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MAO TAKONGMO, Charles Olivier

University of Ottawa

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Charles Olivier MAO TAKONGMO†

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Abstract

One important question in the Keynesian literature is whether we should detrend data when estimating the parameters of a Keynesian model using the moment method. It has been common in the literature to detrend data in the same way the model is detrended. Doing so works relatively well with linear models, in part because in such a case the information that disappears from the data after the detrending process is usually related to the parameters that also disappear from the detrended model. Unfortunately, in heavy non-linear Keynesian models, parameters rarely disappear from detrended models, but information does disappear from the detrended data. Using a simple real business cycle model, we show that both the moment method estimators of parameters and the estimated responses of endogenous variables to a technological shock can be seriously inaccurate when the data used in the estimation process are detrended. Using a dynamic stochastic general equilibrium model and U.S. data, we show that detrending the data before estimating the parameters may result in a seriously misleading response of endogeneous variables to monetary shocks. We suggest building the moment conditions using raw data, irrespective of the trend observed in the data.

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†Assistant Professor, University of Ottawa, 120 University, Social Sciences Building, Room 9005, Ottawa, Ontario, Canada, K1N 6N5; e-mail: cmaotako@uottawa.ca Web page: ht-tps://sites.google.com/site/maotakongmocharles/

1 Introduction

It is common practice to estimate the parameters in a dynamic stochastic equilibrium (DSGE) model using detrended data (see, for example, L. Christiano, Eichenbaum & Rebelo, 2011; Drautzburg & Uhlig, 2011; Del Negro, Schorfheide, Smets & Wouters, 2007; Smets & Wouters, 2007; Ireland, 2004; McGrattan, Rogerson & Wright, 1997; L. J. Christiano & Eichenbaum, 1992). In this paper, we use two simple models to show that removing the trend before estimating the parameters may lead to seriously inaccurate estimators and misleading policy recommendations.

The first simple model is a basic real business cycle (RBC) model that can be solved by hand (and therefore rules out the possibility of computation errors) that we use to assess the effect of the technological shock on the aggregate product. Using that basic RBC model, we identified a huge discrepancy between the true impulse response function and the impulse response function estimated using detrended data.

The second model is a simple DSGE model, used to assess the response of variables to monetary shocks. In our DSGE model, we identified a seriously misleading response of outputs to a monetary shock, attributed to detrending the data before estimating the impulse response functions. This paper suggests estimating Keynesian models using raw data, irrespective of the trend observed in the data.

It is not easy to understand the nature of the trend that is actually driving the data. In practice, researchers usually trust their intuition and choose one filter. However, different filters emphasise different frequencies in the data and different facts about macroeconomic time series (see, Singleton, 1988; L. J. Christiano & Den Haan, 1996; Burnside, 1998; Gorodnichenko & Ng, 2010). As a result, using one filter instead of another may lead to different results.

Focusing on this problem in the context of Keynesian models is interesting. In these models, there is usually not enough information in the data to estimate all parameters of interest (stochastic singularity problem), and removing an inappropriate trend may exacerbate the problem, especially if the filter removes crucial information that is necessary to estimate some important parameters. If parameters that drive the dynamics of the model are affected, this problem may result in a misleading response of variables to shocks.

When dealing with seasonality, for example, Wallis (1974) shows that the effect of seasonal adjustment on the lag relationship of variables disappears when the same filter is applied to each variable. Facing an unknown trend when using the method of moments to estimate the relations between variables, many researchers show that applying the same filter to both the data and the model variables reduces the effect of trend misspecification on the relations between variables (see, for example, Gorodnichenko & Ng, 2010; Burnside, 1998). We argue that doing so cannot solve the problem of identification of some parameters due to detrending in a heavy non-linear Keynesian model. When we detrend the data, some information disappears. In the linear model, by detrending the model in the same way, the parameter related to the information that is no longer present in the detrended data also disappears from the model; thus, all parameters remain identifiable in the model. We ague that in the case of heavy non-linear models (e.g., models that focus on government policies during the Zero Lower Bound on the nominal interest rate (see, L. Christiano et al., 2011; Mao Takongmo, 2017)), some parameters that are no longer identifiable may remain in the model even if the same filter is applied to both data and model. We also argue that the effect of detrending on the dynamic relation between variables in non-linear Keynesian models can be very large as a result of stochastic singularity problems, coupled with identification problems induced by detrending.

We could calibrate the parameters that are known to be unidentified and estimate those remaining. However, in a non-linear setting, it is not easy to know which parameters are not identifiable. Estimating an unidentified parameter with others in the same model may worsen the estimation of all parameters. We argue that, instead of removing the trend, it is a good idea to model it.

Even if variables display a trend, it may be the case that a linear or non-linear combination of some of those variables is stationary at the true value of the parameter, or at least has finite moments. Cogley (2001) was the first to observe that if non-stationary variables enter as a stationary growth rate or great ratio and if the moments are correctly specified, then the Hansen theorem can be applied. The Cogley (2001) suggestion assumes that we should take a stand on the stationarity of the ratio of variables, which is difficult to verify in a finite sample. We also need to have all variables in our model be stationary or written as a stationary ratio of variables. This puts many restrictions on the model. We think that this is one of the reasons why researchers have not taken Cogley (2001)'s suggestion into account. Cogley (2001) did not elaborate further on his suggestion because the aim of his paper was not to test the accuracy of parameter estimation in the possible presence of a trend but rather to test the rational expectations in the case of mis-specified trends. In our paper, we instead focus on the effect of removing the true trend and the effect of a mis-specified trend in a non-linear Keynesian model.

We suggest using the first-order condition of the Keynesian model of interest to build the moment conditions without having to take a stand on the stationarity of any variable. In fact, it is well known that the first-order condition of many DSGE models can be represented as a linear or non-linear relation between variables that are equal to zero or equal to the error terms, which are usually stationary by construction, at the true value of the parameters. Thus, we do not have to worry about knowing the true trend that drives each variable: We can simply take the non-linear relations between variables that are constant or equal to a stationary error to build our moment conditions.

We restrict ourselves to the filter methods most frequently used in the literature: the Hodrick-Prescott (HP) filter, the first difference (FD) filter and the linear filter.

Our first exercise is a Monte Carlo analysis that aims to measure the cost of detrending the data in a simple model. The data-generating process is an RBC model that allows for a deterministic or stochastic trend. Mean square errors (MSEs) are used to estimate the discrepancy between the true and estimated parameters. We show that the MSE is smaller when raw data are used to estimate the parameters of interest, irrespective of the kind of trend displayed by the data-generating process. We also present the implied discrepancies between the true impulse response function and the impulse response function estimated using detrended variables.

In the second exercise, we use U.S. data and a simple medium-scale DSGE model to assess differences in terms of the estimated responses of outputs to a monetary shock that can be attributed to the data detrending process. In our estimated model, we show that when data used in the estimation process are not detrended, the responses of variables to monetary shock are similar to those observed in the literature (e.g., consumption responds positively to a positive monetary shock). However, when data used in the estimation process are detrended, the responses of variables to a monetary shock are no longer consistent with those observed in the literature (e.g., the consumption response to a positive monetary shock is now negative).

The rest of this article is organized as follows: In Section 2, we present a simple description of a solution method for Keynesian models. Section 3 presents the detrending methods used in the paper. Section 4 presents the method of moments. Section 5 presents our Monte Carlo analysis, using an RBC model. Section 6 presents the empirical analysis, and Section 7 concludes the article.

2 DSGE Models and Detrending

2.1 Solving Keynesian Models

In the DSGE context, a system of equilibrium equations is usually derived from each agent that maximises an objective dynamic function, subject to various constraints in the context of uncertainty. This system of equations can be represented by equation (1):

$$E_t[L(y_{t+1}, y_t, y_{t-1}, x_{t+1}, x_t, u_t, u_{t+1}; \theta)]{=}0 \tag{1}$$

where E_t is the expectation operator, conditional on information up to time t; L is a function; θ is a set of parameters; y is the set of variables of interest; x the set of predetermined variables; and u is the vector of shocks. The agents know the value of predetermined variables at time $t-1$ and observe the shock at time t. Their decisions are based on beliefs that relate the variables y_{t+1} to available information.

The aim is usually to forecast the short-run impact of a shock (or a policy) on the variable of interest. The common procedure to assess the short-run effect is to rewrite each variable in deviation from its trend (see Kydland & Prescott, 1982; King, Plosser & Rebelo, 1988; Uhlig, 1995; Smets & Wouters, 2003). The trend represents the long-run macroeconomic dynamic of the variables. The modified model can be written as follows:

$$E_t[f(\hat{y}_{t+1}, \hat{y}_t, \hat{y}_{t-1}, \hat{x}_{t+1}, \hat{x}_t, u_t, u_{t+1}; \gamma)]=0 \tag{2}$$

where $\hat{y} = y - \overline{y}$; $\hat{x} = x - \overline{x}$, $\overline{y}$ and $\overline{x}$ represent the trend of y and x, respectively. The set of parameters, γ, is usually estimated from the data. The function f defines the set of equilibrium equations with detrended variables, $\hat{y}$ is the vector defining the set of detrended variables to predict, $\hat{x}$ is the set of detrended predetermined variables and u is the vector of shocks.

After removing the hypothetical trend in each model variable, the system of equations obtained is rewritten analytically or approximated with a numerical method as an autoregressive representation. Researchers then study how all variables of interest can fluctuate around the trend in response to economic policies or an unpredicted shock. Equations (3 to 4) represent the solution or policy function of our Keynesian model, described in equation (1).

The policy function is a set of relationships between current variables, the predetermined variables, and shocks that satisfy the equation (1) and that define the stochastic equilibrium conditions of our model. Solving for policy function is the same as finding two functions, g and h, such that

$$\hat{y}_t = g(\hat{x}_t; \tau) \tag{3}$$

$$\hat{x}_t = h(\hat{x}_{t-1}, u_t; \tau) \tag{4}$$

where τ is the set of new parameters implied by the transformation. To obtain the functions g and h, we can replace equation (3) and equation (4) in equation (2). This leads to the following equation:

$$F(\hat{x}_t) = E_t[f\ (g(h(\hat{x}_t, u_{t+1}; \tau)), g(\hat{x}_t; \tau), h(\hat{x}_t, u_{t+1}; \tau), \hat{x}_t, u_t, u_{t+1}; \gamma)]=0.$$

One way to solve for g and h is to write the Taylor expansion for g and for h in the chosen order, n, around the steady state and then find the coefficients of the nth-order polynomials considered(see, Collard & Juillard, 2001; Schmitt-Grohé & Uribe, 2004, for more details). Note that F and its derivatives in any order are zero at all points.

It is easy to see that the policy functions g and h are directly affected by the estimated value of related γ. If detrending leads to an inaccurate estimator of γ, the functions g and h will be inaccurate as well and will lead to incorrect responses of endogenous variables to a shock (policy functions).

2.2 Identification Problems and Calibration

2.2.1 The Concept of Identification

Identification problems are usually a situation in which the empirical implications of some model parameters are undetectable or indistinguishable from the implications of other parameters. Formally, let Y represent a random vector in $\mathbb{R}^n$. Let $A \subset \mathbb{R}^m$ represent the space of parameters. For each $\alpha \in A$, let $f(y, \alpha)$ be the density function, which is known for each parameter α. Following Rothenberg (1971), a parameter $\alpha^0 \in A$ is said to be identifiable if there is no other parameter $\alpha \in A$ such that $f(y, \alpha^0) = f(y, \alpha)$ for all $y \in Y$. More formally, $\alpha^0 \in A$ is identifiable if $\forall \alpha \in A$

$$\alpha \neq \alpha^0 \Longrightarrow f(y, \alpha) \neq f(y, \alpha^0), \quad \forall y \in Y \tag{5}$$

A parameter $\alpha^0 \in A$ is said to be locally identifiable if there exists an open neighborhood of α^0 where α^0 is identifiable.

Rothenberg (1971) proves that a general condition for identification of parametric models is that the information matrix must be non-singular at the true value of the parameter. More formally, $\alpha^0 \in A$ is locally identifiable if $I(\alpha^0)$ in equation (6) is non-singular.

$$I(\alpha^0) = \left[r_{ij}(\alpha^0)\right] = E\left[\frac{\partial \log f(y, \alpha^0)}{\partial \alpha_i} \frac{\partial \log f(y, \alpha^0)}{\partial \alpha_j}\right] \tag{6}$$

For a non-likelihood-based approach, the general condition for identifiability is that the Hessian matrix of the objective function has a full rank.

2.2.2 Identification in DSGE and Inaccurate Policy Functions

Computing the Hessian matrix of a Keynesian representation can be very difficult. DSGE models are sometime heavily non-linear; in general, except for some simple versions of RBC models (Kydland & Prescott, 1982), it is not possible to go from the heavily non-linear model in equation (1) to a possibly linear equation (2) without running the risk of losing information. G. D. Hansen (1985) proposes one such simple RBC model with an indivisible labour. Going from equation (2) to equation (3) and (4) is usually done numerically, except for the basic G. D. Hansen (1985)-type models.

Rothenberg (1971) provides an alternative method to determine identification in the parametric method when computing the Hessian matrix may be difficult. The approach is based on the relationship between parameters of interest and the characteristics of the probability distribution. It is related to the question of the uniqueness of the solution of the system of equations (see, Rothenberg, 1971; Iskrev, 2008, 2010). Formally, let the function g represent the link between two parameters γ and θ ($\gamma = g(\theta)$). Suppose that the density of Y depends on the parameter vector θ only through the parameter γ, and assume that γ is globally identifiable. Then a structure θ_0 is locally identifiable if the Jacobian $H = \frac{\partial g}{\partial \theta}$, evaluated at θ_0, has a full column rank.

If the mapping from θ to γ is defined, for example, by implicit function $f(\theta, \gamma) = 0$, then if γ is globally identifiable, θ_0 is locally identifiable if the Jacobian $f_\theta(\theta_0) = \frac{\partial f(\theta_0, \gamma)}{\partial \theta}$ has a full column rank.

In linear models, when it is possible to know that a parameter is not identified and if, additionally, that parameter has an economic interpretation, we could calibrate it based on previous studies. However, in non-linear models, it is difficult to know if a parameter is weak or not identifiable. As pointed out by Lubik & Schorfheide (2004), it is difficult to directly detect identification problems in large DSGE models because the mapping from the vector of structural parameters into the state-space representation that determines the joint probability distribution of Y is highly non-linear and typically can only be evaluated numerically.

If researchers are dealing with parameters that have no clear economic interpretations and therefore are difficult to calibrate and if information regarding those parameters are contained in the trend, some parameters will no longer be identifiable with detrended data. Assuming that we do not really know whether a parameter is identifiable, we may still use its estimated value for economic analysis. As a result, our policy function could be seriously affected.

3 Detrending Methods

3.1 First-order Differences

In this case, we assume that y_t is a random walk with no drift. The trend is the lag value of the series and is not correlated with the cycle, represented by the first difference of the series, which is assumed to be stationary.

$$y_t = y_{t-1} + (y_t - y_{t-1})$$

$$y_t = \eta_t + c_t$$

Thus, the trend is defined as $\eta_t = y_{t-1}$, and the cyclical component is $c_t = y_t - y_{t-1}$

3.2 The Hodrick-Prescott Filter

The stochastic trend is assumed to be smooth over time and independent of the cycle. The HP filter (see, Hodrick & Prescott, 1997; King, Plosser & Rebelo, 1988) is an optimal trend, η_t, obtained by minimising

$$\min_{\{\eta_t\}_{t=1}^{T}} \left[\sum_{t=1}^{T} (y_t - \eta_t)^2 + \lambda \sum_{t=3}^{T} \left((\eta_t - \eta_{t-1}) - (\eta_{t-1} - \eta_{t-2}) \right)^2 \right]$$

$$\hat{c}_t = y_t - \eta_t^{HP}.$$

The expression

$$\sum_{t=1}^{T} (y_t - \eta_t)^2$$

measures the goodness of fit of the trend to the series, and

$$\sum_{t=3}^{T} ((\eta_t - \eta_{t-1}) - (\eta_{t-1} - \eta_{t-2}))^2$$

measures the degree of smoothness of the trend. λ is the parameter that penalises variation in the growth rate of the trend. For example, if $\lambda = 0$, then $\eta_t^{HP} = y_t$ and $c_t = 0$. By increasing λ, the variability of the trend decreases, and the secular component becomes smoother. When λ tends to infinity, the variability of the trend tends to zero and the trend becomes log linear.

3.3 Polynomial Function of Time

Let y_t be the variable of interest. We want to decompose y_t into a trend and a cyclical component. In a polynomial function of time decomposition, the trend and the cycle are assumed to be uncorrelated, and the trend (η_t) of the series can be approximated with a polynomial function of time:

$$y_t = \eta_t + c_t$$

The trend component η_t is the predicted value of a regression, and the cyclical component c_t is the residual (see, Canova, 1998).

4 The Method of Moments

In this paper, we use a particular case of the generalized method of moment (GMM) (see, L. P. Hansen, 1982; Hall, 2005). More precisely, the number of independent moment conditions is exactly equal to the number of parameters to be estimated; there is no instrument in our model; and the weighted matrix is the identity matrix. We do not allow any instruments when building our moment conditions because we do not want results to depend on the instruments used. The number of independent moment conditions is equal to the number of parameters because we also do not want results to depend on the weighted matrix used.

It is important to note that the aim of the first part of this paper is to analyze the discrepancy, resulting from detrending, between the estimators and the true parameters and between the true impulse response functions and the impulse response functions. We do not address the asymptotic properties of those estimators in this paper; in another paper, we will focus entirely on the asymptotic properties of moment estimators when variables used to build the moments conditions are not detrended and the combination of those variables is stationary.

The GMM estimation is based on the population moment and is usually preferred over estimators such as maximum likelihood when the distribution of the data is not fully available. Formally, let Y_t be the vector of random variables; $\theta_0 \subset \mathbb{R}^k$ the vector of the true unknown parameters; and $g(.)$ the vector of functions that will exclusively come from the first-order conditions of our Keynesian models. Let equation (7) represent the population moment condition.

$$E\left[g(Y_t, \theta_0)\right] = 0 \tag{7}$$

For identification to be successful, it is assumed that on the parameter space Θ

$$E\left[g(Y_t, \theta)\right] \neq 0 \;\; \forall \theta \in \Theta \;\; \theta \neq \theta_0 \tag{8}$$

If conditions (7 and 8) hold, then θ_0 is said to be identified.

4.1 The Moment Estimator

4.1.1 Definition

The data are a finite number of realizations of the process $\{Y_t\}_{t \geq 1}$. The moment estimators are constructed using the sample analogue of the population moment conditions. Formally, let $\frac{1}{T}\sum_{t=1}^{T} g(Y_t, \theta)$ be the sample analogue of the population moment, $E\left[g(Y_t, \theta)\right]$. The moment estimator $\hat{\theta}_T$ is the solution to the following problem:

$$\hat{\theta}_T = \arg\min_{\theta \in \Theta} \left(\frac{1}{T}\sum_{t=1}^{T} g(Y_t, \theta)\right)' W_T \left(\frac{1}{T}\sum_{t=1}^{T} g(Y_t, \theta)\right) \tag{9}$$

where W_T is a positive-definite matrix(L. P. Hansen, 1982).

In this paper, because the number of independent moment conditions is equal to the number of parameters, the estimator will not depend on W_T (see, L. P. Hansen, 1982). W_T is therefore chosen to be the identity matrix.

We can build moment conditions using a combination of raw variables that are stationary at the true value of the parameter.

5 Monte Carlo Analysis

5.1 The Data-Generating Process

Our framework is a simple version of the RBC model that can be solved by hand, as proposed by G. D. Hansen (1985). We choose a model that can be solved by hand to avoid computational errors. We modify the G. D. Hansen (1985) model to allow for stochastic and deterministic trends. The model assumes that the planner selects the set of consumption, c_t, and capital, k_{t+1}, to maximize $E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t)$ subject to the resource constraint, $c_t + k_{t+1} = k_t^{\alpha} z_t$. The production technology is $y_t = k_t^{\alpha} z_t$, where $z_t = \exp(\zeta \times t) \times \exp(e_t)$, $e_t = \rho e_{t-1} + u_t$ and $|\rho| \leq 1$.

The optimal policy functions are

$$c_t = (1 - \alpha\beta) z_t k_t^{\alpha} \tag{10}$$

$$k_{t+1} = \alpha\beta z_t k_t^{\alpha} \tag{11}$$

$$y_t = z_t k_t^{\alpha} \tag{12}$$

$$z_t = \exp(\zeta \times t) \times \exp(e_t) \tag{13}$$

$$e_t = \rho e_{t-1} + u_t \tag{14}$$

z_t is the level of technology, and u_t is an innovation in technology, $u_t \sim N(0,1)$. The model has a deterministic trend when $\zeta > 0$, and a stochastic trend when $\rho = 1$. The unobserved variables are z_t and e_t. The unknown parameters are the physical capital share in the production function α; the discount factor β; and the parameters ζ and ρ.

5.2 Monte Carlo Technique

The vectors c, y, and k are simulated 1000 times. Each vector contains $T = 100$ observations. The simulated data are used to estimate the moment estimators of α, β, ζ, and ρ. Mean square errors are used to estimate the discrepancy between the true and estimated parameters. We consider three different cases.

Case 1: No Trend on the Data-Generating Process In this case, $\zeta = 0$, $\rho = 0.5$, $\beta = 0.95$, $\alpha = 0.33$, $k_0 = 1$, $e_0 = 0$ and $u_t \sim N(0,1)$.

Case 2: Deterministic Trend on the Data-Generating Process In this case, $\zeta = 0.0488$, $\rho = 0$, $\beta = 0.95$, $\alpha = 0.33$, $k_0 = 1$, $e_0 = 0$ and $u_t \sim N(0,1)$.

Case 3: Stochastic Trend on the Data-Generating Process In this case, $\rho = 1$, $\zeta = 0$, $\beta = 0.95$, $\alpha = 0.33$, $k_0 = 1$, $e_0 = 0$ and $u_t \sim N(0, 1)$.

To build our moment conditions, we use stationary relations between variables. For example, the moment condition in equation (15) is built using equation (10), and the moment condition in equation (16) is built using equation (11). Because we do not observe z and e, the moment conditions in equations (17) and (18) are built using a combination of equations (12), (13) and (14) and using the information about the mean and variance of the shock u_t.

5.3 Moment Conditions

$$E\left(D\left[\log(c_t)\right] - \log(1-\alpha\beta) - D\left[\log(y_t)\right]\right) = 0 \tag{15}$$

$$E\left(D\left[\log(k_{t+1})\right] - \log(\alpha\beta) - D\left[\log(y_t)\right]\right) = 0 \tag{16}$$

$$E\left(\left[D\left[\log(y_t)\right] - \alpha D\left[\log(k_t)\right] - \zeta t\right] - \rho\left[D\left[\log(y_{t-1})\right] - \alpha D\left[\log(k_{t-1})\right] - \zeta(t-1)\right]\right) = 0 \tag{17}$$

$$\left\{Var\left(\left[D\left[\log(y_t)\right] - \alpha D\left[\log(k_t)\right] - \zeta t\right] - \rho\left[D\left[\log(y_{t-1})\right] - \alpha D\left[\log(k_{t-1})\right] - \zeta(t-1)\right]\right) - 1\right\} = 0 \tag{18}$$

The operator D is defined such that $D[x] = x$ in the case that the moment conditions are built with raw data; $D[x]$ is the HP filter cyclical component for x in the case that the moment conditions are constructed with HP-filtered data; $D[x]$ is the first difference of data x in the case that moment conditions are constructed with data in first differences; and $D[x]$ is the cyclical component for the regression on the polynomial function of time in the case that the moment conditions are constructed using the polynomial function of time. Results for each case are presented in Tables 1, 2 and 3.

5.4 Comments on the Results

Based on MSE[1], our results show that it is always better to estimate parameters using moment conditions built with raw data, irrespective of whether the data-generating process exhibits a stochastic trend, a deterministic trend or no trend. Filtering the data loses part of the information that would have been useful in estimating the discount factor, β, and the production function parameter, α. Those parameters play a role in the dynamic of the model.

[1] $MSE(\hat{\theta}|\theta) \equiv \mathbb{E}\left((\hat{\theta}-\theta)^2\right) = Bias(\hat{\theta})^2 + Var(\hat{\theta})$.

Table 1: Monte Carlo results without deterministic trend on the model (Case 1)

Parameters	β	α	ζ	ρ
True values (case 1)	0.95	0.33	0	0.5
Estimation with raw data				
mean	0.9590	0.3269	-0.0002	0.5695
std	0.0134	0.0046	0.0040	0.0134
MSE	0.0002	0.0000	0.0000	0.0050
Estimation with data in difference				
mean	1.0047	0.4976	0.0000	0.4400
std	0.0096	0.0048	0.0005	0.0140
MSE	0.0030	0.0281	0.0000	0.0037
Estimation with HP-filtered data				
mean	1.0072	0.4956	0.0000	0.4382
std	0.0060	0.0031	0.0004	0.0124
MSE	0.0033	0.0274	0.0000	0.0039
Estimation using data detrended with the polynomial function of time				
mean	1.0080	0.4948	0.0000	0.4373
std	0.0058	0.0031	0.0005	0.0119
MSE	0.0034	0.0271	0.0000	0.0040

Notes: This table shows the estimated parameters, the standard deviation and mean square error of those estimators when they are estimated respectively using raw data and detrended data (i.e., using the difference method, the HP-filtered method and the polynomial function of time). The true data generating process here does not allow for any trend ($\zeta = 0$ and $\rho < 1$; Case 1). The true values of the parameters are also displayed in the first rows for comparison. We obtain better results when we use raw data to estimate our parameters.

5.5 Impulse Response Function

5.5.1 The Solution of Our Simple Model

Our model can be solved by hand. The solution of our simple problem (represented by equations 10 to 14) for $\rho = 1$ and $\zeta \geq 0$ is as follows:

$$\triangle e_t = u_t \equiv E_t$$

$$Z_t = \exp\left(\zeta + E_t\right)$$

$$K_{t+1} = Z_t \left(K_t\right)^{\alpha}$$

$$C_t = Y_t = Z_t \left(K_t\right)^{\alpha}$$

where $Z_t \equiv \frac{z_t}{z_{t-1}}$; $K_t \equiv \frac{k_t}{k_{t-1}}$;$C_t \equiv \frac{c_t}{c_{t-1}}$; and $Y_t \equiv \frac{y_t}{y_{t-1}}$.

5.5.2 Discrepancy of Impulse Response Function due to Detrending (Case 3).

Figure 1 represents the response of the gross growth rate (y_t/y_{t-1}) to the productivity shock, estimated using raw and detrended data. Figure 1 also plots the true response to the shock, for

Table 2: Monte Carlo results with deterministic trend on the model (Case 2)

Parameters	β	α	ζ	ρ
True values (case 2)	0.95	0.33	0.0488	0
Estimation with raw data				
mean	0.9215	0.3406	0.0512	0.0768
std	0.0353	0.0130	0.0016	0.0124
MSE	0.0020	0.0002	0.0000	0.0060
Estimation with data in difference				
mean	1.0022	0.5049	0.0007	0.0903
std	0.1112	0.0543	0.0002	0.0457
MSE	0.0151	0.0335	0.0023	0.0102
Estimation with HP-filtered data				
mean	1.1389	0.4385	0.0000	0.0520
std	0.0282	0.0102	0.0002	0.0061
MSE	0.0364	0.0118	0.0023	0.0027
Estimation using data detrended with the polynomial function of time				
mean	1.1747	0.4235	-0.0007	0.0613
std	0.0503	0.0183	0.0002	0.0047
MSE	0.0530	0.0090	0.0024	0.0037

Notes: This table shows the estimated parameters, the standard deviation and mean square error of those estimators when they are estimated respectively using raw data and detrended data (i.e., using the difference method, the HP-filtered data method and the polynomial function of time). The true data generating process here does display a deterministic trend ($\zeta > 0$, Case 2). The true values of the parameters are displayed in the first rows for comparison. We obtain better results when we use raw data to estimate our parameters.

comparison. The results show that the impulse response function estimated using raw data is the same as the true impulse response function. However, there is a serious discrepancy between the true impulse response function and the function estimated using detrended data. For example, at the beginning of the period, the true response of growth rate$\left(\frac{y_t - y_{t-1}}{y_{t-1}} \times 100\right)$ to the shock[2] is 32.27 % . When the data are detrended, the estimated response of the growth rate to the shock increases to 51.13 % (see figure 1). Moreover, the estimated response to the shock lasts longer. Figures 2 and 3 confirm the large discrepancy in level.

Our first finding is that moment estimators of the parameters of a Keynesian model can be very inaccurate when the moment conditions are built using detrended data. In this analysis, for example, the production parameter α and the discount factor β that play a role in the dynamic of the model are both weakly identified due to detrending. Thus, we cannot expect the impulse response function to be accurate when using detrended data to estimate those parameters. We suggest building the moment conditions using raw data, irrespective of the trend observed in the data.

[2]The true gross growth rate (y_t/y_{t-1}) is 1.3227.

Table 3: Monte Carlo results with stochastic trend on the model (Case 3)

Parameters	β	α	ζ	ρ
True values (case 3)	0.95	0.33	0	1
Estimation with raw data				
mean	0.9720	0.3227	0.0172	1.0155
std	0.0291	0.0093	0.0504	0.0246
MSE	0.0013	0.0001	0.0028	0.0008
Estimation with data in difference				
mean	0.9783	0.5113	-0.0200	1.0053
std	0.0202	0.0104	0.0248	0.0240
MSE	0.0012	0.0329	0.0010	0.0006
Estimation with HP-filtered data				
mean	0.9817	0.5088	-0.0213	1.0053
std	0.0238	0.0122	0.0236	0.0219
MSE	0.0015	0.0321	0.0010	0.0005
Estimation using data detrended with the polynomial function of time				
mean	0.9762	0.5111	-0.0210	0.9986
std	0.0301	0.0175	0.0250	0.0306
MSE	0.0015	0.0331	0.0010	0.0009

Notes: This table shows the estimated parameters, the standard deviation and mean square error of those estimators when they are estimated respectively using raw data and detrended data (i.e., using the difference method, the HP-filtered data method and the polynomial function of time). The true data generating process here displays a stochastic trend ($\rho = 1$, Case 3). The true values of the parameters are also displayed in the first rows for comparison. We obtain better results when we use raw data to estimate our parameters.

6 Empirical Analysis

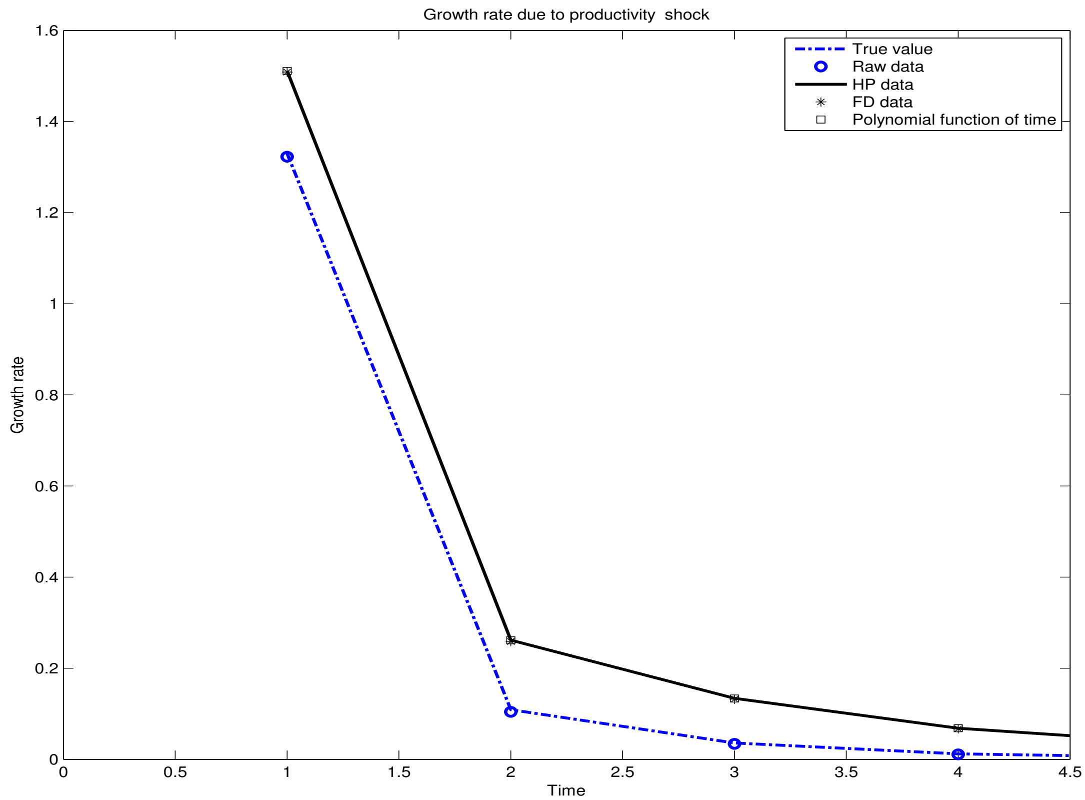
This empirical example illustrates how our suggestion to use raw data instead of detrended data can be implemented in the more completed DSGE model. In this section we also illustrate how detrending may lead to misleading responses of variables to monetary shocks. Our framework is a version of the Amano, Ambler & Rebei (2007) model, in which we assess the short-run effect of monetary supply shocks. We present the estimated parameters and the response of variables to shocks in two cases: In the first case, the parameters are estimated using raw data; and in the second case, the parameters are estimated using detrended data.

In this economy, there is a representative household with a lifetime utility function. In a good market, there is monopolistic competition in the sense of Dixit & Stiglitz (1977), prices are sticky in the sense of Calvo (1983), labour markets are perfectly competitive and there is no capital accumulation (see, Amano, Ambler & Rebei, 2007; Mao Takongmo, 2017, for more details).

6.1 The Households

The representative household derives utility from leisure, consumption and real money balances. The representative household can save by buying one-period nominal bonds or directly by saving

Figure 1 : Growth rate of the gross domestic product response to a technological shock



Notes: This figure plots the impulse response of the gross growth rate of the gross domestic product (y_t/y_{t-1}), to the productivity shock. The blue dashed dotted line shows the true impulse response. The line with "o" markers shows the impulse response function estimated with raw data. The solid black line is the impulse response function estimated using the HP-filtered data. The line with "*" markers is the impulse response function using data in first difference. The line with "□" markers is the impulse response function using data detrended with the polynomial function of time. As shown in the figure, the impulse response function using raw data is similar to the true impulse function. However, there is a huge discrepancy between the impulse response functions using detrended data and the true impulse response function.

money. The household solves the following dynamic problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\gamma}{\gamma-1} \ln \left[C_t^{\frac{\gamma-1}{\gamma}} + b_t \left(\frac{M_t}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right] + \eta \ln(1-h_t) \right)$$

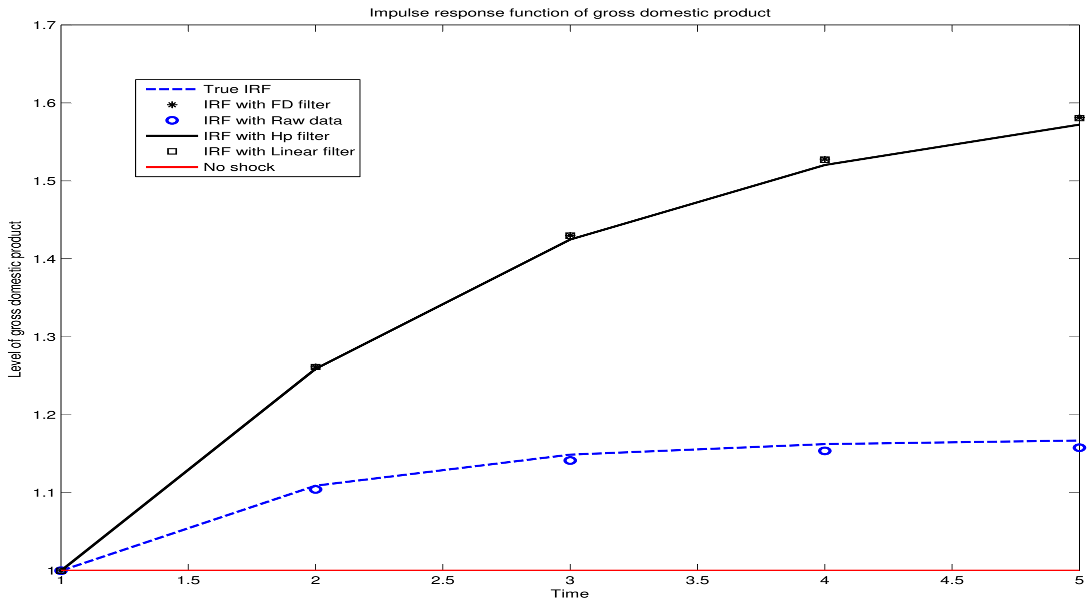
subject[3] to a dynamic budget constraint

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t R_t} \leq \frac{W_t}{P_t} h_t + \frac{M_{t-1}}{\pi_t P_{t-1}} + \frac{B_{t-1}}{\pi_t P_{t-1}} + \frac{T_t}{P_t} + \frac{D_t}{P_t} \tag{19}$$

plus a solvency constraint $\lim_{l\to\infty} E_t \left\{ \beta^l \left(\frac{\wedge_{t+l}}{\wedge_t} \right) \frac{B_{t+l}+M_{t+l}}{P_{t+l}} \right\} \geq 0$, where E_0 is the expectation operator conditional on the information at time 0, C_t the consumption at the end of period t and M_t is the net amount of currency held by the agent at the end of the period t. P_t is the price index

[3]In nominal terms, the constraint is defined as $P_t C_t + M_t + \frac{B_t}{R_t} = W_t h_t + M_{t-1} + B_{t-1} + T_t + D_t$

FIGURE 2 : Gross domestic product response to a technological shock-case 3



Notes: This figure plots the impulse response of the gross domestic product y to the productivity shock. The blue dashed dotted line shows the true impulse response. The line with "o" markers shows the impulse response function estimated with raw data. The solid black line is the impulse response function estimated using the HP-filtered data. The line with "*" markers is the impulse response function using data in first difference. The line with "□" markers is the impulse response function using data detrended with the polynomial function of time. As shown in the figure, the impulse response function using raw data is similar to the true impulse function. There is a huge discrepancy between the impulse response functions using detrended data and the true impulse response function.

at time t, and h_t denotes the number of hours worked. The parameter $\beta \in (0, 1)$ is a subjective discount factor; $\gamma > 0$ is the elasticity of substitution between consumption and real balances; the parameter $\eta > 0$ is the weight on leisure; and b_t is a money demand shock. The shock b_t follows an autoregressive process[4]. W_t is the nominal wage, B_t is the quantity of one-period nominal bonds, R_t is the short- term gross interest rate on a nominal bond, D_t is the nominal dividend received from firms and T_t is the money transfer from the central bank. The domestic price index is denoted by P_t.

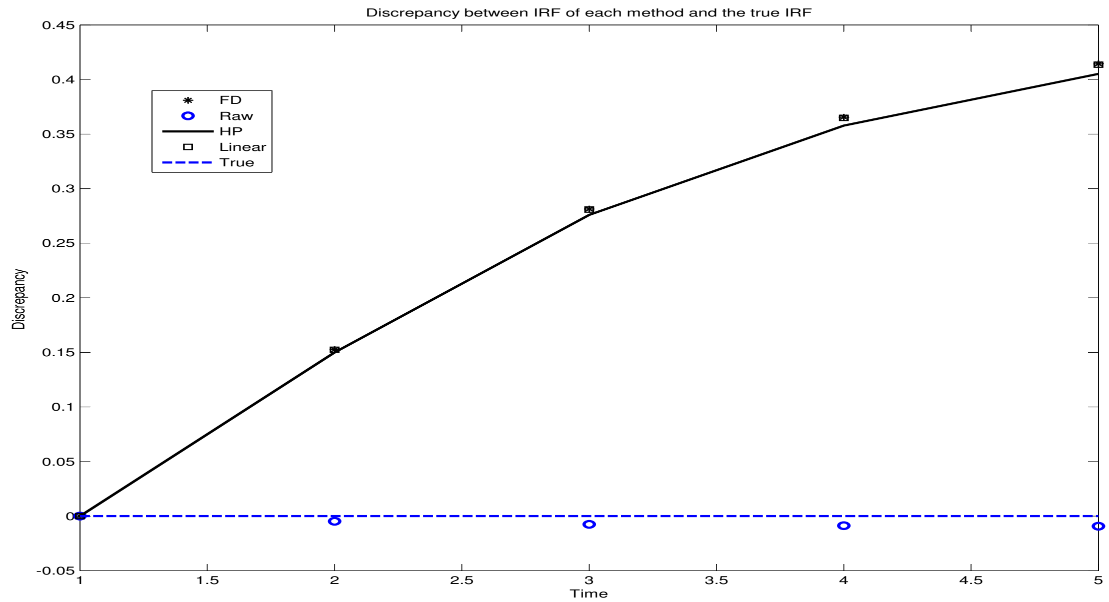
Let $\wedge$ denote the Lagrange multiplier for the household problem. The household's first-order conditions[5] with respect to consumption, money purchases of one-period bond and hours worked, respectively, are written as

$$\frac{b_t C_t^{\frac{-1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + b_t \left(\frac{M_t}{P_t}\right)^{\frac{\gamma-1}{\gamma}}} = \wedge_t \tag{20}$$

[4] $\log(b_t) = (1 - \rho_b)\log(b) + \rho_b \log(b_{t-1}) + \epsilon_{bt}$

[5] It is consistent to divide the two sides of the budget constraint by domestic price index $P_t(i)$ when writing the Lagrangian equation.

Figure 3 : Disparity between the true and estimated responses of the gross domestic product to the technological shock-case 3



Notes: This figure plots the disparity between the true and estimated responses of the gross domestic product to a technological shock (i.e., the difference between any estimated impulse response to a technological shock and the true response to the same technological shock). The blue dashed dotted line shows the true impulse response, normalized at zero. The line with "o" markers shows the impulse response function estimated with raw data, compared to the true impulse response function. The solid black line is the impulse response function estimated using the HP-filtered data, compared to the true impulse response function. The line with "*" markers is the impulse response function using data in first difference, compared to the true impulse response function. The line with "□" markers is the impulse response function using data detrended with the polynomial function of time, compared to the true impulse response function.

$$\frac{b_t\left(\frac{M_t}{P_t}\right)^{\frac{-1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}}+b_t\left(\frac{M_t}{P_t}\right)^{\frac{\gamma-1}{\gamma}}}=\wedge_t-\beta E_t\left[\frac{1}{\pi_{t+1}}\wedge_{t+1}\right] \tag{21}$$

$$\frac{\wedge_t}{R_t}=\beta E_t\left[\frac{1}{\pi_{t+1}}\wedge_{t+1}\right] \tag{22}$$

$$\frac{\eta}{1-h_t}=\frac{W_t}{P_t}\wedge_t \tag{23}$$

6.2 The Final Good

There is perfect competition in the final good market. This market can be represented by a representative firm that uses intermediate goods to produce a final good. The representative firm maximises its profit by choosing the quantity of each intermediate good, subject to the production function. The problem of the representative firm is as follows:

$$\max_{\{Y_t(i)\}} P_tY_t - \int_0^1 P_t(i)Y_t(i)di$$

subject to the following production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}} \tag{24}$$

The parameter $\theta > 1$, in the production function, represents the elasticity of substitution between two differentiated intermediate goods. The variable $Y_t(i)$ represents the quantity of an intermediate good i. The first-order conditions provide the demand function for the intermediate good

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$$

and the price index for the final good

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta}di\right)^{\frac{1}{1-\theta}}.$$

6.3 Intermediate Goods

There is Dixit & Stiglitz (1977) monopolistic competition in the intermediate goods market. A continuum of firms, each with low market power, produces a continuum of differentiated intermediate goods. Intermediate firms set prices in a Calvo (1983) framework. Following a Calvo (1983) price setting, a fraction $(1-d)$ of firms reset their prices, whereas others keep their prices unchanged. The price index in the period t can be written as

$$P_t = \left[d(P_{t-1})^{1-\theta} + (1-d)(\tilde{P}_t)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{25}$$

where $\tilde{P}_t$ is the price index set by firms that can readjust their prices in period t.

6.3.1 Optimal Price Setting

The variable $\wedge$ denotes the Lagrange multiplier for the household problem. It can also represent the marginal utility of wealth.

A firm producing the intermediate good indexed by i solves the following problem:

$$\max_{\left\{\tilde{P}_t(i),h_t(i)\right\}} E_t\left[\sum_{l=0}^{\infty}(d\beta)^l\left(\frac{\wedge_{t+l}}{\wedge_t}\right)\frac{\left(\tilde{P}_t(i)Y_{t+l}(i) - W_{t+l}h_{t+l}(i)\right)}{P_{t+l}}\right],$$

subject to the following intermediate-firm production function:

$$Y_{t+l}(i) = Ah_{t+l}(i)^{1-\alpha}$$

and the demand

$$Y_{t+l}(i) = \left(\frac{\tilde{P}_t(i)}{P_{t+l}}\right)^{-\theta} Y_{t+l}. \tag{26}$$

where $\tilde{P}_t(i)$ is the price set by firms i when it is possible to readjust the prices in period t, A is the common technology and W_t is the nominal wage rate. Letting $\xi_{t+l}(i)$ denote the Lagrange multiplier associated with intermediate production function constraints, the first-order conditions with respect to $h_{t+l}(i)$ and $\tilde{P}_t(i)$ are given respectively by

$$\frac{W_t}{P_t} = \xi_t(i)(1-\alpha)\frac{Y_t(i)}{h_t(i)} \tag{27}$$

$$\tilde{P}_t(i) = \left(\frac{\theta}{\theta-1}\right)\frac{E_t \sum_{l=0}^{\infty}(d\beta)^l \left(\frac{\wedge_{t+l}}{\wedge_t}\right)\xi_{t+l}(i)Y_{t+l}(i)}{E_t \sum_{l=0}^{\infty}(d\beta)^l \left(\frac{\wedge_{t+l}}{\wedge_t}\right)\left(\frac{Y_{t+l}(i)}{P_{t+l}}\right)} \tag{28}$$

6.4 Aggregation

All intermediate firms that can reset their prices face the same marginal cost and solve the same problem; thus, in equilibrium, their optimal prices are the same ($\tilde{P}_t(i) = \tilde{P}_t$). Integrating the two sides of the demand equation yields

$$Y_t^s = \int_0^1 Y_t(i)di = Y_t^d \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di = Y_t^d \left[d\left(\frac{P_{t-1}}{P_t}\right)^{-\theta} + (1-d)\left(\frac{\widetilde{P}_t}{P_t}\right)^{-\theta}\right] \tag{29}$$

where Y_t^s stands for aggregated supply production and Y_t^d for aggregated demand. The aggregated demand is expressed by

$$Y_t^d = C_t. \tag{30}$$

Aggregation can also be expressed in terms of aggregated labour supply as

$$Y_t^s = \int_0^1 Y_t(i)di = Y_t^s = \int_0^1 A_t h_t^{1-\alpha}(i)di = A_t \left[\int_0^1 h_t^{1-\alpha}(i)di\right]^{\frac{1-\alpha}{1-\alpha}}$$

that is

$$Y_t^s = A_t h_t^{1-\alpha} \tag{31}$$

where

$$h_t = \left[\int_0^1 h_t^{1-\alpha}(i)di \right]^{\frac{1}{1-\alpha}}. \tag{32}$$

Our variables will be normalised as follows:

$m_t = M_t/P_t$, $\tilde{p}_t = \tilde{P}_t/P_t$, $\pi_t = P_t/P_{t-1}$, $w_t = W_t/P_t$, $tr_t = T_t/P_t$

6.5 The Monetary Policy

We assume that the dynamics of money supply follow an autoregressive process:

$$\log(\mu_t) = (1 - \rho_\mu)\log(\mu) + \rho_\mu \log(\mu_{t-1}) + \epsilon_{\mu t}$$

where $\epsilon_{\mu t}$ is an i.i.d shock specific to the money supply, which we assume to have zero mean and variance σ_μ. The growth rate of the money supply at time t is $\mu_t = M_t/M_{t-1}$, and μ is the growth rate of the money supply in the steady state.

The money transfer from the central bank to the household is given by

$$M_{t+1} - M_t = T_t$$

6.6 The Summary of Our General DSGE Model

The following equations summarize the equilibrium dynamic of the model. The variable C_t is consumption, M_t the net amount of currency, P_t is the price index, h_t is the number of hours worked, W_t is the nominal wage, R_t is the short-term gross interest rate on a one-period nominal bond and T_t is the lump-sum transfer. ξ_t is the Lagrange multiplier associated with the production function constraint, $\wedge_t$ is the marginal utility of wealth, Y_t^s is the aggregated supply and Y_t^d represents aggregate demand. The variable $\epsilon_{\mu t}$ is an i.i.d shock specific to money supply with zero mean and variance σ_μ. Variables with no subscript represent steady state values. Equations (33) and (35) lead to the Euler equation; (34) is the money demand; (36) is the labour supply; (37) is the price set by firms that can reset their prices; (38) is the dynamic of price in the economy; (39) is the labour demand; (40) is the dynamic of the money supply; and equation (41) describes the discrepancy between the demand and supply of goods in the short run.

Household

$$\frac{b_t C_t^{\frac{-1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + b_t \left(\frac{M_t}{P_t}\right)^{\frac{\gamma-1}{\gamma}}} = \wedge_t \tag{33}$$

$$\frac{b_t\left(\frac{M_t}{P_t}\right)^{\frac{-1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}}+b_t\left(\frac{M_t}{P_t}\right)^{\frac{\gamma-1}{\gamma}}}=\wedge_t-\beta E_t\left[\frac{1}{\pi_{t+1}}\wedge_{t+1}\right] \tag{34}$$

$$\frac{\wedge_t}{R_t}=\beta E_t\left[\frac{1}{\pi_{t+1}}\wedge_{t+1}\right] \tag{35}$$

$$\frac{\eta}{1-h_t}=\frac{W_t}{P_t}\wedge_t \tag{36}$$

Price dynamic and firms

$$\tilde{P}_t=\left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{l=0}^{\infty}(d\beta)^l\left(\frac{\wedge_{t+l}}{\wedge_t}\right)\xi_{t+l}Y_{t+l}}{E_t\sum_{l=0}^{\infty}(d\beta)^l\left(\frac{\wedge_{t+l}}{\wedge_t}\right)\left(\frac{Y_{t+l}}{P_{t+l}}\right)} \tag{37}$$

$$P_t=\left[d(P_{t-1})^{1-\theta}+(1-d)(\tilde{P}_t)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{38}$$

$$h_t\frac{W_t}{P_t}=\xi_t(1-\alpha)Y_t \tag{39}$$

Monetary authority

$$\log(\mu_t)=(1-\rho_\mu)\log(\mu)+\rho_\mu\log(\mu_{t-1})+\epsilon_{\mu t} \tag{40}$$

$$\mu_t=M_t/M_{t-1}$$

$$M_{t+t}-M_t=T_t$$

Aggregation

$$Y_t^s=Y_t^d\left[d\left(\frac{P_{t-1}}{P_t}\right)^{-\theta}+(1-d)\left(\frac{\widetilde{P}_t}{P_t}\right)^{-\theta}\right] \tag{41}$$

$Y_t^d=C_t$ and $Y_t^s=A_th_t^{1-\alpha}$

6.7 Estimation

6.7.1 Moment Conditions

Because of the well-known problem of stochastic singularity, all parameters cannot be identified at the same time. It is common in the DSGE literature to calibrate some parameters and estimate others (see, for example L. Christiano, Eichenbaum & Rebelo, 2011). The set of parameters we choose to estimate includes $\Theta = \{\gamma, \mu, \rho_\mu, \sigma_\mu\}$. For the moment conditions that will help us to estimate γ, we divide equation (34) by equation (33) and combine the result with equation (35) to obtain

$$\left(\frac{M_t}{P_tC_t}\right)^{-\frac{1}{\gamma}} = 1 - \frac{1}{R_t}. \tag{42}$$

We can apply the logarithm function to equation (42) to build the following moment condition:

$$E\left[-\frac{1}{\gamma}\log\left(\frac{M_t}{P_tC_t}\right) - \log\left(1-\frac{1}{R_t}\right)\right] = 0 \tag{43}$$

The sample moment of the value in brackets obviously exists. Our moment conditions can be completed by those related to μ, ρ_μ and σ_μ as follows:

$$E\left[\log(\mu_{t+1}) - \log(\mu_t) - \log(\mu)\right] = 0 \tag{44}$$

$$E\left[(\log(\mu_{t+1}) - \log(\mu)) - \rho_\mu\left(\log(\mu_t) - \log(\mu)\right)\right] = 0 \tag{45}$$

$$\left\{Var\left[(\log(\mu_{t+1}) - \log(\mu)) - \rho_\mu\left(\log(\mu_t) - \log(\mu)\right)\right] - \sigma_\mu^2\right\} = 0 \tag{46}$$

6.7.2 Moment Conditions Using First Difference Data and HP-Filtered Data

The variables that can be detrended are M, P, C, and W. By applying the first difference filter to equation (42), we have the following moment:

$$E\left[-\frac{1}{\gamma}\left(\triangle\log\left(\frac{M_t}{P_tC_t}\right)\right) - \triangle\log\left(1-\frac{1}{R_t}\right)\right] = 0 \tag{47}$$

whereas for the HP filter, the moment becomes

$$E\left[-\frac{1}{\gamma}\left(hp\log\left(\frac{M_t}{P_tC_t}\right)\right) - hp\log\left(1-\frac{1}{R_t}\right)\right] = 0 \tag{48}$$

where $hp(x)$ stands for the cyclical component of the HP filter of the variable x. Other moments remain the same. First, by removing the trend in this way, it is assumed that there is still a trend on the composite variable $\left(\frac{M_t}{P_tC_t}\right)$, and if this is not the case, we are removing a trend that does not exist. In other words, we are removing more information from the data that may weaken the identification of our parameters.

6.8 Data

We use the United States quarterly economic data from 1977:I to 2015:I. C_t represents personal consumption expenditures, h_t represents the employment rate, P_t is the GDP deflator, M_t is $M1$, W_t represents the wage and salary compensation of employees and R_t is the gross federal funds rate. The series comes from economic data from the Federal Reserve Bank of St. Louis.

6.9 Calibrated and Estimated Parameters

The calibrated set of parameters is $\Omega = \{d, \alpha, \eta, \theta, \beta\}$. The probability of no price adjustment, d, is set at 0.75. This means that on average, the price remains fixed for four quarters. The labour share in the production function $(1-\alpha)$ is set at the standard value of 0.667. The weight on leisure, η, is set at 2.67. The elasticity of substitution between the intermediate good, θ, is calibrated at 2.95; and the discount factor β is set at 0.95.

A summary of calibration of parameters is presented in Table (4) and the estimated parameters in Table (5)

Table 4: Model Calibration

Parameter	Value	Description
d	0.75	Probability of no price adjustment (Amano, Ambler & Rebei, 2007)
		(firms adjust price after three quarters)
$1-\alpha$	0.667	Labour share in the production function
η	2.67	Leisure weight
θ	2.95	Elasticity of substitution between the intermediate good
β	0.95	Discount factor

Notes: This table presents the values of the parameters used in the quantitative analysis. The descriptions of those parameters are presented in the right column.

Table 5: Estimated parameters

Parameters	γ	μ	ρ_μ	σ^2_μ
Stationary combination of raw data				
	0.0666***	1.0006***	0.8185***	0.0002***
	(0.0194)	(0.0000)	(0.0107)	(0.0000)
Data in difference				
	0.3929	1.0001***	0.8176	0.0003
	(0.2922)	(0.0013)	(65.186)	(0.0265)
HP-filtered data				
	0.0963	1.0000***	1.0162	0.0001
	(14249000)	(0.0000)	(1751.1)	(0.3169)

Notes: This table reports the moment estimators of the parameters of our model using raw data, the data in difference, the HP-filtered data and data detrended using the polynomial function of time. Standard errors are in parentheses.
***: significant at 1% ; **: significant at 5% ; *: significant at 10% .

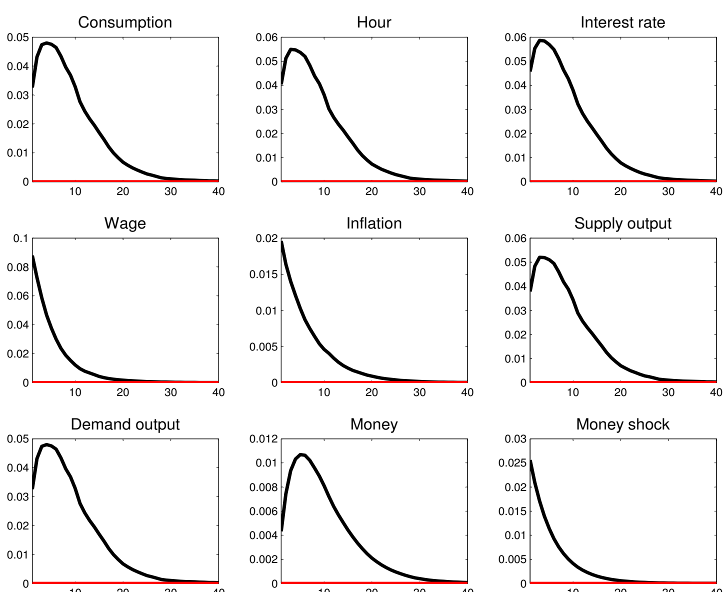
6.10 Impulse Response Function

We use the perturbation method implemented in Dynare for impulse response functions (see, Collard & Juillard, 2001; Schmitt-Grohé & Uribe, 2004; Judd, 1998, for more details). We present the response of variables to a monetary shock, estimated respectively using raw data (Figure 4), data in first difference (Figure 5) and data detrended with the HP filter (Figure 6).

The results obtained when using raw data to estimate the response are similar to those reported in the literature. For example, because of price rigidity, the consumption, output and wages react positively to a money supply shock (Figure 4). In contrast, when we use detrended data to estimate the response of variables, the results obtained become inconsistent with what is common in the literature. For example, when using data in first difference for estimation (Figure 5) or data filtered using the HP filter (Figure 6), consumption reacts negatively to monetary expansion.

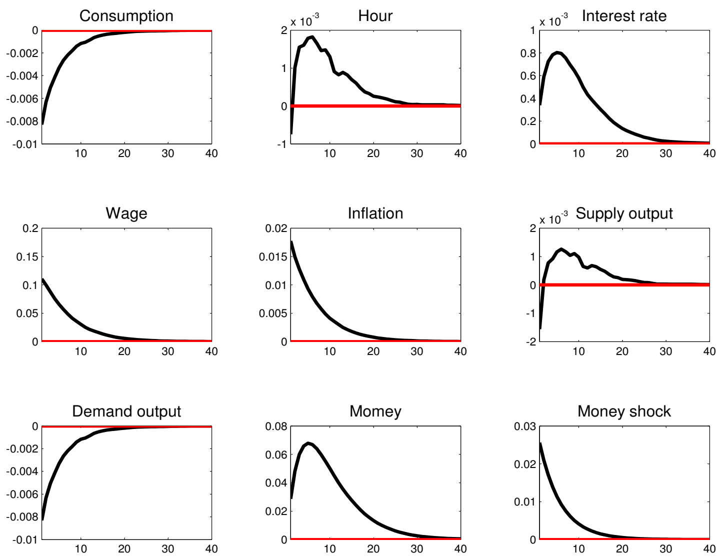
The misleading estimated responses when using data filtered by the first difference filter (Figure 5) may be due to the inaccuracy of the estimated value of γ, (see Table 5), which measures the elasticity of substitution between consumption and the real money balance. The misleading response in the case in which data are detrended using the HP filter may come from the inaccuracy of the estimated value of γ, coupled with the inaccuracy of the estimated value of ρ_{μ}, which represents the persistency of the monetary shock.

FIGURE 4 : Impulse response function using raw data



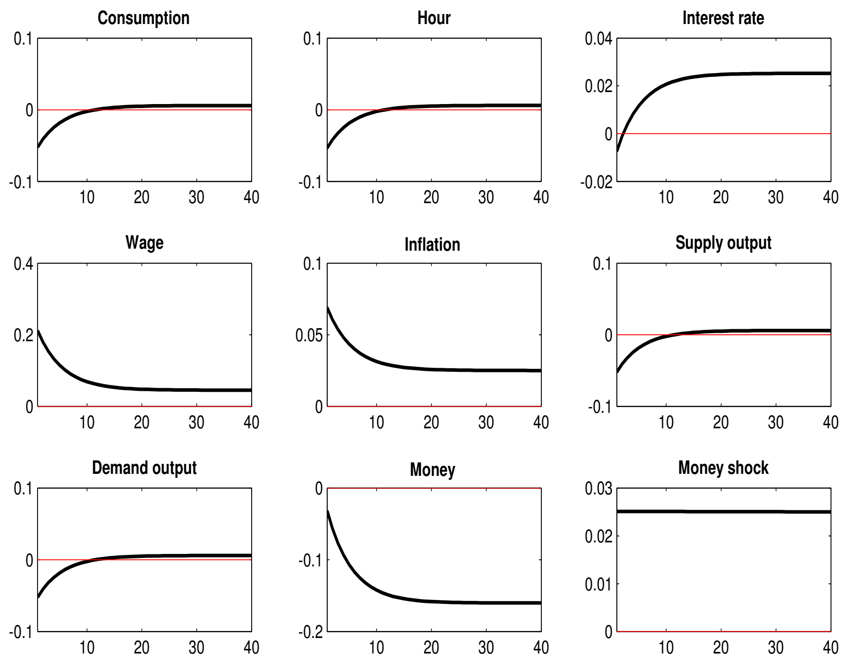
Notes: This figure plots the impulse response of consumption, hours, interest rate, wages and inflation to a monetary policy shock, estimated with raw data. The responses are similar to those in the literature. For example, consumption responds positively to the positive monetary shock.

FIGURE 5 : Impulse response function using first-difference data



Notes: This figure plots the impulse response of consumption, hours, interest rate, wages and inflation to a monetary policy shock, estimated with data in first difference. The responses are no longer consistent with those in the literature. For example, consumption now responds negatively to the monetary shock. Assume that the researcher did not know a priori the response of the policy to consumption (e.g., when a new framework is added in the model) or the researcher is testing a new policy with no a priori result. Detrending the data before estimating the parameters may lead to a seriously misleading policy recommendation.

Figure 6 : Impulse response function using HP-filtered data



Notes: This figure plots the impulse response of consumption, hours, interest rate, wages and inflation to a monetary policy shock, estimated with HP-filtered data. Once again, the responses are not consistent with those in the literature. Consumption responds negatively to the monetary shock. Detrending the data before estimating the parameters may lead to misleading policy recommendation.

It is obvious that the researcher will question the results obtained if the implication of the model goes against fundamental economic theory, especially if new economic conditions have not been added in the model. However, researchers are usually more interested in results that go against existing theory, especially when a new context is added in an existing model (e.g., L. Christiano, Eichenbaum & Rebelo (2011) shows that the government spending multiplier can be very large (close to four) when the zero lower bound context is added in the DSGE model). If a researcher adds to a model a new economic condition that makes him or her believe that monetary expansion will lead to a reduction in consumption and output and if that prediction is exactly what the researcher expects, the researcher may reach a seriously misleading conclusion, even if the main explanation for the result is the fact that data are detrended.

7 Conclusion

In this paper, we show that moment estimators for Keynesian model parameters may not be accurate when moment conditions are built using detrended data. We show that using detrend data when estimating Keynesian models can also create serious discrepancies between the true impulse response functions and the estimated impulse response function. We suggest building the moment conditions using raw data, irrespective of the trend observed in the data. The empirical analysis proposed in this paper also highlights the fact that detrending may lead to a seriously misleading response of variables to monetary supply shocks.

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