Dynamics and Interactions of Monetary Policy and Macroeconomic Variables: Empirical Investigation in the UK Economy with Bayesian VAR

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3 September 2018

MSc Economics Dissertation

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Date: September 3, 2018

Abstract

Applying the MCMC algorithm for time varying Bayesian VAR model, I have estimated the impulse response, stochastic volatility and forecast error variance decomposition. The model allows both parameters and stochastic volatility to vary. The impulse response of unemployment, inflation and interest rate to the interest rate shock using the UK data during 1971:Q1-2016:Q4 has taken place for a horizon of 40 quarters. The obtained result indicates that the response to interest rate shock does not decay entirely and the shocks have substantial effect on error variance in the forecast horizon, and the stochastic volatility has reduced over time except unemployment.

Acknowledgement

Thanks to Allah Almighty Who created and enabled me to dedicate myself to study and conduct this research.

I am grateful to my supervisor, Professor Haroon Mumtaz for agreeing to supervise me and for his support throughout the research period. His expertise in the area and kindness gave me the courage to choose such a challenging topic for my research. I am also thankful to the School of Economics and Finance, Queen Mary, University of London for all the support I have received in my academic year 2017-18.

I remain thankful to the Foreign and Commonwealth Office (FCO) for providing me with 'Chevening Scholarship', without what I would not be able to pursue the MSc in the UK and hence to do this dissertation.

I remember the love and patience of my parents that they have showered on me. At this point, I pray for the departed soul of my father, and happy and healthy life of my mother. I also like to extend my acknowledgement to my other family members, friends, teachers and people in my surroundings who have always been well-wisher and source of strength for me.

Table of Contents

List of Figures and Tables

Appendices

List of Abbreviation

BoE	………..………………... Bank of England
FEVD	…………………..…….. Forecast Error Variance Decomposition
IRF	………………………... Impulse Response Function
IT	…………………...……. Inflation Targeting
MCMC	…........................…….Markov Chain Monte Carlo
MH	………………………… Metropolis Hastings
NAIRU	……………….…..….. Non-accelerating Inflation Rate of Unemployment
NRU	………………………… Natural Rate of Unemployment
OECD	………………………… Economic Cooperation and Development
UK	……………….……….... United Kingdom
US	………………...……..… United States
VAR	………………………… Vector Autoregression

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1. Introduction

In the macroeconomics research, different types of vector autoregression (VAR) model have been using extensively. Contemporary development in the field of empirical research and application is the Bayesian econometric tools and the use of Bayesian VAR has been increasing in this area. In the VAR model, all variables are depended on all other variables and their lag including its own lag. Therefore, VAR models need to estimate a large number of parameter, which can make the estimate imprecise. Bayesian VAR model has to estimate the prior information into the estimation process that makes the estimates more precise. I have applied Bayesian VAR in modelling and conducting the empirical analysis of this study for the United Kingdom (UK) economy data of unemployment, inflation and interest rate.

The relationship between unemployment and inflation has gone through loads of scrutiny from both theoretical and empirical perspectives in the economics literature. A long-standing debate exists about the effectiveness of monetary policy to stimulate the macroeconomic performance, to be specific, determining the relationship between unemployment and inflation, still this is speculative idea (Tobin, 1972; Mankiew, 2001). Thankfully, the world has been able to accept the core principles of monetary policy as a working consensus (Goodfriend, 2007; Arestis and Sawyer, 2008). However, this is neither a real agreement nor a conclusive result on the rule of monetary policy, and relationships between unemployment and inflation. Empirical macroeconomic researchers have been using different econometric tools including reduced form regression, VAR, correlation analysis etc. for a better understanding of the relationship of unemployment and inflation, and the role of monetary policy to obtain the best outcome. Recent literature has been trying to explore unemployment and inflation relationship applying the Bayesian methodology (e.g., Cogley and Sargent, 2001; Cogley and Sargent, 2005; Primiceri, 2004; Sim and Zha, 2006; Benati, 2008).

Due to the drifting coefficients or other potential nonlinearities, monetary policy shock in the time series variables evolves over time. Upon these criticisms of invariant parameters in the model a few empirical researchers, for example, Cogley and Sargent (2005), Primiceri (2004), have started to analyse VAR allowing time varying parameter and at the same time variation in the stochastic volatility. My interest in this paper is to understand the small UK economy case applying the similar identification of the model, as most of the analyses in literature have used the data of the United States (US) economy. I have chosen the variables and model in such a way that keep the number of parameters small as calculation of a large number of parameters is troublesome and cover the theoretical base of Philips curve.

VAR model with stochastic volatility is difficult to estimate because this makes the likelihood function intractable. Bayesian inference with Markov Chain Monte Carlo (MCMC) algorithm methods overcome this difficulty of intractability (Nakajima, 2011). Geyer (1992) endorses that in the simulation of stochastic processes whose probability densities are known up to a constant proportionality, MCMC method is a general statistical application. However, to date, the macroeconomic researchers have paid most focus on Bayesian applications. To compute posterior densities of the model encompassing the drifting coefficients and stochastic volatilities, I have applied the Bayesian VAR with the MCMC algorithm.

In the estimation process of the impulse response function in this model, I have considered the unemployment and inflation rate variables in the non-policy block, and the interest rate in the policy block. I mainly concentrate to the shock to the interest rate, as this is considered to be a policy variable. The reason of counting the interest rate in the policy block is that the monetary policy is intimately connected with the new consensus in macroeconomics where this, as a key policy instrument, has become firmly based on the use of interest rate (Arestis and Sawyer, 2008; Goodfriend, 2007). I have presented the posterior densities of the forecast horizon for the variables applying time varying Bayesian VAR model. The reported results are the impulse response of the unemployment, inflation and interest rate to the interest rate shock, estimated stochastic volatility and forecast error variance decomposition (FEVD) due to the interest rate shock.

The empirical findings of the research, presented in the three-dimensional graph, mostly go with the view of the non-neutrality of monetary policy in the long run. The monetary policy shock does not entirely die out, and it has some implications to interpret the forecast horizon of unemployment and inflation relations as well as interest rate itself. However, the response and the relationship cannot be confirmed as I did not conduct the significance test. The methodological and empirical development of the model is the primary focus of this research than the empirical confirmation of a theory. The related interest of this research is to understand the role of monetary policy, to be specific, the interest rate in forecasting the unemployment and inflation, and determining their relationship. To understand the pertinence of a monetary policy variable in explaining the relationship between unemployment and inflation, I measured the impulse response of shock to the interest rate. The data used in this research on the UK economy are quarterly data for inflation, unemployment and bank rate as a proxy of interest rate. This research mainly contributes to the following key issues for the UK economy case during 1971: Q1 to 2016: Q4. Design Bayesian framework using MCMC algorithm for the above variables in the UK economy and present of the effect of the shock to the monetary policy on unemployment and inflation as studies have been done on this topic using the UK data is few.

I have organized the paper as follows. Section 1 has introduced the topic and methodology, and summed up the core findings. Section 2 covers the review of the literature on concepts, theoretical and methodological development. The third section presents the theoretical framework of the model. Section 4 is on the data source and description, and section 5 gives the empirical framework. The penultimate section contains the research result, and the last section concludes this study.

2. Literature Review

2.1. Unemployment, inflation and monetary policy

Unemployment, inflation and interest rate are some of the very core issues in the Macroeconomic extent. The study on the relationship among them has developed a rich set of literature in the range of both theoretical and empirical aspect. Friedman (1977) remarks that the professional view of the relationship between unemployment and inflation has passed through three stages. First, acceptance of a stable Phillips curve, that is a stable trade-off between unemployment and inflation. Second, the introduction of expectation in the unemployment-inflation model. Third, the development of empirical evidence favouring an apparent positive relationship between unemployment and inflation. Since then quite a few structural shifts have taken place, and studies have endeavoured to capture them. Taylor (1997) to summarise the findings of research of monetary policy, clusters the research issues and results in four blocks. First, attaining the long run neutrality feature of monetary policy, that is, the result does not find the evidence of the long run trade-off between the unemployment and inflation rate. Second, the finding that confirms that monetary policy is non-neutral in the short-run. Third, the collection of researches that are concerned about the function of the monetary policy if it should be viewed as a contingency plan or a policy rule, the focus of these research is the credibility, time inconsistency and rational expectations of monetary policy. Fourth, the studies that deal with determining the broad characteristics that sound monetary policy should contain. Though draws no specific conclusion about how should monetary policy be used to influence unemployment and inflation, Parkin (1998) analysing DGE models for The US, UK and Canadian data opines favouring the consensus rule that arises from an accumulation of evidence and theory which has led most macroeconomists to describe the world in terms of four propositions as follows. First, as changes in the demand for money is not predictable, it is better to target directly the inflation than to target a monetary aggregate. Second, contingency rule has taken as superior to discretion, as the credibility and time consistency problems remain. Third, though with the constant inflation at the natural rate of unemployment a trade-off exists between the unemployment and inflation, there is no long-run trade-off between them. While independent of monetary policy the natural rate of unemployment may varies for many reasons, so targeting zero inflation incurs no permanent costs but brings enduring benefits. Fourth, the research cannot fine-tune unemployment and inflation, as the time lags in the effects of monetary policy extend beyond its forecast horizon. Referring to the historic quote in Hume's 1752 easy, Ball and Mankiw (2002) point that even though two centuries passed, professional of economics could not reach a consensus about the reason of trade-off between unemployment and inflation while classical theory takes money as neutral. According to Dobrescu, Paicu and Iacob (2011), in the post-war era the analysis of the trade-off between unemployment and inflation has undergone numerous stages— the acceptance of the 1957 hypothesis of A.W. Phillips and the succeeding Phillips curve; contributions of Friedman by revealing a long-run vertical Phillips curve and introducing the natural rate of unemployment. Mankiew (2001) stipulates that unemployment and inflation trade-off is not about deriving a stable downward-sloping Phillips curve by plotting these two variables in scatter diagram or not running specific regression of these variables with well-fitted data and producing precise coefficients. At its heart, the idea of trade-off between unemployment and inflation is about the influence of monetary policy that claims that these two variables go opposite directions in the short-run with the changes in monetary policy variable.

Empirical researchers have conducted a bunch of study on unemployment, inflation and monetary policy. Applying different methods for the semi-annual US dataset, Karanassou and Sala (2010) argue for the trade-off between unemployment and inflation. However, considering as a superior indicator, their focus is on the money growth than the federal fund rate in getting the response of unemployment and inflation to the shocks. Blanchard and Galí (2010) show that from the normative point of view when frictions and real wage rigidities are a presence in the labour market, monetary policy that tends to stabilize inflation strictly is not the best. They conclude that monetary policy implies some accommodation of inflation, but limited fluctuations in unemployment. Coibion (2012) finds, for US data during 1970-1996, variation in results based on different methodology. While standard VAR results give a minimal fluctuation in unemployment and inflation accrued from the shock to the monetary policy, the Romer and Romer methodology implies substantial fluctuations in both unemployment and inflation determined by the monetary policy. Medium sized real effects of shocks to the monetary policy have been found from Taylor rules, which also indicate the significant historical contribution to real fluctuations due to the monetary policy shocks. Several factors influence the strengths of the monetary policy shocks. These include the pattern of contraction, lag structure, the period for a target, which are responsible for the consistent and most likely medium size effects of the monetary policy shocks from the different approach. Louis and Balli (2013) attempt to explore if the distinctive unemployment rate in organization for economic cooperation and development (OECD) with the US is the result of unanticipated deviation of the short-term OECD rates from the fund rate and to understand if monetary policy target for low-inflation is responsible for the financial crisis. In the very short-run, they do not find any effects of interest rate differential shocks on unemployment, but there is effect in the long-run. Drastic deviation from the US monetary policy incur a higher cost and persistent unemployment in the OECD countries, on average the inflation targeting countries incur higher cost. Applying an identification scheme for the non-accelerating inflation rate of unemployment (NAIRU) estimate allowing regime shifts in the structural shocks, Kajuth (2014) concludes that there are significant statistical and economic relationships between unemployment and inflation in Germany, and justify the trend of the unemployment rate as NAIRU. To understand the unemployment and inflation relation, Bhattarai (2016) analyse the data of OECD economies with panel data using fixed and random effect, and panel VAR models for the period 1990-2014. In the country specific regressions, out of 35 of OECD economies 28 observe empirically significant Phillips curve phenomena.

Gerlach, Lydon and Stuart (2016) study the determination of inflation between 1926 and 2012 in Ireland to test the overarching view about the unemployment-inflation relationship that the small economy Ireland does not have the Phillips curve relationship. They incorporated unemployment relative to the NAIRU and import prices to estimate a backwards looking Phillips Curve and found that unemployment has a statistically significant effect on inflation.

Literature has also been dealing with different analytical dimensions. According to Karanassou, Sala and Snower (2009), as the conventional wisdom accepts that no long-run relationship exists between unemployment and inflation, the evolution of unemployment and inflation can be sufficiently modelled by separate branches of economics. Hence, the macro branch with inflation dynamics accepts the existence of the natural rate of unemployment (NRU) and estimates the unemployment rate compatible with inflation stability (NAIRU), and the labour macro branch accepts the existence of the NAIRU and attempts to identify the real economic driving forces of NRU. They recommend adopting a holistic framework to be able to model inflation dynamics jointly, estimate the unemployment-inflation trade-off and determine the factors that are responsible for the movements of the equilibrium unemployment rate in the long run. Beyer and Farmer (2007) study the co-movements in unemployment, inflation and federal funds rate in the US and observe that these can be well described as non-stationary but cointegrated variables from 1970 through 1999. They find stable cointegrating equation over the entire period that links unemployment with inflation and the evidence through the neo Keynesian model leads to be sceptical of the theories that maintain super neutrality assumption so the 'natural rate doubts'. Berentsen, Menzio and Wright (2011) investigate the long-run relationships between inflation or nominal interest as a measure of monetary policy and unemployment as a measure of labour market performance for the quarterly US data during the period 1955-2005. The paper documents that in the low-frequency, inflation and interest rates have a positive relationship with the unemployment rate. Using the search-and-bargaining approach to model labour markets and goods markets, they find that monetary policy as a sole driving force accounts for quite a lot of unemployment behaviour.

2.2.Relevant VAR and Bayesian literature

Stock and Watson (2001) use two related structural VAR model to incorporate the various identifying assumption of the causal relationship monetary policy with unemployment, inflation and interest rate. Using the quarterly US data for 1960:1-2000: IV, they find the VAR model to be instrumental in four macroeconometrics tasks—for describing the data, for forecasting, for structural inference and policy analysis. Claiming natural alternative methods to estimating the impulse response from VAR, Jorda (2005) recommends computing the impulse response of a given VAR model for each period's local projections but not for extrapolating into horizon progressively. Ribba (2006), identifying a small structural vector error correction model by using the long-run and short-run combination of restrictions for the period 1980-2001, studies the dynamic interactions at various frequencies among unemployment, inflation and federal fund rate. The study finds that in the short run unemployment and inflation move in opposite directions due to aggregate demand and monetary policy shocks. While analysing the pre and post 1980 data for the US economy using the VAR model, Boivin and Giannoni (2006) find fewer effects of the shock to the monetary policy during the post-1980 period. Lanne, Lutkepohl and Maciejowska (2010) estimate two model to understand the applicability of a Markov regime switching property of structural VAR analysis in identifying shocks when the matrix of reduced form error covariance varies across states. According to the study, to obtain identification under general conditions assumption of the shocks to be orthogonal and of the impulse responses to be invariant across regimes is enough. In case of regimes more than two, the regime invariance condition is advised to test. D'Agostino, Gambetti and Giannone (2013) using a VAR of time varying coefficients with stochastic volatility, produce real time out of sample forecast up to 3 years ahead for the unemployment rate, inflation rate and a short term interest rate for the US economy. The findings confirm the time varying VAR as the only model that delivers precise forecasts systematically for these variables. Therefore, the paper concludes that for forecasting structural economic change is vital to take into account and as the incorporation of the prominent features of a time varying economy in a time varying VAR is flexible but parsimonious, it is a reliable tool for real time forecasting. However, Clark and Ravazzolo (2015) estimate the Bayesian autoregressive and VAR models by incorporating a different form of time varying volatility including stochastic volatility models coupled with random walk, stationary AR process and fat tails, and GARCH and mixture of innovation models. They find that conventional stochastic volatility in the autoregression and VAR specifications dominate other specifications of volatility mainly in density forecasting and to some extent in point forecasting.

A well set of analysis has been done applying the Bayesian VAR with stochastic volatility to explain and forecast the interactions among unemployment, inflation and monetary policy. Cogley and Sargent (2001) apply nonlinear stochastic state space model with time varying parameters in Bayesian VAR to detect the features of unemployment and inflation dynamics of US economy in the post World War II period and describe the evolving nature of the law of motion for inflation. They use time variant coefficients and unknown time invariant innovation covariance matrix for error variance that conflicts with quite a few existing literature, which used changing VAR innovation over time. The model predicts that the evidence goes against the natural rate hypothesis of Philips curve when the observations of lower and more stable inflation are accumulated. Upon controversies with some contemporary literature and criticisms, Cogley and Sargent (2005) conduct a similar type of analysis with same data set. However, to address some of the criticisms, they use VAR allowing both coefficients and the volatilities (R) to vary. They apply the MCMC methods to present posterior densities for numerous objects that are pertinent for monetary policy. They use posterior distributions in evaluating the strength of some tests, which they do for testing the null hypothesis that the autoregressive coefficients of VARs are time invariant against the alternative hypothesis of time varying coefficients. After taking into account the stochastic volatility, the results accept the drifting variances within the new specification and preserves the previous evidence for drifting systematic parts. However, evidence of having drifting coefficients is a concern as detecting evidence for movements in the systematic part is more challenging than detecting the stochastic volatility of a VAR. Cogley, Morozov and Sargent (2005) also estimate a Bayesian VAR for the UK economy allowing coefficients to drifted and the stochastic volatilities during the period 1957-2002. To compute the posterior densities, they used the MCMC algorithm. When they illustrated the model by constructing different fan charts for the UK inflation for 2003-2008, they find that to forecast inflation parameter uncertainty and drift matter greatly for the longer forecast horizon while only slightly matter for one or two years ahead. Primiceri (2004) also uses multivariate stochastic model for US data from 1953:I to 2001:III to model time varying VAR allowing both coefficients and variance covariance matrix to vary over time. The study considers literature as divided into two categories in explaining the strong evidence volatility, those focuses the heteroschedasticity of the exogenous shocks, and those emphasise the ways of responses of macroeconomic variables to the shocks. Specifically, the variability of the monetary policy over time has the potential effect on the transmission of innovation, and the rational and forward looking agents induce additional modifications in the transmission when the macroeconomic system with the rational agent is dynamic and interconnected. The paper confirms the non-policy exogenous shocks to be more dominant than the interest rate policy to explain the occurrence of unemployment and inflation in the considered recent time in the US economy. To address the dynamic evolution of variables with interconnection among them, VAR with time varying parameters performs better (Primiceri, 2004; Cogley and Sargent, 2005). Applying a structural VAR model that allows coefficients and stochastic volatility to vary over time for post war the US, Benati and Surico (2008) report having a strong negative correlation of the persistence and predictability evolution with the long-run coefficient evolution of inflation in the monetary rule. Canova and Gambetti (2009), using 1959-1967 data for prior calibration and 1967-2006 data for estimating the model for US economy, find that in time varying model with robust sign restriction, monetary policy shock is responsible for a little amount of the level and variation of volatility and persistence in the inflation and output growth. They observe little significant evidence of the interest rate response to inflation in the long run. Cogley, Primiceri and Sargent (2010) estimating VAR with drifting coefficients and stochastic volatility, investigate if any change has taken place in the persistence of US inflation. They use similar VAR model as Cogley and Sargent (2005) and Primiceri (2004), but unlike those previous studies, they allow stochastic volatility in the state innovations (Q) as like the VAR innovation (R). Nakajima (2011) estimate a similar model of time varying VAR for Japan economy's inflation rate, output and interest rate. The study suggests that the time varying VAR model with incorporation of stochastic volatility indicates a great potential to be considered as a very flexible toolkit in analysing the structure of evolution in the modern economy. However, the study makes a few cautions about applying this model in case of a negative real interest rate. Blake and Mumtaz (2012) present a very comprehensive book of empirical Bayesian econometrics along with calibration in matlab code.

In this article, I have heavily depended on the content of Cogley and Sargent (2005) and Primicery (2004) for the theoretical framework, and Blake and Mumtaz (2012) for the empirical analysis.

3. Theoretical Framework

I have developed a multivariate stochastic volatility VAR model for the law of motion of variance covariance matrix of the observation equation to observe the response of unemployment and inflation to the monetary policy shock. The model is structural VAR in the sense that it captures well-developed economic theory. For numerical evaluation of the model with drifting coefficients and multivariate stochastic volatility, I evaluate the posterior of the parameter by using MCMC algorithm.

The model contained in this study is a multivariate state space model with observation equation and transition equation. The typical use of this type of model is in detecting structural changes in the relationships and extracting unobserved components from the data in time series set up. Since early 1990s, a considerable amount of literature has been developed that employs VAR model in the attempt of identifying and measuring the effects of monetary policy innovations on unemployment, inflation and other macroeconomic variables (Bernanke and Blinder, 1992; Sims, 1992; Bernanke, Boivin and Eliasz, 2005; Nakajima, 2011). I have used both time varying coefficients and time varying variance covariance matrix of the additive innovations in observation equation in this state space VAR model. The idea of stochastic volatility has been used extensively in the recent empirical literature, particularly in finance since Black (1976) originally proposed the idea (Black, 1976 cited in Nakajima, 2011). Currently, the empirical macroeconomic researcher has been using the stochastic volatility concept (Cogley and Sargent, 2005; Primiceri, 2004). Drifting coefficients and shocks of stochastic volatility are a common observance in the data generating process of economic variables. The consideration of time varying coefficients but a constant volatility in disturbances ignoring possible variation of the volatility raises a possible misspecification bias, so VAR model with both time varying coefficient and time varying stochastic volatility is used (Nakajima, 2011). Time varying coefficients and time varying variance covariance matrix together allow the data to determine where the time variation of the linear structure is derived from—if it is from the changes in the size of the impulse shocks or from the response to the shock, that is changes in the propagation mechanism (Primiceri, 2004). In the considered model, the devised drifting coefficients are supposed to capture the possible nonlinearities or time variation in the lag structure and the multivariate stochastic volatility to capture the possible heteroschedasticity of the shocks and nonlinearities in the simultaneous relations of the variables.

3.1.The model: general specification of the state space model

Observation equation of the state space VAR model is

$Y_t = c_t + \sum_{j=1}^{p} \beta_{j,t} Y_{t-j} + v_t$ …………………………….. (1)

Here, Y_t is a vector of endogenous variables of observed data (U, I, B)

c_t is a vector of constant terms

$\beta_{j,t}$ is a vector of VAR parameters; $\beta_t = \{\beta_{1,t} \dots\dots\dots \beta_{p,t}\}$

Y_{t-j} is a lag of Y_t

v_t is a vector of heteroschedastic unobservable shocks

This VAR can be written in a compact form as follows

$Y_t = X_t\beta + v_t$

$X_t = \{c_t, Y_{it-1}, \dots\dots\dots Y_{it-j}\}$

There are identical regressors in each equation of the VAR so that it can I rewrite as

$Vec(Y_t) = (I_N \otimes X_t)\ Vec(\beta_t) + Vec(v_t)$ for each time period t =1….. T.

$\otimes$ (Kronecker product) allows to present the transition equation in terms of VAR coefficients in vectorised form at each point in time.

The residuals, v_t is conditionally normal with mean zero and covariance matrix R;

Time varying coefficients β_t in the equation (2) below are formulated to follow the first order condition of random walk process, which allows both temporary and permanent shifts in the parameters. Following Cogley and Sargent (2005), I assume that the state vector β_t evolves as driftless random walk subject to reflecting barriers.

$\beta^T = (\beta_1', \ldots\ldots\ldots, \beta_T')'$ represents history from dates 1 to T for the VAR parameters. A joint prior of the driftless random walk component can be represented by

$p(\beta^T, Q) = p(\beta^T | Q)\, p(Q) = p(Q) \prod_{t=0}^{T-1} p(\beta_{t+1} | \beta_t, Q)$

β_t evolves in the transition equation as follows

$\beta_t = \beta_{t-1} + e_t$……………………….(2)

The innovation e_t is normal with mean zero and variance Q; $e_t \sim N(0, Q)$; $Cov(v_t, e_t) = 0$.

In this model both β_t and covariance matrix R_t are allowed to vary.

The drifting coefficient is supposed to capture a possible nonlinearity like a gradual change or structural break; in practical estimation, this assumption implies a possibility that the time varying coefficients capture some spurious movements as well as the actual movement as β_t can move freely under the random walk assumption. To avoid such situation it is advised to assume the time varying coefficients to be stationary while downside of this formulation is that even if there is structural change or permanent shift exists it is difficult to estimate (Nakajima, 2011). However, we need to specify the model in such a way that is suitable for the data set and goes with the policy prioritization. The stability condition for the VAR reflects an apriori belief about the implausibility of explosive representations for unemployment, inflation and interest rate. The stability prior follows from the belief that the Bank of England (BoE) has some purpose when sets monetary policy rules. When the BoE observes a loss function that penalizes the variance of inflation, it will take a policy rule that results compensate the loss and will not allow a unit root in inflation to end up with an infinite loss. β^T is drawn for $p(\beta^T|R, Q, Y_t)$ but ensure the stability of VAR in each point of time (Cogley and Sargent, 2005).

The reflecting barrier for stability can be encoded as an indicator function as below

$I(\beta^T) = \prod_{s=1}^{T} I(\beta_s)$

The value of the function $I(\beta_s)$ varies between 0 and 1, when the roots of the associated VAR polynomial are inside the unit circle the value is 0 and it is equal to 1 otherwise. The stability restriction truncates and renormalises the random walk prior as follows

$p(\beta^T,Q) \propto I(\beta^T)\, f(\beta^T,Q)$.

The covariance matrix of the error term (v_t) of time varying VAR, R_t has the time varying elements as I am allowing the time varying stochastic volatility. Following Blake and Mumtaz (2012), I prefer to present R_t in a simplistic way as follows

$R_t = A_t^{-1} H_t A_t^{-1'}$…………………………(3)

Here, $a_{ij,t}$ are the elements of a lower triangular matrix A_t and $h_{i,t}$ are the diagonal elements of a diagonal matrix H_t. For my three variables model, A_t and H_t can be presented as below

$$A_t = \begin{pmatrix} 1 & 0 & 0 \\ a_{12,t} & 1 & 0 \\ a_{13,t} & a_{23,t} & 1 \end{pmatrix}$$

Here, $a_{ij,t} = a_{ij,t-1} + V_t$,…………………(4)

$Var(V_t) = D$

$$H_t = \begin{pmatrix} h_{1,t} & 0 & 0 \\ 0 & h_{2,t} & 0 \\ 0 & 0 & h_{3,t} \end{pmatrix}$$

I assume that diagonal elements of H_t are independent, univariate stochastic volatilities. The elements evolve as driftless geometric random walks

$\ln h_{it} = \ln h_{it-1} + z_{i,t},$ ……………………..(5)

$Var\ (z_{i,t}) = g_i$

$z_{i,t} = \sigma_i \eta_{it}$, for i = 1, ….. 3.

Therefore, β_t and $a_{ij,t}$ are two sets of time varying coefficients in the model and the diagonal elements h_{it} entail a stochastic volatility model for in the matrix H_t. It is assumed that the standard deviations (σ_i) evolve as geometric random walks in the stochastic volatility model, which establishes a substitute for ARCH models (Primiceri, 2004).

For clarification purpose, it is worth noting the following relationship

$A_t v_t = \varepsilon_t$

Where, $Var\ (\varepsilon_t) = H_t$

For the VAR consisting of three variables the relationship implies the following set of equations

$$\begin{pmatrix} 1 & 0 & 0 \\ a_{12,t} & 1 & 0 \\ a_{13,t} & a_{23,t} & 1 \end{pmatrix} \begin{pmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \end{pmatrix} = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix}$$

It is convenient to expand the above relationship as follows

$v_{1,t} = \varepsilon_{1,t}$………………………………………………………….. (6)

$v_{2,t} = - a_{12,t} v_{1,t} + \varepsilon_{2,t}$; $Var\ (\varepsilon_{2,t}) = h_{2,t}$ ………………………….…… (7)

$v_{3,t} = - a_{13,t} v_{1,t} - a_{23,t} v_{2,t} + \varepsilon_{3,t}$; $Var\ (\varepsilon_{3,t}) = h_{3,t}$…….……….... (8)

$a_{12,t} = a_{12,t-1} + V_{1t}$, Var $(V_{1t}) = D_1$ ……………………..………. (9)

$$\begin{pmatrix} a_{13,t} \\ a_{23,t} \end{pmatrix} = \begin{pmatrix} a_{13,t-1} \\ a_{23,t-1} \end{pmatrix} + \begin{pmatrix} V_{2t} \\ V_{3t} \end{pmatrix}, \text{ Var} \begin{pmatrix} V_{2t} \\ V_{3t} \end{pmatrix} = D_2 \text{ …………………... (10)}$$

Therefore, $a_{ij,t}$ are the time varying coefficients on regressions involving the VAR residuals. Here is the main difference between the specification of this paper and the one by Cogley and Sargent (2005) paper. They take A_t as constant over time while I am taking At as changing over time. It is crucial to allow the matrix A_t to vary in a time varying structural VAR because otherwise an innovation to the i^{th} variable would make a time invariant effect on the j^{th} variable. Hence, to model time variation in a simultaneous equations set up, among variables simultaneous interactions are necessary (Primiceri, 2004).

For permanent shifts in the innovation variance, the random walk specification is devised. Independent of one another and of the other shocks in the model, the volatility innovations $z_{i,t}$ are standard normal random variables. σ_i is a scale free parameter in the volatility innovations to determine their magnitude. R_t is a positive definite confirmed by the factorisation in equation (3) and log specification in equation (5). Correlation among the elements of v_t is allowed because of the free parameters in A_t. The matrix A_t orthogonalizes v_t, however, this is not considered as an identification scheme.

Cogley and Sargent (2005) found that with the stability condition imposed the ordering of the variables in the VAR model has little evidence of differences in posterior estimate of Q and hence shifting drift in β_t. As specification of model in this paper is mostly similar to that of Cogley and Sarget (2005), I have not cared much about the ordering of the variables.

In drawing posterior, I cannot apply Carter and Kohn algorithm only because observation equation in my state space model is non-linear in the state variable h_{it}. As stochastic volatility h_{it} makes the observation equation of the state space model, I need to use independence MH method as Jacquier, Polson, and Rossi (2004) suggested to apply this algorithm at each point in time to sample from the conditional distribution of h_{it} rather than the carter and Kohn algorithm. MH method offers an alternative algorithm to Gibbs sampling when conditional distribution of the parameter is not available in the closed form. Thus this VAR model can be estimated by combining the Carter and Kohn algorithm to draw β_t and $a_{ij,t}$, the independence Metropolis Hastings (MH) algorithm for the stochastic volatility. Time varying coefficients $a_{ij,t}$

in the regression of VAR residual can be sampled similarly as β_t as contain similar characteristics.

3.2.Forecast using the model: IRF and FEVD

The purpose of forming the model and algorithm is to calculate the impulse response function (IRF) of the three macroeconomic variables to the shock to policy variable and decompose the error variance. An impulse responses trace out the response of each of the variables, that is unemployment, inflation and interest rate, to a one-unit change in the current value of interest rate in the VAR errors. In calculating the impulse response, it is assumed that all other errors are equal to zero and the interest rate error returns to zero in subsequent periods. For calculating the impulse response the coefficients and error obtained from the above model for the selected variables can be presented as follows

$$\underset{(Z_t)}{\begin{pmatrix} Ut \\ It \\ Bt \end{pmatrix}} = \underset{(c)}{\begin{pmatrix} \widehat{C1} \\ \widehat{C2} \\ \widehat{C3} \end{pmatrix}} + \underset{(A)}{\begin{pmatrix} \widehat{\beta 11} & \widehat{\beta 12} & \widehat{\beta 13} \\ \widehat{\beta 21} & \widehat{\beta 22} & \widehat{\beta 23} \\ \widehat{\beta 31} & \widehat{\beta 32} & \widehat{\beta 33} \end{pmatrix}} \underset{(Z_{t-1})}{\begin{pmatrix} Ut-1 \\ It-1 \\ Bt-1 \end{pmatrix}} + \underset{(E_t)}{\begin{pmatrix} v1 \\ v2 \\ v3 \end{pmatrix}}$$

$$\text{Var}\begin{pmatrix} v1 \\ v2 \\ v3 \end{pmatrix} = \sum = A_0'A_0$$

Before shock occurring assume Z_0 is zero and $\hat{c}$ is zero.

Because E_t are correlated rewrite the model in terms of orthogonalized shock to distinguish the shock effect.

$$\begin{pmatrix} v1 \\ v2 \\ v3 \end{pmatrix} = \underset{(\text{Square root of } \sum,\ A_0)}{\begin{pmatrix} A11 & A21 & A31 \\ A12 & A22 & A32 \\ A13 & A23 & A33 \end{pmatrix}} \quad \underset{(U_t\text{: Uncorrelated/ orthogonal shocks})}{\begin{pmatrix} \mu 1t \\ \mu 2t \\ \mu 3t \end{pmatrix}}$$

So the VAR model become

$$\begin{pmatrix} U1 \\ I1 \\ B1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \widehat{\beta 11} & \widehat{\beta 12} & \widehat{\beta 13} \\ \widehat{\beta 21} & \widehat{\beta 22} & \widehat{\beta 23} \\ \widehat{\beta 31} & \widehat{\beta 32} & \widehat{\beta 33} \end{pmatrix} \begin{pmatrix} I0 \\ U0 \\ B0 \end{pmatrix} + \begin{pmatrix} A11 & A21 & A31 \\ A12 & A22 & A32 \\ A13 & A23 & A33 \end{pmatrix} \begin{pmatrix} \mu 1 \\ \mu 2 \\ \mu 3 \end{pmatrix}$$

Or $Z_t = AZ_{t-1} + A_0U_t$ ……………………(11)

In the first period $Z_0 = \begin{pmatrix} I0 \\ U0 \\ B0 \end{pmatrix}$ is assumed to be 0.

So if shock is given in period 0 it is transmitted as

$$\begin{pmatrix} U1 \\ I1 \\ B1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \widehat{\beta 11} & \widehat{\beta 12} & \widehat{\beta 13} \\ \widehat{\beta 21} & \widehat{\beta 22} & \widehat{\beta 23} \\ \widehat{\beta 31} & \widehat{\beta 32} & \widehat{\beta 33} \end{pmatrix} \begin{pmatrix} U0 \\ I0 \\ B0 \end{pmatrix} + \begin{pmatrix} A11 & A21 & A31 \\ A12 & A22 & A32 \\ A13 & A23 & A33 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} A31 \\ A32 \\ A33 \end{pmatrix}$$

The given shock in period 0 is transmitted to period 2 as

$$\begin{pmatrix} U2 \\ I2 \\ B2 \end{pmatrix} = \begin{pmatrix} \widehat{\beta 11} & \widehat{\beta 12} & \widehat{\beta 13} \\ \widehat{\beta 21} & \widehat{\beta 22} & \widehat{\beta 23} \\ \widehat{\beta 31} & \widehat{\beta 32} & \widehat{\beta 33} \end{pmatrix} \begin{pmatrix} U1 \\ I1 \\ B1 \end{pmatrix}$$

$$\begin{pmatrix} U2 \\ I2 \\ B2 \end{pmatrix} = \begin{pmatrix} \widehat{\beta 11} & \widehat{\beta 12} & \widehat{\beta 13} \\ \widehat{\beta 21} & \widehat{\beta 22} & \widehat{\beta 23} \\ \widehat{\beta 31} & \widehat{\beta 32} & \widehat{\beta 33} \end{pmatrix} \begin{pmatrix} A31 \\ A32 \\ A33 \end{pmatrix} = \begin{pmatrix} A31\widehat{\beta 11} + A32\widehat{\beta 12} + A33\widehat{\beta 13} \\ A31\widehat{\beta 21} + A32\widehat{\beta 22} + A33\widehat{\beta 23} \\ A31\widehat{\beta 31} + A32\widehat{\beta 32} + A33\widehat{\beta 33} \end{pmatrix}$$

and so on

Therefore, the impulse response for shock in period 0 is cumulated in future as

Period	Unemployment (U)	Inflation (I)	Interest rate (B)
1	A31	A32	A33
2	$A31\widehat{\beta 11} + A32\widehat{\beta 12} + A33\widehat{\beta 13}$	$A31\widehat{\beta 21} + A32\widehat{\beta 22} + A33\widehat{\beta 23}$	$A31\widehat{\beta 31} + A32\widehat{\beta 32} + A33\widehat{\beta 33}$
……	…….	…….	…….

FEVD traces out the percentage of the variance of the error made in forecasting a variable due to a specific shock to the error term in the bank rate equation at a given horizon, for this model the horizon is 10 years or 40 quarters. Error variance decomposition can be represented as follows for a two period simplified version.

The two period ahead forecast, for example, is presented as

$\hat{Z}_{t+1} = c + AZ_t$

$\hat{Z}_{t+2} = c + A\hat{Z}_{t+1} = c + A\,(c + AZ_t) = c + Ac + A^2Z_t$

The true value Z_{t+2} and the estimated value $\hat{Z}_{t+2}$ will be different because of future shock U_2. The FEVD calculates the degree of importance of the shocks in driving the variance of the forecast error. We need to calculate the actual Z_{t+2} for calculating the FEVD.

$Z_{t+1} = c + AZ_t + A_0U_{t+1}$

$Z_{t+2} = c + AZ_{t+1} + A_0U_{t+2} = c + A\,(c + AZ_t + A_0U_t) + A_0U_{t+2} = c + Ac + A^2Z_t + AA_0U_{t+1} + A_0U_{t+2}$

……………………

$Z_{t+40} =$ …………….

The forecast error is for two period

$\zeta = Z_{t+2} - \hat{Z}_{t+2} = AA_0U_{t+1} + A_0U_{t+2}$

Var (ζ)= $[AA_0]^2$ Var (U_{t+1})+ $[A_0]^2$ Var (U_{t+2}) =[AA_0(IRF periodd2) $]^2$+ [A_0 (IRF period1)$]^2$
Var (ζ (i)) is the cumulative sum of the square of the impulse response to shock i
Mean square error (mse) is the total variance in the model due to all the shocks in in the model.
mse= Var (ζ(I)) + Var (ζ(U)) + Var (ζ(B))
FEVD here is the variance of the error made in forecasting three variables due to a specific shock to the error term in all variables at the given 40 quarters horizon.
Therefore, the FEVD for i variable is
FEVD (i) = Var (ζ(i)) / mse

3.3.Bayesian inference

This paper estimates the state space model with time varying state variables. I have used Bayesian methods to evaluate the posterior distributions of the parameters to deal with unobservable components. A Bayesian approach is the commonly used when the parameters and shocks are less clearly distinctive than in other situations. Primiceri (2004) mentions three other factors favouring the suitability of Bayesian methods for estimating this type of models and prefers Bayesian to classical approach. a) The classical maximum likelihood estimator of the variance generates pile-up problem that gives a point mass at zero in case of small variance of the time varying coefficients; b) There is high chance of getting a likelihood with multiple peaks in classical model when there are high dimensionality and nonlinearity in the parameter. In addition to the possibility of the peaks being uninteresting or implausible regions of the parameter space, the likelihood may reach particularly high values if peaks are very narrow. Consequently, the representative may not fit on a wider and interesting parameter region; c) Though in principle it is possible to write up the likelihood of the model, in reality maximising the functions over such a high dimensional space is a difficult task. In a Bayesian setting the use of uninformative priors of the parameter space on reasonable regions helps effectively to rule out these misbehaviours. Bayesian method splits the original estimation problem in smaller and simpler one, and hence is efficient to deal with the high dimensional parameter space and the nonlinearities.

In this study, I have used Gibbs sampler for numerical evaluation of posterior of the parameters. Due to the non-linearity of state space model derived from the stochastic volatility maximum likelihood estimation entails heavy computational burden to repeat the filtering for many times in evaluating the likelihood function for each set of parameters until reach at maximum. In such situation MCMC method provides a precise and efficient estimation. A particular variant of MCMC methods, Gibbs sampling draws from lower dimensional conditional posteriors as opposed to the high dimensional joint posterior of the whole parameter set. Being a smoothing method MCMC generates smoothed estimates, that is, the estimates of the parameters is based on the entire available set of data. However, the appropriateness of smoothed estimates compared to filtered ones cannot be established apriori, but depends on the specific problem we are dealing with. For constructing model diagnostics or forecasting evaluation, filtered estimates are more appropriate but smoothed estimates are more efficient for investigating the true evolution of the unobservable states over time. In such cases, filtered estimates are inappropriate but smoothed one as they may have transient variation even if the model is time invariant.

The history of a variables M^T in generic vector up to a time period T can be expressed as

$M^T = [m'_1, \ldots\ldots\ldots\ldots\; m'_T]'$

In this model the representation is as follows

$Y^T = [y'_1, \ldots\ldots\ldots\ldots\; y'_T]'$ and

$$H^T = \begin{pmatrix} h_{11} & h_{21} & h_{31} \\ h_{12} & h_{22} & h_{32} \\ \cdots & \cdots & \cdots \\ h_{1T} & h_{2T} & h_{3T} \end{pmatrix}$$

Y^T and H^T represent the history of data and stochastic volatilities up to date T respectively.

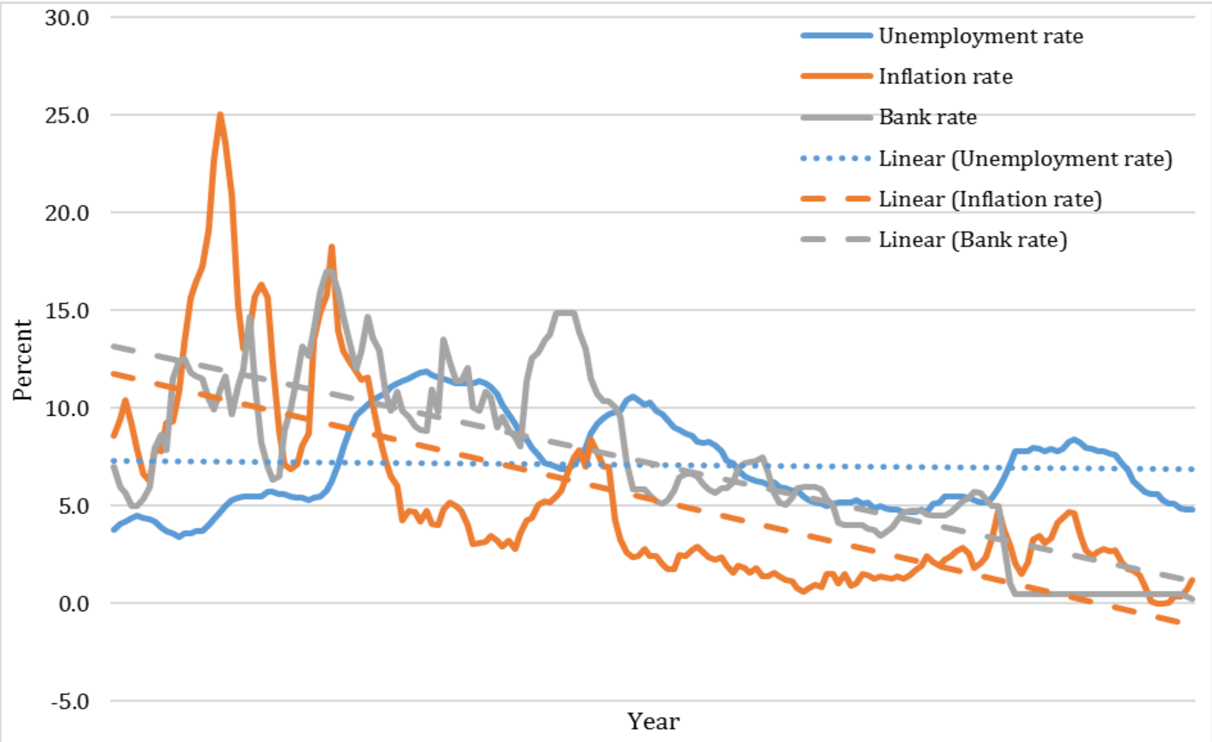
I shall use MCMC methods to simulate the posterior density. η_{it} and ε_t are assumed to be independent in this stochastic volatility model to simplify the algorithm greatly for simulating the posterior distribution.

4. Data and Sources (1971:Q1-2016: Q4)

For this research, I have collected data for all three variables from the Central Bank, BoE. The interest rate variable is captured by the quarterly average of monthly bank rate series. The Bank rate is considered as the single most important interest rate in the UK, which is set usually eight times a years by the Monetary Policy of Committee of the Central Bank. The BoE adjusts bank rate in order to reach the inflation target (BoE website). The inflation is captured as CPI inflation rates for wedge adjustment, and the unemployment is labour force survey unemployment rate. All variables are quarterly for the period 1971-2016, from first quarter 1971 to fourth quarter 2016. Therefore total number of observation is 184.

Figure1 presents the trend of the variables from first quarter of 1971 to fourth quarter of 2016. The figure shows that in the first half of the sample period inflation was very high at some points and more volatile. Unemployment and interest rate were also high with comparatively more volatility than the second half of the observation period. While over the whole period unemployment trend is nearly unchanged, both interest rate and inflation followed downward trends, shown in the dotted linear trend line. In the first half of the period, it seems unemployment and inflation broadly followed opposite direction while in the latter half observes similar direction. The naked eye observation in this figure is that the unemployment in general follows a standard natural rate path that is not much volatile and inflation follows the similar path of the interest rate.

Figure 1: Data plotting and linear trend line for 1971Q1-2016Q4



Source: Author's presentation from the dataset

Descriptive statistics of the variables are presented in the Table1. For the 184 observations, interest rate observed the highest mean followed by unemployment and inflation. The largest difference between maximum and minimum value is observed for inflation (25.06 percentage point), the difference for interest rate is 16.75 percentage point and for unemployment is 8.50 percentage point. In the same token the largest standard deviations is for inflation followed by interest rate and unemployment. The highest unemployment was in 1984Q2 and lowest in 1973Q4, the highest inflation rate was observed in 1973Q3 and lowest in 2015Q2. Since 2009Q2 bank rate has been fixed at 0.50.

Table 1: Descriptive Statistics of the Variables

Variables	Observation	Mean	Std. Dev.	Min	Max
Unemployment rate	184	7.08	2.33	3.40	11.90
Inflation rate	184	5.34	5.09	- 0.02	25.04
Interest rate	184	7.17	4.45	0.25	17.00

Source: Authors calculation from dataset

5. Estimation Framework

For estimating the time-varying parameters in the VAR model, the Gibbs and MH algorithm consists of the follows steps.

Step1: Setting prior and obtaining starting values using the training sample

Step2: Drawing β_t conditional on A_t, H_t, and Q using the Carter and Kohn algorithm; $\beta_t| A_t, H_t, Q$.

Step3: Sampling Q using the draw of β_t; $Q_t| A_t, H_t, \beta_t$.

Step4: Drawing $a_{ij,t}$ the elements of A_t applying the Carter and Kohn algorithm; $A_t| H_t, \beta_t, Q_t$

Step5: Calculating the residuals V_{1t}, V_{2t}, and V_{3t}

Step6: Calculating $A_t v_t = \varepsilon_t$

Step7: Drawing gi

Estimation framework in this section is mostly in line with Blake and Mumtaz (2012) as I have used heavily the contents of this book.

5.1. Priors and ordering

In Bayesian inference process, the priors are chosen depending on intuitiveness and convenience in the application. If the unknown parameter is θ, for example, the prior density need to be specified for the parameters., for instance, π(θ)

$$\pi(\theta|y) = \frac{f(y|\theta)\pi(\theta)}{\int f(y|\theta)\pi(\theta)d\theta}$$

π(θ|y) is posterior distribution, f(y|θ) is the likelihood function for the data y.

General practice is that the prior information concerning the unknown parameter is updated by observing the data y (Nakajima, 2011). Assuming the independence of initial states for the coefficients, covariance and log volatilites, and hyperparameters is convenient (Primiceri, 2004). I assume the hyperparameters and initial states are independent across blocks, as a result, the joint prior can be expressed as the product of marginal priors as follows

$p(\beta_0, h_{10}, h_{20}, h_{30})$

$=p(\beta_0)\ p(h_{10})\ p(h_{20})\ p(h_{30})\ p(Q_0)\ p(\sigma_1)\ p(\sigma_2)\ p(\sigma_3)$

Normal distribution is assumed for the priors for initial states of the time varying coefficients $p(\beta_0)$, simultaneous relations $p(a_0)$, and standard errors $p(\sigma_0)$. These assumptions together with (3), (4) and (5) imply normal priors on the entire sequences of the β's, a's and σ's (conditional on Q, D and g).

As a typical practice in setting the prior, I calibrate the prior distributions using the observations of first 10 years (40 observations, from 1971: Q3 to 1981: Q2) as a training sample. β_0 is estimated from a standard fixed coefficient VAR applying OLS for the initial small sample, $\beta_0 = (X'_{0t}X_{0t})^{-1}(X'_{0t}X_{0t})$ and the coefficient covariance matrix is given by $P_{0|0} = \sum_0 \otimes (X'_{0t}X_{0t})^{-1}$, where $X_{0t} = \{Y_{0t-1},\ldots Y_{0t-p}, 1\}$, $\sum_0=\frac{(Y0t-Xot\beta_0)'(Y0t-Xot\beta_0)}{T0-K}$. I set the prior for Q, D and $h_{ij,t}$ as follows.

I begin with setting a prior for Q and starting values for the Kalman filter. The marginal prior p(Q) for a multivariate model makes Q an inverse-Wishart variate, $p(Q) \sim IW(Q_0,T_0)$. The prior of Q influences the amount of time variation in the VAR model, a large value of the scale matrix Q_0 would mean more fluctuation in the β_t. Therefore, it is critical to fix prior in a prudential way. Inverse-Wishart prior distributions need the degrees of freedom and scale matrices of the hyperparameters. The scale matrix Q_0 is set equal to $P_{0|0}{*}T_0{*}\tau$, here τ is a scaling factor. Sometimes a small number $\tau=3.5*10^{-4}$ is used to reflect the fact the training sample in typically short and the resulting estimates of $P_{0|0}$ may be imprecise. Varying τ can control the amount of time-variation in the model. Set the starting value for Q, the initial state is set equal to $\beta_{0|0} = vec(\beta_0)'$ and the initial state covariance is given by $P_{0|0}$.

After Q, I set the prior for D_1 and D_2; the prior for D_1 is inverse Gamma $p(D_1) \sim IG\ (D_{10}, T_0)$ and the prior for D_2 is inverse Wishart $p(D_2) \sim IW\ (D_{20}, T_0)$. Following Benati and Mumtaz (2006) I use D_{10}=0.001 and $D_{20}=\begin{pmatrix} 0.001 & 0 \\ 0 & 0.001 \end{pmatrix}$. I take $C = \sum_0^{1/2}$ and C_0 as the inverse of the matrix C that has the diagonal elements normalised to 1. The initial state is $a_{ij,0|0}$ for $a_{ij,t}$ values. The $a_{ij,t}$ is the non-zero elements of C_0 and as in the Benati and Mumtaz (2006), the variance of the initial state set equal to $abs(a_{ij})*10$. Set a starting value for $a_{ij,t}$.

I obtain a starting value for $h_{i,t}$, i=1..3 and t=0….T as $\widehat{vit}^2$. I set the prior $\overline{ui}$, $\bar{\sigma}$. Here, $\overline{ui}$ can be set equal to the log of the ith element of the diagonal of $\sum_0$ and $\bar{\sigma}$ to a large number. Set an inverse Gamma prior for g_i like p(gi) ~ IG(g0, v0). Set the starting value for gi.

Usually, the degrees of freedom are set to the dimension of each matrix plus one. Degrees of freedom are chosen differently because for proper inverse-Wishart distribution the degrees of freedom need to exceed the dimension respectively to g and the blocks of A. Because a slightly tighter prior is necessary in order to avoid implausible behaviours of the time varying coefficients, I take the size of the initial subsample 40 as the degrees of freedom Q. Following the literature (Cogley and Sargent, 2005, Cogley, 2003, Primicery, 2004), I have chosen the scale matrices, Q, gi, D_1 and D_2, as constant fractions of the variances of the corresponding OLS estimates on the initial subsample.

5.2.Simulation method and model estimation

Given the data, the model is estimated by simulating the distribution of the required parameters. Applying the MCMC algorithm a sample from the joint posterior of β^T, A^T and H^T has been generated. In exploiting the blocking structure of the unknowns Gibbs sampling has been used in order. Gibbs sampling is carried out in four steps, drawing time varying coefficients (β^T), simultaneous relations (A^T), volatilities (H^T) and hyperparameters (Q and R) conditional on the observed data and the rest of the parameters in turn. The following steps has been followed to draw the posterior distribution for the parameter of interest

Draw β_t conditional on A_t, H_t and Q using the Carter and Kohn algorithm. The only difference of the algorithm in drawing β_t from the VAR with and without time varying stochastic volatility is that in time varying case the variance of v_t changes at each point in time and this needs to be taken into account when run the Kalman filter. The detailed algorithm for drawing β_t from Carter and Kohn algorithm is presented in Appendix A1.

Using the draw for β_t, I calculate the residuals of the transition equation $\beta_t - \beta_{t-1} = e_t$. Then with the help of the scale matrix $e_t'e_t + Q_0$ and degrees of freedom $T + T_0$, sample the Q from the inverse-Wishart distribution. The innovation variance Q in the unrestricted transition equation for VAR parameters is influential in determining the movement of β_t, larger values Q imply rapid movements in β_t, smaller values mean a slower rate of drift, and Q=0 gives a time-invariant model. The stability constraint pushes the system away from the unit root boundary and amplifies evidence for drift in β_t. With imposition of stability condition, posterior mean estimates for Q are smaller, and withdrawal of the stability prior increases the rate of drift in β_t. I have imposed the stability condition in the estimation process.

Conditional on β_t, H_t, D_1 and D_2, I draw the elements of A_t, $a_{ij,t}$, using the Carter and Kohn algorithm. The state space formulation for $a_{12,t}$ are (7), (9) and the state space formulation for $a_{13,t}$ and $a_{23,t}$ are (8), (10). These two formulations are time varying regressions in the residuals and the Carter and Kohn algorithm is applied similarly as β_t to each separately for drawing $a_{12,t}$ $a_{13,t}$ and $a_{23,t}$.

Conditional on a draw for $a_{12,t}$ $a_{13,t}$ and $a_{23,t}$ calculate the residuals V_{1t}, V_{2t} and V_{3t}. Draw D_1 from the inverse-Gamma distribution with scale parameter $\frac{V1t`V1t+D1,0}{2}$ and degrees of freedom $T+T_0$. Draw D_2 from the inverse-Wishart distribution with scale matrix $V_{2t}'V_{2t} + D_{20}$ and degrees of freedom $T+T_0$.

Using the draw of A_t calculate $\varepsilon_t = A_t v_t$ where $\varepsilon_t = \begin{pmatrix} \varepsilon 1t \\ \varepsilon 2t \\ \varepsilon 3t \end{pmatrix}$. ε_t are contemporaneously uncorrelated. We can therefore draw $h_{i,t}$ for i=1…3 separately by merely applying the independence MH algorithm described above for each ε_t conditional on a draw for g_i. The detailed distribution and algorithm are presented in Appendix A2.

Conditional on a draw for $h_{i,t}$ for i = 1..3, draw gi from the inverse Gamma distribution with scale parameter $\frac{(lnhi,t-lnhi,t-1)`(lnhi,t-lnhi,t-1)+g0}{2}$ and degrees of freedom $\frac{T+v0}{2}$. This is the combination of MH and Gibbs sampling algorithm.

The simulations are based on 50000 iterations of the Gibbs sampler, discarding the first 49000 for convergence. From the total draw, the last 1000 draws provide an approximation to the marginal posterior distributions of the model parameters. The recursive means of the retained draws for the time-varying parameter VAR has been considered for checking convergence of the Gibbs sampler. The result is presented as three-dimensional surface diagram for the impulse response and for error variance decomposition. The samples run from 1971:Q1 to 2016:Q4. I have used two lags for the estimation. After taking 2 lags and 40 training sample the posterior density is drawn from 1981: Q3 to 2016: Q4.

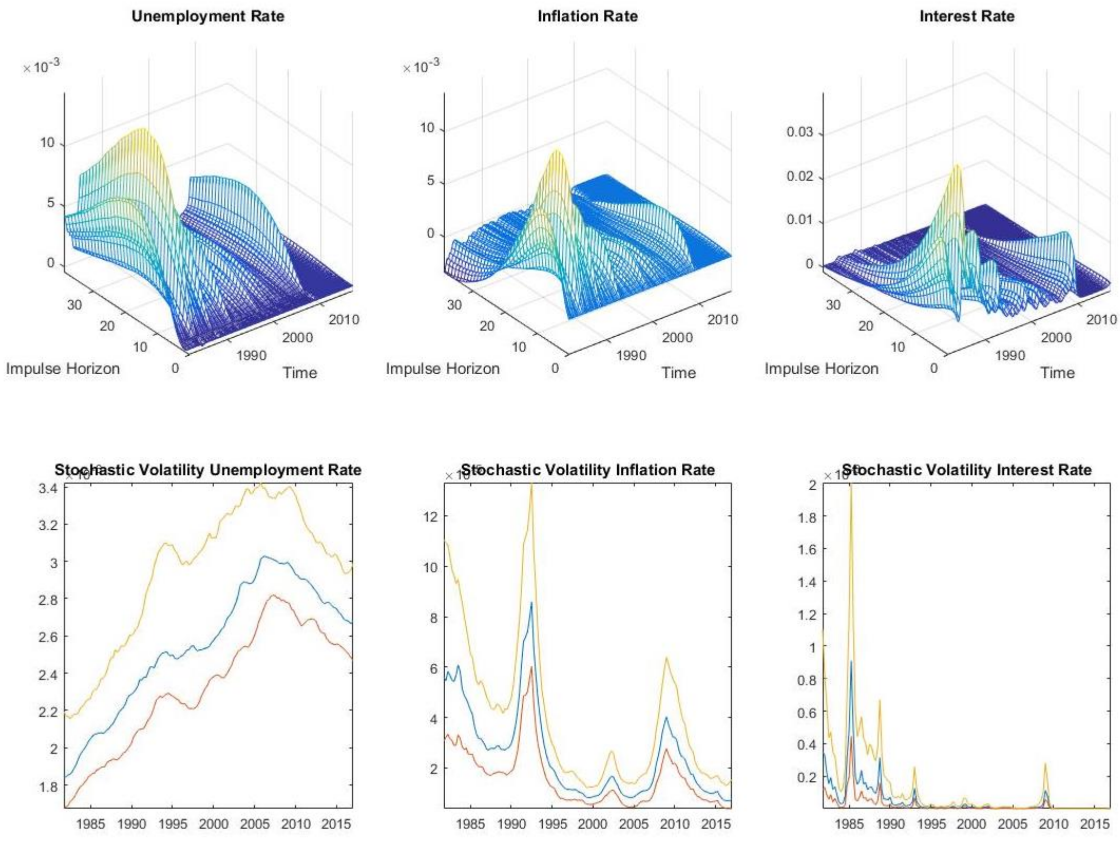
6. Results

Measuring the impulse response of the variables to monetary policy shocks is the primary focus in this research, so I reported the impulse response of the variables. The estimated impulse response to the shock and the estimated stochastic volatility are plotted in Figure 2.

The surface diagram from Monte Carlo simulation gives some interesting response results. The impulse responses do not die out toward zero in the long forecast horizon ever after 10 years (40 quarters). This is more applicable for the unemployment rate, which never shows any indication of zero response. Interest and inflation rate response show a signal to end up with zero response in the later part of the impulse horizon. According to Mankiew (2001), for long-run neutrality the estimated impulse response need to die out toward zero. The impulse response result generates draws some interesting observations here. Bernanke and Mihov (1998) do not highlight the large impact of monetary policy even after ten years on GDP in point estimates because the standard errors rise with the time horizon and thus for far enough the estimated impact becomes statistically insignificant. However, considering that if one does not approach the data with a prior view favouring long-run neutrality, one would not leave the data with that posterior, Mankiew (2001) emphasise the data's best guess that monetary shocks leave permanent scars on the economy. This research has been approached the data with a prior view.

This result could be aligned with some well-supported category of research findings that there is no long run trade-off between unemployment and inflation, there is long run trade-off, there is short term relationship or even not. No trade-off view comes from one group of commonly accepted literature that takes monetary policy as insignificant in the long run. In this paper monetary policy shock is only responded by the unemployment rate and has little influence on the long run inflation rate. The co-movement of unemployment and inflation response to shock is in same direction up to certain horizon then inflation starts to fall but not unemployment. While current day researchers almost agreed on no long-run trade off between unemployment and inflation, most cases they accept the case of short run trade-off. However, at present this topic has been puzzling the researcher when both unemployment and inflation are low. The BoE adopted the principle of inflation targeting (IT) in October 1992 and the changes introduced in May 1997. The strategy has been successful in terms of keeping the UK inflation rates within the targets, however, other countries including USA, non-IT country has also been successful in this regards (Angeriz and Arestis, 2007; Goodfriend, 2007). In bare eye, I can see the relationship in different dimensions but whether they are significant or not is unknown at this point. So it might not be the case why monetary policy can directly influence inflation or not. From the point that people set their expectation for inflation which influence the natural rate of unemployment, it is good to give an expectation benchmark for inflation through credible monetary policy target. As Primiceri (2004) concludes, even though there is little evidence for a causal link between changes in systematic responses of interest rate, and the unemployment and inflation episodes, this is not a statement about neutrality of monetary policy. However, in this research, I do not make any specific conclusion about their relation, as I have not done any test for significance.

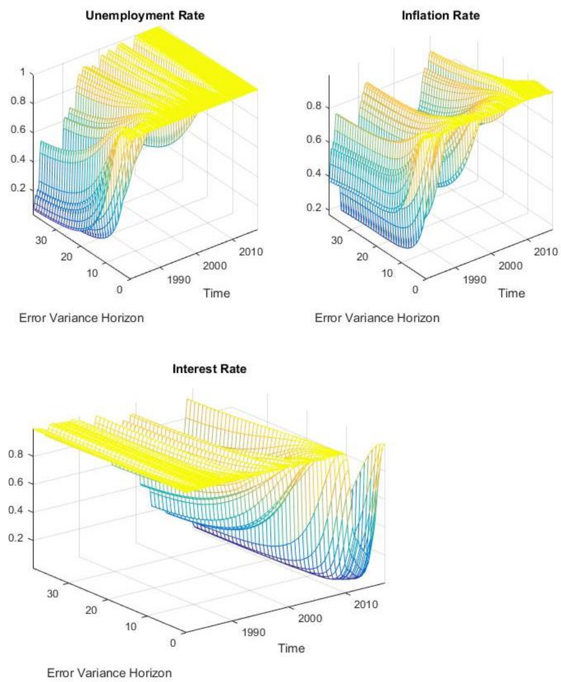
Figure 2: Impulse response to an interest rate shock (top panel) and the estimated stochastic volatility (bottom panel)



The bottom panel in the figure 2 presents the estimated stochastic volatility in the variables. Since 1990s volatilities in all the variables reduced profoundly except unemployment. The highest volatility in the unemployment is observed around 2010 which had been increasing since 2000s. Similar volatility appearances for inflation too. Highest volatility in interest rate is found in around 1985 and there is a small peak in recent time. For all three variables stochastic volatilities featured highest peak in recent time is around 2010, which is very much the result of 2008 financial crisis. The estimated stochastic volatility has been highly reduced for interest rate and somewhat for the inflation. In contrary opposite has happened for unemployment. These pattern of stochastic volatility may makes response of unemployment to shock comparatively less reliable than the response of inflation. One of the reason of reducing stochastic volatility of inflation could be the monetary policy activism of targeting the inflation, while unemployment is responsive to different policy indirectly.

FEVD is the related concepts to the impulse response function. FEVD represents the percentage of variance of the error made in forecasting a variable due to shock to the specific variable at the horizon of 10 years. The result suggests considerable interaction among the variables. FEVD shows that there is substantial part of error variation in the model due to the shocks even in the distant horizon, particularly in case of the interest rate. This indicates that the shocks are influential in the error variance in the forecast horizon. This variance decomposition is consistent with the variance decomposition of the Federal Funds interest rate in Stock and Watson (2001).

Figure 3: Forecast error variance decomposition



The MCMC algorithm is applied to calculate posterior using 50000 iterations and discarding first 49000 as burn-in. The recursive means presented in the figure (Appendix B) has been calculated every 20 draws for the retained draw for β_t, h_{it} and $a_{ij,t}$. The X-axis of each panel represents these parameterised vectorised, the Y-axis represents the draws. The recursive means suggest convergence has almost taken place for β_t and a_{ij}, as there is tiny fluctuations in the figure. However, the recursive mean for h_{it} shows more fluctuations, probably needs more iteration for convergence. This is because here Jacquier, Polson, and Rossi (2004) suggested algorithm has been used which is a single-move algorithm so requires a large number of draws before convergence occurs.

7. Conclusions

By developing the MCMC algorithm for Bayesian VAR with stochastic volatility for the UK data between 1971 and 2016, I have calculated the impulse response of the variables to monetary policy shock. Estimated stochastic volatility and FEVD are useful as a supplement of the impulse response to understand the data behaviour and importance of shock to different variables in the forecast horizon. Bayesian VAR gives more precise estimation result compared to the classical VAR because it incorporates the prior in producing posterior. I allow both parameter and stochastic volatility to vary to capture the possible non-linearity or drifting coefficients in the model. To calibrate the state space model with both time varying parameter and stochastic volatility, I have applied the Carter and Kohn algorithm for parameters and MH algorithm for stochastic volatility in producing posterior densities.

The calculated impulse response of unemployment, inflation and interest rate to the interest rate shock shows that the impulse response does not die out entirely toward zero at the 10 year horizon. Although estimated stochastic volatility is downward trending with some high picks for inflation and interest rate, this is upward trending for the unemployment. FEVD indicates that shock to the variables have a substantial effect on the error variance of the variables in the forecast horizon.

However, as the methodological issue is the primary focus of this research, I have not conducted any significance test for the obtained result so do not draw the specific conclusion. Ample scope remains to apply and develop further this method in forecasting, determining relationships, exploring more the issues concerning methods, country experience and observation period. With some additional test, this research can be used to explain the effect of the shock to the economy and specify the relationship between unemployment and inflation.

APPENDICES

Appendix A1: Drawing β_t (conditional on A_t, H_t and Q) applying Kalman filter and Carter and Kohn algorithm

We can compute the mean ($\beta_{t|t}$) and the variance ($P_{t|t}$) using the Kalman Filter. Kalman filter consists of the following equations which are evaluated recursively through time starting from an initial $\beta_{0|0}$ and $P_{0|0}$

$\beta_{t|t-1} = \mu + F\beta_{t-1|t-1}$

$P_{0|0}= FP_{t-1|t-1}F` + Q$

$\eta_{t|t-1}= Y_t - H\beta_{t|t-1}$

$f_{t|t-1}= HP_{t|t-1}H`+R$

$\beta_{t|t} = \beta_{t|t-1} + K\eta_{t|t-1}$

$P_{t|t} = P_{t|t-1} + KHP_{t|t-1}$

$K=P_{t|t-1}H`f^{-1}_{t|t-1}$

These equations for t=1,…….,T deliver $\beta_{T|T}$ and $P_{T|T}$ at the end of recursion.

The conditional distribution of the state variable

$P(\beta^T|Y^T) = P(\beta^T|Y^T) \prod_{t=1}^{T-1} f(\beta_{t|}\beta_{t+1})$

$\beta^T \sim N(\beta_{T|T}, P_{T|T})$

$\beta_t|\beta_{t+1} \sim N(\beta_{t|t,\beta_{t+1}}, P_{t|t,\beta_{t+1}})$

$\beta^T \sim N\left(\beta_{T|T}, P_{T|T}\right)$is already presented by Kalman filter. The computation of the mean and variance in $N(\beta_{t|t,\beta_{t+1}}, P_{t|t,\beta_{t+1}})$ requires

$\beta_{t|t}, \beta_{t+1} = \beta_{t|t} + P_{t|t}F`(FP_{t|t}F`+Q)^{-1}(\beta_{t+1}-\mu-F\beta_{t|t})$

$P_{t|t}, \beta_{t+1} = P_{t|t} - P_{t|t}F`(FP_{t|t}F`+Q)^{-1}FP_{t|t}$

These are computed going backwards in time from period from t−1 to 1.

Similar approach as β_t in A1, just need to do separately for $a_{12,t}$ (conditional on β_t, H_t, D_1), and $a_{13,t}$ and $a_{23,t}$ (conditional on β_t, H_t, D2)

Appendix A2: Drawing h_{it} of H_t applying MH algorithm

Conditional distribution of h_t

$f(h_t | h_{-t}, y_t)$ (-t is all other dates than t)

As the transition equation of the model is a random walk, the knowledge of h_{t+1} and h_{t-1} contains all relevant information of h_t. Therefore, h_t can be simplified as

$f(h_t | h_{-t}, y_t) = f(h_t | h_{t-t}, h_{t+1}, y_t)$

The density is

$f(h_t | h_{t-t}, h_{t+1}, y_t) = h_t^{-0.5} \exp(-y_t^2/2h_t) * h_t^{-1} \exp\{(-\ln h_t - \mu)^2/2\sigma_h\}$

$\mu = (\ln h_{t+1} + \ln h_{t-1})/2$

$\sigma_h = g/2$

Simplifying some algorithm, acceptance probability can be obtained

$\alpha = \{h_{t,new}^{-0.5} \exp(-y_t^2/2h_{t,new})\} / \{h_{t,old}^{-0.5} \exp(-y_t^2/2h_{t,old})\}$ (see Blake and Mumtaz, 2012)

Initial value of h_t is h_0, the prior $\ln h_0 \sim N(\bar{u}, \bar{\sigma})$ and posterior for $\ln h_0$ is

$f(h_0 | h_1) = h_0^{-1} \exp\{(-\ln h_0 - \mu_0)^2/2\sigma_0\}$

here $\sigma_0 = \bar{\sigma}g/(\bar{\sigma}+g)$

$\mu_0 = \sigma_0 \{(\bar{u}/\bar{\sigma}) + (\ln h_1/g)\}$

Modified candidate generating density

$q(\Phi^{G+1}) = h_t^{-1} \exp\{(-\ln h_t - \mu)^2/2\sigma_h\}$

$\mu = \ln h_{t-1}$

$\sigma_h = g$

The steps of the MH algorithm for the stochastic volatility model consists of the following

Step 1: Obtain a starting value for h_t, t = 0....T as $\hat{\varepsilon}^2$ and set the prior $\bar{u}, \bar{\sigma}$. Set an inverse Gamma prior for g. Set starting value for g.

Step 2: Time 0, Sample the initial value of h_t denoted by h_0 from the log normal density

$f(h_0 | h_1) = h_0^{-1} \exp\{(-\ln h_0 - \mu_0)^2/2\sigma_0\}$

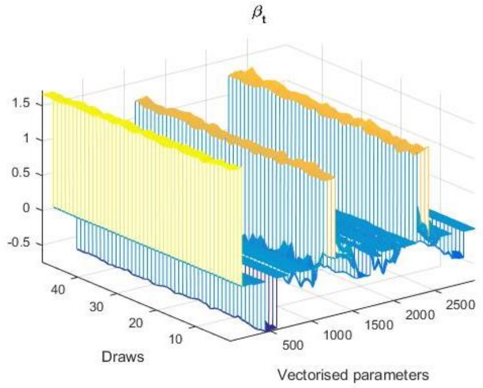
here

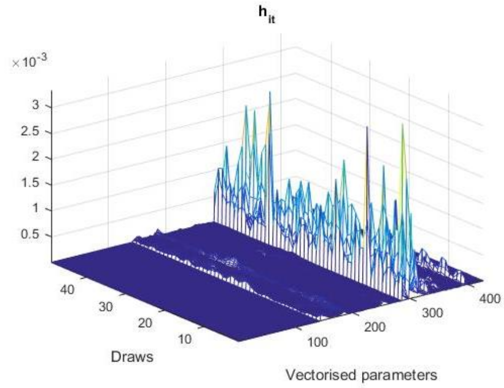
Time 1 to T-1, For each date t=1 to T-1 draw a new value for h_t from the candidate density $q(\Phi^{G+1}) = h_t^{-1} \exp\{(-\ln h_t - \mu)^2/2\sigma_h\}$ and compute the acceptance probability

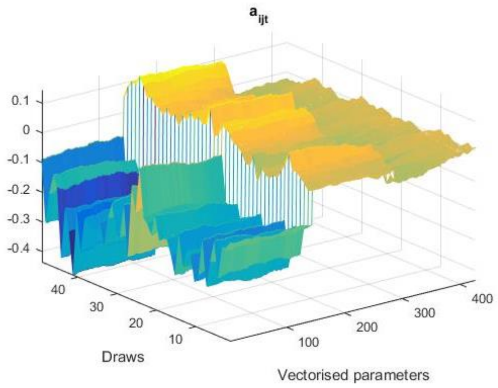
Draw $\mu \sim U(0,1)$. If $\mu < \alpha$ set $h_t = h_{t,new}$, otherwise retain old draw.

Time T:for the last period t=T compute $\mu = \ln h_{t-1}$ and $\sigma_h = g$ and draw $h_{t,new}$ from the candidate density and compute the acceptance probability. Draw $\mu \sim U(0,1)$. If $\mu < \alpha$ set $h_t = h_{t,new}$, otherwise retain old draw.

Appendix B: Recursive mean for the parameters in the model (Convergence)







Appendix C: Matlab code

```
clear
addpath('G:\MSc Economics 2017-18\Semester 3 Dissertation\Estimation\Practice4\functions');

data=xlsread('Practice4\UnemInfBrate.xlsx')/100;

N=size(data,2);
L=2;
Y=data;
X=[lag0(Y,1) lag0(Y,2) ones(size(Y,1),1) ];
Y=Y(3:end,:);
X=X(3:end,:);

%step 1 set starting values and priors using a pre-sample of 10 years
T0=40;
y0=Y(1:T0,:);
x0=X(1:T0,:);
b0=x0\y0;
e0=y0-x0*b0;
sigma0=(e0'*e0)/T0;
V0=kron(sigma0,inv(x0'*x0));

%priors for the variance of the transition equation
Q0=V0*T0*3.5e-04;
P00=V0;
beta0=vec(b0)';

%priors and starting values for aij
C0=chol(sigma0);
C0=C0./repmat(diag(C0),1,N);
C0=inv(C0)';
%aij,0|0
a10=C0(2,1);
a20=C0(3,1:2);
%the variance of the initial state set equal to abs(aij)*10
pa10=abs(a10)*10;
pa20=diag(a20)*10;
D10=10^(-3);
D20=10^(-3)*eye(2);

%remove intial Sample
Y=Y(T0+1:end,:);
X=X(T0+1:end,:);
T=rows(X);


%priors and starting values for the stochastic volatility
hlast=(diff(Y).^2)+0.0001;
hlast=[hlast(1:2,:);hlast];
g=ones(3,1);
g0=0.01^2;
Tg0=1;
mubar=log(diag(sigma0));
sigmabar=10;

%initialise parameters
Q=Q0;
D1=D10;
D2=D20;
a1=repmat(a10,T,1);
```

```
a2=repmat(a20,T,1);

%Gibbs sampling algorithm Step 2
reps=50000;
burn=49000;
mm=1;

for m=1:reps
m
%%Step 2a Set up matrices for the Kalman Filter

ns=cols(beta0);    %from here the time varying coefficients conditional on R and Q using Carter and Cohn algorithm
F=eye(ns);
mu=0;
beta_tt=[];           %will hold the filtered state variable
ptt=zeros(T,ns,ns);     % will hold its variance
beta11=beta0;
p11=P00;

%%Step 2b run Kalman Filter

for i=1:T
   x=kron(eye(N),X(i,:));

a=[a1(i) a2(i,:)];
A=chofac(N,a');
H=diag(hlast(i+1,:));
R=inv(A)*H*inv(A)';

%Prediction
beta10=mu+beta11*F';
p10=F*p11*F'+Q;
yhat=(x*(beta10)')';
eta=Y(i,:)-yhat;
feta=(x*p10*x')+R;

%updating
K=(p10*x')*inv(feta);
beta11=(beta10'+K*eta')';
p11=p10-K*(x*p10);
ptt(i,:,:)=p11;
beta_tt=[beta_tt;beta11];

end

%%end of Kalman Filter%%

%step 2c Backward recursion
chck=-1;
while chck<0
beta2 = zeros(T,ns);
wa=randn(T,ns);
error=zeros(T,N);
roots=zeros(T,1);

i=T;
p00=squeeze(ptt(i,:,:));
beta2(i,:)=beta_tt(i:i,:)+(wa(i:i,:)*chol(p00));
error(i,:)=Y(i,:)-X(i,:)*reshape(beta2(i:i,:),N*L+1,N);
```

```
roots(i)=stability(beta2(i,:)',N,L);
%periods t-1..to .1
for i=T-1:-1:1
pt=squeeze(ptt(i,:,:));
bm=beta_tt(i:i,:)+(pt*F'*inv(F*pt*F'+Q)*(beta2(i+1:i+1,:)-beta_tt(i,:)*F')')';
pm=pt-pt*F'*inv(F*pt*F'+Q)*F*pt;
beta2(i:i,:)=bm+(wa(i:i,:)*chol(pm));
error(i,:)=Y(i,:)-X(i,:)*reshape(beta2(i:i,:),N*L+1,N);
roots(i)=stability(beta2(i,:)',N,L);
end

if sum(roots)==0
    chck=1;
end
end

% step 3 sample Q from the IW distribution
errorq=diff(beta2);
scaleQ=(errorq'*errorq)+Q0;
Q=iwpQ(T+T0,inv(scaleQ));

%step4 sample aij using the carter kohn algorithm
v3=error(:,3);
v2=error(:,2);
v1=error(:,1);

[a1,trash]=carterkohn1(a10,pa10,hlast(:,2),D1,v2,-v1);
[a2,trash]=carterkohn1(a20,pa20,hlast(:,3),D2,v3,[-v1 -v2]);

%step 5 sample D1 and D2
a1errors=diff(a1);
D1=IG(T0,D10,a1errors);
a2errors=diff(a2);
scaleD2=(a2errors'*a2errors)+D20;
D2=iwpQ(T+T0,inv(scaleD2));

%step 6 sample h_i seperately for i=1,3
 %step 6a calculate epsilon=A*v
 epsilon=[];
 for i=1:T
     a=[a1(i) a2(i,:)];
     A=chofac(N,a');
     epsilon=[epsilon;error(i,:)*A'];
 end

 %sample stochastic vol hit, i=1...3 for each epsilon using the MH algorithm
  hnew=[];
  for i=1:N
      htemp=getsvol(hlast(:,i),g(i),mubar(i),sigmabar,epsilon(:,i));
      hnew=[hnew htemp];
  end

  hlast=hnew;

%step 7 Sample G for IG distribution
for i=1:N
    gerrors=diff(log(hnew(:,i)));
g(i)=IG(Tg0,g0,gerrors);  %draw from the inverse Gamma distribution
end
```

```
if m>burn
    out1(mm,1:T,:)=beta2;
    out2(mm,1:T,1:N)=hlast(2:end,:);
    out3(mm,1:N*(N*L+1),1:N*(N*L+1))=Q;
    out4(mm,1:T,1:(N*(N-1))/2)=[a1 a2];
    out5(mm,1)=D1;
    out6(mm,1:2,1:2)=D2;
    out7(mm,1:N)=g';
    mm=mm+1;
end
end
%save results
save tvp.mat out1 out2 out3 out4 out5 out6 out7
%compute irf to a policy shock
horz=40;%
irfmat=zeros(size(out1,1),T,horz,N);
for i=1:size(out1,1);
    for j=1:size(out1,2)
       H=diag(squeeze(out2(i,j,:)));
       a=squeeze(out4(i,j,:));
        A=chofac(N,a);
        sigma=inv(A)*H*inv(A)';
        A0hat=chol(sigma);
%shock to the bank rate
       shock=[0 0 1];
        btemp=squeeze(out1(i,j,:));
        btemp=reshape(btemp,N*L+1,N);
        zz=irfsim(btemp,N,L,A0hat,shock,horz+L);
        irfmat(i,j,:,:)=zz;
    end
end
horz=40;% impulse response horizon
irfmat1=zeros(size(out1,1),T,horz,N);
for i=1:size(out1,1);
    for j=1:size(out1,2)
       H=diag(squeeze(out2(i,j,:)));
       a=squeeze(out4(i,j,:));
        A=chofac(N,a);
        sigma=inv(A)*H*inv(A)';
         A0hat=chol(sigma);
        %shock to the inflation
        shock=[0 1 0];
        btemp=squeeze(out1(i,j,:));
        btemp=reshape(btemp,N*L+1,N);
        zz=irfsim(btemp,N,L,A0hat,shock,horz+L);
```

```
        irfmat1(i,j,:,:)=zz;

    end
end

  horz=40;
irfmat2=zeros(size(out1,1),T,horz,N);


for i=1:size(out1,1);

    for j=1:size(out1,2)

       H=diag(squeeze(out2(i,j,:)));
       a=squeeze(out4(i,j,:));
        A=chofac(N,a);
        sigma=inv(A)*H*inv(A)';
           A0hat=chol(sigma);
           %shock to the unemployment
        shock=[1 0 0];
        btemp=squeeze(out1(i,j,:));
        btemp=reshape(btemp,N*L+1,N);
        zz=irfsim(btemp,N,L,A0hat,shock,horz+L);
        irfmat2(i,j,:,:)=zz;

    end
end

TT=1981.75:0.25:2017.00;
HH=0:horz-1;
%for shock to bank rate
irf1=squeeze(median(irfmat(:,:,:,1),1));  %unemployment
irf2=squeeze(median(irfmat(:,:,:,2),1));  %inflation
irf3=squeeze(median(irfmat(:,:,:,3),1));  %bank rate

%for shock to inflation rate
irf4=squeeze(median(irfmat1(:,:,:,1),1));
irf5=squeeze(median(irfmat1(:,:,:,2),1));
irf6=squeeze(median(irfmat1(:,:,:,3),1));

%for shock to unemployment rate
irf7=squeeze(median(irfmat2(:,:,:,1),1));
irf8=squeeze(median(irfmat2(:,:,:,2),1));
irf9=squeeze(median(irfmat2(:,:,:,3),1));


%Forecast error variance decomposition
%shock to bank rate
irfv1=irf1.^2;
var1=cumsum(irfv1,2);

irfv2=irf2.^2;
var2=cumsum(irfv2,2);

irfv3=irf3.^2;
var3=cumsum(irfv3,2);

%shock to inflation
irfv4=irf4.^2;
var4=cumsum(irfv4,2);
```

```
irfv5=irf5.^2;
var5=cumsum(irfv5,2);

irfv6=irf6.^2;
var6=cumsum(irfv6,2);

%shock to unemployment
irfv7=irf7.^2;
var7=cumsum(irfv7,2);

irfv8=irf8.^2;
var8=cumsum(irfv8,2);

irfv9=irf9.^2;
var9=cumsum(irfv9,2);

mseB=var3+var6+var9
mseI=var2+var5+var8
mseU=var1+var4+var7

c1=var3./mseB
c2=var5./mseI
c3=var7./mseU

%%Figure
figure(1)
subplot(2,3,1);
mesh(TT,HH,irf1')
ylabel('Impulse Horizon');
xlabel('Time');
axis tight
title('Unemployment Rate');

subplot(2,3,2);
mesh(TT,HH,irf2')
ylabel('Impulse Horizon');
xlabel('Time');
axis tight
title('Inflation Rate');

subplot(2,3,3);
mesh(TT,HH,irf3')
ylabel('Impulse Horizon');
xlabel('Time');
axis tight
title('Interest Rate');


temp=(squeeze(out2(:,:,1)));
subplot(2,3,4);
plot(TT,prctile(temp,[50 16 84],1));
axis tight
title('Stochastic Volatility Unemployment Rate')

temp=(squeeze(out2(:,:,2)));
subplot(2,3,5);
plot(TT,prctile(temp,[50 16 84],1));
axis tight
title('Stochastic Volatility Inflation Rate')
```

```
temp=(squeeze(out2(:,:,3)));
subplot(2,3,6);
plot(TT,prctile(temp,[50 16 84],1));
axis tight
title('Stochastic Volatility Interest Rate')

%recursive means
%%Figure of the recursive means calculated every 20 draws for betat, hit, and aij,t
mbeta=rmean1(out1,20);
mh=rmean1(out2,20);
ma=rmean1(out4,20);

figure(2)
subplot(2,2,1);
mesh(mbeta);
axis tight
title('\beta_{t}');
xlabel('Vectorised parameters');
ylabel('Draws')

subplot(2,2,2);
mesh(mh);
axis tight
title('h_{it}');
xlabel('Vectorised parameters');
ylabel('Draws')

subplot(2,2,3);
mesh(ma);
axis tight
title('a_{ijt}');
xlabel('Vectorised parameters');
ylabel('Draws')


figure (3)
subplot(2,3,1);
mesh(TT,HH,c3')
ylabel('Error Variance Horizon');
xlabel('Time');
axis tight
title('Unemployment Rate');

subplot(2,3,2);
mesh(TT, HH, c2')
ylabel('Error Variance Horizon');
xlabel('Time');
axis tight
title('Inflation Rate');

subplot(2,2,3);
mesh(TT,HH,c1')
ylabel('Error Variance Horizon');
xlabel('Time');
axis tight
title('Interest Rate');
```

References

Angeriz, A. and Arestis, P. (2007). 'Monetary policy in the UK'. *Cambridge Journal of Economics*, 31, pp. 863–884.

Arestis, P. and Sawyer, M. (2008). 'A critical reconsideration of the foundations of monetary policy in the new consensus macroeconomics framework'. *Cambridge Journal of Economics*, 32, pp. 761–779.

Ball, L. and Mankiw, N. G. (2002). 'The NAIRU in Theory and Practice'. *Journal of Economic Perspectives*, 16 (4), pp. 115–136.

Bernanke, B. S. and Mihov, I. (1998). 'The liquidity effect and long-run neutrality', *NBER Working Paper*, No. 6608.

Benati, L. (2008). 'Investigating Inflation Persistence across Monetary Regimes'. *The Quarterly Journal of Economics,* 123(3), pp. 1005–1060.

Benati, L. and Surico, Paolo (2008). 'Evolving U.S. Monetary Policy and the Decline of Inflation Predictability'. *Journal of the European Economic Association*, 6 (2/3).

Berentsen, A. Menzio, G. and Wright, R. (2011). 'Inflation and Unemployment in the Long Run'. *The American Economic Review,* 101(1), pp. 371-398.

Bernanke, B. and Blinder, A. (1992). 'The Federal Funds Rate and the Channels of Monetary Transmission'. *American Economic Review*, LXXXII, pp. 901–921.

Bernanke, B. S., Boivin, J. and Eliasz, P. (2005). 'Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach'. *The Quarterly Journal of Economics,* 120 (1), pp. 387–422.

Beyer, A., and Farmer, R. E. A. (2007). 'Natural rate doubts'. *Journal of Economic Dynamics & Control*, 31, pp. 797–825.

Bhattarai, K. (2016). 'Unemployment–inflation trade-offs in OECD countries'. *Economic Modelling*, 58, pp. 93–103.

Blake, A. P. and Mumtaz, H. (2012). "Applied Bayesian econometrics for central bankers," Technical Books, *Centre for Central Banking Studies,* Bank of England, edition 1, number 4, March.

Black, F. (1976). 'Studies of Stock Market Volatility Changes'. *Proceedings of the American Statistical Association*, pp. 177–181.

Blanchard, O. and Galí, J. (2010). 'Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment'. *American Economic Journal: Macroeconomics*, 2 (2), pp. 1-30

BoE website. https://www.bankofengland.co.uk/monetary-policy/the-interest-rate-bank-rate . Bank of England. (accessed on 01 August 2018).

Boivin, Jean and Giannoni, M. P. (2006). 'Has Monetary Policy Become More Effective?'. *The Review of Economics and Statistics*, 88(3), pp. 445–462.

Canova, F. and Gambetti, L. (2009). 'Structural changes in the US economy: Is there a role for monetary policy?'. *Journal of Economic Dynamics & Control*, 33, pp. 477–490.

Clark, T. E. and Ravazzolo, F. (2015). 'Macroeconomic Forecasting Performance under Alternative Specifications of Time-Varying Volatility'. *Journal of Applied Econometrics J. Appl. Econ.,* 30, pp. 551–575.

Cogley, T. and Sargent, T. J. (2001). 'Evolving Post-World War II U.S. Inflation Dynamics'. *NBER Macroeconomics Annual*, 16, pp. 331-373.

Cogley, T. and Sargent, T. J. (2005). 'Drifts and volatilities: monetary policies and outcomes in the post-WWII US'. *Review of Economic Dynamics,* 8, pp. 262–302.

Cogley, T., Morozov, S. and Sargent, T. J. (2005). 'Bayesian fan charts for U.K. inflation: Forecasting and sources of uncertainty in an evolving monetary system'. *Journal of Economic Dynamics & Control,* 29, p.p. 1893–1925

Cogley, T., Primiceri, G. E. and Sargent, T. J. (2010). 'Inflation-Gap Persistence in the US'. *American Economic Journal: Macroeconomics*, 2 (1), pp. 43-69.

Coibion, O. (2012). 'Are the Effects of Monetary Policy Shocks Big or Small?'. *American Economic Journal: Macroeconomics*, 4(2), pp. 1–32.

D'Agostino, A., Gambetti, L. and Giannone, D. (2013). 'Macroeconomic Forecasting And Structural Change'. *Journal of Applied Econometrics J. Appl. Econ.* 28, pp. 82– 101.

Dobrescu, M., Paicu, C. and Iacob, S. (2011). 'The Natural Rate of Unemployment and its Implications for Economic Policy'. *Theoretical and Applied Economics*, XVIII (2(555)), pp. 181-194.

Friedman, M. (1977). 'Nobel Lecture: Inflation and Unemployment'. *Journal of Political Economy,* 85 (3), pp. 451-472.

Gerlach, S., Lydon, R. and Stuart, R. (2016). 'Unemployment and inflation in Ireland: 1926–2012'. *Cliometrica*, 10, pp. 345–364.

Geyer, C. J. (1992). 'Practical Markov Chain Monte Carlo'. *Statistical Science*, 7 (4), pp. 473-483.

Goodfriend, M. (2007). 'How the World Achieved Consensus on Monetary Policy'. *Journal of Economic Perspectives*, 21 (4), pp. 47–68.

Jacquier, E., Polson, N. G. and Rossi, P. E. (2004). 'Bayesian analysis of stochastic volatility models'. *Journal of Business and Economic Statistics*, 12, pp. 371—418.

Jorda, O. (2005). 'Estimation and Inference of Impulse Responses by Local Projections'. *The American Economic Review*, 95 (1), pp. 161-182.

Kajuth, F. (2014). 'NAIRU Estimates for Germany: New Evidence on the Inflation–Unemployment Tradeoff'. *German Economic Review*, 17(1), pp. 104–125.

Karanassou, M. and Sala, H. (2010). 'The US inflation–unemployment trade-off revisited: New evidence for policy-making'. *Journal of Policy Modeling,* 32, pp. 758–777.

Karanassou, M., Sala, H. and Snower, D. J. (2010). 'Phillips Curves and Unemployment Dynamics: a Critique and a Holistic Perspective'. *Journal of Economic Surveys*, 24(1), pp. 1–51.

Lanne, M., Lutkepohl, H. and Maciejowska, K. (2010). 'Structural vector autoregressions with Markov switching'. *Journal of Economic Dynamics & Control*, 34, pp. 121–131.

Louis, R. J. and Balli, F. (2013). 'Low-inflation-targeting monetary policy and differential unemployment rate: Is monetary policy to be blamed for the financial crisis? — Evidence from major OECD countries'. *Economic Modelling*, 30, pp. 546–564.

Mankiw, N. G. (2001). 'The Inexorable and Mysterious Tradeoff between Inflation and Unemployment'. *The Economic Journal,* 111 (471), Conference Papers (May, 2001), pp. C45-C61.

Nakajima, J. (2011). 'Time-varying Parameter VAR model with Stochastic Volatility: an Overview of Methodology and Empirical Applications'. *Monetary and Economic Studies*, 29, pp. 107 – 142 .

Parkin, M. (1998). 'Unemployment, inflation, and monetary policy'. *The Canadian Journal of Economics / Revue canadienne d'Economique,* 31 (5), pp. 1003-1032.

Primiceri, G. E. (2004). 'Time Varying Structural Vector Autoregressions and Monetary Policy'. *Review of Economic Studies*, 72, pp. 821–852.

Ribba, A. (2006). 'The joint dynamics of inflation, unemployment and interest rate in the United States since 1980'. *Empirical Economics,* 31, pp. 497–511.

Sims, C. (1992). 'Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy'. *European Economic Review,* XXXVI, pp. 975–1000.

Sims, C. A. and Zha, T. (2006). 'Were There Regime Switches in U.S. Monetary Policy?'. *The American Economic Review,* 96(1), pp. 54-81.

Stock, J. H. and Watson, M. W. (2001). 'Vector Autoregressions'. *Journal of Economic Perspectives*, 15 (4), pp. 101–115.

Taylor, J. B. (1997). *Inflation, Unemployment and Monetary Policy*, in Taylor, J. B. and Robert M. S.. MIT Press, 1997.

Tobin, J. (1972). 'Inflation and Unemployment'. *The American Economic Review*, 62 (1), pp. 1-18.