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Abstract: We examine a model of Bertrand competition with non-rigid capacity constraints, so that by incurring an additional cost, firms can produce beyond capacity. We find that there is an interval of prices such that a price can be sustained as a pure strategy Nash equilibrium if and only if it lies in this interval. We then examine the properties of this set as (a) the number of firms becomes large and (b) the capacity cost increases.

Key words: Bertrand competition, capacity constraint.

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1 Introduction

We consider price competition between capacity constrained firms. Capacity constraints are not rigid though, and, by incurring an additional per unit cost of capacity expansion, firms can produce beyond capacity. As pointed out by Boccard and Wauthy (2000, 2004), allowing for such non-rigid capacity costs is important and, in their generalization of Kreps and Scheinkman (1983), they explicitly follow this approach.[1] Further, we focus on Bertrand competition, i.e. price competition where firms supply all demand. This framework can be traced back to Chamberlin (1933) and is appropriate when the costs of turning away customers are high (see, for example, Vives (1999)).[2]

We find that there is an interval of prices such that any price in this interval can be sustained as a symmetric Nash equilibrium. Moreover, no other price can be sustained as an equilibrium. We then examine some comparative statics properties of this interval. Even in the limit as the number of firms tend to infinity, the set of equilibrium prices turn out to be a non-degenerate interval. While the competitive price is the lowest price in this set, it contains other prices also. Further, as the cost of capacity expansion increases, the maximal price that is sustainable in equilibrium increases.

Turning to the literature, this paper complements Boccard and Wauthy (2000, 2004), who also examine non-rigid capacity costs, though for the case where firms are free to supply less than the quantity demanded. Another interesting class of models examines capacity constrained firms with rigid capacity where, given prices, firms are willing to supply till capacity. Papers in this framework include, among others, Allen and Hellwig (1986), Dasgupta and Maskin (1986), and Vives (1986). The present paper clearly traces its ancestry to both these streams of the literature.

[1] Other papers to allow for non-rigid capacity constraints include Dixit (1980).

[2] The literature on Bertrand competition is a relatively recent one. It includes, among others, Dastidar (1995), Maggi (1996), Novshek and Roy Chowdhury (2003), Ray Chaudhuri (1996), Roy Chowdhury and Sengupta (2004), etc.

2 The Model

The market comprises n (≥ 2) identical firms, all producing a single homogeneous good, and having the same cost function, $C(q)$. The firms are capacity contsrained with a (non-rigid) capacity level of k. Thus the cost function

$$C(q) = \begin{cases} cq, & \text{if } 0 \leq q \leq k, \\ ck + c'(q-k), & \text{if } q > k, \end{cases}$$

where $c' > c > 0$ and $(c' - c)$ represents the per unit cost of capacity expansion. Interestingly the cost function is somewhat non-standard in that it is not only discontinuous at $q = k > 0$, but is convexo-concave (i.e. can be approximated by convexo-concave functions).

The market demand function $f(p)$ satisfies the following assumption.

Assumption 1. (i) $f : [0, \infty) \to [0, \infty)$ is twice differentiable. Moreover, $f(p)$ is negatively sloped and weakly concave $\forall p$ such that $f(p) > 0$.

(ii) Also, $\lim_{p\to\infty} f(p) < c$ and $f^{-1}(0) > c'$.

For ease of exposition, we assume that $\frac{f(c')}{n} < k$.[3]

Let $D_i(p_1, \cdots, p_i, \cdots, p_n)$ denote the demand facing firm i as a function of the announced price vector $(p_1, \cdots, p_i, \cdots, p_n)$ so that

$$D_i(p_1, \cdots, p_i, \cdots, p_n) = \begin{cases} 0, & \text{if } p_i > p_j, \text{ for some } j, \\ \frac{f(p_i)}{m}, & \text{if } p_i \leq p_j, \ \forall j, \text{ and } \#(l : p_l = p_i) = m. \end{cases}$$

Thus the profit of the i-th firm

$$\pi_i(p_1, \cdots, p_n) = (p_i - AC(D_i(p_1, \cdots, p_n)))D_i(p_1, \cdots, p_n).$$

We examine a game of Bertrand competition where the firms simultaneously announce their prices and supply all demand. We solve for pure strategy Nash equilibria in prices.

[3] As we argue later though, the analysis goes through qualitatively even if $\frac{f(c')}{n} \geq k$.

3 The Analysis

We begin by characterizing the set of Nash equilibria. Let us introduce some notations.

Definition. Let $\pi(p)$ denote the profit of any firm that undercuts all other firms by charging p, where $c \leq p \leq c'$. Formally,

$$\pi(p) = f(p)p - ck - c'[f(p) - k]. \tag{1}$$

Lemma 1 below summarizes some properties of $\pi(p)$. The proofs of all the lemmas are in the Appendix.

Lemma 1.
(i) $\pi(p)$ *is increasing in* p, $\forall p \in [c, c']$.
(ii) $\pi(c) = (k - f(c))(c' - c) < 0$ *and* $\pi(c') = k(c' - c) > 0$.

We require another

Definition. Let $\pi(p, n)$ denote the profit of any firm if all firms charge the same price p, where $c \leq p \leq c'$. Formally,

$$\pi(p,n) = \begin{cases} \frac{f(p)}{n}(p - c), & \text{if } \frac{f(p)}{n} \leq k, \\ \frac{pf(p)}{n} - ck - c'[\frac{f(p)}{n} - k], & \text{if } \frac{f(p)}{n} > k. \end{cases} \tag{2}$$

In case $\frac{f(c)}{n} > k$, let $\underline{p}$ solve $\frac{f(p)}{n} = k$ ($\underline{p}$ is clearly well defined). We then discuss some properties of $\pi(p, n)$.

Lemma 2.
(i) $\frac{\partial \pi(p,n)}{\partial p}|_{p=c} > 0$.
(ii) $\pi(p, n)$ *has a unique maximum and is increasing (respectively decreasing) to the left (respectively right) of this maximal price.*
(iii) $\pi(c', n) = \frac{f(c')}{n}(c' - c) > 0$.
(iv) $\pi(c,n) = \begin{cases} 0, & \textit{if } \frac{f(c)}{n} \leq k, \\ (k - \frac{f(c)}{n})(c' - c) < 0, & \textit{if } \frac{f(c)}{n} > k. \end{cases}$

Given Lemmas 1 and 2 we have the following

Lemma 3.
(i) $\pi(c') > \pi(c', n)$.
(ii) $\pi(c, n) > \pi(c)$.
(iii) $\frac{\partial \pi(p,n)}{\partial p} < \frac{\partial \pi(p)}{\partial p}$, $\forall p \in [c, c']$.

Given the preceding lemmas we have the following definitions.

Definition. Let p'' be the unique p that satisfies $\pi(p, n) = \pi'(p)$.[4]

Further, from Lemmas 3(i) and 3(ii), we have that $c < p'' < c'$.

Definition. For $\frac{f(c)}{n} > k$, let $\tilde{p}$ define the minimum p such that $\pi(p, n) = 0$.[5]

Definition. Finally, let

$$p' = \begin{cases} c, & \text{if } \frac{f(c)}{n} \leq k, \\ \tilde{p}, & \text{otherwise.} \end{cases}$$

The next lemma follows from the preceding discussion.

Lemma 4.
(i) $\tilde{p}$ *solves* $\frac{pf(p)}{n} - ck - c'[\frac{f(p)}{n} - k] = 0$.
(ii) $\tilde{p}(n)$ *is decreasing in* n.
(iii) $p''(n)$ *is decreasing in* n.
(iv) For all $p \in [c, c']$, $\pi(p, n) \geq \pi(p)$ *if and only if* $p \in [c, p'']$.

We are now in a position to characterize the set of equilibrium prices. Proposition 1 below demonstrates that there is an interval such that any price in this interval can be sustained as a symmetric Nash equilibrium.

Proposition 1. *Let Assumption 1 hold. Any price* $p \in [p', p'']$ *can be sustained as a symmetric Nash equilibrium. No other price can be sustained as an equilibrium.*

Proof: (a) We first demonstrate that any price $p \in [p', p'']$ can be sus-

[4] That such a p'' exists follows from Lemma 3(i), 3(ii) and the intermediate velue theorem. Uniqueness follows from Lemma 3(iii).

[5] From Lemma 2(iii) and 2(iv), $\tilde{p}$ is well defined.

tained as a symmetric equilibrium where all firms charge the price p.

We argue that no firm can deviate by charging a higher price compared to p, and gain. From the definition of p', all firms make zero profits at p'. We then argue that $\pi(p,n) \geq 0$ for all $p \in [c, p'']$. Suppose to the contrary, there exists some $p \in (c, p'']$ for which $\pi(p,n) < 0$. Given Lemma 2(ii), this implies that $\pi(p'', n) < 0$. This in turn implies that $\pi(c', n) < 0$, which contradicts Lemma 2(iii). Whereas any firm that deviates and charges a higher price has no demand, and thus obtains a zero profit.

We next argue that any firm that undercuts and charges $p - \epsilon$, $\epsilon > 0$, has a lower profit. This follows since

$$\pi(p - \epsilon) < \pi(p) \leq \pi(p, n),$$

where the first inequality follows as $\pi(p)$ is increasing in p (Lemma 1(i)), and the second inequality follows from Lemma 4(iv).

(b) We then argue that no other price can be sustained as an equilibrium.

We first demonstrate that no price $p < p'$ can be sustained. If $p' = c$ this is obvious. So let $p' > c$ and suppose to the contrary that a price less than p' can be sustained. But then the number of firms charging this price must be less than n (otherwise they have a loss). But this implies that $\tilde{p}$ is increasing in n, a contradiction to Lemma 4(i).

Finally, consider some price $c' \geq p > p''$. In case all n firms charge this price, one of the firms can undercut by a small enough amount and gain, since at $p > p''$ we have that $\pi(p) > \pi(p, n)$ (Lemma 4(iv)). Whereas if the number of firms charging this price is strictly less than n, then one of the firms who charges a higher price, can undercut by a small enough amount and get a strictly positive profit. This follows from the fact that since $\pi(p'') = \pi(p'', n) > 0$ and $\pi(p)$ is increasing, $\pi(p)$ is strictly positive for all $c' \geq p \geq p''$. Finally, if $p > c'$, then any firm that undercuts makes a gain. ■

We then examine some interesting comparative statics properties. We first examine the limit properties of the equilibrium set, i.e. $[p'(n), p''(n)]$, as n goes to infinity. We need one final assumption.

Assumption 2. There exists a choke-off price $\hat{p}$, such that $f(p) = 0$ $\forall p \geq \hat{p}$ and $f(p) > 0$ $\forall p < \hat{p}$.

Interestingly, even for n arbitrarily large, the limit equilibrium set turns out to be a non-degenerate interval. Interpreting c as the competitive price, we find that the competitive price is only one member of the limit equilibrium set. This is interesting since in models with rigid capacity, Allen and Hellwig (1986) and Vives (1986), both find that for n large, the equilibrium prices converge, at least in distribution, to the competitive price.

Proposition 2. *Let Assumptions 1 and 2 hold. Then* $\lim_{n\to\infty}[p'(n), p''(n)] = [c, \overline{p}]$, *where* $c' > \overline{p} > c$.

Proof. (i) Note that for n large $\frac{f(c)}{n} \leq k$. Consequently, $p''(n)$ solves

$$\frac{f(p)}{n}(p-c) = pf(p) - ck - c'(f(p) - k).$$

Given Assumptions 1 and 2, $f(p)(p-c)$ is bounded above by $f(0)(\hat{p} - c)$. Therefore as n becomes large, the LHS goes to zero. Thus in the limit we must have

$$f(\overline{p})(c' - \overline{p}) = k(c' - c).$$

Hence if $\overline{p} = c$, then from the preceding equation, $f(c) = k$, which is a contradiction. Finally note that if $\overline{p} > c'$, then for p close to $\overline{p}$ the firms can profitably undercut.

(ii) For n large, $\frac{f(c)}{n} < k$, so that from Lemmas 2 and 3, $\pi(c, n) > \pi(c)$, and $\pi(c, n) = 0$. Thus $p' = c$. ■

We finally examine the effect of an increase in the capacity costs, i.e. c', on the equilibrium outcome. Interestingly, as the capacity costs increase, higher prices can be sustained in equilibrium.

Proposition 3. *Let Assumption 1 hold. If capacity costs c' increases then*
(a) p'' increases, and
(b) p' increases if $\frac{f(c)}{n} > k$ and remains unaffected otherwise.

Proof. From equation (1), $\pi(p)$ is decreasing in c'. Whereas $\pi(p,n)$ is decreasing in c' if $\frac{f(p)}{n} > k$. Otherwise, $\pi(p,n)$ does not depend on c'. Thus, with an increase in c', p'' increases. If $\frac{f(c)}{n} > k$, then $p' = c$, otherwise $p' = \tilde{p}$. The result now follows from Lemma 4(i). ∎

Remark. How critical is the assumption that $\frac{f(c')}{n} < k$? In case $\frac{f(c')}{n} \geq k$, it is easy to see that Proposition 1 goes through with the modification that p'' is replaced by c'. Thus the only result that is qualitatively affected is that the maximal price becomes independent of the number of firms as well as the demand parameters, though still dependent on the costs of capacity expansion.

4 Conclusion

We examine a model of Bertrand competition with non-rigid capacity constraints, so that by incurring an additional cost, firms can produce beyond capacity. We find that there is an interval of prices such that a price can be sustained as a pure strategy Nash equilibrium if and only if it lies in this interval. We then examine the properties of this set as (a) the number of firms becomes large and (b) the capacity cost increases. For n large, the limit equilibrium set is a non-degenerate interval, with the competitive price being the lowest price in this interval. Further, as the capacity cost increases we find that the maximum possible price that can be sustained as an equilibrium increases.

5 Appendix

Proof of Lemma 1. (i) $\frac{\partial \pi(p)}{\partial p} = f'(p)(p - c') + f(p) > 0$ (since $p \leq c'$).

(ii) Follows from (1). ■

Proof of Lemma 2. (i) $\frac{\partial \pi(p,n)}{\partial p}|_{p=c}$ equals $\frac{f(c)}{n} > 0$ if $\frac{f(c)}{n} \leq k$ and equals $\frac{f'(c)}{n}(c - c') + \frac{f(c)}{n}$ otherwise.

(ii) For $\frac{f(c)}{n} \leq k$, we have that $\frac{f(p)}{n} \leq k \ \forall p \geq c$. Thus

$$\pi''(p,n) = 2f'(p) + f''(p)(p - c) \leq 0.$$

Next consider $\frac{f(c)}{n} > k$. First note that for $p \in [c, \underline{p}]$, $\pi'(p,n) = \frac{1}{n}[f(p) + f'(p)(p - c')] > 0$, so that $\pi(p,n)$ is increasing over the whole interval $[c, \underline{p}]$. From the first part of the argument, $\pi(p,n)$ is concave over $[\underline{p}, c']$. Finally, note that the left hand derivative of $\pi(p,n)$ at $\underline{p}$, i.e. $\frac{1}{n}[f(\underline{p}) + f'(\underline{p})(\underline{p} - c')]$, is greater than the right hand derivative at $\underline{p}$, i.e. $\frac{1}{n}[f(\underline{p}) + f'(\underline{p})(\underline{p} - c)]$. Hence the claim follows.

(iii) and (iv) Follows from equation (2). ■

Proof of Lemma 3. (i) Note that $\pi(c') = k(c' - c)$. Further, since $\frac{f(c')}{n} < k$, $\pi(c', n) = \frac{f(c')}{n}(c' - c)$. Thus $\pi(c') > \pi(c', n)$.

(ii) Note that $\pi(c) = (c' - c)(k - f(c)) < 0$. Moreover, if $\frac{f(c)}{n} \leq k$, then $\pi(c, n) = 0 > \pi(c)$. Whereas, if $\frac{f(c)}{n} > k$, then $\pi(c, n) = (c' - c)[k - \frac{f(c)}{n}] > \pi(c)$.

(iii) Follows straightaway from euqations (1) and (2). ■

Proof of Lemma 4. (i) Suppose $\frac{f(c)}{n} > k$. The result follows since $\pi(p,n)$ is increasing over $[c, \underline{p}]$ and

$$\pi(\underline{p}, n) = \frac{f(\underline{p})}{n}(\underline{p} - c) > 0.$$

(ii) Follows since $\tilde{p}$ solves

$$k(c' - c) = \frac{f(p)}{n}(c' - p).$$

(iii) Since $\frac{f(p'')}{n} < k$, $\pi(p'', n) = \frac{f(p'')}{n}(p'' - c)$.

(iv) Follows from Lemma 1, 2 and 3. ■

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