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Delis, Manthos and Iosifidi, Maria

Montpellier Business School, Montpellier Business School

12 February 2019

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Manthos D. Delis

Montpellier Business School

Maria Iosifidi

Montpellier Business School

We are indebted to Shaqib Jafarey for various comments and suggestions. Previous versions of the paper were presented at the Conference on Research on Economic Theory and Econometrics, and in seminars at Athens University of Economics and Business, Montpellier Business School, Surrey Business School, and University of Glasgow. For further comments and suggestions, we are grateful to George Economides, Sarantis Kalyvitis, Iftekhar Hasan, Steven Ongena, Theodore Palivos, and Apostolis Philippopoulos. Both authors are at Montpellier Business School, 2300 Avenue des Moulins, 34080 Montpellier, France. E-mail addresses m.delis@montpellier-bs.com (Delis) and m.iosifidi@montpellier-bs.com (Iosifidi).

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Abstract

The rising environmental awareness induces a changing landscape for policymakers and real economic prospects. We examine the properties of a general equilibrium model with endogenous household preferences (for labor, consumption, and environmental quality) and a negative environmental externality. The endogeneity of labor creates an additional channel of substitution between environmental quality and labor, besides the channel of substitution between environmental quality and consumption. We show that a key requirement for improved output following a positive shock in the weight of environmental quality (household environmental awareness) is that environmental awareness trades off the weight on labor and not the weight on consumption. An interesting feature of the model is that the existence of the environmental externality gives a non-zero capital tax in the long run.

Keywords: Environmental awareness; Environmental quality; Labor; Consumption; Real outcomes

1. Introduction

What is the role of household environmental awareness on their consumption and labor decisions? What are the implications of these decisions for the real economy? The answer to these questions are important in light of increased pressure for environmental quality and environmental awareness. For example, 94% of European citizens say, as of 2017, that protecting the environment is important to them, 81% suggest agree that that environmental issues have a direct effect on their daily life and their health, and 87% suggest that they have a personal role to play (Special Eurobarometer, 2017). Similar results emerge from relevant questions in the six waves of the World Values Surveys. From a more practical viewpoint, the Fukushima nuclear disaster in 2011 strengthened considerably the share of people opposing the use of nuclear power (BBC, 2011). The German government decided to shut down all nuclear plants by 2022, despite the obvious impact of this decision on output and employment, especially given the surging economic turmoil in the European Union during the same period. Such decisions place inevitably the role of environmental awareness in a central position within the economic decisions of households, the related fiscal decisions of governments, and the end results for real economic outcomes.

We focus on the the weight economic agents place on environmental quality (henceforth environmental awareness) vis-a-vis the respective weights on labor and consumption. Our setup augments the Ramsey-type frameworks of Chamley (1986) and Judd (1985), henceforth Chamley-Judd, by adding an environmental externality to an economy in which labor, consumption, and environmental quality are determined endogenously.

The representative economy consists of a large number of identical infinitely-lived households, whose utility depends on private consumption, labor, and the stock of environmental quality. The households consume, save, and produce a single good. Output produced yields environmental pollution and this worsens environmental quality, which is assumed to be a public good. In other words, private agents do not internalize the effects of their actions on environmental quality. The decentralized equilibrium is inefficient and policy intervention is justified. A Ramsey-type planner (government) intervenes and chooses the best competitive equilibrium for this problem.

The main novelty of our model is that both the labor-leisure decision of households and their consumption are included in the consumer preferences. Indeed, to the best of our knowledge, there is no other study that examines the interplay between an environmental externality and labor-leisure decisions in a model similar to that of Chamley-Judd. Our novelty is important for two main reasons.

First, our analysis allows examining the response of labor supply to changes in the beliefs and attitudes of households with respect to environmental awareness. This is quite important in light of the developments in many countries against production activities that are particularly harmful for the environment. The response of many European countries to the Fukushima disaster and the response of multiple labor unions even from the 1960s further motivate our theoretical model, as they are suggestive of a reduced labor supply to environmentally harmful jobs. This is a key finding of the recent empirical literature (Iosifidi, 2016).

Second, the relation between labor supply and environmental awareness is possibly related to the employees' social status, i.e. the nature of the job is placing the employee in a particular cast. This idea is central in theories of social stratification and class at least since the times of Marx and Weber. In other words, as environmental awareness increases, the labor supply linked to environmentally harmful activities is lower because of the lower social status given to such production activities.

The Chamley-Judd result, which is particularly relevant for our analysis, states that in a steady state there should be no wedge between the intertemporal rate of substitution and the marginal rate of transformation, i.e. the optimal tax on capital is zero. In our framework, individuals face two types of trade-offs, one between consumption and environmental quality and another between labor-leisure and environmental quality. This is mostly observed in the real business cycle (RBC) literature, where labor is endogenous, and creates an additional choice for intratemporal substitution (see e.g., Kydland and Prescott, 1982; Plosser, 1989).

Our economy yields a unique steady state, which we shock to obtain the paths of our endogenous variables. We are mainly interested in the parameters characterizing the effect of household environmental awareness on environmental quality, as well as the overall effect on output and welfare. We find that an increase in environmental awareness always leads to higher environmental quality, irrespective of whether higher environmental awareness comes at the expense of less weight on consumption or labor. For the effect on output, the results are quite intriguing. We find that output (and consumption and labor) decreases when the increases in environmental awareness comes at the expense of the wight on consumption. In contrast, when environmental awareness increases at the expense of the weight on labor, output increases.

We also show that the capital tax is determined in equilibrium, among others, by the environmental parameter of our model related to pollution. More specifically, in the case where the pollution externality is zero, the capital tax is also zero and our result is identical to the Chamley-Judd result. In contrast, in the presence of a negative environmental externality, the tax on capital is positive and the Chamley-Judd result does not hold. We could say that in our model with an environmental externality, we obtain a second-order Chamley-Judd result, where the capital tax is always positive. This result is evident only in the literature imposing constraints on the government to impose taxes. In our model, the mere existence of an environmental externality where labor, consumption, and environmental quality are endogenous yields this result. The empirical implication of this result is that a positive

The rest of the paper is organized as follows. In the next section we provide the background literature implications on which we build our model. describe our model. In Section 3 we describe the economy. In Section 4 we solve for the decentralized competitive equilibrium, check for its stability, and compare our model with the equivalent model with endogenous labor using impulse responses and stylized facts. In Section 5 we solve for the planner's problem and check for its stability. Moreover, we compare our result with the Chamley-Judd result, offer some numerical examples, and illustrate the dynamic responses to permanent shocks in the parameters of interest. Section 6 concludes.

2. Background

Our work is related to a flourishing literature on growth and environmental quality. In this section, we aim to place our work within the most relevant macro literature. In a seminal contribution, Bovenberg and Smulders (1995) were among the first to explore the link between environmental quality and economic growth in an endogenous growth model that incorporates pollution-augmenting technological change. Their model includes environment as a renewable resource. In particular, they model how technological improvements enable production to occur with lower levels of pollution and with more effective use of renewable resources. They show that environmental quality and cleaner technology represent good reasons for policy intervention, as both have a public good character. Further, the revenues from pollution taxes (or pollution permits) exceed public expenditures on the development of pollution-enhancing technology and the optimal size of the government budget tends to increase when environmental awareness increases.

Several other papers consider general equilibrium frameworks with pollution taxation. Angelopoulos, Economides, and Philippopoulos (2013) rank different environmental policy instruments under uncertainty. Xepapadeas (2005) proposes relevant models to study the effects of environmental concerns on economic growth. The important assumptions in these models relate to the choice of emissions in an optimal way and to the devotion of resources to pollution abatement. Dioikitopoulos, Kalyvitis, and Vella (2015) study the allocation of tax revenues between infrastructure and environmental investment in a general-equilibrium growth model with endogenous subjective discounting. An interesting finding of this paper is that, when environmental awareness increases, it is optimal for governments to perform green spending reforms.

A common characteristic of these papers is that the utility function is independent from the labor/leisure decision of households. This is quite important in our view, given the movement over at least the last fifty years of households to demand higher environmental quality and relevant jobs (Special Eurobarometer, 2017; previous versions). This implies increased environmental awareness, which in turn can have a bearing on the labor/leisure decision of households, in addition to the consumption decisions that are studied in the literature. In fact, in the only empirical study on this issue, Iosifidi (2016) shows that the link between environmentally-aware households and their labor supply is tighter compared to the link between environmental awareness and consumption.

In the endogenous-growth literature with fiscal policy (but without environmental quality) many studies indeed treat labor supply as inelastic. This treatment limits certain aspects of fiscal policy (Turnovsky, 2000). More precisely, De Hek (2006) studies an endogenous growth model with physical capital and suggests that the flexibility of the labor supply induces agents to spend more or less time on leisure activities, depending on the relative sizes of the substitution and income effects. Flores and Graves (2008) argue that exogeneity of labor generally results in undervaluation of utility due to increases in the provision of a public good. Phrased differently, if the labor supply is exogenously fixed, the Le Chatelier-Samuelson principle holds. Intuitively, this follows from the fact that an increase in the cost of the public good will result in a higher marginal valuation of ordinary private goods, as their quantities are reduced to pay for the public good, and this in turn will result in a higher marginal cost of leisure.

Our model is also related to the literature predicting the existence of a zero capital tax rate in the long run. The seminal contributions in this literature are the studies by Chamley (1986) and Judd (1985). In similar Ramsey-type settings, these studies show that if an equilibrium has an asymptotic steady state, then the optimal policy is to set the capital tax rate equal to zero. In other words, any positive capital income tax does not help in any efficiency or redistributive goals in the steady state.

However, a more recent literature shows that the optimal factor taxation may involve positive tax rates on both capital and labor incomes (e.g., Correia, 1996; Stiglitz, 1987; Jones, Manuelli, and Rossi, 1997; Acemoglu, Golosov, and Tsyvinski, 2011). Debortoli and Gomes (2012), examine the case where the choice between capital vs. labor income taxation can be intrinsically related to the allocation of expenditure across different public goods. In their model, taxing profits constitutes a way to extract the private rents generated by public capital. As a result, corporate taxes are positive also in the long-run, as opposed to the optimality of zero capital taxation in Judd (1985) and Chamley (1986). Evidently, in all of these papers, a non-zero capital tax arises due to constraints on the government to impose taxes. In our model these constraints are not needed; the mere existence of an environmental externality where labor, consumption, and environmental quality are endogenous yields this result.

3. Description of the economy

We describe our basic framework, placing particular emphasis on the fact that, besides environmental quality, both labor and consumption decisions are endogenous in the individual's preferences and their weight in the utility function is proportional to the weight placed on environmental quality. Subsequently, we describe the decisions of firms, the laws of motion of natural resources, the resources constraint, and we close the model with the government budget constraint. We model a Ramsey-type economy, assuming intertemporal utility-maximizing households, and perfectly-competitive profit-maximizing firms (Beltratti, 1996; Xepapadeas, 2005).[1]

3.1. Households

We assume that the population size is constant and equal to one. The representative infinitely-lived household maximizes the intertemporal utility

$$\sum_{t=0}^{\infty} \beta^t U(c_t, lt, Q_t), \tag{1}$$

where c is the private consumption, l is leisure, Q is the stock of environmental quality,

[1] Instead, we could use an endogenous growth model. In this case, the endogenously-determined growth rates can remain positive if the productivity of capital does not approach zero in the long run or, in a model with human capital, the production of knowledge is characterized by decreasing returns (see e.g., Smulders, 2000). The results from such a model do not change our key implications.

and $\beta \in (0,1)$ is the time discount factor. The utility function has the form[2]:

$$U(c_t, l_t, Q_t) = \frac{[(c_t)^{\mu_1}(l_t)^{\mu_2}(Q_t)^{1-\mu_1-\mu_2}]^{1-\sigma}}{1-\sigma}, \tag{2}$$

where $\mu_1, \mu_2, \mu_3 = 1 - \mu_1 - \mu_2 \in (0,1)$ are preference parameters that assign weights to consumption, leisure, and environmental quality, respectively, and $\sigma \geq 0$ is the intertemporal elasticity of substitution. We define the weight on Q as "environmental awareness." The household is endowed with one unit of time that can be used for leisure l_t or labor n_t, thus $n_t + l_t = 1$. Each household can save in the form of capital k_t, receiving a rate of return r_t. Also, households supply labor services and receive labor income $w_t n_t$. Further, they receive dividends π_t. Each household has to pay a portion of its income to the government in the form of linear taxes. τ_t^k is the tax on capital income and τ_t^l is the tax on labor income. The flow budget constraint of the household is

$$k_{t+1} - (1-\delta^k)k_t + c_t = y_t = (1-\tau_t^l)w_t n_t + (1-\tau_t^k)r_t k_t + \pi_t, \tag{3}$$

where k_{t+1} is the end-of-period capital stock, k_t is the beginning-of-period capital stock, and $\delta^k \in$[0,1] is the rate of capital depreciation.

It follows that the household's problem is to

$$\max_{\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{[(c_t)^{\mu_1}(1-n_t)^{\mu_2}(Q_t)^{1-\mu_1-\mu_2}]^{1-\sigma}}{1-\sigma},$$
$$s.t. k_{t+1} - (1-\delta^k)k_t + c_t = (1-\tau_t^l)w_t n_t + (1-\tau_t^k)r_t k_t + \pi_t,$$

taking w_t, r_t, Q_t, and the policy as given. The problem expressed in a Langrangian form

[2]This is a Constant Relative Risk Aversion (CRRA) utility function which is broadly used by the relevant literature, as it is increasing and concave in consumption, labor, and environmental quality to ensure interior solutions (see Xepapadeas, 2005; Angelopoulos, Economides, and Philippopoulos, 2013; Dioikitopoulos, Kalyvitis, and Vella, 2015).

is given by:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{ \frac{[(c_t)^{\mu_1}(1-n_t)^{\mu_2}(Q_t)^{1-\mu_1-\mu_2}]^{1-\sigma}}{1-\sigma}$$
$$+ \lambda_t[(1-\tau_t^l)w_t n_t + (1-\tau_t^k)r_t k_t + (1-\delta^k)k_t + \pi_t - c_t - k_{t+1}]\}.$$

The FOCs for this problem with respect to c_t, n_t, and k_{t+1} respectively are

$$U_{c_t} = \lambda_t, \tag{4}$$
$$\frac{c_t}{1-n_t} = \frac{\mu_1}{\mu_2}(1-\tau_t^l)w_t, \tag{5}$$
$$U_{c_t} = \beta U_{c_{t+1}}[(1-\tau_{t+1}^k)r_{t+1} + 1 - \delta^k]. \tag{6}$$

The first equation gives the marginal utility of consumption and the second equation is the FOC with respect to labor. The last equation is the Euler equation for capital. It tells us that along an optimal path, the marginal utility from consumption at any point in time is equal to the opportunity cost of consumption. More specifically, the Euler equation says that, on the one hand, the household must be indifferent between consuming one more unit today and, on the other, saving that unit and consuming in the future. If the household consumes today, it gets the marginal utility of consumption today, i.e. the left-hand side of the equation, U_{c_t}. If, in contrast, the household saves that unit, it gets to consume $[(1-\tau_{t+1}^k)r_{t+1} + 1 - \delta^k]$ units in the future, each giving him $U_{c_{t+1}}$ extra units of utility. Because this utility comes in the future, it must be discounted by the weight β. That's the right side of the Euler equation. The fact that these two sides must be equal is what guarantees that the household is indifferent to consuming today versus in the future.

3.2. Firms

The production function of the representative firm is a neoclassical function with constant returns to scale of the form

$$y_t = Ak_t^a n_t^{1-a} = f(k_t, n_t), \tag{7}$$

where $a \in (0,1)$ is the output elasticity of private capital and $1-a \in (0,1)$ is the private elasticity of labor.[3] A is total factor productivity or the index of production technology, which is assumed to be constant. In each period, the representative firm takes w_t and r_t as given[4] and uses capital and labor services from households. The objective of the firm is to

$$\max_{\{l_t, k_{t+1}\}_{t=0}^{\infty}} \pi_t = y_t - w_t n_t - r_t k_t. \tag{8}$$

The FOCs for this problem are

$$r_t = a\frac{y_t}{k_t}, \tag{9}$$

$$w_t = (1-a)\frac{y_t}{n_t}, \tag{10}$$

so that $\pi = 0$.

3.3. Laws of motion of natural resources

The evolution of the stock of environmental quality is given by

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - p_t + \nu g, \tag{11}$$

where $\bar{Q} \geq 0$ is the environmental quality without pollution, p_t is the current pollution flow, and $\delta^q \in$[0,1] is the degree of environmental persistence. Moreover, g is the exogenous public spending that includes spending on abatement activities and $\nu \geq 0$ shows how

[3]In our model, pollution does not enter the production function. There is a large literature that introduces pollution in the production function by assuming that pollution or environmental quality affects amenities and productivity (Brock, 1973; Xepapadeas, 2005; Aznar and Ruiz-Tamarit, 2005).

[4]As firms are price takers, our model assumes perfect competition in the product market. This is a usual assumption in the relevant literature provided in section 2; however, one can alternatively assume an imperfectly competitive product market in the fashion of Bilbiie, Ghironi, and Melitz (2012). Models of imperfect competition are also very common in the micro literature involving environmental regulation (e.g., Fowlie, 2009).

public abatement spending is transformed into units of renewable resources. The flow of pollution is caused by the production of output and is given by

$$p_t = \phi A k_t^a n_t^{1-a}, \tag{12}$$

where ϕ is an index of pollution technology and reflects the emissions per unit of output.[5]Note that we assume a linear relation among economic activity, pollution, cleanup policy, and the change in natural resources (e.g., John and Pecchenino, 1994; Jouvet, Michel, and Rotillon, 2005).

3.4. Government budget constraint

The government collects revenues from the taxes on labor and capital.[6] On the expenditure side, it finances an exogenous stream of government purchases, $\{g_t\}_{t=0}^{\infty}$, that include spending on abatement policy. Assuming a balanced budget, we have

$$g_t = A k_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l]. \tag{13}$$

3.5. Resource constraint (technology)

Output can be consumed by households, used to increase the capital stock, and/or used by the government. Therefore, the resource constraint is

$$c_t + g_t + k_{t+1} = y_t + (1-\delta^k)k_t. \tag{14}$$

[5]We assume that the index of pollution technology is a parameter. Instead, we can assume that it depends on private or public investment in greener technology, or to follow a stochastic process. Our inferences do not change and we aim for the simplest approach.

[6]We could additionally assume that the model includes government debt. This would imply that on the expenditure side of the government budget constraint there would be a term associated with the reimbursement of the debt contracted in the previous period (the rate of debt times the level of the debt), while on the revenue side there would be a term associated with the financing of new debt. In addition, the household's budget constraint (3) will be formatted accordingly (Ljungqvist and Sargent, 2004).

4. Decentralized competitive equilibrium (DCE)

We solve the problem described in Section 3 for a Decentralized Competitive Equilibrium (DCE) in which (i) households maximize welfare, (ii) firms maximize profits, (iii) all constraints are satisfied and, (iv) all markets clear. The DCE of the above economy is given by the following equations:

$$\frac{c_t}{1-n_t} = \frac{\mu_1}{\mu_2}(1-\tau_t^l)w_t, \tag{15}$$

$$U_{c_t} = \beta U_{c_{t+1}}[(1-\tau_{t+1}^k)r_{t+1} + 1 - \delta^k], \tag{16}$$

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - \phi A k_t^a n_t^{1-a} + \nu g_t, \tag{17}$$

$$g_t = A k_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l], \tag{18}$$

$$c_t + k_{t+1} = A k_t^a n_t^{1-a} - g_t + (1-\delta^k)k_t. \tag{19}$$

This is a four-equation system in $\{c_t, n_t, Q_{t+1}, k_{t+1}\}_{t=0}^{\infty}$. The DCE holds for given initial conditions for the stock variables k_0 and Q_0, the FOCs of the representative firm's problem, the exogenous variables A and ϕ, for given policy (which is summarized by the tax rates τ^l, τ^k), and provided that $r_t = aAk_t^{a-1}n_t^{1-a}$, $w_t = (1-a)Ak_t^a n_t^{-a}$. Therefore, we have a system of five equations in $\{c_t, n_{t,} k_{t+1}, Q_{t+1}\}_{t=0}^{\infty}$.

We can obtain the long-run DCE if we simply drop the time subscripts:

$$\begin{aligned}
\frac{c}{1-n} &= \frac{\mu_1}{\mu_2}(1-\tau^l)(1-a)Ak^a n^{-a} \\
1 &= \beta[(1-\tau^k)aAk^{a-1}n^{1-a} + 1 - \delta^k] \\
0 &= (1-\delta^q)\bar{Q} - (1-\delta^q)Q - \varphi A k^a n^{1-a} + \nu g \\
c + \delta^k k &= Ak^a n^{1-a} - g.
\end{aligned}$$

4.1. Steady state

We solve the above system for c^*, n^*, Q^*, k^*, where the asterisk denotes the steady state value of each variable.

$$\begin{aligned}
\frac{c^*}{1-n^*} &= \frac{\mu_1}{\mu_2}(1-\tau^l)(1-a)A(k^*)^a(n^*)^{-a} \\
1 &= \beta[(1-\tau^k)aA(k^*)^a(n^*)^{1-a}+1-\delta^k] \\
0 &= (1-\delta^q)\bar{Q}-(1-\delta^q)Q^*-\varphi A(k^*)^a(n^*)^{1-a}+\nu g \\
c^*+\delta^k(k^*)^a &= A(k^*)^a(n^*)^{1-a}-g.
\end{aligned}$$

Therefore, we have that

$$c^* = \frac{\mu_1}{\mu_2}(1-\tau^l)(1-a)AX^{\frac{a}{a-1}}(1-X^{\frac{1}{1-a}}k^*), \tag{20}$$

$$n^* = X^{\frac{1}{1-a}}k^*, \tag{21}$$

$$Q^* = \bar{Q}-k^*\frac{AX}{(1-\delta^q)}[\phi-\nu a\tau^k-\nu(1-a)\tau^l], \tag{22}$$

where

$$k^* = \frac{\frac{\mu_1}{\mu_2}(1-a)AX^{\frac{a}{a-1}}(1-\tau^l)}{\delta^k-[\frac{\mu_1}{\mu_2}(1-a)+a(1-\tau^k)+(1-a)(1-\tau^l)]AX} \tag{23}$$

and

$$X = \frac{(1-\beta+\beta\delta^k)}{a\beta A(1-\tau^k)}. \tag{24}$$

4.2. Linearization

By substituting Eq. (18) in the rest of the equations of the DCE, the DCE becomes:

$$c_t+k_{t+1} = Ak_t^an_t^{1-a}-Ak_t^an_t^{1-a}[a\tau_t^k+(1-a)\tau_t^l]+(1-\delta^k)k_t, \tag{25}$$

$$\frac{c_t}{1-n_t} = \frac{\mu_1}{\mu_2}(1-\tau_t^l)(1-a)Ak_t^an_t^{-a}, \tag{26}$$

$$U_{c_t} = \beta U_{c_{t+1}}[(1-\tau_{t+1}^k)aAk_{t+1}^{a-1}n_{t+1}^{1-a}+1-\delta^k], \tag{27}$$

$$Q_{t+1} = (1-\delta^q)\bar{Q}+\delta^qQ_t-\phi Ak_t^an_t^{1-a}+\nu Ak_t^an_t^{1-a}[a\tau_t^k+(1-a)\tau_t^l]. \tag{28}$$

We linearize the system of Eqs. (25)-(28) around the steady state, using Taylor's theorem. We assume that the exogenous stream of government spending, $\{g_t\}_{t=0}^{\infty}$, and the exogenous tax rates, τ^l and τ^k, in the long run take the values from the respective Ramsey optimization problem. We find that the model is stable (for the proof, see Appendix A).

4.3. Impulse responses

We provide inferences based on impulse responses. Before moving to the planner's problem on optimal taxation, we shock the DCE to show the substitution effects between the weights on environmental awareness and the weights on labor/leisure and consumption. We take the parameter values from the literature (e.g., Economides and Philippopoulos, 2008; Angelopoulos, Economides, and Philippopoulos, 2013; King and Rebelo, 1999), which we report in Table 1. The value used for the capital share in production, α, is 0.33 and the annual depreciation rate of capital is 0.1 (equivalent to 0.025 on a quarterly basis). For the curvature parameter in utility function, σ (i.e., the intertemporal elasticity of substitution), we use a value equal to 2. There is considerable uncertainty regarding the true value of σ, with Hansen and Singleton (1983) estimating it to be between 0 and 2. Our results are qualitatively the same when using different values in that range. The time discount factor is set equal to 0.97, a value obtained by setting the long-term government bond yield, r_b, equal to 0.03, which is the approximate value for the U.S. economy at the end of 2013. Next, we obtain β from the formula $r_b = (1-\beta)/\beta$ and set the long-run total factor productivity, A, equal to 1 (e.g., King and Rebelo, 1999).

Regarding the parameters in the motion for environmental quality, we choose a relatively high-persistence parameter, $\delta^q = 0.9$, and normalize the level of environmental quality without economic activity, $\bar{Q}$, to equal 1 (e.g., Angelopoulos, Economides, and Philippopoulos, 2013). Using a much lower value equal to 0.15 (Dioikitopoulos, Kalyvitis, and Vella, 2015) the model produces qualitatively similar results. Moreover, we set $\phi = 0.5$. Based on OECD statistics, the CO_2 emissions (kg per PPP\$ of GDP) are equal to 0.4 for the U.S. economy in the period 2009-2013. Given that this concerns only the CO_2 emissions and not other emissions, we believe that our value off 0.5 is quite realistic.

We assume that $\nu[\alpha\tau^k + (1-\alpha)\tau^l] - \phi < 0$, which is a non-trivial solution area (when $\nu[\alpha\tau^k + (1-\alpha)\tau^l] - \phi < 0$, we have a "too good to be true" economy in the sense that effective cleanup policy, $\nu[\alpha\tau^k + (1-\alpha)\tau^l]$, is stronger than the polluting effect of production, ϕ). We study various values for ν, which reflect different levels of public sector efficiency with respect to abatement policy. For example, we set $\nu = 0.7$, 0.75, and 1, and the results are qualitatively the same. Finally, we assume that the weight on environmental quality is equivalent to that of the previous literature on public goods (e.g., Debortoli and Gomez, 2012) and equal to 0.4, while we give an equal weight of 0.3 to consumption and leisure. We carry out an extensive sensitivity analysis in this respect, with results being qualitatively similar.

To see how the endogeneity of labor affects the equilibrium results, in unreported graphs we compare the responses due to permanent unitary changes in the weights of the variables in the utility function for the models with exogenous and endogenous labor. For the model with exogenous labor, where labor is set equal to 1, there are two variables in the utility function and two respective weights, one on consumption and one on environmental quality. Under the assumption of constant returns to scale, a permanent unitary increase (decrease) in the weight on environmental quality results to an equivalent decrease (increase) in the weight on consumption. This has a permanent positive (negative) effect only on welfare because this weight is not included in the steady state equations characterizing the rest of the variables. The rest of the parameters of our model, i.e. ϕ, ν, σ, and β, affect all the endogenous variables.

By introducing endogenous labor in the utility function an extra channel of substitution is created between environmental quality and the leisure-labor decision. This allows studying the impact of changes in the respective weights on households' decision variables for both consumption-environmental awareness and labor-environmental awareness. Given that these weights now affect all endogenous variables in our model, we can examine the relevant impulse responses. We take all three possible combinations when consumption and leisure are substitutes (i.e., an increase in the one variable decreases the other). Initially, all variables are at their steady-state levels.

Figures 1 to 3 show how the DCE reacts to a (i) 1% increase in the weight on environmental quality with a simultaneous 1% decrease in the weight on consumption (labor remains unchanged), (ii) 1% increase in the weight on environmental quality with a simultaneous 1% decrease in the weight on labor-leisure (consumption remains unchanged), and (iii) 1% increase in the weight on consumption with a simultaneous 1% decrease in labor-leisure (environmental awareness remains unchanged).

We find that an increase in the weight on environmental quality (environmental awareness) with a relative decrease in the weight on consumption (leisure-labor decision), keeping the third weight steady, leads to a higher (lower) environmental quality and lower (higher) output (Figures 1 and 2, respectively). In the case where we change the weights on consumption and leisure-labor decision in opposite directions, all endogenous variables increase except from environmental quality and welfare. Given these baseline findings, we move to the optimal tax problem, where the economy moves to a better state given policy action.

5. Optimal tax with an environmental externality

There are many competitive equilibria indexed by different government policies and the planner's problem is to choose the one that maximizes

$$\sum_{t=0}^{\infty} \beta^t \frac{[(c_t)^{\mu_1}(1-n_t)^{\mu_2}(Q_t)^{1-\mu_1-\mu_2}]^{1-\sigma}}{1-\sigma},$$

subject to the DCE. Therefore, the planner chooses the best competitive equilibrium, taking as given $\{g_t\}_{t=0}^{\infty}$, k_0, Q_0, and bounds on taxes, i.e. $0 \leq \tau_t^k < 1$ and $0 \leq \tau_t^l < 1$. Moreover, the period zero tax rates, $0 \leq \tau_0^k < 1$ and $0 \leq \tau_0^l < 1$ are also taken as given, otherwise the government would be able to impose lump-sum taxes, which would make the policy problem first-best.

Optimal taxation provides a compelling argument against taxing capital income in the long run in dynamic macroeconomic models. Following Chamley (1986), we replace

r_t and w_t with net factor prices $\widetilde{r}_t$ and $\widetilde{w}_t$, where

$$\widetilde{r}_t = (1 - \tau_t^k) r_t, \tag{29}$$

$$\widetilde{w}_t = (1 - \tau_t^l) w_t. \tag{30}$$

In this way, the four instruments $\tau_t^k, \tau_t^l, r_t, w_t$ reduce to two.[7] Thus, the DCE is given by

$$\frac{c_t}{1 - n_t} = \frac{\mu_1}{\mu_2} \widetilde{w}_t, \tag{31}$$

$$U_{c_t} = \beta U_{c_{t+1}} (\widetilde{r}_{t+1} + 1 - \delta^k), \tag{32}$$

$$Q_{t+1} = (1 - \delta^q)\bar{Q} + \delta^q Q_t - \phi A k_t^a n_t^{1-a} + \nu g, \tag{33}$$

$$g = A k_t^a n_t^{1-a} - \widetilde{w}_t n_t - \widetilde{r}_t k_t, \tag{34}$$

$$c_t + k_{t+1} - (1 - \delta^k) k_t + g = A k_t^a n_t^{1-a}. \tag{35}$$

The planner's problem in Langrangian form becomes

$$\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{ & U(c_t, n_t, Q_t) \\
& + \lambda_t (\frac{\mu_1}{\mu_2} \widetilde{w}_t - \frac{c_t}{1 - n_t}) \\
& + \psi_t [\beta U_{c_{t+1}} [\widetilde{r}_{t+1} + 1 - \delta^k] - U_{c_t}] \\
& + \zeta_t [(1 - \delta^q)\bar{Q} + \delta^q Q_t - \varphi A k_t^a n_t^{1-a} + \nu g - Q_{t+1}] \\
& + \xi_t (A k_t^a n_t^{1-a} - \widetilde{w}_t n_t + \widetilde{r}_t k_t - g) \\
& + \chi_t [A k_t^a n_t^{1-a} - c_t - k_{t+1} + (1 - \delta^k) k_t - g] \},
\end{aligned}$$

where $\{\lambda_t, \psi_t, \zeta_t, \xi_t, \chi_t\}_{t=0}^{\infty}$ are sequences of Langrange multipliers (or the the shadow prices associated with the household's first order condition with respect to capital), the Euler equation, government budget constraint, household budget constraint, and law of motion of environmental quality, respectively. The FOCs of this problem with respect to c_t, n_t,

[7] This approach, where tax rates are the government decision variables, is known as the dual approach. The primal approach would be to do the exact opposite, i.e. eliminate all prices and taxes so that the government could use quantities as controls (Jones, Manuelli, and Rossi, 1997). Both approaches yield the same results for policies and allocations (Economides, Philippopoulos, and Vassilatos, 2008).

Q_{t+1}, k_{t+1}, $\tilde{r}_t$, $\tilde{w}_t$, λ_t, ψ_t, ζ_t, ξ_t, and χ_t are

$$U_{c_t} = \frac{1}{1-n_t}\lambda_t + \chi_t - \partial U_{c_t}/\partial c_t[\psi_{t-1}(\widetilde{r}_t + 1 - \delta) - \psi_t], \tag{36}$$

$$U_{n_t} = \frac{c_t}{(1-n_t)^2}\lambda_t - (1-a)Ak_t^a n_t^{1-a}(\xi_t - \zeta_t\phi + \chi_t) \tag{37}$$
$$+\xi_t\widetilde{w}_t + \partial(U_{c_t}/\mu_1)/\partial n_t[\psi_t - \psi_{t-1}(\widetilde{r}_t + 1 - \delta)],$$

$$U_{Q_t}[\psi_t(\widetilde{r}_{t+1} + 1 - \delta^k) - \psi_{t+1}] = \frac{\zeta_t}{\beta} - U_{Q_{t+1}} - \zeta_{t+1}\delta^q, \tag{38}$$

$$\chi_t = \beta[\chi_{t+1}(f_{k_{t+1}} + 1 - \delta^k) + \xi_{t+1}(f_{k_{t+1}} - \widetilde{r}_{t+1}) - \zeta_{t+1}\phi f_k], \tag{39}$$

$$\xi_t k_t = \psi_{t-1}U_{c_t}, \tag{40}$$

$$\lambda_t\frac{\mu_1}{\mu_2} = \xi_t n_t, \tag{41}$$

$$\frac{\mu_1}{\mu_2}\widetilde{w}_t = \frac{c_t}{(1-n_t)}, \tag{42}$$

$$U_{c_t} = \beta U_{c_{t+1}}[\widetilde{r}_{t+1} + 1 - \delta^k], \tag{43}$$

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - \phi Ak_t^a n_t^{1-a} + \nu g_t, \tag{44}$$

$$Ak_t^a n_t^{1-a} - \widetilde{w}_t n_t - \widetilde{r}_t k_t = g_t, \tag{45}$$

$$c_t + k_{t+1} = Ak_t^a n_t^{1-a} + (1-\delta^k)k_t - g_{t.} \tag{46}$$

Some considerations are in order. Eq. (39), the Euler equation, tells us that a marginal increase of capital investment in period t increases the amount of available goods in period $t+1$ by $(f_k + 1 - \delta)$, with social marginal value χ_{t+1}. Moreover, tax revenues increase by $(f_k - \widetilde{r}_{t+1})$, which enables the government to reduce its debt on other taxes by the same amount. This increase has a social marginal value equal to ξ_{t+1}, which is interpreted as the extra burden imposed to the society due to the existence of distortionary taxation. β is the discount factor in period $t+1$ and χ_t is the social marginal value of the investment good in period t. Therefore, χ_t and ξ_t are positive for all t. Finally, the increase of capital investment worsens environmental quality by ϕf_k, with social marginal value ζ_{t+1}.

We obtain the long-run conditions by dropping the time subscripts. To simplify the FOCs, we set $\sigma = 1$ in the utility function $U(c_t, l_t, Q_t)$, which then limits to

$$U(c_t, l_t, Q_t) = \mu_1 \ln(c_t) + \mu_2 \ln(l_t) + (1 - \mu_1 - \mu_2) \ln(Q_t). \tag{47}$$

As we did with the DCE, we linearize the system of Eqs. (36)-(46) around the steady state using Taylor's Theorem. We use the same values for the parameters and we find that the model is stable (for details see Appendix B). Once again, there is a unique equilibrium and the economy converges to this through a saddle path.

5.1. The Chamley-Judd approach to the planner's problem

Eq. (39) reduces in the long run to

$$\beta[(r - \widetilde{r})\xi + (r + 1 - \delta)\chi - r\phi\zeta] = \chi. \tag{48}$$

From Eq. (43), it holds in the long run that $(1 - \delta) = \frac{1}{\beta} - \widetilde{r}$. By replacing this result into (48) and rearranging we have

$$(r - \widetilde{r})(\chi + \xi) - r\phi\zeta = 0. \tag{49}$$

We now consider two cases, where $\phi = 0$ and $\phi \neq 0$. In the first case, the environmental externality is zero and Eq. (49) becomes

$$\tau^k(\chi + \xi) = 0. \tag{50}$$

The marginal social value of goods χ is strictly positive and the marginal social value of reducing government taxes ξ is nonnegative; therefore, r must equal to $\widetilde{r}$, so that τ^k is equal to zero. This is the result of the papers by Chamley (1986) and Judd (1985).

We can see this result using a simple numerical example. In Table 1 we provide the parameter values (same as in the DCE shocks) and in Column 1 of Table 2 the results. The findings show that $\tau^k = 0$ and the discounted welfare for $t = 100$ is

$$U^*(c,n,Q) = \frac{(1-\beta^t)}{(1-\beta)}U(c,n,Q) = \frac{(1-\beta^{100})}{(1-\beta)}\frac{\left(c^{\mu_1}(1-n)^{\mu_2}Q^{(1-\mu_1-\mu_2)}\right)^{(1-\sigma)}}{(1-\sigma)}$$
$$= -53.76943282$$

In the case, where $\phi \neq 0$, the first term of Eq. (49) is exactly the same with the Chamley-Judd result. The second term of Eq. (49) appears because of the positive environmental externality. By substituting $\widetilde{r}$ with $r(1-\tau^k)$ and by rearranging the terms we have that

$$\tau^k = \frac{\phi\zeta}{\chi+\xi}, \tag{51}$$

which is always positive. It must hold that $\tau^k < 1 \Leftrightarrow \frac{\phi\zeta}{\chi+\xi} < 1$, or $\phi\zeta < \chi + \xi$.[8]

In Column 2 of Table 2 we provide the results from the numerical example where ϕ is positive and equal to 0.5. The values of the parameters are as before. Evidently, τ^k is positive and discounted welfare in this case for $t = 100$ is given by

$$U^*(c,n,Q) = \frac{(1-\beta^t)}{(1-\beta)}U(c,n,Q) = \frac{(1-\beta^{100})}{(1-\beta)}\frac{\left(c^{\mu_1}(1-n)^{\mu_2}Q^{(1-\mu_1-\mu_2)}\right)^{(1-\sigma)}}{(1-\sigma)}$$
$$= -86.12491269,$$

The presence of the environmental externality worsens environmental quality. Taxes increase and this leads to a lower level of utility, compared to the case where the environmental externality is equal to zero.

In our model with an environmental externality, taxing capital constitutes a way for the government to extract revenues generated by a polluting activity and use these revenues for abatement policy to improve the public good, i.e. the environmental quality. Thus, we obtain a second-order Chamley-Judd result, where the capital tax is always positive.

[8]This result remains the same even if we assume that the weight on environmental quality in the utility function of the agents is equal to zero.

5.2. Impulse response functions and stylized facts

In this section, we illustrate the dynamic response of the economy to permanent unitary increases in certain parameters of our model. We begin by the equivalent shocks to the ones we present for the DCE in Section 4.3. Moreover, we study the responses due to a 1% increase in the weight on the pollution parameter ϕ and a 1% increase in the abatement technology ν.

Figure 4 shows how the economy responds to a 1% increase in environmental awareness with a simultaneous 1% decrease in the weight on consumption. We observe that, in the long-run, output, consumption, and labor decrease. Therefore, there is a channel of substitution running from consumption to environmental quality. Further, to finance the exogenous government spending, there is an increase in the labor tax.

In turn, Figure 5 shows how the economy responds to a 1% increase in environmental awareness with a simultaneous 1% decrease in the weight on labor-leisure. We observe that, in the long-run, consumption falls and labor increases. Importantly, and in contrast with Figure 4, capital and output increase along with environmental quality.

Two main results become apparent from these exercises. First, as expected, an increase in the households' environmental awareness always leads to a higher environmental quality, reflecting on the actions of the government. Second, comparing inferences from Figures 4 and 5, we note (as in the DCE) that the substitution between environmental awareness and consumption lowers output (Figure 4), whereas the substitution between environmental awareness and labor increases output (Figure 5). In other words, the economy is better off when the rising environmental awareness goes as far as mitigating households labor supply toward polluting activities and related jobs. This provides further evidence that endogenizing labor decisions is important to study the effect of environmental awareness on households' decisions and real economic outcomes. Our results are in line with the empirical findings of Iosifidi (2016), who shows that the environmental awareness-labor supply nexus is stronger compared to the environmental awareness-consumption nexus.

An interesting issue related to environmental awareness is the case in which this leads to an improvement in the abatement technology ν. A positive 1% shock to this parameter implies that public abatement spending is transformed more effectively into units of renewable resources. Based on the results presented in Figure 6, households consume more, but labor remains approximately constant. The production of the polluting output increases, but the improvement of abatement technology completely offsets this negative effect, without raising the capital tax. Environmental quality increases and the economy moves to a higher-welfare steady state.

6. Conclusions

This paper studies a dynamic general equilibrium model with an environmental externality and optimal taxation. In our model the households decide between consumption, labor, and environmental quality. Thus, there are two channels of substitution for environmental quality: that of consumption (as in previous literature) and that of labor-leisure. We posit that this distinction is important given empirical facts showing a strong effect of environmental awareness on labor supply.

Our model predicts that an increase in households' environmental awareness improves environmental quality and the same also holds when environmental awareness remains constant and the weight on consumption increases at the expense of the weight on labor. Importantly, an increase in environmental awareness yields lower output when it is accompanied by a decrease in the weight on consumption, *ceteris paribus*. In contrast, an increase in environmental awareness has the exact opposite effect on output when it is accompanied by a decrease in the weight on labor, *ceteris paribus*. Phrased differently, an increase in environmental awareness can have both positive or negative effects on output based on whether it trades off labor (positive effect) or consumption (negative effect). This finding is consistent with recent evidence on the effect of environmental awareness on labor supply and related government actions to improve environmental quality via lower polluting units (and jobs).

We also find that the optimal capital tax in the long run is non-zero. This happens because capital tax constitutes a way for the benevolent Ramsey planner to extract revenues generated by a polluting activity and use these revenues for abatement policy to improve environmental quality. As pollution decreases, the government reduces the capital tax rate to extract a smaller fraction of the rents. When the pollution externality is zero, the model is equivalent to the standard model of optimal dynamic taxation. In that case, there are no capital rents produced by polluting activities, so the optimal steady-state capital tax rate is zero. Thus, our model yields a second-order Chamley-Judd result, where the capital tax is always positive.

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Table 1

Parameter values for the numerical example

Parameter	Description	Value
a	Capital share in production	0.33
δ^k	Capital depreciation rate	0.1
σ	Curvature parameter in utility function	2
β	Time discount factor	0.97
μ_1	Consumption weight in utility function	0.3
μ_2	Leisure weight in the utility function	0.3
$\bar{Q}$	Environmental quality without pollution	1
δ^q	Persistence of environmental quality	0.9
A	Long-run total factor productivity	1
ϕ	Long-run pollution externality	0.5
ν	Transformation of spending into units of nature	0.75

Table 2

Long-run values when

Variable name	1 ($\phi = 0$)	2 ($\phi = 0.5$)
c	0.187	0.081
n	0.288	0.312
Q	2.146	0.928
k	1.146	0.241
τ^k	0.000	0.664
τ^l	0.502	0.617
λ	0.507	$0.337*10^{-2}$
ψ	0.160	$0.165*10^{-3}$
ζ	1.636	6.521
ξ	0.878	$0.712*10^{-2}$
χ	0.349	5.328

Figure 1: Response of the DCE with endogenous labor to an increase in μ_3, decrease in μ_1, with steady μ_2

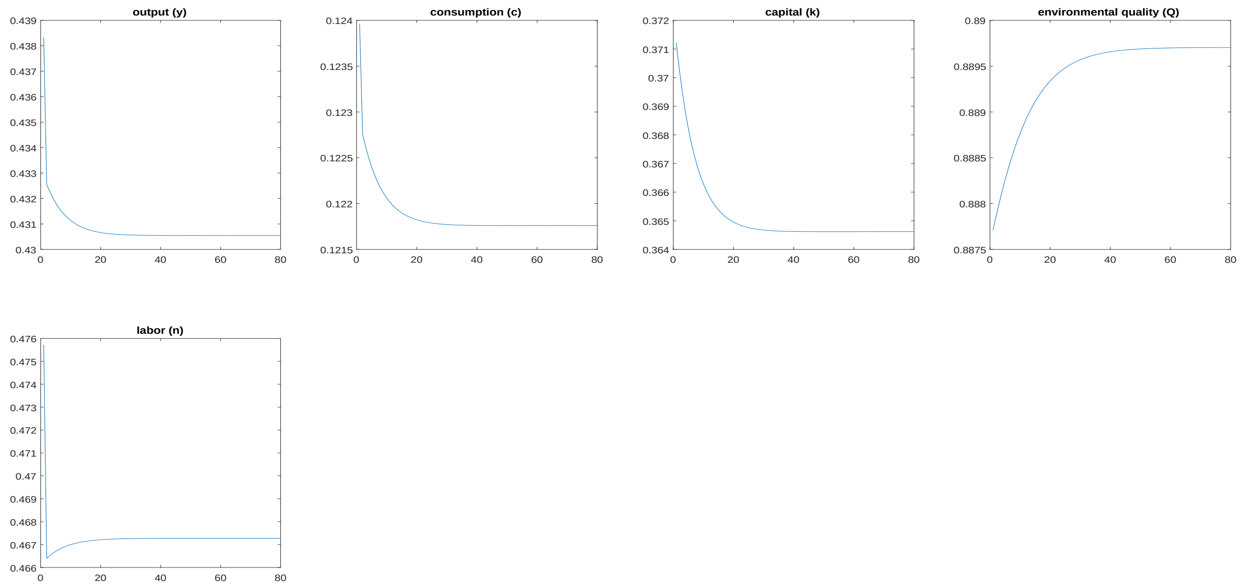


Figure 2: Response of the DCE with endogenous labor to an increase in μ_3, decrease in μ_2, with steady μ_1

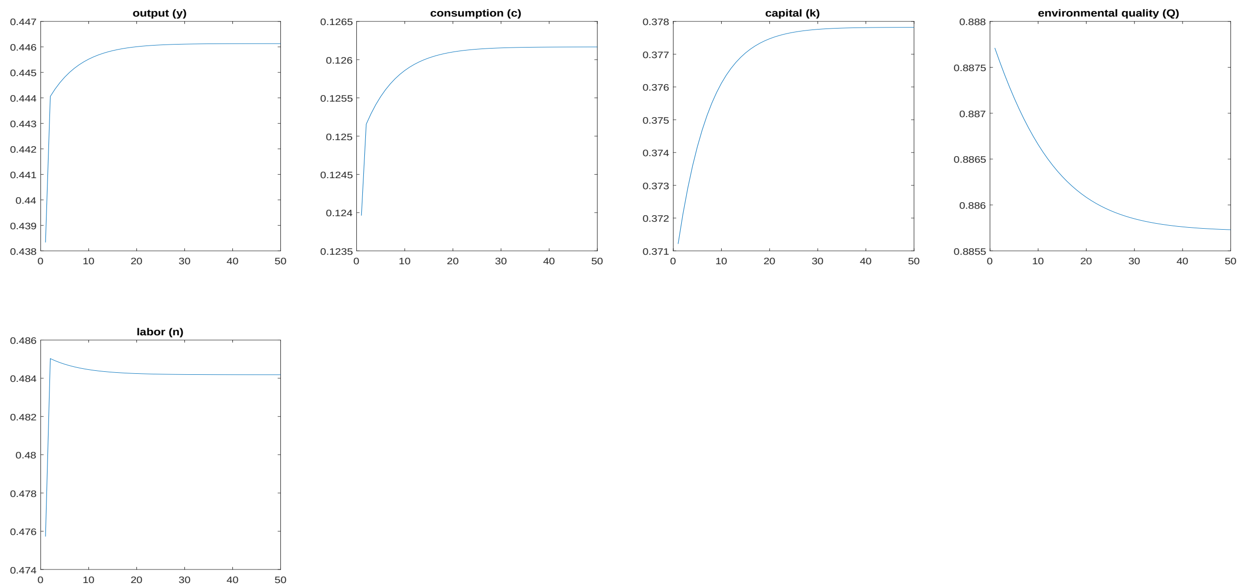


Figure 3: Response of the DCE with endogenous labor to an increase in μ_1, decrease in μ_2, with steady μ_3

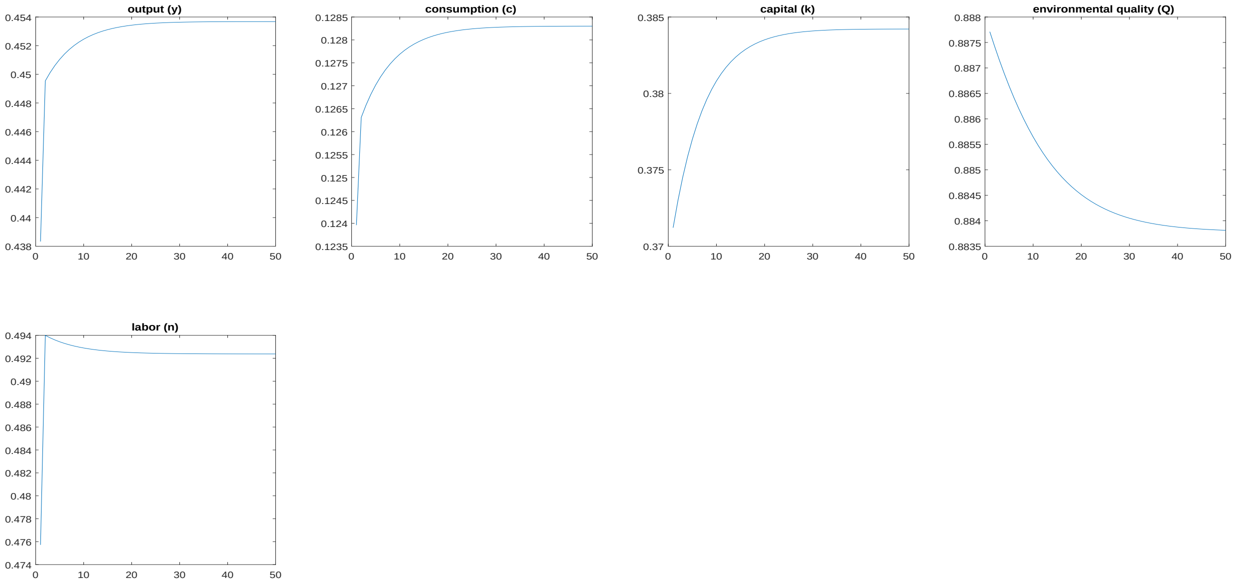


Figure 4: Response of the economy to an increase in μ_3, decrease in μ_1, with steady μ_2

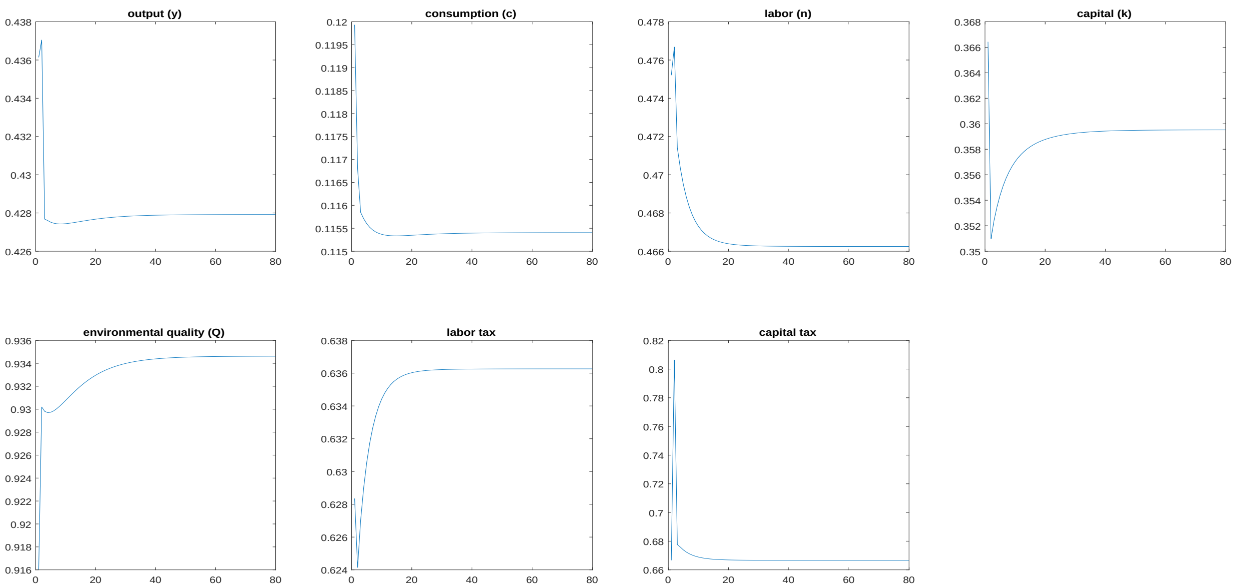


Figure 5: Response of the economy to an increase in μ_3, decrease in μ_2, with steady μ_1

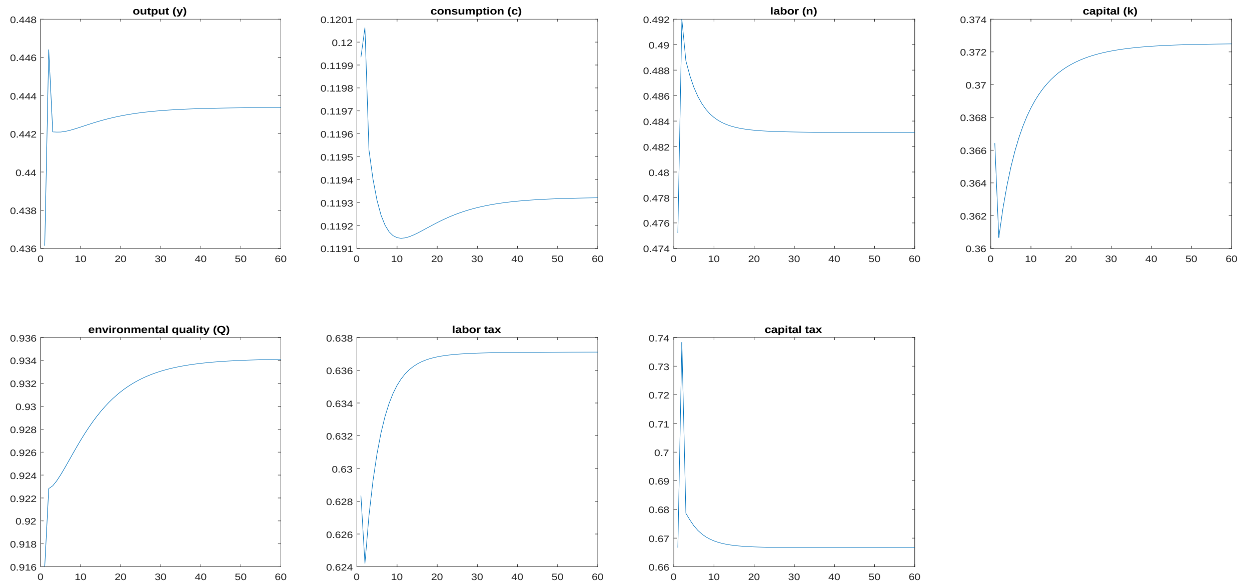
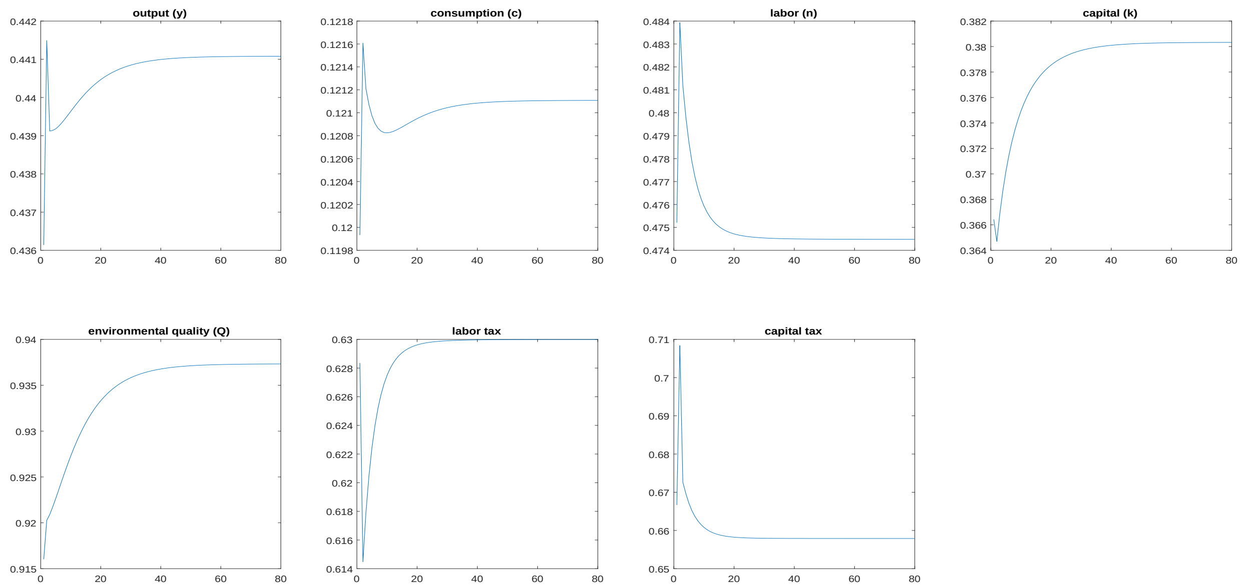


Figure 6: Response of the economy to 1% increase in the abatement technology



Appendix A: Linearization of the DCE

Eq. (26) becomes

$$f(c_t, k_{t+1}, k_t, n_t) = \hat{c}_t + \hat{k}_{t+1} + f_a \hat{n}_t + f_b \hat{k}_t = 0, \tag{A1}$$

where for any variable x of the system it holds that $\hat{x}_t = x_t - x^*$, with x^* being the steady state value of the variable and

$$f_a = f_{n_t}(\cdot) = [-A(1-a)(k^*)^a (n^*)^{-a}[1 - a\tau^k - (1-a)\tau^l]], \tag{A2}$$

$$f_b = f_{k_t}(\cdot) = [-[aA(k^*)^{a-1}(n^*)^{1-a}[1 - a\tau^k - (1-a)\tau^l] + (1-\delta^k)]]. \tag{A3}$$

Eq. (27) becomes

$$g(c_t, k_t, n_t) = \mu_2 \hat{c}_t + g_a \hat{n}_t + g_b \hat{k}_t = 0, \tag{A4}$$

where

$$g_a = g_{n_t}(\cdot) = [\mu_1 a A(k^*)^a (n^*)^{-a-1} + \mu_1 (1-a) A(k^*)^a (n^*)^{-a}](1-\tau^l)(1-a), \tag{A5}$$

$$g_b = g_{k_t}(\cdot) = [-\mu_1 (1-n^*)(1-a) a A(k^*)^{a-1} (n^*)^{-a} (1-\tau^l)]. \tag{A6}$$

Eq. (28) becomes

$$\begin{aligned} h(c_{t+1}, n_{t+1}, Q_{t+1}, k_{t+1}, c_t, n_t, Q_t) &= h_a \hat{c}_{t+1} + h_b \hat{n}_{t+1} + h_c \hat{Q}_{t+1} \\ &+ h_d \hat{k}_{t+1} + h_e \hat{c}_t + h_f \hat{n}_t + h_g \hat{Q}_t = 0, \end{aligned} \tag{A7}$$

where

$$h_a = h_{c_{t+1}}(\cdot) = -[\beta[\mu_1(1-\sigma)-1](c^*)^{\mu_1(1-\sigma)-2}[(1-n^*)^{\mu_2}(Q^*)^{1-\mu_1-\mu_2}]^{1-\sigma}] \tag{A8}$$
$$[(1-\tau^k)aA(k^*)^{a-1}(n^*)^{1-a}+1-\delta^k],$$

$$h_b = h_{n_{t+1}}(\cdot) = [\beta\mu_2(1-\sigma)(c^*)^{\mu_1(1-\sigma)-1}(1-n^*)^{\mu_2(1-\sigma)-1}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)} \tag{A9}$$
$$[(1-\tau^k)aA(k^*)^{a-1}(n^*)^{1-a}+1-\delta^k]-\beta(1-a)(c^*)^{\mu_1(1-\sigma)-1}$$
$$[(1-n^*)^{\mu_2}(Q^*)^{1-\mu_1-\mu_2}]^{1-\sigma}[(1-\tau^k)aA(k^*)^{a-1}(n^*)^{-a}],$$

$$h_c = h_{Q_{t+1}}(\cdot) = -[\beta(1-\mu_1-\mu_2)(1-\sigma)(c^*)^{\mu_1(1-\sigma)-1} \tag{A10}$$
$$(1-n^*)^{\mu_2(1-\sigma)}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)-1}][(1-\tau^k)aA(k^*)^{a-1}(n^*)^{1-a}+1-\delta^k],$$

$$h_d = h_{k_{t+1}}(\cdot) = [-\beta(c^*)^{\mu_1(1-\sigma)-1}(1-n^*)^{\mu_2(1-\sigma)}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)} \tag{A11}$$
$$(1-\tau^k)a(a-1)A(k^*)^{a-2}(n^*)^{1-a}],$$

$$h_e = h_{c_t}(\cdot) = [[\mu_1(1-\sigma)-1](c^*)^{\mu_1(1-\sigma)-2}[(1-n^*)^{\mu_2}(Q^*)^{1-\mu_1-\mu_2}]^{1-\sigma}, \tag{A12}$$

$$h_f = h_{n_t}(\cdot) = -[\mu_2(1-\sigma)(c^*)^{\mu_1(1-\sigma)-1}(1-n^*)^{\mu_2(1-\sigma)-1}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)}], \tag{A13}$$

$$h_g = h_{Q_t}(\cdot) = [(1-\mu_1-\mu_2)(1-\sigma)(c^*)^{\mu_1(1-\sigma)-1}(1-n^*)^{\mu_2(1-\sigma)}(Q^*)^{(1-\mu_1-\mu_2)(1-\sigma)-1}]. \tag{A14}$$

Finally Eq. (29) becomes

$$m(Q_{t+1}, n_t, Q_t, k_t) = \delta^q\hat{Q}_t + m_a\hat{n}_t + m_b\hat{k}_t - \hat{Q}_{t+1} = 0, \tag{A15}$$

where

$$m_a = m_{n_t}(\cdot) = -A(1-a)(k^*)^a(n^*)^{-a}[v[a\tau^k+(1-a)\tau^l]-\phi], \tag{A16}$$

$$m_b = m_{k_t}(\cdot) = -aA(k^*)^{a-1}(n^*)^{1-a}[v[a\tau^k+(1-a)\tau^l]-\phi]. \tag{A17}$$

The 4 by 4 system in matrix notation is

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -h_a & -h_b & -h_d & -h_c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{c}_{t+1} \\ \hat{n}_{t+1} \\ \hat{k}_{t+1} \\ \hat{Q}_{t+1} \end{bmatrix} = \begin{bmatrix} f_a & 1 & f_b & 0 \\ \mu_2 & g_a & g_b & 0 \\ h_f & h_e & 0 & h_g \\ m_a & 0 & m_b & \delta^q \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{n}_t \\ \hat{k}_t \\ \hat{Q}_t \end{bmatrix} \Longleftrightarrow A\hat{X}_{t+1} = B\hat{X}_t.$$

One way to check the stability of equilibrium is with the approach of Blanchard and Kahn (1980). We observe that the second equation is a static equation. We substitute this equation into the other three equations of the system and the system becomes

$$\begin{bmatrix} 0 & 1 & 0 \\ h_1 & h_2 & h_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{Q}_{t+1} \end{bmatrix} = \begin{bmatrix} f_1 & f_2 & 0 \\ h_3 & h_4 & -h_g \\ m_1 & m_2 & \delta^q \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{Q}_t \end{bmatrix} \Leftrightarrow E\hat{X}_{t+1} = F\hat{X}_t \Leftrightarrow \hat{X}_{t+1} = FE^{-1}\hat{X}_t \Leftrightarrow \hat{X}_{t+1} = C\hat{X}_t.$$

Using the parameter values in the paper of Angelopoulos, Economides, and Philippopoulos (2013), we find that there are two eigenvalues with module smaller than 1 for the backward looking variables $\hat{k}_t$ and $\hat{Q}_t$, and one eigenvalue with module larger than 1 for the forward looking variable $\hat{c}_t$. When we solve the 4 by 4 system using Dynare we find that the eigenvalue of n_t, which is a forward looking variable too, has module larger than 1. The Blanchard-Kahn conditions are satisfied and the model is stable. The steady state of the system is a saddle path, therefore it has a unique equilibrium.

Given that in the initial 4 by 4 system the matrix A is singular, we can also check its stability using the approach of Klein (2000). We first recover the generalized Schur decomposition of (A, B). We get the matrices of complex numbers Q and Z, such that $S = QAZ$ and $T = QBZ$ are upper triangular, and $QQ' = ZZ' = I$. Then the dynamics equation can be rewritten as

$$AZZ'X_{t+1} = BZZ'X_t. \tag{A18}$$

Let us define $\varpi_t = Z'X_t$ to get

$$AZ\varpi_{t+1} = BZ\varpi_t \tag{A19}$$

and pre-multiply both sides by Q

$$QAZ\varpi_{t+1} = QBZ\varpi_t, \tag{A20}$$

which is equal to

$$S\varpi_{t+1} = T\varpi_t. \tag{A21}$$

$\frac{T_{ii}}{S_{ii}}$ are the generalized eigenvalues of the system. We find that we have two stable eigenvalues with modulus below unity, which are associated with the variables k_t and Q_t, and two unstable eigenvalues with modulus greater than unity, which are associated with the variables c_t and n_t. Therefore, the model is stable, the steady state of the system is a saddle path and it has a unique equilibrium.

Appendix B: Linearization of the optimal tax model

For

$$U(c_t, l_t, Q_t) = \mu_1 \ln(c_t) + \mu_2 \ln(l_t) + (1 - \mu_1 - \mu_2) \ln(Q_t), \tag{B1}$$

the FOCs of the Ramsey problem become:

$$\mu_1 c_t - \frac{c_t^2 \lambda_t}{1-n_t} + \beta\psi_t \frac{\mu_1}{c_t^2} - c_t^2\chi_t - \psi_{t-1}\frac{\mu_1}{c_t^2}[(1-\tau_t^k)Ak_t^a n_t^{1-a} + 1 - \delta^k] = 0, \tag{B2}$$

$$\begin{gathered}(1-a)Ak_t^a n_t^{-a}[\chi_t - \zeta_t\phi + \xi_t[a\tau_t^k + (1-a)\tau_t^l]] \\ +\psi_{t-1}\frac{\mu_1}{c_t}(1-\tau_t^l)a(1-a)Ak_t^{a-1}n_t^{-a} \\ -\frac{c_t\lambda_t}{(1-n_t)^2} - \lambda_t(1-\tau_t^l)\frac{\mu_1}{\mu_2}a(1-a)Ak_t^a n_t^{-1-a} = 0,\end{gathered} \tag{B3}$$

$$\begin{gathered}\psi_t\frac{\mu_1}{c_{t+1}}(1-\tau_{t+1}^k)a(a-1)Ak_{t+1}^{a-2}n_{t+1}^{1-a} - \frac{\chi_t}{\beta} + \lambda_{t+1}(1-\tau_{t+1}^l)\frac{\mu_1}{\mu_2}a(1-a)Ak_{t+1}^{a-1}n_{t+1}^{-a} \\ +aAk_{t+1}^{a-1}n_{t+1}^{1-a}(\chi_{t+1} - \zeta_{t+1}\phi + \xi_{t+1}[a\tau_{t+1}^k + (1-a)\tau_{t+1}^l]) + \chi_{t+1}(1-\delta^k) = 0,\end{gathered} \tag{B4}$$

$$-\zeta_t Q_{t+1} + \beta(\frac{1-\mu_1-\mu_2}{Q_{t+1}} + \zeta_{t+1}\delta^q) = 0, \tag{B5}$$

$$\xi_t k_t - \psi_{t-1}\frac{\mu_1}{c_t} = 0, \tag{B6}$$

$$\xi_t n_t - \lambda_t\frac{\mu_1}{\mu_2} = 0, \tag{B7}$$

$$\frac{\mu_1}{\mu_2}(1-a)Ak_t^a n_t^{-a}(1-\tau_t^l) - \frac{c_t}{1-n_t} = 0, \tag{B8}$$

$$\beta\frac{\mu_1}{c_{t+1}}[(1-\tau_{t+1}^k)aAk_{t+1}^{a-1}n_{t+1}^{1-a} + 1 - \delta^k] = \frac{\mu_1}{c_t}, \tag{B9}$$

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - \phi Ak_t^a n_t^{1-a} + \nu g, \tag{B10}$$

$$g = Ak_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l], \tag{B11}$$

$$c_t + k_{t+1} = Ak_t^a n_t^{1-a} - g + (1-\delta^k)k_t. \tag{B12}$$

From the Eqs. $(B2)-(B12)$ we can eliminate ζ_t, ψ_t and ψ_{t-1} by substituting $(B11)$,

$(B5)$ and $(B6)$ at time t and $t+1$, so that Eqs. $(B2)-(B12)$ be written as

$$\mu_1 c_t - \frac{c_t^2\lambda_t}{1-n_t} + \beta\xi_{t+1}k_{t+1}c_{t+1} - c_t^2\chi_t - \xi_t k_t c_t[(1-\tau_t^k)Ak_t^a n_t^{1-a} + 1 - \delta^k] = 0, \quad \text{(B13)}$$

$$(1-a)Ak_t^a n_t^{-a}[\chi_t(1-\frac{\varphi}{\nu}) + \xi_t[a\tau_t^k + (1-a)\tau_t^l - \frac{\varphi}{\nu}]] - \frac{\mu_2}{1-n_t} + \xi_t(1-\tau_t^l)a(1-a)Ak_t^a n_t^{-a} \quad \text{(B14)}$$

$$-\frac{c_t\lambda_t}{(1-n_t)^2} - \lambda_t(1-\tau_t^l)\frac{\mu_1}{\mu_2}a(1-a)Ak_t^a n_t^{-1-a} = 0,$$

$$\xi_{t+1}k_{t+1}(1-\tau_{t+1}^k)a(a-1)Ak_{t+1}^{a-2}n_{t+1}^{1-a} - \frac{\chi_t}{\beta} + \lambda_{t+1}(1-\tau_{t+1}^l)\frac{\mu_1}{\mu_2}a(1-a)Ak_{t+1}^{a-1}n_{t+1}^{-a} \quad \text{(B15)}$$

$$+aAk_{t+1}^{a-1}n_{t+1}^{1-a}[\chi_{t+1}(1-\frac{\varphi}{\nu}) + \xi_{t+1}[a\tau_{t+1}^k + (1-a)\tau_{t+1}^l - \frac{\varphi}{\nu}]] + \chi_{t+1}(1-\delta^k) = 0,$$

$$-(\chi_t + \xi_t)\frac{1}{\nu}Q_{t+1} + \beta\frac{1-\mu_1-\mu_2}{Q_{t+1}} + \frac{\beta\delta^q}{\nu}(\chi_{t+1} + \xi_{t+1}) = 0, \quad \text{(B16)}$$

$$\xi_t n_t - \lambda_t\frac{\mu_1}{\mu_2} = 0, \quad \text{(B17)}$$

$$\mu_2 c_t - \mu_1(1-n_t)(1-a)Ak_t^a n_t^{-a}(1-\tau_t^l) = 0, \quad \text{(B18)}$$

$$\beta\mu_1 c_t[(1-\tau_{t+1}^k)aAk_{t+1}^{a-1}n_{t+1}^{1-a} + 1 - \delta^k] = \mu_1 c_{t+1}, \quad \text{(B19)}$$

$$(1-\delta^q)\bar{Q} + \delta^q Q_t - \varphi Ak_t^a n_t^{1-a} + \nu Ak_t^a n_t^{1-a}[a\tau_t^k + (1-a)\tau_t^l] - Q_{t+1} = 0, \quad \text{(B20)}$$

$$c_t + k_{t+1} - Ak_t^a n_t^{1-a}[1 - a\tau_t^k - (1-a)\tau_t^l] - (1-\delta^k)k_t = 0. \quad \text{(B21)}$$

In this way we have a system with nine equations in $\{c_t, n_t, k_{t+1}, Q_{t+1}, \tau_t^k, \tau_t^l, \lambda_t, \chi_t, \xi_t\}_{t=0}^{\infty}$. We linearize Eqs. $(B13)-(B21)$ around the steady state to analyze the system's behavior. By using Taylor's theorem we expand the functions of the system around the steady state.

Eq. $(B13)$ becomes

$$f_1(c_t, n_t, \lambda_t, \xi_{t+1}, k_{t+1}, c_{t+1}, \chi_t, \xi_t, k_t, \tau_t^k) = f_{1a}\hat{c}_t + f_{1b}\hat{n}_t + f_{1c}\hat{\lambda}_t + f_{1d}\hat{\xi}_{t+1} + f_{1e}\hat{k}_{t+1} \quad \text{(B22)}$$

$$+f_{1f}\hat{c}_{t+1} + f_{1g}\hat{\chi}_t + f_{1h}\hat{\xi}_t + f_{1i}\hat{k}_t + f_{1j}\hat{\tau}_t^k,$$

where

$$f_{1a} = f_{1c_t}(c^*, n^*, \lambda^*, \xi^*, k^*, c^*, \chi^*, \xi^*, k^*, \tau^{k*}) = [\mu_1 - \frac{2c^*\lambda^*}{1-n^*} - 2c^*\chi^* - \xi^* k^*[(1-\tau^{k*}) \tag{B23}$$

$$A(k^*)^a(n^*)^{1-a} + 1 - \delta^k]],$$

$$f_{1b} = f_{1n_t}(c^*, n^*, \lambda^*, \xi^*, k^*, c^*, \chi^*, \xi^*, k^*, \tau^{k*}) = [-[\frac{(c^*)^2\lambda^*}{(1-n^*)^2} \tag{B24}$$

$$+\xi^* k^* c^*(1-\tau^{k*})(1-a)A(k^*)^a(n^*)^{-a}]],$$

$$f_{1c} = f_{1\lambda_t}(\cdot) = [-\frac{(c^*)^2}{1-n^*}], \tag{B25}$$

$$f_{1d} = f_{1\xi_{t+1}}(\cdot) = \beta k^* c^*, \tag{B26}$$

$$f_{1e} = f_{1k_{t+1}}(\cdot) = \beta \xi^* c^*, \tag{B27}$$

$$f_{1f} = f_{1c_{t+1}}(\cdot) = \beta \xi^* k^*, \tag{B28}$$

$$f_{1g} = f_{1\chi_t}(\cdot) = [-(c^*)^2], \tag{B29}$$

$$f_{1h} = f_{1\xi_t}(\cdot) = [-[k^* c^*[(1-\tau^{k*})A(k^*)^a(n^*)^{1-a} + 1 - \delta^k]]], \tag{B30}$$

$$f_{1i} = f_{1k_t}(\cdot) = [-\xi^* c^*[(1-\tau^{k*})A(k^*)^a(n^*)^{1-a} + 1 - \delta^k] \tag{B31}$$

$$-\xi^* c^*(1-\tau^{k*})Aa(k^*)^a(n^*)^{1-a}],$$

$$f_{1j} = f_{1\tau_t^k}(\cdot) = \xi^* c^*(1-\tau^{k*})A(k^*)^{a+1}(n^*)^{1-a}. \tag{B32}$$

Eq. $(B14)$ becomes

$$f_2(k_t, n_t, \chi_t, \xi_t, \tau_t^k, \tau_t^l, c_t, \lambda_t) = f_{2a}\hat{k}_t + f_{2b}\hat{n}_t + f_{2c}\hat{\chi}_t + f_{2d}\hat{\xi}_t + f_{2e}\hat{\tau}_t^k + f_{2f}\hat{\tau}_t^l + f_{2g}\hat{c}_t + f_{2h}\hat{\lambda}_t, \tag{B33}$$

where

$$f_{2a} = f_{2k_t}(\cdot) = [(1-a)aA(k^*)^{a-1}(n^*)^{-a}[\chi^*(1-\frac{\phi}{\nu}) \tag{B34}$$
$$+\xi^*[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\phi}{\nu}]] + \xi^*(1-\tau^{l*})a^2(1-a)A(k^*)^{a-1}(n^*)^{-a}$$
$$-\lambda^*(1-\tau^{l*})\frac{\mu_1}{\mu_2}a^2(1-a)A(k^*)^{a-1}(n^*)^{-1-a}],$$

$$f_{2b} = f_{2n_t}(\cdot) = [-a(1-a)A(k^*)^a(n^*)^{-a-1}[\chi^*(1-\frac{\phi}{\nu}) \tag{B35}$$
$$+\xi^*[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\phi}{\nu}]] - \frac{\mu_2}{(1-n^*)^2} - \xi^*(1-\tau^{l*})a^2(1-a)A(k^*)^a(n^*)^{-a-1}$$
$$-\frac{2c^*\lambda^*(1-n^*)}{(1-n^*)^4} + \lambda^*(1-\tau^{l*})\frac{\mu_1}{\mu_2}a(1-a)(a+1)A(k^*)^a(n^*)^{-a-2}],$$

$$f_{2c} = f_{2\chi_t}(\cdot) = (1-a)A(k^*)^a(n^*)^{-a}(1-\frac{\phi}{\nu}), \tag{B36}$$

$$f_{2d} = f_{2\xi_t}(\cdot) = [(1-a)A(k^*)^a(n^*)^{-a}, \tag{B37}$$

$$[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\phi}{\nu}]] + (1-\tau^{l*})a(1-a)A(k^*)^a(n^*)^{-a}], \tag{B38}$$

$$f_{2e} = f_{2\tau_t^k}(\cdot) = (1-a)A(k^*)^a(n^*)^{-a}\xi^*a, \tag{B39}$$

$$f_{2f} = f_{2\tau_t^l}(\cdot) = [(1-a)^2A(k^*)^a(n^*)^{-a}\xi^* - \xi^*a(1-a)A(k^*)^a(n^*)^{-a} \tag{B40}$$
$$+\lambda^*\frac{\mu_1}{\mu_2}a(1-a)A(k^*)^a(n^*)^{-a-1}],$$

$$f_{2g} = f_{2c_t}(\cdot) = [-\frac{\lambda^*}{(1-n^*)^2}], \tag{B41}$$

$$f_{2h} = f_{2\lambda_t}(\cdot) = [-\frac{c^*}{(1-n^*)^2} - (1-\tau^{l*})\frac{\mu_1}{\mu_2}a(1-a)A(k^*)^a(n^*)^{-a-1}]. \tag{B42}$$

Eq. $(B15)$ becomes

$$f_3(\xi_{t+1}, k_{t+1}, \tau_{t+1}^k, n_{t+1}, \chi_t, \lambda_{t+1}, \tau_{t+1}^l, \chi_{t+1}) = f_{3a}\hat{\xi}_{t+1} + f_{3b}\hat{k}_{t+1} + f_{3c}\tau_{t+1}^k + f_{3d}\hat{n}_{t+1} \tag{B43}$$
$$+f_{3e}\hat{\chi}_t + f_{3f}\hat{\lambda}_{t+1} + f_{3g}\hat{\tau}_{t+1}^l + f_{3h}\hat{\chi}_{t+1},$$

where

$$f_{3a} = f_{3\xi_{t+1}}(\cdot) = [(1-\tau^{k*})a(a-1)A(k^*)^{a-1}(n^*)^{1-a} \tag{B44}$$
$$+aA(k^*)^{a-1}(n^*)^{1-a}[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\phi}{\nu}]],$$

$$f_{3b} = f_{3k_{t+1}}(\cdot) = [\xi^*(1-\tau^{k*})a(a-1)^2A(k^*)^{a-2}(n^*)^{1-a} \tag{B45}$$
$$+\lambda^*(1-\tau^{l*})\frac{\mu_1}{\mu_2}a(1-a)^2A(k^*)^{a-2}(n^*)^{-a}$$
$$+a(a-1)A(k^*)^{a-2}(n^*)^{1-a}[\chi^*(1-\frac{\phi}{\nu}) + \xi^*[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\phi}{\nu}]]],$$

$$f_{3c} = f_{3\tau^k_{t+1}}(\cdot) = [a^2A(k^*)^{a-1}(n^*)^{1-a}\xi^* - \xi^*a(a-1)A(k^*)^{a-1}(n^*)^{1-a}], \tag{B46}$$

$$f_{3d} = f_{3n_{t+1}}(\cdot) = [-\xi^*(1-\tau^{k*})a(a-1)^2A(k^*)^{a-1}(n^*)^{-a} \tag{B47}$$
$$-\lambda^*(1-\tau^{l*})\frac{\mu_1}{\mu_2}a^2(1-a)A(k^*)^{a-1}(n^*)^{-1-a}$$
$$+a(1-a)A(k^*)^{a-1}(n^*)^{-a}[\chi^*(1-\frac{\phi}{\nu}) + \xi^*[a\tau^{k*} + (1-a)\tau^{l*} - \frac{\phi}{\nu}]]],$$

$$f_{3e} = f_{3\chi_t}(\cdot) = [-\frac{1}{\beta}], \tag{B48}$$

$$f_{3f} = f_{3\lambda_{t+1}}(\cdot) = (1-\tau^{l*})\frac{\mu_1}{\mu_2}a(1-a)A(k^*)^{a-1}(n^*)^{-a}, \tag{B49}$$

$$f_{3g} = f_{3\tau^l_{t+1}}(\cdot) = [a(1-a)A(k^*)^{a-1}(n^*)^{1-a}\xi^* - \lambda^*\frac{\mu_1}{\mu_2}aA(k^*)^{a-1}(n^*)^{-a}], \tag{B50}$$

$$f_{3h} = f_{3\chi_{t+1}}(\cdot) = [aA(k^*)^{a-1}(n^*)^{1-a}(1-\frac{\phi}{\nu}) + 1 - \delta^k]. \tag{B51}$$

Eq. $(B16)$ becomes

$$f_4(\chi_t, \xi_t, Q_{t+1}, \chi_{t+1}, \xi_{t+1}) = f_{4a}\hat{\chi}_t + f_{4b}\hat{\xi}_t + f_{4c}\hat{Q}_{t+1} + f_{4d}\hat{\chi}_{t+1} + f_{4e}\hat{\xi}_{t+1}, \tag{B52}$$

where

$$f_{4a} = f_{4\chi_t}(\cdot) = -\frac{Q^*}{\nu}, \tag{B53}$$

$$f_{4b} = f_{4\xi_t}(\cdot) = -\frac{Q^*}{\nu}, \tag{B54}$$

$$f_{4c} = f_{4Q_{t+1}}(\cdot) = [(\chi^* + \xi^*)\frac{1}{\nu} - \frac{\beta(1-\mu_1-\mu_2)}{(Q^*)^2}], \tag{B55}$$

$$f_{4d} = f_{4\chi_{t+1}}(\cdot) = \frac{\beta\delta^q}{\nu}, \tag{B56}$$

$$f_{4e} = f_{4\xi_{t+1}}(\cdot) = \frac{\beta\delta^q}{\nu}. \tag{B57}$$

Eq. $(B17)$ becomes

$$f_5(\xi_t, n_t, \lambda_t) = f_{5a}\hat{\xi}_t + f_{5b}\hat{n}_t + f_{5c}\hat{\lambda}_t, \tag{B58}$$

where

$$f_{5a} = f_{5\xi_t}(\cdot) = n^*, \tag{B59}$$

$$f_{5b} = f_{5n_t}(\cdot) = \xi^*, \tag{B60}$$

$$f_{5c} = f_{5\lambda_t}(\cdot) = [-\frac{\mu_1}{\mu_2}]. \tag{B61}$$

Eq. $(B18)$ becomes

$$f_6(c_t, n_t, k_t, \tau_t^l) = \mu_2\hat{c}_t + f_{6a}\hat{n}_t + f_{6b}\hat{k}_t + f_{6c}\hat{\tau}_t^l, \tag{B62}$$

where

$$f_{6a} = f_{6n_t}(\cdot) = [\mu_1(1-a)A(k^*)^a(n^*)^{-a}(1-\tau^{l*}) \tag{B63}$$

$$+a\mu_1(1-n^*)(1-a)A(k^*)^a(n^*)^{-a-1}(1-\tau^{l*})],$$

$$f_{6b} = f_{6k_t}(\cdot) = [-\mu_1(1-n^*)(1-a)aA(k^*)^{a-1}(n^*)^{-a}(1-\tau^{l*})], \tag{B64}$$

$$f_{6c} = f_{6\tau_t^l}(\cdot) = \mu_1(1-n^*)(1-a)A(k^*)^a(n^*)^{-a}. \tag{B65}$$

Eq. $(B19)$ becomes

$$f_7(c_t, \tau^k_{t+1}, k_{t+1}, n_{t+1}, c_{t+1}) = f_{7a}\hat{c}_t + f_{7b}\tau^k_{t+1} + f_{7c}\hat{k}_{t+1} + f_{7d}\hat{n}_{t+1} + [-\mu_1]\hat{c}_{t+1}, \tag{B66}$$

where

$$f_{7a} = f_{7c_t}(\cdot) = \beta\mu_1[(1-\tau^{k*})aA(k^*)^{a-1}(n^*)^{1-a} + 1 - \delta^k], \tag{B67}$$

$$f_{7b} = f_{7\tau^k_{t+1}}(\cdot) = [-\beta\mu_1 c^* aA(k^*)^{a-1}(n^*)^{1-a}], \tag{B68}$$

$$f_{7c} = f_{7k_{t+1}}(\cdot) = \beta\mu_1 c^*(1-\tau^{k*})a(a-1)A(k^*)^{a-2}(n^*)^{1-a}, \tag{B69}$$

$$f_{7d} = f_{7n_{t+1}}(\cdot) = \beta\mu_1 c^*(1-\tau^{k*})a(1-a)A(k^*)^{a-1}(n^*)^{-a}. \tag{B70}$$

Eq. $(B20)$ becomes

$$f_8(Q_t, k_t, n_t, \tau^k_t, \tau^l_t, Q_{t+1}) = \delta^q\hat{Q}_t + f_{8a}\hat{k}_t + f_{8b}\hat{n}_t + f_{8c}\hat{\tau}^k_t + f_{8d}\hat{\tau}^l_t - \hat{Q}_{t+1}, \tag{B71}$$

where

$$f_{8a} = f_{8k_t}(\cdot) = [-\phi aA(k^*)^{a-1}(n^*)^{1-a} + \nu Aa(k^*)^{a-1}(n^*)^{1-a}[a\tau^{k*} + (1-a)\tau^{l*}]], \tag{B72}$$

$$f_{8b} = f_{8n_t}(\cdot) = [-\phi(1-a)A(k^*)^a(n^*)^{-a} + \nu(1-a)A(k^*)^a(n^*)^{-a} \tag{B73}$$

$$[a\tau^{k*} + (1-a)\tau^{l*}]],$$

$$f_{8c} = f_{8\tau^k_t}(\cdot) = \nu A(k^*)^a(n^*)^{1-a}a, \tag{B74}$$

$$f_{8d} = f_{8\tau^l_t}(\cdot) = \nu A(k^*)^a(n^*)^{1-a}(1-a). \tag{B75}$$

Eq. $(B21)$ becomes

$$f_9(c_t, k_{t+1}, k_t, n_t, \tau^k_t, \tau^l_t) = \hat{c}_t + \hat{k}_{t+1} + f_{9a}\hat{k}_t + f_{9b}\hat{n}_t + f_{9c}\hat{\tau}^k_t + f_{9d}\hat{\tau}^l_t, \tag{B76}$$

where

$$f_{9a} = f_{9k_t}(\cdot) = [-[aA(k^*)^{a-1}(n^*)^{1-a}[1 - a\tau^{k*} - (1-a)\tau^{l*}] + (1-\delta^k)]], \tag{B77}$$

$$f_{9b} = f_{9n_t}(\cdot) = [-A(1-a)(k^*)^a(n^*)^{-a}[1 - a\tau^{k*} - (1-a)\tau^{l*}]], \tag{B78}$$

$$f_{9c} = f_{9\tau_t^k}(\cdot) = A(k^*)^a(n^*)^{1-a}a, \tag{B79}$$

$$f_{9d} = f_{9\tau_t^l}(\cdot) = A(k^*)^a(n^*)^{1-a}(1-a). \tag{B80}$$

We observe that the three of the equations in the system are static. We substitute these equations into the other six equations of the system. More specifically, we solve f_2, f_5 and, f_6 with respect to $\hat{\tau}_t^k$, $\hat{n}_t$, and $\hat{\tau}_t^l$ respectively:

$$\begin{bmatrix} f_2(k_t, n_t, \chi_t, \xi_t, \tau_t^k, \tau_t^l, c_t, \lambda_t) = 0 \\ f_5(\xi_t, n_t, \lambda_t) = 0 \\ f_6(c_t, n_t, k_t, \tau_t^l) = 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \hat{\tau}_t^k = \hat{k}_t(\frac{f_{2f}}{f_{2a}} - \frac{f_{2a}}{f2e}) + \hat{\xi}_t(\frac{f_{2b}}{f_{2a}}\frac{n^*}{\xi^*} - \frac{f_{2d}}{f_{2a}} - \frac{f_{2f}}{f2e}f_{6a}\frac{n^*}{\xi^*}) + \hat{\lambda}_t(\frac{f_{2b}}{f_{2a}}\frac{f_{5a}}{\xi^*} - \frac{f_{2f}}{f_{2a}}f_{6a}\frac{f_{5a}}{\xi^*} - \frac{f_{2h}}{f_{2a}}) \\ +\hat{\chi}_t(-\frac{f_{2c}}{f_{2a}}) + \hat{c}_t(\frac{f_{2f}}{f_{2a}}\mu_2 - \frac{f_{2g}}{f_{2a}}) \\ \hat{n}_t = [-\frac{n^*}{\xi^*}]\hat{\xi}_t + [-\frac{f_{5a}}{\xi^*}]\hat{\lambda}_t \\ \hat{\tau}_t^l = -\mu_2\hat{c}_t - f_{6b}\hat{k}_t + f_{6a}\frac{n^*}{\xi^*}\hat{\xi}_t + f_{6a}\frac{f_{5a}}{\xi^*}\hat{\lambda}_t \end{bmatrix}.$$

Therefore, the system becomes

$$\begin{bmatrix} f_{1f} & f_{1e} & 0 & 0 & 0 & f_{1d} \\ f_{35} & f_{32} & 0 & f_{33} & f_{34} & f_{31} \\ 0 & 0 & f_{4c} & 0 & f_{4d} & f_{4e} \\ f_{75} & f_{71} & 0 & f_{73} & f_{74} & f_{72} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{Q}_{t+1} \\ \hat{\lambda}_{t+1} \\ \hat{\chi}_{t+1} \\ \hat{\xi}_{t+1} \end{bmatrix} = \begin{bmatrix} f_{11} & -f_{15} & 0 & -f_{13} & -f_{14} & -f_{12} \\ 0 & 0 & 0 & 0 & -f_{3e} & 0 \\ 0 & 0 & 0 & 0 & -f_{4e} & -f_{4b} \\ -f_{7a} & 0 & 0 & 0 & 0 & 0 \\ f_{85} & f_{83} & \delta^q & f_{82} & f_{84} & f_{81} \\ f_{91} & f_{92} & 0 & f_{94} & f_{95} & f_{93} \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{Q}_t \\ \hat{\lambda}_t \\ \hat{\chi}_t \\ \hat{\xi}_t \end{bmatrix}$$

$$\Leftrightarrow D\hat{X}_{t+1} = E\hat{X}_t \Leftrightarrow \hat{X}_{t+1} = ED^{-1}\hat{X}_t \Leftrightarrow \hat{X}_{t+1} = F\hat{X}_t.$$

By using the parameter values in the paper of Angelopoulos, Economides, and Philippopoulos (2013), we find that the three eigenvalues of F have absolute value smaller than one, while the other three have absolute value larger than one. The Blanchard-Kahn conditions are satisfied and the model is stable. The steady state of the system is a saddle path, therefore it has a unique equilibrium.