$\left(\frac{nf''}{f - nf'} - \frac{2\beta' f + \beta'' fn - \beta f'' n}{\beta f + \beta' nf - \beta f' n}\right) > 0. \tag{13}$$

From (11) and (12), (13) is equivalent to (6), which is assumed to be satisfied. QED

The intuition behind Proposition 1 is the following. When the degree of specialization increases, the fixed cost of production increases, and the marginal cost of production decreases. From the second order necessary condition for a firm's profit maximization, the net effect is that the average cost decreases. Therefore the prices of goods go down because each firm earns zero profit. As the nominal wage rate does not change, the real wage rate increases. As a result, the real wage rate is positively related to the degree of specialization.

If the average cost of production is used to measure labor productivity, then labor productivity increases as the degree of specialization increases.

Young [22] provides an example illustrating Proposition 1. He observes that both the degree of specialization and the wage rate are higher in the United States than in the United Kingdom.

For a given commodity, let x_{-i} denote the total output produced by all firms other than firm i, then $Lc_i = x_i + x_{-i}$. From (3), it can be shown that

$$p_i = \frac{u'\left(\frac{x_i + x_{-i}}{L}\right)}{\lambda}. \tag{14}$$

When the total number of goods is very large, the effect of changing a firm's price of product i on a consumer's shadow price of income can be ignored.[8] As firms engage in Cournot competition, a firm will take other firms' output as given when it chooses its own quantities of production. By differentiating (14) with respect to x_i, it can be shown that

$$\frac{\partial p_i}{\partial x_i} = \frac{u'' p_i}{u' L}. \tag{15}$$

From the definition of ε_i, (15) is equivalent to

$$\frac{\partial p_i}{\partial x_i} = -\frac{p_i}{Lc_i \varepsilon_i}. \tag{16}$$

Plugging (16) into (7), it is clear that

$$p - \frac{xp}{Lc\varepsilon} = \beta w. \tag{17}$$

Now there is a system of five equations (5), (8)-(10), and (17). These equations define the five unknowns, w^r, m, c, x, and n. After eliminating w^r and m from the system, I get the following three equations, (18a)-(18c), defining three variables, c, x and n. The two exogenous variables are L and N.

$$\frac{f}{n} = f' + n\beta' x, \tag{18a}$$

$$\frac{Nc(f + \beta nx)}{nx} = 1, \tag{18b}$$

$$1 - \frac{x}{Lc\varepsilon} = \beta Nc. \tag{18c}$$

[8] See Dixit and Stiglitz [6], and Krugman [10] for similar arguments.

The intuition behind equation system (18) is the following. Equation (18a) comes from (5) and (10). Decreasing the number of different goods produced by one will save a firm the fixed cost of producing this good, which is f/n units of labor. Decreasing the number of different goods produced by one will also change the fixed and the marginal cost of producing the remaining goods, and this cost change is $f'+n\beta' x$ units of labor. Thus equation (18a) says that the marginal revenue equals the marginal cost when a firm's degree of specialization is optimally chosen. Equation (18b) comes from (8)-(10). The average production cost of each unit of output is $(f+\beta nx)/nx$ units of labor input. If a consumer consumes Nc units of products, her total demand for labor would be $Nc\ (f+\beta nx)/nx$, which is the left side of equation (18b). The right side of the equation is her supply of labor. In equilibrium, these two terms should be equal. Equation (18c) comes from (10) and (17). The left side of equation (18c) is the marginal revenue denoted in labor units as the quantity of production changes. The real wage rate equals Nc in equilibrium and each unit of product requires β units of labor. Thus the right side of equation (18c) is the marginal cost of production. Equation (18c) requires that the marginal revenue equals the marginal cost when the production quantity is optimally chosen.

4. The Mutual Dependence between the Division of Labor and the Extent of the Market

In this section, the mutual dependence between the division of labor and the extent of the market is formally established.

For ease of presentation, equation system (18) is transformed into the following equivalent system (19).

$$R_1 \equiv f - nf' - n^2\beta' x = 0\,, \tag{19a}$$

$$R_2 \equiv Nc(f+\beta nx) - nx = 0\,, \tag{19b}$$

$$R_3 \equiv 1 - \beta Nc - \frac{x}{Lc\varepsilon} = 0\,. \tag{19c}$$

Differentiating (19a) with respect to n and x, it can be shown that

$$\frac{\partial R_1}{\partial n}dn + \frac{\partial R_1}{\partial x}dx = 0\,. \tag{20a}$$

Differentiating (19b) with respect to c, n, x, and N, it can be shown that

$$\frac{\partial R_2}{\partial n}dn + \frac{\partial R_2}{\partial x}dx + \frac{\partial R_2}{\partial c}dc = -\frac{\partial R_2}{\partial N}dN\,. \tag{20b}$$

Differentiating (19c) with respect to c, n, x, L, and N, it can be shown that

$$\frac{\partial R_3}{\partial n}dn+\frac{\partial R_3}{\partial x}dx+\frac{\partial R_3}{\partial c}dc=-\frac{\partial R_3}{\partial N}dN-\frac{\partial R_3}{\partial L}dL\,. \tag{20c}$$

Equation system (20) can be expressed as

$$\begin{vmatrix} \frac{\partial R_1}{\partial n} & \frac{\partial R_1}{\partial x} & 0 \\ \frac{\partial R_2}{\partial n} & \frac{\partial R_2}{\partial x} & \frac{\partial R_2}{\partial c} \\ \frac{\partial R_3}{\partial n} & \frac{\partial R_3}{\partial x} & \frac{\partial R_3}{\partial c} \end{vmatrix} \begin{vmatrix} dn \\ dx \\ dc \end{vmatrix} = \begin{vmatrix} 0 \\ -\frac{\partial R_2}{\partial N}dN \\ -\frac{\partial R_3}{\partial N}dN-\frac{\partial R_3}{\partial L}dL \end{vmatrix}. \tag{21}$$

The following proposition gives the relationship between a firm's production quantity of each good and the extent of the market.

Proposition 2 *A firm's quantity of production of each good in its production set increases with the total population.*

Proof: From (21), it can be shown that

$$\frac{dx}{dL}=\frac{\partial R_1}{\partial n}\frac{\partial R_2}{\partial c}\frac{\partial R_3}{\partial L}/\Delta\,. \tag{22}$$

In (22), Δ is the determinant matrix on the left side of equation (21). It is assumed that the equilibrium is locally strictly stable, thus $\Delta<0$.

From (19), the following inequalities can be established,

$$\frac{\partial R_1}{\partial n}=-n(f''+(2\beta'+\beta''n)x)<0\,, \tag{23}$$

$$\frac{\partial R_2}{\partial c}=N(f+\beta nx)>0\,, \tag{24}$$

$$\frac{\partial R_3}{\partial L}=\frac{x}{L^2c\varepsilon}>0\,. \tag{25}$$

From (22)-(25), it is clear that $dx/dL>0$. QED

The intuition behind Proposition 2 is the following. When the total population increases, each consumer receives a lower share of a firm's output. If a firm's output is fixed, its marginal revenue will increase. To regain equilibrium, a firm's production of each good increases.

Young [22] is aware of the impact of total population on the size of production. He observes that the scale of production is much higher in the United States than in the United Kingdom as he writes "Mr. Ford's methods would be absurdly uneconomical if his output were very small, and would be unprofitable even if his output were what many other manufacturers of automobiles would call large" (p530).

Proposition 3 *A firm's degree of specialization is positively related to the extent of the market.*

Proof: From (21), it can be shown that

$$\frac{dn}{dL} = -\frac{\partial R_1}{\partial x}\frac{\partial R_2}{\partial c}\frac{\partial R_3}{\partial L} / \Delta . \tag{26}$$

From (19), it can be shown that

$$\frac{\partial R_1}{\partial x} = -n^2 \beta' < 0 . \tag{27}$$

From (24)-(27), it is clear that $dn / dL < 0$. QED

The intuition behind Proposition 3 is the following. It has been shown in Proposition 2 that a firm's production quantity of each good increases when the total population increases. As the production quantity increases, a firm's degree of specialization increases because a more specialized technology has a lower marginal cost and is more suitable for larger scale production.

Here is an example about the relationship between the degree of specialization and the extent of the market. Baumgardner [3] studies the degree of doctors' specialization in providing medical services. A doctor may provide general services or specialize in some services. Specialists may also differ in their degree of specialization. In Baumgardner's study, the extent of the market is measured by the local population. He finds that the degree of specialization of doctors is strongly related to local population (p. 967).

From Propositions 1 and 3, the real wage rate increases with the total population. Without increasing returns to scale in production, the real wage rate may not change with the total population. Thus specialization leads to a positive relationship between the real wage rate and the total population.

The level of the real wage rate may be interpreted as the wealth of a country. The real wage rate as a measure of the extent of the market is endogenously determined in this model and is shown to increase with the division of labor in Proposition 1. In Proposition 3, it has been shown that the division of labor increases with the extent of the market. Thus the mutual dependence between the division of labor and the extent of the market has been formally established.

5. Comparative Statics

In this section, I conduct some comparative static studies on some interesting issues, such as the relationship between market structure (as measured by the number of firms producing the same product) and the extent of the market.

The following proposition studies the relationship between a consumer's consumption of each good and the total population.

Proposition 4 *An increase in the extent of the market will cause an increase in the per capita consumption of each good.*

Proof: From (21), it can be shown that

$$\frac{dc}{dL} = -\frac{\partial R_3}{\partial L}\left(\frac{\partial R_1}{\partial n}\frac{\partial R_2}{\partial x} - \frac{\partial R_1}{\partial x}\frac{\partial R_2}{\partial n}\right)/\Delta . \tag{28}$$

From (19), it is clear that

$$\frac{\partial R_2}{\partial x} = n(Nc\beta - 1) , \tag{29}$$

$$\frac{\partial R_2}{\partial n} = Nc(f' + \beta' nx + \beta x) - x . \tag{30}$$

From (6), $\partial R_1 / \partial n < 0$. From (19c), $1 - \beta Nc > 0$, thus $\partial R_2 / \partial x < 0$. From (19a) and (19b), $\partial R_2 / \partial n = 0$. Therefore, from (23) and (28)-(30), $dc / dL > 0$. QED

From equation (1) and Proposition 4, a consumer's utility increases with the total labor force. Thus, a country with a larger population may enjoy a higher standard of living than a country with a smaller population even though the same production technologies are available to both countries. This is consistent with Young's view [22]. As the United States has a larger population than Britain, a higher living standard will be achieved in the United States if domestic demand plays a dominant role in a country's total demand. Population growth will increase each consumer's welfare. If transportation costs are not significant, then the opening of international trade will increase consumers' welfare. This provides an explanation of the observation that countries are always looking for new markets.

There is some empirical evidence supporting the results in this paper. Ades and Glaeser [1] conducted some empirical research concerning the relationship between the division of labor and the extent of the market. In their paper, the increasing returns to scale come from the fixed costs of production. They conclude that the division of labor is positively related to the extent of the market, and that the division of labor is important for development. Frankel and Romer [7] show that international trade raises a country's income. After controlling for international trade, they show that within country trade also raises income. Large countries have more opportunities for trade within their borders and therefore have higher incomes.

When there are more firms producing the same good, each firm has less market power. One interesting question is the following: How does the market structure, as measured by the number of firms producing the same product, change with the total population? There are three factors affecting m when L increases. First, as the number of consumers increases, the demand for goods increases even if each consumer consumes the same amount of goods. Second, each consumer's consumption of each good increases. Third, each firm produces a larger amount of each good in its production set when L increases. The first two factors tend to increase m, while the third one tends to decrease m. It can be shown that the third effect always dominates the second one, but the total effect is not clear.[9]

For all of the n different goods in a firm's production set, x units of each good are produced. Thus nx can be interpreted as firm size. What happens to firm size as the population goes up? It has been shown that x increases with L and n decreases with L. These two forces

---

[9] Suppose the utility function is specified as $U = \sum_{i=1}^{N} \ln c_i$. For this utility function, $\varepsilon = 1$. The fixed cost and the marginal cost of production are specified as $f(n) = \sqrt{n}$, $\beta(n) = \sqrt{n}$ respectively. Let an asterisk mark denote a variable's equilibrium value. By plugging the utility function and the cost functions into equation system (19), it can be shown that $n^* = \frac{16N^2}{L^2}$, $x^* = \frac{L^2}{16N^2}$, $c^* = \frac{L}{8N^2}$, $(w^r)^* = \frac{L}{8N}$, $m^* = 2$. In this example, m does not change with L.

work in opposite directions, and in general there is no monotonic relationship between the size of firms and the total population.[10]

My result that there is no monotonic relationship between the size of firms and the extent of the market is consistent with Liu and Yang [11]'s study. They show that there is no monotonic relationship between the size of the firm, specialization, and productivity growth.

How does a firm's degree of specialization change with the total number of goods produced in the economy? From (20), it can be shown that

$$\frac{dn}{dN} = -\frac{n^2 \beta' x(f + \beta nx)(\varepsilon + c\varepsilon')}{Lc\varepsilon^2} / \Delta . \tag{31}$$

From (31), dn/dN has the same sign as $\varepsilon + c\varepsilon'$. Following the same strategy, it can be shown that dx/dN has the opposite sign as $\varepsilon + c\varepsilon'$. Thus for constant elasticity utility functions, a firm's degree of specialization decreases with the total number of goods produced in the economy. A firm's production quantity of a given commodity decreases with the total number of goods produced in the economy.

In this paper, an increase in the total number of goods produced in the economy may decrease each consumer's welfare. This result is different from Krugman 's [10] model. In his model, the number of goods is endogenously determined, and an increase in the variety of goods increases a representative consumer's welfare. In this model, the number of goods produced in the economy is exogenously given, and there are two effects on a consumer's welfare when the total number of goods increases. First, as the consumption of any given good suffers from diminishing marginal utility, increasing the total number of goods spreads the consumption over more goods and raises a consumer's welfare. Second, as the total number of goods increases, the quantity of production for any given good decreases. Because of the existence of increasing returns to scale in production, this decrease means that the average cost of production increases and a consumer's welfare decreases. These two effects work in opposite directions, and the second effect may dominate the first one.[11]

---

[10] For the example in footnote 9, nx does not change with L.

[11] For the example in footnote 9, $U = N \ln\left(\frac{L}{8N^2}\right)$. As a result, $\frac{dU}{dN} = \ln\left(\frac{L}{8N^2}\right) - 2$. From this equation, we see that when the population size is much larger than the number of goods, a consumer will benefit from an increase in the total number of goods produced in the economy. In this example, if in equilibrium each firm produces at least one product, an increase in the total number of goods produced in the economy will decrease a representative consumer's welfare.

How will a firm's degree of specialization be affected by the elasticity of demand? When this elasticity is constant, this question can be answered in a clear way.

Proposition 5 *For constant elasticity utility functions, an increase in the elasticity of demand causes an increase in a firm's degree of specialization.*

Proof: Differentiating (19c) with respect to c, n, x, L, N, and ε, it can be shown that

$$\frac{\partial R_3}{\partial n}dn + \frac{\partial R_3}{\partial x}dx + \frac{\partial R_3}{\partial c}dc = -\frac{\partial R_3}{\partial N}dN - \frac{\partial R_3}{\partial L}dL - \frac{\partial R_3}{\partial \varepsilon}d\varepsilon . \qquad \text{(20c)'}$$

From (20a), (20b), and (20c)', it can be shown that

$$\frac{dn}{d\varepsilon} = -\frac{\partial R_1}{\partial x}\frac{\partial R_2}{\partial c}\frac{\partial R_3}{\partial \varepsilon}/\Delta . \qquad (32)$$

Differentiating (19c) with respect to ε, it can be shown that

$$\frac{\partial R_3}{\partial \varepsilon} = \frac{x}{Lc\varepsilon^2} > 0 . \qquad (33)$$

From (24), (27), (32), and (33), it is clear that $dn/d\varepsilon < 0$. QED

The intuition behind Proposition 5 is the following. When the elasticity of demand increases, a consumer's marginal utility increases, assuming that the amount of consumption does not change. As consumers are more willing to pay, a firm's marginal revenue increases, and the production of each good increases. This leads to an increase in the degree of specialization since a more specialized technology is more suitable for larger scale production.

Following the same method used in proving Proposition 5, it can be shown that a consumer's consumption of each good and a firm's production of each good also increase with the elasticity of demand.

6. Conclusion

This paper examines the mutual dependence between the division of labor and the extent of the market. The crucial assumption in this paper is that the fixed cost of production for each good increases and the marginal cost of production decreases when a firm becomes more specialized. It has been shown that international trade increases welfare as more specialized technology is adopted and the average cost of production for each good decreases. There is no monotonic relationship between the size of firms and the extent of the market.

The setup in this paper is different from that of Krugman [10]. In Krugman [10], a society may produce any number of goods. In this paper, the total number of goods produced in the economy is exogenously given. An examination of the implications of the assumption of a fixed number of goods on trade and growth may be an interesting topic for future research. Alternatively, incorporating the introduction of new goods through research and development into the current model may be an interesting avenue for future research. In this paper, the population is the only factor affecting the supply of labor. In real world situations, the quality of labor, such as human capital accumulated through learning by doing, may also be important in determining the effective supply of labor. Incorporating learning by doing into the model seems to be a promising topic for future research.

References

1. Ades, A., Glaeser, E.: Evidence on Growth, Increasing Returns, and the Extent of the Market. Quarterly Journal of Economics **114**, 1025-1045 (1999)
2. Baumgardner, J.: Physician's Services and the Division of Labor across Local Markets. Journal of Political Economy **96**, 948-982 (1988)
3. Baumgardner, J.: The Division of Labor, Local Markets, and Worker Organization. Journal of Political Economy **96**, 509-527 (1988)
4. Ciccone, A.: Input Chains and Industrialization. Review of Economic Studies **69**, 565-587 (2002)
5. Ciccone, A., Matsuyama, K.: Starting Cost and Pecuniary Externalities as Barriers to Economic Development. Journal of Development Economics **49**, 33-59 (1996)
6. Dixit, A., Stiglitz, J.: Monopolistic Competition and Optimum Product Diversity. American Economic Review **67**, 297-308 (1977)
7. Frankel, J., Romer, D.: Does Trade Cause Growth? American Economic Review **89**, 379-399 (1999)
8. Hotelling, H.: Stability in Competition. Economic Journal **39**, 41-57 (1929)
9. Kim, S.: Labor Specialization and the Extent of the Market. Journal of Political Economy **97**, 692-705 (1989)

10. Krugman, P.: Increasing Returns, Monopolistic Competition, and International Trade. Journal of International Economics **9**, 469-479 (1979)
11. Liu, P., Yang, X.: The Theory of the Irrelevance of the Size of the Firm. Journal of Economic Behavior and Organization **42**, 145-165 (2000)
12. Locay, L.: Economic Development and the Division of Production between Households and Markets. Journal of Political Economy **98**, 965-982 (1990)
13. Romer, P.: Endogenous Technological Change. Journal of Political Economy **98**, S71-S102 (1990)
14. Rosen, S.: Substitution and Division of Labor. Economica **45**, 235-250 (1978)
15. Rosen, S.: Specialization and Human Capital. Journal of Labor Economics **1**, 43-49 (1983)
16. Salop, S.: Monopolistic Competition with Outside Goods. Bell Journal of Economics **10**, 141-156 (1979)
17. Smith, A.: The Wealth of Nations. Chicago: University of Chicago Press (1976)
18. Stigler, G.: The Division of Labor is Limited by the Extent of the Market. Journal of Political Economy **59**, 185-193 (1951)
19. Weitzman, M.: Monopolistic Competition with Endogenous Specialization. Review of Economic Studies **61**, 45-56 (1994)
20. Yang, X., Borland, J.: A Microeconomic Mechanism for Economic Growth. Journal of Political Economy **99**, 460-482 (1991)
21. Yang, X., Ng, S.: Specialization and Division of Labor: A Survey. in Increasing Returns and Economic Analysis, edited by Arrow, K., Ng, K., and Yang, X., New York: St. Martin Press (1998)
22. Young, A.: Increasing Returns and Economic Progress. Economic Journal **38**, 527-542 (1928)