Equalized Factor Price and Integrated World Equilibrium

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Abstract – This paper derives a general equilibrium of the Heckscher-Ohlin model. The equalized factor price at the equilibrium is just the price that Dixit and Norman (1980) described in their Integrated World Equilibrium (IWE), i.e. that equalized factor price and common commodity price remain the same when the allocation of factor endowments changes within the IWE box. This is the first analytical solution that presents an equalized factor price for the Heckscher-Ohlin model. The study demonstrates that giving an IWE box, there is only one equalized factor price with the property of the IWE price. The equilibrium solution is a joint statement of the Heckscher-Ohlin theorem and the factor-price equalization theorem.

Keywords:
Factor content of trade; H-O model; trade; factor price equalization; Heckscher-Ohlin; General equilibrium of trade; IWE; World price

1. Introduction

Essentially the Heckscher-Ohlin theorem and the factor-price equalization (FPE) theorem paved foundations of general equilibrium of the Heckscher-Ohlin model. The general equilibrium and the FPE are the same issues by different angles. McKenzie (1955)'s cone of diversification of factor endowments is a good concept to understand FPE and trade from production supply constraints. He provided a mathematical demonstration of the existence of the FPE for many factors and many goods.

Vanek(1968)'s HOV model promoted the usability of Heckscher-Ohlin theories on empirical trade analyses. The share of GNP in the HOV model engaged prices with trade and consumption. It also resulted in the application issue how to convert the assumption of homothetic taste into consumption balance. Woodland(2013) summarized the general equilibriums of trade and reviewed all important parts about trade equilibriums.

Dixit and Norman (1980) and their Integrated World Equilibrium (IWE) provided a strong clue for what a price-trade equilibrium should be and what an equalized factor price is. It draws out unique characteristic of equalized factor price at IWE box. Helpman and Krugman (1985) normalize the assumption of integrated equilibrium, which presented equilibrium analyses in a simple way. Deardroff (1994) derived the conditions of the FPE for many goods, many factors, and

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many countries by using the IWE approach. He discussed the FPE for all possible allocations of factor endowments.

Guo (2005) initialed his studies for the price structure of the FPE by a very specific condition. Guo (2015) introduced boundaries of commodity prices and the boundaries of shares of GNP in the price-trade equilibrium.

Is it possible to present a specific equalized factor price for a giving IWE box? This study derived a price-trade equilibrium for the Heckscher-Ohlin model and demonstrated that the equalized factor price and common commodity price at the equilibrium depended directly on world factor endowments. This means that the whole IWE box shares the same equalized factor price no matter where factor endowments allocate inside the box. The result is consistent with the insight inference that Dixit and Norman made four decades ago.

This is the first study to try to answer what equalized price is for a giving IWE box, although the factor-price equalization theorem and many studies had proved that there exists equalized-factor price with continuous free trade.

This paper is divided into five sections. Section 2 introduces the general equilibrium of trade and presents the structure of equalized factor prices. It also discusses a new logic of international trade that world factor resources determine world prices. Section 3 inferences autarky prices and presents a gain-from-trade triangle on IWE box. Section 4 is a numerical example to show equalized factor price at equilibrium, gains from trade, changes of allocation of factor endowments, changes of trade directions by reallocations of factor endowments. Section 5 provides a way to state the Heckscher-Ohlin model and theory briefly in algebra.

2. The general equilibrium of the Heckscher-Ohlin model

We take the following normal assumptions of the Heckscher-Ohlin model in this study: (1) identical technology across countries, (2) identical homothetic taste, (3) perfect competition in the commodities and factors markets, (4) no cost for international exchanges of commodities, (5) factors are completely immobile across countries but that can move costlessly between sectors within a country, (6) constant return of scale and no factor intensity reversals (7) full employment of factor resources.

We present the Heckscher-Ohlin model in the following way, for the convenience of analyses of trade equilibriums of this paper.

a. The production constraint of full employment of resources are

$$AX^h = V^h \qquad (h = H, F) \qquad (1)$$

where A is the 2x2 technology matrix, X^h is the 2 x1 vector of commodities of country h, V^h is the 2x1 vector of factor endowments of country h. The elements of matrix A is $a_{ki}(r, w), k = K, L, i = 1,2$. We assume that A is not singular.

b. The zero-profit unit cost condition

$$A'W^h = P^h \qquad (h = H, F) \qquad (2)$$

where $W^h$is the 2x1 vector of factor prices, P^h is the 2x1 vector of commodity prices.

Factor price equalization means (assuming it was equalized completely),

$$P^* = P^H = P^F \tag{3}$$

$$W^* = W^H = W^F \tag{4}$$

$$A'W^* = P^* \qquad (h = H, F) \tag{5}$$

Both P^* and W^* are world prices when factor price equalization reached.

c. The definition of the share of GNP of country h to world GNP,

$$s^h = P'\,X^h / P'\,X^W \qquad (h = H, F) \tag{6}$$

d. The export specification for the home country is

$$T^H = (1-s)X^H - sX^F \tag{7}$$

e. The factor content of trade for the home country is

$$F^H = (1-s)V^H - sV^F \tag{8}$$

f. The trade balance condition is

$$P'\,T^h = 0 \qquad (h = H, F) \tag{9}$$

or

$$W'\,F^h = 0 \qquad (h = H, F) \tag{10}$$

where $T^h$is the 2x1 vector of commodity export, F^h is the 2x1 vector of factor content of trade.

g. The constraint of the cone of diversification of factor endowments

$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} > \frac{a_{K2}}{a_{L2}} \quad , \quad \frac{a_{K1}}{a_{L1}} > \frac{K^F}{L^F} > \frac{a_{K2}}{a_{L2}} \tag{11}$$

This condition makes sure that the commodity outputs obtained from equation (1) are positive.

h. The constraint of commodity price limits[2]

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \tag{12}$$

This condition will make sure that the factor rewards from equation (2) are positive.

2.1 The determination of the equilibrium solution

Equations (1), (5), and (8) are available conditions for equilibrium solution directly; other equations are related conditions. Embedded in the Heckscher-Ohlin system represented by (1),(5), and (8), there are seven equations with nine endogenous variables in the model, they are p_1^*, p_2^*, w^*, r^*, x_1^H, x_2^H, x_1^F, x_2^F, and s (there are four exogenous variables as K^H, L^H, K^F, and L^F). It is not determined. We need to find another two conditions to solve the equilibrium. By Walras' equilibrium, we can drop one of these market-clearing conditions, such as we can take one price as the numeraire and set its value to unity as 1. That will leave only one uncertain condition for the equilibrium. If we result in that one, we will solve the equilibrium from algebra view.

We select to solve the share of GNP by using welfare analyses as an extra condition.

2.2 Engaging price with trade through the share of GNP

The HOV studies show that the GNP share of the home country is with boundaries by

$$s_b^H = \frac{K^H}{K^F + K^H} > s^H \tag{13}$$

[2] This condition will guarantee all possible factor prices are positive. We may refer (7) to constraint of cone of commodity prices. See section 3.

$$s^H > s_a^H = \frac{L^H}{L^F + L^H} \tag{14}$$

where s_b^H is the upper boundary and s_a^H is the lower boundary. We suppose here that country home is capital abundant as

$$\frac{K^H}{L^H} > \frac{K^F}{L^F} \tag{15}$$

Trade will redistribute national welfares, which are measured by GNP.

The boundaries of the share of GNP correspond commodity price boundaries as

$$s_b^H(p) = s(\left(\frac{a_{K1}}{a_{K2}}, 1\right)) = \frac{K^H}{K^F + K^H} \tag{16}$$

$$s_a^H(p) = s(\left(\frac{a_{L1}}{a_{L2}}, 1\right)) = \frac{L^H}{L^F + L^H} \tag{17}$$

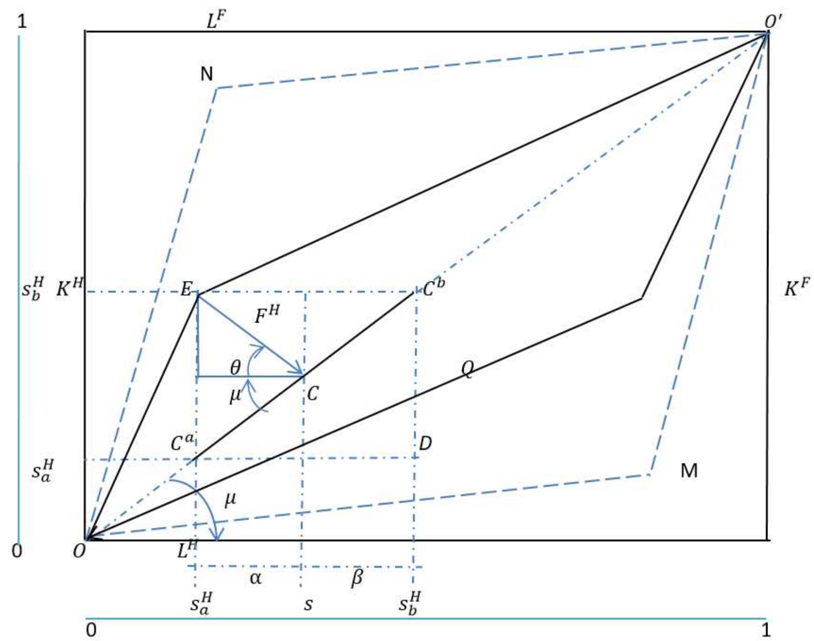

Figure 1 IWE with HOV GNP Share

Figure 1 is the IWE diagram with the GNP share boundaries added by this study. The dimensions of the box represent world factor endowments. The origin for country home is the lower left corner, for country foreign is the right upper corner. Any point in the world endowment box $ONO'M$ measures a set of available endowments of two countries. Suppose that the allocation of the factor endowments is at point E, where the home country is capital abundant.

The shares of GNP s_b^H and s_a^H are indicated on both the vertical unity axis and on the horizontal unity axis.

We refer to the area EC^aDC^b as the trade box. The point C is the trade equilibrium point. It measures the factor contents of trade as

$$F_K^H = K^H - sK^w \tag{18}$$
$$F_L^H = L^H - sL^w \tag{19}$$

By trade balance (10), there is

$$\frac{r}{w} = -\frac{L^H - sL^w}{K^H - sK^w} \tag{20}$$

It is the term of factor contents of trade.

The share of GNP of the home country s divides the trade box into two parts. Their lengths are α and β separately. When α increases, the share of GNP of the home country increases; the share of GNP of the foreign country decreases, and vice versa. In trade competitions, the both countries want to reach their maximum GNP share through free trade.

We notice that only trade box is the part of GNP redistribution area for the two countries. Outside the box, they are fixed and not redistributable by trade.

2.3 Settling the share of GNP

From Figure 1, the length of β can be expressed as

$$\beta = s_b^H - s_a^H - \alpha \tag{21}$$

We set a utility function as the product of α and β as

$$U = \alpha\beta \tag{22}$$

We maximize function U subject to the constraint of equation (22), to determine the values of α and β. The optimal solution is

$$\alpha = \beta = \frac{1}{2}(s_b^H - s_a^H) \tag{24}$$

With this simple competitive solution, both countries reach their maximum values of GNP shares in the box. Again, we emphasize that α and β are redistributable parts of GNP. This is a key point for the solution.

We now obtain the share of GNP of the home country by equations (13) and (24) as

$$s = \frac{1}{2}\frac{K^H L^w + K^w L^H}{K^w L^w} \tag{25}$$

It is a weighted average of factor endowments of two countries. It is just the middle point of the GNP boundaries (13). We can also interpret the result that the best welfares of two countries should avoid the hurts of extreme trades at the share of GNP of s_b^H or s_a^H, as far as possible. When taking the share of GNP as s_b^H, then $w^* = 0$; and when taking share of GNP as s_a^H, then $r^* = 0$. The middle point is the best position to reward both factors fairly based on existing factor endowment supplies.

Substituting (25) into (21), we get the rental-wage ratio as

$$\frac{r^*}{w^*} = \frac{L^w}{K^w} \tag{26}$$

Assuming the wage value equal to 1, we obtain the equilibrium solution of the Heckscher-Ohlin model as

$$r^* = \frac{L^w}{K^w} \tag{27}$$

$$w^* = 1 \tag{28}$$
$$p_1^* = a_{k1}\frac{L^w}{K^w} + a_{L1} \tag{29}$$
$$p_2^* = a_{k2}\frac{L^w}{K^w} + a_{L2} \tag{30}$$

We can also obtain factor content of trade and trade volumes,

$$F_K^h = \frac{1}{2}\frac{K^h L^w - K^w L^h}{L^w} \quad , F_L^h = \frac{1}{2}\frac{K^h L^w - K^w L^h}{K^w} \quad , \quad (h = H, F) \tag{31}$$

$$T_1^h = x_1^h - \frac{1}{2}\frac{K^h L^w + K^w L^h}{K^w L^w} x_1^w, \qquad T_2^h = x_2^h - \frac{1}{2}\frac{K^h L^w + K^w L^h}{K^w L^w} x_2^w \ , \quad (h = H, F) \tag{32}$$

Definitely, there are are

$$X^H = A^{-1}V^H \tag{33}$$
$$X^F = A^{-1}V^F \tag{34}$$

The equations (27) through (34) are the general equilibrium solution.

From (31) we can observe that when $\frac{K^H}{L^H} > \frac{K^w}{L^w}$, then $F_K^H > 0$. This is just the content of the Heckscher-Ohlin theorem.

The equalized factor price (26) display that he relative factor price, in reversely, is proportional to their world factor endowments. It does not relate to technologies. Moreover, it does not relate to commodity prices.

The Dixit-Norman factor price is not sensitive for factor mobile across countries. Dixit and Norman (1980) illustrated that when the allocation of the factor endowments changes, the factor price and the commodity price will remain the same. Their major argument is that the new allocation of factor endowments of the two countries leaves the same world supply of goods and, hence incomes unchanged and so supplies will still match the unchanged world demand.

In the algebra expression of the Heckscher-Ohlin model in (1) and (5), we did not use the assumption of factors mobile across countries. The world prices (27) through (30) themselves satisfy the Dixit-Norman IWE price inference. The equalized factor price and the Dixit-Norman IWE price are mutually ascertained each other in the solution. We see that world prices (27) through (30) as the PFE prices are confirmed by Dixit-Norman Inference.

The changes of allocations of factor endowments within the IWE box will cause changes of shares of GNP and the changes of trade volumes of two countries. This does not affect world commodity output and world prices.

The equilibrium adds another story to the IWE prices, that even the cone of diversification of factor endowments changes, the allocation change of factor endowments in the updated IWE box (world resource does not change) will not cause the change of relative factor price[3].

2.4 Allocation of Factor Endowments and Free Trade

[3] Commodity prices will change.

Why the above solution does not matter for the changes in the allocation of factor endowments? What is the meaning behind the mathematical expression?

The answer here is that it settled down the size issue. The size is a bothering issue to trade analysis. To make it safe, some studies add an assumption that the system is two small economies or two similar size economies. This can exclude the dominations of a country with a size advantage.

Inequalities (12) and (13) have limited trades on a reasonable range of all allocations of equilibriums. Therefore, no negative payment for factor endowments may occur. This outlines a basic relationship between the size of factor endowments and commodity price. This is just the beginning condition of equilibrium by the inside logic of Heckscher-Ohlin model.

When the relative commodity price closes to a_{K1}/a_{K2}, the home country, which is capital abundant, dominated the trade, there is no reward for labor. This is a hurt for both countries. The two countries only exchange commodity 1, which was produced intensively using capital.

On the contrary, When the relative commodity price closes to a_{L1}/a_{L2}, , the foreign country, which is labor abundant, dominated the trade, there is no reward to rent for the both countries. This is also a hurt for both countries.

When the share of GNP s moves toward the middle from the left, another factor, labor, begins to get its reward and begins to play a role in determining the world price.

In the middle point C, two factors symmetrically play equal roles fully to determine prices. Only at this point, prices are a function of world resources, as

$$p_i^* = p_i(L^w, K^w) \tag{35}$$

$$r^* = r(L^w, K^w) \tag{36}$$

At all other points of trade, two countries' factor endowments play roles separately and unevenly (unsymmetrically) as

$$p_i^* = p_i(L^H, K^H, L^F, K^F) \tag{37}$$

$$r^* = r(L^H, K^H, L^F, K^F) \tag{38}$$

Free trade is a fair trade. The relative factor price will not change if the allocation of factor endowments changes. This means that there is no room to adjust factor rewards. It also means that the size does not matter and that world resources matter for world prices.

2.5 World Factor Resources Determine World Price

Comparing the approach of achieving this equilibrium, the result of the equilibrium itself is more important[4]. For a giving IWE box, there is only one equilibrium point, which satisfies the IWE price

[4] The author had known the share of GNP (25) and the price solution (26) around 2008 by data simulation and paid some attentions for its symmetric characteristic. The author thought that it might be too simple to be acceptable as a FPE structure. The author did not realize that the solution was inferenced by Dixit and Norman in their IWE diagram, until 2015, in one of his manuscript submitted (not accepted). Without the IWE price inference, there is no delivery of this solution. The IWE provided a reference for the property of the FPE.

under mobile factor endowments. When world supply and demand of factor endowments remains the same, even factor endowments are mobile across countries, if they are in full employment, world prices will remain the same.

Thinking that the equilibrium price remaining same with mobile factor endowments before knowing structure details of prices is remarkable. Dixit and Norman did it with their IWE diagram[5].

3. Autarky Price and Comparative Advantage

3.1 Autarky price

It is hard to know autarky prices for countries before free trade. Therefore, it is not easy to show comparative advantages and gains from trade for the Heckscher-Ohlin model. We now propose a way to estimate autarky prices.

By the logic, that world factor resource determines world price, we can image of an autarky country, an isolated market, but with certain domestic trade, its "autarky" price can be determined by its "autarky" factor endowment.

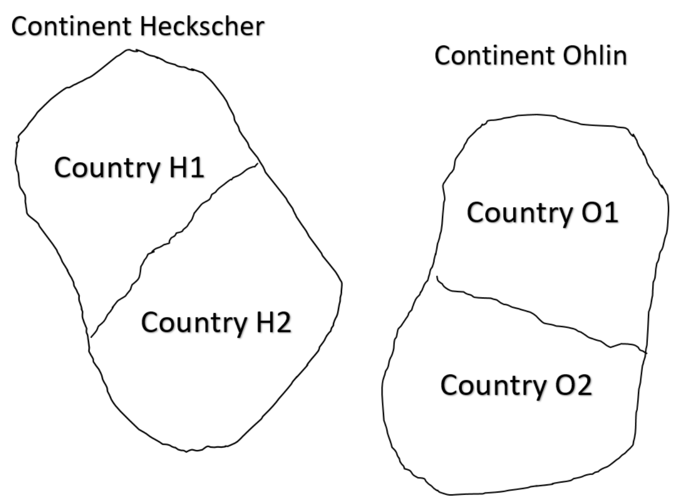

Figure 2 Autarky Price Formation

A good case to explain the estimation of autarky price is by Figure 2. There are two geographic continents, Heckscher and Ohlin, separated by an ocean. Continent Heckscher is with two free trade countries, H1 and H2. In addition, Continent Ohlin is with two free trade countries O1 and O2. Two continents start to free trade by no-cost shipping. Knowing the total factor endowments of each continent, we can estimate the prices of each continent by the expression of world price (27) through (30).

[5] Samuelson (1949) thought using an angel geographer-recording device dividing an isolated kingdom then everything remains same. Dixit and Norman built the device as the IWE diagram.

The IWE itself supports and proves the logic of autarky factor resource determining autarky price analytically. Assuming that one country shrinks to very small, another country's autarky price is then the world price of the current trade. Mathematically, when $V^H \to 0$, inside the IWE box, then $V^F \to V^W$ and $P^* \to P^{Fa}$. So that, we proved the autarky price mathematically.

Based on the above discussion, we present the autarky prices of countries that participate in free trade as

$$r^{ha} = \frac{L^h}{K^h} \qquad (h = H, F) \tag{40}$$

$$w^{ha} = 1 \qquad (h = H, F) \tag{41}$$

$$p_1^{ha} = a_{k1}\frac{L^h}{K^h} + a_{L1} \qquad (h = H, F) \tag{42}$$

$$p_2^{ha} = a_{k2}\frac{L^h}{K^h} + a_{L2} \qquad (h = H, F) \tag{43}$$

where superscript ha is used to indicate the autarky price of country h.

Assuming the home country is capital abundant, we immediately have:

$$\frac{p_1^{Ha}}{p_2^{Ha}} = \frac{a_{k1}L^H + a_{L1}K^H}{a_{k2}L^H + a_{L2}K^H} < \frac{p_1^*}{p_2^*} = \frac{a_{k1}L^W + a_{L1}K^W}{a_{k2}L^W + a_{L2}K^W} < \frac{p_1^{Fa}}{p_2^{Fa}} = \frac{a_{k1}L^F + a_{L1}K^F}{a_{k2}L^F + a_{L2}K^F} \tag{44}$$

$$\frac{w^{Ha}}{r^{Ha}} = \frac{K^H}{L^H} > \frac{w^*}{r^*} = \frac{K^W}{L^W} > \frac{w^{Fa}}{r^{Fa}} = \frac{K^F}{L^F} \tag{45}$$

Inequalities (44) and (45) are the necessary and sufficient condition of gains from trade, separately. They show the trade reason and the source of comparative advantage. Moreover, inequality (45) is the price definition of capital abundant. Appendix A is the proof of Inequality (44).

The Heckscher-Ohlin model brings another source of comparative advantage, differences in factor endowments across countries. Inequalities (44) and (45) soundly confirm the comparative advantages from differences of factor endowments as

$$-W^{ha\prime}F^h > 0 \qquad (h = H, F) \tag{46}$$

$$-P^{ha\prime}T^h > 0 \qquad (h = H, F) \tag{47}$$

$$-(P^{ha} - P^W)'T^h > 0 \qquad (h = H, F) \tag{48}$$

We add the negative sign in inequalities above since we expressed trade by net export, T^h . In most other literatures, they express trade by net import. Appendix B is the proof of inequality (46).

The analyses of this section demonstrate that the world prices at the equilibrium will ensure the gains from trade for both countries, by the autarky prices inference.

The result of gains from trade is just a good side effect of the equilibrium of trade. It is one property of the equilibrium and the FPE. We just realized it.

3.2 Triangle of gains from trade.

Another definition of a share of GNP is

$$s^h = W'\,V^h / W'\,V^W \qquad (h = H, F) \tag{49}$$

Substituting autarky factor prices (40) and (41) into the GNP definition (49), we get another two boundaries of the share of GNP,

$$s_a^{Ha} = \frac{w^{Ha}L^H + r^{Ha}K^H}{w^{Ha}L^w + r^{Ha}K^w} \quad (50)$$

$$s_b^{Fa} = \frac{w^{Fa}L^H + r^{Fa}K^H}{w^{Fa}L^w + r^{Fa}K^w} \quad (51)$$

The trade points by above shares of GNP are at N and at M separately, in Figure 3. The triangle ENM is a gains-from-trade triangle, a trade vector must fall on the diagonal line NM. This is a sufficient condition that both countries obtain gain from trades.

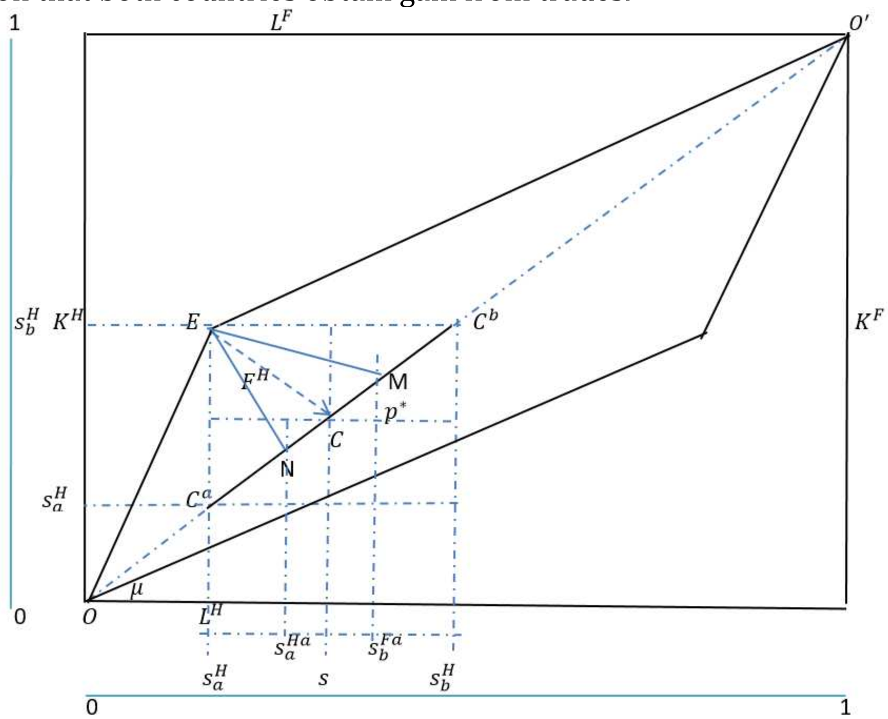

Fig. 3 IWE with Gains-from-Trade Triangle

If trade falls in line $C^a N$, the home country does not have a chance of obtianing its gains from trade. It even loses from trade $(-P^{ha\prime}T^h < 0)$. The foreign country will get all of the gains from trade with extra. The sector with a comparative advantage in production, of the home country, will not be rewarded well. This is a situation of the gains from trade for a country and the loss from trade for another country. Similarly, If trade falls in line MC^b , the foreign country will not have a chance to obtain the gain from trade.

The boundaries of autarky prices are useful to identify gains from trade and useful to identify loses from trade.

Theorem – The IWE compare advantage theorem
Behind the commodity trade, world factor resources determine world price, which ensures the gains trade of countries participating trade.

Proof

The solution (27) through (30) shows how the world prices are determined and why it remains the same with mobile factor endowments in the IWE box. The solution is unique for a giving IWE box.

Figure 2 illustrates the proposition by Edgeworth diagram. Mobile factor endowments will not change world commodity output, so the supply and demand of commodity do not change, world price will not change.

Appendix B proved the gains from trade as equation (46).

End Proof

4. Numerical examples

Let see some numerical examples, which displays the trade equilibrium, autarky price, gains from trade, and reallocation of factor endowments.

4.1 Equalized Factor Price and trade equilibrium

Consider two countries, home and foreign, two commodity, 1 and 2, two factors, capital and labor. The technological matrix is

$$\begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L2} \end{bmatrix} = \begin{bmatrix} 2.5 & 1.1 \\ 1 & 2 \end{bmatrix} \tag{52}$$

The factor endowments in two countries are

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 2400 \\ 1700 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 3600 \\ 4000 \end{bmatrix} \tag{53}$$

The outputs of the two countries by full employment are

$$\begin{bmatrix} x_1^H \\ x_2^H \end{bmatrix} = \begin{bmatrix} 751.28 \\ 474.94 \end{bmatrix}, \qquad \begin{bmatrix} x_1^F \\ x_2^F \end{bmatrix} = \begin{bmatrix} 717.94 \\ 1641.02 \end{bmatrix} \tag{54}$$

Commodity 1 is K-intensive and commodity 2 is L-intensive. The factor abundant ranking is that home country is capital-abundant and foreign country is labor-abundant. The trade direction is that home country exports commodity 1 and foreign country exports commodity 2.
The share of GNP of the home country is calculated as 0.3491, based on factor endowments across countries. The consumption, export, and prices, under free trade, reach the following equilibrium:

$$\begin{bmatrix} c_1^H \\ c_2^H \end{bmatrix} = \begin{bmatrix} 512.94 \\ 738.52 \end{bmatrix}, \qquad \begin{bmatrix} c_1^F \\ c_2^F \end{bmatrix} = \begin{bmatrix} 956.28 \\ 1376.85 \end{bmatrix}$$

$$\begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} 238.34 \\ -264.17 \end{bmatrix}, \qquad \begin{bmatrix} T_1^F \\ T_2^F \end{bmatrix} = \begin{bmatrix} -238.34 \\ 264.17 \end{bmatrix}$$

$$\begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} 305.26 \\ -290.00 \end{bmatrix}, \qquad \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = \begin{bmatrix} -305.26 \\ 290.00 \end{bmatrix}$$

$$\begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 3.3750 \\ 3.0450 \end{bmatrix}, \ \begin{bmatrix} r^* \\ w^* \end{bmatrix} = \begin{bmatrix} 0.9500 \\ 1.0000 \end{bmatrix} \tag{55}$$

4.2 Gains from trade

The autarky prices of two countries are estimated as

$$\begin{bmatrix} p_1^{Ha} \\ p_2^{Ha} \end{bmatrix} = \begin{bmatrix} 2.7708 \\ 2.7791 \end{bmatrix}, \begin{bmatrix} r^{Ha} \\ w^{Ha} \end{bmatrix} = \begin{bmatrix} 0.7083 \\ 1.0000 \end{bmatrix}$$
$$\begin{bmatrix} p_1^{Fa} \\ p_2^{Fa} \end{bmatrix} = \begin{bmatrix} 3.7777 \\ 3.2222 \end{bmatrix}, \begin{bmatrix} r^{Fa} \\ w^{Fa} \end{bmatrix} = \begin{bmatrix} 1.1111 \\ 1.0000 \end{bmatrix}$$

The gains from trade for the two countries are

$$-[p_1^{Ha} \quad p_2^{Ha}] \begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = -[r^{Ha} \quad w^{Ha}] \begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = 73.77$$
$$-[p_1^{Fa} \quad p_2^{Fa}] \begin{bmatrix} T_1^F \\ T_2^F \end{bmatrix} = -[r^{Fa} \quad w^{Fa}] \begin{bmatrix} F_K^F \\ F_L^F \end{bmatrix} = 49.18$$

4.3 The changes of allocation of factor endowments

We suppose the reallocation of factor endowments from (53) as

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 2400 - 2200 \\ 1700 - 1500 \end{bmatrix} = \begin{bmatrix} 200 \\ 200 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 3600 + 2000 \\ 4000 + 1500 \end{bmatrix} = \begin{bmatrix} 5600 \\ 5500 \end{bmatrix}$$

Now the home country is Labor abundant. The foreign country is a relatively large size of the economy. The share of GNP of the home country is 0.0342. Consumptions and trade are

$$\begin{bmatrix} c_1^H \\ c_2^H \end{bmatrix} = \begin{bmatrix} 50.26 \\ 72.36 \end{bmatrix}, \qquad \begin{bmatrix} c_1^F \\ c_2^F \end{bmatrix} = \begin{bmatrix} 1418.96 \\ 2043.01 \end{bmatrix}$$
$$\begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} -4.109 \\ 4.55 \end{bmatrix}, \qquad \begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} -5.263 \\ 5.00 \end{bmatrix}$$

And commodity price and factor price remain the same as the values in equation (55).

4.4 changes of the cone of diversification of factor endowments

Assuming that the technological matrix changed as

$$\begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L2} \end{bmatrix} = \begin{bmatrix} 2.2 & 1.2 \\ 1 & 2 \end{bmatrix}$$

We still use the allocation of factor endowment in equation (53). The share of GNP will remain the same, the factor price will remain the same. The output, trade, and commodity price will change as

$$\begin{bmatrix} x_1^H \\ x_2^H \end{bmatrix} = \begin{bmatrix} 862.50 \\ 418.75 \end{bmatrix}, \qquad \begin{bmatrix} x_1^F \\ x_2^F \end{bmatrix} = \begin{bmatrix} 750.00 \\ 1625.00 \end{bmatrix}$$
$$\begin{bmatrix} T_1^H \\ T_2^H \end{bmatrix} = \begin{bmatrix} 299.53 \\ -294.76 \end{bmatrix}, \qquad \begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} 305.26 \\ 290.00 \end{bmatrix}$$
$$\begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 3.090 \\ 3.140 \end{bmatrix}$$

5. Presenting basic Heckscher-Ohlin theories briefly

It is said that the Heckscher-Ohlin theories are a proportional theory. This is true. With help of the price-trade equilibrium, we provide the following chain inequalities to tell the Heckscher-Ohlin story in algebra briefly.

Before trade, we tell the characteristics of the model by

$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} > \frac{K^H - F_K^H}{L^H - F_L^H} \geq \frac{K^H + K^F}{L^H + L^F} \geq \frac{K^F - F_K^F}{L^F - F_L^F} > \frac{K^F}{L^F} > \frac{a_{K2}}{a_{L2}} \tag{56}$$
$$\frac{a_{K1}}{a_{k2}} > \frac{p_1^{Fa}}{p_2^{Fa}} > \frac{p_1^{Ha}}{p_2^{Ha}} > \frac{a_{L1}}{a_{L2}} \tag{57}$$

Alter trade, we tell it by

$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} > \frac{K^H - F_K^H}{L^H - F_L^H} = \frac{K^w}{L^w} = \frac{w^*}{r^*} = \left|\frac{F_K^H}{F_L^H}\right| = \frac{K^F - F_K^F}{L^F - F_L^F} > \frac{K^F}{L^F} > \frac{a_{K2}}{a_{L2}} \quad (58)$$

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^{Fa}}{p_2^{Fa}} > \frac{p_1^*}{p_2^*} > \frac{p_1^{Ha}}{p_2^{Ha}} > \frac{a_{L1}}{a_{L2}} \quad (59)$$

The chain of inequalities covers the Heckscher-Ohlin theorem, the Leamer theorem, the Factor Price Equalization theorem, and the Dixit and Norman IWE price. This will be a comprehensive brief description of the theories.

Conclusion

The paper attained the general equilibrium of trade in the 2 x 2x 2 standard Heckscher-Ohlin model and draw pictures to try to tell basic Heckscher-Ohlin theories briefly and clearly.

It argued that the equalized factor price from the IWE equilibrium is theoretically consistent with Heckscher-Ohlin theorem, the Factor-price equalization theorem, and the Dixit-Norman price.

The study abstracted one new logic of international economics that world factor resources determine world prices.

The paper made an inference of autarky prices by using the principle that world factor resources determining world price. The study examined the necessary and sufficient condition of comparative advantage for the Heckscher-Ohlin model. It identified the gains-from-trade triangle in the IWE diagram.

Appendix A

We just proof the following

$$\frac{p_1^{Ha}}{p_2^{Ha}} = \frac{a_{k1}L^H + a_{L1}K^H}{a_{k2}L^H + a_{L2}K^H} < \frac{p_1^*}{p_2^*} = \frac{a_{k1}L^w + a_{L1}K^w}{a_{k2}L^w + a_{L2}K^w} \quad \text{(A-1)}$$

It can be rewritten as

$$(a_{k1}L^H + a_{L1}K^H)(a_{k2}L^w + a_{L2}K^w) < (a_{k1}L^w + a_{L1}K^w)(a_{k2}L^H + a_{L2}K^H) \quad \text{(A-2)}$$

It can be simplified as

$$a_{L1}K^H a_{k2}L^w + a_{k1}L^H a_{L2}K^w < a_{L1}K^w a_{k2}L^H + a_{k1}L^w a_{L2}K^H \quad \text{(A-3)}$$

or

$$a_{L1}K^H a_{k2}L^w - a_{L1}K^w a_{k2}L^H < a_{k1}L^w a_{L2}K^H - a_{k1}L^H a_{L2}K^w \quad \text{(A-4)}$$

Rewrite it as

$$a_{L1}a_{k2}\,(K^H L^w - K^w L^H) < a_{K1}a_{L2}\,(K^H L^w - K^w L^H) \quad \text{(A-5)}$$

Inside the inequality, we know

$$K^H L^w - K^w L^H > 0 \quad \text{(A-6)}$$

(A-5) can be simplified as

$$a_{L1}a_{k2} < a_{K1}a_{L2} \tag{A-7}$$

This is true. So (A-1) is true.

Appendix B

Using (31) and (32), we rewrite (46) as

$$-\frac{1}{2}\frac{K^H L^w - K^w L^H}{L^w} r^{Ha} > \frac{1}{2}\frac{K^H L^w - K^w L^H}{K^w} w^{Ha} \tag{A-8}$$

By (A-6), we simplify (A-8) as

$$-\frac{K^w}{L^w} > \frac{w^{Ha}}{r^{Ha}} \tag{A-9}$$

Using $\frac{w^{Ha}}{r^{Ha}} = \frac{K^H}{L^H}$, (A-9) yields,

$$-\frac{K^w}{L^w} > \frac{K^H}{L^H} \tag{A-10}$$

We obtain

$$\frac{K^w}{L^w} < \frac{K^H}{L^H} \tag{A-11}$$

Therefore, inequality (46) is true.

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