# INFORMATIVE ADVERTISING IN MONOPOLISTICALLY COMPETITIVE MARKETS

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ABSTRACT. In their seminal paper Grossman and Shapiro (1984) find that informative advertising is socially excessive in an oligopoly (entry is also socially excessive). However, the analysis assumed that all consumers receive at least one advertisement. Christou and Vettas (2008), among others, present counter-examples in alternative settings, showing when the assumption does not hold, the equilibrium advertising may, instead, be inefficiently low. Christou and Vettas (2008) also show there may be non-existence due to discontinuities from undercutting, that quasiconcavity may not hold, and present examples in which the equilibrium does not exist as firms would deviate to a higher price. We revisit the question by modeling firms (like consumers) as a continuum, which mitigates the discontinuity that exists in both papers and allows the general analysis to include the cases when some consumers receive no advertisements. As a result, we are able to derive explicit and intuitive conditions for an equilibrium. More importantly, we find, instead, that advertising is socially *insufficient* regardless of the fraction of the consumers who receive an ad, including when all consumers receive at least one ad. We also find that there is insufficient entry instead of excess entry. We provide intuition for the difference between our and previous results.

**Keywords:** informative advertising, product differentiation, monopolistic competition, welfare.
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# 1. Introduction

Advertising is ubiquitous in print, airwaves and digital media with over a half-trillion dollars (about Belgium's GDP) being spent globally on advertising each year (www.statista.com). With advertising, a firm can persuade consumers to buy its product by communicating to them its characteristics and prices. This also allows the firm to distinguish its product from its competitors' and to enter into new markets, including those in which consumers are already aware of its rivals' products.

Given the large expenditures on advertising, its potential informational role and its effect on consumption decisions, determining the benefit these expenditures create for society has been a focus of economist, policy makers and the public. Seminal, classic work by Butters (1977) and Grossman and Shapiro (1984) introduced the modeling to investigate how informative (truthful) advertising benefits society through *demand creation* in the former –consumers learn of the existence of the market – and *matching effect* in the latter –consumers learn a product's characteristics relative to rival products.[1] Grossman and Shapiro (1984) model the match effect by using the Salop (1979) circular-city model: transportation costs represent society's lost from poor product matches.

While society and firms incur the same cost from advertising, the benefits to society can differ from the individual firm's benefit even with truthful advertising. That is, the market could have excessive or insufficient advertising from the social perspective.[2] Society's benefit from the demand creation tends to be greater than the firm's as a firm usually does not capture the consumer's entire benefit from learning about the product (i.e., consumer surplus).[3] However, the relationship between society's benefit from the matching effect (the consumer gets a product closer to their most preferred, which the firm does not internalize) versus the firm's benefit (it steals a sale from a rival, which has no benefit to society if costs are identical –*business stealing* effect) is ambiguous (Bagwell, 2007) though in most models including Salop (1979) the firm's benefit exceeds society's benefit.

A key result in Grossman and Shapiro (1984) is that for a given number of firms, advertising is socially excessive, though they note the demand creation effect is not present in their model as they assume that the number of firms is sufficiently large[4] in equilibrium so that all consumers receive at least one ad.[5] Thus, their analytical results turn only on whether the business stealing effect is greater than the matching effect. However, in the appendix they provide examples showing that with few firms in the market, advertising is still excessive. Christou and Vettas (2008, Fn. 29), however, find that if there are few firms, numerical examples show that the Grossman and Shapiro (1984) result "is not generally correct." They also show through examples that there can be insufficient advertising in a random utility model and recall that Tirole (1988) had noted this possibility and demonstrated it in a Hotelling model.

The main point of Christou and Vettas (2008), though, is that there may not be an equilibrium in these models: "Quasi-concavity of profits may fail, as each firm may prefer to deviate to a high price, targeting consumers who only become informed about its own product." Technically, Grossman and Shapiro (1984, Fn. 8) restrict the firm to prices below this price arguing the restriction "amounts to putting an upper bound on $v$," but they do not explicitly derive a condition on $v$. Unfortunately, providing a condition for the existence of the symmetric equilibrium in Christou and Vettas (2008) is not feasible. The main issue is that profits are discontinuous in price: a firm may choose to deviate from the equilibrium to

[1] However, as these –and our– models assume unit demand and covered market, a third beneficial "quantity-demand effect" resulting from the lower prices induced by advertising does not exist in these models.

[2] Given the assumption in all of these (and our) models of unit demand and "covered market," *i.e.*, all consumers are served in equilibrium, the market price is irrelevant to the social optimum.

[3] Shapiro (1980) shows that a monopolist under-advertises for this reason.

[4] Grossman and Shapiro (1984, Fn. 10) "The approximation is most accurate for $n$ large."

[5] E.g., the demand function Grossman and Shapiro (1984, eq. 7) assumes all consumers receive at least one ad.

a lower price to not only capture the customers between the firm and its nearest neighbor but also its next nearest neighbors –the supercompetitive price in Salop (1979), leading to a discrete increase in demand. To avoid this possibility Grossman and Shapiro (1984) restrict their analysis to prices greater than the Salop supercompetitive price.[6]

We return to the question of informative advertising in an oligopoly, but with a different approach to resolve the problems with discontinuity, allow for the demand creation effect, and obtain existence conditions so that the equilibrium can be fully characterized without an approximation. We do this by also using the Salop (1979) model but take Grossman and Shapiro (1984) assumption that the number of firms is "large" to the limit: we assume a continuum of firms just as Butters (1977) implicitly assumes,[7] which also mirrors the assumption on consumers. This assumption could also be viewed as merging of the two classic models.[8] *A priori* the assumption would not appear to change the underlying market mechanisms, and indeed our equilibrium price and profits expressions are consistent with Grossman and Shapiro (1984) and have all the same comparative static.[9] With this assumption there is no price decrease that creates a discrete increase in demand and as a result we can achieve our goals. Finally, this assumption allows us to focus on the effect entry has on advertising without conflating entry's (welfare beneficial) effect on reducing transportation costs as with a continuum entry solely increases the mass of firms.[10]

The first gain from our approach is that we derive explicit conditions for the existence of a unique symmetric equilibrium. This is not just a technical curiosity as the conditions give insights as to when there will be a monopolistically competitive equilibrium, which are missing from the previous work. Specifically, there are two conditions, and they ensure that a firm does not have an incentive to deviate to the monopoly price. The first (18) is that the consumer that a firm reaches is sufficiently likely to receive an advertisement from another firm (about a half chance): if this condition did not hold a firm is unlikely to face competition, and so would want to set instead a higher price. This condition effectively requires that marginal advertising costs are low relative to transport costs. The second condition (19) is that the monopoly price (or more precisely, the consumers' potential reservation value $v$) cannot be too much greater than the competitive price: even if a firm faces a good chance of having at least one rival, if the equilibrium price is too close to marginal cost (and thus earns effectively zero), then the monopoly price is attractive even if it unlikely to succeed. For this reason, the second condition is that the monopoly price cannot be too much greater than the equilibrium price interacting with the probability that a firm is in a monopoly position.

To provide more details to the second condition, recall that in Hotelling/Salop type models there is a disconnect between the competitive price and the monopoly price, as the former depends on transportation costs and the number of rival firms, but is independent of the consumers' reservation value. Thus, the second condition requires an upper bound on consumers' potential reservation value $v$ relative to transportation costs: a greater $t$ both lowers the monopoly price ($v - \frac{t}{2}$) and raises the monopolistically competitive price, the direct effect from transportation costs. The need for an upper bound is noted in Grossman and Shapiro (1984, Fn. 8), but not derived. The second condition also depends on transportation and advertising costs through their effect on aggregate advertising, but aggregate advertising has are two opposing effects. On one hand, as aggregate advertising increases

[6]Specifically, given rivals all setting $\overline{p}$ Grossman and Shapiro (1984, Fn 8) assume that the firm would choose a $p \in \left(\overline{p} - \frac{t}{n}, \overline{p} + \frac{t}{n}\right)$. This restriction may have additional importance as, e.g., in their base case simulation, it would seem a deviation to the supercompetitive price is more profitable than their equilibrium price. Thus, letting firms set the supercompetitive price may well change the analysis.

[7]Butters (1977) assumes the number of firms approaches infinity so that the Poisson distribution can be used to approximate the probability that any given consumer receives an ad.

[8]Though we do not study this, in the limit, as transportation costs go to zero ($t \to 0$), the model becomes the Butters (1977) model.

[9]Not all comparative statics in Christou and Vettas (2008) are the same as in Grossman and Shapiro (1984).

[10]In Grossman and Shapiro (1984), entry reduces aggregate transportation costs directly.

the monopolistically competitive price decreases and so deviating to the monopoly price appears more attractive. On the other hand, as aggregate advertising increases, the probability of being *ex post* a monopolist decreases and so deviating to the monopoly price is less attractive. As a result, the relationship between aggregate advertising and the upper bound on the consumers' reservation value is non-monotonic. For very low levels of aggregate advertising, an increase in aggregate advertising causes the upper-bound on consumers' reservation value to decrease (so that the monopoly price is lower and less attractive relative to the monopolistically competitive equilibrium). However, beyond a point, as aggregate advertising increases, the probability of the firm ever being in the monopoly position goes to zero, and the largest monopoly price compatible with equilibrium also increases.

These opposing effects appear with changes in advertising costs. As advertising costs increase, aggregate advertising decreases (a firm is more likely in a monopoly position), but the equilibrium price increases (monopoly price is less attractive). As a result, there is a non-monotonicity: starting at low costs, the former dominates and so the condition for an equilibrium tightens as advertising costs increase. As they increase further, the latter can dominate, thereby relaxing the condition. In an example reported in Section 7, with a specific cost function, for some values of $v$ there is an equilibrium for low advertising costs, but as advertising costs increase, the equilibrium no longer exists, but as costs increase even further the equilibrium exists again, and finally if it increases even further an equilibrium no longer exists (because of the first condition).

The second condition, an upper bound on consumers' potential reservation value, is interesting for another reason as it points to the conflict between the *covered market* assumption and the monopolistically competitive equilibrium. Specifically, as in Grossman and Shapiro (1984, Fn. 9), we make the standard covered market assumption: the consumers' potential reservation value $v$ is sufficiently large that all consumers that receive an advertisement, will chose to buy. However, our condition (and in Grossman and Shapiro (1984, Fn. 8)) shows that $v$ cannot be too large relative to advertising and transportation cost, or else the firm will deviate to the higher monopoly price. These two restrictions could in principle rule out existence of an equilibrium, both here and possibly in Grossman and Shapiro (1984); however, values of $v$ compatible with equilibrium existence always exists in our model.

Another gain from our approach is that we can have monopolistically competitive equilibria in which a large fraction of consumers receives no advertisements, while Grossman and Shapiro (1984) results are derived under the assumption that all consumers receive at least one ad. Further, while they do provide examples that have a small number of firms ($n$), it is still true that the probability that every consumer receives at least one ad is roughly 1. E.g., for the benchmark case, the probability is 0.9999. The lowest probability in any of the examples is 0.9945. As discussed above, several have noted that if some consumers receive no advertisements, advertising can be socially insufficient. We confirm this in our setting. However, we find an even stronger result: that advertising is socially insufficient even if all consumers receive an ad, and that therefore *advertising is always socially insufficient* (Proposition 4). Moreover, since in our model, as with the previous models, there is no individual "quantity demand effect" from the lower price that increased advertising induces (all the models have unit demand), our model may underestimate the social benefit from truthful advertising and how much advertising is socially under-provided in the market.

The intuition for our result is as follows. When aggregate advertising is low, and each customer is unlikely to receive multiple advertisements, there is a substantial demand creation effect. In this case, as the firm does not capture all the surplus, its private return is less than the social return, a well-known effect. As aggregate advertising increases, it becomes more likely that a consumer which the firm reaches also receives other advertisements. The gain to society becomes smaller as it is more about a better match (saving in transportation costs), than demand creation, but the equilibrium price decreases so the firm's gain is also smaller. However, a key difference between a full information environment such as, e.g., Salop (1979) and the environment in Grossman and Shapiro (1984) or in the present paper is that "competition is no longer localized" (Grossman and Shapiro, 1984, p. 76); the firm potentially competes for all consumers, including those the furthest away from it, who may have an offer from a firm that has a product very close to that consumer's ideal. In contrast, with compete information the firm only competes for consumers that are immediate neighbors. Thus, with incomplete information there is downward pressure on the price as it induces the furthest consumer to buy while the price need not do this with complete information. So, the firm's return is smaller. On the other hand, society cares about the expected savings in transportation costs. Thus, the gain from the business stealing effect is smaller than in the Salop model and society's gain: there is too little advertising.

As to the differences between our results and the results of Grossman and Shapiro (1984), recall that Christou and Vettas (2008, Fn. 29) find examples in which when the demand creation effect is strong there is insufficient advertising in the Grossman and Shapiro (1984) model. Secondly, the approximation on which the analysis of Grossman and Shapiro (1984) rely does more than just eliminate the demand creation effect; Equation (6) in Grossman and Shapiro (1984) overstates the marginal revenue (quantity sold) from an increase in advertising and so overstates the equilibrium level of advertising. The approximation also underestimates the welfare benefit from the *matching effect* unless the number of firms and/or the fraction of consumers reached are small – which is ruled out by their approximation. Further, the approximation effectively requires that consumers receive advertisements from at least two firms, which of course implies a large chance the consumer receives an advertisement from more than two firms.[11] Since the consumer is likely to receive several advertisements, the welfare gain from the expected reduction in transportation costs from another advertisement is reduced. Thirdly, as Butters (1977) with effectively a continuum of firms finds advertising to be socially optimal with homogeneous goods, the divergence in results would not seem to turn on the assumption of a continuum of firms. Fourthly, in the Salop (1979) model with a discrete number of firms, there is an "excessive entry" effect as the private return from entry exceeds the social benefit from the reduction in transportation cost, and this may be driving the excess advertising. Without the demand creation effect, the marginal increase in advertising can be thought of as "entering" a new Salop market that already has firms in it (i.e., to the consumer reached, the other firms that have reached them) in which we know the incentives to enter are excessive. Thus, one advantage of our approach is that while there is "business stealing" – one firm's sale may come at the expense of another – there is not the effect that induces excess entry with the Salop model, which may muddle the analysis of the direct effect of advertising.

We also consider how social and private returns differ as the product become more differentiated. Christou and Vettas (2008) provide examples with a fixed number of firms in which advertising can be either insufficient of excessive, from a social point of view, depending on whether the support of the consumers' valuation is small or large, namely depending on whether product differentiation is weak or strong. In our model, we find too that, for fixed number of firms, as transport cost increase the private value increases relative to the social value because the equilibrium price increases. However, this points to why having the equilibrium conditions is important: while for large enough $t$ the private value of advertising would exceed the social value *if it were a monopolistically competitive equilibrium*, with such a large $t$ a monopolistically competitive equilibrium does not exist.

We then endogenize entry and again find that our comparative statics are in line with Grossman and Shapiro (1984).[12] In particular, we find, as they do, that an increase in the

[11] Specifically, equation (6) in Grossman and Shapiro (1984) has a term $(1-\overline{\phi})^{n-1}$ that is set to zero as well as a term $(1-\overline{\phi})^{n}$. The probability that a consumer receives advertisements from at least two firms is $1-(1-\overline{\phi})^{n}-n(1-\overline{\phi})^{n-1}$. Even in their simulations, the probability of receiving advertisements from *at least four* firms is high. For example, in the benchmark case it is 0.96 with $n=14$.

[12] Because of intractability, the entry equilibrium cannot be characterized in Christou and Vettas (2008).

cost of advertising could induce entry as it increases profits for given number of firms. We provide an alternative condition for this: a proportional increase in marginal cost does this (for example, an ad-valorem tax based on marginal costs). We present an example showing that with quadratic cost of advertising (in which case, an increase in the coefficient of marginal cost causes a proportionate increase in marginal cost), that equilibrium profits for a fixed number of firms increases. The intuition is that advertising is a prisoner's dilemma –the cooperative solution has less advertising– so increases in the cost of only the last units of advertising can increase profits by inducing the cooperative solution of less advertising.

Finally we show that socially there is too little entry: the planner would have more firms enter the market while in Grossman and Shapiro (1984) the planner would reduce entry. This result occurs despite our model not having the social benefit of entry reducing transportation cost as it does in Grossman and Shapiro (1984). The reason for the difference may partly turn on the earlier discussion regarding localized competition, but also on that in the Salop model there is inherently excessive entry. In contrast with a continuum Dixit and Stiglitz (1977) find there can be too much or too little entry. Thus, Grossman and Shapiro's analysis of entry may be combining the effects of advertising (the question at hand) with the effect of strategic competition and the direct effect of transportation cost. Intuitively, since our model does not have the excess entry effect nor a direct effect on transportation costs, entry is solely a question of its effect on advertising (which has an indirect effect on transportation costs). As we had already seen that for fixed number of firms there is too little advertising, then that there is too little entry is no longer surprising. In some sense the planner's problem is to choose the level of aggregate advertising while minimizing cost through the choice of entry and advertising levels.

The remaining part of the paper is organized as follows. Section 2 presents the model, and Section 3 characterizes the conditions guaranteeing surplus maximization. Sections 4, 5 and 6 characterize the sellers' pricing, advertising and entry equilibrium decisions. Section 7 presents an example that highlights some important points, with particular reference to the case of free entry, and Section 8 contains some concluding remarks. The proofs of the main results are in Appendix A; the results related to the model's comparative statics, along with the respective proofs, are in Appendix B.

## 2. The model

We model preferences and production as in the standard Grossman and Shapiro (1984); Salop (1979) setting, with a unit continuum of buyers, endowed with a preference parameter that is uniformly distributed around a circle with unit circumference. Each buyer demands at most one unit of one of the products. The distance between any two arbitrary addresses $x$ and $s$ in $[0, 1)$ is defined as

$$d(s, x) = \min\{|s - x|, 1 - |s - x|\}, \tag{1}$$

where $|.|$ is the absolute value operator. A buyer with preference parameter (or "address") $x$ who purchases and consumes the product of seller $s$ at the price $p_s$ receives utility

$$u(x, s) = v - t\, d(s, x) - p_s,$$

where $v \in \mathbb{R}_{++}$ and $t \in \mathbb{R}_{++}$ respectively express the payoff from consumption of the "ideal" product and the pace at which the payoff decreases as $d(s, x)$ increases. To dispense with tedious qualifications related to possible corner solutions, we follow Grossman and Shapiro (1984)[13] and make the standard "covered market" assumption, expressed in this case by $t < v$. Under this assumption, each buyer would be willing to pay the monopoly price $v - \frac{t}{2}$ even for her least preferred product.

There is also a set $\Sigma$ of potential, ex-ante identical sellers. By default, all sellers are inactive. Each seller who decides to be active chooses an arbitrary address in the circle and

[13] See Grossman and Shapiro (1984, Fn. 9).

produce as many units of the product as she wishes, upon demand, at the constant unit cost of 0. The buyers who do not purchase a product and the sellers who remain inactive realize a payoff equal to 0. The size of the set of the sellers who choose to be active is denoted by $m$.

The buyers may in principle know about the existence of the sellers and the products, but they are only able to purchase the products - if any - for which they receive advertisements from the respective sellers, The advertisements truthfully inform the buyers about the characteristics and price of the products, and the number of advertisements delivered by each seller is drawn from a Poisson distribution with parameter $\alpha \in \mathbb{R}_+$, whose value is chosen by the seller and coincides with the expected number of advertisements delivered.

In connection with the delivery process, we can envisage our circle as a projection of a vertical cylinder with unit height, with the buyers uniformly distributed on its side surface and each vertical line identifying buyers with identical preferences. The identities of the buyers who receive the advertisements are then randomly drawn from the uniform distribution on the cylinder's side-surface. The draws are thus independent, both for each seller and across sellers; the addresses of the potential trading partners faced by each seller are uniformly distributed around the circle, and the probability of multiple advertisements delivered by a seller to any given buyer is equal to 0.

The advertising cost faced by each seller is linked to the intensity variable $\alpha$ by the function $c : \mathbb{R}_+ \to \mathbb{R}$, which is at least twice differentiable, strictly increasing and strictly convex over $(0, \infty)$. We conveniently denote the first and the second derivative of $c(\alpha)$ by $c'(\alpha)$ and by $c''(\alpha)$. We assume that $c'(\alpha)$ grows without bounds as $\alpha$ increases, namely $\lim_{\alpha\to\infty}\{c'(\alpha)\} = \infty$, and that the limit $\lim_{\alpha\to 0}\{c(\alpha)\}$, denoted by $F$, is strictly positive. We also identify the choice of $\alpha = 0$ with the decision to remain inactive, and set $c(0) = 0$, i.e. we effectively assume that each seller will only bear the fixed cost if her advertising intensity is positive with no loss of generality. Our assumptions imply

$$\lim_{\alpha\to 0}\left\{\frac{c(\alpha)}{\alpha}\right\} = \lim_{\alpha\to\infty}\left\{\frac{c(\alpha)}{\alpha}\right\} = \infty, \tag{2}$$

and guarantee existence of a unique positive advertising intensity $\underline{\alpha}$, featuring equality between the average cost

$$\underline{c} \equiv \frac{c\left(\underline{\alpha}\right)}{\underline{\alpha}} \tag{3}$$

and the marginal cost corresponding to it, which minimizes the cost per unit of advertising intensity.[14]

We also assume a relatively efficient advertising technology, in the sense that

$$\left(v - \frac{t}{4}\right)\underline{\alpha} - c\left(\underline{\alpha}\right) > 0. \tag{4}$$

This assumption guarantees that a set of active sellers with positive measure choosing the most efficient level of advertising intensity can generate a surplus, net of the advertising cost. Given (3) and convexity of $c(\alpha)$, (4) guarantees that

$$\left(v - \frac{t}{4}\right)\alpha - c'(\alpha) > 0. \tag{5}$$

[14]Existence of $\underline{\alpha}$ follows from continuity of $\frac{c(\alpha)}{\alpha}$ over $\mathbb{R}_{++}$ and (2). As to uniqueness, if (3) holds both if $\underline{\alpha} = \alpha'$ and if $\underline{\alpha} = \alpha''$, then strict convexity of $c(\alpha)$ over $(0, \infty)$ implies that for any $\lambda \in (0, 1)$ we must have

$$\begin{aligned} c\left(\lambda\,\alpha' + (1-\lambda)\,\alpha''\right) &\le \lambda\, c\left(\alpha'\right) + (1-\lambda)\, c\left(\alpha''\right) \\ &= \lambda\,\alpha'\,\underline{c} + (1-\lambda)\,\alpha''\,\underline{c} \\ &= \left(\lambda\,\alpha' + (1-\lambda)\,\alpha''\right)\underline{c}. \end{aligned}$$

If $\alpha' \neq \alpha''$, the inequality would be strict, and the choice of any advertising level $\lambda\,\alpha' + (1-\lambda)\,\alpha''$ would be associated with an average cost lower than the minimum cost $\underline{c}$. Hence, $\alpha'$ must be equal to $\alpha''$.

Convexity of $c(\alpha)$ is not sufficient to guarantee existence of a cost-minimizing scale of advertising. For example, a cost-minimizing scale does not exist if $c(\alpha) = k + \alpha + \frac{1}{\alpha+1}$, for any $k \in \mathbb{R}_{++}$.

holds for values of $\alpha$ in a right-neighborhood of 0 essentially, a positive margin over the variable cost at low scales of production is required to cover the fixed cost.

Trade takes place subject to mutual agreement. The timing is as follows:

(1) The sellers choose their advertising intensity $\alpha$ - and thus also choose whether to be active, or not, respectively if they set $\alpha > 0$ or $\alpha = 0$. Each active seller also chooses her address $s \in [0, 1)$.
(2) The sellers set their prices.
(3) Consumers receive the sellers' advertisements, and decide whether to purchase one of the products for which they received an offer - if any - or to exit the market without purchasing any product.

Decisions made within each stage are simultaneous.

All agents are risk-neutral, and maximize their expected payoffs. We focus on pure strategy Perfect Bayesian Equilibria ("PBEs"), in which the sellers are uniformly distributed around the circle,[15] and all active sellers choose identical advertising intensities and prices. As we show in Proposition 1, in Section 3, the uniform distribution of the sellers is a necessary condition for surplus maximization. Definition 1 provides the operational definition of equilibrium.

**Definition 1.** *An equilibrium is:*

*(1) An acceptance strategy, used by all buyers, such that a buyer with address $x$ who receives offers from a (finite) set of sellers $S$ accepts the offer of an arbitrary seller in the set*

$$S' \equiv \{ s' \in S | v - t\, |s' - x| - p^E \geq 0 \text{ and for each } s \in S, \\ v - t\, |s' - x| - p^E \geq v - t\, |s - x| - p^E \},$$

*including those among the sellers in $S$ whose offers yield the largest positive net surplus, and to forgo purchasing a product if $S'$ is empty, namely if she does not receive any offers that would leave her with a positive net surplus.*

*(2a)* In the case of a given mass of active sellers - *A profit maximizing ordered triple $(\alpha_s, i_s, p_s) \in (\mathbb{R}_+ \times [0, 1) \times \mathbb{R}_+)$, for each seller $s \in \Sigma$, such that $\alpha_s > 0$ and $p_s$ are identical across sellers, the choices of $i_s$ make the sellers' population uniformly distributed over* $[0, 1)$*, and each seller realizes a non-negative profit.*

*(2b)* In the case of free entry - *The same requirements as in (2a), for the active sellers, augmented with the requirement that the mass of the active sellers $m$ makes the maximized expected profit equal to* 0*, and each seller is therefore indifferent between being active or inactive.*

In the case of a given number of active sellers, we effectively assume that the size of the sellers' population $\Sigma$ is sufficiently small that all sellers can realize a non-negative profit if they choose to be active. The analysis of free entry covers the cases in which, by contrast, $\Sigma$ is sufficiently large that the expected profit for the active sellers is equal to 0, and the sellers are therefore indifferent between being active or inactive.

In our analysis, we generally focus on the strategies of the *active* sellers. The identical choices of the advertising intensity and the price of the active sellers and their uniform distribution allow us to consider a simplified representation of their strategies, namely the ordered pair $(\alpha, p)$, where $\alpha > 0$.

## 3. Surplus maximization

We first consider the problem of assigning to each address an advertising intensity and a set of active sellers to maximize the expected surplus generated by the exchanges. The assignments are expressed by two measurable functions $\rho : [0, 1) \to \mathbb{R}_+^{[0,1)}$ and

[15] This is also assumed in Grossman and Shapiro (1984) and equivalently in Christou and Vettas (2008).

$\mu : [0,1) \to \mathbb{R}_+^{[0,1)}$. We denote by $\phi_\rho(x,y)$ the density function of the event that the closest seller from whom a buyer at $x$ receives an advertisement - the one whose product the buyer should consume, if any - is at a distance $y$ from her; we assume that $\phi_\rho$ has the extended set $\left[0, \frac{1}{2}\right] \cup \left\{\frac{v}{t}\right\}$ as its support, and has a mass of $1 - \phi_\rho\left(x, \frac{1}{2}\right)$ at $y = \frac{v}{t}$, corresponding to a utility level of 0. By convexity of $c(\alpha)$ and Jensen's inequality, the sellers operating at any address $x$ should choose the same advertising intensity $\frac{\rho_x}{\mu_x}$ - hypothetical differences between the aggregate intensities at different points should only be matched by a suitable distribution of the sellers. We can then directly write the objective function as

$$W(\rho,\mu) = \int_0^1 \left( \int_0^{\frac{1}{2}} \phi_\rho(x,y)\,(v - t\,y)\,dy - \mu_x\, c\left(\frac{\rho_x}{\mu_x}\right)\right) dx, \tag{6}$$

with the convention that $c\left(\frac{\rho_x}{\mu_x}\right) = 0$ holds if $\mu_x = 0$.

Proposition 1 provides necessary conditions for surplus maximization, along with a refined expression for the total surplus.

**Proposition 1.** *If the distribution of the sellers over the circle is a choice variable, then surplus maximization is guaranteed if the following conditions are both verified.*

*I. For any given mass of active sellers m, (i) the sellers are uniformly distributed around the circle, and (ii) (almost) all sellers choose an identical advertising intensity.*

*II. The first- and second-order conditions for maximization of*

$$\begin{aligned} w(\alpha,m) &= \int_0^{\frac{1}{2}} 2\,\alpha\, m e^{-2\,\alpha\, m\, x}\,(v - t x)\,dx - m\,c(\alpha) \\ &= \left(1 - e^{-\alpha\, m}\right)\, v + \frac{\left((\alpha\, m + 1)\, e^{-\alpha\, m} - 1\right)\, t}{2\,\alpha\, m} - m\,c(\alpha), \end{aligned} \tag{7}$$

*w.r.t. the choice variable(s) - α and possibly m - are verified.*

Proposition 2 characterizes the solution to the surplus maximization problem if the social planner can choose both $m$ and $\alpha$, and if the planner only chooses the advertising intensity for each seller, taking the number of the active sellers as given.[16] In both cases, considering Proposition 1, we focus directly on cases featuring a uniform distribution of the sellers.

**Proposition 2.**

*I. In the solution of the surplus maximization problem*

$$\max_{(\alpha,m)\in\mathbb{R}_+^2} \{w(\alpha,m)\}, \tag{8}$$

*the mass of the active sellers is equal to $m_w$, defined as the value of m that solves*

$$e^{-\underline{\alpha}\, m}\left(v + \frac{t\,\left(e^{\underline{\alpha}\, m} - \underline{\alpha}^2\, m^2 - \underline{\alpha}\, m - 1\right)}{2\,\underline{\alpha}\, m^2}\right) - c\left(\underline{\alpha}\right) = 0, \tag{9}$$

*and (almost) all active sellers choose an advertising intensity equal to $\underline{\alpha}$.*

*II. In the solution of the surplus maximization problem*

$$\max_{\alpha\in\mathbb{R}}\{w(\alpha,m)\}, \tag{10}$$

[16] Grossman and Shapiro (1984) obtain a similar condition: the weighted marginal cost must be equal to the average cost. In the limit as each firm's market share goes to zero - as is always the case here - the two conditions are identical.

*for a given mass $m > 0$ of active sellers, almost all sellers choose an advertising intensity $\alpha_m$, defined as the value of $\alpha$ that solves*

$$e^{-\alpha m}\left(\upsilon m + \frac{t\left(e^{\alpha m} - a^2 m^2 - \alpha m - 1\right)}{2\alpha^2 m}\right) - m c'(\alpha) = 0. \tag{11}$$

*$\alpha_m$ is increasing in $\upsilon$ and is either increasing or decreasing in $t$, depending respectively on whether $\alpha_m$ is smaller or greater than the value of $\alpha$ that solves $e^{\alpha m} - a^2 m^2 - \alpha m - 1 = 0$; this equation is verified if $\alpha m \approx 1.79$, which corresponds to a probability of approximately $\frac{1}{6}$ that any given consumer receives no advertisements.*

Essentially, if the planner can choose both $\alpha$ and $m$, $\alpha$ is set at the level that minimizes the unit cost of advertising intensity, and the number of the sellers is chosen based on the first- and second-order conditions; in the case of an optimum conditional on a given value of $m$, the advertising intensity of each seller is chosen based on the first- and second-order conditions. In the latter case, the optimal advertising intensity may be smaller or greater than $\underline{\alpha}$, depending respectively on whether $m$ is greater or smaller than $m_w$.

The result that $\alpha_m$ increases with $\upsilon$ follows directly from the greater expected payoff from each advertisement delivered. As to the response of $\alpha_m$ to changes in $t$, there are two effects: An increase in $t$ *reduces* the expected value of the exchanges induced by a given advertising intensity, because the distance between a buyer and a seller is positive with probability 1; however, it also *increases* the gain from a greater advertising intensity, by potentially allowing each buyer to locate a closer seller. As a result of this tension, the response of the advertising intensity to changes in the transportation cost $t$ depends on the initial level of aggregate advertising. For very low levels, the consumers are unlikely to receive multiple advertisements; hence, advertising largely translates into "demand creation", and welfare is decreasing in transport costs. However, as aggregate advertising increases, the chances of a consumer receiving a second advertisement increases and so the better match reduces transportation cost.

The turning point corresponds to a probability of about $\frac{1}{6}$ that any given consumer receives no advertisements (at approximately $\alpha m = 1.79$). The key is whether the marginal advertisement is likely to be demand creating (so larger $t$ means greater cost/less value from demand creation), or match enhancing (so larger $t$ means greater cost saving from better match). This turned on whether the probability a consumer received an advertisement was less or greater than $\frac{1}{6}$ ($\alpha m$ is less or greater than approximately 1.79). So, for example, holding $m$ constant, if advertising becomes costlier so that the optimal $\alpha$ decreases lowering optimal aggregate advertising below 1.79, the marginal welfare benefit goes from increasing in $t$ to decreasing in $t$. Then, an increase in $t$ reduces the optimal advertising intensity $\alpha$ further.

## 4. The equilibrium in the pricing game

In this Section, we establish existence of an equilibrium with symmetric prices of the game played in a scenario in which all sellers choose the same advertising intensity $\widehat{\alpha}$. We focus on a "reference seller" with address 0 - with no loss of generality, given symmetry - and assume that the seller's competitors all chose the same price $\widehat{p}$. The optimal pricing strategy is independent of the seller's own advertising intensity $\alpha$, as far as it is positive (any price would trivially be an optimal price if $\alpha = 0$).

We denote by

$$p_M \equiv \upsilon - \frac{t}{2}. \tag{12}$$

the price set by a hypothetical monopolist, under our assumption that $\upsilon$ is sufficiently large that every buyer is potentially willing to purchase the seller's product, regardless of her address. We also introduce two further prices to which we repeatedly refer:

$$\underline{p} \equiv \widehat{p} - \frac{t}{2}, \tag{13}$$

$$\overline{p} \equiv \widehat{p} + \frac{t}{2}. \tag{14}$$

If we disregard the possibility of values of $\underline{p}$ and $\overline{p}$ greater than the monopoly price $p_M$, $\underline{p}$ is the greatest price such that all buyers - except possibly the buyer with address $\frac{1}{2}$, faced with an offer for her ideal product - would prefer the offer of our reference seller to that of any other seller. By contrast, $\overline{p}\left(\leq p_M\right)$ is the lowest price such that an offer would only be possibly accepted by the buyers who receive no other offers - labeled *captive buyers*.

In Lemma 1, we establish that the price chosen by the reference seller must be strictly positive and is also subject to a further, possibly tighter lower bound.

**Lemma 1.** *The price that maximizes the expected profit of a seller whose competitors choose the advertising intensity $\widehat{\alpha}$ and the price $\widehat{p}$ is strictly positive, and is no smaller than $\underline{p}$ in* (13).

The results in Lemma 1 essentially follow from the fact that each buyer is a captive buyer with a positive probability, and from the inelastic demand faced by the seller for prices lower than $\underline{p}$.

It is readily verified that the buyers with address $\beta > \frac{v-p}{t}$, where $p$ is the price of the reference seller, would not be willing to purchase the seller's product, regardless of whether or not they receive other offers. If $p \geq v$, then the set of the buyers who do purchase the seller's product has measure 0, and we can further restrict attention to prices strictly lower than $v$. Let us then consider any buyer with address $\beta \in \left[0, \frac{v-p}{t}\right]$ - and thus focus on one of the two semi-circles delimited by the address of the reference seller and by its antipodal address, to fix ideas - and set

$$x_\beta^L\left(p, \widehat{p}\right) = \frac{\widehat{p} - p}{t},$$

$$x_\beta^R\left(p, \widehat{p}\right) = 2\,\beta - \frac{\widehat{p} - p}{t}.$$

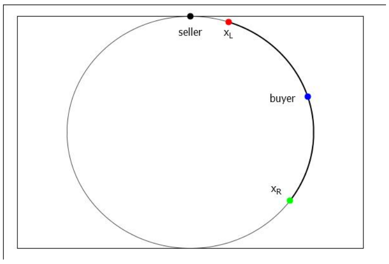
If $p \leq \widehat{p}$, then a buyer would accept an offer from the reference seller if she received no offers from sellers with addresses in the set

$$T_\beta^-\left(p, \widehat{p}\right) = \begin{cases} \emptyset, & \text{if } \beta < \frac{\widehat{p}-p}{t}, \\ \left(x_\beta^L\left(p, \widehat{p}\right), x_\beta^R\left(p, \widehat{p}\right)\right), & \text{if } \beta \geq \frac{\widehat{p}-p}{t}. \end{cases} \tag{15a}$$

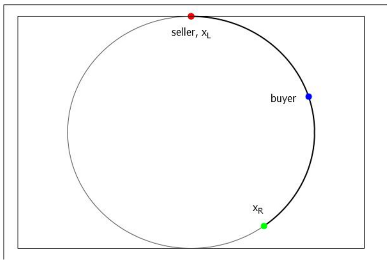
Basically, if the reference seller charges a price lower than that of her competitors, the buyers with higher valuations for her product would choose her offer regardless of what other offers, if any, they receive (first line of (15a)). For values of $p$ greater than $\underline{p}$, some buyers would rather purchase the products of sellers closer to them, the price differential notwithstanding (second line of (15a)); the number of these sellers increases with the buyer's distance from the reference seller. In the case of buyers with addresses in $\beta \in (\frac{1}{2}, 1)$, the values of the variables corresponding to $x_\beta^L$ and $x_\beta^R$ are the same as those in (15a), except for the sign. Similar remarks apply in the case of (15b) below. Notice that because the probability of an offer from a seller with any given address is equal to 0, we consider open intervals of addresses with no consequence on the results.

If $p \geq \widehat{p}$, then a buyer with address $\beta$ would accept the offer of our reference seller if she received no offers from sellers with addresses in the set

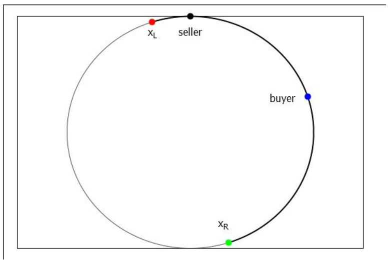
$$T_\beta^+\left(p, \widehat{p}\right) = \begin{cases} \left(x_\beta^L\left(p, \widehat{p}\right), x_\beta^R\left(p, \widehat{p}\right)\right), & \text{if } \beta \leq \frac{1}{2} - \frac{p-\widehat{p}}{t}, \\ [0, 1), & \text{if } \beta > \frac{1}{2} - \frac{p-\widehat{p}}{t}. \end{cases} \tag{15b}$$



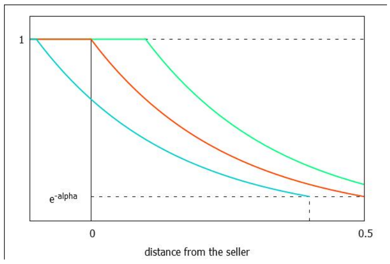

(A) The case of $p < \widehat{p}$.




(B) The case of $p = \widehat{p}$.




(C) The case of $p > \widehat{p}$.




(D) Probability of trade as a function of the distance between the seller and the buyer.

FIGURE 1. Parts 1a, 1b and 1c illustrate the sets of the addresses of the sellers whose products, offered at $\widehat{p}$, would be preferred to the product of a seller located at the top of the circle. Part 1d illustrates the probability of trade as a function of the distance between the seller and the buyer, in an example with $\alpha = 2$, $m = 1$ and $t = 1$.

The interval in first line of (15b) is identical to that in the second line of (15a). In the case of the second line of (15b), only the captive buyers would accept an offer from the reference seller, given the distance and the high price of the product.

Figures 1a, 1b and 1c illustrate the different possible scenarios. Unless the interval with the potentially preferred sellers engulfs the whole circle, its boundaries are symmetric around the buyer's address. If $p = \widehat{p}$, then the length of the interval is equal to twice the distance between the buyer and the reference seller. If $p < \widehat{p}$, the interval is shorter, and its length is equal to 0 for buyers suitably close to the seller; conversely, if $p > \widehat{p}$, the interval is generally longer, and fully covers circle in the case of buyers far away from the seller.

The probability $q_y$ that our reference buyer will receive no offers from sellers in an interval $(-y, 0]$ or $[0, y)$, where $y \in \left[0, \frac{1}{2}\right)$, obeys the differential equation

$$\frac{d q_y}{d y} = -\alpha\, m\, q_y.$$

By integrating both sides of the equation and using the boundary condition $q_0 = 1$, we can immediately establish that the complementary event of receiving advertisements from one or more sellers at a distance smaller than $y$ is exponentially distributed, with parameter $\alpha\, m$.

By using (15), we can then write the "twin" demand functions if $\widehat{p} < p_M$ and $p \in \left(\underline{p}, \overline{p}\right)$ - so that the reference seller can potentially sell her product even to the antipodal buyers who do not receive better offers - as:

$$D^{-}\left(\widehat{\alpha},m,p,\widehat{p}\right)=2\alpha\left(\int_{0}^{\frac{\widehat{p}-p}{t}}d\beta+\int_{\frac{\widehat{p}-p}{t}}^{\frac{1}{2}}\exp\left(-2\widehat{\alpha}\,m\left(\beta-\frac{\widehat{p}-p}{t}\right)\right)d\beta\right)$$
$$=\alpha\left(\frac{2}{t}\left(\widehat{p}-p\right)+\frac{1}{\widehat{\alpha}\,m}\left(1-\exp\left(\frac{2\,\widehat{\alpha}\,m}{t}\left(\widehat{p}-p\right)-\widehat{\alpha}\,m\right)\right)\right),\tag{16a}$$

if $p\in\left[\overline{p},\widehat{p}\right)$ and $p>0$, and as

$$D^{+}\left(\widehat{\alpha},m,p,\widehat{p}\right)=2\alpha\left(\int_{0}^{\frac{1}{2}-\frac{p-\widehat{p}}{t}}\exp\left(-2\widehat{\alpha}\,m\left(\beta-\frac{\widehat{p}-p}{t}\right)\right)d\beta+\int_{\frac{1}{2}-\frac{p-\widehat{p}}{t}}^{\frac{1}{2}}\exp\left(-\widehat{\alpha}\,m\right)d\beta\right)$$
$$=\alpha\left(\frac{1}{\widehat{\alpha}\,m}\left(e^{\frac{2\,\widehat{\alpha}\,m}{t}\left(\widehat{p}-p\right)}-e^{-\widehat{\alpha}\,m}\right)+\frac{2e^{-\widehat{\alpha}\,m}}{t}\left(p-\widehat{p}\right)\right).\tag{16b}$$

if $p\in\left[\widehat{p},\min\left\{\overline{p},p_M\right\}\right]$.

The reason why we include $\widehat{p}$ in the domains of both $D^{-}\left(\widehat{\alpha},m,p,\widehat{p}\right)$ and $D^{+}\left(\widehat{\alpha},m,p,\widehat{p}\right)$ - similarly to what we do in the case of the profit equations (17) below - is that we ultimately want to obtain a candidate optimal, interior price that coincides with $\widehat{p}$, when we investigate existence of a symmetric equilibrium. Figure 1d can help us to interpret the expressions for the probability of acceptance of an advertisement as a function of the distance between the seller and the buyer, used in the integrands of (16a) and in (16b). The blue, the orange and the green curve are referred to the cases of $p>\widehat{p}$, $p=\widehat{p}$ and $p<\widehat{p}$. The expression for the expected profit of our reference seller at an interior equilibrium is then

$$\pi\left(\alpha,\widehat{\alpha},m,p,\widehat{p}\right)=\begin{cases}\pi^{-}\left(\alpha,\widehat{\alpha},m,p,\widehat{p}\right), & \text{if } p\in\left[\overline{p},\widehat{p}\right) \text{ and } p>0,\\ \pi^{+}\left(\alpha,\widehat{\alpha},m,p,\widehat{p}\right), & \text{if } p\in\left[\widehat{p},\min\left\{\overline{p},p_M\right\}\right].\end{cases}$$

where

$$\pi^{-}\left(\alpha,\widehat{\alpha},m,p,\widehat{p}\right)=p\,D^{-}\left(\alpha,\widehat{\alpha},m,p,\widehat{p}\right),\tag{17a}$$
$$\pi^{+}\left(\alpha,\widehat{\alpha},m,p,\widehat{p}\right)=p\,D^{+}\left(\alpha,\widehat{\alpha},m,p,\widehat{p}\right),\tag{17b}$$

and the problem correspondingly faced by our reference seller is

$$\max_{p\in\mathbb{R}}\left\{\pi\left(\alpha,\widehat{\alpha},m,p,\widehat{p}\right)\right\},$$

where $\alpha$, $\widehat{\alpha}$, $m$ and $\widehat{p}$ are all taken as given in the pricing game.

Lemma 2 establishes that the optimal price set by a seller responding to the choice of $\widehat{p}$ by her competitors is also bounded above by the monopoly price $p_M$.

**Lemma 2.** *A seller who were forced to choose a price no smaller than* $\min\left\{\overline{p},p_M\right\}$*, if all other sellers choose an advertising intensity* $\widehat{\alpha}$ *and a price* $\widehat{p}<p_M$*, would maximize her profit by choosing the monopoly price* $p_M$*.*

Lemma 2 allows us to conclude that a symmetric equilibrium price can be equal either to $p_M$, or to a price strictly lower than $p_M$. It also justifies both the assumption of prices no greater than the monopoly price, and therefore the procedure used to derive (15), (16) and (17).

We are now ready to characterize the symmetric price equilibrium, in Proposition 3.

**Proposition 3.** *If*

$$\widehat{\alpha}\,m>\log(2)\tag{18}$$

*and $v \leq \mathring{v}$, where*[17]

$$\mathring{v} = \frac{t}{2}\left(1 + \frac{\exp\left(\widehat{\alpha}\, m\right) - 1}{\widehat{\alpha}^2\, m^2}\right), \tag{19}$$

*then the symmetric equilibrium price and profit gross of advertising costs are*

$$\mathring{p}\left(\widehat{\alpha}, m\right) = \frac{t}{2\,\widehat{\alpha}\, m}, \tag{20}$$

$$\mathring{\pi}\left(\alpha, \widehat{\alpha}, m\right) = \frac{\alpha\, t}{2\widehat{\alpha}^2\, m^2}\left(1 - \exp\left(-\widehat{\alpha}\, m\right)\right). \tag{21}$$

Condition (18) is perhaps more easily interpreted as the probability a given consumer receives an advertisement is no smaller than $e^{-2\frac{\log(2)}{2}} = \frac{1}{2}$. The optimal price (20) is equivalent to the price in (11a) in Grossman and Shapiro (1984). As in Salop (1979); Grossman and Shapiro (1984)), the monopolistically competitive price does not increase with $v$, albeit the monopoly price does.

The two conditions for a monopolistically competitive equilibrium (18) and (19) rule out deviating to the monopoly price as a best response. (18) ensures a sufficiently large probability that any customer receives multiple advertisements, and thereby a sufficiently intense level of competition. (19) guarantees that the largest valuation $v$ and therefore the monopoly price is not too much greater than the equilibrium price, and features two opposite effects. Increases in advertising do in fact translate into more intense competition, and thereby cause the equilibrium price to decrease; however, they also make it less likely that a consumer reached by a firm is not reached by any other firm, and will thus be a captive customer. The former and the latter effect respectively make deviating to the monopoly price more and less attractive for the firms. As it turns out, the largest value of $v$ compatible with an interior equilibrium in (19) is decreasing and increasing in aggregate advertising $\alpha\, m$, depending respectively on whether $\alpha\, m$ is smaller or greater than approximately 1.59.

As one goal of this paper is to extend Grossman and Shapiro (1984) and gain new insights, a critical question is whether or not our assumptions are introducing spurious results via effects that do not exist in Grossman and Shapiro (1984). One check is how the equilibrium prices compare and indeed the price in (20) has the same limiting properties. As the advertising intensity $\alpha$ becomes large, and we ideally approach perfect information, the price approaches the competitive price; an increase in the mass of the active firms produces the same effect. The profit *gross* of advertising costs in (21) shares the same properties.

We close the present Section by recalling that similarly to previous work, the covered market and unit demand assumptions make the symmetric equilibrium price in (20) consistent with surplus maximization.

## 5. The equilibrium in the advertising game

We now move back to the stage in which the mass $m$ of firms simultaneously choose their advertising intensities, correctly expecting a monopolistically competitive equilibrium in the pricing stage. Given (21), we can express each firm's expected profit and first order condition for an interior optimum as

$$\Pi\left(\alpha, \widehat{\alpha},\, m\right) = \mathring{\pi}\left(\alpha, \widehat{\alpha}, m\right) - c(\alpha)$$

and as

$$\frac{\partial \Pi\left(\alpha, \widehat{\alpha},\, m\right)}{\partial \alpha} = \frac{t\left(1 - e^{-\widehat{\alpha}\, m}\right)}{2\left(\widehat{\alpha}\, m\right)^2} - c'(\alpha) = 0. \tag{22}$$

[17] It is straightforward to show there always exist values of $v$ such that $t \leq v \leq \mathring{v}$, i.e., that the covered market assumption plus the upper bound on $v$ does not eliminate the monopolistically competitive equilibrium. The condition simplifies to $\left(\widehat{\alpha}\, m\right)^2 + 1 < e^{\widehat{\alpha}\, m}$, which is always verified.

Because the expected revenue per buyer is independent of $\alpha$, convexity of the cost function $c(\alpha)$ guarantees an optimum for the firm. Because the cost function is strictly convex, its first derivative admits an inverse $\left(c'\right)^{-1}$, and we can conveniently express (22), with a slight abuse of notation, as $\alpha = \left(c'\right)^{-1}\left(\widehat{\alpha},\, m\right)$. For any given value of $m$ inherited from the previous stage, a candidate equilibrium advertising level is thus a value of $\widehat{\alpha}$ such that

$$\widehat{\alpha} = \left(c'\right)^{-1}\left(\widehat{\alpha},\, m\right).$$

In equilibrium, $\alpha = \widehat{\alpha}$, so $\widehat{\alpha}$ is implicitly defined by the function[18]

$$H\left(\widehat{\alpha}; t, m\right) \equiv \frac{t}{2\left(\widehat{\alpha}\, m\right)^2}\left(1 - e^{-\widehat{\alpha}\, m}\right) - c'\left(\widehat{\alpha}\right) \equiv 0. \tag{23}$$

For uniqueness of equilibrium (and to use the Implicit Function Theorem in our analysis), we need the first partial derivative w.r.t. $\widehat{\alpha}$,

$$H_{\widehat{\alpha}}\left(\widehat{\alpha}; t, m\right) = \frac{tme^{-\widehat{\alpha}\, m}}{2\left(\widehat{\alpha}\, m\right)^2} - \frac{t\, 4\widehat{\alpha}m^2}{(2\left(\widehat{\alpha}\, m\right)^2)^2}(1 - e^{-\widehat{\alpha}\, m}) - c''\left(\widehat{\alpha}\right).$$

to have negative sign - namely, we need the sellers' advertising intensities to be strategic substitutes, in a symmetric setting. Lemma 3 shows that this is indeed the case.

**Lemma 3.** $H_{\widehat{\alpha}}\left(\widehat{\alpha}; t, m\right) < 0$.

Finally, again because the expected revenue per buyer is independent of $\alpha$, the cross-partial of profit evaluated at $\widehat{\alpha}$ and $\mathring{p}$ is zero, and so profits are locally concave in the two variables.[19] Thus, for any $m$, if the conditions for the monopolistically competitive pricing equilibrium are met, we have a unique price and advertising pair.

**Corollary 1.** *If $\widehat{\alpha}\, m > \log(2)$ and $v < \mathring{v}$, then there is a unique symmetric monopolistically competitive advertising and price pair $\left(\widehat{\alpha}, \mathring{p}\right)$.*

We then turn to the comparative static properties of the equilibrium, in particular how the exogenous parameters affect aggregate advertising $\widehat{\alpha}\, m$.

### 5.1. Comparative statics for exogenous $m$.

In addition to giving intuition regarding the mechanics of the model, our comparative statics results summarized in Table 1 help us to interpret the existence conditions (18) and (19) in terms of the exogenous variables advertising and transportation costs. Deriving the comparative statics also serves a purpose by allowing a different check that our modeling is consistent with Grossman and Shapiro (1984). Indeed the sign of all the comparative statics are identical, suggesting that the underlying mechanisms at work are the same; we thus refer to Grossman and Shapiro (1984) for a general discussion and simply present the summary of the comparative statics for exogenous $m$ (labeled " the oligopoly model" in Grossman and Shapiro (1984)) below with the derivations in Appendix B.

To ease comparison, recall that our advertising level $\alpha$ is denoted by $\phi$ in Grossman and Shapiro (1984). For cost, we focus on two specific, standard types of changes to give insight. First, we consider a unit increase in marginal cost independent the level of marginal cost $c'(\alpha)$. One could also view this as asking the effect of a small per-unit tax on advertising. Second, we specify costs as $\tilde{c} \equiv kc(\alpha)$ and consider the effect of a change in $k$ on $k\, c'(\alpha)$. The latter case includes cost functions such as $\tilde{c} = F + \frac{k\alpha^2}{2}$, which we consider in Section 7 below. In this case, since marginal costs are increasing, a unit increase in $k$ results in a

[18]This is equivalent to condition (11b) in Grossman and Shapiro (1984), once one substitutes for their price in (11a).

[19]That is, at $\widehat{\alpha}$ and $\mathring{p}$, $\pi_{\alpha\, p} = 0$ (as $\alpha$ enters linearly in demand, i.e. demand could be written as $D = \alpha\, \tilde{D}$, where $\tilde{D}$ is expected demand per buyer, so $\pi_{\alpha\, p} = \alpha\left(\tilde{D} + p\partial\tilde{D}/\partial p\right)$), so $\pi_{\alpha\alpha}\pi_{pp} - \pi_{\alpha p}^2 > 0$.

*greater* increase in marginal cost the higher the level of advertising. One could also view this as the effect of a small ad-valorem (based on the marginal cost) tax on each unit of advertising. These two forms have very different implications on profits and so entry.

| | | Endogenous variables | | | |
|---|---|---|---|---|---|
| | | $p$ | $\hat{\alpha}$ | $\hat{\alpha}\, m$ | $\pi$ |
| Exogenous | $F$ | 0 | 0 | 0 | $-$ |
| | $t$ | $+$ | $+$ | $+$ | $+$ |
| variables | $c'(\alpha)$ | $+$ | $-$ | $-$ | $-$ |
| | $k$ | $+$ | $-$ | $-$ | ? |
| | $m$ | $-$ | $-$ | $+$ | $-$ |

TABLE 1. Comparative statics analysis with exogenous values of $m$. The question mark indicates that if marginal cost is relatively large, compared to the total cost, then profit increases (a possibility noted by Grossman and Shapiro (1984)); otherwise they decrease. For example, with a quadratic variable cost, an increase in $k$ *increases* profits.

From the comparative statics we see that as increases in transportation costs increase both the equilibrium price and aggregate advertising, it relaxes both equilibrium condition (18) and (19).[20] Note that while by itself an increase in aggregate advertising would decrease the equilibrium price, the direct effect of transportation cost dominates. As a result, the second condition (19) increases monotonically in $t$.

The effect of an increase in advertising cost tightens condition (18) as it reduces aggregate advertising. The effect on condition (19) is more complicated, reflecting the non-monotonicity aggregate advertising has on the condition, but not having a direct effect on price unlike transportation costs. That is, because increases in advertising costs decrease aggregate advertising, and decreases in aggregate advertising first tighten and then relaxes the second condition (19), so too does increases in advertising costs first tighten and then relaxes the second condition (19). Instances of both of these points are shown in the example in Section 7 below.

5.2. **Welfare comparison in the advertising stage.** Having established that our model is consistent with Grossman and Shapiro (1984), we turn to examine how the monopolistically competitive outcome for given mass of firms $m$ compares to the social outcome. We find instead that advertising is always socially insufficient (recall that there is unit demand and the market covered assumption, so the price has no welfare effect though in that sense the following proposition is a second-best statement). That is, the welfare gain from better matching always outweighs the capture effect even when the demand creation effect is negligible.

**Proposition 4.** *For a fixed value of m, the symmetric monopolistically competitive equilibrium has* too little *advertising compared to the social optimum.*

**Remark 1.** *The proof of Proposition 4 actually establishes a more general result: for a fixed value of m, the marginal social benefit per firm from increasing advertising is greater than the marginal private benefit of each firm. Thus, e.g., at the socially optimal level of entry there is too little advertising.*

Given the findings in Grossman and Shapiro (1984) – who find that there is always *excessive* adverting – our result of *insufficient* advertising is surprising. As Grossman and Shapiro (1984) make clear, though, their results are for when *every* consumer receives a least one advertisement so there is no demand creation effect there (as there is here). As a

[20] Of course, it eventually binds against the covered market assumption.

firm does not capture the entire surplus from demand creation, having the demand creation effect could be sufficient to reverse the results and have society wanting the firms to advertise more and indeed Christou and Vettas (2008) report that when there is demand creation in Grossman and Shapiro (1984) (i.e., not all consumers receive at least one ad), then there can be socially insufficient advertising.

However, our result is for all equilibria including ones in which there is very little demand creation (i.e., each consumer receives at least one advertisement with probability close to one). There are intuitive reasons to expect this result, since as Grossman and Shapiro (1984) noted, with incomplete information each firm is potentially competing not only with nearby consumers as in the complete information Salop model, but also for far away consumers who would find the firm's product a bad match. This competitive pattern forces down the price and the firms' private return to advertising. Yet Grossman and Shapiro (1984) find excessive advertising. There are several possible factors that together could explain the discrepancy. First, the assumption in Grossman and Shapiro (1984) does more than eliminate the demand creation effect, as it overstates the firm's demand conditioned on the consumers receiving an advertisement from the firm – making the private marginal return to advertising for a firm *greater* than it actually is, so the true market equilibrium level is less than their approximation yields. Second, their assumption implies that each consumer receives advertisements from at least two firms, which in turn implies the probability of receiving an advertisement from at least three firms, etc., is large.[21] As a result, the marginal advertisement does not have a large improvement in match/expected transportation costs and the marginal reduction in transportation cost from another advertisement is smaller in the approximation than the exact expression when their assumption holds (i.e., the number of firms is large or the probability of receiving an advertisement is large). That is, even ignoring the elimination of the demand creation effect, their assumption makes the social return to advertising *smaller* than it actually is. Third, in their model increase advertising is similar to entry in the Salop model: by advertising more a firm reaches a consumer who already knows the location of at least one other firm. As is well known, entry is socially excessive in the Salop model and this may be driving the cases of socially excessive advertising in Grossman and Shapiro (1984). In contrast, Butters (1977) finds socially optimal advertising. Further, the Dixit and Stiglitz (1977) type modeling of a continuum of firms does not inherently have excessive entry and for this reason, our model may be closer to capturing the pure advertising effects. Finally, the analysis in Grossman and Shapiro (1984) restricts the firms' deviation prices away from both the supercompetitive and monopoly price. As we shown here, the condition for the latter to hold here is not irrelevant and so including it in the analysis of the Grossman and Shapiro (1984) model could affect the conclusions there. Likewise, including the firm's ability to deviate to the supercompetitive price in Grossman and Shapiro (1984) could affect their conclusions. For example, in the benchmark case of the numerical analysis in Grossman and Shapiro (1984), it would seem a deviation to the supercompetitive price is more profitable than their equilibrium price.

## 6. The entry stage

In this section we move back to the beginning of the game to determine the measure of the set of the active firms under free entry. Given our assumption of a large set of potential sellers, equilibrium then holds if profits given $\widehat{m}$ firms entering are zero - that is, given $\widehat{m}$ the resulting equilibrium advertising $\widehat{\alpha}$ in (23) and equilibrium price $\mathring{p}$ in (20). The mass of sellers $m$ compatible with free entry is defined by[22].

$$\pi(m) = \frac{t}{2\, a(m)\, m)^2}(1 - e^{-a(m)\, m}) - c(a(m)) - F = 0. \tag{24}$$

[21] See footnote 11 for details.

[22] This is condition (11c) in Grossman and Shapiro (1984).

| | | Endogenous variables | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Exog. $m$ | | | | Endog. $m$ | | | |
| | | $p$ | $\widehat{\alpha}$ | $\widehat{\alpha}\, m$ | $\pi$ | $p$ | $\widehat{\alpha}$ | $m$ | $\widehat{\alpha}\, m$ |
| Exogenous | $F$ | 0 | 0 | 0 | $-$ | $+$ | $+$ | $-$ | $-$ |
| | $t$ | $+$ | $+$ | $+$ | $+$ | $+$ | 0 | $+$ | $+$ |
| variables | $c'(\alpha)$ | $+$ | $-$ | $-$ | $-$ | $+$ | 0 | $-$ | $-$ |
| | $k$ | $+$ | $-$ | $-$ | $?^\star$ | $+?$ | $?^\blacklozenge$ | $?^\star$ | $-?$ |
| | $m$ | $-$ | $-$ | $-$ | $-$ | | | | |

TABLE 2. Comparative statics analysis with endogenous values of $m$. Question mark indicate that if marginal cost is relatively large to total cost, then profit increases (a possibility noted by Grossman and Shapiro (1984)); otherwise they decrease. For example, with a quadratic variable cost, an increase in $k$ *increases* profits.
$^\star$ If marginal cost is relatively large to total cost, then positive.
$^\blacklozenge$ If marginal cost is relatively large to total cost, then this decreases. For the case of $k$ on $p$ and $\widehat{\alpha}$, even examples with profits increasing have aggregate advertising decreasing (and so price increasing). For changes in costs, if the marginal cost of advertising does not increase uniformly for all levels of $\alpha$ (the $k$ case), then it is possible in which an increase in advertising cost increases profits (as Grossman and Shapiro (1984) note)

Denote $\hat{m}$ as the entry equilibrium defined by (24) and so $a\left(\hat{m}\right)$ is the corresponding market equilibrium of advertising. From Lemma 13 in Appendix B , profits are decreasing in entry, namely $\frac{d\pi}{dm} < 0$ , the equilibrium is unique. With this, the comparative statics when entry is endogenous can be derived.

**6.1. Comparative Statics.** Table 2 summarizes our results concerning the comparative statics with endogenous $m$, which again are in line with those of Grossman and Shapiro (1984). The proofs are in Appendix B.[23] For convenience, we also include the results already reported in Table 1 for exogenous values of $m$.

**6.2. Welfare comparison with entry.** With the characterization of the entry equilibrium we turn to how the market entry equilibrium compares to the welfare optimal. That is, given the market level of entry and corresponding equilibrium advertising level, would the planner encourage or discourage entry? Despite entry not having the positive welfare effect of reducing the average distance for consumers, we find that there is always *insufficient* entry, much like there was insufficient advertising for fixed $m$ in Proposition 4.[24] As with Proposition 5, the proof is more general: for any fixed $\alpha$ (including the one selected by the planner), the market would have less entry than the planner would choose.

**Proposition 5.** *The symmetric monopolistically competitive equilibrium has too* little *entry compared to the social optimum.*

Although Proposition 5 may be initially surprising, the proof reveals that the underlying mechanism is the same as for Proposition 4. This is because the planner's problem could be viewed as choosing the optimal *aggregate* advertising ($\alpha m$) in (8) subject to minimizing

[23] Our $\alpha$ – the advertising level – corresponds to $\phi$ in Grossman and Shapiro (1984). Recall that for cost, we consider two types of changes on the cost function. First, a unit increase in marginal cost independent the level of marginal cost ($c'(\alpha)$). The second is to specify costs as $\tilde{c} \equiv kc(\alpha)$ and consider the effect of a change in $k$ on $k\, c'(\alpha)$. For example, if costs are $\tilde{c} = \frac{k\alpha^2}{2}$, the effect is expressed by the derivative with respect to $c$.

[24] Recall that with unit demand and covered market assumptions, price plays no welfare role. More generally, Proposition 5 could be understood as a second-best statement (i.e., given the market price).

costs through entry and advertising levels since entry has no direct effect on the average distance consumers incur in buying their product. As the costs are the same to the firms as to society, the question is only if the social benefit of the marginal increase in aggregate advertising is greater or less than the private. Since the social benefit was greater when the choice was advertising level for fixed entry, then it is greater when the choice is entry with fixed advertising level, as that too increases aggregate advertising.

A different way in which Proposition 5 may seem initially surprising is that generally in strategic models there is excessive rather than insufficient entry (Mankiw and Whinston, 1986). However, here as there is a continuum of firms, while the result for excessive entry in Mankiw and Whinston (1986) requires each firm to have measure. That is, while there is business stealing here (each firm's gain is at another firm's loss), there is not the strategic effect needed. Indeed, in the Dixit and Stiglitz (1977) model there could be too little entry depending on the consumers' preferences.

As in the case of Proposition 4, the proof of Proposition 5 actually establishes a more general result: For fixed $\alpha$, the marginal social benefit from entry is greater than the marginal private benefit of entry. This implies that if the firms were forced to advertise at the efficient level, profits would be negative and there would be fewer active firms.

## 7. An example

To provide some further understanding of the results, we investigate an example in which the advertising cost, at the firm level, is expressed by the function

$$c_\chi(\alpha) = \frac{\chi\,\alpha^2}{4} + F,$$

for given parameters $\chi$ and $F$ in $\mathbb{R}_{++}$. In a symmetric equilibrium, each firm then chooses its advertising intensity $\alpha$ to maximize its profit

$$\Pi_\chi\left(\alpha, \widehat{\alpha}, m\right) = \frac{\alpha\, t}{2\widehat{\alpha}^2 m^2}\left(1 - e^{-\widehat{\alpha}\, m}\right) - \frac{\chi\,\alpha^2}{4} - F.$$

Convexity of the cost function guarantees that the first order condition for an optimum in (22), which reads

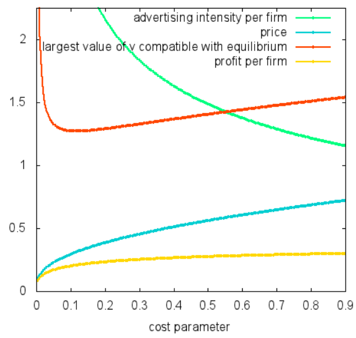
$$\begin{aligned}\frac{d\Pi_\chi(\alpha)}{d\alpha} &= \frac{t}{2\widehat{\alpha}^2 m^2}\left(1 - e^{-\widehat{\alpha}\, m}\right) - \frac{\chi\,\alpha}{2}\\ &= 0,\end{aligned}$$

identifies an optimal choice of $\alpha$ for each firm. Setting $\alpha = \widehat{\alpha}$, we can then obtain the condition for the candidate monopolistically competitive equilibrium value of the advertising intensity:

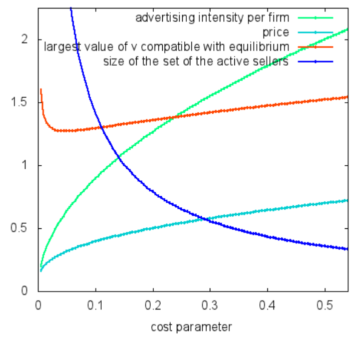
$$\frac{t}{2\widehat{\alpha}^2 m^2}\left(1 - e^{-\widehat{\alpha}\, m}\right) - \frac{\chi\,\widehat{\alpha}}{2} = 0. \tag{25}$$

Inspection of the LHS of (25) reveals that for any given value of $m$, there exists a unique, positive value of $\widehat{\alpha}$ which is potentially compatible with an equilibrium, and does qualify as an equilibrium if (18) and (19) are verified.

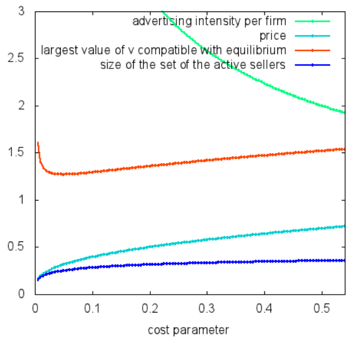
In Figure 2a, we plot the values of the main endogenous variables for values of the variable cost parameter $\chi$ between 0 and 0.9 - which approximately coincides with the largest value of $\chi$ for which the aggregate advertising intensity is no smaller than log(2), as required by (18). Larger values of the cost parameter correspond to a lower advertising intensity and thus to higher prices, given the greater probability that any buyer reached by a seller will not receive other advertisements. The balance of the effect of the reduced competition and the lower marked coverage is such that the sellers' profits increase with the advertising cost parameter. The greatest value of the preference parameter $\upsilon$ compatible with equilibrium existence, $\mathring{\upsilon}$, is only affected by changes in the cost parameter via the equilibrium advertising intensity, and its response follows the pattern indicated in the discussion of (19).



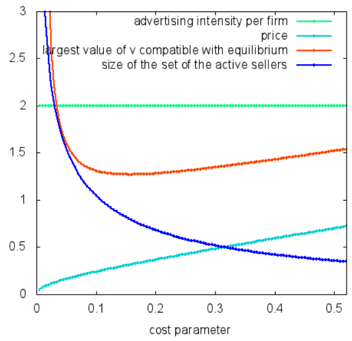

(A) Different values of the (variable) cost parameter with an exogenous mass of active sellers.




(B) Different values of the fixed cost parameter with an endogenous mass of active sellers.




(C) Different values of the variable cost parameter with an endogenous mass of active sellers.




(D) Different values of *both* the fixed and the variable cost parameter with an endogenous mass of active sellers.

FIGURE 2

In the case of free entry, under our assumption that the number of sellers is sufficiently large that some sellers choose not to be active, an equilibrium is a pair $(\widehat{\alpha}, m)$ that solves the system comprised of (25) and the zero profit-condition

$$\Pi_\chi \left(\alpha, \widehat{\alpha}, m\right)\Big|_{\alpha=\widehat{\alpha}} = 0, \tag{26}$$

Figures 2d, 2b and Figure 2c respectively illustrate the responses of the main endogenous variables in cases of changes of the fixed cost $F$, changes of the variable cost parameter $\chi$ and proportional changes of the two parameters. As in the case of Figure 2a, the plot covers a set of values of the parameters such that (18) is verified. The non-monotonic response of $\mathring{v}$ persists in the face of changes of both $\alpha$ and $m$ induced by changes of the cost parameters with free entry. If only one of the cost parameters changes, then the changes in $\alpha$ and $m$ take opposite directions. Identical proportional increases of both parameters, do not affect the advertising intensity, in the specific case of a quadratic cost function, and reduce the size of the set of the active firms.

## 8. Conclusion

We have introduced a tractable model that captures the essence of Grossman and Shapiro (1984), without approximations or the requirement that essentially every consumer receives at least one advertisement, which was needed for their general results. The model also rectifies the issues of existence of the monopolistically competitive equilibrium (Christou and Vettas, 2008) from either deviating up to the monopoly price or deviating down to the supercompetitive price of Salop. Tractability, achieved by modeling the sellers' population as a continuum, as that of the buyers, allowed us to provide explicit conditions for the monopolistically competitive equilibrium to hold.

In addition to the theoretical contributions, we obtain new insights. In particular, one inference of Grossman and Shapiro (1984) is that when in equilibrium nearly all consumers receive at least one ad, there is excessive advertising. However, we show that in our model even when nearly all consumers receive at least one ad, there is still socially too little advertising. In fact, there always is insufficient socially insufficient advertising in the monopolistically competitive equilibrium rather than excessive advertising. The intuition comes from Grossman and Shapiro (1984): with incomplete information competition is no longer localized, as is the case in the full information setting of Salop (1979). That is, here the firm not only competes for consumers with its nearest neighbor (as in Salop) but also for consumers far away, and so may face a rival with a good match for the consumer. This extra competition reduces the equilibrium price. As our model (like its predecessors) assumes unit demand and covered market, it eliminates a potentially positive effect from advertising: more advertising results in lower prices (what Grossman and Shapiro (1984) call the quantity demanded effect). This situation suggests, more generally (e.g., with downward sloping demand), that there may be insufficient informative advertising in an oligopoly setting.

We then endogenize entry, finding that there is socially too little entry rather that socially excessive entry as found in Grossman and Shapiro (1984) even though entry does not have the direct social benefit of reducing transportation cost (i.e., a better match) that exists in Grossman and Shapiro (1984). This result is in contrast to the standard result of excessive entry even though *business stealing* –one firm's sale often comes at the expense of another firm– is present. However, one difference is with a continuum of firms, each firm has zero measure, which violates a requirement for the excess entry result (Mankiw and Whinston, 1986). Indeed, in the seminal monopolistic-competitive model Dixit and Stiglitz (1977) find there can be too little or too much entry. The reason is because advertising has convex costs, entry is a second tool to "produce" advertising and so the planner chooses entry and advertising levels to minimize costs for a given level of aggregate advertising. That is, just like the firm does not capture the full social value of the marginal increase in advertising, the firm does not capture the full social value of entry on aggregate advertising. As a result, our results may be closer to the pure question of the role of advertising.

Finally, tractability of our model allows for new questions to be asked within the framework, which can therefore be a valuable tool for further analysis. From this point of view, the framework can allow us to extend the analysis of the paper in future research without encountering the difficulties that are present in Grossman and Shapiro (1984), whereby the main results are derived by using approximations that among other things rule out demand creation. Likewise, although Christou and Vettas (2008) provide many insights, their analysis could not be extended to endogenizing entry nor explicit conditions derived for the equilibrium.

## Appendix A. Proofs

### A.1. Proof of Proposition 1.

*Part* I. We first show that if the assignment $\rho$ were subject to the constraint of a *given* total advertising intensity $\boldsymbol{R} \in \mathbb{R}_{++}$, written as

$$\int_0^1 \rho(s)\, ds \leq \boldsymbol{R}, \tag{A.1}$$

and we disregarded the advertising cost, then surplus maximization would require an identical advertising intensity across (almost) all addresses in $[0, 1)$. Let $\Psi_\rho(x, s)$ denote the probability that a buyer with address $x$ will *not* receive an advertisement from any sellers with address in $(x - s, x + s)$, under a generic assignment $\rho$; if necessary, all distributions considered here can be completed by including a point with positive mass that corresponds to a utility level of 0, as in the case of $\phi_\rho$ in (6). For each buyer, receiving advertisements from multiple sellers at the same distance $s$ has probability 0, and the dynamics of $\Psi_\rho(x, s)$ is expressed by the differential equation

$$\frac{d\Psi_\rho(x, s)}{ds} = -(\rho(x - s) + \rho(x + s))\, \Psi_\rho(x, s).$$

By integrating both sides of the previous equation and using the initial condition $\Psi_\rho(x, x) = 1$, we obtain

$$\Psi_\rho(x, \xi) = \exp\left( - \int_x^\xi (\rho(x - s) + \rho(x + s))\, ds \right). \tag{A.2}$$

Under the given uniform distribution of the buyers, convexity of $\Psi_\rho(x, \xi)$, viewed as a function of the integral in (A.2), implies via Jensen's inequality that the probability of receiving no advertisements from sellers at distances no greater than $\xi$, averaged across buyers, is minimized if $\Psi_\rho(x, \xi)$ is independent of the buyer's address $x$. As (A.1) should hold as an equality, this requirement is in turn satisfied if the advertising intensity is equal to $\boldsymbol{R}$ for (almost) every $s \in [0, 1)$. Setting $\overline{\Psi}_\rho(\xi) = \int_0^1 \Psi_r(x, \xi)\, dx$, and using $\rho = r$ to denote the uniform assignment, we thus have

$$\overline{\Psi}_r(\xi) \leq \overline{\Psi}_\rho(\xi) \tag{A.3}$$

where strict inequality holds in the presence of differences with positive measure between $\rho$ and $r$ over any subinterval of $\left[0, \frac{1}{2}\right)$.

Because receiving and not receiving an advertisement from a seller within any distance $y$ are complementary events, we can set $\phi_\rho(x, s) = -\psi_\rho(x, s)$, where $\psi_\rho(x, s) = \frac{d\Psi_\rho(x,s)}{ds}$ is the density function associated with $\Psi_\rho(x, s)$, and $\overline{\psi}_\rho(s) = \int_0^1 \psi_\rho(x, s)\, dx$, and rewrite the expected total surplus as

$$\begin{aligned}
\int_0^1 \left( \int_0^{\frac{1}{2}} \phi_\rho(x, s)\, (v - t\, s)\, ds \right) dx &= \int_0^{\frac{1}{2}} \left( \int_0^1 \phi_\rho(x, s)\, (v - t\, s)\, dx \right) ds \\
&= \int_0^{\frac{1}{2}} \left( \int_0^1 \left( -\psi_\rho(x, s) \right) (v - t\, s)\, dx \right) ds \\
&= \int_0^{\frac{1}{2}} \left( -(v - t\, s) \int_0^1 \psi_\rho(x, s)\, dx \right) ds \\
&= - \int_0^{\frac{1}{2}} (v - t\, s)\, \overline{\psi}_\rho(s)\, ds.
\end{aligned}$$

Integration by parts allows us to further work out the previous expression as follows:

$$
\begin{aligned}
-\int_0^{\frac{1}{2}} (v - t\,s)\,\overline{\psi}_\rho(s)\,ds &= -\,(v - t\,s)\,\overline{\Psi}_\rho(s)\Big|_0^{\frac{1}{2}} - t\int_0^{\frac{1}{2}} t\,\overline{\Psi}_\rho(s)\,ds \\
&= v - \left(v - \frac{t}{2}\right)\overline{\Psi}_\rho\left(\frac{1}{2}\right) - t\int_0^{\frac{1}{2}} \overline{\Psi}_\rho(s)\,ds,
\end{aligned}
$$

where $\overline{\Psi}_\rho(s) = \int \overline{\psi}_\rho(s)\,ds$. We can then establish that the surplus under the uniform assignment $r$ is no smaller than the surplus under a generic assignment $\rho$ by noting that $\overline{\Psi}_r\left(\frac{1}{2}\right) \le \overline{\Psi}_\rho\left(\frac{1}{2}\right)$ and (A.3) imply

$$
\begin{aligned}
&\int_0^1 \left(\int_0^{\frac{1}{2}} \phi_r(x,s)\,(v - t\,s)\,ds\right)dx - \int_0^1 \left(\int_0^{\frac{1}{2}} \phi_r(x,s)\,(v - t\,s)\,ds\right)dx \\
&= \left(v - \frac{t}{2}\right)\left(\overline{\Psi}_\rho\left(\frac{1}{2}\right) - \overline{\Psi}_r\left(\frac{1}{2}\right)\right) + t\left(\int_0^{\frac{1}{2}} \overline{\Psi}_r(s)\,ds - \int_0^{\frac{1}{2}} \overline{\Psi}_\rho(s)\,ds\right) \\
&\ge 0.
\end{aligned}
$$

To minimize the total advertising cost with a given mass $m$ of active sellers, all sellers should choose the same advertising intensity. The minimized total cost of any admissible assignment $\rho$ for a given mass $m$ of active sellers is then achieved by setting $\mu_x = \frac{m\,\rho_x}{R}$ for each address $x \in [0,1)$, and is equal to $R\,c\left(\frac{R}{m}\right)$. Optimality of the uniform assignment therefore persists even if the surplus levels *net* of the minimized cost are considered.

*Part* II. As we know from Part I, the cost of implementing the uniform assignment $r$ is minimized if $\mu_x = \frac{m\,R}{R} = m$ holds for every $x \in [0,1)$. Setting $\rho_x = \alpha\,m$ in (A.2), for every $x \in [0,1)$, we can express $1 - \Psi_r^*(x,\xi)$, the cumulative distribution of the distance from the seller with whom any buyer is matched, as an exponential distribution with parameter $2\,\alpha\,m$, which in turn allows us to obtain (7) from (6).

A.2. **Proof of Proposition 2.** As mentioned in the text we build on Proposition 1 and focus directly on cases in which the sellers are uniformly distributed over $[0,1)$.

*Part* I. (9) expresses the first order condition for maximization of $w\left(\underline{\alpha}, m\right)$ w.r.t. $m$ if each seller chooses the optimal scale of production $\underline{\alpha}$, which is independent of $m$. The LHS of (4) is the limit of the LHS of (9) as $m \to 0$, and its positivity guarantees existence of a value of $m$ at which both (9) and the second order-condition for an optimal choice are verified.

*Part* II. (11) expresses the first order condition for maximization of $w\,(\alpha, m)$ w.r.t. $\alpha$ for a given mass $m$ of sellers, and (5) guarantees positivity of the limit of its LHS for conveniently small values of $\alpha$. Continuity of $\frac{\partial w(\alpha,m)}{\partial \alpha}$ allows then us to conclude that there must then exist a positive value of $\alpha$ for which the first and second order conditions for the planner's problem are verified.

A.3. **Proof of Lemma 1.** As each buyer who receives an advertisement is a captive buyer with probability $\exp\left(-\widehat{\alpha}\,m\right)$, the reference seller can realize an expected profit equal to $\alpha\,\exp\left(-\widehat{\alpha}\,m\right)p' > 0$ by charging a price $p' \in \left(0, p_M\right)$. Both the optimized profit and the price allowing the seller to achieve it must therefore be positive as well.

Moreover, if the reference seller set her price equal to $\underline{p} > 0$, then each advertisement delivered would lead to a transaction, and the seller's expected profit would be equal to $\alpha\,\underline{p}$. By contrast, any alternative (positive) price $p'' < \underline{p}$ would yield an expected profit of $\alpha\,p'' < \alpha\,\underline{p}$, and its choice would therefore be dominated by the choice of $\underline{p}$.

A.4. **Proof of Lemma 2.** We consider separately the cases of $\overline{p} \le p_M$ and $\overline{p} > p_M$. If $\overline{p} \le p_M$, then a price no lower than $\overline{p}$ can only possibly be accepted by the captive buyers. Hence, the result follows from the fact that $p_M$ is the unique solution to the monopoly pricing problem that the seller would correspondingly face.

If $\overline{p} > p_M$, then the reference seller could out-compete the more distant sellers with a positive probability, albeit the buyers at a distance greater than $\frac{v-p}{t}$ would not purchase her product even if they were captive buyers. The expected demand if $p \in \left(p_M, \overline{p}\right)$ and its first derivative are then

$$\begin{aligned}\overline{D}(p) &= 2\alpha\left(\int_0^{\frac{1}{2}-\frac{p-\hat{p}}{t}} \exp\left(-2\,\hat{\alpha}\,m\left(\beta-\frac{\hat{p}-p}{t}\right)\right)d\beta + \int_{\frac{1}{2}-\frac{p-\hat{p}}{t}}^{\frac{v-p}{t}} \exp\left(-\hat{\alpha}\,m\right)d\beta\right)\\ &= \dots, \end{aligned} \tag{A.4}$$

$$\frac{\partial\overline{D}(p)}{\partial p} = -\frac{2\alpha}{t}\exp\left(\frac{2\hat{\alpha}}{t}\left(\hat{p}-p\right)\right).$$

Since the derivative is negative, increases of $p$ above $p_M$ lead to a lower profit, and also in this case $p_M$ is an optimal choice in the interval $\left[p_M, \overline{p}\right]$.

A.5. **Proof of Proposition 3.** The first derivatives of the profit equations in (17) are

$$\frac{\partial\pi^-}{\partial p} = \alpha\left(\frac{2}{t}\left(\hat{p}-2p\right)+\frac{1}{\hat{\alpha}}+e^{\frac{2\hat{\alpha}}{t}(\hat{p}-p)-\hat{\alpha}}\left(\frac{2p}{t}-\frac{1}{\hat{\alpha}}\right)\right), \tag{A.5}$$

$$\frac{\partial\pi^+}{\partial p} = \alpha\left(e^{-\hat{\alpha}}\left(\frac{2}{t}\left(2p-\hat{p}\right)-\frac{1}{\hat{\alpha}}\right)+e^{\frac{2\hat{\alpha}}{t}(\hat{p}-p)}\left(\frac{1}{\hat{\alpha}}-\frac{2p}{t}\right)\right). \tag{A.6}$$

(A.5) and (A.6), evaluated at $p = \hat{p}$, are respectively the left- and the right-derivative of the seller's profit at the symmetric equilibrium price, and the necessary conditions for $p = \hat{p}$ to be a best response to the competitors' choice of $\hat{p}$ are

$$\left.\frac{\partial\pi^-}{\partial p}\right|_{p=\hat{p}} = 0, \tag{A.7}$$

$$\left.\frac{\partial\pi^+}{\partial p}\right|_{p=\hat{p}} = 0. \tag{A.8}$$

Both (A.7) and (A.8) are verified iff $\hat{p}$ is equal to $\mathring{p}$ in (20). It is also readily verified that $\hat{\alpha}\,m > \log(2)$ is equivalent to negativity of the second derivative of $\pi^+$ at $p = \mathring{p}$; $\hat{\alpha}\,m > \log(2)$ also guarantees a negative second derivative of $\pi^-$, and thereby ensures local profit maximization.

As to other possible maximizers of $\pi^-$ and $\pi^+$, separately considered, if the seller's competitors set their prices equal to $\mathring{p}$, (A.7) also holds at $p = p^-$, where

$$p^- = \frac{t}{2\hat{\alpha}\,m}\left(1-\hat{\alpha}\,m-\log(2)\right), \tag{A.9}$$

This price can however be disregarded, as (A.9) and $\hat{\alpha}\,m > \log(2)$ imply

$$p^- < \frac{t}{2\hat{\alpha}\,m}\left(1-2\,\log(2)\right) < 0.$$

(A.8) also admits a further real solution at $p = p^+$, where

$$p^+ = \frac{t}{2\hat{\alpha}\,m}(1+\hat{\alpha}\,m-\log(2)), \tag{A.10}$$

However, evaluation of the second derivative reveals that if $\hat{\alpha}\,m > \log(2)$, then $p^+$ actually corresponds to a local *minimum* of $\pi^+$.

Even if $\hat{\alpha}\,m > \log(2)$, we must still consider the possibility of non-interior best responses to the competitors' choice of $\mathring{p}$ before concluding that $\mathring{p}$ is a symmetric equilibrium price. For prices below $\mathring{p}$, the fact that $p^-$ in (A.9) is negative and differentiability of $\pi^-$ imply that

$\frac{\partial \pi^-}{\partial p}$ is negative between 0 and $\mathring{p}$, and therefore rules out any price lower than $\mathring{p}$ as a best response to $\mathring{p}$.

For prices above $\mathring{p}$, it is readily verified that the minimum point $p^+$ in (A.10) satisfies $p^+ \in \left(\mathring{p}, \overline{p}\right)$. If $\overline{p} \le p_M$, and the profit function is expressed by (17b) over $\left[\mathring{p}, \overline{p}\right]$, then the seller's profit achieves a local maximum over the same interval at $p = \overline{p}$. If $\overline{p} > p_M$, then the profit is expressed by (17b) over $\left[\mathring{p}, p_M\right]$, and by (A.4) over $\left[p_M, \overline{p}\right]$; if $p_M > p^+$, then $p_M$ could in principle dominate $\mathring{p}$, from the seller's point of view. In both scenarios, Lemma 2 guarantees that the choice of $p_M$ dominates the choice of $\overline{p}$, regardless of the ranking of the two prices. The critical value of $v$ in (19) is then obtained by comparing the expected profits realized by charging $\mathring{p}$ and by charging $p_M$, respectively expressed by $\mathring{\pi}$ in (21) and by

$$\widehat{\pi}_M = \alpha \exp\left(-\widehat{\alpha}\, m\right)\left(v - \frac{t}{2}\right).$$

A.6. **Proof of Lemma 3.** The first two terms of $H_{\widehat{\alpha}}$ collected yields

$$H_{\widehat{\alpha}}(\widehat{\alpha}; t, m) = t\frac{(2 + \widehat{\alpha}\, m)e^{-\widehat{\alpha}\, m} - 2}{2\,\widehat{\alpha}^3 m^2} - c''\left(\widehat{\alpha}\right).$$

The numerator of the first term, $(2 + \widehat{\alpha}\, m)e^{-\widehat{\alpha}\, m} - 2$ is decreasing in $\widehat{\alpha}$ and the numerator is therefore maximized at $\widehat{\alpha} = 0$. The numerator evaluated at $\widehat{\alpha} = 0$ is negative and so the first term is always non-positive and the entire expression negative.

A.7. **Proof of Proposition 4.** The monopolistically competitive equilibrium is defined by (23)

$$\frac{t}{2\left(\widehat{\alpha}\, m\right)^2}(1 - e^{-\widehat{\alpha}\, m}) - c'\left(\widehat{\alpha}\right) = 0.$$

Multiplying both sides by $m$ we have

$$\frac{mt}{2\,\left(\widehat{\alpha}\, m\right)^2}(1 - e^{-\widehat{\alpha}\, m}) - m\, c'(\alpha) = 0. \tag{A.11}$$

The marginal effect of increased advertising on welfare is expressed by the LHS of (11). As the private and social cost are identical, whether the social optimum features a greater or smaller advertising intensity than the symmetric monopolistically competitive equilibrium, depends on whether the social marginal benefit is greater than the private benefit at $\widehat{\alpha}$, namely on the direction of the inequality

$$e^{-\widehat{\alpha}\, m}\left(vm + \frac{t\,\left(e^{\widehat{\alpha}\, m} - \widehat{\alpha}^2\, m^2 - \widehat{\alpha}\, m - 1\right)}{2\,\widehat{\alpha}^2\, m}\right) - \frac{mt}{2\,\left(\widehat{\alpha}\, m\right)^2}(1 - e^{-\widehat{\alpha}\, m}) \gtreqless 0 \tag{A.12}$$

The LHS of (A.12) is positive at $t = 0$ and decreasing in $t$ with a root at $t = \frac{2\,\widehat{\alpha}\, m\, v}{1 + \widehat{\alpha}\, m}$. However, for any $t$ greater than $\frac{2\,\widehat{\alpha}\, m\, v}{1 + \widehat{\alpha}\, m}$ there is no longer a symmetric monopolistically competitive equilibrium as the price required for such an equilibrium would be greater than the monopoly price $v - t/2$ (and so firms would deviate from that price).

A.8. **Proof of Proposition 5.** The monopolistically competitive free entry equilibrium is defined by (24)

$$\pi(m) = \frac{t}{2\widehat{\alpha}(m)\, m)^2}(1 - e^{-a(m)\, m)}) - c(a(m)) - F = 0.$$

The marginal effect of entry for fixed $\alpha$ was given by the LHS of (9)

$$e^{-\alpha\, m}\left(v + \frac{t\,\left(e^{\alpha\, m} - \alpha^2\, m^2 - \alpha\, m - 1\right)}{2\,\alpha\, m^2}\right) - c\left(\alpha\right) - F.$$

Thus, if we fixed $\alpha$ at the entry equilibrium $\alpha$, $a\left(\widehat{m}\right)$, the expressions represents the marginal effect of entry when $\alpha = a\left(\widehat{m}\right)$. A the private and social cost are the same ($F$), whether the social optimum would increase entry over the symmetric monopolistically competitive equilibrium, depends on whether the social marginal benefit is greater than the private benefit or if

$$e^{-a(\widehat{m})\,m}\left(v+\frac{t\left(e^{a(\widehat{m})\,m}-a\left(\widehat{m}\right)^2 m^2-a\left(\widehat{m}\right)m-1\right)}{2\,a\left(\widehat{m}\right)m^2}\right)-\frac{t}{2\,a\left(\widehat{m}\right)\,m^2}(1-e^{-a(\widehat{m})\,m}) > 0 \quad \text{(A.13)}$$

As with expression (A.12), expression (A.13) is decreasing in $t$ with a root at $t = v\frac{2,\widehat{\alpha}\,m}{1+\widehat{\alpha}\,m}$. However, as in the proof for Proposition 4, for any $t$ greater than $v\frac{2\,\widehat{\alpha}\,m}{1+\widehat{\alpha}\,m}$ there is no longer a symmetric monopolistically competitive equilibrium as the price required for such an equilibrium would be greater than the monopoly price $v - t/2$ (and so firms would deviate from that price).

## Appendix B. Comparative Statics

Below is a summary table for the derivations in this appendix.

| | | Endogenous variables | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Exog. $m$ | | | | | Endog. $m$ | | | |
| | | $p$ | $\widehat{\alpha}$ | $\widehat{\alpha}\,m$ | $\pi$ | \| | $p$ | $\widehat{\alpha}$ | $m$ | $\widehat{\alpha}\,m$ |
| Exogenous | $F$ | 0 | 0 | | $-$ | \| | $+$ | $+$ | $-$ | $-$ |
| | $t$ | $+$ | $+$ | $+$ | $+$ | \| | $+$ | 0 | $+$ | $+$ |
| variables | $c'(\alpha)$ | $+$ | $-$ | $-$ | $-$ | \| | $+$ | 0 | $-$ | $-$ |
| | $k$ | $+$ | $-$ | $-$ | $?^{\star}$ | \| | $+?$ | $?^{\blacklozenge}$ | $?^{\star}$ | $-?$ |
| | $m$ | $-$ | $-$ | $+$ | $-$ | \| | | | | |

$^{\star}$ If marginal cost is relatively large to total cost, then this increases.
$^{\blacklozenge}$ If marginal cost is relatively large to total cost, then this decreases
For endogenous $m$, the effect of $k$ on $p$, $\widehat{\alpha}$ and $m$, even examples with profits increasing have aggregate advertising decreasing (and so price increasing). For cost, if the marginal cost of advertising does not increase uniformly for all levels of $\alpha$ (the $k$ case), then it is possible to create examples of profits increasing with costs.

### B.1. Exogenous m.

In this section, $t$, $m$ and the function $c(\alpha)$ are exogenous. In the next section, $m$ is endogenized and entry costs $F$ is added as an exogenous variable.

#### B.1.1. *Comparative statics on advertising.*

Given its optimal price ($p = t/(2\,a\left(\widehat{m}\right)\,m)$), we can express the profit as

$$\pi\left(\alpha, \widehat{\alpha}, m\right) = \frac{\alpha t\left(1-e^{-\widehat{\alpha}\,m}\right)}{2\left(\widehat{\alpha}\,m\right)^2} - c(\alpha).$$

Using primes to denote derivatives, we can write the first order condition for an interior optimum as

$$\frac{\partial \pi\left(\alpha, \widehat{\alpha}, m\right)}{\partial \alpha} = \frac{t\left(1-e^{-\widehat{\alpha}\,m}\right)}{2\left(\widehat{\alpha}\,m\right)^2} - c'(\alpha) = 0; \qquad (22)$$

because the expected revenue per buyer is independent of $\alpha$, a well-defined optimum for the firm is guaranteed by a convex cost function. Because the cost function is monotonic, it admits an inverse $\left(c'\right)^{-1}$, and we can rewrite (22) as

$$\alpha = \left(c'\right)^{-1}\left(\widehat{\alpha},\, m\right).$$

An equilibrium advertising level is thus a fixed point of $\left(c'\right)^{-1}\left(\widehat{\alpha},\, m\right)$.

In equilibrium, $\alpha = \widehat{\alpha}$, so $\widehat{\alpha}$ is implicitly defined by the function

$$H(\widehat{\alpha}; t, m, c) \equiv \frac{t}{2\,(\widehat{\alpha}\, m)^2}(1 - e^{-\hat{\alpha}\, m}) - c'\left(\widehat{\alpha}\right) \equiv 0. \tag{23}$$

To use the implicit function we need $H_{\widehat{\alpha}} < 0$, which was proved in Lemma 3. However, one useful fact from the proof is that $(2 + \widehat{\alpha}\, m)e^{-\hat{\alpha}\, m} - 2$ is decreasing in $\widehat{\alpha}$ and is maximized at $\widehat{\alpha} = 0$ and so the expression (which appears in various forms below) is always negative.

B.1.2. *Comparative statics on equilibrium advertising.* With $H_{\widehat{\alpha}} < 0$ we can use the implicit function theorem to derive the effect of $t$, marginal cost $c'(\alpha)$, and $m$ on $\widehat{\alpha}$. That is, on equilibrium advertising and not just on one firm's advertising level. We begin with the effect of transport cost $t$ on advertising when the number of firms is fixed.

**Lemma 4.** *With exogenous m, increase in transportation cost increases equilibrium advertising:* $\frac{d\widehat{\alpha}}{dt} > 0$.

*Proof.* As $\frac{d\widehat{\alpha}}{dt} = -H_t/H_{\widehat{\alpha}}$, we need to calculate $H_t$:

$$H_t(\widehat{\alpha}; t, m, c) = \frac{1}{2(\hat{\alpha}(t,m)m)^2}(1 - e^{-\hat{\alpha}\, m}) > 0.$$

Thus, $\frac{d\widehat{\alpha}}{dt} = -H_t/H_{\widehat{\alpha}} > 0$:

$$\frac{d\widehat{\alpha}}{dt} = -\frac{H_t}{H_{\hat{\alpha}}} = \frac{-\widehat{\alpha}\,(1 + \widehat{\alpha}\, m)e^{-\hat{\alpha}\, m})}{t((2 + \widehat{\alpha}\, m)e^{-\hat{\alpha}\, m} - 2)) - 2\hat{\alpha}^3 m^2 c''\left(\widehat{\alpha}\right)} > 0 \tag{B.1}$$

□

We next turn to the effect of a change in marginal cost and a change on the coefficient of marginal cost $k$ on the equilibrium of advertising, which are straightforward.[25]

**Lemma 5.** *A unit decrease in marginal cost or the coefficient of the marginal cost of advertising increases the equilibrium level of advertising:* $\frac{d\widehat{\alpha}}{dc'} < 0$ *and* $\frac{d\widehat{\alpha}}{dk} < 0$.

*Proof.* As $\frac{d\widehat{\alpha}}{dc'} = -H_{c'}/H_{\widehat{\alpha}}$, we need to calculate $H_{c'}$:

$$H_{c'}\left(\widehat{\alpha}; t, m, c\right) = -\frac{d[c'(\alpha)]}{d[c'(\alpha)]} < 0.$$

Thus, $\frac{d\widehat{\alpha}}{dc'} = -H_{c'}/H_{\widehat{\alpha}} < 0$:

$$\frac{d\widehat{\alpha}}{dc'} = -\frac{H_{c'}}{H_{\hat{\alpha}}} = \frac{2\,\widehat{\alpha}^3 m^2}{t((2 + \widehat{\alpha}\, m)e^{-\hat{\alpha}\, m} - 2)) - 2\hat{\alpha}^3 m^2 c''\left(\widehat{\alpha}\right)} < 0.$$

For when it is a change in $k$ for cost function $kc(\alpha)$, $H_k = -c'(\alpha) < 0$ and the proof follows analogously. □

We now turn to the effect of entry on the equilibrium of advertising is derived.

**Lemma 6.** *Entry reduces the equilibrium level of advertising:* $\frac{d\widehat{\alpha}}{dm} < 0$

[25]Recall that for the second case we are considering cost functions of the form $kc(\alpha)$.

*Proof.* By the implicit function theorem $\frac{d\widehat{\alpha}}{dm} = -H_m/H_{\widehat{\alpha}}$ and so, to establish the sign of $\frac{d\widehat{\alpha}}{dm}$, the sign of $H_m$ needs to be established.

$$H_m\left(\widehat{\alpha}; t, m, c\right) = \frac{t\,\widehat{\alpha}\,e^{-\widehat{\alpha}\,m}}{2\,\left(\widehat{\alpha}\,m\right)^2} - \frac{t\,4\,\widehat{\alpha}^2 m}{(2\,\left(\widehat{\alpha}\,m\right)^2)^2}(1 - e^{-\widehat{\alpha}\,m}).$$

This RHS can be rewritten as

$$\frac{t\,\widehat{\alpha}\,e^{-\widehat{\alpha}\,m}}{2\,\left(\widehat{\alpha}\,m\right)^2} - \frac{t\,4\,\widehat{\alpha}^2 m}{(2\,\left(\widehat{\alpha}\,m\right)^2)^2}(1 - e^{-\widehat{\alpha}\,m}) = t\frac{(2 + \widehat{\alpha}\,m)e^{-\widehat{\alpha}\,m} - 2}{2\,\widehat{\alpha}^2 m^3} < 0,$$

where the last inequality follows as it was already shown that $(2 + \widehat{\alpha}\,m)e^{-\widehat{\alpha}\,m} - 2$ is negative for positive $\widehat{\alpha}$. Thus,

$$\frac{d\widehat{\alpha}}{dm} = -\frac{H_m}{H_{\widehat{\alpha}}} = \frac{-\widehat{\alpha}\,t((2 + \widehat{\alpha}\,m)\,e^{-\widehat{\alpha}\,m} - 2)}{m[t((2 + \widehat{\alpha}\,m)e^{-\widehat{\alpha}\,m} - 2)) - 2\,\widehat{\alpha}^3\,m^2 c''\left(\widehat{\alpha}\right)]} < 0.$$

□

The result that entry reduces advertising is intuitively straightforward: if with entry firms initially maintain the same level of advertising, the representative firm's marginal benefit from advertising has decreased (as total advertising by all of its rivals has increased) and so it is optimal to reduce its advertising.

Finally, the effect of entry on aggregate advertising $(\widehat{\alpha}\,m)$ is considered.

**Lemma 7.** *Entry increases aggregate advertising:* $\frac{d(a(m)\,m)}{dm} > 0$.

*Proof.* Using the derivation for $\frac{d(a(m)\,m)}{dm}$ above we have

$$\begin{aligned}\frac{d(a(m)\,m)}{dm} &= \frac{d\,a(m)}{dm}\,m + a(m)\\ &= \frac{-2c''\left(\widehat{\alpha}\right)\,\widehat{\alpha}^4\,m^2}{t(\left(2 + \widehat{\alpha}\,m\right)\,e^{-\widehat{\alpha}\,m} - 2) - 2c''\left(\widehat{\alpha}\right)\,\widehat{\alpha}^3\,m^2}\\ &> 0,\end{aligned}$$

with the denominator negative because, as has been shown, $(2 + \widehat{\alpha}\,m)\,e^{-\widehat{\alpha}\,m} - 2 > 0$. □

At first glance, the result is somewhat surprising that entry *always* increases aggregate advertising. With just some pause, it becomes obvious, as if it did not, then equilibrium price would increase with entry.

Finally, while obvious, it should be noted that entry costs cannot have a direct effect on advertising levels as they are fixed costs (of course, they will indirectly affect advertising through their effect on entry level $m$).

B.1.3. *Comparative statics on price.* With the effect of entry, advertising cost and transportation cost on advertising levels, we can now compute the effect of these parameters on the equilibrium price, $p(t, m, c) = t/[2\hat{\alpha}(t, m, c)m]$. The effect of either type of change in marginal cost on price is obvious: since by Lemma 5 advertising is decreasing in marginal cost, then from the definition of $p(t, m, c)$ we have immediately

**Lemma 8.** *With exogenous m, a unit increase in marginal cost or in the coefficient of marginal cost of advertising increases the equilibrium price:* $dp/dc' > 0$ *and* $dp/dk > 0$.

The first non-trivial question is the effect of transportation cost on the equilibrium price when the number of firms is fixed. There is is an indirect and direct effect. The direct effect raises price, but the indirect effect reduces it: an increase in $t$ causes $\widehat{\alpha}$ to increase, which causes the price to fall. Again the direct effect dominates:

**Lemma 9.** *If $m$ is exogenous, an increase in transport cost increases the equilibrium price: $dp/dt > 0$.*

*Proof.*

$$\begin{aligned}\frac{dp}{dt} &= \frac{1}{2\left(\widehat{\alpha}\, m\right)} - \frac{t}{2\,\widehat{\alpha}^2 m}\frac{da(t)}{dt}\\ &= \frac{1}{\widehat{\alpha}\, m}\frac{t(1-(1+\hat{\alpha}\, m)e^{-\widehat{\alpha}\, m})+2c''(\alpha)\hat{\alpha}^3 m^2}{t(2-(2+\hat{\alpha}\, m)e^{-\hat{\alpha}\, m})+2c''(\alpha)\hat{\alpha}^3 m^2}\\ &> 0,\end{aligned}$$

where again the inequality follows because $1-(1+\widehat{\alpha}\, m)e^{-\hat{\alpha}\, m}$ is increasing in either $\alpha$ or $m$ and at zero value for either is zero, i.e., the expression is non-negative. Therefore, the numerator and denominator are positive: increased transportation costs raises the equilibrium price. □

The next question is the effect of entry ($m$) on price and given the equilibrium price. Like with transport cost, there is a direct and indirect effect By the direct effect, entry reduces the price – there is more competition. However, there is an indirect effect: entry also reduces advertising, and lower advertising increases price. However, above it was shown that aggregate advertising increases with entry ($d(\hat{\alpha}(t,m)m)/dm > 0$) so, the direct effect dominates: the price deceases.

**Lemma 10.** *The equilibrium price decreases with entry: $dp/dm < 0$.*

*Proof.*

$$\frac{dp}{dm} = \frac{t}{2[a(m)\, m]^2}\frac{-d[a(m)\, m]}{dm} < 0,$$

where $d(\hat{\alpha}\, m)/dm > 0$ by Lemma 7. □

B.1.4. *Comparative statics on equilibrium profit.* Profit gross of entry costs are

$$\pi\left(\widehat{\alpha}\right) = \frac{t}{2\,\widehat{\alpha}\, m^2}(1-e^{-\widehat{\alpha}\, m}) - c\left(\widehat{\alpha}\right). \tag{B.2}$$

It is perhaps worthwhile to note that this is equilibrium profit. The importance is that changes in entry or transportation costs will affect profits indirectly through the *equilibrium* level of advertising ($\hat{\alpha}(t,m)$) and so the envelope theorem from the firm's first order condition does not explain the entirety of how changes in advertising affect profits.

We begin again with the effect of an increase in transport cost $t$ on profits when the number of firms is fixed. Differentiating equilibrium profits with respect to transport cost ($t$) obtains

$$\frac{d\pi(\hat{\alpha}(t),t)}{dt} = \frac{\partial\pi(\hat{\alpha}(t),t)}{\partial t} + \frac{\partial\pi(\hat{\alpha}(t),t)}{\partial\hat{\alpha}(t)}\frac{da(t)}{dt}. \tag{B.3}$$

In this case it is easy to see from examining the profit expression that the direct effect is positive. For the indirect effect, the second term was shown above to be positive: transportation costs increase the equilibrium levels of advertising. However, intuitively the first part of the second term is negative as greater equilibrium advertising reduces the price, demand from a contacted consumer (as there is a greater possibility they received an advertisement from a better match) and raises cost (this can be easily checked by differentiating (B.2) with respect to $\hat{\alpha}$). Thus, again the indirect effect runs contrary to the direct effect. However, as in the previous case the expressions simplify, and it is easy to show that increase in transportation costs increase profits: the direct effect dominates the indirect effect.

**Lemma 11.** *For exogenous $m$, equilibrium profits are increasing in transport cost: $d\pi/dt > 0$.*

*Proof.* Using (B.1) for $d\hat{\alpha}/dt$, and differentiating equilibrium profits with respect to $\hat{\alpha}$ and $t$, the expression simplifies to

$$\frac{d\pi(\hat{\alpha}(t),t)}{dt} = \frac{(1-e^{-\hat{\alpha}\,m})\hat{\alpha}^2\,c''(\alpha)}{t(2-(2+\hat{\alpha}\,m)e^{-\hat{\alpha}\,m}) + 2c''(\alpha)\,\hat{\alpha}^3 m^2} > 0, \tag{B.4}$$

where again the first order condition (22) helps to simplify the numerator of the second term on the RHS of B.3. □

We know turn to the effect of the two types of change in marginal cost of advertising on profits. Considering the first and differentiating equilibrium profits with respect to a unit increase in marginal cost ($c'$) obtains

$$\frac{d\pi(\hat{\alpha}(c'),c')}{dc'} = \frac{\partial\pi(\hat{\alpha}(c'),c')}{\partial\hat{\alpha}(c')}\frac{d\hat{\alpha}(c')}{dc'} + \frac{\partial\pi(\hat{\alpha}(c'),c')}{\partial c'}. \tag{B.5}$$

In this case it is easy to see from examining the equilibrium profits that the direct effect is negative; a firm is worse off from its marginal cost increasing. For the indirect effect, the second term was shown above to be negative: marginal cost decreases the equilibrium levels of advertising. However, as noted when examining the effect of transportation costs on equilibrium profits (Lemma 11), the first part of the second term intuitively is also negative. Once again, the indirect effect runs contrary to the direct effect. However, as in the previous case the expressions simplify, and it is straightforward to show that increase in marginal costs decreases profits.

**Lemma 12.** *For exogenous $m$, equilibrium profits are decreasing in a uniform increase of marginal cost of advertising (i.e., a unit increase for all levels of $\alpha$): $d\pi/dc' < 0$.*

*Proof.* It is useful to rewrite the equilibrium profit expression as

$$\pi(\hat{\alpha},c') = \frac{t}{2\hat{\alpha}(c')m^2}(1-e^{-\hat{\alpha}(c')m}) - \int_0^{\hat{\alpha}(c')} c'(\alpha)d\alpha$$

Differentiation of the equilibrium profit w.r.t. the marginal advertising cost yields

$$\frac{d\pi(\hat{\alpha}(c'),c')}{dc'} = \left[\frac{te^{-\hat{\alpha}\,m}}{2m\hat{\alpha}} - \frac{t(1-e^{-\hat{\alpha}\,m})}{2(\hat{\alpha}\,m)^2} - c'(\hat{\alpha}(c',m)\right]\frac{d\hat{\alpha}(c',m)}{dc'} \\ - \int_0^{\hat{\alpha}(c',m)} dc'(\alpha)d\alpha \tag{B.6}$$

The first term (the indirect effect) is positive and the second is negative. Now assuming that the change across marginal cost is independent of $\alpha$ (i.e., $dc'(\alpha) = 1$ for all $\alpha$) yields

$$\frac{d\pi(\hat{\alpha}(c'),c')}{dc'} = \frac{-2\hat{\alpha}^4 m^2\,c''(\alpha)}{t(2-(2+\hat{\alpha}\,m)e^{-\hat{\alpha}\,m}) + 2\hat{\alpha}^3\,m^2\,c''(\alpha)} < 0. \tag{B.7}$$

□

The assumption that the effect on marginal cost is independent of $\alpha$ is equivalent to the case of $\beta = 1$ in Grossman and Shapiro (1984). Specifically, if their $\beta$ (the elasticity of the marginal effect of the shift parameter on cost with respect to the proportion of consumers reached) equals one (unit elastic), then an increase in the shift parameter has zero effect on the advertising level (Grossman and Shapiro, 1984, Table 1), which effectively is the case here.[26] Since $\eta > 0$ there (and here since $c''(\alpha) > 0$), we have the same sign as Grossman and Shapiro (1984) do for this case: a unit increase in marginal cost reduces equilibrium profits. It should also be clear that other assumptions could yield equilibrium profits increasing. For example, if the increase in cost were only on the last units of $\alpha$ and sufficiently small, then the last term of the RHS of (B.6) is smaller and the expression could

[26]Specifically here a unit increase in marginal cost increase advertising costs in our model by $\alpha$, and the derivative of that with respect to $\alpha$ is of course 1.

be positive. An easier way to see this is to consider the second change to the cost coefficient $k$ (where costs are $kc(\alpha)$). In this case, equilibrium profits are

$$\pi(\hat{\alpha}, k)) = \frac{t}{2\hat{\alpha}(k)m^2}(1 - e^{-\hat{\alpha}(k)m}) - kc(\hat{\alpha}(k))$$

Differentiating profits with respect to $k$ instead of $c'$ is a slight modification to (B.5) with the same general direct (higher $k$ increases costs) and indirect (higher costs ($k$) reduces equilibrium advertising which benefits the firm) effects. Specifically,

$$\frac{d\pi(\hat{\alpha}(k), k)}{dk} = \left[\frac{te^{-\hat{\alpha}\, m}}{2m\hat{\alpha}} - \frac{t(1 - e^{-\hat{\alpha}\, m})}{2(\hat{\alpha}\, m)^2} - kc'(\hat{\alpha}(k)\right]\frac{d\hat{\alpha}(k)}{dk} - c(\hat{\alpha}(k)) \tag{B.8}$$

This simplifies to

$$\frac{d\pi(\hat{\alpha}(k), k)}{dk} = \frac{t(1 - (1 + \hat{\alpha}\, m)e^{-\hat{\alpha}\, m}) + 2\hat{\alpha}^2 m^2 kc'}{t(2 - (2 + \hat{\alpha}\, m)e^{-\hat{\alpha}\, m}) + 2\hat{\alpha}^3 m^2\, k\, c''(\alpha)}\hat{\alpha}c' - c(\hat{\alpha}(k)), \tag{B.9}$$

and its sign depends on the relative size of the marginal cost to total costs. If the former is sufficiently large, then profits could increase with an increase in $k$, as was noted by Grossman and Shapiro (1984). In Example 7, we show that with "large" $\alpha\, m$ in equilibrium, quadratic cost of the form $k\alpha^2$ create such a scenario of an increase in $k$ increases equilibrium profits and so inducing entry.

We conclude this section by examining the effect of entry on equilibrium profits. Differentiating the RHS of (B.2) with respect to the size of the sellers' population $m$ yields

$$\frac{d\pi(\hat{\alpha}(m), m)}{dm} = \frac{\partial\pi(\hat{\alpha}(m), m)}{\partial m} + \frac{\partial\pi(\hat{\alpha}(m), m)}{\partial\hat{\alpha}(m)}\frac{d\hat{\alpha}(m)}{dm}. \tag{B.10}$$

The first term, the direct effect of entry on profits, can be shown to be, as expected, negative. The second term has the indirect effect of entry through its effect on equilibrium levels of advertising. As noted just before Lemma 11, the first part of the second term, the effect of an increase in equilibrium advertising on profits, intuitively is negative as the effect from all the rivals increasing their advertising reduces the firm's profit. However, as we know from Lemma 6, entry *reduces* the equilibrium level of advertising; hence, the second part of the second term is also negative, and the second term is positive. The entire expression is unclear unless costs are strictly convex (a sufficient condition).

**Lemma 13.** *Profits are decreasing in entry:* $\frac{d\pi}{dm} < 0$.

*Proof.* The RHS of (B.10) simplifies to

$$\frac{d\pi}{dm} = \frac{-t(2 - (2 + \hat{\alpha}\, m)e^{-\hat{\alpha}\, m})}{2\hat{\alpha}\, m^3}\frac{t(1 - e^{-\hat{\alpha}\, m}) + 2(\hat{\alpha}\, m)^2[c''(\alpha)\,\hat{\alpha} - c']}{t(2 - (2 + \hat{\alpha}\, m)e^{-\hat{\alpha}\, m}) + 2c''(\alpha)\,\hat{\alpha}^3 m^2}.$$

The first order condition (22) can be written as $t(1 - e^{-\hat{\alpha}\, m}) - c'(\alpha)2(\hat{\alpha}\, m)^2 = 0$ and so the above simplifies to

$$\frac{d\pi}{dm} = \frac{-t[2 - (2 + \widehat{\alpha}\, m)\, e^{-\widehat{\alpha}\, m}]\,\widehat{\alpha}^2\, c''\left(\widehat{\alpha}\right)}{m[t(2 - (2 + \hat{\alpha}\, m)e^{-\hat{\alpha}\, m}) + 2\, c''\left(\widehat{\alpha}\right)\,\widehat{\alpha}^3 m^2]} < 0 \tag{B.11}$$

as $c''(\alpha) > 0$. □

B.2. **Endogenous m.** We now consider the effect of transportation costs $t$, advertising costs $c$, and entry costs $F$ with the entry level $m$ endogenous, or what Grossman and Shapiro (1984) refer to as the "monopolistically competitive" model. With this we have another condition that profits are zero:[27]

$$J(t, F, c) \equiv \frac{t}{2\hat{\alpha}\, m^2}(1 - e^{-\hat{\alpha}\, m)}) - c(\hat{\alpha}) - F = 0. \tag{24}$$

[27]This is condition (11c) in Grossman and Shapiro (1984).

With this and the condition on the equilibrium level of advertising given $m$ we can derive the comparative statics with the implicit function theorem.

B.2.1. *Long run effects on entry.* We begin by determining the effect of entry costs and transportation on the entry equilibrium: $dm/dF$ and $dm/dt$. Using the implicit function theorem requires that $J_m < 0$ and that profits are decreasing in $m$, which has already been shown in (B.11).

Starting with the effect of entry costs, the effect is immediate:

**Lemma 14.** *Entry is decreasing in entry costs: $dm/dF < 0$.*

*Proof.* $J_F = -1$. From (B.11), $J_m < 0$, so $dm/dF = -J_F/J_m < 0$. □

In this case, the expression simplifies neatly to

$$\frac{dm}{dF} = -\frac{J_F}{J_m} = \frac{m[t(2-(2+\hat{\alpha}\,m)e^{-\hat{\alpha}\,m}) + 2c''(\alpha)\,\hat{\alpha}^3 m^2]}{-t[2-(2+\hat{\alpha}\,m)e^{-\hat{\alpha}\,m}]\hat{\alpha}^2 c''(\alpha)} < 0.$$

Next consider the effect of transportation cost $t$ on entry. We have already seen that an increase in transportation cost increases profit ($J_t > 0$) from (B.4), and so the effect is immediate:

**Lemma 15.** *Entry increases with transportation cost: $dm/dt > 0$.*

*Proof.* From (B.4) we have $J_t > 0$ and from (B.11) we have $J_m < 0$ and so $dm/dt = -J_t/J_m > 0$. □

In this case too, simplification yields a clean expression:

$$\frac{dm}{dt} = -\frac{J_t}{J_m} = \frac{(1-e^{-\hat{\alpha}\,m})m}{t(2-(2+\hat{\alpha}\,m)e^{-\hat{\alpha}\,m})} > 0. \tag{B.12}$$

Turning to the effect of constant increase in marginal cost $c'$ on entry, it too follows almost immediately.

**Lemma 16.** *Entry decreases with a uniform increase of marginal cost of advertising (i.e., a unit increase for all levels of $\alpha$): $dm/dc' > 0$.*

*Proof.* From expression (B.7) we have $J_{c'} < 0$ and from (B.11) we have $J_m < 0$ and so $dm/dc' = -J_{c'}/J_m < 0$. □

With a little algebra, this expression also simplifies cleanly:

$$\frac{dm}{dc'} = \frac{2\hat{\alpha}^2 m^3}{-t[2-(2+\hat{\alpha}\,m)e^{-\hat{\alpha}\,m}]} < 0 \tag{B.13}$$

As noted in the preamble this could also be interpreted as the effect of a per-unit tax on advertising, with it decreasing entry.

Finally, turning to to the effect of a unit increase in marginal cost proportional to the marginal cost. We have already seen from (B.9) that a proportional increase in marginal cost (e.g., with costs are $k\alpha^2$, $dc/dk$), could result in an increase in equilibrium profits if marginal costs are relatively large (i.e., steeply sloped) on the margin. As a result, the effect of a change in $k$ could either increase or decrease entry, and in the example below with quadratic cost, increases entry. To summarize,

**Lemma 17.** *Entry may increase with a proportionate increase of marginal cost of advertising (e.g., when $c(\alpha) = k\alpha^2$, $dc/dk$), if the marginal cost on the margin is relatively large.*

B.2.2. *Long run effects on advertising.* With the effects on entry from transportation, marginal advertising costs and entry costs, we can establish the entry equilibrium effects of these costs (transport, advertising and entry) on advertising. Specifically, we can use the first order condition in equilibrium (23), but now with entry level $m$ as a function of transportation and fixed costs

$$H(\hat{\alpha}; t, c, m(t, F, c)) \equiv \frac{t}{2(\hat{\alpha}\, m)^2}(1 - e^{-\hat{\alpha}\, m}) - c'(\hat{\alpha}) \equiv 0$$

to determine the effects of these two costs on advertising in long run (free entry) equilibrium.

We begin with the effect of transportation costs on advertising levels. There is, as usual, two opposing effects. First, the direct effect of increasing transportation cost for fixed $m$ (positive, established in Lemma 4) leads to more advertising. However, there is also the indirect effect: increases in $t$ induce entry, which has a negative effect on advertising levels. These two effects, however, cancel out.

**Lemma 18.** *Increases in transportation costs have no effect on a firm's advertising level in the long run:* $d\hat{\alpha}/dt = 0$.

*Proof.* In Lemma 3 it was established that $H_{\hat{\alpha}} < 0$. With the calculation of $dm/dt$ (B.12),

$$H_t = \frac{t(e^{-\hat{\alpha}\, m}(2 + \hat{\alpha}\, m) - 2)}{2\hat{\alpha}^2 m^3} \frac{dm(t, F)}{dt} + \frac{1 - e^{-\hat{\alpha}\, m}}{2\hat{\alpha}^2 m^2} = 0.$$

Thus,

$$\frac{d\hat{\alpha}}{dt} = \frac{-H_t}{H_{\hat{\alpha}}} = 0.$$

□

Since increases in transportation cost do not change any one firm's advertising in the long run, but does induce entry, it follows that

**Corollary 2.** *Increase in transportation costs, increase aggregate advertising.*

The effect of entry costs $F$ is immediate: since entry costs $F$ reduce entry $m$, but has no effect on advertising directly, we have

**Lemma 19.** *Increased entry costs result in greater advertising per firm in the long run:* $\frac{d\hat{\alpha}}{dF} > 0$.

As it has already been shown in Lemma 7 that entry increases aggregate advertising, and since entry costs do not effect advertising levels directly, it follows

**Corollary 3.** *Increased entry costs decrease aggregate advertising.*

Next we turn to the effect of changes in the costs of advertising on advertising in the long run. Beginning with a uniform unit increase in marginal cost for advertising (or equivalents a per-unit tax on advertising), there are two counter effects. On one hand, for fixed number of firms an increase in marginal cost, reduces equilibrium advertising. However, it also reduces profits which induces exit, which has a positive effect on advertising, so the net effect is unclear. It turns out that the effects exactly offset themselves, and so a unit increase in marginal cost has no effect on the long run equilibrium advertising per firm.

**Lemma 20.** *A unit increase in the marginal cost of advertising has no effect on a firm's advertising level in the long run:* $d\hat{\alpha}/dc' = 0$.

*Proof.* In Lemma 3, it was established that $H_{\hat{\alpha}} < 0$. With the calculation of $dm/dc'$ (B.13),

$$H_{c'} = \frac{t(e^{-\hat{\alpha}\, m}(2 + \hat{\alpha}\, m) - 2)}{2\hat{\alpha}^2 m^3} \frac{dm}{dc'} - 1 = 0,$$

Thus,

$$\frac{d\hat{\alpha}}{dc'} = \frac{-H_c'}{H_{\hat{\alpha}}} = 0.$$

□

While the result may initially seem surprising, it too is in line with what Grossman and Shapiro (1984) found, because as discussed after Lemma 12, $\beta = 1$ in this case. Finally, although the advertising per firm does not change, since the increase in marginal cost induced exit, aggregate advertising has decreased - witness Lemma 4.

**Corollary 4.** *A per unit-tax on advertising (a unit increase in marginal cost) decreases aggregate advertising.*

Thus, a per-unit tax on advertising level would have the expected effect of reducing aggregate advertising.

Finally turning to the effect of a proportionate increase in marginal cost $k$, not surprisingly given Lemma 17, such a change in cost could increase or decrease a firm's advertising level in the long run, depending on the relative size of the marginal cost on the margin: if the marginal cost is relatively small, then it is possible a firm's advertising increases. The intuition follows from Lemma 17: if on the margin, marginal cost is relatively small, this induces exit, which increases the equilibrium level of advertising (Lemma 6). This effect has to be large enough to offset the direct effect from the increase in marginal cost. For aggregate advertising, this becomes even more muddled as exit on its own would decrease aggregate advertising. However, for the quadratic cost example presented in Examle **?**, for "large" $\hat{\alpha}\, m$, an proportionate increase in marginal cost, decreases aggregate advertising. That is, the direct effect on an individual firm in reducing its advertising overwhelms the entry effect on aggregate advertising. Given that quadratic cost produces otherwise some counter-intuitive results, this suggests that generally a proportionate increase in all marginal cost will decrease aggregate advertising.

B.2.3. *Long run effects on price.* The long-run effect of different variables on price is primarily through the effect on aggregate advertising ($\hat{\alpha}\, m$), and so the above corollaries on the long-run effects on aggregate advertising determines the effect on price (with the exception of transport cost). We begin with the most straight-forward by considering the effect of entry costs $F$ on the equilibrium price, which is recall

$$p = \frac{t}{2\hat{\alpha}\, m}. \tag{B.14}$$

Because entry increases aggregate advertising (which decreases price), increases in fixed cost increase the price.

**Lemma 21.** *The long run price is increasing in entry costs: $dp/dF > 0$.*

*Proof.* Differentiation the long run equilibrium price (B.14) with respect to the fixed cost $F$ we have

$$\frac{dp}{dF} = \frac{t}{2}\frac{-d\hat{\alpha}(m)m}{dm}\frac{dm}{dF}\frac{1}{(\hat{\alpha}(m)m)^2} > 0.$$

The term $d(\hat{\alpha}\, m)/dm$ was established as positive in lemma 7. The term $dm/dF$ was determined to be negative in lemma 14. Thus, the total effect is for fixed cost to increase the equilibrium price because aggregate advertising decreases. □

Next we consider the effect of a unit increase in all marginal costs ($c'$). From Corollary 4, we know that a unit increase in all marginal cost decreases aggregate advertising, and so from (B.14) it increases the long-run price $dp/dc' > 0$.

**Corollary 5.** *The long run price is increasing in a uniform increase in marginal cost (or a small per-unit tax on advertising).*

Turning to the effect of a proportionate increase in marginal cost, as the effect on aggregate advertising is ambiguous, the effect on the long-run price is ambiguous. Again, turning to the example of "large" $\hat{\alpha}\, m$, given that aggregate advertising decreases with the proportionate increase in marginal cost, the long-run price is increasing in this case.

The final comparative static is of transport cost on the equilibrium price. In this case, the relationship is more complicated. On one hand, it was noted above that

$$\frac{dp}{dt} = \frac{1}{2\hat{\alpha}\, m} - \frac{t}{2(\hat{\alpha}\, m)^2}\left(\frac{\partial[\hat{\alpha}(t,m)m]}{\partial m}\frac{dm}{dt} + \frac{\partial\hat{\alpha}(t,m)}{\partial t}m\right). \tag{B.15}$$

The first term is the direct effect from an increase of transportation cost on the price as is positive. The second term is the indirect effect from transportation cost increasing. This induces more entry, which we have seen increases aggregate advertising thereby reducing the price, running counter to the direct effect. There is also the effect transportation costs have directly on advertising levels, but we have seen that nets out to zero: the last term $(d\hat{\alpha}/dt)$ was shown to be zero in Lemma 18. The other terms prove to be positive.

**Lemma 22.** *The long run price is increasing in transportation cost.*

*Proof.* Collecting the remaining terms we have

$$\frac{dp}{dt} = \frac{t(e^{-\alpha\, m}(2+\alpha\, m)-2)^2 - 2\, c''(\alpha)\, \hat{\alpha}^3 m^2 t(e^{-\alpha\, m}(1+\alpha\, m)-1))}{(e^{-\alpha\, m}(2+\alpha\, m)-2)[t(e^{-\alpha\, m}(2+\alpha\, m)-2)-2c\hat{\alpha}^3 m^2]} > 0.$$

The inequality follows because as shown before both $e^{-\alpha\, m}(1+\alpha\, m)-1$ and $e^{-\alpha\, m}(2+\alpha\, m)-2$ are non-positive. □

All of the comparative statics here have the same sign as in Grossman and Shapiro (1984).

## References


Bagwell, K., 2007. The Economic Analysis of Advertising. *Handbook of Industrial Organization* 2, Chapter 28, 1701-1844.

Butters, G., 1977. Equilibrium distributions of sales and advertising prices. Review of Economic Studies 44, 465-491.

Christou, C. and N. Vettas, 2008, On informative advertising and product differentiation, International Journal of Industrial Organization, 26, 92-112.

Dixit, A., and J. Stiglitz, 1977, Monopolistic Competition and Optimum Product Diversity, The American Economic Review, 67(3), 297-308.

Fishman, A., Levy, N., 2015. Search Costs and Investment in Quality. Journal of Industrial Economics 63, 625-641.

Grossman, G.M., Shapiro, C., 1984. Informative Advertising with Differentiated Products. Review of Economic Studies, 51, 63-81.

Mankiw, N.G., Whinston, M.D., 1986, Free Entry and Social Inefficiency. RAND Journal of Economics 17, 48-58.

Salop, S.C., 1979. Monopolistic Competition with Outside Goods. Bell Journal of Economics 10, 141-156.

Shapiro, C., 1980, Advertising and Welfare: Comment, The Bell Journal of Economics, 12(2), 749-75 1.

Tirole, J, 1988, The Theory of Industrial Organization, MIT Press: Cambridge, MA.