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ABSTRACT

This paper studies volumetric risk hedging strategies for solar power under incomplete market settings with a twofold proposal of temperature-based and solar power generation-based models for solar power derivatives and discusses the basis risk arising from solar power volumetric risk hedge with temperature. Based on an indirect modeling of solar power generation using temperature and a direct modeling of solar power generation, we design two types of call options written on the accumulated non cooling degree days (ANCDDs) and the accumulated low solar power generation days (ALSPGDs), respectively, which can hedge cool summer volumetric risk more appropriately than those on well-known accumulated cooling degree days. We offer the pricing formulas of the two options under the good-deal bounds (GDBs) framework, which can consider incompleteness of solar power derivative markets. To calculate the option prices numerically, we derive the partial differential equations for the two options using the GDBs. Empirical studies using Czech solar power generation and Prague temperature estimate the parameters of temperature-based and solar power generation-based models, respectively. We numerically calculate the call option prices on ANCDDs and ALSPGDs, respectively, as the upper and lower price boundaries using the finite difference method. Results show that the call option prices based on a solar power generation process are bigger than the call option prices based on a temperature process. This is consistent with the fact that the solar power generation approach takes into account more comprehensive risk than the temperature approach, resulting in the bigger prices for the solar power generation approach. We finally show that the basis risk premiums, i.e., solar power generation-based call option prices minus temperature-based call option prices, decrease in line with initial temperature greater than around 25 °C. This may be because the uncertainty in solar power generation by temperature decreases due to the cancellation between the increase in solar power generation due to the increase in solar radiation and the decrease in solar power generation due to the decrease in solar panel efficiency.

Key words: Solar power, weather risk, temperature model, basis risk, good-deal bounds
JEL Classification: G13, L94, Q42

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1. Introduction

Renewable energy has recently been promoted in order to achieve decarbonized power generation all over the world. Since renewable energy is affected by the weather, it is constantly exposed to weather risk. It is important to conduct risk management of renewable energy for these promotions of renewables. This paper handles solar power generation in order to think of renewable energy risk management as one of the most popular renewables. The sources of renewable energy power generation come from natural phenomena. Since solar power gets energy from solar radiation, the output from solar power generation is usually calculated using the amount of solar radiation. Benth and Ibrahim (2017) propose a stochastic model for solar power production using cyclical solar radiation intensity. That is why it is natural to conduct volumetric risk management of solar power generators based on solar radiation or solar power generation. However, risk takers of solar power generators like financials are not familiar with the behavior of solar radiation or solar power generation, rather familiar with temperature because they often incorporate temperature-based weather risk products like weather derivatives in the portfolios. It is known that solar radiation or solar power generation has a positive linear relation to temperature as in e.g., Ibrahim, Daut, Irwan, Irwanto, Gomesh, and Farhana (2012). So an indirect temperature-based model of volumetric risk management for solar power will work well from the perspective of the existence of both the risk takers and the hedgers. In contrast, when solar power generation is directly modeled for the volumetric risk, the hedging errors will be smaller than the hedging errors from a temperature-based model. A solar power generation-based model will also be effective from the perspective of the risk hedgers. In this way, in considering the risk management of solar power generation, there is a risk due to mismatch between the hedged item and the hedging instrument, that is, a basis risk. Detailing the characteristics of basis risk is important in developing hedging instruments for solar power volumetric risk management.

Weather derivative pricing has a long list of literature. Cao and Wei (2000) calculate the price of weather derivatives based on the stochastic discount factor (SDF) obtained from the utility function and the optimal consumption of the estimated representative agent. Davis (2001) conducts the derivative pricing written on accumulated heating degree days using the SDF of an agent with a log utility function whose optimal consumption is proportional to the payoff of the derivatives. Platen and West (2004) propose a fair pricing of weather derivatives where the growth optimal portfolio is used as a benchmark or numeraire. Brockett, Wang, Yang, and Zou (2006) apply the indifference pricing approach to the valuation of weather derivatives. Kanamura and Ōhashi (2009) apply the good-deal bounds of Cochrane and Saa-Requejo (2000) to summer day options as the incomplete market pricing. Lee and Oren (2009) derive an equilibrium pricing model for weather derivatives in a multi-commodity setting. These studies are quite interesting in the sense of new development of weather derivative pricing. Nevertheless, the applications to renewable energy, in particular solar power, are limited as long as we know.

Regarding solar power risk hedge, Bhattacharya, Gupta, Kar, and Owusu (2016) develop a framework to construct explicit cross hedging strategies of solar power for mitigation of the identified weather risk using temperature-based weather derivatives and find that temperature derivatives-based hedges are effective for summer. However, it is unfortunate that Bhattacharya, Gupta, Kar, and Owusu (2016) do not consider the incompleteness of weather derivatives, which is one of the most important and tough issues for the derivative pricing.[1]

This paper studies volumetric risk hedging strategies for solar power under incomplete market settings with a twofold proposal of temperature-based and solar power generation-based models for solar power derivatives and discusses the basis risk arising from solar power volumetric risk hedge with temperature. Based on an indirect modeling of solar power generation using temperature and a direct modeling of solar power generation, we design two types of call options written on the accumulated non cooling degree days (ANCDDs) and the accumulated low solar power generation days (ALSPGDs), respectively, which can hedge cool summer volumetric risk more appropriately than those on well-known accumulated cool-

[1] Härdle and Cabrera (2012) show the incompleteness of weather derivative markets by inferring the market price of risk from traded futures-type weather derivative contracts.

ing degree days. Then we offer the pricing formulas of the two options under the good-deal bounds (GDBs) framework, which can take into account incompleteness of solar power derivative markets. To calculate the option prices numerically, we derive the partial differential equations for the two options using the GDBs. Empirical studies using Czech solar power generation and Prague temperature estimate the parameters of temperature-based and solar power generation-based models, respectively. We numerically calculate the call option prices on ANCDDs and ALSPGDs, respectively, as the upper and lower price boundaries using the finite difference method. Results show that the call option prices based on a solar power generation process are bigger than the call option prices based on a temperature process. This is consistent with the fact that the solar power generation approach takes into account more comprehensive risk than the temperature approach, resulting in the bigger call option prices for the solar power generation approach. We finally show that the basis risk premiums, i.e., solar power generation-based call option prices minus temperature-based call option prices, decrease in line with initial temperature greater than around 25 °C. This may be because the uncertainty in solar power generation by temperature decreases due to the cancellation between the increase in solar power generation due to the increase in solar radiation and the decrease in solar power generation due to the decrease in solar panel efficiency.

The remainder of this paper is organized as follows. Section 2 proposes twofold models to hedge the volumetric risk on solar power generation: temperature-based and solar power generation-based models and offers the GDBs pricing for the corresponding call options. Section 3 conducts empirical studies of ANCDDs and ALSPGDs call option prices for cool summer risk hedge and discusses the basis risk premiums for temperature hedging. Section 4 concludes.

2. The Model

2.1. Solar power generation models

As solar power generation models, we use an indirect modeling of a linear relation between solar power generation and temperature and a direct modeling of solar power generation. We start with the indirect modeling of solar power generation using temperature. Solar power generation is affected by the panel temperature. The standard power generation of a solar power plant is generally designed at 25 °C. When panel temperature increases in ambient temperature, it is known that the efficiency is reduced, i.e., temperature negatively affects solar power generation. In contrast, recent technological advance of solar power develops solar power with high efficiency and heat resistance including advanced cadmium telluride (CdTe) thin film photovoltaic modules. In this case, solar power generation increases in line with temperature from solar radiation. Considering that the impact of temperature on solar power generation is characterized by the balance of the two effects, solar power generation V_t is given by using temperature T_t:

$$V_t = f(T_t), \tag{1}$$

$$T_t = X_t^T + \bar{T}_t \tag{2}$$

where f is a temperature impact function on solar power generation. Temperature stems from two components of yearly cyclical trend $\bar{T}_t$ and the deviation from the trend X_t^T referring to the formulation by Cao and Wei (2000). It is historically known that the deviation mean reverts.

Next, we directly model a process of solar power generation V_t on date t using the sum of the normal level of solar power generation $\bar{V}_t$ on date t and the deviations X_t^V from the normal level, i.e.,

$$V_t = X_t^V + \bar{V}_t, \tag{3}$$

from the analogy to the modeling of temperature.

To estimate the normal levels $\bar{i}_t$ for $i = T, V$ as a sum of a linear trend and a cyclical trend of temperature and solar power generation, respectively, we formulate the normal levels:

$$\bar{i}_t = \kappa_1^i + \kappa_2^i t + \kappa_3^i \sin(\omega t) + \kappa_4^i \cos(\omega t) \tag{4}$$

where κ_1^i, κ_2^i, κ_3^i and κ_4^i for $i = T, V$ are constant and $\omega \equiv \frac{2\pi}{365}$.

Since X_t^is for $i = T, V$ represent the deviations from the normal levels, it should exhibit mean-reverting property; if temperature abnormally rises (or drops), it tends to go back down (or up, resp.) to the normal level. In many cases including Cao and Wei (2000) as well as the empirical results later, we can find the presence of such mean-reversion. We also assume that the temperature modeling is applicable to the solar power generation modeling for the similarity. Thus, we formulate the deviation X_t^i for $i = T, V$ to evolve in the following way:

$$dX_i^i = (m_i - \lambda_i X_t^i)dt + \sigma_i dv_t^i, \tag{5}$$

$$dv_t^i = \rho_i dw_t + \sqrt{1 - \rho_i^2} dz_t^i \tag{6}$$

where m_i, λ_i, σ_i and ρ_i are constant, and v_t^i is a standard Brownian motion. Note that dz_t^i is the orthogonal part of dv_t^i to dw_t, implying that dz_t^i represents the orthogonal risk associated with temperature or solar power generation risk to stock market risk dw_t.

Note that $di_t = d\bar{i}_t + dX_t^i$ for $i = T, V$. Define $\theta_1^i = m_i + \lambda_i \kappa_1^i + \kappa_2^i$, $\theta_2^i = \lambda_i \kappa_2^i$, $\theta_3^i = \lambda_i \kappa_3^i - \omega_i \kappa_4^i$, and $\theta_4^i = \omega_i \kappa_3^i + \lambda_i \kappa_4^i$. The evolutions of temperature and solar power generation satisfy

$$di_t = (\mu_i(t) - \lambda_i i_t)dt + \sigma_i dv_t^i, \tag{7}$$

$$\mu_i(t) = \theta_1^i + \theta_2^i t + \theta_3^i \sin(\omega t) + \theta_4^i \cos(\omega t) \text{ for } i = T, V. \tag{8}$$

The component of the mean part $\mu_i(t)$ is expressed by the sum of annually sinusoidal seasonal functions $\theta_3^i \sin(\omega t) + \theta_4^i \cos(\omega t)$ and a linear long-term trend for time $\theta_2^i t$. We use the two formulations of temperature and solar power generation in our analyses.

2.2. Call option designs for solar power generation

We first design accumulated non cooling degree days (ANCDDs) call option for solar power generation based on temperature modeling. We assume that the solar power generation is represented by a linear function of temperature for simplicity based on Ibrahim, Daut, Irwan, Irwanto, Gomesh, and Farhana (2012).

$$V_t = f(T_t) = p + qT_t \tag{9}$$

Since we consider that temperature positively affects solar power generation like advanced CdTe type solar power, i.e., $q > 0$, the generation loss due to the decrease in temperature is represented by $\max\left[f(T_B) - f(T_t), 0\right]$ where T_B is the benchmark temperature. Assuming a constant selling price of solar power generation as P_0 for simplicity, the cash loss I_t^T is given by

$$I_t^T = P_0 \int_0^t \max[q(T_B - T_\tau), 0] d\tau \tag{10}$$

where T_τ is the temperature at date τ. Note that I_t^T is referred to as "the accumulated non cooling degree days (ANCDDs)" for solar power from time 0 to t in that I_t^T represents the opposite directional index to ordinary CDDs in the sense of temperature. Accumulated cooling degree days (ACDDs) may be familiar with weather derivative market participants. However, ACDDs cannot capture the loss of solar power generation due to the decrease of temperature appropriately. That is why this paper introduces ANCDDs rather than ACDDs.

Using the number of ANCDDs I_t^T as the index, the payoff $g(I_M^T)$ of ANCDD European call option with the strike K at the maturity M is given by

$$g(I_M^T) = \max(I_M^T - K, 0). \tag{11}$$

Note that since I_t^T represents how much temperature is lower than the benchmark, the call option price is expected to decrease in line with temperature.

Secondly, we design another call option for solar power based on solar power generation modeling. Assuming selling price as P_0, cash loss I_t^V is given by

$$I_t^V = P_0 \int_0^t \max[V_B - V_\tau, 0] d\tau \tag{12}$$

where V_τ is the power generation at date τ where V_B is the benchmark solar power generation. Note that I_t^V is referred to as "the accumulated low solar power generation days (ALSPGDs)" for solar power from time 0 to t. Using the number of ALSPGDs I_t^V as the index, the payoff $g(I_M^V)$ of ALSPGDs European call option with the strike K at the maturity M is given by

$$g(I_M^V) = \max(I_M^V - K, 0). \tag{13}$$

Note that since I_t^V represents how much solar power generation is lower than the benchmark, the call option price is expected to decrease in line with solar power generation.

2.3. Good-deal bounds pricing for solar power generation call options

Following Kanamura and Ōhashi (2009), we derive the good-deal bounds pricing formulas for solar power generation call options. We assume that the complete market asset is a stock whose price follows a simple lognormal process:

$$\frac{dS_t}{S_t} = \mu_s dt + \sigma_s dw_t \tag{14}$$

where both μ_s and σ_s are constant and w_t is a standard Brownian motion. Note that under this formulation, the market price of risk ϕ for w_t is given by $\phi = \frac{\mu_s - r}{\sigma_s}$.

The lower boundaries of the ANCDDs option and ALSPGDs option for solar power generation $\underline{C}_t^i$ for $i = T, V$ are obtained as

$$\underline{C}_t^i = \min_{\{\Lambda_s, t \le s \le T\}} E_t \left[\int_t^T \frac{\Lambda_s}{\Lambda_t} x_s^c ds + \frac{\Lambda_T}{\Lambda_t} x_T^c \right], \tag{15}$$

$$\text{s.t. } \frac{d\Lambda_t}{\Lambda_t} = \frac{d\Lambda_t^*}{\Lambda_t^*} - v dz_t^i, \tag{16}$$

$$\frac{d\Lambda_t^*}{\Lambda_t^*} = -r dt - \phi dw_t, \tag{17}$$

$$\frac{1}{dt} E_t \left[\frac{d\Lambda_t^2}{\Lambda_t^2} \right] \le A^2. \tag{18}$$

Similarly, the upper boundaries of the prices are obtained by replacing the minimization with the maximization in Eq. (15).

Suppose that the maximum Sharpe ratio after introducing a new derivative is given by A. Suppose also that stock price S_t and temperature and solar power generation i_t for $i = T, V$ are given by Eqs (14) and (7), respectively. Denote by I^i for $i = T, V$ the temperature and solar power generation indices that determine the derivative payoffs $g(I_M^i)$ at maturity M. Then, the GDBs upper and lower price boundaries of European ANCDDs and ALSPGDs for $i = T, V$ are given by the solutions of the following partial differential equations:

$$\begin{aligned} & -r\underline{C}^i + \frac{\partial \underline{C}^i}{\partial t} + \frac{1}{2}\sigma_s^2 S^2 \frac{\partial^2 \underline{C}^i}{\partial S^2} + \frac{1}{2}\sigma_i^2 \frac{\partial^2 \underline{C}^i}{\partial i^2} + \rho_i \sigma_s \sigma_i S \frac{\partial^2 \underline{C}^i}{\partial i \partial S} + \frac{dI^i}{dt} \frac{\partial \underline{C}^i}{\partial I^i} \\ = & -rS \frac{\partial \underline{C}^i}{\partial S} + \left(\frac{\mu_s - r}{\sigma_s} \rho_i \sigma_i - \mu_i(t) + \lambda_i + k \sqrt{A^2 - \left(\frac{\mu_s - r}{\sigma_s} \right)^2} \sigma_i \sqrt{1 - \rho_i^2} \operatorname{sgn}\left(\frac{\partial \underline{C}^i}{\partial i} \right) \right) \frac{\partial \underline{C}^i}{\partial i} \end{aligned} \tag{19}$$

with the terminal payoff

$$\underline{C}^i(S, i, I, M) = g(I_M^i) \tag{20}$$

where $k = +1$ and -1 generate the upper and lower price boundaries, respectively.

3. Empirical Studies

3.1. Data

We use the data between ambient temperature in Prague in Czech Republic (°C) and the solar power generation volume (MWh) to price solar power generation derivatives numerically. The data of solar power generation and daily average temperature at Prague Clementinum observatory are obtained from the website of ČEPS company and Czech meteorology institute. The data covers from January 1, 2012 to December 31, 2016. We also use daily stock price index (PX) data in Czech Republic obtained from the website of Prague Sock Exchange.

3.2. Temperature-based model and call option pricing

To obtain the latest relationship between temperature and solar power generation, we use the year 2016 data of ambient temperature in Prague in Czech Republic and the solar power generation volume (MWh), whose relation is reported in Figure 1. As we can see, the solar power generation volume increases in line with temperature from solar radiation. Here we linearly regress solar power generation on temperature using least squares.

$$V_t = p + qT_t + \varepsilon_t \tag{21}$$

The estimation results are reported in Table 1. The estimated variables of p and q are statistically significant from the comparisons with the corresponding standard errors. In particular, q is positive, implying that recent technological advance of solar power proceeds towards solar power generation with high efficiency and heat resistance.

As can be seen from Figure 1, the uncertainty in the relationship between the temperature and the solar power generation increases as the temperature rises. On the other hand, when the temperature exceeds near 25 °C, it can be seen that the uncertainty is decreasing. When the air temperature exceeds this level, the increase in solar power generation due to the increase in solar radiation and the decrease in the solar power generation due to the decrease in solar panel efficiency are canceled. As a result, the uncertainty of the relationship between the temperature and the solar photovoltaic power generation is reduced.

[INSERT FIGURE 1 ABOUT HERE]

[INSERT TABLE 1 ABOUT HERE]

We conduct the temperature-based model parameter estimation. The discretized stochastic process of the stock prices is given by

$$\Delta \log S_t = (\beta_0 - \frac{1}{2}\sigma_1^2) + \varepsilon_{1t}, \tag{22}$$

and the discretized model of temperature for $i = T$ is given by

$$\Delta i_t = \alpha_0^i + \alpha_2^i t + \alpha_3^i \sin(\omega t) + \alpha_4^i \cos(\omega t) - \alpha_1^i i_t + \varepsilon_{2t}^i. \tag{23}$$

Note that $\varepsilon_i = (\varepsilon_{1t}, \varepsilon_{2t}^i) \sim N(\mu_\varepsilon^i, \Sigma_\varepsilon^i)$, $\mu_\varepsilon^i = (0,0)$, and $\Sigma_\varepsilon^i = \begin{pmatrix} \sigma_1^2 & \rho_0^i \sigma_1 \sigma_2^i \\ \rho_0^i \sigma_1 \sigma_2^i & (\sigma_2^i)^2 \end{pmatrix}$.

We simultaneously estimate the parameters by the maximum likelihood method. The results are reported in Table 2. According to the standard errors in Table 2, σ_1, α_0^T, α_1^T, σ_2^T, α_2^T, α_3^T, and α_4^T are statistically significant.

[INSERT TABLE 2 ABOUT HERE]

Note that the result captures the mean reversion of the temperature deviation because the estimate of α_1^T (0.279) is greater than 0 and less than 1. It shows the long-term upward trend of temperature, which often describes global warming, because the estimate of α_2^T (1.560E-04) is positive and statistically significant. It also shows the annually sinusoidal trend of temperature by the statistically significant α_3^T (-0.530) and α_4^T (-2.719).

The parameters (μ_s, σ_s, λ_T, σ_T, ρ_T, θ_1^T, θ_2^T, θ_3^T, and θ_4^T) of the continuous-time models in Eqs. (14) and (7) for $i = T$ are obtained by integrating Eqs. (14) and (7) from t to $t+1$ and comparing the coefficients with the corresponding discrete-time models. We have the following results for $i = T$:

$$
\begin{aligned}
&\mu_s = \beta_0, \qquad\qquad \sigma_s = \sigma_1, \\
&\lambda_i = -\ln(1-\alpha_1^i), \;\; \sigma_i = \sigma_2^i\sqrt{\frac{2\ln(1-\alpha_1^i)}{(1-\alpha_1^i)^2-1}}, \;\; \rho_i = \frac{\sigma_1\sigma_2^i}{\sigma_s\sigma_i}\frac{\lambda_i}{1-e^{-\lambda_i}}\rho_0^i, \\
&\theta_1^i = \frac{\theta_2^i}{\lambda_i} + (\lambda_i\alpha_0^i - \theta_2^i)\frac{1}{1-e^{-\lambda_i}}, \;\; \theta_2^i = \frac{\lambda_i}{1-e^{-\lambda_i}}\alpha_2^i, \\
&\begin{pmatrix} \theta_3^i \\ \theta_4^i \end{pmatrix} = \frac{1}{B_{1i}^2+B_{2i}^2}\begin{pmatrix} \lambda_i B_{1i}+\omega B_{2i} & -(\omega B_{1i}-\lambda_i B_{2i}) \\ \omega B_{1i}-\lambda_i B_{2i} & \lambda_i B_{1i}+\omega B_{2i} \end{pmatrix}\begin{pmatrix} \alpha_3^i \\ \alpha_4^i \end{pmatrix}.
\end{aligned} \tag{24}
$$

Note that $B_{1i} = \cos(\omega) - e^{-\lambda_i}$ and $B_{2i} = \sin(\omega)$. Table 3 reports the conversion results.

[INSERT TABLE 3 ABOUT HERE]

We numerically calculate ANCDDs call option prices for cool summer risk hedge as the upper and lower price boundaries using the finite difference method. The benchmark temperature is assumed as 20 °C. We assume that $\mu_S = 0.0002$ to avoid a negative Sharpe ratio, selling price for solar power is $P_0 = 30$ EUR/MWh, and risk free rate is 0.00004 (1%/year). The strike is set as $P_0 * q * \Delta Temp * \Delta Days$ which implies cool summer risk exceed $\Delta Days = 5$ Days and $\Delta Temp = 5$ °C from 20 °C based on the five year historical data. The results are reported in Figures 2 and 3. Note that Figure 2 demonstrates the lower boundary surface of the option prices using the twice Sharpe ratio and Figure 3 demonstrates the upper and lower boundaries of the option prices at an initial stock price of 1050. We calculate ANCDDs call option prices on on August 1, 2017, which covers the period of late summer from August 2, 2017 to September 13, 2017 as an example. Figures 2 and 3 demonstrate that the call option prices decrease in line with initial temperature because I_t^T represents how much solar power generation is lower than the benchmark due to cool summer risk.

[INSERT FIGURE 2 ABOUT HERE]

[INSERT FIGURE 3 ABOUT HERE]

3.3. Solar power generation-based model and call option pricing

We conduct the solar power generation-based model parameter estimation. The discretized stochastic process of the stock prices is given by Eq. (22) and the discretized model of solar power generation is given by Eq. (23) for $i = V$. We simultaneously estimate the parameters by the maximum likelihood method. The results are reported in Table 4. According to the standard errors in Table 4, σ_1, α_0^V, α_1^V, σ_2^V, α_2^V, α_3^V, and α_4^V are statistically significant.

[INSERT TABLE 4 ABOUT HERE]

Note that the result captures the mean reversion of the power generation deviation because the estimate of α_1^V (0.622) is greater than 0 and less than 1. It shows the long-term upward trend of solar power generation, which may partly describe an indirect global warming effect, because the estimate of α_2^V (1.114E-01) is positive and statistically almost significant. It also shows the annually sinusoidal trend of solar power generation by the statistically significant α_3^V (541.774) and α_4^V (-2531.868).

The parameters (μ_s, σ_s, λ_V, σ_V, ρ_V, θ_1^V, θ_2^V, θ_3^V, and θ_4^V) of the continuous-time models in Eqs. (14) and (7) for $i = V$ are obtained by Eq. (24) for $i = V$ integrating Eqs. (14) and (7) from t to $t+1$ and comparing the coefficients with the corresponding discrete-time models. The results are reported in Table 5.

[INSERT TABLE 5 ABOUT HERE]

We numerically calculate the call option prices of ALSPGDs for cool summer risk hedge as the upper and lower price boundaries using the finite difference method. Here we use the same conditions to the ANCDDs call option pricing based on temperature. The benchmark solar power generation is assumed the power generation corresponding to 20 ^{o}C using Eq. (9). We assume that $\mu_S = 0.0002$ to avoid negative Sharpe ratios, selling price for solar power is $P_0 = 30$ EUR/MWh, and risk free rate is 0.00004 (1%/year). The strike is set as $P_0 * q * \Delta Temp * \Delta Days$ which implies cool summer risk exceed $\Delta Days = 5$ Days and $\Delta Temp = 5$ ^{o}C from 20 ^{o}C based on the five year historical data. We calculate ALSPGDs call option prices on August 1, 2017, which covers the period of late summer from August 2, 2017 to September 13, 2017 as an example. The results are reported in Figures 4 and 5. Note that Figure 4 demonstrates the lower boundary surface of the option prices using the twice Sharpe ratio and Figure 5 demonstrates the upper and lower boundaries of the option prices at an initial stock price of 1050. Figures 4 and 5 demonstrate that the call option prices decrease in line with initial solar power generation because I_t^V represents how much solar power generation is lower than the benchmark.

The call option prices in Figure 5 based on a solar power generation process approach are bigger than the call option prices in Figure 3 based on a temperature process approach. This is consistent with the fact that the solar power generation approach takes into account more comprehensive risk than the temperature approach as shown in Figure 1, resulting in the bigger call option prices for the solar power generation approach.

[INSERT FIGURE 4 ABOUT HERE]

[INSERT FIGURE 5 ABOUT HERE]

3.4. Basis risk premium for temperature hedging

Since in general temperature modeling is easier than solar power generation modeling, volumetric risk hedging strategies with temperature as the underlying asset are heavily used in practice. However, because of the mismatch between the hedged item and the hedging instrument, there is a basis risk between temperature and solar power generation hedging. By focusing on the basis risk, we empirically examine how much the prices of the two call options based on temperature and solar power generation are different. We define by $\underline{BRP}_t$ the lower (or upper) boundary of the basis risk premium arising from the mismatch between solar power generation and temperature hedging as follows:

$$\underline{BRP}_t = \underline{C}^G - \underline{C}^T \tag{25}$$

where $\underline{C}^G$ and $\underline{C}^T$ represent the GDBs lower (or upper, resp.) price boundaries of the European call options on solar power generation and temperature, respectively. Figure 6 reports the basis risk premiums for temperature hedging for the upper and lower boundaries with the twice Sharpe ratio.

From Figure 6, when the temperature rises from 0 ^{o}C, the basis risk premium is increasing. This is because the uncertainty in the relationship between the temperature and the solar power generation increases as the temperature rises in Figure 1. This is considered to be the reason of a deviation between temperature-based hedging and solar power generation-based hedging. More importantly, when the temperature exceeds near 25 ^{o}C in Figure 6, the basis risk premiums tend to decrease in line with the temperature. When the temperature exceeds this level, the increase in solar power generation due to the increase in solar radiation and the decrease in solar power generation due to the decrease in solar panel efficiency are canceled. As the result of the decrease of the uncertainty in solar power generation by temperature, the divergence between temperature-based hedging and solar power generation-based hedging diminishes.

[INSERT FIGURE 6 ABOUT HERE]

4. Conclusions

This paper studied volumetric risk hedging strategies for solar power under incomplete market settings with a twofold proposal of temperature-based and solar power generation-based models for solar power derivatives and discussed the basis risk arising from solar power volumetric risk hedge with temperature. Based on an indirect modeling of solar power generation using temperature and a direct modeling of solar power generation, we designed two types of call options written on the ANCDDs and the ALSPGDs, respectively, which can hedge cool summer volumetric risk more appropriately than those on well-known accumulated cooling degree days. We offered the pricing formulas of the two options under the GDBs framework, which can take into account incompleteness of solar power derivative markets. To calculate the option prices numerically, we derived the partial differential equations for the two options using the GDBs. Empirical studies using Czech solar power generation and Prague temperature estimated the parameters of temperature-based and solar power generation-based models, respectively. We numerically calculated the call option prices on ANCDDs and ALSPGDs, respectively, as the upper and lower price boundaries using the finite difference method. Results showed that the call option prices based on a solar power generation process are bigger than the call option prices based on a temperature process. This is consistent with the fact that the solar power generation approach takes into account more comprehensive risk than the temperature approach, resulting in the bigger call option prices for the solar power generation approach. We finally showed that the basis risk premiums, i.e., solar power generation-based call option prices minus temperature-based call option prices, decrease in line with initial temperature greater than around 25 °C. This may be because the uncertainty in solar power generation by temperature decreases due to the cancellation between the increase in solar power generation due to the increase in solar radiation and the decrease in solar power generation due to the decrease in solar panel efficiency.

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Figures & Tables

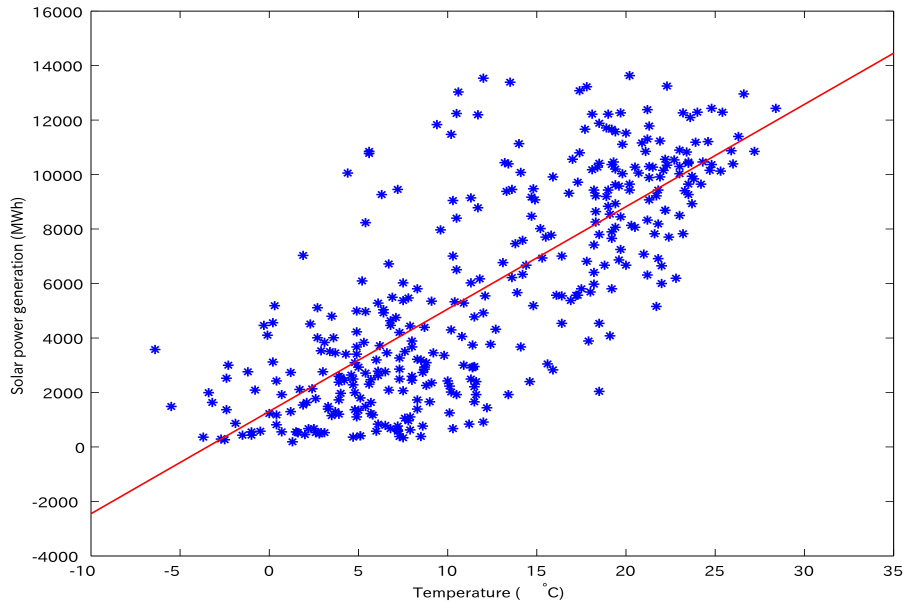

Figure 1. Scatter plots between ambient temperature and solar power generation in Czech solar power plants: We use the year 2016 data of ambient temperature in Prague in Czech Republic (°C) and the solar power generation volume (MWh). As we can see, the solar power generation volume increases in line with temperature from solar radiation.

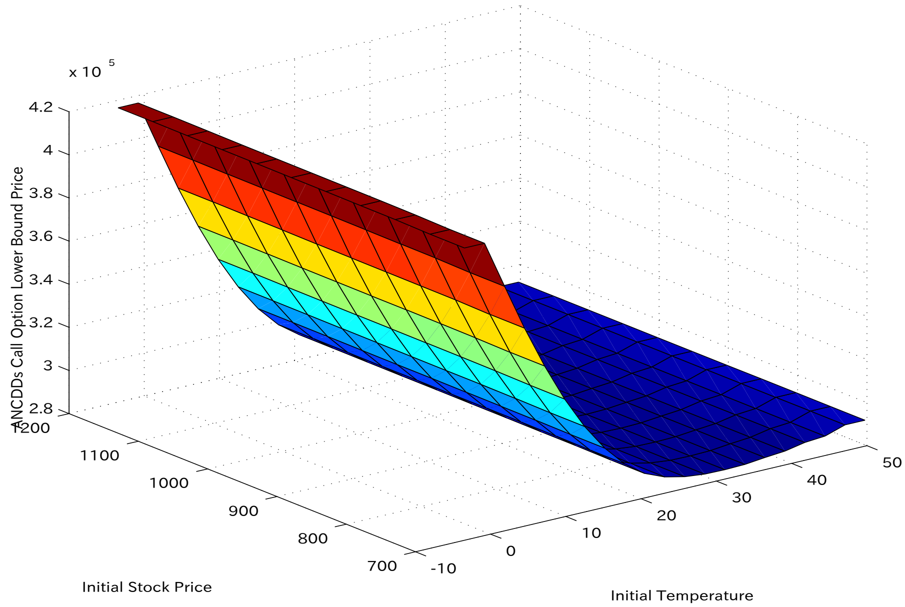

Figure 2. ANCDDs Call Option Price Lower Boundaries on August 1, 2017, which covers the period of late summer from August 2, 2017 to September 13, 2017: It demonstrates the lower boundary surface of the option using twice Sharpe ratio.

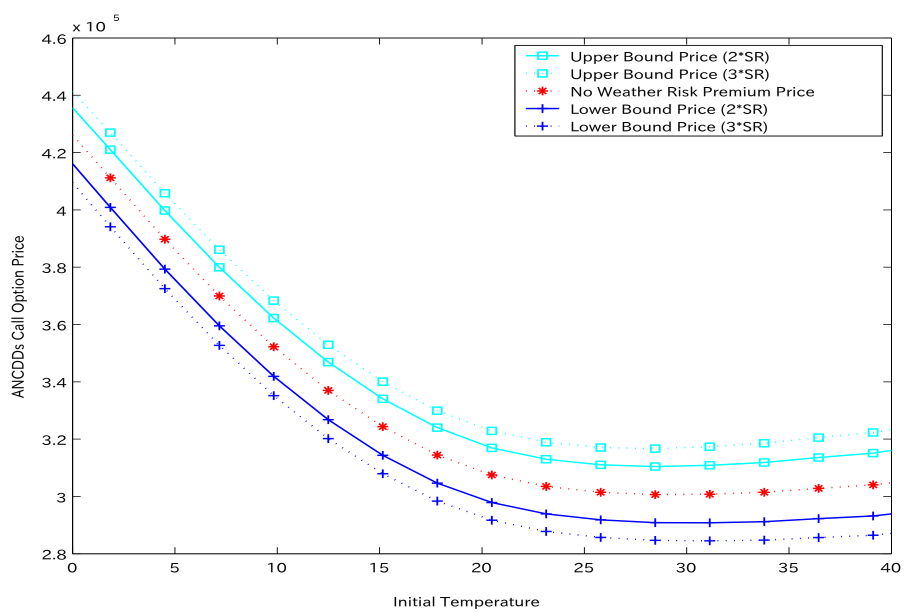

Figure 3. ANCDDs Call Option Price Boundaries on August 1, 2017, which covers the period of late summer from August 2, 2017 to September 13, 2017: It demonstrates upper and lower boundaries at an initial stock price of 1050.

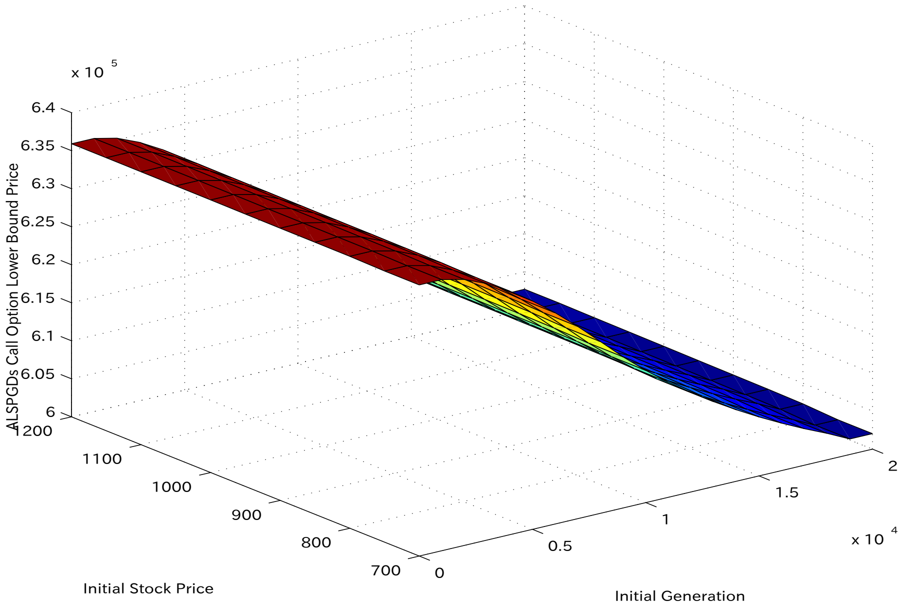

Figure 4. ALSPGDs Call Option Price Lower Boundaries on August 1, 2017, which covers the period of late summer from August 2, 2017 to September 13, 2017: It demonstrates the lower boundary surface of the option using the twice Sharpe ratio.

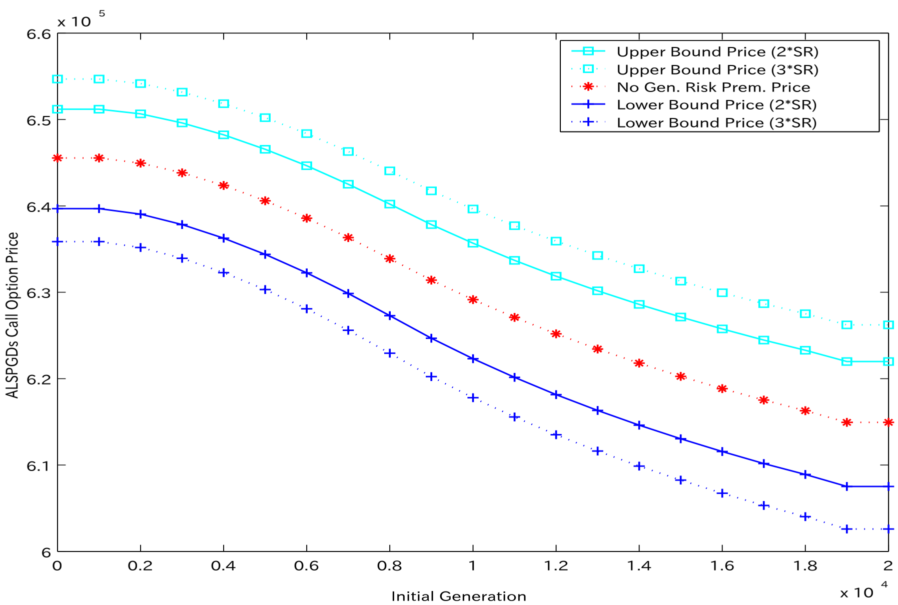

Figure 5. ALSPGDs Call Option Price Boundaries on August 1, 2017, which covers the period of late summer from August 2, 2017 to September 13, 2017: It demonstrates upper and lower boundaries at an initial stock price of 1050.

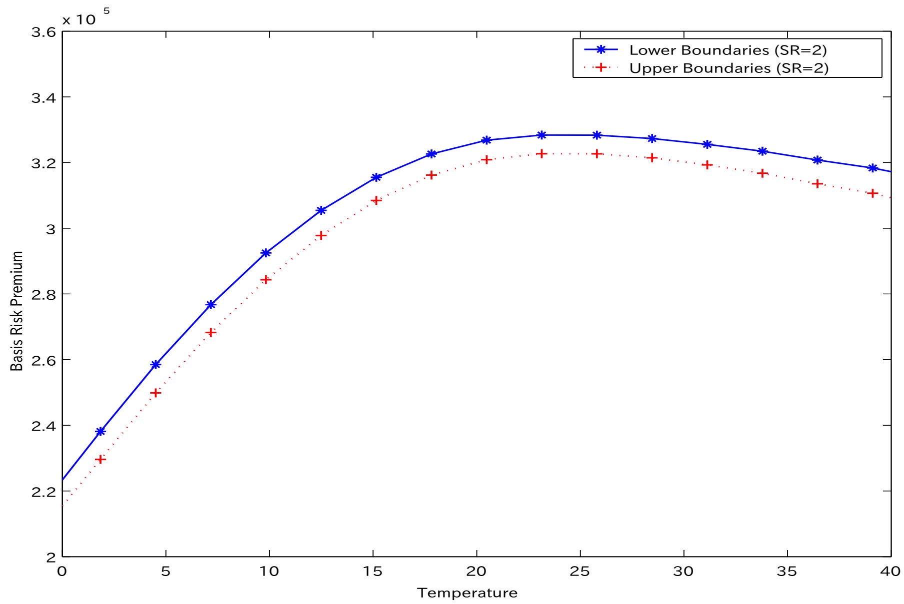

Figure 6. Basis Risk Premium for Temperature Hedging: Note the upper and lower boundaries with the twice Sharpe ratio. The basis risk premiums increase in line with initial temperature. When the temperature exceeds near 25 ^{o}C, the basis risk premiums tend to decrease in line with the temperature.

Variable	p	q
Coefficient	1306.090	375.711
Std. Error	233.012	16.415

Table 1. The parameter estimation regressing solar power generation on temperature: The estimated variables of p and q are statistically significant from the comparisons with the corresponding standard errors.

	β_0	σ_1	α_0^T	α_1^T	σ_2^T	ρ_0^T	α_2^T	α_3^T	α_4^T
Estimates	0.0000	0.010	3.154	0.279	2.555	0.048	1.560E-04	-0.530	-2.719
Std Errs	0.0003	0.000	0.072	0.005	0.054	0.029	6.699E-05	0.096	0.104
Loglik	1.07E+03								
AIC	-2.12E+03								
SIC	-2.14E+03								

Table 2. The model parameter estimation for temperature and stock price index in Czech Republic: According to the standard errors, σ_1, α_0^T, α_1^T, σ_2^T, α_2^T, α_3^T, and α_4^T are statistically significant.

μ_S	σ_S	λ_T	σ_T	ρ_T	θ_1^T	θ_2^T	θ_3^T	θ_4^T
0.0000	0.0097	0.3274	2.9832	0.0484	3.6979	1.8294E-04	-0.6506	-3.1831

Table 3. The continuos-time model parameters for temperature and stock price index in Czech Republic: The parameters (μ_s, σ_s, λ_T, σ_T, ρ_T, θ_1^T, θ_2^T, θ_3^T, and θ_4^T) of the continuous-time models in Eqs. (14) and (7) for $i = T$ are obtained by integrating Eqs. (14) and (7) from t to $t+1$ and comparing the coefficients with the corresponding discrete-time models.

	β_0	σ_1	α_0^V	α_1^V	σ_2^V	ρ_0^V	α_2^V	α_3^V	α_4^V
Estimates	0.0000	0.010	3413.418	0.622	2249.196	0.008	1.114E-01	541.774	-2531.868
Std Errs	0.0003	0.000	43.846	0.005	42.578	0.016	4.400E-02	65.276	53.805
Loglik	-7.41E+03								
AIC	1.48E+04								
SIC	1.48E+04								

Table 4. The model parameter estimation for solar power generation and stock price index in Czech Republic: According to the standard errors, σ_1, α_0^V, α_1^V, σ_2^V, α_2^V, α_3^V, and α_4^V are statistically significant.

μ_S	σ_S	λ_V	σ_V	ρ_V	θ_1^V	θ_2^V	θ_3^V	θ_4^V
0.0000	0.0097	0.9732	3389.2200	0.0082	5339.5477	1.7422E-01	807.9376	-3968.9388

Table 5. The continuos-time model parameters for solar power generation and stock price index in Czech Republic: The parameters (μ_s, σ_s, λ_V, σ_V, ρ_V, θ_1^V, θ_2^V, θ_3^V, and θ_4^V) of the continuous-time models in Eqs. (14) and (7) for $i = V$ are obtained by Eq. (24) for $i = V$ integrating Eqs. (14) and (7) from t to $t+1$ and comparing the coefficients with the corresponding discrete-time models.