Using Value-at-Risk for effective energy portfolio risk management

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23 January 2019

Online at https://mpra.ub.uni-muenchen.de/91674/
MPRA Paper No. 91674, posted 23 January 2019 21:38 UTC
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Abstract
It is evident that the prediction of future variance through advanced GARCH type models is essential for an effective energy portfolio risk management. Still it fails to provide a clear view on the specific amount of capital that is at risk on behalf of the investor or any party directly affected by the price fluctuations of specific or multiple energy commodities. Thus, it is necessary for risk managers to make one further step, determining the most robust and effective approach that will enable them to precisely monitor and accurately estimate the portfolio’s Value-at-Risk, which by definition provides a good measure of the total actual amount at stake. Nevertheless, despite the variety of the variance models that have been developed and the relative VaR methodologies, the vast majority of the researchers conclude that there is no model or specific methodology that outperforms all the others. On the contrary, the best approach to minimize risk and accurately forecast the future potential losses is to adopt that specific methodology that will be able to take into consideration the particular characteristic features regarding the trade of energy products.

Keywords: Energy commodities, Risk Management, Value-at-Risk (VaR).

JEL Classifications: C01; C58; D8; G3; O13; P28; Q43; Q47; Q58.
1. Introduction

Under normal circumstances, in a competent and balanced energy market the dimension of discrepancy among supply and demand would be simply the reason affecting the change in the price level. But there may be more than a few other aspects disturbing energy market regularity, with the most crucial factors driving energy price volatility being structural and having a long term impact. Additionally, unexpected events such as geopolitical instability and distortion and severe environmental problems can seriously affect the global economic climate and consequently necessitate the supply capability for some of the most extensively used energy products, like oil and natural gas. These events may raise in an even higher degree the amount of risk that is taken by the investors involved in the energy market.

Energy commodity producers are heavily depended industrial firms, traders, refiners, energy investors have focused on developing the essential technical tools to regularly monitor and minimize their overall price risk market exposure. At the same time they have build the optimal strategy which would allow them to maximize their profitability given a certain acceptable amount of risk. As a result, many financial consulting firms and researchers were motivated to get closely involved and find ways to manage with the certain issue.

The basic approach behind all future research done in this field was to appropriately modify traditional financial risk management tools, in order to take account for the unique characteristic features of the energy commodities’ market. A fundamental concern was to find an accurate and scientifically approved way to measure energy price risk exposure for a certain strategy and portfolio. In this case, risk analysts based their research on minimum variance, using it as a key indicator of the total price risk of a portfolio containing energy assets. In these lines a high number
of studies make one step further trying to get an estimate of the actual potential loss, using the well-known Value-At-Risk (hereafter VaR) methodology.

VaR was originally introduced by Markowitz (1952) and Roy (1952) in an attempt to optimize profit for corresponding specific levels of risk. In its current form, VaR was presented in 1989 by JP Morgan in their risk management tool called the RiskMetrics. Since then it is widely used in finance, especially by financial control institutions, as well as investment and commercial banks and private investment funds, to estimate current risk exposure. Based on the strictness for the acceptable level of risk exposure, VaR and its alternations can be estimated for a 1%, 5% or 10% confidence level. Dependent on the examined time horizon (daily, weekly, monthly or even yearly) VaR can be calculated indicating the probability of suffering a certain loss or more given the mixture of a certain investment portfolio.

The aim of the existent study is to extend the work of Halkos and Tsirivis (2018), trying to offer a complete presentation on the most representative models and methodologies that have been developed to help investors in the energy market to obtain valuable information and precise monitoring of the total amount of capital which is at risk based on their portfolio assets, through the most scientifically appropriate and accurate estimation of VaR.

2. The VaR approach as risk management tool in energy commodity market

The economic growth, which was accompanied by an extreme expansion of the financial markets and the offered financial products and services during the 1970’s and the 1980’s made risk management and risk quantification tools an absolute necessity for everyone involved. VaR methodology was particularly appreciated by superior regulatory authorities in the banking sector, as it enables financial institutions
to examine and report their overall risk exposure daily or for a predetermined time period.

A key moment for the future development and implementation of VaR by financial market participants was in 1996 when the Basel II accord, which was an agreement between the Basel Committee on Banking Supervision (BCBS) and the presidents of the national central banks of the most powerful global economies and several other countries, decided to incorporate and establish VaR as the official measurement of risk exposure for banking institutions. Basel II also encouraged financial institutions to estimate their minimum capital requirements based on their own VaR calculations for their total market risk. This development was urged by the collapse of the British Barings bank at the same year, which was a huge hit and shock for the global banking sector, creating severe turbulence and increase high uncertainty for both investors and financial markets.

Already by 1989 the private investment bank J. P. Morgan started to incorporate VaR in its risk exposure valuations, while in 1996 developed its econometric risk management tool called the Risk Metrics, which was built by integrating the VaR methodology to their risk management concept. Another important moment towards the establishment of VaR as one of the most important and scientifically approved methods for monitoring risk exposure was also when the US Securities and Exchange Commission in 1997 allowed public traded corporations to use VaR in order to report any potential losses regarding their earnings or cash flows.

As far as concerning risk management in the energy sector, the special features of this particular market which is characterized by complex return and price distributions, non-normality and outliers, as well as strong mean reversion and high and unstable volatility and correlations, urge the need for extra risk management tools
and methodologies except from the traditional approaches borrowed from financial markets, with VaR being perhaps the most popular and characteristic example in this domain of economics.

Despite its simplicity the VaR concept can be a rather valuable tool for all banks, corporations and investors to monitor their portfolios’ and investments’ total market risk exposure, providing an important guidance for their current accepted level of risk and giving them the opportunity either to reduce their exposure to speculative-high risk investments or become less conservative if allowed by their given position. Based on Jorion (2001), VaR represents the worst possible expected loss for an examined time horizon and a pre-specified confidence interval under normal market conditions. According to Hendricks (1996), Saunders and Allen (2002) and Holton (2003) VaR is able to reveal the market price risk exposure of a financial asset or portfolio in the event of a statistically bad day. Similarly, as a result, if a corporation reports a 1% one-day-VaR of a million euro, it basically means that the corporation given its current investment portfolio mix and under normal conditions, 99% of the times it will suffer a loss of a million euro or less in a single day, while there is a 1% chance that the corporation will lose more than a million euro in a single day.

Assuming that the financial asset’s returns are normally distributed the Value-at-Risk can be estimated by the following formula:

$$\text{VaR} = a \cdot \sigma \cdot \Phi^{-1}(\alpha)$$  \hspace{1cm} (1)

While the Value-at-Risk for a portfolio of assets, again assuming that the returns are normally distributed can be estimated as follows:

$$\text{VaR} = a \cdot \sqrt{X^t \Sigma X}$$  \hspace{1cm} (2)

Due to the assumed normal distribution of returns the $\alpha$ variable corresponds to the examined level of confidence (2.33 for 99% VaR and 1.65 for 95% VaR
respectively), $\sigma$ is the converted standard deviation for the specific examined VaR (1-day-VaR, 5-day-VaR, 20-day-VaR ETC.), $V_0$ is the initial market value of the financial asset and $\Delta t$ denotes the examined time horizon, while $X$ and $\Sigma X$ are the diagonal matrix of the asset’s returns and the covariance matrix respectively.

This basic methodology for estimating VaR corresponds to the Delta-Normal approach, which despite its simplicity is quite popular among risk management analysts as it requires a low computational effort, while it provides a quick and rough approximate of the firm’s risk exposure.

The Basel II regulations regarding the necessary capital requirements of banks was based on the aforementioned concept of VaR, as the Basel II Accord imposes that banks need to report their 99% daily-VaR estimate to the relative supervision authority and then compare it to their actual risk exposure calculated at the end of the same trading day. The capital framework of the Basel II agreement specifies that a bank’s capital requirement is equal to the highest amount appointed by the previous day actual VaR or the average of the actual VaR values of the 60 previous trading days, times a scaling factor accounting and penalizing the bank for the number of VaR violations during the past 250 trading days.

The daily capital need for a banking institute is calculated as follows:

$$ (DCC_{t+1}) = \text{Max} \{(3+k)V_{\text{VaR}_{60}}, \text{VaR}_t\} $$

Where $DCC_{t+1}$ is the designated capital charges at time $t+1$; $\text{VaR}_t$ represents the 99% VaR at day $t$ estimated with use of GARCH class models and long-position trading data, and $V_{\text{VaR}_{60}}$ denotes the average of the past 60 VaR values. The scaling factor, depending on the number of violations of the 99% daily-VaR during the past 250 trading days is getting values from $3 \leq (3+k) \leq 4$ (Table 1). As a result, the most
appropriate GARCH model for the calculation of VaR is the one that provides VaR estimates with the lowest possible violations according to the Basel II capital regulations.

**Table 1:** Basel committee’s penalty zones

<table>
<thead>
<tr>
<th>Zone</th>
<th>Number of exceedances</th>
<th>Multiplier k</th>
<th>Cumulative probability assuming q’=0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>0</td>
<td>3.00</td>
<td>0.0811</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.00</td>
<td>0.2858</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.00</td>
<td>0.5432</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.00</td>
<td>0.7581</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.00</td>
<td>0.8922</td>
</tr>
<tr>
<td>Yellow</td>
<td>5</td>
<td>3.40</td>
<td>0.9588</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3.50</td>
<td>0.9863</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>3.65</td>
<td>0.9960</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>3.75</td>
<td>0.9989</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3.85</td>
<td>0.9997</td>
</tr>
<tr>
<td>Red</td>
<td>10 or more</td>
<td>4.00</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

The proposed approach of the Basel II regulatory framework forms the basis behind the established risk management methodologies incorporating a GARCH-VaR procedure, as instead of the scaling factor several test have been developed which penalize the examined GARCH type models whenever they under or over-estimate VaR revealing the appropriate model for the proposed risk management research and the data sample.

2.1 *Alternations of the basic Value-at-Risk approach*

Except from the previously analyzed traditional VaR concept, several other approaches have been developed by researchers and risk management analysts in order to cover the need for examining the potential risk concerning the expected earnings or cash flows of a financial institution or corporation, as well as investigating the actual amount of the expected loss in case the loss surpasses the estimated VaR value.
One of these variations of VaR concerning the uncertainty of future earnings is the Earnings-at-Risk approach (EAR), which is widely used for risk management analysis in energy commodities like Doris and Dunn (2001) and Denton et al. (2003). The EAR emphasizes on measuring the variability in the accumulated earnings regarding both physical deliveries and financial contracts over a defined time period. The EAR is mostly implemented on an entire fiscal year in order to gain an extensive view of any potential effects on earnings, due to changes in energy commodity prices, exchange rates and investments. EAR is a valuable tool for managers to plan their firm’s management, investment and hedging strategies for a broad time horizon.

Another popular variation of VaR incorporated in several risk management researches in the energy sector such as Guth and Sepetys (2001) and Stein et al. (2001) is the Cash-flow-at-Risk (CFaR). This is used to measure the variability in the expected cash flows regarding both physical deliveries and financial contracts taking into consideration the cash-flow timing of the relative settled trades or deliveries. Both EAR and CFaR can prove to be very useful management tools, however in order to obtain accurate estimates an extremely regular monitoring of price processes and a comprehensive and detailed analysis of every single contract is absolutely necessary.

Finally, a third and perhaps most important alternation of VaR is the Conditional VaR (CVaR) introduced by Artzner et al. (1999), which was developed as a risk management tool for estimating the downside risk of an investment or portfolio, whenever the VaR value is exceeded. CVaR is basically the interval of the loss function, providing valuable information regarding the actual amount of loss in case the losses cross the estimated boundary set by VaR for the specific examined confidence level. As a result, CVaR offers more specific and accurate estimations for relatively less computational effort than VaR, therefore CVaR is widely used by
academic researchers in portfolio optimization with some of the most characteristic examples being the papers of Rockafellar and Uryasev (2000), Bertsimas et al. (2004) and Harris and Shen (2006).

2.2 Quantification methodologies for Value-at-Risk

Even supposing one of the most important advantages of VaR is that it is based on a rather simplistic theoretical concept, practical estimation of VaR can prove to be a quite challenging statistical problem. As a result, several researches have made an attempt to discover a methodology that will manage to address this issue ensuring both the accuracy and efficiency of the process and providing the researcher with an analytical tool that will produce robust results. The developed methodologies vary in terms of input data requirements, conditions, as well as the way they may be applied and the degree of complexity of the necessary computations.

But, even though all the models used for the calculation of VaR employ a variety of different methodologies, they all share a common general procedure consisting of a three stage structure. The first stage includes the market valuation of the current price of the portfolio (Market-to-market). The second stage refers to the estimation or selection of the distribution of the portfolio returns, which consists the major difference between the various VaR methodologies. Finally the third stage follows the actual calculation of VaR.

Traditionally, depending on the approach that is incorporated to estimate or select the portfolio return distribution during the second stage and hence predict the potential changes of its value, economists categorize the developed methodologies for calculating VaR into three main groups. These include the Historical-simulation Approach (HAS), the Analytical methodology and the Monte-Carlo simulation
method. Nevertheless, many researchers in the most recent years tend to identify the following four different VaR calculation methodologies:

1. The *parametric*, including the Risk Metrics, GARCH and Markov-Switching GARCH approaches

2. The *non-parametric*, including the Historical Simulation approach

3. The *semi-parametric*, including the Historical Simulation ARMA Forecasting (HSAF) approach and the Extreme Value Theory (EVT) approaches

4. *Monte-Carlo simulation* methodologies

2.2.1 Parametric methodologies

2.2.1.1 Risk Metrics

All the developed parametric methodologies are based on selecting the most appropriate probability curve that will best fit the examined data sample and from this curve extract VaR. One of the oldest, simplest and perhaps most representative parametric methodologies to calculate VaR is the Risk Metrics model introduced by J.P. Morgan (1996). The Risk Metrics uses a normal distribution to fit the portfolio returns and return innovations, where the conditional variance is calculated using an Exponentially Weighted Moving Average (EWMA):

\[ \sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda)r_{t-1}^2, \]  

where \( \lambda \) denotes the decay factor getting values from 0 to 1. Risk Metrics is basically equal to an IGARCH model with an assumed normal distribution, however it is not necessary to calculate any unknown parameters in the conditional variance equation as J. P. Morgan proposes a pre-determined value for \( \lambda \) of either 0.94 or 0.97 depending on the research purpose, with Fleming et al. (2001) supporting that a value of 0.97 for the decay parameter enables the model to produce quite sufficient
predictions of the one-day VaR. Using the Risk Metrics model for a pre-specified time period and confidence level $\alpha$ and under the normality assumption, the VaR value is obtained by multiplying $\alpha$ with the 1-$\alpha$ quantile of the standard normal distribution.

In general, although the Risk Metrics approach is a relative crude way to calculate VaR, it is rather popular among risk managers as a simple and quick method to get satisfactory volatility predictions for short-time horizons. Still the Risk Metrics methodology carries all the disadvantages of a normal IGARCH model, as it assumes a normal distribution for the returns and the innovations distribution, while there are strong empirical evidence reported by many researchers, from both financial and energy markets, showing persistent signs of kurtosis and skewness, which are way off the properties of a symmetric normal distribution.

Additionally, contrary to other GARCH type models it is unable to allow for asymmetry and hence account for phenomena like the leverage volatility effect (Halkos and Tsirivis, 2018). Finally, as one of the most characteristic early parametric approaches it contains the false assumption of independent and identically distributed (IID) return data, which is far from the evidence coming from the real financial markets (e.g. Brooks et al., 2005 and Bali and Weinbaum, 2007) and especially the energy commodity market.

2.2.1.2 The GARCH methodology

In an attempt to overcome the disadvantages of the traditional parametric approach, researchers focused on more sophisticated models to capture the various volatility effects observed in both the returns and prices of financial and energy products. The most popular models, especially in the research area of energy commodities, are the GARCH family volatility models. The most easily implemented
and widely used GARCH model to estimate VaR is the basic GARCH(1,1) model with normal distribution.

The VaR value using a GARCH model with an assumed normal distribution can be estimated as follows:

\[ \text{VaR}_t = \mu_t + z * \sigma_t, \]  

(5)

Where \( \mu_t \) and \( \sigma_t \) represent the conditional mean and volatility of the financial product or energy commodity returns, which are calculated with the use of a GARCH class model. In order to estimate the VaR for a long market position, \( z \) denotes \( z_{\alpha} \), which symbolizes the left end quantile corresponding to the \( \alpha \) per cent confidence level of the assumed distribution, which in this case is the typical Gaussian (Normal) distribution. Similarly, when estimating VaR for a short market position, \( Z \) denotes \( z_{1-\alpha} \), corresponding to the right end quantile of the \( \alpha \) per cent confidence level of the assumed model distribution.

Nonetheless, energy commodity returns exhibit even more intense signs of skewness and kurtosis than the vast majority of financial products. A plethora of researchers, among them Clewlow and Strickland (2000), strongly support that assuming a normal return and innovations distribution can prove to be rather problematic when forming a firm’s strategy regarding energy risk management and particularly when trying to get a robust and accurate estimate of VaR relative to individual or a portfolio of energy commodities. As a result, using a normal return distribution it is high likely to produce VaR calculations that will substantially underestimate the potential loss.

Therefore, trying to model effectively energy data and accurately measure VaR based on GARCH family models, academic researchers experimented by
replacing the traditional normal distribution of the basic model with a variety of
density functions such as the student-t, skewed student-t, heavy tailed distributions
and others\textsuperscript{1}. Additionally, to overcome some of the weaknesses of the basic GARCH
model researchers use instead other GARCH type models like EGARCH, GJR-
GARCH, FIGARCH, FIAPARCH and HYGARCH and others to account for
asymmetric and long memory volatility effects.

Apart from the above single-regime GARCH models, there is interest to
capture the changes in the market behavior of energy products under the influence of
extreme economic and geopolitical events, when there is high chance that these
volatile assets will behave in a completely different way than under normal
conditions. For this reason researchers adopt models that combine the Markov-
Switching regime methodology with several GARCH family models. In this
alternative approach, by incorporating a regime-switching variable, the original
GARCH model that is used is modified to account for a probable regime switch in the
estimated parameters of the variance process (Halkos and Tsirivis, 2018).

In general, combining different GARCH type models with various return
distributions and deciding through a number of evaluation methodologies for the most
accurate and efficient model, can provide a successful approach for getting
trustworthy approximations of VaR, however the whole procedure will still suffer
from the fact that a fixed random distribution is selected in advance in order to predict
the future distribution of risk factors.

\textsuperscript{1} See Halkos and Tsirivis (2018)
2.2.2 Non parametric methodologies

2.2.2.1 Standard historical simulation methodology

The standard historical simulation is by far the fastest and most simplified methodology to estimate VaR, as it uses past time series return data for the current portfolio asset mixture, in order to build a hypothetical return distribution based on which the future possible returns will be predicted. The VaR value in this case is obtained by sorting in ascending arrangement the asset or portfolio returns of the considered past time framework. The observation that fulfils the requirement of having X% of the observations beneath and (1-X%) of observations above represents the VaR value for the initially determined confidence level (95%, 99%, etc).

In contrast with the traditional parametric approaches, in the historical simulation no assumptions are being made about the return distribution being similar to a normal distribution or other fixed form distribution. Even though, the procedure to estimate VaR by ascending the observations would make someone reasonably think that this leads to normal return distribution, in reality it produces a random distribution. This allows the historical simulation to account for fat tails, skewness and other phenomena that are frequently observed in financial markets and are particularly more intensively present in the energy commodity markets.

Nevertheless, as every other method despite its advantages, the historical simulation also has some important weaknesses. Namely, it requires a large sample of past historical data to ensure a reliable result, while the sample may contain the effect of rare and unexpected events which are not likely to take place again in the future or may lack the effect of events that researchers might want to consider in their research. At last, it gives the same weight to each observation regardless of whether they are
being old or more recent, so the final result is equally affected by both very distant and latest returns.

On the other hand, although historical simulation may be computationally demanding and show a number of disadvantages, the basic concept of the method still remains very popular in risk management researches requiring a VaR calculation. Over the years, many researchers including the historical simulation in their studies contributed in the relative academic literature by modifying and extending the original method, in an attempt to overcome its shortcomings and make it more suitable for risk management analyses in highly volatile economic environments such as the financial and energy markets. Some of the most characteristic examples of such studies is the paper of Boudoukh et al. (1998), in which the historical simulation is combined with the Risk Metrics model putting less weight on distant observation and more weight on the recent ones, as well as the paper of Hull and White (1998) in which the authors create a link between the historical return and volatility changes, updating the historical return by multiplying it with a volatility ratio of historical and current volatility, both estimated through a GARCH model.

2.2.3 Semi–Parametric methodologies

2.2.3.1 Historical simulation ARMA forecasting (HSAF) approach

Specifically, in the field of risk management regarding energy commodities, perhaps the most important development in the original concept of historical simulation is the paper of Cabedo and Moya (2003), in which a modification of the initial methodology is presented. This new methodology involves the use of an ARMA model, based on which the return distribution is derived, instead of using the past returns’ distribution as in the original historical simulation method or making a normality or other type of distributional assumption for the return distribution as in
the GARCH framework. The HSAF as presented by Cabedo and Moya (2003) consists of a four stage process, during which the sample of past asset or portfolio returns is initially being tested for both stationarity and autocorrelation using a relative statistical test such as the Augmented Dickey Fuller and the Ljung-Box test respectively. In case the assumption of stationarity is rejected by the relative test and the examined return sample is characterized by non-stationarity, then the first difference or the lowest possible difference that exhibits stationarity is used. On the other hand, if the autocorrelation test reveals that the hypothesis for autocorrelation in the sample is not statistically significant, then the HSAF becomes equal to the standard historical simulation methodology. Nevertheless, to proceed in the next stages requires a statistically significant sample autocorrelation to be determined.

The second step includes the estimation of an ARMA model for the original past return sample, when this meets all the above requirements or the produced sample containing all the necessary adaptations. Next, again in this stage the Ljung-Box test is used to check for autocorrelation, and if present to control for the number of lags that are needed to remove it.

During the third stage, the forecasting errors that are produced from the relative forecasts for the estimated coefficients of the previous stage are used to form the statistical distribution upon which the percentile associated with the desired likelihood level will be determined after conducting the necessary statistical analysis.

Finally, in the fourth and last stage of the procedure the quantification of VaR is made. Based on the ARMA model that was used during the second step of the method, forecasts for the possible future returns are being made this time, adjusted for the percentile that was estimated previously. These forecasts provide the VaR value for a statistical likelihood level equal to the percentile decided earlier.
The HSAF is an improved methodology compared to the traditional historical simulation, providing the researcher with a more flexible and efficient VaR estimate, which takes into consideration the persistent return fluctuations. As a result, the HSAF was used and further developed in a number of studies over the years such that of Sadeghi and Shavvalpour (2006), in which they concluded that the HSAF is the most appropriate methodology among a number of others, to estimate VaR using a high confidence level (e.g. 99%) to manage risk in the oil market. The HSAF is very often used in the academic literature as one of the competing alternative methodologies, which are used by researchers trying to determine the most appropriate methodology to estimate VaR for specific energy markets or energy commodities they examine in terms of risk management.

2.2.3.2 The Extreme Value Theory (EVT) approach

The EVT approach is mainly used by researchers to deal with the fact that the standardized residuals in the majority of volatility models, trying to estimate VaR in portfolios containing financial assets and especially assets related to energy products, exhibit strong signs of fat tails and asymmetry. Calculating VaR following the traditional procedure through a volatility model, it is most likely to lead to an underestimated VaR value for high confidence levels. EVT emphasizes on the extreme returns in the tail of the return distribution, which can also be considered and examined as a smaller independent distribution. Even though extracting a safe conclusion regarding the tail structure of the return distribution (i.e. extreme events) it is quite reasonable to require a large sample of historical data.

Nevertheless, having to take into consideration such long series of data is making more difficult to make a forecast for the near future, as the model will not be capable to put the necessary weight towards the recent market fluctuations. In
addition, the EVT approach is based on the assumption that the examined extreme returns in the tail distribution are IID, an assumption which is most likely to be proved wrong, as the stochastic volatility and the long memory volatility effect which are present in the returns of highly volatile assets, suggest that an extreme and rare event is very probable to be followed by another.

McNeil and Frey (2000) are trying to address all the above issues in their relative academic study proposing a two stage process. During the first stage, they suggest using an appropriate GARCH class model as a filter for the examined return series in order for the GARCH residuals that will be produced to appear as close as possible to a match with IID observations. In the second stage the authors employ the EVT methodology to the residuals previously estimated, hence this hybrid model manages to incorporate both time-varying volatility as well as the important characteristic of fat tails in the return distribution, as the GARCH residuals will still maintain this characteristic from initial returns. Moreover, McNeil and Frey (2000) support that both conditional volatility and marginal distribution are individually modelled for the left tail. This provides the advantage to researchers to limit their studies only to the left tail of the return distribution as this is the only important part of the distribution for estimating VaR.

Empirical results in numerous academic papers confirm the success of the McNeil and Frey (2000) two stage approach to deal with the previously mentioned shortcomings of implementing the EVT in the returns of portfolios containing highly volatile assets. Especially, in risk management analysis involving VaR estimation regarding energy products, the vast majority of the researchers such as Byström (2004) and Chan and Gray (2006) and a high number of others, based their studies on
the aforementioned paper including an EVT implementation similar to the one proposed by McNeil and Frey (2000).

The EVT methodology incorporates alternative distributions to investigate the behavior of the returns belonging to the tail of the return distribution. For the implementation of EVT two main basic models have been developed based on a variety of different distributions, the Block Maxima model using the Generalized Extreme Value (GEV) distribution and the Peaks Over Threshold (POT) model using the General Pareto distribution.

2.2.3.2.1 The Block Maxima (BM) Model

In the BM model the Frechet, Weibull and Gumbel extreme value distributions are combined constructing the Generalized Extreme Value (GEV) distribution. The BM concept relies on the idea of dividing the examined data series into fixed size blocks focusing on the extreme values that appear during the individual time periods. These extreme observations represent extreme events forming what is called a block maximum. The BM model guides the researcher to make critical decisions regarding the time interval \( n \) and the data block which is contained in that interval.

Given that \( X_1, \ldots, X_n \) represent a group of IID observations forming a cumulative distribution \( F(x) \) with maximum loss equal to \( M_n = \max \{ X_1, \ldots, X_n \} \), the cumulative distribution function of \( M_n \) in this case can be described as follows (McNeil et al, 2005):

\[
P(M_n \leq x) = P(X_1 \leq x, \ldots, X_n \leq x) = \prod_{t=1}^{n} P(X_t \leq x) = F^n(x),
\]

Concerning \( F^n(x) \) the asymptotic approach suggests that it relies on the maximum standardized value:

\[
Z_n = \frac{M_n - \mu_n}{\sigma_n}
\]
With $\mu_n$ denoting the location and $\sigma_n$ a positive constant parameter. According to the theorem of Fisher and Tippett (1928) if $Z_n$ approaches a non-degenerated distribution then this is the GEV distribution, which can be mathematically represented as:

$$H_{\xi, \mu, \sigma}(x) = \begin{cases} \exp\left(-\xi + \frac{\xi (x - \mu)}{\sigma} \right) & \xi \neq 0 \text{ with } \left(-1 + \frac{\xi (x - \mu)}{\sigma} \right) > 0, \\ \exp(-e^{-\xi}) & \xi = 0 \end{cases}, \quad (8)$$

For which applies that $\sigma > 0$, $-\infty < \mu < +\infty$ and $-\infty < \xi < +\infty$, with all three $\xi$, $\sigma$ and $\mu$ parameters being estimated using the maximum likelihood approach. Nevertheless, the above GEV distribution is basically a generalized representation of the subsequent three distributions:

**Frechet:** \[ \Phi_{\alpha}(x) = \begin{cases} 0 & x \leq 0 \\ e^{-\alpha x^{-\alpha}} & x > 0 \end{cases} \text{ with } \alpha > 0, \quad (9) \]

**Weibull:** \[ \Psi_{\alpha}(x) = \begin{cases} e^{-(\alpha x)^{\alpha}} & x \leq 0 \\ 1 & x > 0 \end{cases} \text{ with } \alpha > 0, \quad (10) \]

**Gumbell:** \[ \Lambda(x) = xe^{-xe^{-x}}, \text{ with } x \in \mathbb{R}, \quad (11) \]

The $\xi$ parameter defines the shape of the distribution, contingent on the value of this parameter H distribution becomes a generalization of the above distributions, for $\xi > 0$ of the Frechet type distribution, for $\xi < 0$ of the Weibull type distribution and for $\xi = 0$ of the Gumbell distribution. The VaR estimation based on the Frechet and Gumbell distributions is made as follows:

$$\text{VaR} = \begin{cases} \mu_n - \frac{\sigma_n}{\xi} [1 - (-n \ln(n))]^{-\xi_n}, & \text{for } \xi > 0, \quad \text{Frechet} \\ \mu_n - \sigma_n \ln(-n \ln(n)), & \text{for } \xi = 0, \quad \text{Gumbell} \end{cases} \quad (12)$$

It is quite usual in the BM model approach that the blocks are being chosen in such order that they match a full economic year with representing the number of observations during that year. Nevertheless, the BM model is not regularly used when
examining samples containing time series data of financial returns as a result of the intense presence of volatility clustering.

2.2.3.2.2 Peaks Over Threshold model

The Peaks Over Threshold (POT) model is by far the most popular model for implementing the EVT approach to estimate VaR in risk management analyses concerning financial assets and energy commodity assets. The main reason is that the POT model is proved to be more practically applicable in such studies providing a more efficient use of the available data regarding extreme prices or returns. The POT model enables a comprehensive use of the entire data sample exceeding a significant threshold, in contrast with the BM which uses only the maximum from a fixed size ‘block’ to estimate the extreme value distribution. Hence, the POT model exhibits an obvious advantage over the BM model.

The POT model is mainly based on the General Pareto Distribution (GPD), attempting to fit the distribution of the data exceeding the predetermined threshold to a GPD distribution. Specifically, given that \((X_1, X_2, \ldots, X_n)\) is a sequence of IID observations denoting financial returns, which form an unknown distribution \(F\), and \(u\) is a predetermined threshold, the POT model examines all the \((Y_1, Y_2, \ldots, Y_{N_u})\) values exceeding this threshold \((Y_i = X_i - u)\), with \(N_u\) representing the \(N\) number of sample observations which exceed \(u\).

In this case, the distribution of losses exceeding the threshold \(u\) can be specified as the following conditional probability:

\[
F_u(Y) = P(X - u / X > u),
\]

\[
F_u(Y) = \left\{
\begin{array}{ll}
\frac{F(x+u)-F(u)}{1-F(u)}, & Y \geq 0 \\
u, & Y < u
\end{array}
\right.
\]
Based on the theorem of Balkema and De Haan (1974) and Pickands (1975) for large values of \( u \) the excess distribution function \( F_u \), can be resembled to a Generalized Pareto Distribution. This GPD distribution is described as follows:

\[
G_{\xi, \sigma}(Y) = \begin{cases} 
1 - \left(1 + \frac{\xi Y}{\sigma}ight)^{-\frac{1}{\xi}}, & \xi \neq 0 \\
1 - \exp\left(-\frac{Y}{\sigma}\right), & \xi = 0
\end{cases}
\]  

(15)

For which it applies that \( Y \geq 0 \) for \( \xi \geq 0 \) and \( Y \in [0, -\frac{\sigma}{\xi}] \) for \( \xi < 0 \). Again, similarly to the BM model the \( \xi \) parameter defines the shape of the extreme value distribution and \( \sigma \) is a scale parameter, while the decided threshold \( u \) represents the location parameter.

Furthermore, likewise the GEV distribution the main GPD distribution is a combination of three other distributions. As a result, when \( \xi \) becomes 0 the estimated GPD distribution is approximated by the normal distribution, with the tails decreasing at an exponential rate. Furthermore, for negative values of the \( \xi \) parameter the GPD distribution is approximated by a beta distribution with finite tails. Finally, for \( \xi \) values exceeding 0 the particular GPD distribution resembles to a Student-t type distribution. By making the assumption that the distribution of extreme values can be fitted to a GPD distribution, the VaR value for a predetermined probability \( p \) can be estimated as follows:

\[
\text{VaR}_p = u + \frac{\sigma}{\xi} \left(\frac{N}{N_{\infty}}(1 - p)\right)^{-\frac{1}{\xi}} - 1.
\]  

(16)

Nevertheless, all the previously discussed EVT methodologies individually lack the ability to take into consideration the phenomenon of volatility clustering, which is exceptionally present in the financial and energy markets. For this cause a conditional VaR estimation is required using the conditional EVT methodology. This
methodology is based on the work of McNeil and Frey (2000) and combines the above unconditional EVT method with GARCH type volatility models. The main advantage that arises from this hybrid approach is that it initially enables the researcher to account for conditional heteroskedasticity in the data sample, as well as to measure and predict future volatility with the use of a single-regime GARCH or a Markov-Switching GARCH model, while at a second stage model the extreme values at the distribution tail using the EVT concept.

In this case the filtering of the sample using a determined GARCH type volatility model produces IID time series data on which the EVT can be directly implemented. Given a stationary return sample and assuming that the residuals \((e_t)\) can be approximated by a GPD distribution \(G_{\xi,\sigma}\), then the conditional VaR for a specific \(\alpha\) quantile can be estimated as follows:

\[
\text{VaR}_\alpha = \mu_t + \sigma_t G_{\xi,\sigma}^{-1}(\alpha),
\]

Where, \(\mu_t\) denotes the conditional mean and \(\sigma_t^2\) the conditional variance, while the \(G_{\xi,\sigma}^{-1}(\alpha)\) is the \(\alpha\) quantile of the specific GPD distribution which can be obtain based on equation (12).

2.2.4 The Monte Carlo methodology

A rather popular methodology for estimating VaR when conducting risk management analyses regarding assets from financial and energy markets is the Monte Carlo methodology. This methodology is built based on the widely used in the academic literature Monte Carlo simulation and relies on the hypothesis that prices or returns track a particular stochastic process. As a result, by incorporating these processes to the Monte Carlo simulation it is possible to form the distribution of a
specific asset or portfolio value for an examined time frame. Furthermore, simulating at the same time a series of important and representative market variables and obtaining their possible future value paths enables the researcher to take account for factors influencing the future performance of the market and hence include in the research any possible volatility jumps or extreme events. Specifically, regarding VaR studies the necessary quantile in the tail of the distribution is being produced straightforward from the random paths.

In general, academic researchers consider the Monte Carlo approach as a rather useful alternative model among others to estimate VaR relative to highly volatile markets and assets like energy commodities, as it can account for several unique features of the market such as volatility clustering and non-normal return distribution. It is exactly this flexibility of the method that led many researchers in this specific field to include it in their studies, despite the fact that in some cases it may prove to be rather computationally intensive.

The most simplified version of the Monte Carlo approach to calculate VaR for a specific time horizon and confidence level, involves simulating \( N \) draws from the return distribution at time \( t + 1 \) and ranking them from the lower to the highest. Then it is necessary to locate the price for the \( \alpha \% \) lowest percentile that corresponds to the initial confidence level for which the VaR is estimated, meaning that there is \( \alpha \% \) probability that the asset value could diminish from this value \( (S_{t+1}^{\alpha \%}) \) to even lower levels. Finally, by deducting the above future asset value from the current value \( (S_t^\alpha S_{t+1}^{\alpha \%}) \), the potential loss that corresponds to the VaR for the specific time interval and confidence level is calculated. The VaR value in the Monte Carlo approach basically
represents the maximum loss from the random return distribution for specific and predetermined time interval and confidence level.

For applications involving a dynamic model with multiple risk factors affecting the asset or portfolio returns and taking into account phenomena like volatility clustering and non-normality, it is necessary first to define the dynamics of the fundamental processes. Then $N$ sample paths are generated illustrating variations in the asset or portfolio value during the examined time frame, while all the details included in the probability distribution must be integrated. Finally, based on the generated sample paths and according to the hypothesized processes, the value of every individual risk factor is estimated and used to define the asset or portfolio value at the specific examined time.

2.2.4.1 Hybrid Monte Carlo and Historical Simulation approach

A rather interesting variation of the traditional Monte Carlo approach is the hybrid model proposed by Andriosopoulos and Nomikos (2013). The authors combined Monte Carlo with the Historical simulation approach in an attempt to build a model that would be capable to provide a more accurate estimation of VaR relative to other competitive models. The main focus of the researchers is to create an appropriate risk management tool suitable to investigate risk particularly in the energy markets, by bridging the gap that is left from other studies which tried to present improved variations of these two methodologies trying to overcome their weaknesses.

Specifically, this hybrid approach provides the ability to the researcher to account for volatility jumps and fat tails in the return distribution, in contrast with most variations of Historical Simulation which focus mostly on capturing any volatility drifts which the original method underestimates or fails to consider. The
hybrid Monte Carlo-Historical Simulation is based on the concept of Historical Simulation by taking advantage of the flexibility given by Monte Carlo simulation.

Particularly, Andriosopoulos and Nomikos (2013) suggest generating an extremely large number $N$ of sample paths for the underlying processes and forecast the spot price for a particular time period in the future. The daily VaR is estimated using the average sample path and based on the method of the rolling window as in the Historical Simulation, rolling the simulated observations forward one by one until the last at the end of the initially determined future time period. This VaR using this Hybrid approach can be mathematically represented as follows:

$$\text{VaR}_{t+1} = \text{Percentile}\left\{ \left\{ \bar{r}_t^s \right\}_{t=1}^{T}, \bar{r}_{t,N} \right\},$$  \hspace{1cm} (18)$$

Where $T$ represents the total return sample including the observations both from the original sample as well as those simulated, while $\bar{r}_t^s = \sum_{\omega=1}^{N} \frac{r_{t}\omega}{N}$ is the average simulated return at time $t$, $r_{t}\omega$ is the simulated price value $\omega$ at time $t$ and $N$ the number of simulations.

2.3 Model evaluation and statistical accuracy of VaR

The wide variety of methodologies and approaches to estimates VaR creates the need to compare and determine which models are appropriate and provide accurate results for a specific risk management analysis. In the relative academic literature, researchers base their examination for the most suitable VaR models on two main statistical tests, the unconditional coverage test and the conditional coverage test developed by Kupiec (1995) and Christoffersen (1998) respectively. The models that are not rejected and fulfill the requirements of both tests for the specific VaR confidence level are then compared using the regulatory loss function (Lopez, 1999 and Sarma et al., 2003).
In general, the backtesting analysis of VaR involves comparing the predicted values of the competing VaR models with the actual losses in the following period. Next according to the results from the unconditional and the conditional coverage tests, it is examined whether the number of violations exceeds the expected number based on the selected confidence level of VaR, as well as whether the violations are independent and randomly distributed.

2.3.1 The Kupiec’s unconditional coverage test

The concept of the Kupiec’s (1995) test relies on estimating the probability of observing a loss exceeding the predicted VaR amount. In an attempt to examine the accuracy and performance of the various VaR models, Kupiec developed a likelihood ratio test (LRuc) which investigates if the failure rate of a particular model is statistically equal to the one expected. For this purpose the following exception indicator need to be determined:

\[ I_{t+1} = \begin{cases} 1, & \text{if } r_{t+1} < \text{VaR}_{(α)} \\ 0, & \text{if } r_{t+1} \geq \text{VaR}_{(α)} \end{cases} \]  

(19)

Given that \( x = \sum I_{t+1} \) is a variable following a binominal distribution \( x \sim B(N,α) \) and denoting the number of exceptions in a sample consisting of \( N \) observations and \( p=\frac{\sum I_{t+1}}{N} \) representing the expected exception frequency or ratio of violations, then in order for VaR\(_{(α)}\) to provide an unconditional coverage of \( α\% \) the null hypothesis that \( H_0: \ p = a \) needs to be confirmed or else the alternative will apply \( H_1: \ p \neq a \). In this case the likelihood ratio statistic of the unconditional coverage test (LRuc) for the specific null hypothesis is approximated by an \( \chi^2 \) asymptotic distribution and is described as follows:

\[
\text{LRuc} = -2\log\{α^x(1-α)^{N-x}\} + 2\{\log p^x(1-p)^{N-x}\} \quad ,
\]

(20)
2.3.2 The Christoffersen’s conditional coverage test

The unconditional coverage test may be able to reject a model that under or over-estimates VaR, however it is unable to detect those VaR models that may provide sufficient unconditional coverage but their VaR violations are not independent. Christoffersen (1998), trying to overcome this weakness of the Kupiec’s test, developed the conditional coverage test (LRcc) which is able to examine at the same time whether the sum of actual VaR violations match with the one predicted by the model and whether these violations are correlated or not through time, providing the advantage to the researcher to discard a model for producing either a very high or very low number of clustered exceptions. In risk management studies regarding highly volatile markets and assets such as the energy markets and energy commodities, it is quite important to examine for conditional coverage as in most cases they exhibit strong signs of volatility clustering.

The unconditional coverage test basically examines the joint hypothesis of both unconditional and independence tests, with an appropriate model providing both sufficient unconditional coverage and serial independence of $I_{t+1}$. Combining the two properties the test statistic can be described as $LR_{cc} = LR_{uc} + LR_{ind}$, with $LR_{cc}$ under the null hypothesis following a $\chi^2(2)$ asymptotic distribution and $LR_{ind}$ representing the likelihood ratio statistic for the null hypothesis of serial independence. Specifically, for $\pi_{ij} = P(I_t = j \mid I_{t-1} = i)$ denoting the transition probability and $n_{ij} = \sum_{t=1}^{N} P(I_t = j \mid I_{t-1} = i)$, with $n_{ij}$ representing the number of observations with value $i$ being followed by value $j$, where $i, j = 0$ or 1, the hypothesis of the independence test is described as follows (Aloui and Mabruik, 2010):

$$H_{0,ind}: \pi_{00} = \pi_{11} = \pi, \pi_{01} = \pi_{10} = 1-\pi,$$ (21)
Finally, the combined null hypothesis for both unconditional coverage and independence of the failure process is tested based on the following likelihood ratio statistic:

\[
LR_{cc} = -2 \log \left[ \frac{\hat{f}_{\alpha}^{n}} {1 - \hat{f}_{\alpha}^{n}} \right] + 2 \log \left[ \frac{\hat{f}_{\alpha}^{n} \hat{n}^{n-1} \hat{f}_{\alpha}^{n} \hat{n}^{n-1}} {\hat{f}_{\alpha}^{n} \hat{n}^{n-1} \hat{f}_{\alpha}^{n} \hat{n}^{n-1}} \right].
\]  

(22)

Where \( \hat{f}_{\alpha}^{n} = \frac{n_{t+1}^{n_{t+1}}}{n_{t+1}^{n_{t+1}} + n_{t-1}^{n_{t-1}}} \), \( n_{t+1}^{n_{t+1}} \), \( n_{t-1}^{n_{t-1}} \), \( n_{t+1}^{n_{t+1}} + n_{t-1}^{n_{t-1}} \), and \( \hat{f}_{\alpha} = \frac{n_{t}^{n_{t}}}{n_{t}^{n_{t}} + n_{t}^{n_{t}}} \).

2.3.3 The regulatory loss function criterion

It is most common both at business but especially at academic level that when a risk management analysis is conducted, including the estimation of an individual or multiple VaR values, that more than one model is used for this purpose. The coverage tests help researchers reject some or the majority of the models for lacking the necessary accuracy to provide a reliable VaR estimate, however there is high chance that the competing models at the end of this procedure is more than one. In that case researchers use an extra model evaluation criterion in order to conclude which of the available models performs better than the others reaching the final stage assessment and this is the regulatory or magnitude loss function. This test was initially developed by Lopez (1999) as an additional tool to compare the models that satisfy the conditions of the coverage tests and it primarily takes into consideration the notification of the Basel Banking Supervision Committee, that both the number of VaR exceptions as well as the magnitude or size of these exceptions should be of equal importance for individual researchers or institutes when pursuing a risk management evaluation. The regulatory loss function is generally described as follows:

\[
Loss_{t+1} = \begin{cases} 
1 + \left( \frac{r_{t+1}}{VaR_{t+1}} - 1 \right)^2, & \text{if } r_{t+1} < VaR_{t+1} \\
0, & \text{otherwise}
\end{cases}
\]

(23)
The proposed quadratic loss function has the advantage of incorporating the extra information regarding the magnitude of the VaR exceptions, creating a stricter and reliable model accuracy and performance criterion penalizing in a more severe way large exceptions.

3. Literature review

3.1 Energy commodity risk management studies based on the VaR concept

Due to the various advantages and unique characteristics of the Value-At-Risk approach, which were thoroughly presented earlier in this paper, VaR is considered both at academic level as well as in business and financial institutions cycles, as a powerful analytical tool to measure and manage risk. Particularly, in the field of risk management regarding the energy commodity market, more and more researchers incorporate VaR in their analyses, trying to present a precise amount that could be lost as a result of the risk exposure to an individual or a portfolio of assets belonging to this highly volatile market.

Academic researchers in energy economics have long ago realized the advantages of VaR, hence a wide range of studies have been published especially over the last 15 years, where the interest for energy commodities has dramatically increased due to the high economic importance of the particular products along with the extreme uncertainty in the energy market. Researchers compare the different methodologies and approaches in order to conclude to the most appropriate way to estimate VaR for a specific energy commodity or portfolio for a specific data sample.

3.1.1 Academic papers using HS, and GARCH based models for VaR forecasting

As already mentioned in the paper, standard historical simulation (HS) approach and its variations have been very popular and it is used by a large number of
researchers as one of the competing methods among others to provide the most accurate and efficient VaR estimation. Cabedo and Moya (2003) present a development of the traditional HS, the historical simulation ARMA forecast (HSAF), which based on an 8-year sample (1992-1999) of daily Brent oil prices, they compare the VaR estimates provided by a basic GARCH model assuming a normal return distribution, with those produced by HS and the HSAF. The authors concluded that the HSAF is the most appropriate methodology of the three to estimate VaR, as it provides a more flexible VaR quantification than HS which better fit the extremely high price volatility, while the standard normal GARCH tends to overestimate VaR.

Sadeghi and Shavvalpour (2006) further confirmed the findings of Cabedo and Moya (2003), as they suggest that the HSAF, within a 6-year sample (1997-2003) of weekly OPEC oil price data, produced the most precise forecasted VaR values, overwhelming the various basic normal ARCH and GARCH models to which it was compared with. They noted that both ARCH and GARCH models again systematically overestimated VaR, nevertheless they claim that no matter which method is used, VaR is a valuable and trustworthy tool to quantify risk regarding oil prices. Fan and Jiao (2006) present another development of the standard HS, the exponential decreased frequency with ARMA forecasts approach (EDFAAF), which was built based on the HSAF concept. Their approach was tested against the HSAF approach using a 12-year sample consisting of weekly observations relative to Brent spot oil prices and it was found that at all times performed better forecasting VaR than HSAF approach.

Zikovic et al. (2015) compare the predicting ability of the standard HS approach together with other HS based approaches such as the Mirrored HS\(^2\) (MHS),

\(^2\) See Holton (1998)
the Filtered HS ³ (FHS), as well as that of the BRW ⁴ simulation, that of a basic GARCH model, Risk Metrics and that of an unconditional EVT (GPD) model and an EVT-GARCH model. The researchers gathered a sample including 9-year (1995-2014) daily return data of one month futures contracts for several key energy commodities such as WTI crude oil, Brent oil, natural gas and heating oil, in order to test the accuracy of the forecasted VaR values of the competing models. The results of the relative tests revealed that the rather simplistic methodology of FHS performed better than any other competing model, noting that the more simple non-parametric models were better able to capture the large number of extreme events included in the sample.

In line with the previous study Huisman et al. (2015) using a 5-year sample (2008-2013) of daily futures returns for crude oil, gas oil, natural gas and coal they also find that among other models such as the HS and Risk Metrics, the FHS model produced the best VaR estimates. Finally, Andriosopoulos and Nomikos (2013) based on a 9-year sample of daily price data concerning a wide range of energy commodities, such as heating oil, crude oil, gasoline, natural gas propane and electricity, they discovered that for the vast majority of the examined energy commodities the models that rely on their proposed hybrid Monte Carlo – HS approach together and the standard Monte Carlo approach, significantly outperform the various competing GARCH type models, in terms of accurately forecasting the 1% daily VaR.

On the other hand, Costello et al. (2008) support that the findings of Cabedo and Moya (2003) and Sadeghi and Shavvalpour (2006), are mainly driven due to the assumption that in the examined ARCH and GARCH models the return distribution is

³ See Barone-Adesi et al. (1999)
⁴ See Boudoukh et al. (1998)
approximated by the normal distribution, an assumption which is empirically rejected by numerous studies for both financial and especially for the even more volatile energy commodity markets. To address this problem the authors incorporate the concept of Barone-Adesi et al. (1999) and modify the basic procedure under which the VaR is estimating using a GARCH based model.

Particularly, Barone-Adesi et al. (1999) trying to improve the poor VaR forecasts of GARCH models assuming a normal return distribution suggest using the historical simulation to determine the future return distribution. In their results it is revealed that the VaR estimates from their suggested hybrid GARCH-HS approach significantly outperform those from standard GARCH setups. Costello et al. (2008) adopt the main idea of the aforementioned semi-parametric procedure and using a 23 year sample (1992-2003) containing daily Brent oil prices, they conclude that the semi-parametric GARCH approach performs better than HSAF of Cabedo and Moya (2003), as it manages to capture changing volatility.

The inability of GARCH family models to provide trustworthy VaR forecasts when assuming a normal innovations distribution, triggered the interest of many researchers both to modify the standard normal GARCH model in such way that it will outperform other competing VaR estimation methodologies, as well as to determine which alternation of the basic model will outperform the other GARCH based models. Fan et al. (2008) propose that GARCH models based on the Generalized Error Distribution (GED) are better able to capture phenomena like fat tails and skewness which are generally accepted to be present in the highly volatile energy commodity returns. The authors using a 20-year (1987-2006) daily spot price sample for WTI oil and Brent oil, they reached the conclusion that both GED-GARCH and GED-TGARCH models are better able to take into consideration the
volatility characteristics of these two commodities, while they outperform the HSAF model at forecasting the daily VaR for a 95% confidence level.

Other researchers focused mostly on determining which specific innovations distribution would alter the standard normal ARCH and GARCH models and produce a better model in terms of VaR forecasting ability. Giot and Laurent (2003) assessed the Risk Metrics, skewed-Student-APARCH and skewed-Student-ARCH models based on their predicted VaR values. Their sample consisted of daily spot prices for rare precious metals and WTI oil and Brent oil, for a 15-year (1987-2002) period, with the researchers determining that the skewed-Student-APARCH model is the more appropriate model to deal with the fat tailed and skewed return distribution of WTI oil and Brent oil and produce the best 1% VaR forecast.

Similarly, Hung et al. (2008) compare the basic GARCH model with normal distribution, the Student-GARCH and the GARCH-HT model, which incorporates the heavy-tailed distribution proposed by Politis (2004). A sample of 10-year (1996-2006) daily spot prices for WTI oil, Brent oil, heating oil, propane and gasoline was used, with the relative accuracy and efficiency tests revealing that GARCH-HT model is by far the best model to forecast VaR, providing very precise estimates for very high confidence levels and thus making it suitable for more conservative risk management analyses.

Furthermore, Aloui and Mabrouk (2010) using a 21-year (1986-2007) sample of daily spot prices for WTI oil, Brent Oil and gasoline, they compare the ability of FIGARCH, FIAPARCH and HYGARCH models to provide accurate future VaR estimations considering three different innovation’s distributions, the Student the skewed-Student and the normal distribution, with the relative tests indicating that the
FIAPARCH model incorporating the Student-t distribution outperforms the other models in terms of out of sample VaR estimates.

Additionally, Cheng and Hung (2011) comparing three GARCH type models developed based on the skewed-generalized-t (SGT) distribution, the generalized-error-distribution (GED) and the normal distribution respectively, they find that according to the relative accuracy tests the GARCH-SGT model outperforms rival models, as it is more capable of taking into account the exhibited fat-tailed and negatively skewed return distribution of the examined energy commodities when forecasting VaR. This result is rather important as the authors used for their analysis a 7-year (2002-2009) sample including the years of financial crisis, which contained observations of daily spot and futures prices for WTI oil, gasoline and heating oil, among other commodities.

Finally, Chkili et al. (2014) using a large sample containing 14-year (1997-2011) data of daily spot and 3-month futures returns for WTI oil, natural gas and precious metals, they examine the future VaR estimates of a group of seven linear and non-linear GARCH family models including GARCH, IGARCH, EGARCH, Risk Metrics, FIGARCH, FIAPARCH and HYGARCH. The accuracy tests used in the particular study show that the FIAPARCH model clearly performs better than the rest models of the group, with the authors pointing that the key reason for this result is the presence of the asymmetric and the long memory volatility effect in the return data.

3.1.2 Academic papers using EVT based models for VaR forecasting

Another popular methodology for estimating and forecasting VaR is the extreme value theory (EVT) approach, which is either used individually or in most cases combined with a GARCH family model in an attempt to lift the restrictions of the standard methodology and accomplish a more precise prediction of VaR.
Specifically, regarding energy commodity risk management there has been a sufficient number of studies with researchers incorporating EVT and the vast majority of them trying to compare EVT models with other models which rely on other competitive methodologies.

Ren and Giles (2010) after finding significant evidence of fat tails and negative skewness in their examined data sample, which consisted of 8-years (1998-2006) of daily returns from the Canadian oil market, they determined that the Peaks-Over-Thresholds (POT) approach with Generalized Pareto Distribution (GPD) is the most appropriate methodology to employ so as to effectively model their data and estimate VaR, after ruling out the existence of conditional heteroskedasticity and hence the necessity of employing a combined GARCH-EVT approach.

Byström (2005) using a 5-year (1996-2000) sample of hourly electricity returns, determined that both the basic GARCH model with normal distribution as well as the GARCH model with Student-t distribution tend to underestimate and overestimate respectively the probability that an extreme return is observed, even though the electricity return distribution was found to be less fat tailed and negatively skewed than this of other energy commodities. The author argues that this is mainly happening due to the fact that GARCH family models are built to model the behavior of the whole return distribution and as a result they are less effective to capture the extreme returns in the tails of the distribution, on the contrary EVT models focus on the tail of the distribution and hence are more suitable.

Furthermore, the presence of skewness in the return distribution makes the models using symmetrical distributions less appropriate for making tail quantile estimations. For all these reasons, Byström (2005) employing the methodology of McNeil and Frey (2000) develops a combined GARCH-EVT model based on the POT
approach with GPD to better forecast the probability of extreme returns, as it was found slightly more accurate than the GARCH-EVT model based on the BM approach.

Similarly, Krehbiel and Adkins (2005) examining the returns for both spot and futures prices for WTI oil, Brent oil, heating oil, gasoline and natural gas they also find evidence of fat tails and negative skewness. Applying the same procedure as Byström (2005) they further agree that a GARCH-EVT model based on the POT method outperforms GARCH family models using symmetrical distribution, when forecasting the probability of occurrence of an extreme price change and measuring risk exposure as in the case of VaR, while it also provides more accurate results than a GARCH-EVT model using the BM method.

Chan and Gray (2006) rely on an AR-EGARCH model to successfully account for seasonality and leverage effects in the conditional volatility in electricity prices. The authors using a large sample of spot prices coming from five major international electricity markets compare the ability of several competing models, such as the HS model, the AR-EGARCH model with normal distribution, the AR-EGARCH model with Student-t distribution and the AR-EGARCH-EVT model based on the POT approach, to provide precise predictions of VaR. The results clearly indicate that the AR-EGARCH-EVT model is by far the most appropriate to model data coming from markets which exhibit high volatility, skewness and kurtosis as well as to perform tail quantile estimates and predict VaR.

Marimoutou et al. (2009) rely on an AR-GARCH model to effectively model a 20-year (1987-2006) sample of daily Brent oil spot returns and perform future projections of VaR. The authors incorporate an AR-GARCH model and modify traditional methods, such as the HS and the conditional EVT approach, developing
models which are then been tested according to their ability to predict VaR. Results in this occasion show that the AR-GARCH-EVT model relying on the POT approach as well as the FHS employing the AR-GARCH, outclass the traditional HS model and the normal AR-GARCH model. Nevertheless, it is found that the AR-GARCH model with Student-t distribution is equally able to provide VaR estimates that capture changes of volatility dynamics and appropriately adjust to them.

On the contrary, Paraschiv et al. (2016) testing a sample of 5-year (2009-2014) hourly electricity returns, they reach the conclusion that the AR-GARCH-EVT relying on the conditional GPD approach, better estimates the probability of observing extreme returns when compared to the basic AR-GARCH model with either normal or Student-t distribution. Additionally, Youssef et al. (2015) following the same concept of the aforementioned studies, they examine the out of sample VaR estimates of a FIAPARCH-EVT model with GPD distribution relative to a similar GARCH-EVT model used as benchmark. The backtesting results in this occasion revealed that the FIAPARCH-EVT model outperformed its benchmark in terms of estimating the one day ahead VaR values, while when the authors incorporated a combination of bootstrap and GPD approach it was found that the model continued to provide reliable VaR estimates for even longer time periods. The two models in this study were built and tested using a 10-year (2003-2012) daily return sample for WTI oil, Brent oil and gasoline.

Finally, a rather interesting finding regarding the use of the GARCH-EVT methodology to estimate VaR in energy portfolios is that presented in the research paper of Nomikos and Pouliaisis (2011), in which the aforementioned methodology is being tested on its ability to accurately predict VaR. The evidence presented based on a data sample containing daily futures prices from 1991 to 2008 for WTI oil, Brent,
Heating oil and Gasoline, reveal that the GARCH-EVT approach consistently provided overestimated values of VaR, which can prove to be very ineffective in terms of capital cost for investor groups with an average risk aversion and a valuable tool for more conservative investors.

4. Conclusion

The energy market is described by extreme ambiguity and price instability due to the high effect of geopolitical and ecological aspects, in addition to the worldwide demand, competition increase and market deregulation. These extremely unbalanced market circumstances demand the consideration and determination of the most suitable approaches and instruments to administer the energy commodities’ disproportionate price risk.

Extending the work of Halkos and Tsirivis (2018), the current paper emphasizes on the importance of Value-at-Risk as a tool for a more effective risk management and for a deeper understanding of the investor’s risk, holding a portfolio of energy assets. The various VaR based methodologies which are thoroughly presented, can provide the researcher who conducts a risk management analysis about an energy commodity with vital information regarding the risk arising from price volatility.

Nevertheless, due to the nature of VaR, models based on this particular methodology can offer a deeper understanding of the total risk involving the investigated or forecasted price shifts of a particular energy product, as they have the ability to quantify this risk and express it in currency units. Though, selecting the most appropriate return distribution still remains of critical importance. In general, a VaR model using the correct return distribution with the lowest possible number of
violations, constitutes a very reliable risk management tool in the hands of energy economists, corporate managers and policy makers.

Lastly, in the present study as emphasized by most of latest studies, there is no particular individual model or methodology that can do better than the others in modeling and accurately predicting the total amount of capital which is at risk in portfolios containing energy products. Similarly to volatility centered researches, economists show the certainty that the appropriateness and therefore the performance of a unique model or approach firmly depends on the precise sample considered together with any particular distinguishing characteristics that influence the trade of the particular energy product.

However, particularly in the case of VaR researches regarding a peculiar and highly risky group of assets like energy commodities, a key factor that could determine which specific methodology is the most appropriate for estimating the VaR value, is the attitude of the portfolio owner towards risk and its development through time. As different models being more suitable for investors becoming more risk averse or risk friendly, providing more conservative and less conservative values respectively relative to the amount of money which is at risk based on the specific portfolio.
References


## Appendix

Table A1: Proposed VaR estimating models for portfolios containing energy commodity assets.

<table>
<thead>
<tr>
<th>Year</th>
<th>Author</th>
<th>Examined Data Set</th>
<th>Examined Energy Commodity</th>
<th>Outperforming Method</th>
</tr>
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<tbody>
<tr>
<td>2003</td>
<td>Cabedo and Moya</td>
<td>Daily Spot</td>
<td>Brent oil</td>
<td>HS-ARMA Forecast (HSAF)</td>
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<td>2003</td>
<td>Giot and Laurent</td>
<td>Daily Spot</td>
<td>WTI oil, Brent oil</td>
<td>Skewed-Student-APARCH</td>
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<tr>
<td>2005</td>
<td>Byström</td>
<td>Hourly Spot</td>
<td>Electricity</td>
<td>GARCH-EVT-POT with GPD</td>
</tr>
<tr>
<td>2005</td>
<td>Krehbiel and Adkins</td>
<td>Daily Spot, Daily Futures</td>
<td>WTI crude oil, Brent oil, Heating oil, Gasoline, Natural gas</td>
<td>GARCH-EVT-POT with GPD</td>
</tr>
<tr>
<td>2006</td>
<td>Sadeghi and Shavvalpour</td>
<td>Weekly Spot</td>
<td>OPEC oil</td>
<td>HS-ARMA Forecast (HSAF)</td>
</tr>
<tr>
<td>2006</td>
<td>Chan and Gray</td>
<td>Daily Spot</td>
<td>Electricity</td>
<td>AR-EGARCH-EVT-POT</td>
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<tr>
<td>2006</td>
<td>Fan and Jiao</td>
<td>Weekly Spot</td>
<td>Brent oil</td>
<td>HS-Exponential Decreased Frequency ARMA forecasts (EDFAAF)</td>
</tr>
<tr>
<td>2008</td>
<td>Costello et al.</td>
<td>Daily Spot</td>
<td>Brent oil</td>
<td>Semi-Parametric GARCH</td>
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<tr>
<td>2008</td>
<td>Hung et al.</td>
<td>Daily Spot</td>
<td>WTI oil, Brent oil, Heating oil Gasoline, Propane</td>
<td>GARCH-HT</td>
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<td>2009</td>
<td>Marimoutou et al.</td>
<td>Daily Spot</td>
<td>Brent oil</td>
<td>AR-GARCH-EVT-POT</td>
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<td>2010</td>
<td>Ren and Giles</td>
<td>Daily Spot</td>
<td>WTI oil, Brent oil</td>
<td>GARCH-EVT-POT with GPD</td>
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<td>2011</td>
<td>Nomikos and Pouliaios</td>
<td>Daily Futures</td>
<td>WTI oil, Brent oil, Heating oil Gasoline</td>
<td>GARCH-EVT</td>
</tr>
<tr>
<td>2011</td>
<td>Cheng and Hung</td>
<td>Daily Spot, Daily Futures</td>
<td>WTI oil, Heating oil Gasoline</td>
<td>GARCH-SGT</td>
</tr>
<tr>
<td>2013</td>
<td>Andriosopoulos and Nomikos</td>
<td>Daily Spot</td>
<td>WTI oil, Heating oil Gasoline, Natural gas Propane, Electricity</td>
<td>Monte Carlo – HS</td>
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<tr>
<td>2014</td>
<td>Chkili et al.</td>
<td>Daily Spot, 3-month Futures</td>
<td>WTI oil, Natural gas</td>
<td>FIAPARCH</td>
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<td>2015</td>
<td>Zikovic et al.</td>
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<td>WTI crude oil, Brent oil, Heating oil, Natural gas</td>
<td>Filtered HS</td>
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<td>2015</td>
<td>Huisman et al.</td>
<td>Daily Futures</td>
<td>WTI oil, Natural gas, Gasoline, Coal</td>
<td>Filtered HS</td>
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<td>2015</td>
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<td>Daily Futures</td>
<td>WTI oil, Brent oil, Gasoline</td>
<td>FIAPARCH-EVT with GPD</td>
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<td>2017</td>
<td>Paraschiv et al.</td>
<td>Hourly Spot</td>
<td>Electricity</td>
<td>GARCH-EVT-POT with GPD</td>
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