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Abstract

We introduce a dynamical model describing the interaction between a three-sectors real economy and a financial market with four assets. Investors and financial mediators have heterogeneous beliefs. The model may be used to investigate interdependence within economic fluctuations and assets volatility.

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JEL: C61, C68, G12, O41

1. Introduction

Economics and finance research deals with the interdependence between stock returns and macroeconomic events. Connecting asset returns to real economy allow to understand how assets volatility influences economic fluctuations and vice-versa. To this aim, a general equilibrium (GE) approach has been defined by Cochrane (2006) as a "largely unexplored new land", indispensable to understand how macroeconomic variables interact with financial market. This paper aim is to provide a theoretical model able to describe the aforementioned relation. We study an economy in which three generations of individuals have no direct access to financial market and select between two mediators depend-

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ing on their risk propensity. Real economy is described by three sectors where firms pay wage to workers and dividends to shareholders. The model verifies the Small Minus Big (SMB) and the High Minus Law (HML) effects described by Fama and French (1996) and may be used to analyse influences within real economy and financial market.

2. The model

2.1. Production

We consider three sectors of economy: primary, secondary and tertiary. Output per worker of each sector is described by the CES production function

$$f^{(n)}(k_t) := \left(l^{(n)} + b^{(n)} k_t^{s^{(n)}}\right)^{\frac{1}{s^{(n)}}},$$

where $t \in \mathbb{N}$ and $n = 1, 2, 3$ refers respectively to the primary, secondary and tertiary sector. For each sector, $l^{(n)} \in [0,1]$ and $b^{(n)} \in [0,1]$ represent the portion of implied labour force and capital, respectively to total quantities $L > 0$ and $K \geq 0$, $k := K/L$ while $s^{(n)} \in (-\infty, 1)$, $s^{(n)} \neq 0$ is the output elasticity of capital. It is assumed $\sum_{n=1}^{3} l^{(n)} = \sum_{n=1}^{3} b^{(n)} = 1$. In each sector, the wage per worker (considering total labour force) is the marginal product of labour. Therefore the average worker's wage is

$$w(k_t) = \sum_{n=1}^{3} \left\{ \left[l^{(n)} + \left(1 - b^{(n)}\right) b^{(n)} k_t^{s^{(n)}}\right] \left(l^{(n)} + b^{(n)} k_t^{s^{(n)}}\right)^{\frac{1}{s^{(n)}} - 1} \right\}. \tag{1}$$

Each sector profit per worker is

$$\pi^{(n)}(k_t) = f^{(n)}(k_t) - w^{(n)}(k_t) = \left[b^{(n)}\right]^2 k_t^{s^{(n)}} \left(l^{(n)} + b^{(n)} k_t^{s^{(n)}}\right)^{\frac{1}{s^{(n)}} - 1} \tag{2}$$

and it is distributed between physical investment per worker $i_t^{(n)} := \left(1 - \nu_t^{(n)}\right) \pi_t^{(n)}$ and dividends per worker $d_t^{w(n)} := \nu_t^{(n)} \pi_t^{(n)}$, where $\nu_t^{(n)} \in [0,1]$ is the ratio of profit distributed as dividends. Following Böhm et al. (2008), we assume the evolution of capital per worker over time as given by

$$k_t = \sum_{n=1}^{3} \left[\left(1 - \delta^{(n)}\right) b^{(n)} k_{t-1}] + i_{t-1}^{(n)}\right], \tag{3}$$

where $\delta^{(n)} \in [0,1]$ is the depreciation rate of capital in the n-th sector.

2.2. Consumption

At time t, 3 generations of consumers exist: young, adults and old. Young and adults earn a wage and have different consumption propensities, $c_2 \in [0, 1]$ and $c_1 \in [0, 1]$ respectively. They invest in 3 risky assets (stocks, i.e. shares of the three sectors defined in Section 2.1) and one risky free asset (bond). The older generation only consumes. The wealth to be invested at time t, expressed in per capita of investors, is given by

$$e_t := \frac{(1 - c_2)w_t + (1 - c_1)\left[w_t + y^o_{t-1}R + {x^o}^\top_{t-1}(p_t + d_t)\right]}{2} \tag{4}$$

where $y^o_{t-1} \in \mathbb{R}$ and $x^o_{t-1} = \left({x^{(1)}}^o_{t-1}, {x^{(2)}}^o_{t-1}, {x^{(3)}}^o_{t-1}\right)$ are the per capita amount (in units of consumption good) invested at time $t-1$ by the current older generation in bond and stocks, respectively. The constant return of the bond is $R > 0$, given exogenously. The stocks price vector is $p_t = \left(p^{(1)}_t, p^{(2)}_t, p^{(3)}_t\right)$ while dividends per share are given by the vector $d_t = L\left(d^{w(1)}_t/x^{(1)}_m, d^{w(2)}_t/x^{(2)}_m, d^{w(3)}_t/x^{(3)}_m\right)$, where $x_m = \left(x^{(1)}_m, x^{(2)}_m, x^{(3)}_m\right)$ is the market portfolio. Behaviours of the three generations at time t are summarized in Table 1.

	Young	Adults	Old
Investment	x	x	
Consumption	x	x	x
Earning from wage	x	x	
Earning from dividend and interest payments		x	x

Table 1: Behaviours at time t, for each generation.

2.3. Financial investment

Following Wenzelburger (2004), it is assumed that investors have no direct access to financial market and they select between two financial mediators (agents) $i = 1, 2$, who manage their portfolios. Given a trading period t,

$\eta_t \in [0,1]$ is the fraction of individuals that employ agent 1. The amount of resources per capita of agents, before trading at time t, are given by $W_t^{(1)} := \eta_t e_t$ and $W_t^{(2)} := (1-\eta_t)e_t$ and their budget constraint is $W_t^{(i)} = y_t^{(i)} + p_t^\top x^{(i)}$ where $\left(y^{(i)}, x^{(i)}\right)$ is the portfolio held by mediator i. The cum-dividend price vector is $q_{t+1} = d_{t+1} + p_{t+1}$. Each agent i has subjective probability of distribution regarding future q, parametrised by the subjective conditional mean values $\bar{q}_t^{(i)}$ and the subjective conditional covariance matrices $V_t^{(i)}$ (assumed to be symmetric and positive definite 3×3 matrices). Mediators have linear mean-variance preferences and they maximise expected utility over future consumption with respect to subjective expectations for future asset prices, i.e. $U^{(i)}(\mu^{(i)}, \sigma^{(i)}) := \mu - \frac{\alpha^{(i)}}{2}\left(\sigma^{(i)}\right)^2$, where $\alpha^{(i)} > 0$ represents a measure of the agent's i risk aversion while $\mu^{(i)}$ and $\sigma^{(i)}$ are, respectively, his expected wealth and standard deviation for the portfolio of stocks $x^{(i)} \in \mathbb{R}^3$. The assets demand function of mediator i is given by

$$x_t^{(i)} = \varphi^{(i)}\left(a_t^{(i)}, \bar{q}_t^{(i)}, V_t^{(i)}, p_t\right) := a_t^{(i)}\left(V_t^{(i)}\right)^{-1}\left(\bar{q}_t^{(i)} - Rp_t\right), \tag{5}$$

where $a_t^{(1)} := \frac{\eta_t}{\alpha^{(1)}}$ and $a_t^{(2)} := \frac{1-\eta_t}{\alpha^{(2)}}$ denote the risk-adjusted market shares for agents 1 and 2 respectively. The market clearing condition at time t is

$$\sum_{i=1}^{2}\left[\varphi^{(i)}\left(a_t^{(i)}, \bar{q}_t^{(i)}, V_t^{(i)}, p_t\right) - x_{t-1}^{(i)}\right] + \xi_t - \xi_{t-1} = 0. \tag{6}$$

where $\xi_t \in \mathbb{R}^3$ is the portfolio of noise traders, i.e. traders whose demand for shares is not driven by a microeconomic decision model. Solving equation (6) for p_t, the evolution of market clearing prices over time may be written as

$$p_t = A_t^{(1)}\bar{q}_t^{(1)} + A_t^{(2)}\bar{q}_t^{(2)} - A_t(x_m - \xi_t) \tag{7}$$

where $A_t := \frac{1}{R}\left[a_t^{(1)}\left(V_t^{(1)}\right)^{-1} + a_t^{(2)}\left(V_t^{(2)}\right)^{-1}\right]^{-1}$ and $A_t^{(i)} := a_t^{(i)} A_t \left(V_t^{(i)}\right)^{-1}$. Novak et al. (2017) showed that the dividend payout ratio $\nu_t^{(n)}$ is close to 0.5 and it varies depending on previous assets prices. Moreover Baker and Farrelly (1988) argued that managers use dividend payments to maintain or increase share prices (higher dividends increase share prices). Following these findings, we assume when prices are stable $\nu_t^{(n)} = 0.5$ otherwise it is increased (decreased) up to 0.2 points, as the following rule states:

$$\nu_t^{(n)} := 0.5 - \frac{0.4}{\pi} \arctan\left(\frac{p_{t-1}^{(n)} - p_{t-2}^{(n)}}{p_{t-2}^{(n)}} \right). \tag{8}$$

2.4. Social and professional heterogeneous beliefs

We assume both individuals and mediators have heterogeneous heuristics and they are introduced in this section.

Financial mediators. Grosshans and Zeisberger (2018) showed that agents make expectations about future returns strongly believing in short-term trend continuation. Therefore we model expectation on returns as follows

$$\bar{q}_t^{(i)} := q_{t-1} + z^{(i)}(q_{t-1} - q_{t-2}), \tag{9}$$

where $z^{(i)} \in (0, 1]$. Agents mainly differ for their willingness to take risk. We assume that agent 1 has a higher risk-taking behaviour, therefore $\alpha^{(1)} < \alpha^{(2)}$. Higher-risk investments may be driven from overconfidence (see Broihanne et al. (2014)). Consequently, we assume

$$V_t^{(i)} := r^{(i)} \begin{bmatrix} q_{t-1}^{(1)} & 0 & 0 \\ 0 & q_{t-1}^{(2)} & 0 \\ 0 & 0 & q_{t-1}^{(3)} \end{bmatrix}, \quad 0 < r^{(1)} < r^{(2)} < 1\,. \tag{10}$$

Equation (10) states that both agents assume stocks as uncorrelated, moreover the subjective belief about volatility on expected returns depends on previous dividends and it is higher for the agent with a lower risk-taking profile.

Society. As widely demonstrated, the willingness to take risk decreases with age. Therefore we assume young and adults select respectively agent 1 and 2. Malmendier and Nagel (2011) found that a personal experience of economic fluctuations affects risk attitudes. We set that some people may switch their risk preference: young generation choosing the agent with higher risk aversion after a bust period in real economy and adults choosing mediator 1 otherwise. Previous considerations are taken into account in the following rule:

$$\eta_t := 0.5 + \frac{1}{\pi} \arctan\left(\frac{k_{t-1} - k_{t-2}}{k_{t-2}}\right). \tag{11}$$

2.5. Final model

Now we can describe how both financial market and real economy evolve over time. The final model is

$$T := \begin{cases} p_t & = g(p_{t-1}, p_{t-2}, p_{t-3}, p_{t-4}, k_{t-1}, k_{t-2}) = \sum_{i=1}^{2}\left(A_t^{(i)}\bar{q}_t^{(i)}\right) - A_t(x_m - \xi_t) \\ k_t & = h(p_{t-2}, p_{t-3}, k_{t-1}) = \sum_{n=1}^{3}\left[\left(1-\delta^{(n)}\right) b^{(n)} k_{t-1} + i_{t-1}^{(n)}\right] \end{cases}, \tag{12}$$

where $A_t^{(i)}$, A_t, $\bar{q}_t^{(i)}$ and $i_{t-1}^{(n)}$ are defined in previous sections. The interdependences between real economy and financial markets are summarised in Figure 1

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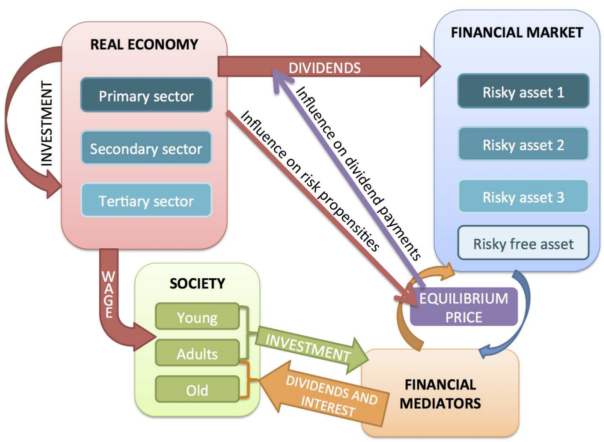

Figure 1: Interdependences between real economy and financial markets

3. Results: challenges for a GE model

Model (12) satisfies most of the requirements discussed by Cochrane (2006) for a GE model: it admits multiple firms, preferences and technologies are consistent with microeconomic investigation; moreover it generates the SMB and HML effects introduced by Fama and French (1996), as demonstrated below. Assume real economy is in equilibrium k^*, noise traders do not exist and firms differ only for their total stock $x_m^{(n)}$ and their capital ratio $b^{(n)}$ (intended as a measure for the firm's *book value*). Returns in equilibrium are given by

$$q^{(n)^*} := \frac{[b^{(n)}]^2 k^{*s} \left(l + b^{(n)} k^{*s}\right)^{\frac{1-s}{s}}}{2 + 2R^{-1}(B x_m^{(n)} - 1)}$$

where $B := \frac{r^{(1)} r^{(2)}}{a^{(1)} r^{(2)} + a^{(2)} r^{(1)}}$. Notice that $\frac{\partial q^{(n)^*}}{\partial x_m^{(n)}} < 0$, therefore firms with less shares for a given price, i.e. small-cap firms, have higher returns in the long run (SMB effect). Moreover $\frac{\partial q^{(n)^*}}{\partial b^{(n)}} > 0$, hence companies with a higher book value for a given market value, i.e. value companies, tend to have higher returns in equilibrium (HML effect).

4. Concluding Remarks

We developed a GE model that can be used to analyse how macroeconomic dynamics influence stock returns and vice-versa. Particularly, a stability analysis may be done to understand which factors generate economic fluctuations and stock volatility. The model verifies the SMB and HML effects discussed in Fama and French (1996).

Declarations of interest: none

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