Fair Disclosure and Investor Asymmetric Awareness in Stock Markets

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Abstract

The US Security and Exchange Commission implemented Regulation Fair Disclosure in 2000, requiring that an issuer must make relevant information disclosed to any investor available to the general public in a fair manner. Focusing on firms that are affected by the regulation, we propose a model that characterizes the behavior of two types of investors—one professional investor and many small investors—in the regimes before and after the regulation, i.e., under selective disclosure and fair disclosure. In particular, we introduce the concept of awareness and allow investors to be aware of relevant information symmetrically or asymmetrically.

We show that with symmetric awareness, fair disclosure induces both a low cost of capital and a low cost of information, therefore making the market efficient. Also, the professional investor collects an equal level of information under fair disclosure than under selective disclosure. However when small investors are not fully aware, fair disclosure still induces a low cost of capital but may induce a high cost of information. The professional investor may deliberately collect less information under fair disclosure than under selective disclosure.

With asymmetric awareness, our theory produces predictions that match the empirical findings by Ahmed and Schneible Jr. (2004) and Gomes, Gorton and Madureira (2006). They find that small and complex firms are negatively affected by the regulation. We also show that fair disclosure improves the welfare of small investors when they are extremely unaware. Such results are not compatible with the standard symmetric awareness assumption.
1 Introduction

In stock markets, information is usually transmitted from issuers to investors through several different channels: 1. mandatory public disclosure by issuers; 2. voluntary public or private disclosure by issuers; 3. private acquisition by investors from sources other than the issuer, such as purchasing research reports from stock analysts, examining the firm’s products or services, and consulting the firm’s competitors.

While most small investors mainly rely on public disclosure, professional investors use all channels. In particular, some professional investors are selected by the issuer to receive material information, for example, through quarterly analyst conference calls. Many issuers favor such selective disclosure for practical reasons, such as concealing information from their competitors. However selective disclosure is viewed with suspicion by the regulatory authorities, since it creates a class of informationally privileged investors. Since selective disclosure creates information asymmetry, it also increases the cost of capital (Verrecchia, 2001).

To curtail selective disclosure, in August 2000, the U.S. Securities and Exchange Commission adopted Regulation Fair Disclosure (henceforth the regulation), also commonly referred to as Reg FD. This ruling requires that an issuer must make material information disclosed to securities market professionals or shareholders available to the general public simultaneously (for intentional disclosures), or promptly (for non-intentional disclosures).

In this paper we model the regulation as requiring an issuer only to disclose information in a public forum where participants (professional investors and small investors) ask questions. This disclosure form is referred to as fair disclosure, in contrast to selective disclosure, where only professionals can ask questions.

At first glance, fair disclosure seems the best remedy for the information asymmetry caused by selective disclosure. It also reduces the incentive of private information acquisition, because

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1In 2002, Korea became the second country introducing Reg FD.

2The debate on the benefit and cost of Reg FD has never stopped since it was proposed by the SEC in December 1999. By the time it was adopted, “more than 6,000 comment letters, mostly from individual investors, were submitted in response to the Reg FD proposal. Individual investors and the media generally favored the proposed regulation, believing that it would level the playing field for the retail investor. Large brokerage firms, on the other hand, generally opposed the rule, predicting that it would lead to a chilling of the information flow from issuers to the marketplace.” Quote from the introduction of the SEC’s Special Study: Regulation Fair Disclosure Revisited. Source link: http://www.sec.gov/news/studies/regfdstudy.htm.
information disclosed by the issuer usually has high quality and is relatively easy to collect. However practitioners have argued that the regulation has produced some undesirable side effects:

1. The ambiguous definition of material information makes issuers reluctant to provide “immaterial” information.

2. Professionals may be unable to obtain information because of ineffective technology utilized in public communications.

3. Professionals “with the most perception, intuition, or experience” are not willing to share their insights with other investors under fair disclosure, so that less information can be revealed.

In this paper, we explore the third argument more in depth. There are two main reasons driving us. First, this argument was both less understood and less emphasized in practice. Second, the superior knowledge of professionals may imply either that uninformed small investors are just unable to acquire the information and aware of their ignorance, or that they are totally unaware of such information; we want to understand whether these assumptions produce different predictions and which one fits the facts better. To isolate it from the first two arguments and make the modeling simple, we assume that the issuer has to answer sincerely to questions posed by investors under both disclosure forms, and that the technology used in public communications is effective.

We analyze the effect of the regulation on the cost of capital, the cost of information, the quantity of information collected by professionals, and the welfare of small investors. We show that the effect differs under different assumptions about investors’ awareness. This result hence suggests reconsidering the traditional economic approach to understanding the regulation; furthermore, it suggests that “asymmetric awareness” may offer important insights in certain fields of economic research, such as information disclosure where some agents are specialists.

We start by defining “asymmetric awareness” and discussing how it differs from “asym-

\footnote{Words from the comment of the Association for Investment Management and Research to SEC regarding the regulation. Source link: http://www.cfainstitute.org/centre/issues/comment/2000/00disclosure.html.}
metric information”. In general, information may have many dimensions. For instance, information that affects a firm’s value may regard general management, finance, marketing, technology, government regulation, and so on. In the literature on epistemic foundations of game theory, these dimensions are called awareness information, or simply awareness. Agents have asymmetric awareness if they have different dimensions of information. They are unaware of some information because either they are not aware of the existence of the information (this is what “unawareness” literally means), or they are not able to incorporate the information in their decision making—for instance, a layman can tell a technical term but can not properly use it. In many cases, imposing symmetric awareness makes an economic theory more appealing, especially for normative purposes. However recent work in game theory (Modica and Rustichini, 1999; Feinberg, 2005; Li, 2006; Heifetz, Meier and Schipper, 2006) develops clear mathematical characterizations for awareness and enables us to understand asymmetric awareness in economic and game theoretical models.

The asymmetric information assumption in “symmetric awareness” models means that the uninformed agents know exactly how informed agents update their beliefs, though they don’t have the same information. In short, they know that they don’t know. On the contrary, with asymmetric awareness, unaware agents have no idea that other agents may update their beliefs upon certain pieces of information. In short, they don’t know that they don’t know. Such “ignorance about ignorance” is a consistency condition imposed on the belief hierarchies of agents in order to characterize unawareness.

Consider the following example. There are two agents A and B with different awareness from a set of dimensions about an object, $S = \{\text{color, size, shape}\}$. Agent A is aware of the set $S_A = \{\text{color, size}\}$ while agent B is aware of the set $S_B = \{\text{size, shape}\}$. Thus A is unaware that shape even is an issue, while B is unaware that the object may be of different colors. Moreover, when it comes to interactive awareness, since A is not aware of the shape, she considers that B is only aware of $S_B \cap S_A = \{\text{size}\}$, and she believes that this is common knowledge (Li, 2006; Heifetz et al., 2006).

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4 A common opinion is that unawareness, like other behavioral assumptions or bounded rationality, has too little regularity to show interesting insights. Another difficulty is that unawareness, implying inconsistent preferences, makes it ambiguous to define and calculate welfare.
Our main premise is that small investors are aware of fewer dimensions than professionals. This is supported by finance and accounting literature on small investors’ behavior. For instance, Malmendier and Shanthikumar (2005) find empirical evidences suggesting that small traders fail to account for the distortion in analyst stock recommendations, while large traders do not. For practical reasons, professionals usually take significant costs to gain not only more detailed information but also more dimensions of the information. Under fair disclosure, when professionals ask these questions about these dimensions of the information, small investors are very likely to know as much as professionals know. This potential free-riding certainly reduces the incentive of professionals to ask all the questions that they have. Before discussing our model, we invite readers to note a comment by one of the professionals six months after the regulation was implemented:\footnote{Comment by Eric Roiter, Sr., VP/General Counsel, Fidelity Management & Research, in Regulation FD Roundtable, on April 24, 2001. Source link: http://www.sec.gov/news/studies/regfdconf.txt}:

\[
\ldots \text{ analysts, even if given an opportunity to ask all of his or her questions in a public forum, will not do so; at least buy side analysts will not do so. And this reflects the fact that the very questions posed by insightful, well-prepared and skilled analysts have value. At times, I would submit, even greater value than any particular answer that a company executive may provide.}
\]

In our model, we consider a general capital financing and stock trading scenario. There are one issuer, many small investors, one professional, and many market makers. The issuer raises capital from small investors in a primary market. Small investors expect to trade upon a potential liquidity shock in a secondary market, where the professional with his private information may trade for speculative purposes. Market makers set prices and execute orders. Since uninformed market makers are aware of the private information of the professional, their prices reflect the information asymmetry and increase small investors’ transaction costs. Having rational expectations, small investors demand from the issuer a return compensating both the initial investment and the transaction costs. This required return determines the cost of capital of the issuer.
The professional has several means to collect information at various costs. He can always privately acquire information, with a cost function increasing in the quantity of information; or, under selective disclosure, he can enter selective forums with a fixed cost as an entry fee; or, under fair disclosure, he can enter public forums at no cost.

We focus on an issuer who is affected by the regulation; namely under selective disclosure the professional uses the issuer’s selective forums instead of private acquisition to acquire information. We define a stock market to be efficient if it maintains a low cost of capital and a low cost of information. Our first result is that, with symmetric awareness, the market is efficient under fair disclosure; but with asymmetric awareness, although the cost of capital is low, the cost of information may be high, hence efficiency is not assured. The reason can be briefly explained as follows.

Suppose small investors have the same set of questions in mind, i.e., they are fully aware. Under fair disclosure, they can ask all the questions in public forums and acquire all information. The professional has no advantage even if he can find answers privately. This leads to zero cost of information and a low cost of capital due to symmetric information. Under selective disclosure, the professional incurs costs of information when he enters selective forums. Meanwhile, small investors with full awareness demand extra return for the information asymmetry, inducing a high cost of capital. Hence the market is efficient under fair disclosure.

What if small investors are not fully aware? Under fair disclosure, the cost of capital remains low because small investors can ask all the questions that they have and believe that information is symmetric. However the professional may not ask all questions that he has, and he may privately search for the answers about unasked questions. Therefore fair disclosure may still induce the cost of information. Under selective disclosure, small investors demand a high return due to the expected information asymmetry. The cost of capital is still high. But the cost of information may be lower than that under fair disclosure.

Our second result is on the quantity of information collected by the professional. Many empirical studies on the regulation have examined market or accounting data related to professionals’ information. We find that with symmetric awareness, when the regime switches from
selective disclosure to fair disclosure, the professional collects equal information. However with asymmetric awareness, when the regime switches, the professional collects less information if the marginal cost of acquiring information is high enough.

To compare our results with empirical findings, we consider the scenario without the fund raising period so that the ownership share of small investors is fixed before and after the regulation. The model with asymmetric awareness matches the data better. It predicts that for firms which use selective disclosure more before the regulation (usually small firms and complex firms), professionals collect less information after the regulation. This is consistent with the findings on the effect of the regulation by Ahmed and Schneible Jr. (2004) and Gomes, Gorton and Madureira (2006). Ahmed and Schneible Jr. (2004) report that the information quality of average investors is worse for small firms and high tech firms; Gomes et al. (2006) report that small firms and complex firms suffered significant losses of analyst following.

At last we address the welfare of small investors. We show that if small investors are very unaware, and the professional does not share any information, then fair disclosure may reduce some losses, though not completely. In that sense, the regulation does help small investors. This effect of the regulation is not revealed with symmetric awareness, because then small investors with rational expectations always break even under both disclosure forms.

The remainder of this paper is organized as follows. In Section 2 we discuss related literature. In Section 3 we present the basic model and a few preliminary results. The symmetric awareness case is analyzed as a benchmark in Section 4. The asymmetric awareness case is analyzed in Section 5. In both cases, we first study selective disclosure then fair disclosure. In Section 6 we discuss the implications on the professional's information and small investors' welfare. Conclusions are in Section 7. All proofs are collected in Appendix A. A generalized model allowing both selling and buying stocks is discussed in Appendix B.
2 Related Literature

Theoretical analyses on Reg FD have been scarce, although many papers\(^6\) empirically test various effects of it.

Our paper is closely related to the literature on the theory of information disclosure in financial markets. The standard paradigm in that literature assumes fully aware and rational agents (see Verrecchia, 2001; Dye, 2001). In contrast, our model emphasizes that unawareness of some agents can have important effects, although we retain the rationality assumption. In the literature on auctions, Milgrom and Weber (1982) allow one bidder to acquire some information and compare the value of information with and without other bidders aware of this activity.

Our paper also contributes to a growing body of literature related to unawareness in different contexts. For example, Gabaix and Laibson (2006) study consumer product markets and Abreu and Brunnermeier (2003) study a stock market. In the first paper, the authors assume some consumers are myopic or unaware of negative attributes of products, and show that even if competitors have zero cost to educate consumers, the education does not happen in equilibrium. In the second paper, the authors assume that less sophisticated traders are unaware of the possibility of burst, and that rational arbitrageurs are not able to coordinate to correct the price because they become sequentially aware that the price has departed from fundamentals and they are not sure whether they learned that early or late relative to other rational arbitrageurs. In such settings the authors show that bubbles can be a unique equilibrium. In more abstract contexts, Modica, Rustichini and Tallon (1998) study the impact of unawareness in the theory of general equilibrium with pure exchange economies, and Kawamura (2004) extends the model to economies with production.

3 A Stock Market Model

This section contains three parts. The first one introduces the agents, their goals, and the rules of the game. The second one discusses the information structures of agents. The third one presents some preliminary results. Our model is based on the model in Section 4 of Verrecchia (2001), which in turn draws on Diamond and Verrecchia (1991) and Baiman and Verrecchia (1996).

3.1 Time Line

There are a monopoly issuer $K$, a measure one of small investors $U$, a professional investor $I$, and many market makers $M$. All agents are risk neutral and no investor has budget constraints. All this information is common knowledge.

![Time line of the game](image)

Figure 1: Time line of the game

The time line is shown in Figure 1. In the fund-raising period, the issuer raise a capital $C$ for a risky project, whose return is a nonnegative real number $v \in V$, as the realization of random variable $V$ with expectation $E(V) > C$. Small investors are the only investors in the primary stock market. For the settings of this period, we assume that: 1. information is symmetric between the issuer and small investors; 2. each small investor $j$ bids $R_j$, the ownership share for the whole project, $R_j \in \left[\frac{C}{E(V)}, 1\right]$; 3. the issuer accepts the lowest $R_j$ and each winning bidder obtains an equal number of shares and invests equally.

In the information acquisition period, relevant information is generated and the professional can do the following: 1. privately acquire information from sources other than the issuer; 2. enter selective forums under selective disclosure; 3. enter public forums under fair disclosure. Small investors however are only able to acquire information under fair disclosure.

In the trading period, both the professional and small investors trade in a secondary market through competitive market makers. We assume that small investors may experience
a negative liquidity shock with probability \( q \in (0, 1) \); when it happens, they always sell all
the shares that they own. The professional trades on private information for profits; when he
knows that the stock is over valued, he shorts \( r \) shares with a cap \( N \leq \frac{C}{E(V)} \). For simplicity,
we do not consider the case when the traders need to buy shares. In the appendix, we show
that the above model can be easily extended to a more general case without changing our
results.

Market makers can not tell from whom they buy and do not acquire private information,
but they anticipate the amount of the shares that small investors or the professional want
to sell and the information structure of the professional. As market makers compete in a
Bertrand style, they set a price \( \lambda \), which is nonnegative, just to break even. In this way, the
realized information of the professional is not revealed in the market equilibrium.

In the liquidation period, the value of the project is realized and the game ends.

3.2 Information Structure

Denote the set of states of the world by \( S \). Its element \( s \) is the realization of a random variable
\( S \). The prior belief is \( P \in \Delta S \). A signal \( \tau^\theta \) is chosen from a family of partitions of \( S \), which
is denoted by \( \{ \tau^\theta \}_{\theta \in \Theta} \), where set \( \Theta = \{ \emptyset \} \cup [0, \bar{\theta}], \bar{\theta} < 0 \). At every \( s \), given signal \( \tau^\theta \), the
realized value of signal is \( \tau^\theta(s) \); an agent’s posterior belief about \( S \) is \( P(s|\tau^\theta(s)) \in \Delta S \) and
the posterior belief about \( v \) is \( F(v|\tau^\theta(s)) \in \Delta V \).

The set of all possible signals, \( \{ \tau^\theta \}_{\theta \in \Theta} \) is a fully ordered set with a larger \( \theta \) referring to a
finer signal. In other words, if \( \theta > \theta' \), then partition \( \tau^\theta \) is finer than partition \( \tau^\theta' \). Signal \( \tau^2 \)
is a null signal representing no information.

Before the trading period, the private information of the professional is denoted by \( \tau^\theta \),
the public information that small investors and market makers have is denoted by \( \tau^\eta \), \( \eta \in \Theta \).

The issuer has the finest information \( \bar{\tau} \equiv \tau^{\bar{\theta}} \) at no cost. Under either form of disclosure,
the issuer commits to answer questions with the finest information that he has. It is equivalent
to assume that given the issuer’s information, he commits to disclose the best information
that investors can possibly acquire by all other means.

\[ \text{That is, he can only short no more than the minimum total amount of the shares.} \]
Under selective disclosure, small investors can not collect information. When the professional chooses to enter the selective forum, the cost of information is $F_s$ ("s" for selective forums), a fixed cost of entry which could be interpreted as an entry fee needed to gain the trust of the issuer. When the professional chooses to privately acquire, the cost is $C_p(\theta)$ ("p" for private acquisition), an increasing function of $\theta$ with $C_p(\theta) = 0$.

Under fair disclosure, both small investors and the professional collect information at no cost when the issuer answers to questions posed by any investor. When the professional privately acquires $\tau^\theta$, the cost depends on both $\tau^\theta$ and $\tau^\eta$. Since the professional knows the public information, $\theta \geq \eta$. Then the cost is a function $C_f(\theta, \eta)$ ("f" for fair disclosure), which increases in $\theta$. Since $\tau^\eta$ substitutes $\tau^\theta$, we assume that the cost $C_f(\theta, \eta) = 0$ if $\theta = \eta$ and decreases in $\eta$.

We define the cost of capital $C_c \equiv RE(V)$. We also define the cost of information $C_i$ as the professional’s cost. Under selective disclosure, if he uses selective forum, it is $F_s$; if he uses private acquisition, its $C_p(\theta)$. Under fair disclosure, it is $C_f(\theta, \eta)$.

Regarding awareness, the issuer, market makers and the professional are aware of $S$ and all above are common knowledge among them. If awareness is symmetric, small investors are also aware of $S$, and all above is common knowledge among all agents.

If awareness is asymmetric, small investors are certainly aware of $V$, but they are only aware of $\tau^\theta$ with $\theta \in \{\theta, 0\}$. In other words, they ignore signals $\tau^\theta$ with $\theta \in (0, \theta)$, which depends on some dimensions that they are unaware of. Using the language of game theory, it is convenient to assume that there are two versions of games $g$ and $G$ in agents’ minds: small investors believe that the small\textsuperscript{8} game $g$ is common knowledge to all; however, the issuer, market makers and the professional believe that the great game $G$ is common knowledge to all except small investors; they also believe that it is common knowledge to all that small investors believe in $g$. \textsuperscript{9}

\textsuperscript{8}In the sense of the dimensionality of games.

\textsuperscript{9}See Feinberg (2005) for a formal model of games with incomplete awareness.
3.3 Preliminary Results

First we briefly restate the model. It is a three-stage game with observed actions. In the fund-raising period, the share of ownership $R$ is determined by the competition among small investors, who have the future periods in mind. Then all other agents know $R$. In the information acquisition period, the professional acquires $\theta$, and at state $s$ observes $\tau^\theta(s)$, while the public observes $\tau^n(s)$. Market makers also observe that the professional acquires $\theta$. In the trading period, simultaneously, small investors sell $R$ shares with probability $q$, the professional sells $r$ shares, and market makers set price $\lambda$.

We begin with small investors and the determination of ownership share $R$. As small investors are homogeneous and compete in a Bertrand style, each small investor would demand an equal minimum share of ownership for the whole project, such that he breaks even. This is an equilibrium behavior because if only one of small investors deviates, he does not gain. Since the mass of small investors is 1, each small investor receives his share $R$ and contributes capital $C$.

Then each small investor knows that, in the case of a negative liquidity shock, he sells all shares he owns; his profit is $E(\lambda)R - C$, where $E(\lambda)$ is the expectation of the prices in the trading period. Otherwise, his profit is $E(V)R - C$. Since the shock will happen with probability $q$, his ex ante expected profit is

$$\pi_U := R[qE(\lambda) + (1-q)E(V)] - C.$$  \hfill (3.1)

Now consider the trading period. We want to show that given information $\tau^\theta$ and $\tau^n$, there is an equilibrium where the price set by market makers and the quantity of shares traded by the professional are determined.

The professional’s ex post profit is $\pi_I = r(\lambda - v)$. His decision problem is to maximize his interim expected profit from trade:

$$\max_{r \in [0,N]} E_\theta(\pi_I | s) := \int_V r(\lambda - v) dF(v | \tau^\theta(s)).$$  \hfill (3.2)
Note that because the professional can always choose not to trade, the optimal profit is nonnegative. Because the public information is not finer than the private information and \( \lambda \) depends on public information, the expression becomes

\[
\int_V r(\lambda - v) dF(v|\tau^\theta(s)) = r[\lambda - \int_V v dF(v|\tau^\theta(s))] = r[\lambda - E_\theta(V|s)].
\]

The optimal choices is, \( r = N \) when the conditional expected value is \( E_\theta(V|s) < \lambda \), and \( r = 0 \) otherwise.

**Fact 1.** Given \( \lambda \) and \( \tau^\theta \), define \( d(\lambda, \theta) \equiv \{ s | E_\theta(V|s) < \lambda \} \). An optimal trade rule of the professional is a function of \( s \), \( \lambda \), and \( \theta \).

\[
r(s; \lambda, \theta) = \begin{cases} N & \text{if } s \in d(\lambda, \theta) \\ 0 & \text{if else.} \end{cases}
\]

When the professional uses the optimal trade rule \( r(s; \lambda, \theta) \), his interim expected profit conditional on the public information is:

\[
E_\eta(\pi_I|s) = \int_S \int_V r(s'; \lambda, \theta)(\lambda - v) dF(v|\tau^\theta(s')) dP(s'|\tau^\eta(s)) = \int_{s' \in d(\lambda, \theta)} N(\lambda - E_\theta(V|s')) dP(s'|\tau^\eta(s)).
\] (3.3)

Therefore \( E_\eta(\pi_I|s) \) is determined by \( (\theta, \lambda) \). It is useful to prove the following results about it.

**Lemma 1.** Given any \( s \in S \), the profit function \( [E_\eta(\pi_I|s)](\theta, \lambda) \) has the following properties:

i It increases in \( \theta \). That is, when \( \theta' > \theta \), so that for all \( s' \in \tau^\eta(s) \), \( \tau^\theta(s') \subset \tau^\theta(s) \), then

\[
[E_\eta(\pi_I|s)](\theta', \lambda) \geq [E_\eta(\pi_I|s)](\theta, \lambda).
\]

ii It increases in \( \lambda \). That is, if \( \lambda' > \lambda \), then

\[
[E_\eta(\pi_I|s)](\theta, \lambda') \geq [E_\eta(\pi_I|s)](\theta, \lambda).
\]

iii If the probability measure of the states such that the conditional expectation of \( V \) is less than \( x \in R_+ \), or \( G(x|s) \equiv \frac{Prob\{s' \in \tau^\eta(s)|E_\theta(V|s') < x\}}{Prob\{s' \in \tau^\eta(s)\}} \), is smooth in \( x \), then \( E_\eta(\pi_I|s) \) is continuous in \( \lambda \).
When the professional trades \( r \) shares and the market price is \( \lambda \), market makers’ aggregate \textit{ex post} profit is \( \pi_M = (Rq+r)(v-\lambda) \). The price is identical for all market makers because they are homogeneous and the competition is Bertrand. Given public information \( \tau^u \), equilibrium price is given by the highest \( \lambda \) (for they are competing for sell orders) which satisfies the following condition:

\[
E_\eta(\pi_M|s) := \int_S \int_V (Rq + r)(v - \lambda)dF(v|\tau^\theta(s'))dp(s'|\tau^u(s)) = 0. \tag{3.4}
\]

In equilibrium, by Condition 3.4 and Equation 3.3, the optimal pricing rule of market makers is given by

\[
\lambda(s; R, \eta, \theta) = \max \left\{ \lambda \geq 0 \left| Rq[E_\eta(V|s) - \lambda] = E_\eta(\pi_I|s) \right. \right\}. \tag{3.5}
\]

We prove the following Lemma in the Appendix.

**Lemma 2.** The following is true for the optimal pricing rule \( \lambda(s; R, \eta, \theta) \).

\( i \) It exists and is unique.

\( ii \) It strictly increases in \( R \), decreases in \( \theta \), and is continuous in \( R \).

\( iii \) The \textit{ex ante} expected price \( (E|\lambda)|(R, \eta, \theta) = \int_{s\in S} \lambda(s; R, \eta, \theta)dP(s) \) strictly increases in \( R \) and decreases in \( \theta \).

Hence by Fact 1 and Lemma 2, given ownership share \( R \), information \( \tau^u \) and \( \tau^\theta \), in the trading period, a pure strategy equilibrium \( (\lambda(s; R, \eta, \theta), r(s; \eta, \theta))_{s\in S} \) exists. The equilibrium \textit{ex ante} expected profit of the professional is

\[
\Pi_I = \int_S \int_V r(s; \eta, \theta)(\lambda(s; R, \eta, \theta) - v)dF(v|\tau^\theta(s))dP(s). \tag{3.6}
\]
4 Symmetric Awareness

Suppose that small investors are aware of $S$, with belief $P$ being common knowledge. First consider selective disclosure. Because there is no public information, namely $\eta = \theta$, expected profit $E_\eta(\pi_I|s) = \Pi_I$ for all $s \in S$, and the stock price is also constant. Denoted the price by $\lambda_1(\theta|R) \equiv (E[\lambda])(R, \theta, \theta)$. Equation 3.5 implies that

$$\Pi_I = Rq(E(V) - \lambda_1(\theta|R)).$$

In the information acquisition period, adjusted by the costs of information, the professional’s expected profit becomes the following: If he collects information through selective forums,

$$\Pi_I - F_s = Rq(E(V) - \lambda_1(\theta|R)) - F_s.$$

By Lemma 2, the optimal $\theta = \bar{\theta}$, because then $\lambda_1(\theta|R)$ is minimized. The optimal profit of the professional is

$$\Pi_{Is} = Rq(E(V) - \lambda_1(\bar{\theta}|R)) - F_s.$$

If he collects information through private acquisition,

$$\Pi_I - C_p(\theta) = Rq(E(V) - \lambda_1(\theta|R)) - C_p(\theta). \quad (4.1)$$

The optimal $\theta$ may not be $\bar{\theta}$ if the marginal cost is too high. Being a function of $R$, the optimal solution is denoted by $\hat{\theta}(R)$. The optimal profit of the professional is

$$\Pi_{Ip} = Rq(E(V) - \lambda_1(\hat{\theta}(R)|R)) - C_p(\hat{\theta}(R)).$$

Assume that for all $R \in \left[\frac{C}{E[V]}, 1\right]$, it is always profitable for the professional to privately collect information. Then when $\Pi_{Is} > \Pi_{Ip}$, or

$$Rq\lambda_1(\hat{\theta}(R)|R) + C_p(\hat{\theta}(R)) > Rq\lambda_1(\bar{\theta}|R) + F_s, \quad (4.2)$$

14
the professional chooses selective forums. When the equality holds, the professional is indifferent. Otherwise, the professional chooses private acquisition.

For the purpose of this paper, we want to restrict our attention to firms affected by the regulation. For such firms, professionals prefer using selective forums to acquire information about them. It is sufficient for Condition 4.2 to hold if the cost $F_s$ is relatively low.

**Lemma 3.** Denote $\hat{\theta}^* = \min\{\hat{\theta}(R) | R \in [\frac{C}{E(V)}, 1]\}$. If

$$F_s < C_p(\hat{\theta}^*) \tag{4.3}$$

then Condition 4.2 holds for all $R \in [\frac{C}{E(V)}, 1]$.

For the rest of this paper, we assume Assumption 4.3 holds. Now we include the fund raising period when the ownership share is determined and show the existence of the equilibrium. Because the competition among small investors pushes the share of ownership to the minimum level where their profits are zero, by Definition 3.1, define

$$\bar{R} \equiv \min \left\{ R \in [\frac{C}{E(V)}, 1] \right\} R = \frac{C}{E(V)(1-q) + q\lambda_1(\hat{\theta}|R)} \right\}. \tag{4.4}$$

If the likelihood of liquidity shock is too large, it is possible that small investors require $R_e > 1$ when they expect a low stock price, which is not acceptable to the issuer. To ensure the initial investment, we assume that this likelihood is relatively small, or $q \leq 1 - \frac{C}{E(V)}$. Using the fixed point argument, we prove the existence of the equilibrium. Here we summarize the results.

**Proposition 1.** Denote an equilibrium by a vector $(R_e, \eta_e, \theta_e, \lambda_e(s), r_e(s))_{s \in S}$. Assume that $q \leq 1 - \frac{C}{E(V)}$. Under selective disclosure, the equilibrium exists and is

$$\begin{pmatrix} R_e = \bar{R}, \eta_e = \bar{\theta}, \theta_e = \hat{\theta}, \lambda_e(s) = \lambda_1(\bar{\theta}|\bar{R}), \ r_e(s) = r(s; \lambda_1(\bar{\theta}|\bar{R}), \bar{\theta}) \end{pmatrix}_{s \in S}.$$

In this equilibrium, $C_c = \bar{R}E(V)$ and $C_i = F_s$.

Now consider fair disclosure. Since small investors have costless access to public forums,
and the issuer answers questions sincerely, the competition among small investors lead to an efficient outcome. The professional’s advantage in private acquisition becomes ineffective. In the Appendix, we prove that full information disclosure can happen in an equilibrium.

**Proposition 2.** Under fair disclosure, there is an equilibrium where full information is disclosed, or

\[
\left( R_e = \frac{C}{E(V)}, \quad \eta_e = \theta_e = \bar{\theta}, \quad \lambda_e(s) = E(V|\bar{\pi}(s)), \quad r_e(s) = 0 \right)_{s \in \mathbb{S}}.
\]

In this equilibrium, \( C_c = C \) and \( C_i = 0 \).

In addition, there may be other equilibria where \( \eta_e = \theta_e < \bar{\theta} \), thus only partial information is disclosed, but it remains true that \( R_e = \frac{C}{E(V)} \), \( C_c = C \) and \( C_i = 0 \).

Note that by Definitions 4.4, \( \bar{R} \geq \frac{C}{E(V)} \). So the cost of capital is low under fair disclosure.

If small investors are not sure what cost it takes for the professional to privately acquire information, then requesting full information disclosure is a dominant strategy for them, because the professional will not have a chance to profit from acquiring private information. Also note that the investment is sure and it is not necessary that the possibility of shock \( q \) has to be small as assumed in Proposition 1. So investment is more likely to happen under fair disclosure.

**Theorem 1.** Suppose awareness is symmetric. Fair disclosure is more efficient than selective disclosure. Namely, both the cost of the capital and the cost of information are greater under selective disclosure.

Meanwhile, the professional’s information does not change under fair disclosure.

**Theorem 2.** Suppose awareness is symmetric. The professional collects the same full information under fair disclosure and under selective disclosure.
5 Asymmetric Awareness

Suppose that small investors are unaware of some dimensions of information. Thus they believe that only signal $\tau^\theta$, $\theta \in \{\theta, 0\}$ matters. For them, the game is similar to the game in the previous section. The only difference is that they think the professional’s information is given by $\dot{\theta} \in \{\theta, 0\}$ and market makers believe so too. Here we use original notations with a dot to denote the variables in small investors’ mind that are different from the fact.

First we consider selective disclosure. Denote small investors’ expected price $\dot{\lambda}(\dot{\theta}|R) \equiv (E[\lambda])(R, \theta, \dot{\theta})$. Then the profit of the professional when he chooses selective forums is

$$Rq(E(V) - \dot{\lambda}(\dot{\theta}|R)) - F_s.$$

By Lemma 2, the optimal $\dot{\theta}$ is 0. The profit of the professional when he privately acquires information is

$$Rq(E(V) - \dot{\lambda}(\dot{\theta}|R)) - C_p(\theta).$$

Assume that for all $R \in [CE(V), 1]$, it is always profitable for the professional to acquire signal $\tau^0$ no matter it is by private acquisition or selective forums. Small investors believe that the professional collects $\tau^0$ in the equilibrium.

**Proposition 3.** Denote $\dot{\lambda}(R) \equiv (E[\lambda])(R, \theta, 0)$. Assume that $q \leq 1 - \frac{C}{E(V)}$. Under selective disclosure, Small investors believe that there is an equilibrium where the professional collects signal $\tau^0$, or

$$\left( R_e = \tilde{R}, \; \eta_e = \theta, \; \dot{\theta}_e = 0, \; \dot{\lambda}_e(s) = \dot{\lambda}(\tilde{R}), \; \dot{r}_e(s) = r(s; \dot{\lambda}(\tilde{R}), 0) \right)_{s \in S},$$

where $\tilde{R} = \min \left\{ R \in \left[ \frac{C}{E(V)}, 1 \right] \left| R = \frac{C}{E(V)(1 - q) + q\lambda(R)} \right. \right\}.$

The proof is omitted for it is similar to the proof of Proposition 1.

On the other hand, given that small investors choose $R_e = \tilde{R}$, the professional’s interim profit from trade is still given by Definition 3.3, with its properties stated in Lemma 1. Denote
the expected price \( \lambda_2(\theta) \equiv \left( E[\lambda]\right)(\bar{R}, \bar{\theta}, \theta) \).

If the professional chooses selective forums, his profit is

\[
\tilde{R}q(E(V) - \lambda_2(\theta)) - F_s.
\]

Then by Lemma 2, the optimal \( \theta \) is \( \bar{\theta} \). The optimal profit of the professional is

\[
\Pi_{Is} = \tilde{R}q(E(V) - \lambda_2(\bar{\theta})) - F_s. \tag{5.1}
\]

If the professional chooses private acquisition, his profit is

\[
\tilde{R}q(E(V) - \lambda_2(\theta)) - C_p(\theta). \tag{5.2}
\]

This profit function is similar to Function 4.1 with symmetric awareness but now \( R \) is fixed as \( \tilde{R} \). Therefore, by Lemma 3, given Assumption 4.3, the professional always prefers using selective forums.

Assume that for the optimal profit 5.1 is positive. Then in equilibrium, the professional chooses selective forums. The result is stated in the following Proposition with the proof omitted.

**Proposition 4.** Under selective disclosure, the professional chooses selective forums, or

\[
\left( R_e = \tilde{R}, \ \eta_e = \frac{\theta}{2}, \ \theta_e = \bar{\theta}, \ \lambda_e(s) = \lambda_2(\bar{\theta}), \ r_e = r(s; \lambda_2(\bar{\theta}), \bar{\theta}) \right)_{s \in \mathcal{S}},
\]

in which the costs \( C_c = \tilde{R}E(V) \), \( C_i = F_s \).

Suppose now fair disclosure is implemented. Small investors expect a game similar to the one with symmetric awareness. By an argument similar to the proof of Proposition 2, there is an equilibrium where small investors request the disclosure of \( \tau^0 \), the full information in their minds, and set \( R_e = \frac{C}{E(V)} \) in the fund raising period.

The professional can also request a disclosure, so that the index of public information is \( \eta \geq 0 \), and he also privately acquires information so that the index is \( \theta \geq \eta \). Denote
\[ \lambda_3(s; \eta, \theta) \equiv \lambda(s; \frac{C}{E(V)}, \eta, \theta), \] with its expectation denoted by \((E[\lambda_3])(\eta, \theta)\). The professional’s \textit{ex ante} expected profit function is

\[ \frac{Cq}{E(V)} [E(V) - (E(\lambda_3))(\eta, \theta)] - C_f(\theta, \eta). \quad (5.3) \]

The equilibrium information \((\bar{\theta}, \bar{\eta})\) must maximize Equation 5.3.

\[ \max_{0 \leq \eta \leq \theta \leq \bar{\theta}} \frac{Cq}{E(V)} [E(V) - (E(\lambda_3))(\eta, \theta)] - C_f(\theta, \eta). \quad (5.4) \]

Suppose everything is differentiable. Then there are two possibilities. One is that given any public information \(\eta \geq 0\), the marginal cost of acquiring private information is no less than the marginal profit from trading on the private information, namely

\[ \left. \frac{\partial C_f(\theta, \eta)}{\partial \theta} \right|_{\theta = \eta} \geq \left. \frac{qC}{E(V)} \frac{\partial (E[\lambda_3])(\eta, \theta)}{\partial \theta} \right|_{\theta = \eta} \quad (5.5) \]

then the professional does not have incentives to acquire more information. In equilibrium, \(\bar{\theta} = \bar{\eta} = 0\), market makers do not charge transaction costs, the professional does not acquire private information and does not trade. The other possibility, is that there are some levels of public information \(\eta > 0\), such that Condition 5.5 does not hold. Then in equilibrium, \(\bar{\theta} > \bar{\eta} \geq 0\). In this equilibrium, the cost of information acquisition is \(C_i = C_f(\bar{\theta}, \bar{\eta})\).

**Proposition 5.** Under fair disclosure, there are two types of equilibria. If Condition 5.5 hold for all \(\eta \geq 0\), then

\[ \left( R_e = \frac{C}{E(V)}, \ \eta_e = \theta_e = 0, \ \lambda_e(s) = E(V|\tau^0(s)), \ r_e(s) = 0 \right)_{s \in S}, \]

In this equilibrium, \(C_c = C\), and \(C_i = 0\).

If not, then

\[ \left( R_e = \frac{C}{E(V)}, \ \eta_e = \hat{\eta}, \ \theta_e = \hat{\theta}, \ \lambda_e(s) = \lambda_3(s; \hat{\eta}, \hat{\theta}), \ r_e(s) = r(s; \lambda_3(s; \hat{\eta}, \hat{\theta}), \hat{\theta}) \right)_{s \in S}, \]

where \(0 \leq \hat{\eta} < \hat{\theta} \leq \bar{\theta}\). In this equilibrium, \(C_c = C\), and \(C_i = C_f(\hat{\theta}, \hat{\eta})\).
To compare the costs under different disclosure forms, we first note that by definition, 
\( \hat{R} \geq \frac{C_c}{E(V)} \), so the cost of capital \( C_c \) is high under selective disclosure. Also we note that under selective disclosure \( C_i = F_s \) and that under fair disclosure, in the second type of equilibrium, \( C_i = C_f(\bar{\theta}, \bar{\eta}) \). If \( F_s \) is small enough, for instance, \( F_s \) is smaller than \( C_f(\bar{\theta}, \bar{\eta}) \), then the cost of information under fair disclosure is also greater than under selective disclosure.

**Fact 2.** Suppose awareness is asymmetric. The cost of capital is lower under fair disclosure than under selective disclosure. On the other hand, the cost of information may be higher than that under selective disclosure if the cost of entering selective forums is low.

We want to emphasize that asymmetric awareness is not the only cause of high information cost. The other possible cause in practice is that the issuer can hide information. Even though small investors have full awareness, the issuer can refuse to answer and the professional may want to privately acquire information without violating the rule. Then the cost of information acquisition is still positive. It implies that, to emphasize the effect of asymmetric awareness on the efficiency issue, the assumption that the issuer always provides information upon request is important. On the other hand, the empirical examinations of Reg FD (Sunder, 2002; Heffin et al., 2003; Choi, 2003) suggest that after the regulation, issuers in general disclose more information on many aspects about the stocks. Therefore we believe that this assumption is plausible and asymmetric awareness should be considered as a cause of high cost of information.

Now we compare the quantity of information collected by the professional. For the first type of equilibrium, it is clear that \( \theta_e = 0 < \bar{\theta} \). Suppose for the second type of equilibrium the optimal solution is interior, \( \bar{\theta} < \bar{\theta} \), then the professional does not collect full information. Hence his information becomes worse if he has collected it through selective disclosure.

**Theorem 3.** Suppose awareness is asymmetric. If under fair disclosure, the equilibrium is of the first type or is of the second type with interior solution, the professional always collects less information than under selective disclosure, namely \( \bar{\theta} < \bar{\theta} \).

In the next section, we discuss the implication of this theorem.
6 Discussion

6.1 Professionals’ Information

Theorems 2 and 3 predict different effects of the regulation on the quantity of information acquired by professionals. These results are obtained in a framework including the fund raising period, hence the public investors’ ownership share \( R \) is endogenous and can vary under different disclosure forms. The empirical examinations of this aspect, however, typically use the data of given firms before and after the regulation; in other words, \( R \) is nearly fixed under different disclosure forms. We discuss our result in this circumstance.

The result of Theorem 2 is unaffected when \( R \) is fixed. Consider the information acquisition period. Under selective disclosure, whatever \( R \) is, the professional acquires full information through selective forums. Under fair disclosure, by requesting full information, small investors’s loss can be minimized to zero, meanwhile the professional also have full information. In conclusion, after the regulation, in some equilibrium, the professional has the same information.

The result of Theorem 3 is also unaffected when \( R \) is fixed. Consider the information acquisition period. Given \( R \), if the professional acquires full information through selective forums before the regulation, then after the regulation, he does not acquire more information and probably less information because of the high marginal cost of private acquisition. We still have the following facts.

**Fact 3.** For a given firm, i.e., for the same \( R \), if the professional acquires information via selective forums before the regulation, then his information decreases after the regulation.

Meanwhile, empirical studies in the literature on information disclosure say the following.

**Assumption 2** Before the regulation, for investors, the cost of privately acquiring information about small firms and complex firms is high. These firms’ entry cost of selective forums is low.

For example, Bushee, Matsumoto and Miller (2004) empirically find more complex firms were more likely to use selective forums in the pre-FD period.
Hence under selective disclosure, Assumption 3 is more likely for small firms and complex firms, so professionals are more likely to acquire their information via selective forums. Putting things together, we have the following.

**Theorem 4.** After the regulation, for small firms and complex firms, the professional collects less information.

This prediction matches the finding by Ahmed and Schneible Jr. (2004) that Reg FD has worsened the information quality of average investors for some firms (particularly small, high tech firms). It is also related to the findings by Gomes et al. (2006). They report the following:\(^{10}\)

1. Small firms on average lost 17 percent of their analyst following while big firms increased theirs by 7 percent, on average.\(^{11}\)

2. More complex firms (using intangible assets as a proxy for complexity) overall are being more adversely affected than less complex firms. Regardless of size, more complex firms suffer a significantly larger loss of analyst following.

If the number of analysts following a stock is a proxy of the quantity of information acquired by professionals, our prediction is consistent with their findings. We should emphasize that the literature on analyst following, such as Bhushan (1989), suggests that the analyst following of a firm is a proxy for the total expenditure investors spend on information, which in turn depends on the demand and supply of information. In this paper we only consider a monopoly issuer, so there is no analysis about the professional ’s choosing what firm’s stock to trade, and firms’ choosing the entry cost of selective disclosure. In addition, we do not model the role of analysts as producers of information. Although further study is needed to understand the full story, it is worth noting that such a simple theory featuring asymmetric awareness provides some clue.\(^{12}\)

\(^{10}\)The authors did not find satisfactory explanations and suggested that “… ‘information in financial markets may be more complicated than current finance theory admits.”

\(^{11}\)Survey data also support this finding: The ABA FD Task Force Survey, which surveyed securities attorneys about their clients’ practices reports that FD had bigger impact on small and midsize companies rather than large companies. Source link: http://www.sec.gov/news/studies/regfdconf.txt.

\(^{12}\)Caveats: Mohanram and Sunder (2006) however finds a different empirical result: In the post-FD period analysts reduce coverage for well followed firms, which mainly are big firms, and increase coverage of firms
6.2 Small Investors’ Welfare

One of the regulators’ major goals is to protect small investors. This issue is discussed in this subsection. We will compare the welfare gain under different disclosure forms by measuring the change of small investors’ profits.

Since small investors are competitive and rational, when they have full awareness, their expected profits are zero. However, when their awareness is limited, although they think the expected profits are zero, their expected real profits are negative. With that said, when the loss under fair disclosure is less than that under selective disclosure, the protection of the regulation is effective.

Recall small investor’s profit function in Definition 3.1 and the equilibrium behavior of the professional under different disclosure forms in Propositions 4 and 5. Under selective disclosure, since the professional chooses selective forms, the actual profit of each small investor is

\[ \pi_{Us} = \hat{R} \left[ qE[\lambda](\hat{R}, \hat{\theta}, \bar{\theta}) + (1 - q)E(V) \right] - C. \]

Under fair disclosure, the actual profit of each small investors is

\[ \pi_{Uf} = C \left[ qE[\lambda]\left(\frac{C}{E(V)}, \tilde{\eta}, \tilde{\theta}\right) + (1 - q)E(V) \right] - C. \]

We prove the following in Appendix.

**Theorem 5.** If small investors are completely unaware, namely signal \( \tau^0 \) is null, and the professional does not request information via public forums after the regulation, namely \( \eta = \theta \), then

\[ \pi_{Uf} \geq \pi_{Us}. \]

This seemingly contradicting result might be explained by different empirical methods applied in two papers. One difference is that Mohanram and Sunder (2006) only covers sample firms that had some analyst following in both the pre- and post-FD period; but Gomes et al. (2006) also use sample firms that totally lost analyst following after Reg. FD. However, we still need to be cautious about missing factors affecting their findings. More investigation shall be taken along the process of our research.

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*that were less followed prior to Reg. FD.*
Hence fair disclosure induces a lower loss than selective disclosure.

Although this result depends on very restrictive assumption, it has an interesting implication. That is, if small investors’ are very unprepared and do not have any chance of free-riding, then fair disclosure may improve their welfare.

7 Conclusions

In this paper we study Regulation Fair Disclosure, a ruling adopted by the SEC to forbid selective disclosure. Using a stock market model with four periods—fund raising, information acquisition, trading, and liquidation—we analyze market participants’ behavior when one professional can acquire information directly from an issuer in selective forums, i.e., under selective disclosure; or when he can not, i.e., under fair disclosure. Focusing on the issuer who would have used selective forums, we address the aspects including the cost of capital, the cost of information, the quantity of information acquired by the professional, and the welfare of small investors.

Our results depend on assumptions about investors’ awareness. We find that, when all investors are equally aware of the relevant information, fair disclosure induces a low cost of capital and a low cost information, therefore making the market efficient. It also induces equally good information collected by the professional. When small investors are unaware, fair disclosure still induces a low cost of capital, but it may induce a high cost of information and less information collected by the professional.

Under the asymmetric awareness assumption, our theory gives predictions which match the empirical findings that the regulation has worsened the information quality of average investors for some firms (particularly small, high tech firms), and negatively affected the analyst following of small and complex firms. We also show that when small investors are extremely unaware, the regulation improves their welfare. Since the asymmetric awareness assumption is not standard in the literature, our analysis suggests an alternative approach to understand the regulation and perhaps other information disclosure related problems.
A Proofs

Proof of Lemma 1.

i. Suppose $\theta' > \theta$, by the definition in Fact 1, we denote the new optimal trade rule by $r(s; \lambda, \theta')$. Then first for each $\tau^{\theta'}(s')$, the optimal expected profit is

\[
\mathbb{E}_{\theta'}(\pi_I|s')|\theta', \lambda' = \int r(s''; \lambda', \theta') [\lambda - E_{\theta'}(V|s'')] dP(s''|\tau^{\theta'}(s'))
\]

\[
= \max_r \int [\lambda - E_{\theta'}(V|s'')] dP(s''|\tau^{\theta'}(s'))
\]

\[
\geq \int r(s''; \lambda, \theta) [\lambda - E_{\theta'}(V|s'')] dP(s''|\tau^{\theta'}(s'))
\]

Because $\tau^{\theta'}$ is finer than $\tau^\theta$, for every $s'$, $E_{\theta'}(V|s') = \int E_{\theta'}(V|s'') dP(s''|\tau^{\theta'}(s'))$ and $r(s''; \lambda, \theta) = r(s'; \lambda, \theta)$ if $s'' \in \tau^{\theta'}(s')$. Now for each $\tau^{\theta}(s)$,

\[
\mathbb{E}_{\theta}(\pi_I|s')|\theta', \lambda' \geq \int \int [\lambda - E_{\theta'}(V|s'')] dP(s''|\tau^{\theta'}(s')) dP(s'|\tau^{\theta}(s)) \quad (A.1)
\]

\[
= \int r(s'; \lambda, \theta) [\lambda - E_{\theta}(V|s')] dP(s'|\tau^{\theta}(s))
\]

\[
= \mathbb{E}_{\theta}(\pi_I|s')|\theta, \lambda
\]

Because $\theta' > \theta > \eta$, integrate the both sides on $s' \in \tau^{\eta}(s)$, then the result follows.

ii. Suppose $\lambda' > \lambda$. By the definition in Fact 1, the new optimal trade rule is

\[
r(s; \lambda', \theta) = \begin{cases} N & \text{if } s \in d(\lambda', \theta) \\ 0 & \text{if else.} \end{cases}
\]

Again for simplicity we denote $d_1 \equiv d(\lambda', \theta)$, $d_2 \equiv d(\lambda, \theta)$. Note that $d_1 \supset d_2$, then
Therefore \( [E_\eta(\pi|s)](\theta, \lambda') \geq [E_\eta(\pi|s)](\theta, \lambda) \).

iii. Suppose the probability measure \( G(x|s) \) is smooth in \( x \in R_+ \). Note that by its definition, \( G(x|s) = \int_{s' \in d(x, \theta)} dP(s'|\tau^n(s)) \). Then the function

\[
[E_\eta(\pi_I|s)](\theta, \lambda) = \int_{s' \in d(\lambda, \theta)} N[\lambda - E_\eta(V|s')]dP(s'|\tau^n(s))
\]

\[
= N\lambda \int_{s' \in d(\lambda, \theta)} dP(s'|\tau^n(s)) - N \int_{s' \in d(\lambda, \theta)} E_\eta(V|s')dP(s'|\tau^n(s))
\]

\[
= N\lambda G(\lambda|s) - N \int_0^\lambda xdG(x|s)
\]

\( G(x|s) \) is smooth so that the second part is well defined and continuous in \( \lambda \). The function is continuous in \( \lambda \) because it is the sum of two continuous functions.

\[
QED.
\]

**Proof of Lemma 2.**

i. Define function

\[
f(\lambda) \equiv R[q[E_\eta(V|s) - \lambda] - E_\eta(\pi_I|s)].
\]

First \( f(\lambda) \) is continuous in \( \lambda \) by the third claim in Lemma 1. Second when \( \lambda = 0 \),
\(d(\lambda, \theta) = \emptyset, \) and \(E_\eta(\pi_I|s) = 0,\) then

\[ f(0) = RqE_\eta(V|s) \geq 0. \]

Also when \(\lambda = E_\eta(V|s),\)

\[ f(E_\eta(V|s)) = -E_\eta(\pi_I|s) \leq 0. \]

Then by Intermediate Value Theorem, there is at least one \(\lambda \in [0, E_\eta(V|s)]\) such that \(f(\lambda) = 0.\)

Hence the pricing rule \(\lambda(s; R, \eta, \theta)\) exists. Since \(E_\eta(\pi_I|s)\) decreases in \(\lambda,\) \(f(\lambda)\) is also strictly decreasing in \(\lambda,\) then it implies the uniqueness.

**ii.** The monotonicity of \(\lambda(s; R, \eta, \theta)\) can be shown by by contradictions. Suppose \(\lambda(s; R, \eta, \theta)\) does not increase in \(R.\) Then if \(R\) increases, the LHS of

\[ Rq[E_\eta(V|s) - \lambda(s; Rq, \eta, \theta)] = E_\eta(\pi_I|s) \]

would increase, while the RHS of it would decrease by Lemma 1. A contradiction.

Similarly we can show that if \(\theta\) increases, \(\lambda(s; R, \eta, \theta)\) can not increase for otherwise the RHS of the equation increases but the LHS decreases, which is a contradiction.

For the continuity of function \(k(R) \equiv \lambda(s; Rq, \eta, \theta)\) in \(R,\) we want to show that \(\lim_{\epsilon \to 0}k(R + \epsilon) = k(R)\) for all \(R \in R_+,\) where \(k(R)\) is given by

\[ Rq[E_\eta(V|s) - k(R)] = [E_\eta(\pi_I|s)](k(R)); \tag{A.2} \]

and \(k(R + \epsilon)\) is given by

\[ (R + \epsilon)q[E_\eta(V|s) - k(R + \epsilon)] = [E_\eta(\pi_I|s)](k(R + \epsilon)). \tag{A.3} \]

Subtracting Equation A.2 from Equation A.3, we have

\[ \epsilon q[E_\eta(V|s) - k(R + \epsilon)] + Rq(k(R) - k(R + \epsilon)) = [E_\eta(\pi_I|s)](k(R + \epsilon)) - [E_\eta(\pi_I|s)](k(R)). \]
Suppose the continuity does not hold. Without loss of generality, we assume \( k(R) - \lim_{\epsilon \to 0} k(R + \epsilon) > a > 0 \), \( a \) being a constant. Then take the limit of both sides of the above equation. For the LHS,

\[
\lim_{\epsilon \to 0} \{\epsilon q[E_{\eta}(V|s) - k(R + \epsilon)] + Rq(k(R) - k(R + \epsilon))\} > Rqa > 0;
\]

for the RHS, because \( E_{\eta}(\pi_I|s) \) is continuous and increases in \( \lambda \),

\[
\lim_{\epsilon \to 0} \left[ E_{\eta}(\pi_I|s) \right] (k(R + \epsilon)) - \left[ E_{\eta}(\pi_I|s) \right] (k(R)) = \left[ E_{\eta}(\pi_I|s) \right] (\lim_{\epsilon \to 0} k(R + \epsilon)) - \left[ E_{\eta}(\pi_I|s) \right] (k(R)) \leq 0.
\]

A contradiction. Hence the continuity must hold.

\iii. Following the last claim, the properties of \((E[\lambda])(R, \eta, \theta)\) are obtained by the definition

\[
(E[\lambda])(R, \eta, \theta) = \int_{s \in S} \lambda(s; R, \eta, \theta) dP(s).
\]

QED.

**Proof of Lemma 3**

By definition, \( \lambda_1(\theta|R) = (E[\lambda])(R, \theta, \theta) \) and \( \lambda_1(\hat{\theta}(R)|R) = (E[\lambda])(R, \hat{\theta}(R), \theta) \). Because \( \theta \geq \hat{\theta}(R) \), by Lemma 2,

\[
\lambda_1(\theta|R) \leq \lambda_1(\hat{\theta}(R)|R).
\]

Then given that \( F_s < C_p(\hat{\theta}^*) \), by definition, for all \( R \in [\frac{C}{E(V)}, 1] \),

\[
Rq \lambda_1(\hat{\theta}(R)|R) + C_p(\hat{\theta}(R)) \geq Rq \lambda_1(\hat{\theta}(R)|R) + C_p(\hat{\theta}^*) \geq Rq \lambda_1(\hat{\theta}|R) + F_s,
\]

which is Condition 4.2.

QED.

**Proof of Proposition 1.**
To ensure the existence of $R$, we use Brouwer’s Fixed Point Theorem. Define function

$$f(R) = \frac{C}{E(V)(1-q) + q\lambda_1(\bar{\theta}|R)}.$$  

Since $R \in \left[\frac{C}{E(V)}, 1\right]$. We want to show that $f(R) \in \left[\frac{C}{E(V)}, 1\right]$ too. Because $f(R)$ is monotone in $\lambda_1(\bar{\theta}|R)$, $\lambda_1(\bar{\theta}|R) \in [0, E(V)]$, and $q \leq 1 - \frac{C}{E(V)}$ as assumed, we have that

$$f(R) \leq \frac{C}{E(V)(1-q) + 0} \leq 1;$$

$$f(R) \geq \frac{C}{E(V)(1-q) + qE(V)} = \frac{C}{E(V)}.$$  

Also by Claim 2 in Lemma 2, $\lambda_1(\bar{\theta}|R)$ is continuous in $R$, so $f(R)$ is also continuous in $R$. Then we can apply the Fixed Point Theorem. By the definition, $\bar{R}$ exists.

The cost of capital and the cost of information in each equilibrium follow by their definitions.

QED.

**Proof of Proposition 2.**

We want to show that it is an equilibrium that small investors request the disclosure of $\bar{r}$.

Suppose small investors commit to request $\bar{r}$. Since full information becomes public, market makers and the professional all know $\bar{r}$. At every $s$, the market makers’ interim expected aggregate profit is

$$E_{\theta}(\pi_M|s) = Rq(E_{\theta}(V|s) - \lambda) + r(E_{\theta}(V|s) - \lambda)$$

$$= [Rq + r](E_{\theta}(V|s) - \lambda)$$

If a market maker sets a price $\lambda < E_{\theta}(V|s)$, then another market maker can take away the deal by a slightly higher price. That is, because of the competition, in the equilibrium $\lambda_e(s) = E_{\theta}(V|s)$. Therefore, by the optimal trade rule in Fact 1, the professional will not trade.
For each small investor, regardless the liquidity shock, the \textit{interim} expected profit is $E_{\theta}(V|s)R - C$. Then the \textit{ex ante} expected profit is $E(V)R - C$. The competition among small investors determines that $R_e = \frac{C}{E(V)}$.

Suppose there is one of small investors who commits not to request the disclosure of $\bar{\tau}$, then it does not matter because under fair disclosure, any information disclosed is public; as long as some other small investors request $\bar{\tau}$, he also knows it. So the ownership share is unchanged. Then the one who deviates does not gain. Neither does it matter how much information that the professional requests to disclose.

Therefore, it is an equilibrium and in it $C_c = C$ and $C_i = 0$.

There may be some other equilibria where small investors only request disclosure of partial information, so that $\eta < \bar{\theta}$. For example, due to the high marginal cost of private acquisition the professional does not profit from privately collecting more information. Then small investors do not worry about the transaction costs charged by market makers. In equilibrium, the professional’s private information is the same as the public information. Since investors’ information is symmetric, the equilibrium share of ownership is still $R_e = \frac{C}{E(V)}$, and $C_c = C$, $C_i = 0$.

QED.

\textbf{Proof of Theorem 5}

Under selective disclosure, when small investors are fully unaware, they do not expect the professional to acquire any private information; then by Proposition 3, $\bar{R} = \frac{C}{E(V)}$. If the professional chooses selective forums, the actual profit of each small investor is

$$\pi_{Us} = \frac{C}{E(V)} \left[ qE[\lambda] \left( \frac{C}{E(V)} q, \bar{\theta}, \bar{\theta} \right) + (1-q)E(V) \right] - C.$$

Under fair disclosure, since the professional does not request information via public forums, $\bar{\eta} = \bar{\theta}$,

$$\pi_{Uf} = \frac{C}{E(V)} \left[ qE[\lambda] \left( \frac{C q}{E(V)}, \bar{\theta}, \bar{\theta} \right) + (1-q)E(V) \right] - C.$$
By Lemma 2, since $\bar{\theta} \leq \tilde{\theta}$,

$$E[\lambda]\left(\frac{C}{E(V)}q, \bar{\theta}, \tilde{\theta}\right) \leq E[\lambda]\left(\frac{C}{E(V)}q, \bar{\theta}, \bar{\theta}\right).$$

Thus $\pi_{Uf} \geq \pi_{Us}$.

QED.

B Trade with Both Buy and Sell Orders

In this section we generalize the original model by allowing both selling and buying in the trading period. We will show that under selective disclosure and with symmetric awareness, there is a similar equilibrium where the professional acquires information from selective forums and small investors’ ownership share reflects the adverse selection. We omit the derivation of other results since following the derivations in the original model, they are relatively easy to see.

B.1 The Model

Now small investors could experience a negative liquidity shock or a positive one, which happen with probability $q$ and $p$ respectively, with $q, p \in (0, 1]$. We assume that small investors sell all their shares at a negative shock and double their shares at a positive shock. The professional can thus mimic them and profit from his private information by selling or buying. Market makers set price $\lambda_q$ for every unit share sold by investors, and price $\lambda_p$ for every unit share bought by investors. Note that $\lambda_q < \lambda_p$. Other notations follow the original model.

Then for a small investor, his \textit{ex ante} profit function is

$$\pi_U = R\left[qE(\lambda_q) - pE(\lambda_p) + (1 - q + p)E(V)\right] - C,$$

where $E(\lambda_i), i \in \{q, p\}$ are the expected bid price and the expected ask price.
The professional’s decision problem is

\[
\max_{r_q, r_p \in [0, N]} E_\theta(\pi_I | s) := \int_{V} r_q(\lambda_q - v)dP(v | \tau^\theta(s)) + \int_{V} r_p(v - \lambda_p)dP(v | \tau^\theta(s))
\]

Define two separated sets \( d(\lambda_q, \theta) \equiv \{ s | E_\theta(V|s) < \lambda_q \} \) and \( d(\lambda_p, \theta) \equiv \{ s | E_\theta(V|s) > \lambda_p \} \).

The new optimal trade rule is that for \( i \in \{ q, p \} \):

\[
r_i(s; \lambda_i, \theta) = \begin{cases} N & \text{if } s \in d(\lambda_i, \theta) \\ 0 & \text{if else.} \end{cases}
\]

Market makers know that the professional uses the optimal trade rule. If a sell order is executed for the professional, conditional on the public information, they expect the professional’s interim expected profit to be

\[
E_\eta(\pi_I q | s) = \int_{s' \in d(\lambda_q, \theta)} N(\lambda_q - E_\theta(V|s))dF(s' | \tau^\eta(s)).
\]

Similarly, if a buy order is executed for the professional, conditional on the public information, market makers expect the professional’s interim expected profit to be

\[
E_\eta(\pi_I p | s) = \int_{s' \in d(\lambda_p, \theta)} N(E_\theta(V|s) - \lambda_p)dF(s' | \tau^\eta(s)).
\]

All the properties states in Lemma 1 can be applied to both cases, additionally \( E_\eta(\pi_I p | s) \) decreases in \( \lambda_p \).

For market makers, when executing a sell order, their aggregate profit is

\[
E_\eta(\pi_M q | s) := Rq[E_\eta(V|s) - \lambda_q] - E_\eta(\pi_I q | s),
\]

when executing a buy order, their aggregate profit is

\[
E_\eta(\pi_M p | s) := Rp[\lambda_p - E_\eta(V|s)] - E_\eta(\pi_I p | s).
\]
Due to the competition among market makers, in the equilibrium, the optimal pricing rule is given by

\[
\lambda_q(s; R, \eta, \theta) = \max \{ \lambda_q \geq 0 | R_q[E_\eta(V|s) - \lambda_q] = E_\eta(\pi_1q|s) \};
\]

\[
\lambda_p(s; R, \eta, \theta) = \min \{ \lambda_p \geq 0 | R_p[\lambda_p - E_\eta(V|s)] = E_\eta(\pi_1p|s) \}.
\] (B.1)

Hence we know that market makers determine the bid and ask prices in a similar way. The results in Lemma 2 can be applied to both. Note that to show the existence of \( \lambda_p(s; R, \eta, \theta) \), we need to assume that the maximum expectation that the professional has, namely \( \bar{v} = \max\{E_\theta(V|s)|\theta \in \Theta, s \in S\} \) is finite, then show that for \( \lambda_p > \bar{v}, f(\lambda_p) \geq 0 \).

There is also a little change for \( \lambda_p \) since it strictly decreases in \( R \) and increases in \( \theta \).

Then we can show that the professional’s information affects his profit from both selling and buying. Without considering the cost, more information is always better. And if \( F_s \) is small, the professional always prefers using selective forums under selective disclosure.

### B.2 The Equilibrium

For small investors, when they determine the ownership share \( R \), they expect it to affect both \( \lambda_i, i \in \{q, p\} \). The difference is that instead of \( \lambda \), the focus is on \( qE[\lambda_q] - pE[\lambda_p] \), which is the price spread adjusted by the probability of liquidity shocks. This expression is continuous in \( R \), then in order to use the fixed point argument, we need that

\[
f(R) := \frac{C}{qE(\lambda_q) - pE(\lambda_p) + (1 - q + p)E(V)} \in \left[ \frac{C}{E(V)}, 1 \right]
\]

Function \( f(R) \) is strictly decreasing in \( qE(\lambda_q) - pE(\lambda_p) \). We can show that if

\[
E(V)(1 - q) + [E(V) - \bar{v}] \frac{p}{1 + p} \geq C, \quad \text{13}
\]

where \( \bar{v} = \max\{E_\theta(V|s)|\theta \in \Theta, s \in S\} \), then the range of the function is within \([\frac{C}{E(V)}, 1]\).

To see this, first, note that since \( E(\lambda_q) \leq E(V) \), \( E(\lambda_p) \geq E(V) \), then when \( E(\lambda_q) = \text{13It implies that both } q \text{ and } p \text{ are small.} \)
\[ E(\lambda_p) = E(V), \] we have that \( \min f(R) = \frac{C}{E(V)} \).

Second, by Definition B.1, \( R_p[\lambda_p - E_\eta(V|s)] = E_\eta(\pi_ip|s), \) so

\[
R_p[E(\lambda_p) - E(V)] = \int_{s \in d(\lambda_p, \theta)} N(E_\theta(V|s) - \lambda_p(s))dP(s)
\leq \int_{s \in d(\lambda_p, \theta)} N(\bar{\nu} - \lambda_p(s))dP(s)
\leq N(\bar{\nu} - E(\lambda_p))
\leq R(\bar{\nu} - E(\lambda_p)).
\]

The first inequality comes from the definition of \( \bar{\nu} \). The second inequality comes from the fact that \( \lambda_p(s) \leq \bar{\nu} \) for all \( s \in S \), otherwise \( R_p[\lambda_p - E_\eta(V|s)] = E_\eta(\pi_ip|s) \) can not hold\(^{14}\). The third inequality comes from the assumption that \( N \leq \frac{C}{E(V)} \).

Therefore it is true that

\[ E_p \leq \frac{\bar{\nu} + pE(V)}{1 + p}. \]

Also not that \( E(\lambda_q) \geq 0 \). We have that

\[
qE(\lambda_q) - pE(\lambda_p) + (1 - q + p)E(V) \geq 0 - p[\frac{\bar{\nu} + pE(V)}{1 + p}] + (1 - q + p)E(V)
= E(V)(1 - q) + [E(V) - \bar{\nu}] \frac{p}{1 + p}
\geq C.
\]

The last inequality comes from the assumption.

Then the Fixed Point Theorem is applied and an equilibrium \( R_e \) exists, namely

\[
R_e = \min \left\{ R \in \left[ \frac{C}{E(V)} \cdot 1 \right] \left| R = \frac{C}{q\lambda_q(\theta|R) - p\lambda_p(\theta|R)) + (1 - q + p)E(V) } \right. \right\},
\]

where \( \lambda_i(\theta|R) = E(\lambda_i), i \in \{q, p\} \) because the professional always chooses selective forums.\(^{14}\) If so, then \( R_p[\lambda_p - E_\eta(V|s)] > 0 \) and \( E_\eta(\pi_ip|s) = 0. \)
References


