Why Are Inflation and Real Interest Rates So Low? A Mechanism of Low and Floating Real Interest and Inflation Rates

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A Mechanism of Low and Floating Real Interest and Inflation Rates

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Abstract

Real interest and inflation rates have been very low in many industrialized countries since the Great Recession. In this paper, a mechanism of low and floating real interest and inflation rates is examined based on the concept a "Nash equilibrium of a Pareto inefficient path" and the law of motion for trend inflation. I show that, because the link between the marginal product of capital and the real interest rate is severed on this path, the real interest rate loses its anchor and therefore floats. In addition, the inflation rate floats together with the real interest rate. There are, however, upper and lower bounds of the floating rates. It is also likely that the real interest rate floats below the marginal product of capital on this path and the inflation rate floats below the target rate of inflation.

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1 INTRODUCTION

Real interest rates have been historically low in many industrialized countries since the Great Recession (International Monetary Fund, 2014; King and Low, 2014; Bean et al., 2015; Council of Economic Advisers, 2015; Hall, 2016). Furthermore, real interest rates have been lower than the usually supposed values of marginal product of capital (MPK), which is theoretically equal to the rate of time preference (RTP) of households at steady state. RTP is often assumed to be about 4% annually (see Harashima, 2016a).

At the same time, inflation has been also very low in many industrialized countries (Kiley, 2015; Arias et al., 2016; Ciccarelli and Osbat, 2017). Furthermore, inflation rates have floated below the target rates (usually 2%) of most central banks. For example, although Japan's central bank has continuously injected a huge amount of money into financial markets to achieve the implicitly or explicitly expressed 2% target rate of inflation, the consumer price index (CPI) of Japan has varied between about 1% and –1% since the 1990s. Theoretically, inflation rates should converge at the target rate of inflation, but they have floated below this theoretically predicted level for more than a decade in many countries.

To explain the observed persistently low real interest rates, Bean et al. (2015) argued that there was a simultaneous rise in the overall propensity to save and a decline in the overall propensity to invest, and that savings (investments) became relatively insensitive to changes in interest. Thwaites (2015) argued that demographics, inequality, and emerging economies played important roles and the common underlying factor is a rise in saving. Hall (2016) argued that the low real interest rate is attributed to a change in the composition of investors from those with higher degrees of risk aversion to those with lower degrees. These arguments seem to be unrelated to each other at first glance, but a common thread is that firms have become less inclined to invest as household savings have piled up.

Low inflation may be explained partly by weak domestic demand and other cyclical factors with some sort of Phillips curve (e.g., Coibion and Gorodnichenko, 2015; Conti et al., 2015; Del Negro, 2015; Arias et al., 2016; Ciccarelli and Osbat, 2017), but the ways used to estimate and interpret the Phillips curve are controversial. Some structural factors may have influenced inflation, causing it to remain low. Of these, demographic factors and technological progress have attracted particular attention. A declining share in the working-age population may have a negative impact on inflation (e.g., Anderson et al., 2014; Deroose and Stevens, 2017), and the spread of e-commerce may also decrease inflation (e.g., Ciccarelli and Osbat, 2017), but the magnitudes of the impacts of these are not well known.

In this paper, a mechanism of low and floating real interest and inflation rates is examined from another perspective. Nevertheless, the mechanism shown here may act in somewhat similar ways as those reported by Bean et al. (2015), Thwaites (2015), and Hall (2016) in that a change in people's investment behaviors plays an essential role. However, the reasons why people changed their behaviors are completely different from those offered by Bean et al. (2015), Thwaites (2015), and Hall (2016).

As noted above, low and floating real interest and inflation rates have been observed since the Great Recession. Hence, the cause of the Great Recession needs to be examined prior to examining the mechanism of floating rates. Harashima (2016a) showed a cause of the Great Recession based on the concept a "Nash equilibrium of a Pareto inefficient path" (NEPIP). The concept of NEPIP reported by Harashima (2004a, 2009, 2012, 2016a, 2016d, 2017, 2018) enables us to explain a mechanism for why households rationally choose a Pareto inefficient path. Because of this choice, phenomena like the Great Recession and the Great Depression can be generated. An important feature of NEPIP is that it does not require a sudden huge technological regression and persisting rigidities in price adjustment processes to explain the Great Recession.

It is also necessary to examine the mechanism by which inflation is determined before examining the mechanism of floating rates. In this paper, I use the model of inflation and the law of motion for inflation described previously (Harashima 2004b, 2006, 2007a, 2007b, 2008a,

2008b, 2013, 2016b, 2016c) because this model offers a micro-founded mechanism of why inflation has a persistent nature (i.e., why it has trends). In this model, the persistence of inflation is explained without including *ad hoc* lagged inflation into the model, unlike the case of the hybrid new Keynesian Philips curve presented by Galí and Gertler (1999).

Here, I show that the link between MPK and the real interest rate is severed on NEPIP. As a result, the real interest rate loses its anchor and therefore floats regardless of MPK. If the central bank is sufficiently independent, however, the inflation (deflation) rate does not accelerate or decelerate; rather, it moves stochastically by the law of motion for trend inflation in the abovementioned model of inflation. If the nominal interest rate is kept near zero, the inflation rate is equal to zero minus the real interest rate. Therefore, the inflation (deflation) and real interest rates are connected and float together regardless of either the target rate of inflation or MPK. However, there are upper and lower bounds for the floating rates. Because of these bounds, it is likely that, on average, the real interest rate is lower than MPK and the inflation rate is lower than the target rate of inflation on NEPIP. In addition, low unemployment, inflation, and real interest rates may continue to coexist for a long period after a severe recession or depression is almost stabilized.

2   FLOATING REAL INTEREST RATES

2.1   *Real interest rates and MPK*

2.1.1   Real interest rate

The real interest rate cannot be directly observed and is usually substituted with the return on government bonds with inflation protection or is indirectly estimated by subtracting the estimated rate of expected inflation or realized rate of inflation from the observed nominal rate of return on government bonds. In this paper, for simplicity, the real interest rate specifically indicates the realized real interest rate (RRIR) calculated by subtracting the realized rate of inflation from the observed nominal rate of return on government bonds. Hence,

$$r_t = i_t - \pi_t \tag{1}$$

where r_t is RRIR, i_t is the observed nominal rate of return on government bonds in period t, and π_t is the realized inflation rate in period t.

Note that if the disturbances in inflation are independently and identically distributed, RRIR is on average equal to the expected real interest rate calculated by subtracting the expected inflation rate from the observed nominal rate of return on government bonds.

2.1.2   Link between RRIR and MPK

MPK is $\dfrac{\partial y_t}{\partial k_t}$ where y_t and k_t are production and capital per capita in period t, respectively. Suppose that the economy is at steady state or on the saddle path to steady state. Standard economic theory tells us that the real interest rate for private sector investments is equal to MPK. In addition, RRIR (i.e., the real interest rate for government bonds), is on average also equal to MPK, such that,

$$r_t = \frac{\partial y_t}{\partial k_t} \ . \tag{2}$$

Note that "on average" is added because RRIR is on average equal to the expected inflation rate. The rationale for equation (2) is as follows. Because the economy is at steady state or on the saddle path, the government issues new bonds not because it wants to deviate from these states, but simply because it wants to replace taxes with borrowings (i.e., reduce current taxes and instead

increase taxes in the future when the bonds are redeemed). Because the economy is at steady state or on the saddle path, however, there is no excess saving. Therefore, if $r_t < \frac{\partial y_t}{\partial k_t}$, there is no incentive for economic agents to switch from private sector investments to new government bonds. On the other hand, if $r_t > \frac{\partial y_t}{\partial k_t}$, excess demand for new government bonds will be generated. The excess demand will be reduced by decreases in r_t, and therefore r_t will eventually stabilize at the level that satisfies equation (2). Hence, government bonds can be sold only when RRIR on average satisfies equation (2). If equation (2) is on average satisfied and government bonds are sold out at this level of RRIR, the government reduces taxes by the amount substituted with the sale of bonds. Because economic agents know about the present tax reduction and future tax increase as well as the steady state conditions, they do not increase consumption even if taxes are reduced, but instead invest money obtained from the reduced taxes in the private sector. As a result, the levels of capital and consumption at steady state and on the saddle path are unaffected and maintained even after the government issues new bonds while equation (2) is on average satisfied. This logic is basically identical to that in the Ricardian equivalence proposition.

Equation (2) indicates that there is a tight link between MPK and RRIR. In other words, MPK plays the role of an anchor for RRIR, and RRIR is constrained to be equal to MPK if the economy is at steady state or on the saddle path.

2.2 Nash equilibrium of a Pareto inefficient path (NEPIP)

Harashima (2004a, 2009, 2012, 2016a, 2016d, 2017, 2018) showed that it is possible that households will rationally choose a Pareto inefficient transition path (i.e., NEPIP) if a shock that changes the steady state occurs. The mechanism of this phenomenon is briefly explained in this section.

It is assumed that households are non-cooperative, risk averse, identical, and infinitely living, and that the number of households is sufficiently large. Each household maximizes its expected utility

$$E \int_0^\infty \exp(-\theta t)\, u(c_t)dt$$

subject to

$$\frac{dk_t}{dt} = f(A, k_t) - c_t$$

where c_t is consumption in period t; A is technology; u is the utility function; $y_t = f(A, k_t)$ is the production function; $\theta\,(>0)$ is RTP; and E is the expectation operator.

Suppose that there is a shock that makes the RTP of the representative household (RTP RH) shift upward (increase) in period $t = 0$. After the shock, the steady state is changed from the prior (original) one to the posterior one. There are two options for each non-cooperative household with regard to consumption just after the shock. The first is a jump option J, in which a household's consumption jumps upward and then proceeds on the posterior Pareto efficient saddle path to the posterior steady state. The second is a non-jump option NJ, in which a household's consumption does not jump but instead gradually decreases from the prior steady state to the posterior steady state. The household that chose the NJ option reaches the posterior steady state in period $s\,(\geq 0)$. The difference in consumption between the two options in each period t is $b_t\,(\geq$

0). The existence of b_t indicates that excess capital exists and needs to be eliminated.

There is a sufficiently high probability that a household chooses the NJ option because option J requires a discontinuous large and sudden increase in consumption but risk-averse households intrinsically dislike a discontinuous change in consumption and want to smooth the stream of consumption. The expected utility of a household after the shock depends on its choice of J or NJ. Let Jalone indicate that a household chooses the J option but the other households choose the NJ option, NJalone indicate that a household chooses the NJ option but the other households choose the J option, Jtogether indicate that all households choose the J option, and NJtogether indicate that all households choose the NJ option. Let $p\ (0 \leq p \leq 1)$ be the subjective probability of a household that the other households choose option J. The expected utility of the household when it chooses option J is,

$$E(J) = pE(Jtogether) + (1-p)E(Jalone),$$

and when it chooses option NJ is

$$E(NJ) = pE(NJalone) + (1-p)E(NJtogether),$$

where $E(Jalone)$, $E(NJalone)$, $E(Jtogether)$, and $E(NJtogether)$ are the expected utilities of the household when choosing Jalone, NJalone, Jtogether, and NJtogether, respectively. A household strategically determines whether to choose the J or NJ option, considering other households' choices. Harashima (2009) proved that, under reasonable conditions, there is a $p^*\ (0 \leq p^* \leq 1)$ such that if $p = p^*$, $E(J) - E(NJ) = 0$, and if $p < p^*$, $E(J) - E(NJ) < 0$; that is, it is possible for a Pareto inefficient path to be rationally chosen by households.

Suppose that there are $H\ (\in N)$ identical households in the economy where H is sufficiently large. Households' strategic choices between the J and NJ options are well described by an H-dimensional symmetric mixed-strategy game. Let $q_\eta\ (0 \leq q_\eta \leq 1)$ be the probability that household $\eta\ (\in H)$ chooses option J. Harashima (2009) showed that strategy profiles

$$(q_1, q_2, \ldots, q_H) = \{(1, 1, \ldots, 1),\ (p^*, p^*, \ldots, p^*),\ (0, 0, \ldots, 0)\}$$

are the Nash equilibria of this game. Furthermore, Harashima (2009) showed that if households have a risk-averse preference in the sense that they avert the worst scenario when its probability is not known, households suppose very low p and select the NJtogether $(0, 0, \ldots, 0)$ equilibrium, which is NEPIP. Because NEPIP is Pareto inefficient and excess capital exists and b_t unutilized resources are successively generated and destroyed, a recession or depression is generated (Harashima, 2004a, 2009, 2012, 2016a, 2016d, 2017). In this situation, as Harashima (2012) showed, the unemployment rate rises in the search and matching process of jobs. Harashima (2014b) described a generation mechanism for a shock on RTP RH. The main underlying factor of this shock is that households need to generate an expected RTP RH under sustainable heterogeneity (Harashima, 2014a, 2014b).

After an upward shift of RTP RH, MPK begins to increase to reach the posterior steady state corresponding to the higher posterior RTP RH. Figure 1 shows the estimated MPK of the United States since 2000. The method of estimation is the same as that used in Harashima (2016a) and is shown in the Appendix. Because the primary focus is fluctuations of MPK and not its absolute level, the value of MPK in 2000 was arbitrarily chosen to be near the 4% value. Figure 1 indicates that MPK actually increased after the Great Recession although RRIR significantly decreased as indicated in the Introduction. Although specific estimates may differ depending on the assumptions used, the trend in Figure 1 seems quite reasonable because investments decreased and the rate of increase in capital stocks correspondingly decreased after the Great Recession.

Figure 1: The estimated MPK of the United States
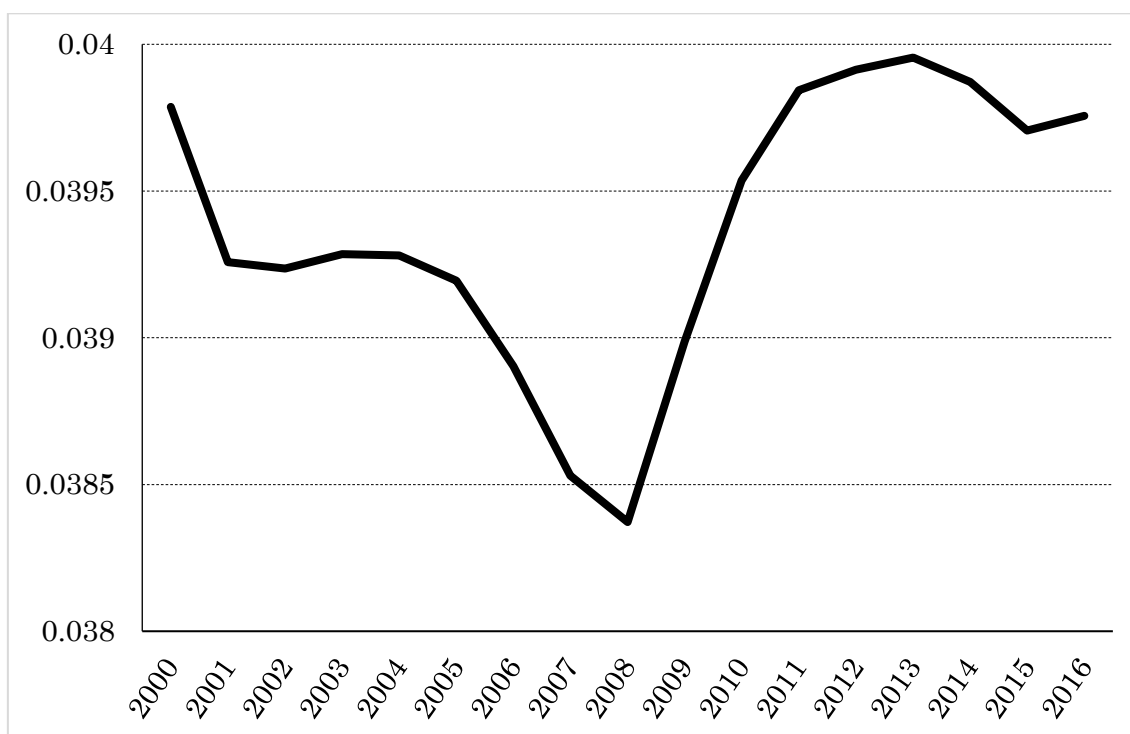
2.3 Rationality

Rational expectations mean model-consistent expectations. Economic agents behave based on optimization solutions in the models they know while utilizing all available information. This behavior indicates that they behave in principle in accordance with the steady state because utilizing all available information requires that the models used be dynamic with an infinite horizon, and the optimization solution in a dynamic model in principle indicates the steady state. Under rational expectations, therefore, economic agents behave in accordance with the steady state and are not affected persistently by temporary disturbances. Because rationality is assumed to be fundamental to human nature, in principle it will prevail in any situation, even on NEPIP.

Behaving in accordance with the steady state even on NEPIP suggests the possibility that on NEPIP (i.e., on a "rationally" chosen Pareto inefficient path), no firm will "rationally" undertake additional investments even if RRIR is almost zero. Furthermore, households will "rationally" deposit their money in banks even if the nominal interest rates of bank deposits are almost zero, if these behaviors are in accordance with the steady state. This possibility is examined in the following sections.

2.4 Floating RRIR on NEPIP
2.4.1 Investments, savings, and loans on NEPIP
2.4.1.1 Investments

Behaving in accordance with the steady state as shown in Section 2.3 means that firms prioritize their sustainability or survivability over present profits because a steady state means persistence or indefinite stability. In a static model, sustainability or survivability does not matter, and a firm can simply be assumed to be an entity that prioritizes current profit maximization. However, in a dynamic model, sustainability or survivability matters greatly to firms. In some cases, earning low profits or even experiencing losses at present may be the best strategy for a firm to survive

for a long period, particularly on NEPIP.

As shown in Section 2.2, excess capital and b_t exist on NEPIP, and capital has to be reduced to the posterior steady state level. In this situation, additional investments will likely fail quickly and generate losses. A firm that undertakes such an eventually unprofitable investment will not survive. Therefore, even if the real lending rates of banks (the nominal lending rates of banks minus the inflation rate) to firms are very low and lower than MPK, a firm will not undertake additional investments on NEPIP. Note that, in reality, firms will undertake investments to replace obsolete machines or introduce the latest technologies even on NEPIP, but in this paper, it is assumed for simplicity that there is no capital depreciation and no technological progress.

Here, suppose that an additional investment generates profits in each period until it fails and generates losses when it fails. Suppose also that firms have to borrow money from banks to undertake investments and the real lending rate of banks is equal to RRIR. A firm will undertake a new investment if

$$E\left[\sum_{t=0}^{v}(\mu_t - r_t) - l\right] > 0 \tag{3}$$

is satisfied, where μ_t is the real return on the investment (the nominal return on the investment minus the inflation rate) in period t, v (> 0) is the period when the investment fails, and l is the initial investment and indicates the loss when it fails in period v. Hence, $E(v)$ indicates the expected length of period before the fail. $E(v) = \infty$ means that the investment is not expected to fail indefinitely and thereby $E(l) = 0$.

In a normal period (i.e., not on NEPIP), if $r_t > \theta_P$ on the saddle path (θ_P indicates RTP RH), there are many investment opportunities that satisfy condition (3) because $E(v)$ will be a large number, and many investments will be undertaken until the economy reaches steady state. However, because excess capital exists on NEPIP, $E(v)$ is far shorter than that in the normal period because economic agents who behave in accordance with the steady state do not purchase products that are not in accordance with the steady state. Put more intuitively, because the overall demand is shrinking on NEPIP, an additional investment will soon fail. Here, "additional" means a deviation from the state that is in accordance with the steady state. Hence, for many investment opportunities on NEPIP,

$$E\sum_{t=0}^{v}(\mu_t - r_t) < E(l)$$

because of the short $E(v)$. It will be difficult to satisfy condition (3) even if the real lending rate of banks (i.e., RRIR [r_t]) is relatively low. As a result, additional investments will be rarely undertaken on NEPIP.

Note that some additional investments may be sustainable even on NEPIP, but only if part of the existing capital with a value equivalent to these additional investments is destroyed.

2.4.1.2 Savings and loans

Because additional investments are rarely undertaken on NEPIP, firms rarely borrow money from banks even if RRIR is very low, and banks cannot easily find new borrowers. On the other hand, if RRIR is very low, the real interest rate for bank deposits also becomes very low because banks still need to earn profits. Even if the real deposit rate is very low, however, households will still deposit money into banks on NEPIP because their consumption levels are lower than the level that satisfies Pareto efficiency, as shown in Section 2.2.

As a result, a large amount of savings is additionally deposited in banks by households,

but the banks can rarely lend money to firms for additional investments. Therefore, a large amount of excess money is generated inside banks. The gap between deposits and loans has to be destroyed (e.g., some banks fail), unless the government intervenes.

Note that the very low real deposit rate indicates that the interest income of households from their deposits in banks is smaller on NEPIP than in a normal period. This means that some of the excess capital that should eventually be destroyed is destroyed through the lower real deposit rate (i.e., the lower RRIR); that is, households' savings in banks are invested in less or unproductive projects (i.e., government bonds) by banks.

2.4.1.3 Government bonds fill the gap between savings and loans

If the government intervenes, however, the situation can change. If the government additionally issues a considerable amount of bonds, banks can survive because they can fill the gap between deposits and loans by purchasing the government bonds. Even if RRIR is very low, it is better for banks to buy the government bonds than to hold money without any return. Conversely, even if RRIR is very low and lower than MPK, the government bonds are strongly demanded by banks. This persistent disparity between RRIR and MPK indicates that they are determined differently on NEPIP.

As Harashima (2017) showed, filling the gap by issuing additional government bonds will significantly delay the economy's arrival at the posterior steady state; that is, the transition period from the prior to the posterior steady state is significantly prolonged. Hence, if the government continues to intervene, the period when RRIR and MPK are determined differently will continue for a longer period.

If sufficient amounts of government bonds are not supplied to financial markets or the central bank absorbs a large part of them, some banks may eventually fail because they cannot obtain sufficient profit by purchasing government bonds. On NEPIP, some central banks may be tempted to purchase a large amount of government bonds to boost their economies (i.e., to implement quantitative easing), but excessive quantitative easing will hurt banks.

2.4.2 Severed link between RRIR and MPK on NEPIP

As shown in Section 2.1, in the normal period, a tight link exists between RRIR and MPK, and RRIR on average equals MPK as equation (2) indicates. However, on NEPIP, RRIR and MPK are determined differently as discussed in Section 2.4.1; that is, the link between them is severed. Hence, MPK no longer acts as an anchor for RRIR, RRIR is not constrained to be equal to MPK on NEPIP, and equation (2) does not hold. In addition, the arguments in Sections 2.4.1.1 and 2.4.1.2 indicate that RRIR is basically lower than MPK on NEPIP; that is,

$$r_t < \frac{\partial y_t}{\partial k_t} \ .$$

2.4.3 Floating RRIR

Without MPK as an anchor, RRIR floats on NEPIP and can take various values depending on the supply and demand for government bonds and the inflation rate in each period. An important point is that whatever values RRIR take, the paths of consumption and investment are not affected by RRIR on NEPIP because economic agents behave in accordance with the steady state, as shown in Section 2.3.

As Harashima (2012, 2017) noted, however, the unemployment rate will decrease as the level of capital approaches the level at the posterior steady state and excess capital and b_t decrease, or as b_t is almost filled by increases in government expenditure financed by new government bonds. However, even if excess capital and b_t decrease to very low levels or b_t is almost filled by increases in government expenditure, additional investments will still be rarely undertaken even though RRIR is very low, as long as any excess capital and b_t remain, as shown in Section 2.4.1.1. As a result, although the unemployment rate decreases as the economy approaches the posterior

steady state or b_t is almost filled by increases in government expenditure, RRIR will continue to float and stay at historically low rates.

In addition, if a government continues to intervene through issuing additional large quantities of government bonds and increasing government expenditures, it will significantly delay the economy's arrival at the posterior steady state, as shown in Section 2.4.1.3 and reported by Harashima (2017), and RRIR will continue to float and be historically low for a long period.

3 FLOATING INFLATION (DEFLATION) RATE

3.1 Law of motion for trend inflation

Harashima (2004b, 2008b, 2016b) presented a model of inflation (deflation) and a law of motion for trend inflation. This law is explained briefly in this section. Because the nominal rate of return on government bonds (i_t) is

$$i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v dv ds + r$$

and because $r = \theta_P$ at steady state, the law of motion is

$$\int_{t-1}^{t} \int_{s}^{s+1} \pi_v dv ds = \pi_t + \theta_G - \theta_P \qquad (4)$$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$ by the simultaneous optimization of government and households, where θ_G and θ_P are the RTPs of the government and representative household, respectively; r is RRIR at steady state; and π_t is the inflation rate, g_t is the per capita real government expenditure, and x_t is the per capita real tax revenue at time t. The mechanism by which the RTP of government is formed is shown in Harashima (2015b). By the law of motion (i.e., by equation [4]), if $\theta_G = \theta_P$ at steady state, inflation (deflation) does not accelerate or decelerate. Note that if $r \neq \theta_P$ but $r = \bar{r}$ where $\bar{r}$ is a constant, the law of motion is modified to

$$\int_{t-1}^{t} \int_{s}^{s+1} \pi_v dv ds = \pi_t + \theta_G - \bar{r} \ , \qquad (5)$$

and if $\theta_G = \bar{r}$ at steady state, inflation (deflation) does not accelerate or decelerate.

A solution of integral equation (4) for given θ_G and θ_P is

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)t^2 \ .$$

Generally, the path of inflation that satisfies equation (4) for $0 \leq t$ is expressed as

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)\exp[z_t \ln(t)] \ ,$$

where z_t is a time-dependent variable. If π_t satisfies equation (4) for $0 \leq t$, and $-\infty < \pi_t < \infty$ for $-1 < t \leq 1$, then

$$\lim_{t \to \infty} z_t = 2 \quad .$$

Harashima (2016b) showed that even in a period of deflation, if the central bank is sufficiently independent, households expect that $\theta_G = \theta_P$ (or $\theta_G = \bar{r}$). Therefore, deflation does not accelerate or decelerate even though the central bank lowers the nominal interest rate to zero.

3.2 Inflation (deflation) on NEPIP

By the law of motion for trend inflation, if the central bank is sufficiently independent and households expect that $\theta_G = \theta_P$ (or $\theta_G = \bar{r}$), inflation (deflation) does not accelerate or decelerate and moves stochastically, whether or not the economy is on NEPIP. However, in a normal period, in addition to equation (4) or (5), equations (1) and (2) also have to be satisfied. Hence, the inflation (deflation) rate moves stochastically but within a very narrow range in which equation (2) is on average satisfied; that is, it moves in the neighborhood of the target rate of inflation (an inflationary steady state) or zero minus MPK (a deflationary steady state). On NEPIP, however, although equation (1) is also satisfied as with the case of a normal period, equation (2) is not necessarily satisfied as described in Section 2.4. Therefore, the inflation (deflation) rate is not constrained to move in the neighborhood of the target rate of inflation or zero minus MPK.

Suppose that the central bank is sufficiently independent and households expect that $\theta_G = \theta_P$ (or $\theta_G = \bar{r}$), and in addition, the central bank makes the nominal rate of return on government bonds zero (i.e., $i = 0$). Hence, by equation (1)

$$r_t = -\pi_t \quad . \tag{6}$$

In a normal period, if equation (6) holds,

$$\frac{\partial y_t}{\partial k_t} = -\pi_t \tag{7}$$

for any t by equation (2); that is, the deflation rate is equal to MPK (i.e., the inflation rate is equal to zero minus MPK). Equation (7) is superficially equivalent to the Friedman rule. On NEPIP, however, equation (7) does not necessarily hold even if equation (6) holds because equation (2) does not hold. The link between the inflation rate and MPK does not exist on NEPIP.

Note that the reason why the central bank makes $i = 0$ is generally to initiate or support the "recovery" of the economy from NEPIP. However, because NEPIP was strategically and rationally chosen by households and the link between RRIR and MPK is severed on NEPIP, households do not deviate from NEPIP even when facing a zero nominal interest rate. That is, monetary policies will be ineffective to improve an economy on NEPIP.

3.3 Floating inflation (deflation) rate

On NEPIP with $i = 0$, the inflation rate is not necessarily equal to zero minus MPK, as shown in Section 3.2, and RRIR is also not necessarily equal to MPK, as shown in Section 2.4. Hence, both the inflation (deflation) rate and RRIR have no anchor and commonly float. Nevertheless, because equation (6) holds even on NEPIP with $i = 0$, the inflation (deflation) rate and RRIR are connected and therefore float together.

As shown in Section 2.4.3 and Harashima (2012, 2017), as the economy approaches the posterior steady state or if b_t is almost filled by increases in government expenditure, the unemployment rate decreases. However, RRIR still floats as long as the excess capital and b_t remain, and the inflation (deflation) rate also floats together with RRIR. In addition, if the government continues to intervene, it will take a long time for the economy to reach the posterior

steady state, as shown in Section 2.4.1.3. As a result, a low unemployment rate, low inflation, and low RRIR may continue to coexist for a long period after a severe recession is almost stabilized. A situation like this occurred with the U.S. economy in the 1950s after the period when the Great Depression had almost been stabilized.

4 RANGE OF FLOATING RATES

Both inflation (deflation) and RRIR float together on NEPIP, but they cannot float without any constraint. Upper and lower bounds exist.

4.1 Upper (lower) bound of RRIR (inflation)

If the deflation rate (i.e., $-\pi_t$) exceeds MPK, the real lending rates of banks would exceed MPK even if the nominal lending rate was zero. Therefore, if this level of deflation continues, many firms will not pay back their bank loans and will go bankrupt, and eventually the economy will collapse. Because households know of this consequence and behave in accordance with the steady state, it is highly unlikely that they generate the expectation that deflation exceeds MPK.

Furthermore, if the economy's collapse was expected because deflation rates were persistently larger than MPK, the government would annul the central bank's independence and behave very myopically. Thereby, the expected θ_G would sharply increase and the deflation would turn to inflation (probably hyper-inflation) by equation (4) or (5). Households also know about this devastating consequence, and therefore will not generate an expected deflation rate larger than MPK.

This means there is the upper bound for the deflation rate on NEPIP (i.e., MPK). Conversely, the lower bound of the inflation rate on NEPIP is zero minus MPK. In addition, for $i = 0$, the upper bound of RRIR on NEPIP is also MPK by equation (6). Note, however, because of initial confusion among people in the initial stage of NEPIP, the deflation rate may temporarily exceed MPK.

4.2 Lower (upper) bound of RRIR (inflation)

Given a sufficiently independent central bank, households will not expect inflation rates higher than the central bank's target rate of inflation (π^*) because if the rate exceeds π^*, the independent central bank must raise the nominal interest rate enough to bring the inflation rate down to π^*. Unlike the zero lower bound in the case of lowering the nominal interest rate, there is no upper bound for raising it, so the central bank can raise it as much as necessary. As described by Harashima (2007b, 2008b), faced with an intentionally raised nominal interest rate, the government is forced to change its behavior to one that is consistent with π^*, and thereby the inflation rate will converge at π^*. Therefore, the upper bound of the inflation rate on NEPIP is the target rate of inflation (π^*). In addition, for $i = 0$, the lower bound of RRIR on NEPIP is zero minus the target rate of inflation ($-\pi^*$) by equation (6).

4.3 Range

Sections 4.1 and 4.2 indicate that RRIR floats between MPK and zero minus the target rate of inflation ($-\pi^*$) on NEPIP, and the inflation rate floats between the target rate of inflation (π^*) and zero minus MPK. For example, suppose that the target rate of inflation is 2%, the MPK at steady state is 4%, and the nominal rate of return on government bonds is zero. In this case, RRIR floats between –2% and 4%, and the inflation rate floats between 2% and –4%. On average, RRIR will be 1% and the inflation rate will be –1% (i.e., 1% deflation) on NEPIP. This example implies that, on average, RRIR is lower than MPK and the inflation rate is lower than the target rate of inflation on NEPIP. This property is consistent with the recently observed low inflation and real interest rates and the empirical estimates presented in Harashima (2016a).

5   CONCLUDING REMARKS

Recently, both RRIRs and inflation rates have been very low in many industrialized countries. Theoretically, RRIR is equal to MPK, and the inflation rate converges at the target rate of inflation set by the central bank, but in reality, RRIR and the inflation rate have been below MPK and the target rate of inflation, respectively. That is, they have both floated for a long period below theoretically predicted levels.

In this paper, a mechanism of low and floating RRIRs and inflation rates was examined based on the concept of NEPIP described by Harashima (2004a, 2009, 2012, 2016a, 2016d, 2017, 2018) and the law of motion for trend inflation presented by Harashima (2004b, 2006, 2007a, 2007b, 2008a, 2008b, 2013, 2016b, 2016c). I showed that, because the link between MPK and RRIR is severed on NEPIP, RRIR loses its anchor and floats. In addition, the inflation (deflation) rate also floats together with RRIR on NEPIP. However, upper and lower bounds exist for the floating rates. Because of these bounds, it is likely that, on average, RRIR is lower than MPK and the inflation rate is lower than the target rate of inflation on NEPIP. In addition, low unemployment rates, low inflation, and low RRIRs may continue to coexist for a long period after a severe recession or depression is almost stabilized, for example, the period in the U.S. economy in the 1950s after the Great Depression was almost stabilized.

Appendix

MPK ($\frac{\partial y_t}{\partial k_t}$) in Figure 1 was estimated using capital stock data and an assumed rate of average technological progress. The production function was assumed to be a Harrod-neutral production function such that $y_t = A_t^\alpha k_t^{1-\alpha}$; thus,

$$\frac{\partial y_t}{\partial k_t} = A_t^\alpha (1 - \alpha) k_t^{-\alpha} \ , \tag{A1}$$

where A_t is technology in period t, and α is a constant that indicates the labor share. I estimated the time-series data of $\frac{\partial y_t}{\partial k_t}$ based on equation (A1), with data for k_t and assumed values of A_t and α.

Data for k_t were derived from the chain-type quantity index for private nonresidential fixed assets in *National Economic Accounts*. α was set at 0.7, which is a typical value for labor share, and A_t^α was assumed to grow at a constant rate of 1.25% annually. This means that technology was assumed to progress constantly. This rate of growth was adopted based on an average annual per capita GDP growth rate of 1.8% because, if sustainable heterogeneity is satisfied, the growth rate of A_t is equal to the growth rate of y_t on a balanced growth path. Therefore, the growth rate of A_t^α is $(1.018^{0.7} - 1) \times 100 = 1.25\%$. Because the primary focus is fluctuations of MPK and not its absolute level, the value of MPK in 2000 was arbitrarily chosen to be near the 4% value.

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