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АНАЛИТИЧЕСКИЕ МЕТОДЫ ПРОЕКТИРОВАНИЯ ТЕХНОЛОГИЧЕСКИХ ТРАЕКТОРИЙ ПРЕДМЕТОВ ТРУДА В ФАЗОВОМ ПРОСТРАНСТВЕ СОСТОЯНИЙ

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АНАЛІТИЧНІ МЕТОДИ ПРОЕКТУВАННЯ ТЕХНОЛОГІЧНИХ ТРАЄКТОРІЇ ПРЕДМЕТІВ ПРАЦІ У ФАЗОВОМУ ПРОСТОРІ СТАНІВ

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ANALYTICAL METHODS OF THE DESIGN OF TECHNOLOGICAL TRAJECTORIES OF THE OBJECT OF LABOR IN A PHASE SPASE OF STATES

Цель. Разработка аналитических методов проектирования технологических траекторий движения предметов труда в пространстве состояний с целью построения замкнутых PDE-моделей, применяемых для описания производственных систем.

Методика. Для вывода уравнения движения предмета труда в фазовом пространстве состояний использован математический аппарат и методы аналитической механики, вариационного исчисления.

Результаты. Получено уравнение движения предмета труда в пространстве состояний и рассмотрены интегралы движения, связанные с однородностью времени и пространства состояний.

Научная новизна. Научная новизна полученных результатов заключается в совершенствовании PDE-моделей производственных систем, используемых для проектирования высокоэффективных систем управления производством. Предложена модель переноса технологических ресурсов на предмет труда, основанная не на традиционном феноменологическом описании стационарных производственных явлений, а на законах сохранения, характеризующих процесс переноса технологических ресурсов на предмет труда и пространственно-временной структуре производственного процесса. Это позволило получить уравнения движения предметов труда по технологическому маршруту с последующим построением на их основе нестационарных уравнений PDE моделей для описания состояния параметров производственного процесса. При выводе уравнения технологической траектории движения предмета труда учтены дифференциальные связи, накладываемые производственной системой на процесс переноса технологических ресурсов на предметы труда в результате взаимодействия их с технологическим оборудованием и между собой при переходе от одной технологической операции к другой.

Практическая значимость. Заключается в том, что методы построения уравнения технологической траектории предмета труда позволяют разработать высокоточные модели переходных процессов производственной системы, которые являются основой для проектирования высокоэффективных систем управления предприятием с поточным методом организации производства

Ключевые слова: предмет труда, технологический процесс, технологическая траектория, PDE-модель

Articulation of the issue. The process of the technology engineering of the product manufacturing presents a search for the technological paths for the parameters of the subjects of labour, defining the process of its production in accordance with construction and technological documentation [1]. A path, containing the points with values of ever changing specified parameters [1] of the subjects of labour in accordance with the specified manufacturing technology is a regulatory technological path of the products manufacturing. (Fig.1), [2]. The technological paths, defining the changes of the parameters of the subjects of labour, are valid (outside the rejection area), if the divergence of parameters falls within the limits of manufacturing tolerance. The selection of the regulatory technological path, which corresponds to the prescribed manufacturing method, if defined both by technico-economic factors of production, characterizing a manufacturing cost, a manufacturing cycle and a manufacturing capacity, and the social factors of production. Each operation is characterized by the equipment, requiring the personnel qualification, consumption criteria and a law concerning the transfer of the resources to the subject of labour. The requirements to the parameters of the subject of labour are defined by the phase space field, where there is carried out the technological transformation of the basic material to the finished product. (Fig.1), [2,3]. The alteration of technical and economic parameters of the production requires the transfer from one production method to another with its own regulatory technological path $S_0(t)$ .In the neighborhood of the regulatory path there are located $S_j(t)$ paths of the production of j- subject of labour ($0 \le j \le N$), being processed at the point of time $t = t^*$ with the intensity of $\mu_j(t)$ (Fig.1). While projecting the high performance control systems, an important attention is paid to the model of the manufacturing process. Severization of the requirements to the quality of manufacturing systems models initiated the development of a new type of model within the last decade, which obtained the following foreign name PDE–models. One of the difficulties of using this type of equipment is the construction of a closed equation system of the production process. As one of the problem-solution methods related to the system of equation closing there is used a constitutive equation of the manufacturing process. However, this approach, having established itself in the course of the construction of the quasistatic models of the manufacturing systems, does not allow describing with the satisfactory accuracy the transitional manufacturing processes. This aspect has defined the topicality of the investigation, which distinguishes the designing method of the technological paths, the equation of which may be used for the provision of the PDE–models closing [1-3].

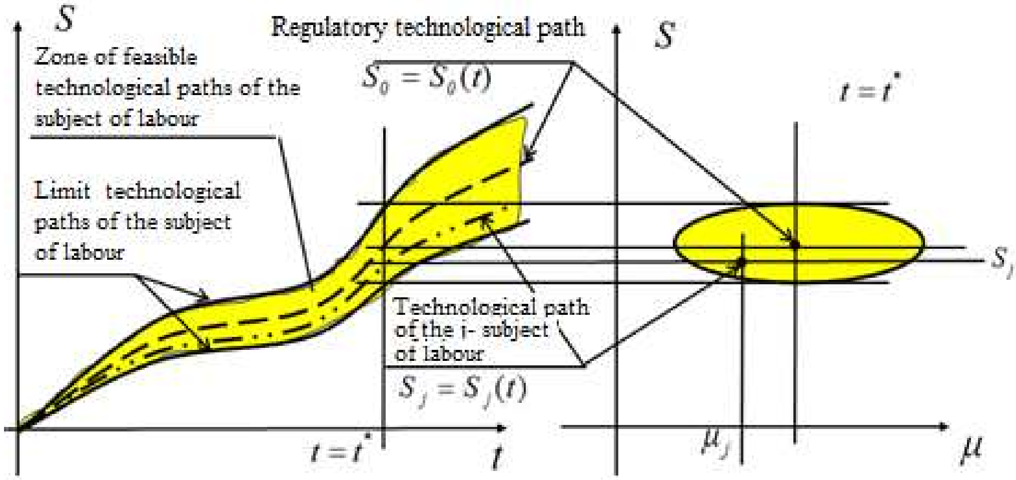


Fig.1. A family of technological paths. Cost S *(UAH) of transferred resources to the subject of labour in case of intensive processing* μ *(UAH/hour) depending on the total processing time* t *(hour)*

The analysis of the recent investigations and definition of the previously unsolved part of general issue related to the construction of the technological paths. Let us consider the geometric locus in technological phase space, the position of which is determined by coordinate value q_j of consecutive states of the subject of labour due to technological transformational change [1-4]. Let us consider that in the analyzed n-dimension technological phase space there has been defined the metric, a square of the length element dG^2 of which represents the following formula

$$dG^2 = \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha,\beta}(q_\alpha, q_\beta) \cdot dq_\alpha \cdot dq_\beta ,$$

where $a_{\alpha,\beta}(q_\alpha, q_\beta)$ - the coordinate functions of technological space.

In order to analyze the manufacturing system state there is used a cost performance, which allows taking into consideration the technical and economic indicators. Herewith it is necessary to dispose a morphological structure of technological manufacturing process [5,p.27]. In order to construct a system of cost performance in general terms the morphological structure is presented as a network graph. In such a case, a cost of the finished product consists of the costs of the resources that were used: components, raw materials, materials, and labour [5,p.75], which gives an opportunity to use for the square of the length element dG^2 of the phase space a square of the resources cost transferred to the subject of labour dS^2 [2,4], which vividly characterize the changes of the cost performance of the subject of labour in the course of processing , (Fig.1), [5,p.75], [6]. There exist the models where there are used the notions of the stage of incompleteness of the product manufacture x in the capacity of variable, defining the state of the subject of labour (Armbruster D., Ringhofer C., Berg V., Lefeber E.), [7-9], $x \in [0,1]$. Such an approach is applicable to motion specification of the subject of labour in one-dimension state space, which is hard to be realized when describing the product manufacturing process, in the course of which there are used several resources, the transfer of each of which to the subject of labour is characterized by its own parameters. Alongside with the use of independent characteristics for the determination of the state of the subject of labour, V.K. Fedyukin [3,p.27]offered to use a term of the product value, which as well as a cost increases in the course of transfer from one operation to another due to the processing of the subject of labour (Fig.1). The resources being transferred to the subject of labour are summed up together with the resources that had been transferred before while performing the previous operations. Such an operation is vividly expressed through the composition of the cost of the transferred resources [5, p.81], and is expressed by means of the following formula in the accounting enterprise balance.

The formula $dS^2 = \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha,\beta}(q_\alpha, q_\beta)\, dq_\alpha\, dq_\beta$ is the result of the natural cost increase of the subject of labour in the course of processing by means of transferring to it the α-type resources in the capacity of dq_α ($\alpha = 1..n$). The transfer process view of the several resources of variable, defining the stage of the production incompleteness of the subject of labour depending on the amount of resources transferred to it q_α, is complicated and difficult to understand. Taking into account the fact that in the course of production there are, usually used, several kinds of technological resources, in this article there is analyzed the technological path engineering of the subject of labour in the phase space with metric, the source of the length element dS^2 which is defined by means of the following formula [4]:

$$dS^2 = \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha,\beta}(q_\alpha, q_\beta)\cdot dq_\alpha \cdot dq_\beta, [UAH^2] \quad (1)$$

If ,while constructing the technological path, to take the amount of materials and raw materials Θ_{CuM} [kg], electrical energy $\Theta_{Э}$ [kW] and the total effective time of processing τ_m [time], transferred to the subject of labour as the coordinates of technological space, then the state of the subject of labour at the point of time t is defined by the following parameter values $q_1 = \Theta_{CиM}(t)$, $q_2 = \Theta_{Э}(t)$, $q_3 = \tau_m(t)$,with coordinate functions, which under ordinary conditions are as follows

$$a_{\alpha,\beta}(q_\alpha, q_\beta) = Z_\alpha(q_\alpha)\cdot Z_\beta(q_\beta),$$

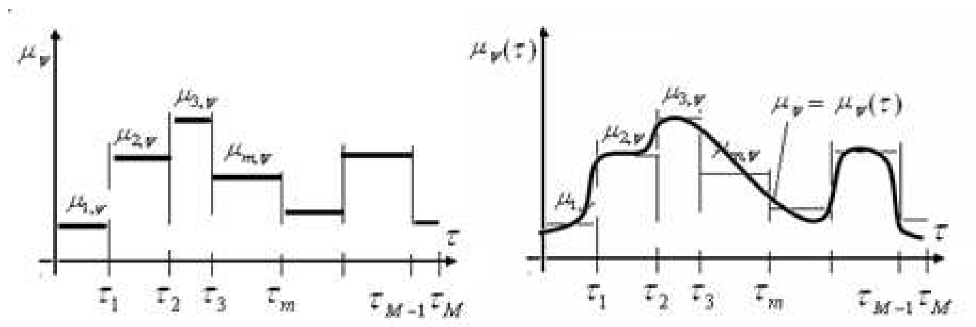
where $Z_1(q_1)$ [UAH/kg] – price per unit of raw material and materials, $Z_2(q_2)$ [UAH/kW] – price per electric energy unit, $Z_3(q_3)$ [UAH/hour] – price of the unit of labour time. A.V.Dabagyan, for the product engineering offered to use the cost of components, cost value of internal activities, assembly and processing costs, in the capacity of the space coordinates [5,p.81].

Coordinate functions of phase technological space in most cases may be presented as the production of unit cost of technological resource or the modification unit of the technological parameter of the subject of labour. The quadratic form (1) is supposed to be quite positive. The size of curve joining two points of technological path $A(q_{1A}, q_{2A},..q_{\alpha A},..,q_{A0})$ and $B(q_{1B}, q_{2B},..q_{\alpha B},..,q_{nB})$

$$\Delta S = \int_A^B \sqrt{\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha,\beta}(q_\alpha, q_\beta)\, dq_\alpha dq_\beta},$$

constitutes the subject of labour change in cost when transferring from one state to another. The distance between two points of technological path of the coordinate space is the change in cost of the subject of labour in the result of technological processing. In the further discourse when designing the paths, there will be used a notion of event. The event is defined by the position in the coordinate technological space, where it occurred and the time when it occurred. The typical state trajectory for the DES-models of the manufacturing systems has been considered by Ramadge P. J. and Wonham W.M. In figure 2 in one dimension space there have been presented the path of motion of the subject of labour along the technological route with discrete and continuous variation of the state parameters. In the capacity of state parameter of the subject of labour on *m*-operation in the course of the effective processing period $\Delta\tau_m = \tau_m - \tau_{m-1}$ there was used the value of the resources intensity transfer for $\mu_{m,\psi}$ where the DES-models and $\mu_\psi(\tau)$ for the models with the continuous variation of state. Another approach when describing the continuous flow lines was presented in the works of Eekelen J.A., Lefeber E., Rooda J.E.. The distinctive feature is the fact that the phase paths are being constructed not for the state parameters of the subject of labour but for the parameters that characterize the workstation condition (Fig.3), which is typical for the fluid models of the manufacturing systems. In the capacity of the state parameters, there are used the following variables: $x_1(t)$ - number of the subjects of labour in the interoperation process stock; number of the subjects of labour in the course of processing $x_2(t)$ with the duration of time $x_3(t)$, necessary for the accomplishment of processing; number of objects $x_4(t)$, which have been processed by the work station during the particular period of time $[t_0, t]$.

Presentation of the basic material For the derivation of an equation of the technological path in the investigation there will be considered two events in the phase technological space (Fig.4), which relate to the change in state of the subject of labour in the course of processing. Let the first event lie in the fact that in the point of time t_0 the subject of labour is under the conditions with the following coordinates $(q_{\alpha 0})$. Let the second event lie in the fact that in the point of time $(t_0 + dt)$ the subject of labour has passed to the point with the coordinates $(q_{\alpha 0} + dq_{\alpha 0})$. On the one hand, over the course of dt the subject of labour has travelled the path in *n*-dimension coordinate space, which equals to



a) б)

Fig.2. Typical path $\mu_{\psi}(\tau)$ *for the state parameters of the subject of labour: a-DES model (Ramadge P. J, Wonham W.M); b-model with the continuous variation of state.*

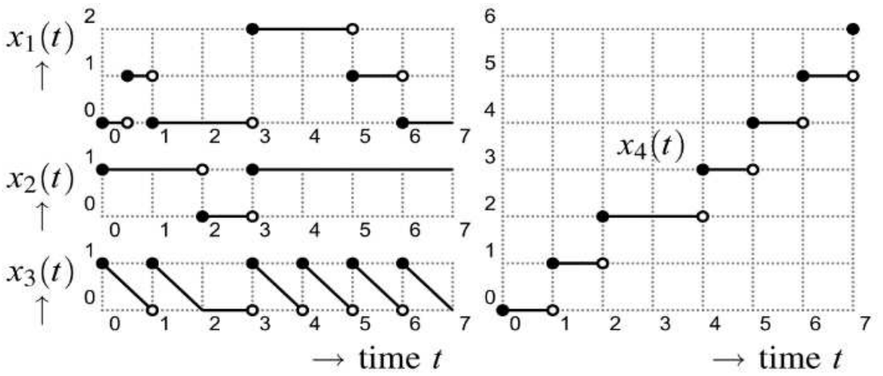


Fig.3. Phase paths for state parameters of the workstation (Eekelen J.A., Lefeber E., Rooda J.E): $x_1(t)$ *- number of the subjects of work in the operational process stock; number of the subjects of labour in the course of processing* $x_2(t)$ *with the duration of time* $x_3(t)$, *necessary for the accomplishment of processing; number of subjects* $x_4(t)$, *processed by the workstation over the period of time* $[t_0, t]$.

$\sqrt{\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha,\beta}(q_\alpha, q_\beta) dq_\alpha dq_\beta}$, on the other hand over the period of dt to the subject of labour in the volume element, limited by the following coordinates $(q_1, q_2, \ldots\ldots q_n)$ and $(q_1 + dq_1, q_2 + dq_2, \ldots\ldots q_n + dq_n)$, there were transferred the technological resources in amount of $\mu_{\psi}(t, q_1, q_2, \ldots\ldots q_\alpha, \ldots\ldots q_n)\cdot dt$. There may be recorded the relation between the coordinates of two events under the consideration in *n*-dimension technological space

$$(\mu_{\psi} \cdot dt)^2 - (dS)^2 \geq 0 \qquad (2)$$

The differential constraint (2) may be used for derivation of an equation of the technological path.

1.Equation of technological relations

Dividing (2) into *dt.* we will receive a unilateral differential constraint $F_\nu\left(t, q_\alpha, \frac{dq_\alpha}{dt}\right) \geq 0$:

$$F_\nu\left(t, q_\alpha, \frac{dq_\alpha}{dt}\right) = \left(\mu_{\psi}(t, q_1, q_2, \ldots\ldots q_\alpha, \ldots\ldots q_n)\right)^2 - \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha,\beta}(q_\alpha, q_\beta)\cdot dq_\alpha \cdot dq_\beta \geq 0, \ \nu = 1..V \qquad (3)$$

The subject of labour motion in the phase space, the unilateral constraints have been applied to, may be split in such a way that on one sections the movement of the subject arises under the unilateral constraint, and on the other sections, its looks as if this constraint does not exist at all. In such a way on the separate sections, the unilateral constraint either is substituted by the bilateral constraint or is rejected. The presence of the bilateral constraint means that all the resources are being transferred to the subject of labour without any loss. The technological parameters, characterizing the process of the resources transfer to the subject of labour, are defined by manufacturing technology and as a rule, in the course of the products manufacture remain unchanged. It follows that the constraint (3) will not explicitly depend on time, that is $\frac{\partial F_\nu}{\partial t} = 0$. Later on, we will consider the independent of time bilateral differential constraints

$$F_\nu\left(q_\alpha, \frac{dq_\alpha}{dt}\right) = \left(\mu_{\psi}(q_1, q_2, \ldots\ldots q_\alpha, \ldots\ldots q_n)\right)^2 - \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha,\beta}(q_\alpha, q_\beta)\cdot dq_\alpha \cdot dq_\beta = 0, \ \nu = 1..V \qquad (4)$$

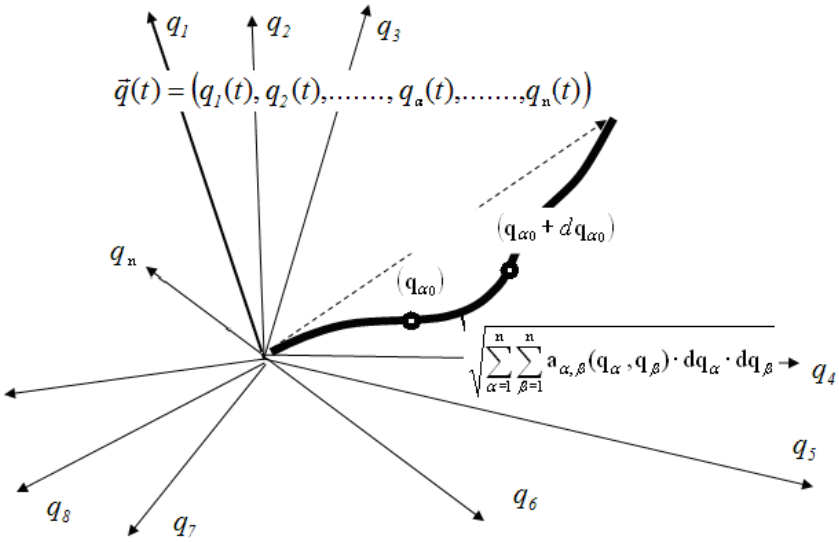


Fig.4. Coordinate technological space

With number of degrees of freedom of the manufacturing system, describing the movement of one subject of labour, $(n-V)$. It is convenient to move from coordinates q_α, describing the movement of the subject of work in the phase space in nature value to the coordinates of cost value S_j [4]. If the coordinate functions are presented as $a_{\alpha,\beta}(q_\alpha, q_\beta) = Z_\alpha(q_\alpha)\cdot Z_\alpha(q_\alpha)$, then putting $dS_j = Z_j(q_j)\cdot dq_j$, we will receive the bilateral differential constraints (4) through the cost coordinates S_j

$$F_\nu\left(S_\alpha, \frac{dS_\alpha}{dt}\right) = \left(\mu_\psi(S_1, S_2, .., S_\alpha, .., S_n)\right)^2 - \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} dS_\alpha \cdot dS_\beta = 0, \quad \nu = 1..V$$

Having differentiated the last equation in time, we receive the formula for the restrictions of imposing technological constraints to the movement of the subject of work

$$\sum_{\alpha=1}^{n} \frac{\partial \mu_\psi}{\partial S_\alpha}\cdot \mu_\alpha - \sum_{\alpha=1}^{n} \frac{d\mu_\alpha}{dt} = 0, \quad \mu_\alpha = \frac{dS_\alpha}{dt} \qquad (5)$$

For the differential bilateral constraints, which definitely depend on time the equation (5) is as follows:

$$\frac{\partial \mu_\psi}{\partial t} + \sum_{\alpha=1}^{n} \frac{\partial \mu_\psi}{\partial S_\alpha}\cdot \mu_\alpha - \sum_{\alpha=1}^{n} \frac{d\mu_\alpha}{dt} = 0, \; \mu_\alpha = \frac{dS_\alpha}{dt}.$$

If the intensity of the resources transfers $\mu_\psi(S_1, S_2, .., S_\alpha, .., S_n)$ to the subject of labour, which is presented as a sum of interdepend intensities of different kinds of resources $\mu_\psi(S_1, S_2, ..., S_\alpha, ..., S_n) = \sum_{\alpha=1}^{n} \mu_{\psi\alpha}(S_\alpha)$, then:

$$F_\alpha\left(S_\alpha, \frac{dS_\alpha}{dt}\right) = \left(\mu_{\psi\alpha}(S_\alpha)\right)^2 - \mu_\alpha^2 = 0,$$

which allows recording the equation of constraint for the projection to axis of reference of the cost value of α-dimension technological resource.

$$\frac{\partial \mu_{\psi\alpha}(S_\alpha)}{\partial S_\alpha}\cdot \mu_{\psi\alpha}(S_\alpha) - \frac{d\mu_\alpha}{dt} = 0,$$

If the α-dimension resource is being transferred to N_m of the subject of labour, located in the process stock of m-dimension technological operation; the equation of the constraint will look as follows

$$\left(\mu_{\psi\alpha}(S_{\alpha,1}, S_{\alpha,2} \ldots\ldots S_{\alpha,N_m})\right)^2 - \sum_{k_1=1}^{N_m}\sum_{k_2=1}^{N_m} \frac{dS_{\alpha,k_1}}{dt}\frac{dS_{\alpha,k_2}}{dt} = 0$$

$$, \mu_{\psi\alpha}(S_{\alpha,1}, S_{\alpha,2} \ldots\ldots S_{\alpha,N_m}) = \sum_{k=1}^{N_m} \mu_{\alpha,k}$$

For the batch consisting of N_m subjects of labour, located in the interoperation process stock of m operation $S_{\alpha,j} \in [S_{\psi m}, S_{\psi m+1}[$, corresponding to the beginning and ending of the operation, let us introduce the following variables $\nu_\alpha = \frac{1}{N_m}\sum_{k=1}^{N_m} \mu_{\alpha,k}$ and $\Theta_\alpha = \frac{1}{N_m}\sum_{k=1}^{N_m} S_{\alpha,k}$, we will receive the equation of the subject of labour movement of batch N_m products with the averaged parameters ν_α and Θ_α

$$\frac{\partial}{\partial \Theta_\alpha}\left(\frac{\mu_{\psi\alpha}(\Theta_\alpha)}{N_m}\right)\cdot \frac{\mu_{\psi\alpha}(\Theta_\alpha)}{N_m} - \frac{d\nu_\alpha}{dt} = 0.$$

We will express the correlation of $\frac{\mu_{\psi\alpha}(\Theta_\alpha)}{N_m} = \frac{\Delta S_{\psi m}}{\Delta \tau_m}\frac{1}{N_m}$ through the density of the subjects of labour $[\chi]_0(t, \Theta_\alpha) = \frac{N_m}{\Delta S_{\psi m}}$, located within the limits of the section $\Delta S_{\psi m}$ and by means of equipment work speed $[\chi]_{1_\psi}(t,\Theta_\alpha)=\frac{1}{\Delta\tau_m}$

$$\frac{\partial}{\partial\Theta_\alpha}\left(\frac{[\chi]_{1_\psi}(t,\Theta_\alpha)}{[\chi]_0(t,\Theta_\alpha)}\right)\cdot\frac{[\chi]_{1_\psi}(t,\Theta_\alpha)}{[\chi]_0(t,\Theta_\alpha)}-\frac{dv_\alpha}{dt}=0 .$$

2. Variation technical path engineering method

The bilateral differential constraints (4) corresponding to the limiting processing case of the subject of labour along the technological route, when the resources are transferred to the subject of labour in full, without any loss. Nevertheless, in the manufacturing activity of the company there are always present the inevitable operations, with the performance of which there always happen the losses of technological resources, which is expressed by the following equation

$$F_\nu\left(q_\alpha,\frac{dq_\alpha}{dt}\right)=\left(\mu_\psi(q_1,q_2,\dots\dots q_\alpha,\dots\dots q_n)\right)^2-$$
$$-\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}a_{\alpha,\beta}(q_\alpha,q_\beta)\cdot\frac{dq_\alpha}{dt}\cdot\frac{dq_\beta}{dt}>0,\ \nu=1..V .$$

It is required for the technological processes of the subject of labour to be fulfilled in accordance with regulatory construction and technological parameters with minimum loss of the resources. We consider that for the manufacturing system under the investigation there exist an interval, which for the subject of labour movemenent according to the technological rouse in accordance with the regulatory path is a minimum

$$\Re_{ab}=\int_a^b\sqrt{\left(\mu_\psi(q_\alpha)dt\right)^2-\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}a_{\alpha,\beta}(q_\alpha,q_\alpha)\,dq_\alpha\,dq_\beta} ,$$

which is taken along the paths between two events «a» and «b» of the technological processing of the subject of labour. The objective functional is presented as follows

$$\Re_{ab}=\int_a^b\left(\sqrt{\left(\mu_\psi(q_\alpha)\right)^2-\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}a_{\alpha,\beta}(q_\alpha,q_\beta)\frac{dq_\alpha}{dt}\frac{dq_\beta}{dt}}\right)dt$$

with the objective function of the technological process $J=\sqrt{\mu_\psi^{\ 2}-\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}a_{\alpha,\beta}\frac{dq_\alpha}{dt}\frac{dq_\beta}{dt}}$, describing the behavior of the subject of work in the technological process of the manufacturing system. The fact that each operation of the above-mentioned manufacturing process is characterized only by the following values ΔS_ψ (UAH) and $\mu_\psi\left(\frac{UAH}{hour}\right)$, not higher derivatives, confirms the fact that the state of technological process is fully defined by the knowledge of coordinates q_j (UAH) and the intensity of their change in the course of the time $\frac{dq_\alpha}{dt}$ $\left(\frac{UAH}{hour}\right)$. From the variation that equals to zero $\delta\,\Re_{ab}=0$ there are derived the Euler's equations for the variational problem, describing the changes of the labour parameters in *n*-dimensional technological space under the effect of the equipment

$$\frac{d}{dt}\frac{\partial J}{\partial\frac{dq_\alpha}{dt}}=\frac{\partial J}{\partial q_\alpha},\qquad i=1..n.$$

While considering the movement of the subject of labour in one dimension technological space (n=1) the objective functional may be recorded as follows:

$$\Re_{ab}=\int_a^b\left(\sqrt{\mu_\psi^{\ 2}-\left(\frac{dS}{dt}\right)^2}\right)dt,\ J=\sqrt{\mu_\psi^{\ 2}-\mu^2},$$
$$\mu_\psi=\mu_\psi(S),\quad \frac{dS}{dt}=\mu .$$

From the variation that equals to zero $\delta\ \Re_{ab}=0$ there is derived the Euler's equation:

$$\frac{d\mu}{dt}=-\left(\frac{\mu_\psi^{\ 2}-2\mu^2}{\mu_\psi}\right)\frac{\partial\mu_\psi}{\partial S} .$$

In the limiting case, when the losses of technological resources tend to zero $\mu\to\mu_\psi$ the last equation takes the form, which is similar to (5)

$$\frac{d\mu}{dt}=\mu_\psi\frac{\partial\mu_\psi}{\partial S} . \qquad (6)$$

While moving along the technological route the subject of labour should be processed strictly in accordance with the specified manufacturing technology (Fig.1), specified regulatory technological path $S_0=S_0(t)$ and the limiting technological paths, being set based on manufacturing tolerance for the performance of operation. Departure from the technology is inadmissible. It leads to adverse effect and results in the defect.

Let us consider the motion integrals related to uniformity of time and space. If the objective function of the manufacturing system does not depend directly on time, then its total derivative by times may be recorded as follows:

$$\frac{dJ}{dt}=\sum_{\alpha=1}^{n}\frac{\partial J}{\partial q_\alpha}\cdot\frac{dq_\alpha}{dt}+\sum_{\alpha=1}^{n}\frac{\partial J}{\partial\frac{dq_\alpha}{dt}}\cdot\frac{d^2q_\alpha}{dt^2},\frac{\partial J}{\partial t}=0 .$$

Let us replace the derivatives $\frac{\partial J}{\partial q_\alpha}$ by $\frac{d}{dt}\frac{\partial J}{\partial\frac{dq_\alpha}{dt}}$ in accordance with the Euler's equations, we will receive:

$$\sum_{\alpha=1}^{n}\frac{d}{dt}\frac{\partial J}{\partial\frac{dq_\alpha}{dt}}-J=\text{const} .$$

This value remains unchanged through time in the course of the subjects of labour movements in *n*-dimension technological space, and specifies the relation between the intensity μ of the resources consumption by the subject of labour and the intensity μ_ψ of the transfer of the technological resources by the manufacturing equipment. The change of the intensity μ is definitely determined by the intensity variation μ_{ψ} .In case of uniformity of the technological space with respect to coordinate the objective function of the manufacturing system J does not implicitly depend on coordinate q_α , $\frac{\partial J}{\partial q_\alpha}=0$. In the virtue of the Euler's equations it follows that

$$\frac{\partial J}{\partial \frac{dq_\alpha}{dt}} = \text{const} .$$

The received motion integral for the system of "the continuous flow line –the subject of labour" may be treated as the constancy of the technological resources consumption rate by the subject of labour in the course of its movement along the technological route.

3. Technological paths engineering with the use of general dynamic equation.

The equation of motion of the subject of labour may be received from their general equation of the subject of labour dynamics in the phase technological space.

$$\sum_{\alpha=1}^{n}\left(Q_\alpha(q_\alpha)-\sum_{\beta=1}^{n}a_{\alpha,\beta}\cdot\frac{d^2q_\beta}{dt^2}\right)\cdot\delta q_\alpha=0,$$

where $Q_\alpha(q_\alpha)$-generalized technological strength, effecting the subject of labour along the coordinate q_α with the purpose of transfer of the technological resources, under which there is performed a work over the subject of labour $\delta A_\alpha = Q_\alpha \cdot \delta q_\alpha$:

$$\delta A=\sum_{\alpha=1}^{n}\delta A_\alpha,\ \sum_{\alpha=1}^{n}\delta A_\alpha-\sum_{\alpha=1}^{n}Q_\alpha\cdot\delta q_\alpha=0,$$

$$\delta A-\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}a_{\alpha,\beta}\cdot\frac{d^2q_\beta}{dt^2}\cdot\delta q_\alpha=0. \qquad (7)$$

The summary with double amount is presented as follows

$$\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}a_{\alpha,\beta}\cdot\frac{d^2q_\beta}{dt^2}\cdot\delta q_\alpha=$$

$$=\frac{d}{dt}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}a_{\alpha,\beta}\frac{dq_\beta}{dt}\delta q_\alpha-\delta\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{a_{\alpha,\beta}}{2}\frac{dq_\beta}{dt}\frac{dq_\alpha}{dt}.)$$

Let us present the received formula in (7) and integrate in accordance with t

$$\int_{t_0}^{t_1}\left(\delta\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{a_{\alpha,\beta}}{2}\cdot\frac{dq_\beta}{dt}\cdot\frac{dq_\alpha}{dt}+\delta A\right)dt$$

$$-\left(\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}a_{\alpha,\beta}\cdot\frac{dq_\beta}{dt}\cdot\delta q_\alpha\right)\Bigg|_{t=t_0}^{t=t_1}=0.$$

As long as the initial and final state has been fixed, then $\delta q_i = 0$ and, consequently

$$\int_{t_0}^{t_1}\left(\delta\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{a_{\alpha,\beta}}{2}\frac{dq_\beta}{dt}\frac{dq_\alpha}{dt}+\delta A\right)dt=0.$$

Let us introduce the notion of potential energy of the system $\Pi(t,q_\alpha)$, $\delta A=-\delta\Pi(t,q_\alpha)$, presenting the objective function as follows

$$J\left(t,q_\alpha,\frac{dq_\alpha}{dt}\right)=\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{a_{\alpha,\beta}}{2}\frac{dq_\beta}{dt}\frac{dq_\alpha}{dt}-\Pi(t,q_\alpha).$$

If the manufacturing technology has been specified and if it is not changed in the course of the time or the time that has to be spent for such manufacturing technology change is much more than the time of the production cycle, then with the sufficient degree of accuracy one may assume the objective function does not implicitly depend on time, which allows recording the first integral

$$\sum_{\alpha=1}^{n}\frac{dq_\alpha}{dt}\cdot\frac{\partial J\left(q_\alpha,\frac{dq_\alpha}{dt}\right)}{\partial\frac{dq_\alpha}{dt}}-J\left(q_\alpha,\frac{dq_\alpha}{dt}\right)=$$

$$=\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{a_{\alpha,\beta}}{2}\cdot\frac{dq_\beta}{dt}\cdot\frac{dq_\alpha}{dt}-\Pi(t,q_\alpha)=h_0=\text{const}$$

with the system potential $\Pi(q_\alpha)$. In case of one-dimension description of the manufacturing process with the objective function $J(S,\mu)=\frac{\mu^2}{2}-\Pi(S)$, the motion integral will take the following form

$$\frac{\mu^2}{2}+\Pi(S)=h_0=\text{const},\quad \frac{dS}{dt}=\mu.$$

If the motion of the subject of labour complies with the bilateral constraint (5) of the kind $\mu^2-{\mu_\psi}^2(S)=0$, then the formula for the potential energy may be received from the following equality ${\mu_\psi}^2(S)=2(h_0-\Pi(S))$. The objective function is defined within the accuracy of the summand, which is the total derivative of the coordinate function and time. It follows thence

$$J(S,\mu)=\mu^2+{\mu_\psi}^2(S)$$

which allows recording the Euler's equation that coincides with the equation (6)

$$\frac{d\mu}{dt}=\mu_\psi(S)\frac{\partial\mu_\psi(S)}{\partial S}.$$

Conclusions, future development prospects and improvement of PDE-model of the manufacturing systems.

The obtained results of the investigation are the basic ones for the development of high performance manufacturing control systems based on PDE-model of the manufacturing systems. As distinguished from the models, where for the obtaining of the closed equation system there is applied the equation of condition, models, which use the equation of the technological path, allow describing the manufacturing processes, functioning under the transient condition. An important and a separate task is the distribution of the obtained results in the event the description of the basic data have been executed in terms of fuzzy mathematics [10]. It allows significantly extend the application field of class PDE-models of the manufacturing systems.

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Мета. Розробка аналітичних методів проектування технологічних траєкторій руху предметів праці в просторі станів для побудови замкнутих PDE-моделей опису виробничих систем.

Методика. Для виведення рівняння руху предмета праці в просторі станів використані математичний апарат і методи аналітичної механіки, варіаційного числення.

Результати. Отримано рівняння руху предмета праці в просторі станів і розглянуті інтеграли руху, пов'язані з однорідністю часу і простору станів.

Наукова новизна. Наукова новизна отриманих результатів полягає в удосконаленні PDE-моделей виробничих систем, використовуваних для проектування високоефективних систем управління виробництвом. Запропоновано модель перенесення технологічних ресурсів на предмет праці, заснована не на традиційному феноменологическом описі стаціонарних виробничих явищ, а на законах збереження, що характеризують процес перенесення технологічних ресурсів на предмет праці і просторово-часову структуру виробничого процесу. Це дозволило отримати рівняння руху предметів праці за технологічним маршрутом з наступною побудовою на їх основі нестаціонарних рівнянь PDE моделей для опису стану параметрів виробничого процесу. При виведенні рівняння технологічної траєкторії руху предмета праці враховані диференціальні зв'язку, що накладаються виробничою системою на процес перенесення технологічних ресурсів на предмет праці в результаті взаємодії їх з технологічним обладнанням і між собою при переході від однієї технологічної операції до іншої.

Практична значимість. Практична значимість полягає в тому, що методи побудови рівняння технологічної траєкторії предмета праці позволяють розробити високоточні моделі перехідних процесів виробничої системи, , які є основою для проектування високоефективних систем управління підприємством з потоковим методом організації виробництва

Ключові слова: предмет праці, технологічний процес, технологічна траєкторія, PDE-модель

Purpose. The development of analytical methods of technological paths engineering of the subjects of labour movements in the state space with the purpose of construction of closed PDE-models, used to describe the manufacturing system.

Methodology. For derivation of an equation of the subject of labour movement in the phase space of states, there has been applied a mathematical tool and the variational calculation methods of analytical mechanics.

Findings. There has been derived the equation of the subjects of labour movement in the state of space and there have been considered the motion integrals, related to the uniformity of time and state space.

Originality. The originality of the obtained results involves the improving of PDE-models of manufacturing systems, used for the engineering of the high performance manufacturing control systems. The offered model of technological resources transfer to the subject of labour is based not on the traditional phenomenological description of the static production phenomena, but on conservation laws, which characterize the transfer process of technological resources to the subject of labour and space-time structure of the manufacturing process. It allowed deriving the equation of the subjects of labour movement along the manufacturing route, followed by the construction on their ground the non-steady-state equations of the PDE models for the description of the parameters status of the manufacturing process. While deriving the equations of the technological path movement of the subject of labour there were taken into consideration the differential constraints, being applied by the manufacturing system to the transfer process of technological resources of the subjects of labour, resulting from their interaction with production equipment and against each other in the course of transfer from one manufacturing operation to another.

Practical value. Lies in the fact the methods of the equation construction of technological path of the subject of labour allow developing high-quality models of the transfer processes of the manufacturing system, which are the basis for the high quality enterprise management system engineering with a straight flow method of industrial organization.

Keywords: the subject of work, manufacturing process, technological path, PDE-model

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