Enhancing Managerial Decision-Making Through Multicriteria Modeling

Gawlik, Remigiusz

Cracow University of Economics

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Politechnika Krakowska
im. Tadeusza Kościuszki

Remigiusz Gawlik

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Wydawnictwo PK, ul. Skarżyńskiego 1, 31-866 Kraków; tel. 012 628 37 25, fax 012 628 37 60
e-mail: wydawnictwo@pk.edu.pl www.wydawnictwo.pk.edu.pl
Postal address: ul. Warszawska 24, 31-155 Kraków

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INTRODUCTION

People make decisions on a daily basis. The decision-making of individuals is rather intuitive, hardly ever scientific methods are employed for this task. The ambit of consequences of individual decisions is usually limited to the person of decision-maker and his or her closest environment (family, friends, surrounding people and private possessions).

Decision-making in business organisations, however, seems to be a distinctively different issue, as the consequences of managerial decisions affect the entire organisation: its assets and liabilities, employees (both horizontally and vertically), clients and business partners. In the short term, faulty decisions may cause multiple, but still manageable costs, whereas in the long term they may result in undermining organisation's growth potential or even threaten its survival. Managerial decision-making focuses on solving managerial problems. Szarucki (2015, p. 363) defines a managerial problem as a difference, defined by a manager between what is perceived and the desired output. In this author's opinion managerial decision-making heavily depends the additional context imposed by the classification of the managerial problems in question. Possible divisions can be related to: (i) the goals of problem solving (defined/undefined); (ii) the level of their structuring (well-structured, ill-structured); (iii) the character of the problem (in the field of human relations/technical issues); (iv); organisational level of problem's occurrence (strategic/operational); (v) management-function specific (human resources, strategy, operations, marketing, production, management, MIS-data processing, external-environmental, communications, customer, accounting); (vi) ways of solving the problem (programmable/non-programmable). Other classification criteria include the source of problem initiation, the causal character of a problem, the conditions under which a problem is solved, the possibility of expressing the problem in numbers (quantification), decision options, the level of individual involvement within problem solving, the management functions to which the problem could be related, problem complexity and organisational level.

Consequently, it can be assumed that managerial decision-making requires a more sophisticated and structured approach than the individual one. The focus of contemplated decision alternatives should encompass the entire business environment of the organisation. It should also anticipate the consequences of made decisions for all agents directly and indirectly involved in the decision problem. Science covers these issues in terms of the Decision Theory, which can be divided into fields that form separate research fields themselves and allow them to formulate their own research problems. The examples include, inter alia, different approaches to decision-making (normative, descriptive, prescriptive), various types of decisions (rational vs. irrational, preference-based vs. prospect-based), considered measures of decision-making effects (maximising utility, minimising risk), reasons for recurring to decision-making enhancement methods (complexity of decisions, environmental uncertainty, intertemporal choice, multicriteriality of decision problems). Even if one's interests are limited to the enhancement of managerial decision-making, the vastness of the research field provides enough justification for continuing its scientific exploration.

Literature review offers numerous methods for the enhancement of decision-making processes, inter alia, combining selected hierarchical and fuzzy models. For example, the Web of Science database returns 5487 results for a "TOPIC=AHP and fuzzy" inquiry, out of which 2825 have been published in the past five years[1]. Despite such an impressive number of publications, a **research gap** can be identified: no procedure for the enhancement of managerial decision-making that would combine the hierarchical and fuzzy approaches in a sequential mode has been found.

Filling in the inventoried research gap in its **scientific aspect** leads to the complementation of the decision theory methodology in the field of complex managerial problems, which is the **scientific purpose** of this monograph. In the **utilitarian aspect** the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) developed by the author provides an additional tool for the decision-makers in solving their managerial problems.

The formulated scientific purpose was accomplished through the application of scientific methodology. The **research problem** is the development of a sequential, qualitative-quantitative method for the enhancement of managerial decision-making, appropriate for multicriteria management problems.

Justification for addressing the research problem: the motivation for addressing the research problem is to fill in the abovementioned inventoried research gap, which is the lack of a procedure for the enhancement of decision-making in the field of multicriteria managerial problems that would sequentially combine the hierarchical and fuzzy approaches.

The hitherto considerations have led to the formulation of **research questions**:

1. What is the decision-making practice of tactical and strategic managers in companies?
2. Do they perceive both measurable and immeasurable factors as relevant for their decision-making practice?
3. What multicriteria methods for enhancing managerial decision-making can be found in management science literature?
4. Is it possible to combine qualitative and quantitative methods of the enhancement of managerial decision-making processes within a single model?

[1] As of November 15^{th}, 2018.

5. As multicriteria managerial decisions hardly ever have a deterministic character, how to design a decision-making model that includes these decision alternatives which slightly vary from the optimal possible solution (e.g. second-best variant)?

One of the characteristics of managerial decision-making processes is their complexity. Therefore to address the main research problem of this monograph a multistage approach was adopted. It is composed of a series of **research tasks.** First, literature review has been performed. Second, the practice of decision-making in enterprises operating in Poland has been analysed through the quantitative research. Its results have been subjected to testing of hypotheses. The goal of this step was to identify and present the actual trends in decision-making practice in enterprises – with regard to the application of tools for the enhancement of decision-making processes. The third research step consisted of the development of a modular multicriteria model for the enhancement of managerial decision-making processes.

The research problem was solved through the Analytic Hierarchy Process (AHP) method and the Fuzzy sets theory. The **original and innovative** aspect of the author's proposal is a decision-making model that offers a direct choice of a decision alternative, encompassing, however, some of the criteria from the close, but rejected decision alternative (second-best solution). Moreover, as no definition of an integral managerial decision can be found in the scientific literature, the author proposes his own definition of this notion. The postulated methodology, sequentially linking the AHP and Fuzzy sets theory, adds a new quality to the modelling of multicriteria decision-making processes. The presented approach also broadens the actual state of knowledge in the field of available managerial decision-making enhancement methods, which constitutes the **author's input to the development of management science**.

In order to develop the **Hierarchical Module** of the proposed decision-making model, measurable and immeasurable determinants of managerial decision-making processes need to be included in the analysis. In practical applications, they form the decision criteria level and serve for the input data for the theoretical model. The output side of the model's Hierarchical Module consists of decision alternatives: Decision A_1, Decision $A_{2.}$ Decision A_3 and Decision A_4, which reflects the available potential solutions of the decision problem. Factual attributes of each decision variant depend on the specificity of a given practical decision problem. For the theoretical form of the model the concretisation of decision alternatives is not required. For the construction of the Hierarchical Module a mathematical toolbox transposing the qualitative criteria into quantitative ones was applied. The significance of particular decision-making criteria was graded through the Analytic Hierarchy Process method. These relevance rankings were obtained in an evaluation process performed by practising managers. As a result, a decision alternative that best conforms to the expert rankings of all the decision criteria was chosen. The Hierarchical Module of the model can also be subject to optimisation with regard to a chosen parent criterion or even one sub-criterion.

Since the practice of managerial decision-making shows that quite often close decision alternatives are taken into consideration, the proposed theoretical tool encompasses a **Fuzzy Module** as well. Its role is to extend the decision field by relevant elements of a second-best decision alternative. The scientific method applied for this purpose is based on the Fuzzy sets theory. After the Fuzzy Module has been applied, an extended decision field is indicated. The Fuzzy Module provides a more realistic measurement of the significance of the AHP-resulting hierarchy of decision alternatives, which expands the decision-maker's awareness of the ongoing decision-making process.

In order to counterbalance any ranking inconsistencies originating in the application of various comparison scales in the Hierarchical Module, as well as to additionally increase the precision of the final decision alternative, a mixed **AHP-Fuzzy Module** – an expansion of two former modules is proposed. By reassessing the decision-making effects of the Hierarchical Module and their fuzzification in the Fuzzy Module, the final and definitive decision alternative was chosen.

Such an analysis was made possible owing to the developed Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes. The created model is assumed to lead towards the best possible results of the decision-making process for actual functioning and future development of an enterprise. Enterprise development is understood here as a process of consecutive changes aiming at improving, reaching a higher level, a more perfect and more complex form (Kaczmarek, 2017, pp. 65-66).

The main advantage of the proposed solution compared with other decision-making methods combining Fuzzy sets theory and AHP (e.g. those described in Mardani, Jusoh & Zavadskas, 2015) is the sequential application of each of the methods (first AHP, then Fuzzy, then mixed AHP-Fuzzy). Subsequent processing of the decision problem by three modules on a one-after-another basis allows elements of the decision alternative rejected in the Hierarchical Module (second-best) to be incorporated into the final managerial decision. Such an approach results in making an integral managerial decision rather than an optimal possible one. The proposed MMUMADEMM tool resolves Simon's satisficing decision (Simon, 1955) and bounded rationality (Simon, 1957) concerns about the quality of final managerial decisions.

A word of explanation of the vocabulary used in the monograph seems necessary. The author is aware that the idea of managerial decisions implies finality, however, for the needs of the presented research the term 'final decision' will mean the definitive decision alternative chosen as a result of the application of all three modules of the MMUMADEMM. This term will help distinguish them from partial decisions obtained from earlier stages of model application, e.g. the decision alternative resulting from the Hierarchical Module.

The above allows the formulation of the **main goal of the monograph**: the presentation of the developed Modular Multicriteria Managerial Decision-Making

Model (MMUMADEMM) for the enhancement of managerial decision-making processes. The main goal of the monograph was achieved through steps enumerated in the **detailed goals of the monograph**:

1. Proposal of a definition of an integral managerial decision in the frame of the prescriptive approach.
2. Identification of practice of managerial decision-making in enterprises operating in Poland.
3. Analysis of multicriteria methods enhancing managerial decision-making in scientific literature.
4. Presentation of methodological bases for the development of a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes.
5. Construction of a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes.

The main research problem together with the main and detailed goals of the monograph lead to the formulation of the following **general research thesis:**

The application of a sequential qualitative-quantitative model for the enhancement of multicriteria managerial decision-making processes allows integral managerial decisions to be made.

The general research thesis will be confirmed or falsified through the application of research methods (discussed further in the text) to the verification of the following **general sub-theses**:

T1. In a dynamically changing environment of companies managerial decision-making processes require the inclusion of a growing number of decision criteria.

T2. The inclusion of the quantitative (measurable) and qualitative (immeasurable) criteria into this analysis can improve the integrity of the final decision.

T3. The joint qualitative-quantitative research method is appropriate to address the scientific purpose of this monograph – the complementation of the decision theory methodology in the field of complex managerial problems by developing a method of the enhancement of managerial decision-making processes through a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM).

1. THE PLACE OF DECISION-MAKING IN MANAGEMENT SCIENCE

1.1. Methodological framework

Czakon (2015, pp. 10-11) introduces the following hierarchy of notions describing the research process:

- methodology is a science of methods, which catalogues research methods, describes their application, characterises the shortcomings and qualities of their application to research problems, as no universal, efficient research method exists; therefore methodology, a science of many available methods, is needed;
- a research method is the composition and hierarchy of stages of research process, repeatable for studies on a given class of research problems, because of its efficiency;
- a research technique is most application connoted, which means a standard procedure for each field of study.

The general scientific method adopted in this monograph is the empirical analytical approach of nomothetic character, as the author's aspiration was to identify and formulate general laws of science rather than to describe separate occurrences. Niemczyk (2015a, p. 21) states that the search for general laws through switching from specific (observation facts) to general (laws of science) is a recommended manner of theory building in economics, including management science. The author also advocates caution and moderation in generalisation of the results of empirical research onto general population by saying that actually such judgements indicate plausibility rather than absolute truth (Niemczyk, 2015a, p. 21).

As management science does not possess its own cognitive nor methodological toolbox (Krzyżanowski, 1999, p. 134), it is necessary to use methods, techniques and research tools of other fields of study. For higher precision of the obtained results the triangulation method was used for the needs of the presented research. Triangulation is a way of consolidating knowledge through the comparison of data gathered with several research methods (Konarzewski, 2000, p. 33). Miles & Huberman (2001, p. 276) see the sense of triangulation in the legitimization of findings through proving that independent measurements confirm them, or at least do not contradict them. In the discussed research task this is accomplished by quantitative (questionnaire) and qualitative research (direct semi-structured interviews) conducted independently, which follows the multiple heterogeneous approach in terms of the triangulation method (Stańczyk, 2015, p. 246-248).

At the same time, in view of the interdisciplinary character of the main research problem and following Sułkowski's (2015, p. 43) postulate, it is justified to apply a pluralist methodology. Such methodology encompasses both pragmatic and understanding approach, with a dominant role of inductive sciences' research toolbox (empirical methods), with elements of formal methods (Ostasz, 1999, pp. 11-17). Moreover, this approach recently gained support of the members of International Symposium of the Analytic Hierarchy Process, a scientific research network grouped around the person of late T.L. Saaty, the inventor of the AHP methodology.

Therefore the study employs the following **research methods and techniques**:

For introductory research tasks

1. **Literature overview** – to assess the presence of the research problem (decision-making) in the management science literature and to identify decision-making methods applied in managerial decision-making in the scientific literature on the subject. Counted between primary research methods.
2. **Black box approach** – the main research approach according to which the attention of the researcher is focused on the interrelations between input and output data of the created analytical model, rather than examining the mechanism of data processing. Counted between primary research methods.

For quantitative research tasks

3. **Structured self-administered questionnaire** – for the collection of quantitative data on the practice of managerial decision-making. The questionnaire included questions of closed-ended format: single and multiple choice, with rankings of importance, 7-point Likert-type scale inquiries and a few open-ended format questions. Counted among primary quantitative research methods.
4. **Mixed-type sampling** – to create a general population of respondents to the questionnaire. They were recruited from among executives of local and international companies operating in Poland. Counted among secondary quantitative research methods.
5. **Testing of hypotheses** – to perform an analysis of the significance of the questionnaire results. The goal of this research method is to find regularities and relations between the key obtained responses and drawing conclusions on the practice of managerial decision-making in enterprises operating in Poland. Counted among primary quantitative research methods.

For qualitative research tasks

6. **Arbitral choice** – for non-random sampling of the composition of experts for the direct semi-structured interviews. Experts were recruited from among practitioners of management of tactical and strategic managerial levels. Used as a primary qualitative research method.

7. **Direct semi-structured interviews** – to additionally test the level of expertise on the decision-making problem of preselected evaluators. Used as a secondary qualitative research method.

Joint qualitative-quantitative research

8. **Analytic Hierarchy Process (AHP)** – for the development of **Hierarchical Module** of Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making. In practical applications this research method will also provide expert evaluations of the determinants of managerial decision-making processes with regard to expected and undesirable decision-making effects. Counted among primary joint qualitative-quantitative research methods.
9. **Fuzzy sets theory** – for the construction of **Fuzzy Module** of Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making. The application of this method is aimed at the creation of a self-correcting auto-refining mechanism. It will include close-to-optimal decision alternatives rejected in the Hierarchical Module of the model. In order to include in the final decision the nuances of the second-best decision alternative, which although lower valued by the decision criteria hierarchy, still could contain elements relevant to the integrity of the final decision. Counted among primary joint qualitative-quantitative research methods.
10. **Both AHP and Fuzzy sets theory** were employed for the construction of the mixed **AHP-Fuzzy Module** of Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making. The combination of these two research methods was employed for further smoothing the emerging decision alternatives and making the final decision – choosing the decision alternative that contributes the most to the attainment of the main goal of decision-making, i.e. making an integral managerial decision. Counted among primary joint qualitative-quantitative research methods.

The research concept presented above was conducted in a series of steps that formed consecutive chapters of this monograph. First, decision-making as a research problem of management science is explored. Second, an overview of multicriteria decision-making methods in the management literature is presented. Third, quantitative research is conducted in order to provide information on the practice of managerial decision-making in companies operating in Poland. Fourth, theoretical and methodological frameworks for the construction of the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making are established. The input side of the model consists of a hierarchy of decision criteria, whereas its output side a set of prioritised decision alternatives. The same chapter contains the construction pattern of the model. As multicriteria decision-making models usually require a separate setup for each analysed decision problem, the construction of MMUMADEMM is illustrated using the applicatory procedure. It is based on an AHP hierarchy of decision criteria with significance ranks based on simulated expert evaluations. The model's application procedure is also presented graphically.

An attempt to place the research problem in the scientific discipline management science follows.

Pieter (1975, p. 5) states that scientific research is a special form of creative work, which usually consists of stating the existence or discovering, establishing, determining or creating the relations between known occurrences, things or their parts, or between the existing concepts or other reflections of phenomena. Therefore it can be stated that the main research problem of this monograph, which is the elaboration of a complex decision-making model that would help to understand the patterns according to which managers make decisions that affect managerial processes of their business organisations, conforms to the general methodology of science. It also forms a research issue, as research or scientific tasks are formulated when the researcher's effort focuses on unveiling facts, describing and explaining phenomena that rely on his/ her own research experience and scientific materials, and on proving the existence of conceptual dependencies (Pieter, 1975, p. 15).

The research task positions itself in terms of philosophy of science, as the decision-making model was created using scientific methods. It was based on the scientific approach to research, i.e. rationalism, criticism, anti-dogmatism, intersubjective communicativeness and verifiability, transparency of research and undisturbed flow of information (Kałuszyńska, 1994, pp. 9-10).

Appreciating the benefits for management derived from interdisciplinary approach to perception, research and solving of management problems, Lichtarski raises an important issue of the pureness of the academic discipline of management science (2015, pp. 16-19). The researcher points at the need of keeping the discipline in two categories of limits:

- external – in order to distinguish between actions of managerial character (including managing) and those without it – administrative ones;
- internal – in order to differentiate such notions as managing, administration, rule, command.

Both categories fall into Zieleniewski's (1982, p. 380) definition of management, which is an action aiming at making things function according to the goal of the one who guides them. At the same time, they show nuanced notional differences. According to Lichtarski (2015, p. 85) what is the most important is the discretionary power (freedom of making decision) of the decision-maker, which is fully granted only to the manager (not the administrator, commander, or even ruler). Then, the enhancement of decision-making processes, which forms the core of the research problem under discussion not only fits into management science, it does not violate the pureness of the academic discipline defended by Lichtarski either.

According to the resolution of the Central Commission for Degrees and Titles of Poland on the Definition of Academic Disciplines and Fields of Study in the Arts and Sciences (Polish Monitor No. 40, pos. 586[2]) and the ordinance of the Minister of Science and Higher Education of Poland of August 8th, 2011 on the List of Areas of Academic Study, Academic Disciplines and Fields of Study in the Arts and Sciences (Polish Official Journal Dz. U. of August 30th, 2011[3]) management is a field of study in the academic discipline of economics in the area of academic study social studies, but also a field of study in the academic discipline of humanities in the area of academic study humanities.

Enhancement of managerial decision-making falls into the first of the abovementioned classifications, which makes it a sub-field of the study of the field of study management, academic discipline of economics, area of academic study social studies. According to the OECD classification, applied also by the UNESCO and considered as a framework for qualification of academic disciplines in the new Law on Higher Education of Poland[4], the main research problem conforms to the field of science 5. Social sciences, academic discipline 5.2 Economics and business, sub-discipline 5.2.c Business and management (OECD 2015).

Following the Journal of Economic Literature (JEL) classification by the American Economic Association the main research problem of this monograph is described by the following codes:

- C. Mathematical and Quantitative Methods:
  - C4. Econometric and Statistical Methods: Special Topics
    - C44. Operations Research • Statistical Decision Theory

Applying the division of management science academic discipline postulated by Cyfert et al. (2014, pp. 41-45), the main research problem falls into the practical stream, sub-discipline enhancement of managerial decisions.

In his analysis of the specificity of research in terms of management science Burnewicz (2007, p. 8) states that its main function, as an applied science, is the projective function. Therefore in management science practical strains dominate. Nevertheless, they still need to include theoretical elements, as without theoretical knowledge the practical one would be superficial. Lichtarski (2015, p. 14) observes that the practical (normative) dimension of management science is its constant, inalienable feature. Therefore, apart from its descriptive value, the applicatory (normative) quality of the presented monograph constitutes an important dimension of this study. The presented research task can be counted among applied research

2 M.P. Nr 40, poz. 586.
3 Dz. U. 2011 nr 179, poz. 1065.
4 so-called Konstytucja dla Nauki [Constitution for Science], in force since 1 October, 2018.

of empirical character, with a reserve on the need for the creation of a necessary methodological framework.

Decision-making as a scientific research problem falls into the scope of management science and economics. In order to provide a conceptual framework for its discussion, a hierarchical approach, from general to specific has been adopted.

Economics as a science deals with business entities – enterprises. These can be divided according to many criteria, e.g. into small and medium or large, national or international, private or state-owned, with few or numerous shareholders, with various forms of ownership, etc. These differences required the adoption of suitable management frameworks, which in turn influenced the evolution of the discipline of economics called management science.

Drucker (1986, p. 19) states that management is an objective function, determined by the tasks, that is, it is a discipline. And yet it is culturally conditioned and subject to the values, traditions, habits of a given society. The same author bolsters that management exists only in contemplation of performance that is delivered through its three primary tasks:

- economic performance;
- making work productive and the worker achieving;
- managing social impacts and social responsibilities (Drucker, 1986, p. 32).

Penc (2001, pp. 29-30) defines management as a set of decision-making tasks that aim at controlling company's resources and processes and guiding them towards achievement of possibly best operational effects. This happens through more effective binding of resources and processes, in the existing environmental conditions (legal, economic, social, ecological, etc.) and according to the rationality of business operations. The same author defines management also as a deliberate and conscious choice of actions that aim at organising human teams cooperating in a given organisational structure, making decisions on what and how should be done and implementing it through the employees (Penc, 2007, p. 7).

Among the multitude of management definitions two inseparable approaches can be distinguished, institutional and functional. In terms of the institutional approach, management exemplifies a group of people who have been entrusted with the ability of giving instructions in the company. Meanwhile, the functional approach encompasses all administrative actions (functions) that are necessary to perform the company's tasks and accomplish its goals. Consequently, management[5] is defined as purposeful decision-making aiming at achieving predefined goals, through the administration of material, human, information and financial resources by the company's directors. (Lichtarski et al., 2001, p. 185).

[5] Attempts of definition of management also bring interesting linguistic issues into scope, e.g. in the Polish language management understood as directing the actions of subordinates and management of company's resources are two different words and separate definitions.

Management is implemented into managerial practice through the functions of management. Durlik (1996, p. 260) enumerates four main managerial functions: planning, organising, guiding & motivating and controlling, expanding the set by an additional one, which accompanies every managerial task – decision-making. The presented set of management functions is a development of Fayol's (1917) original concept. Table 1.1 shows their scope with regard to three main levels of management – indicated by the number of "x"s.

Table 1.1.

Relevance of management functions at various levels of management

Levels of management	**Management functions**				
	Planning	Organising	Guiding & Motivating	Controlling	Decision-making
Strategic	xxx	x	x	xx	xxx
Tactical	xx	xxx	xx	xxx	xxx
Operational	x	x	xxx	xxx	xxx

Source: Compiled on the basis of Ackoff (1970, pp. 5-42), Durlik (1996, p. 260) & Waldman & Yammarino (1999).

It is clearly seen that the decision-making function studied in this monograph is vital at each level of management.

Durlik (1996, p. 260) divides management functions into features common for all managerial processes. Although this division was originally worked out for the needs of production engineering, the core of management functions remains similar for other types of enterprises. The structure of management functions consists of:

1) Planning:
   - formulation of a management strategy;
   - forecasting of needs of clients, costs, prices, environmental and operating conditions;
   - determination of risk acceptance;
   - design of production processes;
   - design and choice of production capacities;
   - planning of localisation of production and other facilities;
   - design of enterprise structures.
2) Organising:
   - design of operations and organising activities;
   - work measurement and standardisation;
   - leading product (or service) development projects;
   - administration of company processes;
   - restructuring of the systems.

3) Guiding and motivating:
   - scheduling of work and production;
   - synchronisation of deliveries of production factors, resources and operations;
   - optimisation of operations.
4) Controlling:
   - inspection of the entire route between company's inputs and outputs;
   - controlling stocks of raw materials, sub-products and final products;
   - quality control of products and processes;
   - control of company's finance;
   - assessment of risk level.

Decision-making is not treated as a separate function of management, but rather as a procedure that is present at every level of management, at any managerial process.

Management science is inseparably related to the notion of enterprise, which origins from Latin prendere (to take), translated into French prendre (to take), which has evolved into entreprendre (to undertake an action) and entreprise (noun version of the verb entreprendre). As many words of French origin, after a slight spelling adaptation it has been incorporated into the English language as enterprise.

The definition of enterprise has developed through years in the economics literature. Lichtarski et al. (2001, pp. 26-39) provides an overview of the main concepts in terms of the theory of firm:

1. Neo-classical theory perceives an enterprise as a "black box"[6] that transforms input, i.e. factors of production (costs) into output, i.e. income, which results from sales of products or services (profit). An enterprise is an entity that aims at optimisation of the ratio between profit maximisation and cost minimisation (Lange, 1959). It is organised according to Weber's (1919) theory of power and bureaucratic organisation (Martyniak, 1996, p. 109), i.e. has a hierarchic organisational structure, division of work, formal procedures for claimants, qualification-based recruitment, separation of work and private matters and property (Zieleniewski, 1969, p. 111). After the Neo-classical theory, the firm follows strictly Fayol's (1917) fourteen principles of management. The criticism of the Neo-classical approach points at its detachment from reality, lack of information of decision-makers, that makes optimisation of managerial decision-making processes impossible, tendency of adopting acceptable instead of optimal solutions and lack of interest in the mechanism of input-output transformation (Galbraith, 1987).
2. Managerial theories of the firm (which are not a unified set of concepts on its nature) derive from the Neo-classical theory, by negating some of its assumptions. Gorynia (1999, p. 537) enumerates them:

[6] The concept of a "black box" model has been elaborated and explained by Cauer (1941).

- the environment of the company is not a perfectly free market, the perfect competition model is hardly present in reality;
- when maximising profit, companies can act "faulty", e.g. by changing costs or demand conditions;
- an enterprise cannot be perceived only through the person of the owner, as in most cases it is a conglomerate of entities that can have different goals;
- the firm is not seen as a "black-box" anymore, as the relations between its internal elements are subject to analysis as well;
- profit is not the only, nor is it always the most important goal of the company. Generally all managerial theories of the firm assume that an enterprise cannot be defined apart from the persons of its managers who, despite the separation of ownership and management, act in the best interest of their companies (Berle & Means, 1933). Baumol (1959) realised that such an assumption can be very misleading, as individuals rather tend to pursue their own goals in the first place (i.e. maximising their own, not corporate profits). Among the managerial approaches Gorynia (1999, pp. 538-540) mentions turnover maximisation (Baumol, 1959), growth rate maximisation (Marris, 1963, 1964) and managers' preference towards some types of costs (Williamson, 1986).

3. The behavioural theory of the firm perceives enterprises rather as fields of an interaction between the aspirations of managers (groups and individuals) and achievements of the company. The emphasis is on the attainability of aspirations and their distance from the achievements (Simon 1955). Behaviourists transplant psychological concepts into managerial scope by underlining that aspirations cannot be too ambitious (too far from accessible achievements) in order not to become destructive for managers and for the enterprise itself. Consequently, enterprises follow multiple goals simultaneously, defined by many decision-makers, whereas the level of the fulfilment of former targets is assessed with regard to the level of particular aspirations of managers. The criticism of the behavioural concept points at the difficulties in defining a hierarchy of aspirations universal for different enterprises, or even the mechanism of defining aspirations (Gorynia, 1999, p. 542), as well as its link with their efficiency on the market.
4. The transaction cost theory assumes the existence of two parallel mechanisms of regulating transactions, namely the market and hierarchy. An enterprise is seen here as a mechanism of resource allocation, which is parallel to the free market mechanism (Coase, 1937). The firm is perceived as a cheaper solution than the free market (fewer transaction costs, i.e.: price creation, signing transaction agreements, no need for the conception of long-term cooperation projects, preferential treatment of intra-enterprise transactions by the government). As a result, the size and structure of an enterprise will reflect the equality between the costs of internalisation of each additional transaction

and its organisation by the market (Lichtarski et al, 2001, p. 31). Therefore an enterprise will create better allocation conditions than the market itself. Apart from the transaction costs it results also from the cooperative character of the employed human resources (Williamson, 1983) as well as consumers' limited rationality and decision-makers' opportunism. The critics of the transaction cost theory emphasise the multiple unanswered issues, e.g. the "no-firms paradox" and market-based resource allocation paradigm (Dietrich, 1994) or empirical verification that is almost impossible[7].

5. The agency theory defines an enterprise as a set of characteristics, behaviours and relationships between company principals (owners, shareholders, creditors, insurers, lenders, etc.) and its agents (employees, debtors, insured, leasers, etc.). The link between these two opposite groups is established in the person of the company manager. In a complex business environment principals and agents are driven by various motivations, they also demonstrate different levels of risk acceptance. Both sides are also subject to uncertainty, random determinants of enterprise's efficiency (i.e. shocks on the financial market) and information asymmetry. Within the framework of the agency theory, a mixture of all these elements can negatively affect company's market results (Ross, 1973), but can also provide a comparative advantage, e.g. to family enterprises, because of lack of control costs (Surdej & Wach, 2010, p. 34). Nevertheless, despite the criticism that points out that the agency theory assumes conflict of interests between principals and agents, this approach provides justification for the need of the implementation of multicriteria decision-making models in managerial activities.
6. The resource-based theory of enterprises integrates various understandings of the nature of companies. It assumes that the enterprise constitutes a set of resources, competences and skills, the combination of which determines its efficiency. The crucial difference lies in the causative aspect. According to this modern theory, it is the enterprise that should adapt to the possessed resources, competencies and skills rather than forcefully acquire the missing ones. Another important change is the inclusion of immeasurable resources, such as brand, image, reputation, clients, knowledge, etc. into the scope of management science, parallel to material ones, e.g. liabilities and assets (Barney, 1991).
7. Gorynia (1998, pp. 44-50) expands Lichtarski's et al. set by adding a definition of the firm by the theory of property rights, which perceives property and derived rights as the main determinant of the behaviour of business entities. According to Gabrié & Jacquier (1994, pp. 296-297), these rights determine

[7] An exhaustive list of allegations to the transaction cost theory can be found in (Gorynia, 1998, pp. 54-56).

the individual initiative, which is of crucial importance for entrepreneurship. Changes in property rights are triggered by the interaction between their initial structure and human search for increased utility. Moreover, they possess social utility, serving for the benchmark for the measurement of achievements of individuals. What is also important, within the theory of property rights the assumption of market non-interference in the production or consumption of a good cannot be sustained (Gorynia, 1998, p. 47).

Borowiecki, Jaki & Kaczmarek (1999, p. 18) see the enterprise as a unique form of investment, as the owners expect their capital not only to provide return on investment, but also an additional growth of value of the created business venture. In a globalized economy the main sources of valuation growth derive from new key success factors, such as a local and global competitive advantage, international configuration and coordination, economies of scale, reach and experience and internationalisation of supply chain (Koźmiński, 1999, pp. 59-96).

An enterprise is a human team that systematically undertakes actions aiming at earning money (Lange, 1959, p. 209). With this in view, it is impossible to perceive enterprises without explaining their human aspect, which directs the research towards two related notions: entrepreneurship and entrepreneurs. Lichtarski et al. (2001, pp. 14--16) enumerates such common features of all enterprises as ownership, entrepreneurship, innovativeness, thriftiness, finance stability, autonomy, profitability and profit maximisation. Other researchers point at such binding aspects as competitiveness (Czaja, 2015; Piątkowski, 2015; Urbaniec, 2015; Wach, 2015; Żur, 2015), human resources (Lange, 1959; Pocztowski, 2008), internationalisation and regionalisation (Wach, 2012; Morawczyński, 2008), continuous presence of risk (Jedynak, Teczke, & Wyciślak. 2001) or creativity (Kosała, 2015).

Rogoda (2013, p. 47) bolsters the idea that entrepreneurship is a multidimensional occurrence, encompassing personal attitudes and characteristics, as well as business activities. Wach (2013, p. 247) confirms that entrepreneurship is a multithreaded research topic, which can be described from various angles:

- as a function of competitive behaviours that guide the market;
- as a function of implementation of new business ventures that lead to market changes;
- as a function of the market itself;
- as a function of an individual entrepreneur;
- as a process of:
  - creation of new market entities;
  - discovering of opportunities;
- as a function of personality;
- as a function of small and medium companies.

The link between all the above functions that define entrepreneurship is the person of **entrepreneur**. According to Lichtarski et al. (2001, p. 48) it is a person who:

- creates the enterprise;
- enhances technical change;
- introduces new products (or services);
- conquers new markets;
- makes organisational decisions.

The same link can also be traced in the definition of management by Grudzewski (2004, p. 9), who sees it as a series of the optimal possible decisions in an organisation. Out of the above a conclusion can be drawn: the notions of management, enterprise, entrepreneurship and entrepreneur possess a common element, which is the decision-making function.

1.2. Decision-making as a research problem

Doyle & Thomason (1999, p. 65) trace the formal studies on decision-making back to the 17th century. They state that these studies reached a point of maturity in the mid and late 1940s with reasonably solid mathematical foundations and practical quantitative, statistical methods. Sopińska & Mierzejewska (2014, pp. 38-39) observe the important and growing share of decision-making-related research topics in research grants attributed in the field of management. This points at the fact that a relatively important number of researchers place their scientific interests in the field of decision-making.

According to the Oxford Dictionary (2018), a decision is “a conclusion or resolution reached after consideration”. Bross (1953, pp. 18-32) emphasises the connection between a decision and an action, saying that a decision requires the choice of a course of action (Bross, 1953, p. 19). Wierzbicki (2018, p. 31) defines the decision as a choice between many possibilities, also called options, decision variants or decision alternatives[8]. An overview of decision definitions in the Polish literature is provided by Szarfenberg (2002b, p. 159). Bodnar (1985, p. 10) defines it as a non-random choice of action, a conscious settlement of a situation of choice, initiating an action Szewczuk (1985, p. 54) describes it as a conscious settlement of a situation of choice. In Woleński’s definition it (1987, p. 19) it initiates an action, it is a non-random choice between different alternatives of actions. According to Koźmiński & Piotrowski (1996, p. 88) it is a choice of possible variants of action in a given situation or a conscious choice of one of identified and perceived as possible variants of action. Jabłoński & Szarucki (2011, p.167) provide a few more definitions. They understand decision as an internal move, a free choice of future actions, being a result of transformation of wishes into goals (after Pszczołowski, 1978, pp. 44-45)

[8] Wierzbicki mentions that in the Polish language the use of word “alternatives” is not fully appropriate, as it means a choice between two options. Nevertheless, the terminology “decision alternative” will be employed throughout the text, as it is widely used in scientific publications on the Analytic Hierarchy Process, which is one of the basic research methods in this monograph.

or a conscious choice of action (from one or many solutions), preceded by a reflection and consideration (after Kozielecki, 1992, p. 155). Goldman's (2016, p. 11) definition requires special attention as short, but precise: a decision is a value judgement on the part of the decision-maker.

A multitude of notions related to decision-making can be found in the scientific literature. A basic set of concepts to be defined first includes such terms as decision, decision-making, decision-making process, decision-maker, decision problem and decision-making enhancement. The word cloud in Figure 1.1 illustrates the terms related to managerial decision-making.

Figure 1.1 Managerial decision-making - word cloud
Source: Compiled on the basis of vocabulary database created by the author, using www.tagxedo.com.

In their discussion of the types of decisions Prusak & Stefanów (2014, pp. 20-24) state that they can be divided into multiple categories out of which most common are:

- the number of decision criteria (single- and multicriteria);
- decision environment (private and professional);
- time of decision-making (impulsive – intuitive, naturalistic and argumentative – factual, systematic);
- the level of repetitiveness (primal and routine);
- character of activity (regulative, controlling, innovative);

- geographic coverage (local, regional, national, international);
- time span (short-, mid- and long-term);
- stability of decision environment (choice under certainty, uncertainty and risk).

It is difficult to find one binding definition of decision-making in the management science literature. Bradley (2017, p. 3) defines the decision-making process through a sequence of its elements. First, a decision problem has to appear. Second, it has to be acknowledged by its agents – the decision-makers. Bradley defines agents as "entities with the resources to represent, evaluate and change the world around them in various different ways, typically within the context of ongoing personal and institutional projects, activities or responsibilities" (Bradley, 2017, p. 3). Third, the decision problem has to occur in a context, the decision-making environment, which provides the limitations to the decision-making. These could come from scarcity of material resources, lack or limited information, and non-existing standards for evaluation of decision-making outcome or source of decision problems. Four, the existence of decision alternatives, i.e. different actions that can be undertaken by decision-makers to answer and, hopefully, solve the decision problem.

A choice of descriptive definitions of decision-making is listed in Table 1.2, out of which the author's original definition proposal is developed. It is interesting to observe that many of these definitions point at the consciousness of the decision-making process and the resulting necessity of initiating an action.

Table 1.2.

Overview of chosen decision-making definitions in management science

Author	Definition
Drucker (1986, p. 91)	Decision-making is a time machine that synchronises into one present a great number of divergent time spans. […] Our approach still tends toward making plans for something we will decide to do in the future, which may be entertaining but is futile. We can make decisions only in the present and yet we cannot make decisions for the present alone; the most expedient, most opportunistic decision – let alone the decision not to decide at all – may commit us for a long time, if not permanently and irrevocably.
Kurnal (1969, p. 368)	Decision-making is a constant situation of choice, congenital for performance of managerial tasks.
Penc (1997, pp. 126-127)	Decision-making can be defined as choice of one of at least two options, solutions (variants), paths or directions of action, desirable from the point of view of the interest (needs) of the system, in frames of which this choice is performed.
Polemarchakis (1998, p. 753)	To decide is to choose from sets of alternatives.

Piotrowski (2009, pp. 21-22)	Decision-making is an inherent element of private and professional life of every human being. At the same time it is one of our most common actions. It can encompass both everyday small activities and large-scale ventures, incorporating organisations and entire states.
Prusak & Stefanów (2014, p. 20)	Final decision is the choice of one of possible decision alternative in a given decision problem. Decision-making is the process of choosing this alternative.
Supernat (2000, p. 16	Making decisions is a special type of choice – conscious and leading to action.
Szarfenberg (2002, p. 2)	Decision-making is a conscious and non-random choice linked with action.

Source: Compiled on the basis of literature research.

The literature survey proves that it is difficult to define an optimal decision. The issue has been widely discussed since Simon published his satisficing decision (Simon, 1955) and bounded-rationality (Simon, 1957) concepts (although he discussed both in his earlier works as well). The lasting actuality of Simon's both concerns is proved by the huge number of most recent publications in the management science that still directly or indirectly address both issues. These include Bashar et al. (2018), Bouzarour-Amokrane, Tchangani & Peres (2015), Chun (2015), Gonzalez-Pachon & Romero (2015), Mitra (2017), Pinto (2016), Sakawa & Matsui (2014), Yano (2017) (satisficing decision) and Clement & Puranam (2018), Cui & Zhang (2018), Eliens et al. (2018), Felin, Foss & Ployhart (2015), Kim & Anand (2018), Mauer et al. (2018), Peysakhovich & Rand (2016), Schlaile et al. (2018), Schweisfurth & Raasch (2018), Sun, Karwan & Kwon (2018) (bounded-rationality).

An attempt at combining definitions from Table 1.2 and the concerns about the notion of optimal decision into one definition of managerial decision-making follows. For the needs of the research presented in this monograph it will be understood as a process that aims at changing the state of a decision problem encountered by the decision-maker in his or her managerial practice, in order to better adapt to changing reality of the business environment, but also leaving the decision-maker with the satisfaction about the final decision. According to the prescriptive approach, in terms of the proposed definition the final decision should be as close to an objective optimum as possible, but also taking into account the manager's (decision-maker's) preferences and limitations. Therefore from now on it will be referred to as the integral managerial decision. It could be defined in the following way: an integral managerial decision is the optimal possible solution to a managerial problem, fuzzified by the preferences and limitations of the decision-maker, resource scarcity and environmental constraints.

The process approach to decision-making comes from Simon (1957), who defined it as a three-stage process composed of: (i) intelligence; (ii) design and (iii) choice, later complemented by (iv) implementation and monitoring. More definitions followed, most of them exploiting and expanding Simon's division and analysing the nature of the decision-making process from different angles. Some authors perceive it as sequential, which means that it is composed of consequent elements, which will keep their order despite the character of the analysed decision problem. After Witte's empirical research a non-sequential perspective on decision-making processes gained preference. Witte (1972, p. 180) observed that the phases of decision-making occur rather in parallel, which results in their various combinations, when making decisions in managerial practice. Table 1.3 presents these different approaches to stage-based definitions of decision-making process that can be found in literature.

Table 1.3.

Stage-based definitions of decision-making process in management science literature

Author of the concept	Type of decision-making process	Stages of the decision-making process in management science
Anderson et al. (2011, p. 3)	sequential	1. Identifying and defining the decision problem (and goal of decision-making). 2. Presentation of possible solutions (decision alternatives). 3. Defining criteria that influence the analysis and choice of solutions (alternatives). 4. Evaluation of particular solutions with regard to alternatives. 5. Choice of best solution. 6. Implementation of chosen solution. 7. Evaluation of implementation results and recognition whether the problem has been solved satisfactorily.
Brim (1962, p. 9)	sequential	1. Identification of the problem. 2. Obtaining necessary information. 3. Generation of possible solutions. 4. Evaluation of possible solutions. 5. Choice of a strategy for performance. 6. Implementation of the decision.
de Caritat Condorcet ([1793] 1847, pp. 342-343)	sequential	1. Discussion of general principles that will serve as the basis for decision. 2. Examination of various aspects of the issue and consequences of different ways to make the decision. 3. Second discussion to combine opinions about the clarified question into a small number of more general opinions.
Dewey (1910, p. 72)	sequential	1. Felt difficulty. 2. Its location and definition. 3. Suggestion of possible solution.

Dewey (1910, p. 72)	sequential	4. Development by reasoning of the bearings of the suggestion. 5. Further observation and experiment leading to its acceptance or rejection.
Gorynia (1999, pp. 544-545)	sequential	1. Problem identification and diagnosis of situation through analysis of former achievements and aspirations of the organisation and its competitors. 2. Elaboration of decision alternatives based on actually available information. 3. Choice of an optimal or satisfactory decision alternative (depending on the adopted theory of the firm) and return to steps 1 & 2 in case of failure.
Hammond, Keeney, Raiffa (2002 - PrOACT)	sequential	1. Problem – identification and formulation of the right decision problem. 2. Objectives – specification of objectives in order to: (i) gather appropriate information; (ii) justify future decisions; (iii) rank the importance of decisions. 3. Alternatives – formulation of real and imaginary decision alternatives (aspirations, constraints, suggestions, experience, modes of operation, timing, assessment of alternatives). 4. Consequences – identification, definition and analysis of the consequences of each decision alternative. 5. Trade-offs – evaluation of trade-offs between objectives. 6. Uncertainty – clarification of level of uncertainty related to each decision alternative: (i) what are the key uncertainties? (ii) what is their provenance? (iii) what are their possible outcomes? (iv) what is the probability of occurrence of each uncertainty? (v) what are the consequences of each outcome? 7. Risk tolerance – assessment of desirability of each consequence, search for consequence balancing options, choice of least risky alternative. 8. Linked decisions – consideration of associated decisions – what new decision problems arise from the performed decision-making process?
Levin & Atkinson Kirkpatrick (1965, p. 105)	sequential	1. Formulation of decision problem, goal of decision-making, decision criteria and limitations. 2. Observation of decision problem's environment (preliminary research). 3. Construction of a mathematical or graphic decision-making model, incorporating all limitations of decision environment. 4. Choice of an appropriate method and data collection. 5. Application and empirical verification of the model. 6. Interpretation of obtained results.

Levin & Atkinson Kirkpatrick (1965, p. 105)	sequential	7. Optimisation and sensitivity analysis with regard to particular parameters of the model. 8. Implementation of results.
Mintzberg, Raisinghani, and Théorêt (1976, pp. 253-266)	non-sequential	• identification – (i) decision recognition – between incoming data; (ii) diagnosis of communication channels to identify the issue; • development – (i) search for ready-made solutions; (ii) design of new solutions or modifying existing ones; • choice – (i) screen – sifting of non-optimal ready-made solutions from too many to analyse; (ii) evaluation-choice between alternatives through judgement, bargaining and analysis; (iii) authorisation of selected alternative by the hierarchy.
Saaty (1996, p. 6)	sequential	1. Planning. 2. Generating a set of alternatives. 3. Setting priorities. 4. Choosing a best policy after finding a set of alternatives. 5. Allocating resources. 6. Determining requirements. 7. Predicting outcomes. 8. Designing systems. 9. Measuring performance. 10. Insuring the stability of a system. 11. Optimisation. 12. Resolving conflict.
Simon (1960, pp. 1-2)	sequential	1. Intelligence – finding occasions for making a decision. 2. Design – finding possible courses of action. 3. Choice – choosing among courses of action.
Practice of managerial decision-making	empirical	• diagnosis activity – consuming some of decision-maker's time; • design activity – consuming most of decision-maker's time; • choice activity – consuming little of decision-maker's time.

Source: Compiled on the basis of literature research.

The person making the decision is called the decision-maker. Wierzbicki (2018, p. 31) points out that such a definition can be misleading, as it suggests that the decision-maker is fully independent and final in his or her judgements. Taking into account the managers' prerogatives of applying managerial decision-making enhancement systems, this simplification need be rejected. One element seems missing, though, namely the responsibility for the results of the decision-making process. Consequently, with regard to particularities of the management science, this definition should take the form: the decision-maker is a person making the decision and bearing the responsibility for its consequences.

Bross (1953, p. 32) perceives the decision-maker as a pragmatic machine which acquires information and transforms it into a course of action. He continues saying that the mechanism consists of three basic components: the Prediction System deals with alternative futures; the Value System handles the various conflicting purposes; the Criterion integrates the other two components and selects an appropriate action.

The first step of any decision-making process consists of defining the decision problem. Bradley (2017, p. 4) perceives it as a situation in which the decision-maker has one or more options to choose from. The exercise of each option is associated with a number of possible consequences. Some of these are desirable from the perspective of the decision-maker's goals, while others are not. Which consequence will result from the exercise of an option depends on the prevailing features of the environment. Therefore a decision problem can be presented as a matrix of problem-solving options and their consequences. According to Prusak & Stefanów (2014, p. 20), a decision problem occurs when a man chooses from one of at least two possibilities. The final choice made depends on the decision criteria, i.e. factors that affect this choice.

Wierzbicki (2018, pp. 33-36) observes that initially decision-making enhancement focused on the computational help in gathering and processing data. With time, since the 1990s it evolved into commercially used Decision Support Systems (DSS), which moved their focus from the very fact of making the final decision towards complex support at earlier stages of decision-making process (i.e. intelligence and design) and learning from them. Also the perception of the role of the decision-maker has evolved, from substituting the human being (via automation of decision-making processes) to incorporating him or her into the decision enhancement tool as an active participant with administrative, creative and analytical attributes. This evolution is reflected by the changing definitions of decision-making enhancement. The narrow definition saw decision support as consisting of any and all data, information, expertise and activities that contribute to option selection (Andriole, 1989, after Wierzbicki, 1993, p. 159). Most current definitions concentrate on the multitude of types of decision problems and the need of adapting the proper definition to the analysed issue. Wierzbicki (2018, p. 37) proposes a definition of decision-making enhancement that is wide enough to incorporate this complexity, namely a toolbox of computational devices that aid the used [decision-maker] in leading an interactive decision-making process. Although this definition encompasses mainly IT enhancement tools, in the author's opinion it can be expanded onto managerial applications.

1.3. Scientific approaches to decision-making

In the past two decades innovative research trends have resulted in the creation of several scientific approaches to decision-making. These include the intertemporal choice theory (Berns, Laibson & Loewenstein, 2007), socio-cognitive engineering, which incorporates the findings of the game theory into decision-making perceived from the perspective of interactions between decision-makers (Sharples et al., 2002) or decision-making under complexity (Bennet & Bennet, 2008; Lusardi & Mitchell, 2009). Nevertheless, owing to the complexity of its scope and the number of publications, it is the decision theory that remains the most common scientific framework for decision-making.

Doyle & Thomason (1999, p. 55) describe the field of decision theory as "the merits and making of decisions […] developed by philosophers, economists, and mathematicians over some 300 years […] which exert major influences over virtually all the biological, cognitive, and social sciences". The interdisciplinary range of decision theory is also raised by Hansson (2005, p. 6), who states that while being a separate research field, decision theory profits from scientific interests of economists, statisticians, psychologists, political and social scientists and philosophers.

According to Okasha (2016, p. 410), the scientists who developed the concepts that became the basis for establishing the decision-making as a field of scientific research and foundations for the decision theory include philosopher Ramsey (1931), mathematician-economist tandem von Neumann and Morgenstern (1944) and statistician Savage (1954). Salo, Keisler & Morton (2011, p. 6) add the works of de Finetti (1937), Pratt, Raiffa & Schlaifer (1965) and Howard (1966).

Similarly, there is no consent among researchers about the founders of modern decision theory. Hammerstein & Stevens (2012, p. 2) refer to Pascal & Fermat, who introduced the notion of expected value in 1654. Okasha (2016, p. 411) points at Bernoulli's (1738) subjective value. Hansson (2005, p. 9) quotes de Caritat Condorcet's (1785, [1793] 1847]) division of the decision-making process, while Szarucki (2016, p. 92) refers to Barnard (1938) and his formalisation of organisational functionality. Porębski (1986, p. 16) highlights the importance of Arrow's (1963) impossibility theorem as a milestone for its development (although with a sociological bias).

Three main research approaches to decision-making in terms of the decision theory can be found in the literature, namely normative, descriptive and prescriptive (Bell, Raiffa & Tversky, 1988; Smith & von Winterfeldt, 2004).

Normative models describe decision-making processes of perfectly rational agents and derive from the concept of decision-maker's preferences and resource allocation according to the maximisation of his or her utility functions, which in turn can be defined as decision-maker's expected satisfaction (Bronfenbrenner et al. 1990). Kenrick et al. state that "utility-based theories of rational decision-making have a number of conceptually useful features". These include the possibility of transposing preferences into quantifiable units and therefore allowing their comparisons with the values of other goods, and create a mathematical basis for preference-based decision-making on resource allocation by humans (2009, p. 765). The same authors also observe that traditionally, microeconomic decisions have been modelled on broad notions of rational choice whereby entities attempt to maximise their utility, or expected satisfaction (Kenrick et al., 2009, p. 764). Rational choice decision-making approaches can be found in almost every economic textbook, e.g. Krugman, Obstfeld & Melitz (2014), Husted & Melvin (2012), Mas-Collel, Whinston & Green (1995), Bronfenbrenner, Sichel & Gardner (1990).

The problem arises when the assumption of decision-maker's rationality is questioned. Managerial decision-making very often involves the need of making decisions about distant future (e.g. long term strategic planning), basing on the current resource availability, technology, knowledge and environmental conditions. Additionally, managers have to anticipate decision problems from various areas (e.g. human resources management, production engineering and finance). Although all these decisions are of microeconomic nature, they require different tools, individual approaches and alternative allocation of resources.

Therefore descriptive decision-making models concentrate on real-life decision-making patterns, putting into doubt the empirical contradictions to decision-makers' perfect rationality paradigm (Tversky & Kahneman, 1974; Kahneman, Slovic, Tversky, 1982) and the similarity of preferences theory (Becker, 1981). Another incentive to question the normative approach was the observation by Simon (1957), who analysed the efficiency of optimisation vs. heuristic decision-making and opted in favour of the last one.

The prescriptive approach to decision-making focuses on the practical side of decision-making, taking into account decision-maker's imperfect and limited rationality and biases (Baron, 2000; Gigerenzer & Selten, 2002), but aiming at the maximal possible rationalisation of the decision-making process through the application of de-biasing tools (Fischoff, 1999; Edwards & von Winterfeldt, 1986). Phillips (1984) explains that besides enhancing the quality of the final decision made, the goal of prescriptive decision-making is to leave the decision-maker with a satisfactory choice that follows his or her preferences and understanding of the decision problem and its environment. This approach has been adopted by the author to develop his definition of managerial decision-making, formulated in sub-chapter 1.2.

Table 1.4 presents a comparison of the three leading streams of the decision theory.

Table 1.4.

Decision theory approaches in management science

Characteristics of decision-making	**DECISION THEORY APPROACHES**		
	Normative	**Descriptive**	**Prescriptive**
Chosen theoretical contributors (alphabetically)	de Finetti, Howard, Levin, Morgenstern, Pratt, Ramsey, Raiffa, Savage, Schlaifer, von Neumann	Allais, Brehmer, Ellsberg, Fischoff, Gigerenzer, Hammond, Kahneman, Klein, Lichtenstein, Phillips, Selten, Simon, Slovic, Stewart, Steinmann, Tversky	Brach, Edwards, Howard, Keeney, Łukasiewicz, Matheson, Saaty, von Winterfeldt, Zadech

Focus	how people should decide with logical consistency	how and why people decide in reality	how to prepare people to make better decisions and help them to do so
Rationality	objective	subjective	subjective, but aiming at enhancing objectivity
Criterion	theoretical adequacy	empirical validity	efficacy and usefulness
Scope	all decisions	already analysed types of decisions	decisions targeted at specific problems
Theoretical framework	axioms of utility theory axioms of probability theory	cognitive sciences psychology (beliefs and preferences)	normative and descriptive theories axioms of decision analysis
Main models	expected utility model (von Neumann & Morgenstern, 1944) subjective expected utility model (Savage, 1954) Bayesian statistics	Allais's paradox (Allais, 1953) Ellsberg's paradox (Ellsberg, 1961) Prospect Theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) Social Judgment Theory (Hammond et al., 1986) Naturalistic Decision Making (Klein, 1999)	Multicriteria decision making (MCDM) models, including the Analytic Hierarchy Process Saaty, 1980) and the Analytic Network Process (Saaty, 1996) Fuzzy logic (Zadech, 1965, 1973) Real Options Model (Brach, 2002) Stanford University School of Decision Analysis (Howard & Matheson, 2004) Value Focused Thinking (Keeney, 1992)
Operational focus	Analysis of alternatives determining preferences	prevention of systematic human errors in inference and decision-making	procedures and processes end-end decision life cycle
Experts	theoretical sages	experimental researchers	applied analysts
Employed methods	quantitative	qualitative	joint (quantitative-qualitative) or mixed (qualitative + quantitative)

Source: Compiled on the basis of Tang (2006, p. 27), Phillips (1984).

In the Polish literature a bi-divisional approach is more common (Szarucki, 2016, p. 92), although no unified terminology has been elaborated. We are offered (i) descriptive and formalised (Zieleniewski, 1969, p. 63); (ii) psychological, process-oriented and formalised theory of choice (Wawrzyniak, 1977, 22-23); (iii) quantitative and qualitative (Targalski, 1977, p. 8); (iv) normative and descriptive (Mesjasz, 2012, p. 86). Regardless of the adopted terminology, the types of determinants of each approach are similar, namely quantitative for normative-like, qualitative for descriptive-like and mixed for prescriptive-like.

The stability of decision environment criterion is the most common division pattern in terms of the decision theory. The difference lies in the consciousness of the decision-maker of the consequences of his or her choice. Decisions made under certainty have known, easily predictable consequences, e.g. hitting a wall at high speed when driving a car will most probably result in death or serious injuries of its driver and passengers. Decisions taken in the context of uncertainty are basically the opposite, as the decision-maker does not know, nor can predict the consequences of his choice of a given decision alternative instead of another one. For example, investing into a highly innovative start-up in an emerging branch of industry does not provide any certainty on return on investment, its time span, or even the guarantee of withdrawing the money invested. In fact, the majority of managerial decisions are taken under risk. Even though every decision taken can result in a variety of consequences, their set is finite, known to the decision-maker (at least to some extent) and the probability of their occurrence can be approximated. Conscious and purposeful counterbalancing of taken risks with expected income is the key mechanism of management in capitalist economies.

It is this need for a structured, scientific and more effective risk management (originating in the business environment) that has created solid grounds for the construction of models that aim at enhancing managerial decision-making. Practical applications of decision theory range from providing mathematical foundations for microeconomics to daily application in a range of fields of practice, including finance, public policy, medicine, and now even automated device diagnosis (Doyle & Thomason, 1999, p. 55)[9]. Until now, the largest juxtaposition of decision analysis applications has been presented by Keefer, Kirkwood and Corner (2004).

Justification of a constant (and growing) need for the creation of decision-making models can also be found in the literature. Ackoff (1967) observes that in most management problems there are too many possibilities to expect experience, judgement, or intuition to provide good guesses, even with perfect information (p. B-150). He also states that most managers receive much more data (if not information) than they can possibly absorb even if they spend all of their time trying to do so. (Ackoff, 1967, p. B-148). Rappaport (1968) draws a conclusion by stating that the fact that managers cannot easily convert data to information underlies the very need and justification for developing management decision models (p. B-136).

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[9] To provide an idea on the extensity of the research field in modern science "decision theory": at the moment of writing this monograph (as of November 15th, 2018) the Web of Science Core Collection returned 75 265 scientific works, out of which 8 890 in Management science, Google Scholar showed 2 030 000 citations and Google Books showed around 131 000 research results for past ten years (2007-2017).

Simon (1955) provides the reasons for a more formalised managerial decision-making process:

- discrepancy between economic rationality and uncertainty of business environment;
- choice of an optimal managerial decision is detached from the reality of business environment;
- limited nature of human perception, which in combination with limited rationality of decision-makers leads to the acceptance of only satisfying rather than optimal possible decision alternatives.

On the basis of the above Simon (1955) draws a conclusion: the focus of managerial decision-making processes should be placed on the development of such decision-making criteria that would guarantee the fulfilment of minimal conditions defining satisfying problem solving.

It is interesting to see how decision-making is perceived in cultures other than the Western one. For example, in Japan decision-making is perceived as an immanent and important part of the organisational development and self-improvement process. Czekaj (2013, p. 23) observes that in Japan modern organisations follow the idea of a constant self-improvement, that derives from the idea of kaizen. The same author describes it as a process of continuous, never ending self-amelioration, that includes a systematic search for reasons of arising problems and introducing changes to the organisation or its elements. This process aims at raising the efficiency of goal achievement and involves all managerial levels and regular employees. One of the tools of this constant self-improvement is such an enhancement of organisational decision-making that would continuously bring new decision determinants into the scope and at the same time develop tools for obtaining faster and more precise final decisions. These reasons seem contradictory only until the decision-maker considers the use of multicriteria decision-making models. These models allow the incorporation of many decision factors of various nature (both quantitative and qualitative) and a smooth passage through the decision-making process, e.g. by providing space for the valuation of external experts, not directly related with the company.

2. MULTICRITERIA DECISION-MAKING METHODS IN MANAGEMENT SCIENCE

2.1. The essence and justification of multicriteria approach to managerial decision-making

Wachowicz (2015, pp. 402-403) defines Multiple Criteria Decision Analysis (MCDA), also called Multiple Criteria Decision Making (MCDM)[10] as a toolbox part of the Operations Research scientific discipline for solving decision problems, when the set of decision alternatives is relatively easily identifiable, but there is a multitude of decision criteria. For this purpose a set of analytical tools is being developed, which help the decision-maker in properly defining the decision problem, understanding its structure, sort and filter available information, hierarchize and prioritize decision-makers' preferences and creating a system for the evaluation of decision alternatives. Wachowicz (2015, p. 402) continues by providing a set of features typical of MCDA-based decision-making:

- a multitude of goals;
- difficulty of clearly pointing at one and only solution (decision alternative);
- serious gravity of the decision problem (after Figuera, Greco & Ehrgott, 2005);
- low tolerance for ineffective or faulty decisions;
- unacceptability of fast heuristics (after Gigerenzer & Todd, 1999).

The importance and growing potential of MCDM concepts have already been noted by Ackoff (1956, p. 265), who wrote that operations research is neither a method nor a technique; it is or is becoming a science and as such is defined by a combination of the phenomena it studies.

There are several reasons for applying the multicriteria approach to managerial decision-making. First, most managerial decision problems are complex – in their field, subject, nature, range, environment and implications. Ackoff (1956, p. 287) supports this thesis by saying that a problem never exists in isolation – it is surrounded by other problems in space and time. Wierzbicki (2018, p. 33) supports this statement by writing that most decision-making processes have a multicriteria and dynamic character. Second, the multicriteria approach allows the quantitative and qualitative determinants of decision-making processes to be incorporated into the scope of analysis. This issue needs further explanation.

It is interesting to observe how the quantitative approach became the predominant research methodology in the academic discipline of economics,

[10] In scientific literature both acronyms, MCDA and MCDM are often used interchangeably, which is not precise, as MCDA defines an entire field of research, whereas MCDM is a result-targeted practical process. Following this reasoning, MCDM would be a part – of crucial importance – of MCDA.

even though its overriding category, the social sciences widely use the qualitative research methods. Burns & Groove (2005, p. 27) define quantitative research as a formal, objective, systematic process in which numerical data are used to obtain information about the World. They also point at the following applications of this type of research methods:

- to describe variables;
- to examine relationships among variables;
- to determine cause-and-effect interactions between variables.

The qualitative methodology, however still not too popular in managerial decision-making, has recently gained some attention. The reasons are:

- constantly growing complexity of economic phenomena;
- need for expanding the spectrum of managerial decision-making by hardly measurable or immeasurable determinants of the socio-economic environment;
- necessity of more precise mapping of multicriteria decision-making processes;
- need of more precise recognition and modelling of their driving forces.

Barkin (2009, p. 211) states that the term 'qualitative' evokes a narrative or analytical richness, a method that brings out more detail and nuance from a case than can be found by reducing it to quantitative measures. Unfortunately, Hoffmann's (2009, p. 188) observation that in practice the term is generally used simply to mean not quantitative is quite often true.

Stemplewska-Żakowicz (2010, p. 88) explains that the qualitative and quantitative research approaches define the sense of research process differently. While the quantitative methodology aims at understanding and controlling the analysed phenomenon, the qualitative one focuses on understanding it through the perspective of its participants. Therefore the knowledge resulting from the qualitative research cannot be objective. Moreover, it is perceived as valuable only if it reflects adequately the subjective senses and perspectives. Nevertheless, Stemplewska-Żakowicz (2010, p. 91) maintains that the qualitative methods are useful when the results of objective tests are not explanatory enough. In that case they should be accepted as methods that deepen the understanding of research done with the quantitative methods. This approach has been adopted in the present monograph.

The qualitative research is sometimes perceived as not relevant, due to low representativeness of the research sample. In fact, this is a common misunderstanding about the nature of this type of methods, as subjectivity provides the respondent with enough representativeness – due exactly to his or her individual perspective. It is important to underline, though, that drawing general conclusions on such a basis would be a methodological error.

To summarise this dualism, the quantitative research approach can be understood as a search for common, repetitive, objective characteristics in a mass of people, whereas the qualitative approach aims at finding unique and subjective features of individuals that co-create this entity. In the managerial decision-making tasks an overattachment to one group of research methods only can cause some threats that will be enumerated below[11].

Goczek (2010, pp. 205-206) points at several problems that result from concentrating exclusively on the quantitative (statistical) analysis, without taking into account the qualitative (contextual) factors:

- concentration on the method of estimation – when the focus of a quantitative researcher on the compliance with statistical rules leads to the omission of the qualitative aspects of the analysed phenomenon; for instance, a simple quantitative estimate of the number of faulty machined products is just a statement; only a qualitative analysis provides the researcher with the knowledge about the causes of these aberrations that can reduce (or eliminate) them in the future;
- partial rejection of quantitative variables for theoretical reasons – some research provides the contextual (qualitative) ground for the rejection of some variables, even though they conform to the quantitative research criteria; an example: replacing nominal values with real ones;
- lack of deeper knowledge on the research object and its environment including the characteristics of decision-making environment, the sense and logic of decision-making process, the definition of variables of decision-making model, etc. can strongly affect the quality of the incoming quantitative data; an example: faulty estimation of the technological quality of products may result in rejecting good products that have been mistakenly assessed as wrong ones; the author observes that data are simply numbers with some context and only this context provides them with significance;
- sometimes the choice of a quantitative method precedes a solid study on the research object, which constitutes a dangerous malpractice, namely, although the obtained results will conform to the quantitative approach criteria, their credibility will be doubtful; this is due to the fact that the main weight has been put on the methodological correctness rather than focusing on the scientifically valid image of reality; an example: the correlation analysis without being preceded by a minutious analysis of the character of compared variables; errors of this type are hard to discern, because the methodological perfection effectively masks shortcomings in the theoretical and contextual bases of the decision-making model in question;
- omitting the endogenousness and identifiability of some model variables i.e. introducing as continuous the variables of a quantitative model determinants that reflect irregular, exponential or discontinuous phenomena (qualitative

[11] A deeper analysis of this problem can be found in Gawlik, R. (2016). Methodological Aspects of Qualitative-Quantitative Analysis of Decision-Making Processes. Management and Production Engineering Review, 7(2), 3-11.

characteristic); they may badly affect the calculations of statistical values, while, on the contrary, they should be seen as random factors only; example 1: the car industry crisis caused by a supply shock on the oil market, without noticeable reasons for changes in car demand; example 2: unexpected machine tool vibrations in some ranges of rotation speed;
- type III error risk (the right answer to a wrong question) – when proper answers lead to wrong conclusions; the researcher obtains a precise image of an occurrence, but not the one initially targeted by research; an example: question: what is a device that performs various operations on objects, such as cutting, drilling, deformation or facing? Correct answer: a machining tool; wrong answer: a lathe; explanation: although a lathe is indeed a machining tool, not all machining tools are lathes.

Meanwhile, concentrating on the qualitative methodology only also generates some limitations. Barkin (2009) enumerates them:
- negative associations of this term in social sciences (qualitative research is perceived as simply non-quantitative, therefore not systematised, scientifically unsound);
- in the eyes of some colleagues, the application of qualitative methods disqualifies the research task as non-scientific, because it impedes an explicit assignment of the research object to a specific branch of science;
- teaching qualitative methods brings counterproductive results – it gives the students a set of fuzzy criteria rather than clear and precise methodological indications (which does not enhance the development of a sound scientific apparatus);
- as the qualitative methods seem to be easier to apply than quantitative ones, they tend to be overused, which is not always correct or possible.

Other types of errors may occur when the context (qualitative variables) becomes more important than mass phenomena (quantitative variables). According to Goczek (2010, pp. 207-208) these include:
- research populism – lack of confirmation of some popular theories in empirical data;
- gaps in researcher's mathematical and statistical apparatus – misunderstanding of data generation and collection processes, amplified by the use of inadequate or outdated methods of statistical analysis;
- logical error – confusion between correlation and causality;
- methodological inadequacy of the researcher – disregarding the stationarity of time series, properties of research instruments, autocorrelation and heteroscedasticity of random residuals;
- hasty conclusions based on result estimates. Incidentally, most of these errors could be avoided by promoting ubiquitous cooperation of researchers and

statisticians (e.g. a compulsory consultation of research plan, methodology and final text of the paper with a statistician, prior to publication).

Piech (2000) warns against employing research approaches prior to the analysis of a more comprehensive research context (e.g. the size of an organisation). He observes that in companies it can be hard to perform qualitative analysis because of "information noises", i.e. singular opinions of employees, which are not confirmed by other people from the company. He also observes that if the qualitative questionnaire covers a larger sample, some recurring opinions can be identified and treated as dominant – producing an image of the company in the eyes of most employees. However, this is not the case of SME, in which case each opinion can be true and refer to a specific field of the company (e.g. someone's worksite).

A good summary of this discussion is provided by Goczek's (2010, p. 204) observation that in Poland there is a tendency to strictly observe the distinction between "quantitative" and "qualitative" methods understood separately. This is difficult to understand, as both groups are indispensable to each other, especially in practical applications in social sciences, including economics and management. At the same time, the analytic hierarchisation methods, with their fuzzy versions are applied rather infrequently. Meanwhile, as already mentioned, limiting the description of managerial environment only to its quantitative determinants restricts its complexity to an unacceptable extent.

A solution to the limitations of both methodologies is the application of a joint qualitative-quantitative approach. Dixon & Reynolds (2005) present the methodology of building quantitative models based on qualitative data in sociology and political sciences, whereas Zaborek (2009) discusses their adaptation to managerial problem solving.

Although the application of the joint qualitative-quantitative approach is justified in multicriteria decision-making models, it shows some limitations as well. In his discussion of the statistical hierarchization in multidimensional models Kukuła (2012) states that there are various methods of standardisation of quantitative attributes. The problem becomes more complicated when both quantitative and qualitative attributes come into question, as when the research sample contains both quantitative and qualitative attributes at the same time. It is then justified to ask which group of methods is particularly exploitable for the managerial decision-making applications. Mikuła & Potocki (1998, p. 71) associate the appropriateness of a selected decision-making method with the types of organisational climate (Table 2.1).

This division implies an evolution of applied decision-making methodology together with decision-maker's growth of awareness, which is reflected in the organisational climate characteristic of a given company. A study on the relation between manager's maturity and his or her decision-making patterns has been done by Gawlik, Grzesik & Kwiecińska (2018). As the majority of criteria that characterise such notions as organisational climate, awareness, maturity and their evolution would be of immeasurable nature, they form another justification for the incorporation of qualitative decision-making factors into the analysis, which in turn is an argument for the development of multicriteria qualitative-quantitative decision-making models.

Table 2.1.

Decision-making method vs. types of organisational climate

Organisational climate	**Authoritarian – autocratic**	**Bureaucratic**	**Innovative – technocratic**	**Gregarious**
Decision-making method	Decision-making is a result of power play between various connections and interrelations; decision-making methods are not employed	Procedure-oriented decision-making, choice of methods according to the situation and respective procedures	Task-oriented decision-making, flexible choice of methods	Employee-oriented, methods aim to protect the team from decision-making effects, and act as counterbalance the social system of the organisation

Source: compiled on the basis of Mikuła & Potocki, 1998, p. 71.

Drucker (1986, pp. 319-329) perceives managerial decision-making as a culture-dependent phenomenon. In his discussion of the differences between the Western and Japanese decision-making patterns, he explores this assumption further by saying that in the West, all the emphasis is on the answer to the question, whereas in Japan the important element in decision-making is defining the question. […] The answer to the question (what the West considers the decision) follows from its definition (Drucker, 1996, p. 320). Therefore the Japanese decision-making method is composed of four essential elements:

- definition of the real question – what is the decision-making all about?;
- pursue of dissenting opinions and formulation of a consensus about the true nature of the problem;
- exploration of the existing decision alternatives, rather than focusing on the right-and-only solution;
- finally, finding out at what managerial level and by whom precisely the decision should be made (Drucker, 1986, p. 322).

The entire process creates an additional added value – through the participation in the decision-making process it builds commitment to the realisation of the final decision and eliminates the need of "selling" it to the employees. One could risk a thesis that this is what makes the decision-making effective. All the proposed stages of Japanese managerial decision-making are dependent on a substantial amount of measurable and immeasurable decision criteria and a multitude of decision-makers – similarly to the Western culture. Therefore, a multicriteria approach seems to be justified regardless of the cultural background of the decision-making process.

Stemplewska-Żakowicz (2010, p. 89) tries to anticipate this problem by proposing a set of criteria to provide the representativeness and reliability of a qualitative research task (after Lincoln & Guba, 1985). Table 2.2 presents such sets for both qualitative and quantitative approach.

Table 2.2.

Criteria of methodological evaluation of quality of research techniques and procedures

Quantitative approach criteria	Qualitative approach criteria
internal accuracy	credibility
external accuracy	transferability
reliability	dependability
objectivity	confirmability

Source: Stemplewska-Żakowicz (2010, p. 89) after Lincoln & Guba (1985).

The same author summarizes the pros and cons of both approaches by saying that compared with the quantitative research, the qualitative approach shows one serious weakness, namely it does not lead to certain and universal knowledge. She explains that "certain knowledge" does not mean truthful always and everywhere (no procedure in social sciences field of science can assure such wisdom), but rather the kind of knowledge for which the applicability requirements and error criteria are known. Moreover, the qualitative approach can lead to such knowledge, but at a cost of serious limitations of its questions and reduction of meanings. Nevertheless, only at such a cost are scientists able to create just and accurate research tools to understand not only mass phenomena, but also their context (Stemplewska-Żakowicz, 2010, p. 90).

All of the above proves that the managerial decision-making processes require the application of a mixed multicriteria qualitative-quantitative approach. Joint research methodology helps the decision-maker to scientifically anticipate the impact of qualitative and quantitative determinants of the decision problem in one decision-making model. Thus, in the analysis of the decision-making processes of managerial nature, it is fully justified to search for the existing mathematical instruments for the enhancement of decision-making and develop innovative ones. The creation of new multicriteria decision-making models can bring innovative contributions to the management science. A brief review of the main multicriteria decision-making methods in scientific literature of management field of study is offered in what follows.

2.2. Overview of multicriteria decision-making methods in scientific literature

Ćwiklicki (2011, pp. 51-53) looks into two problems that arise when trying to structure managerial methods: different precision levels of the characteristics of the same method provided by different researchers and the evolution of the method with time. The first issue is due to:

- the description subjectivity and the perspective of the scientist who employs original or second hand sources for his studies;
- the fact that some companies perceive their managerial and organisational methods as a source of competitive advantage and tend to keep it private (after Czekaj, 2000, p. 7);
- the ownership of some methods that belongs to consulting companies and research institutions (after Teczke, 1993, p. 65).

The second issue emerges from the evolutionary changes in the method itself, which can cause an obstruction related to its particulars, ignorance or quasi-evolutionary applications of the already known method to new research tasks. Nevertheless, being aware of these limitations, an attempt to present a review of scientific multicriteria decision-making methods that can be applied to managerial problem solving will be undertaken.

Triantaphyllou (2000, p. 4) declares that the most commonly used multicriteria decision-making methods are the Weighted Sum Model (WSM), the Weighted Product Model (WPM), the Analytic Hierarchy Process (AHP), the Revised Analytic Hierarchy Process, the ELECTRE Method and the TOPSIS method. However, being aware of the constant dynamic development of this field of study, the author has decided to employ Trzaskalik's (2014, pp. 241-249) division of multicriteria decision-making methods into seven groups[12]:

- additive methods;
- methods of analytic hierarchization and related;
- verbal methods;
- ELECTRE methods;
- PROMETHEE methods;
- methods based on reference points;
- interactive methods.

The first group, the additive methods have a common denominator in the modelling of decision-making process through additive linear functions. The choice of a decision alternative is based on the highest weighted sum of evaluations or the highest utility rank. The ranking is based on the changing level of criteria fulfilment from least to most desirable. Particular methods from this group differ mainly in the procedure of evaluation of decision alternatives, i.e. the calculation of matrices of normalised evaluations or the sum of ranks. The examples of additive multicriteria decision-making methods include SAW (Simple Additive Weighting Method), F-SAW (Fuzzy Simple Additive Weighting Method), SMART (Simple Multi-Attribute Ranking Technique) and SMARTER (Simple Multi-Attribute Ranking Technique Exploiting Ranks).

[12] This discussion can also be found in Gawlik, R. (2016), *op. cit.*

The second group, the methods of analytic hierarchization base on the Analytic Hierarchy Process (AHP), which is characterised by the creation of a vector of scale through pairwise comparisons of the decision-making criteria (each with each). The selected decision alternative is the one that maximally meets all the criteria simultaneously. Wierzbicki (2018, p. 33) points at the fact that such an approach to decision-making is quite close to reality, when usually a compromise between several indexes needs to be found in the course of the decision-making process. He goes even further by stating that this search for a compromise is the main aspect of decision-making (Wierzbicki, 2018, p. 33).

The other analytic hierarchization methods are developments of the AHP elaborated in response to its criticism. They include alternative judgement scales for ranking the decision-making criteria and additional measures to assure the composition of criteria hierarchies. Other examples of multicriteria decision-making methods based on analytic hierarchization include REMBRANDT (Ratio Estimation in Magnitudes or deciBells to Rate Alternatives which are Non-DominaTed), F-AHP (Fuzzy Analytic Hierarchy Process), ANP (Analytic Network Process), F-ANP (Fuzzy Analytic Network Process) and MACBETH (Measuring Attractiveness by a Categorical Based Evaluation TecHnique). Dytczak & Wojtkiewicz (2010, p. 399) expand this set by two more methods: MUZ (Pl.: Metoda Unitaryzacji Zerowanej [Zero Unitarisation Method]) and DEMATEL (DEcision-MAking Trial and Evaluation Laboratory), Cluster Analysis (Pl.: taksonomia wrocławska [Wrocław Taxonomy]). Góralski & Pietrzak (2011, pp. 63-64) describe MUZ as one of the methods that allow normalisation of diagnostic variables through the analysis of a characteristic's range. Kukuła (1999, p. 17) adds that MUZ is a universal method that can be applied for the normalisation of various variables, independent of their type, sign, size, unit. Yang et al. (2008, pp. 161-162) describe DEMATEL as a tool for the formation and analysis of causal links between evaluation criteria. Lin & Tzeng (2009) discuss its application to derive schemes of interdependencies between decision criteria. ĆwiąkałaMałys & Nowak (2005, pp. 56-63) describe the cluster analysis as a method that can be successfully used for linking objects (variables) into homogenous groups in respect of *n*-characteristic (dimensions).

The third group, the verbal methods concentrate on the qualitative variables only (disregarding the quantitative variables). For the reasons presented in the preceding chapter, their application for managerial decision-making seems to be too restrictive. The main verbal methods are ZAPROS (Rus.: ЗАмкнутые ПРоцедуры у Опорных Ситуаций [Closed Procedures for Reference Situations]) and its development ZAPROS III.

The fourth group, ELECTRE (French: ELimination Et Choix Traduisant la REalité) methods base the analysis of significance rankings of decision-making criteria on four preference levels: strong, weak, equal and incomparable. The incomparability of criteria provides an argument in favour of the analytic hierarchization methods, where independence of criteria is a condition sine qua non for the construction of a hierarchical model. It is important to add that the lack of relation between the decision-making criteria does not have to imply their incomparability. The ELECTRE methods anticipate this problem by introducing equivalence thresholds and preferences of grouped (mutual) relations, as well as the rule of limited compensation. Subsequent versions of ELECTRE method (ELECTRE I, ELECTRE Iv, ELECTRE Is, ELECTRE III, ELECTRE TRI, ELECTRE I+SD, ELECTRE III+SD) differ mainly in the way of defining thresholds of evaluations of the decision-making criteria and how to clarify the ambiguities (with or without participation of a decision-maker/expert). Budziński (2010, p. 20) extends this set by ELECTRE's evolutions: QUALIFLEX, REGI-ME, ORESTE, ARGUS, EVAMIX, TACTIC and MELCHIOR.

The fifth group of multicriteria methods for the enhancement of managerial decision-making is PROMETHEE (Preference Ranking Organisation METHod for Enrichment Evaluations) tools. For each decision criterion the preference function is derived from the differences in evaluations of significance of decision alternatives. A serious difference means a strong preference for a given decision alternative. Particular methods from this group (PROMETHEE I, PROMETHEE II, EXPROM – EXtension of the PROMethee method, EXPROM II, PROMETHEE II+veto, EXPROM II+veto, PROMETHEE II+veto+SD, EXPROM II+veto+SD) differ mainly in the way of the calculation of outranking flows (the extent to which one alternative outranks others in the eyes of decision-makers). The methods with a "veto" or "veto+SD" mark are combinations of the base PROMETHEE methods with adequate ELECTRE methods.

The sixth group of methods is those based on reference points. In this group, the essence of calculations is to determine the extremities – ideal and anti-ideal solutions, which are the reference points in question. The next step is the measurement of distance of each decision alternative from both extremities. The alternative closest to the ideal solution is acknowledged as the highest attainable optimum. Particular methods differ in the measurement of this distance. The multicriteria decision-making methods based on reference points include TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), F-TOPSIS (Fuzzy Technique for Order Preference by Similarity to Ideal Solution), VIKOR (Sr-SP: VIsekrzterijumska Optimizacija i Kompromisno Resenje [Multicriteria Optimization and Compromise Solution]), BIPOLAR and its evolution – modified BIPOLAR. In practical applications, combinations of the reference-point-based methods are used, e.g. DEMATEL+ANP+VIKOR or BIPOLAR+SD. In this case each decision-making level is managed by a single method.

The seventh group, the interactive methods are based on individual evaluations of decision alternatives or their groups by the decision-maker. His or her preferences provide a basis for the calculations that arrange decision alternatives in respect to their distance from the ideal solution. If needed, the process is repeated until a satisfactory approximate solution is reached (the subjective assessment of the decision-maker). Therefore the application of a selected interactive method for a given decision task will change at the very moment of decision-maker's intervention. Also his or her satisfaction at fulfilling a given sub-criterion or a parent criterion is always subjective and can only be approximated. The interactive multicriteria methods encompass STEM-DPR (STEp Method for Discrete Decision-making Problems under Risk), INSDECM (INteractive Stochastic Decision-making) and ATO-DPR (Analysis of Trade-Offs for Discrete Decision-making Problems under Risk).

Budziński (2010, p. 20) expands this set by an eighth group, namely PCCA (Pairwise Criterion Comparison Approach) methods. These methods employ a two-stage approach: first, the decision criteria are clustered with a fuzzy binary preference index; second, the partial indexes are aggregated into a global index that expresses fuzzy preferences for each pair of alternatives. PCCA methods include PAC CA, MAPPAC, PRAGMA, IDRA and PACMAN versions of this approach.

Some authors postulate the use of statistic methods for the assessment of results of decision-making, e.g. the quantitative analysis of decision and information streams (Woźniak, 1980), multidimensional comparative analysis (Wydymus, 1984); random segment variables (Smaga, 1980), stochastic models (Kot, 1984). However, the goal of this monograph is the enhancement of the decision-making process since its early stages. A post-factum evaluation of decision-making is just one of the considered steps, whereas the quoted group of methods concentrates mainly on this aspect.

The multicriteria character of managerial decision-making induces the need for the application of methods that enable the quantification of qualitative criteria. At the same time, most managerial applications allow the hierarchization of decision criteria. Therefore out of the eight types of multicriteria decision-making methods presented above the analytic hierarchization group of methods has been selected for further analysis.

2.3. Choice of method for the managerial decision-making processes enhancement

Prusak & Stefanów (2014, p. 25) pose a vital question – what types of decisions require the application of MCDM methods? They answer it by providing two tables, one grouping professional, private, routine and essential decisions, and the second one grouping essential, routine, impulsive and factual decisions. In their opinion professional-essential and essential-factual decisions automatically require the application of the multicriteria decision-making enhancement tools. In other cases the following rules need to be adopted:

- private-routine decisions never require multicriteria decision-making framework or enhancement;

- most private-essential and routine-factual decisions neither require multicriteria decision-making support, however in some special cases exceptions to this rule can occur;
- professional-routine and essential-impulsive decisions do not fall under multicriteria decision-making support rigour, however a fast check of the available decision alternatives is usually necessary (Prusak & Stefanów, 2014, pp. 24-27).

Regardless of which method for the enhancement of multicriteria decision-making process is being considered, a necessary, but not always sufficient condition for the successful choice is an accurate preliminary observation of the decision problem environment. A special attention should be paid to the proper structuring of the decision problem, which counts for an important part of decision-making process efficiency. The predominant approach in the literature is the CAUSE framework (Criteria, Alternatives, Uncertainties, Stakeholders, Environment) proposed by Belton & Stewart (2002, pp. 44-46).

Prusak & Stefanów (2014, p. 20) observe that none of the decision-making enhancement methods can guarantee fully optimal and objective results, due to several reasons:

- decision-making models are mere simplifications of reality;
- not all the determinants of decision-making process are known;
- decisions of other agents that could affect the process are unknown;
- assessment of particular elements of decision-making models is subjective and dependent on individual preferences, motivations and intuition of the evaluator;
- experts not always possess credible information about the decision environment;
- imperfections of gathering of data or evaluations;
- uncertainty about the consequences of made decision.

Even if complete objectivity and ideal choice of a decision support method are not possible, it is important to determine the key success factors for appropriate structuring of decision problems. These include:

- a clear definition of the decision problem under inspection;
- an assessment of decision environment as complete as possible;
- a deliberate choice of the set of input variables;
- identification of threats and uncertainties originating from possible solutions;
- choice of a research method adequate to the nature of the analysed phenomenon;
- possibilities of application of the selected method according to its rules and limitations;
- practical applicability of output variables;
- implementation limitations;
- attempt at prediction of new problems generated by solving the actual decision problem.

Once the above criteria have been fulfilled, it is possible to properly select the methods for the managerial problem solving enhancement. Nevertheless, owing to the uncertainty and turbulence of business environment, it will always constitute an approximation.

Wachowiak (2002, pp. 52-67) proposes an overview of the most popular techniques applied to decision-making in business organisations. Some questions, however, remain unanswered, i.e. which method is the appropriate one to help solve a given decision problem? What makes a decision-making enhancement method the proper one? “A better final decision” would be the easiest answer. Even if intuitively hard to contradict, this answer does not provide any insight into the decision-making process. Additionally, the word “better” should be defined separately for each decision, in order to provide a possibility for comparison.

The goal of decision-making support is not only the probability of making an optimal possible, or an integral decision. It should also expand decision-maker’s understanding of the decision problem, its environment, measurable and immeasurable decision criteria, costs and potential gains from the available decision alternatives and new problems arising from the accomplished decision-making process. Using Howard’s (1988, pp. 685-686) decision quality concept, an appropriate decision-making method should cover proper decision framing, informational excellence, creativity, significantly different decision alternatives, clear values, integration and evaluation with logic, balance of basis (reasonable allocation of effort) and commitment to action. This in turn requires the choice of a method that will ensure a committed and purposeful decision-maker, a realistic framework, appropriate decision alternatives, necessary information, clear preferences of all stakeholders and strict decision procedures. Budziński (2010, p. 23) adds one important element, namely a properly selected method for decision-making support should also provide the decision-maker with a possibility of justification and clarification of motivation of the decisions taken formerly.

For the needs of the presented research the choice of research methods for the development of the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes is derived from Lisiński’s (1992, pp. 29-31) categorisation of approaches to the design of solutions to management science problems, cf. Table 2.3.

Table 2.3.

Approaches to the design of solutions to management science problems

Proposed or postulated by:	Design approaches	Predominant types of applied research techniques
Nadler (1967); **Trzcieniecki** (1979)	diagnostic; prognostic	diagnostic: observation, registration, description, measurement, testing of hypotheses, explanation of causes; prognostic: analysis & synthesis, modelling, diagnosis, expeditious and base design;

Martyniak (1976)	classical; enhancing; systemic; prognostic.	classical: observation, registration; enhancing: analysis & synthesis, description, explanation; systemic: modelling, design of organisational structures; prognostic: statistical testing of hypotheses, modelling;
Martyniak (1987)	descriptive-enhancing; functional-patterning; diagnostic-functional.	descriptive-enhancing: organisational cycle, ILO labour registration, classical organisational research, diagnostic methods; functional-patterning: operations research, technical systems value analysis, systemic organisational behaviour analysis; diagnostic-functional: description and analysis of factual state + idealisation of searched solutions;
Grelik & Wawrzyniak (1982)	classical; sociological; systemic.	classical: observation, registration; sociological: analysis & synthesis, description, explanation; systemic: modelling, design of organisational structures;
Sikorski (1988)	mechanistic; humanistic; systemic; situational.	mechanistic: observation, registration, description, enhancement of procedures; humanistic: analysis & synthesis, description, explanation; systemic: modelling, design of complex organisational structures & systems; situational: mixed approach that adopts mostly systemic research techniques to managerial problem solving.

Source: Compiled on the basis of (1992, pp. 29-31).

The approach adopted for the development of the model is close to Martyniak's (1987) diagnostic-functional approach, but also shows traits of Sikorski's (1988) situational design method. An initial observation, description and analysis of decision environment is performed in order to formulate a hierarchy of decision criteria. Next, in terms of the Hierarchical Module, the optimal possible decision is made, which conforms to the idealisation of the potential decision alternatives. The idea behind Fuzzy and mixed AHP-Fuzzy Modules goes beyond Martyniak's proposal, which consists in a methodological innovation in the problem solving approaches in the management science. The research techniques selected to achieve the main goal of this monograph are the Analytic Hierarchy Process and Fuzzy sets theory.

For the decision problems with a hierarchical structure the Analytic Hierarchy Process (AHP) is the appropriate basis for a complete representation of the hierarchical decision-making structures (Trzaskalik, 2014, p. 242). Saaty (2001, p. 23) points at its utility for decision-making tasks in a highly uncertain environment, i.e. management. Kłos & Trebuna (2014, p. 15) explain that by reducing complex decisions to a series of pairwise comparisons, and then synthesizing the results, the AHP method helps to capture both subjective and objective aspects of a decision. The AHP application is mostly justified when the manager confronts a decision-making problem of high complexity. It can be applied when the problem can be presented in a hierarchical structure and when higher hierarchy elements do not interact, nor interfere with lower ones. AHP should be considered when the optimal solution is selected from among many variants based on the subjective criteria. Last, but not least, the AHP decision-making models can be relatively easily optimized with regard to a predefined sub-criterion or parent criterion. Therefore AHP will be treated as the main research method for the development of a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes. A large number of successful AHP applications for the analysis of interdependencies between input and output variables of decision-making models in such fields of science as technology, biotechnology, medicine and physics is an additional argument in favour of AHP application for this purpose.

In order to incorporate managerial problems with a priorities fuzzy hierarchy and for situations where multicriteriality is not the only crucial feature of the decision-making system, a fuzzy development of the Hierarchical Module of the model is proposed. This is followed by a mixed AHP-Fuzzy extension. The selected research method for the construction of the Fuzzy Module is the Fuzzy sets theory and its development, the fuzzy logic. Mardani, Jusoh & Zavadskas, 2015 present an exhaustive discussion of methods combining Fuzzy sets theory and AHP. Most recent applications can be found in inter alia Awasthi, Govindan & Gold (2018), Ayhan (2018), Deptuła & Rudnik (2018) or Mosadeghi et al. (2015). The reason for a mixed approach is the possibility of including the second-best decision alternative, rejected by the hierarchical version of the model. This would enrich the proposed solution by introducing also the interrelations and feedbacks between particular decision criteria, also those ranked as less relevant ones. The optimal possible decision that emerges from the Hierarchical Module will not be put in doubt, on the contrary, it will have its range expanded and smoothened in the form of the resulting integral decision. A justification follows.

The first argument in favour of the above statement is the diverse character of the decision criteria, both qualitative and quantitative. Secondly, their number in multicriteria models makes the decision-maker anticipate an important amount of determinants of decision-making process simultaneously. The feature that predestines fuzzy methods for the modelling of decision-making processes is the nonlinearity of environmental occurrences. Economists who apply mathematical apparatus for their research generally use linear modelling, which is not inherently wrong. Many economic trends show linear characteristics, which in turn makes prognostics easier and allows basing them on previous trends. Linear models can be optimised gracefully as well, which allows a relatively simple analysis of the impact of input data on output parameters. Meanwhile, the linear description requires significant simplification of reality. But today's global economy is complex in character, with shorter economic cycles and an acceleration of sequence of events happening in parallel. As a result, input data sets for linear models condense, which lowers the probability of an accurate prognosis. Moreover, it will cover a shorter time period in the future. Another argument in favour of the application of Fuzzy sets theory for the enhancement of managerial decision-making is the ease of application. It can be assumed that the application of the Fuzzy Module will require a relatively low involvement of decision-maker's time, but deeper exploration of the decision alternatives achieved through fuzzy logic can yield highly enriching results for the integrity of the final decision. All of the above provides motivation for the development of a fuzzy expansion of the Hierarchical Module.

The idea is further substantiated by the compliance of the adopted methodology for the construction of the model with Szarucki's (2016) methodological framework for a scientifically justified choice of methods for the solution of managerial problems. It encompasses a proposal of a cascade of steps necessary for an appropriate execution of this task (Szarucki, 2016, pp. 123-130). Table 2.4 shows the common points of this concept and the pattern of choosing methods for the enhancement of the managerial decision-making processes adopted in this monograph.

Table 2.4.

Comparison of Szarucki's concept with adopted research methodology

Szarucki's methodology for choice of decision-making methods for solving managerial problems	**Corresponding field of adopted pattern for choice of managerial decision-making enhancement methods**
1. Problem formulation: 1.1. designation of the problematic situation; 1.2. problem extraction; 1.3. choice of specific method; 1.4. choice of the task team.	1. Identification of decision environment: 1.1. formulation of the main goal of decision-making; 1.2. formulation of the decision problem; 1.3. choice of decision-making method (AHP & fuzzy logic); 1.4. choice of experts for AHP evaluations.
2. Variant elaboration: 2.1. gathering of information; 2.2. analysis of information; 2.3. invention of ideas; 2.4. formulation of variants; 2.5. choice of optimal variant.	2. Formulation of the AHP decision hierarchy: 2.1. formulation of a hierarchy of decision criteria; 2.2. expert evaluations of decision criteria; 2.3. identification of decision alternatives; 2.4. expert evaluations of decision alternatives; 2.5. choice of the decision alternative (Hierarchical Module → Fuzzy Module → mixed AHP-Fuzzy Module).
3. Implementation of selected variant: 3.1. realisation planning; 3.2. realisation; 3.3. adjustive verification; 3.4. assessment and recapitulation of results.	3. Implementation of selected decision (post-applicatory): 3.1. planning the implementation of made decision; 3.2. decision implementation; 3.3. verification of decision-making impacts; 3.4. formulation of the next decision problem, resulting from accomplished decision-making process.

Source: Compiled on the basis of Szarucki (2016, p. 125).

The implementation phase (item 3 of the list in Table 2.4) is a step executed following the application of the decision-making model, which leaves it out of the scope of the discussed research task.

Romanowska (2008, pp. 119-124) categorises the obstacles to rational decision-making in business organisations into competence, organisational and informational barriers. In the managerial practice each of the mentioned types can constitute a serious impediment to the choice of an appropriate method for the enhancement of decision-making processes. Therefore, when a direct choice of the decision-making method is impossible or too difficult, a negational approach to this task is possible. For this purpose Cyfert's (2009, p. 167) list of limitations to improving the efficiency of organisational processes can be employed. The list addresses organisational processes in general, but as decision-making is one of the key processes in organisation management, a modified version of the list will be used for the choice of the decision-making enhancement method. The negational approach means that the refutation of Cyfert's statements provides arguments in favour of the application of chosen research methods. A decision-making modification of Cyfert's (2009, p. 167) list includes:

- the chosen decision-making method does not allow a consistent set of actions to be defined;
- the applied method does not allow a consistent set of measures of decision-making efficiency to be defined;
- the employed decision-making method reverses the motivation for solving the decision problem – from strategic goals of the organisation towards the efficiency of the decision-making procedure itself;
- the applied decision-making method encompasses only singular decision criteria, rather than the entire decision hierarchy;
- the chosen decision-making method concentrates on the operational managerial level only;
- the chosen method allows changes in the criteria hierarchy during active decision-making process;
- the adopted decision-making method focuses on partial goals of the decision-making instead of its main goal;
- the chosen decision-making method optimizes singular stages of decision-making rather than the entire decision-making process;
- the adopted method does not contain self-correcting, smoothening mechanisms.

Because of the modular structure of the decision-making model in question, the proposed choice of AHP and fuzzy logic methods, together with their mixed AHP-Fuzzy version do fulfil the above requirements.

Piotrowski (2009, pp. 53-98) proposes a complex algorithm for the choice of appropriate decision-making method, according to the analysed decision problem and to the compiled database of the existing applications of MCDA methods. As a result of the application of the algorithm, out of seven decision problems where the use of AHP was advised four are of strictly managerial nature (management, choice of production equipment) and one in the field of politics, which has an important number of characteristics similar to management. Two more applications postulated the use of a combined methodology (AHP+TOPSIS and AHP+PROMETHEE) for the choice of production technique (with some decision criteria from the scientific discipline of management) and directly management again. Piotrowski's empirical research results together with his findings from the application of the algorithm prove that the choice of AHP and fuzzy logic methods, as well as their combined AHP-Fuzzy version for the enhancement of managerial decision-making processes is justified. One can also note that the choice of research methods follows Lisiński's (2016, p. 17) three premises on the choice of research methods for the solution of managerial problems.

3. PRACTICE OF DECISION-MAKING IN ENTERPRISES – MANAGERS' PERSPECTIVE

3.1. Quantitative research design

From the statistical perspective, managerial decision-making problems demonstrate traits of a random experiment, as they can produce varying results, even if repeated under apparently identical conditions. They should be interpreted with mathematical schemes involving one or more parameters without fixed values, which should be calculated from experimental data and upon the assumption that the mathematical model of the experiments is correct (Neyman, 1937, pp. 333-334). Therefore, prior to the construction of a decision-making model quantitative research on the practice of decision-making should be done. Such research will reveal the habits of managers of enterprises operating in Poland in their everyday decision-making routine, with special regard to the use of tools for the enhancement of their decision-making.

The exploration of the above issue was also prompted by the fact that some researchers feel serious concerns about whether the decision-making patterns of management practitioners is in some way rooted in scientific methods. Goldman (2012) exploits this issue with an analysis of whether at least some strategic decision-making in practice is not merely simple and uncritical following the opinions expressed by people perceived in the society as management "gurus". There is yet another question – how knowledgeable are managers about the availability of tools for the enhancement of decision-making and to what extent are they willing to employ these tools in their everyday professional decision-making processes?

In the presented research these concerns are addressed by the quantitative research phase, which is perceived as the introductory phase to a broader research task executed by the main goal of this monograph. Both the main research problem addressed in this monograph and the adopted research design require the coverage of the quantitative research to be limited to enterprises operating in Poland, with Polish and international capital, operating locally and internationally.

The quantitative research phase was conducted with the following research tools:

- structured self-administered questionnaire;
- mixed-type sampling;
- testing of hypotheses.

A structured self-administered questionnaire was employed for the collection of the quantitative data on the practice of managerial decision-making in enterprises. The predominant type of questions was of closed-ended format, including single and multiple choice questions, inquiries on the importance (rankings) and 7-point Likert-type scale inquests. A few open-ended questions gave the respondents an opportunity to expand the set of answers with their own comments and explanations.

The questionnaire is one of two methods considered as primary in the quantitative research phase of investigation.

Goldman & Bounds (2015, p. 16) rightly observe that the use of a 6-point Likert-type scale in questionnaires discourages the notions of central tendency from respondents. Being aware of this concern, the author decided to employ a 7-point Likert-type scale in order to assure to lowest possible bias of answers, but also to give the respondents an opportunity to differentiate their answers more than 5-point scales allow. This choice is based on the assumption that if the employed scale offers enough differentiation, outliers will be attributed with caution. Dyduch (2015, p. 315) is in favour of the 7-point Likert-type scale because it:

- increases measurement accuracy;
- is continuous;
- is not very complex;
- gathers responses relatively fast;
- produces direct numerical results, ready for a statistical analysis.

The same author also quotes the results of his own research, and points at too low an extent of the 5-point scale and too large an extent of the 10-point one. Moreover, the latter one does not have the "middle" answer, which enhances the risk of bias resulting from the scale structure. However, he also lists some negative aspects of the 7-point scale application related to the quality of the gathered data, namely high subjectivity and superficial character (Dyduch, 2015, p. 315).

Jabłońska & Sobieraj (2013, p. 43) write that the key success factor of empirical research is the appropriate composition of the research sample. Czakon (2014, p. 54) observes that without doubt the best research results are provided by observing the entire population, with a research sample composed of 100% of research objects. However, due to the high costs, data availability, access to respondents, lack of time and many other aspects this is not feasible. Therefore some limitations of the sample size need to be taken into account. The same researcher does not exclude small samples or even singular instances, provided they are still representative for a larger population, or not treated as a basis for generalisation (Czakon, 2006, p. 10). Faber & Fonseca (2014, p. 29) support this opinion stating that although very small samples undermine the internal and external validity of a study, very large samples tend to transform small differences into statistically significant differences, which adds them false significance.

Silverman (2008, p. 169) states that sample composition procedures adequate for quantitative research are usually unreachable in qualitative research designs. This argument is backed by Mason (1996, p. 91), who states that even if a representative sample can be created, its size excludes the possibility of an intensive, qualitative analysis. This observation is of crucial importance from the perspective of the qualitative-quantitative research methodology adopted in the discussed research task.

There are several factors that need to be taken into account when composing the research sample. The most important ones are:

- type of model research (quantitative or qualitative);
- type of variables (categorical or continuous);
- sample size (big vs. small samples);
- sample composition (independent or interfering responses);
- threshold magnitude – whether expanding the sample by an additional respondent will significantly change the obtained results or increase their precision;
- probability of occurrence of hypothesis testing errors (type I – rejection of true null hypothesis; type II – retention of false null hypothesis; type III – the right answer to the wrong question) related to research sample composition.

All the mentioned factors affect the research outcome, its efficiency and representativeness. Researchers also need to consider the ethical aspect of the conducted research and the ability (or possibility) of drawing binding conclusions or generalising their findings onto a larger population.

In their discussion on the choice of sampling methods Morse & Niehaus (2009) point at the need of defining the focus and aim of sample composition. If the validity and generalisation of research results are the main concern, the quantitative research should be employed, together with sampling methods that focus on the representativeness of sample population. When the research task requires a deeper insight into a singular event, the qualitative research and sampling methods that facilitate sampling efficiency maximisation can be considered.

Jabłońska & Sobieraj (2013, p. 43) distinguish the following sampling methods:

- purposive sampling – non-probabilistic sampling technique; respondents present a significant level of expertise in the field of research; their choice is based on researcher's judgement; applied to research tasks that aim to provide answers to a specific, precisely formulated research problem;
- random sampling – the probability of any sample from the general population being selected is the same; applied for research tasks with the ambition of providing information about the entire general population;
- snowball sampling – an initially chosen group of respondents forwards the research task to their acquaintances;
- stratified sampling – the sample population is divided into mutually exclusive and collectively exhaustive strata; then a probabilistic (simple random or systematic) sampling is conducted separately within each sub-population; the entire sample is composed of the sum of samples from each stratum;
- quota sampling – a non-probabilistic version of the stratified sampling in which elements still need to be mutually exclusive, but only quotas from the entire population are selected – they are not collectively exhaustive; a non-random sample selection is applied.

In order to adopt the most efficient sampling combination, particular types of sampling will be briefly analysed.

Patton sees the value of purposive sampling in selecting information-rich cases for in-depth study (2002, p. 230). The choice of purposive sampling (the expert sampling type of this method, to be more precise) in the presented research can be justified by the fact that managers who form the only group of respondents to the questionnaire are perceived as experts in managerial decision-making. Therefore expanding the group of respondents onto the general population makes little sense. Moreover, it would bias the results, as it is impossible to expect a high level of expertise in managerial decision-making from respondents who were never exposed to practical execution of managerial tasks, duties and responsibilities. The application of expert purposive sampling is further supported in cases of no direct evidence and a level of uncertainty inherent in managerial decision-making. Another argument in favour is the nature of the research task, which is a random experiment. The fact that the same managerial decision-making task executed under identical conditions can result in different final decisions leads to the conclusion that the person of the decision-maker, the manager, is crucial for the efficiency of this process. Therefore preselection of potential respondents reduces the risk of result bias originating from a low level of expertise in management. One can realise that this argument acts also in favour of the choice of the stratified sampling method.

Marcus et al. (2017, p. 636) declare that snowball sampling has become very popular in organisational research, especially for recruiting informant samples for multisource studies. The method was discussed by Biernacki & Waldorf (1981), Penrod et al. (2003) and, from the perspective of the management science, by Wheeler et al. (2014). Methodological improvements, answering mainly the sample filtration issue (when the first respondents filter the composition of future samples by sending it to selected people only), were presented by Heckathorn, (1997), Platt et al., (2008). The method needs to be employed with caution, especially in the field of composing the initial research sample and supervising the forwarding process; the discussion of the reasons can be found in Marcus et al. (2017). Being aware of its limitations, the author has decided to use snowball sampling as one of the two main approaches to research sample composition because of its high efficiency of reaching respondents who would not be interested in the research project without personal notification from their business partners[13].

In order to reduce the risk of societal bias of the research sample (e.g. since people tend to socialise with people with similar views, attitudes, background, habits, etc.), that is likely to occur with snowball sampling, some elements of stratified sampling were applied.

[13] Difficulty of reaching respondents from other business organizations than international companies was already raised by Martyniak (1999) and despite the ease of access to online questionnaires, poses a growing problem in conducting scientific research in Management.

Fernandes et al. (2018) state that stratified sampling belongs to probability samples family and consists in dividing the entire population or study object in different subgroups or different strata. The use of stratified sampling is applied to increase the estimation precision through the minimisation of the errors stemming from the sample composition, as well as the reduction of respondent recruitment cost. Although the entire general population was not divided into strata, the author drew up a set of features prior to sending the questionnaire. The first group of respondents was explicitly informed that all further respondents should be active managers at any managerial level, with at least one-year managerial experience. Together with the invitation to the research project this information was sent further to each next respondent with the invitation email. It was also included in the online questionnaire. The aim of such a procedure was to make sure that the questionnaire answers were provided only by the persons who make managerial decisions on a daily basis.

Another facet of the stratified sampling method applied in research sample design was that the general population of managers was divided into three levels (operational, tactical, and strategic). However, as there was no equal representation of each stratum, no general conclusions could be drawn on the basis of the management level criterion. The author is aware that this is one of the limitations of the presented research.

Siegel (2016, p. 195) defines random sampling as a way of distinguishing a smaller population that has been selected for observation in order to learn about a larger population that is too large to analyse in its entity. The survey results will be an approximation of the traits of the general population, but they will be close enough to assure the researcher's comfort, ethics, and allow a cautious generalisation on a larger population. In the discussed research task sampling randomness is partially ensured by the choice of the initial group of managers – they were chosen randomly, through mailing companies chosen at random. They were also made aware of Heckathorn's (1997) sample filtration issue, with a request to forward the questionnaire to a random group of managers, without selecting their composition.

The quota sampling is defined by Gschwend (2005, p. 89) as a cheap and convenient method to generate a sample quickly and without any special sampling frame. In the presented research it was disregarded on purpose, as it implies a risk of bias that was judged too important for the reliability of the findings.

Palinkas et al. (2015, p. 533) observe that for some types of research combining sampling strategies may be more appropriate and more consistent with recent developments in sampling methods.

Therefore for the needs of the presented research mixed sampling was employed in order to compose an accessible and representative research sample. The composition of sampling methods is a combination of purposive and snowball sampling, with elements of stratified and random sampling, i.e. mixed sampling was chosen.

The respondents to the questionnaire were found among executives of local and international companies operating in Poland. Mixed-type sampling is counted among secondary quantitative research methods, as it only serves as a tool for composing the group of questionnaire respondents.

Testing of hypotheses was employed to perform an analysis of the significance of answers to the questionnaire. Pieter (1957, p. 70) defines a hypothesis as a scientific supposition on the existence of a relation between events under examination and other phenomena, or on the link between notions in relation to notional values of established meaning. Stachak (2006, p. 91) provides a set of requirements posed to scientific hypotheses:

- hypotheses have to be answers to posed research questions;
- a hypothesis should be an unambiguous, precise and simple answer to a given research question;
- hypotheses should above all be related to causative relationships between researched facts;
- each hypothesis has to be verifiable, i.e. directly or indirectly linked with observed facts;
- conclusions resulting from a hypothesis need to conform to the existing knowledge on researched occurrences;
- there has to be a probability that the formulated hypothesis could be the right answer to the question it researches.

The mentioned features are verified in the process of testing of hypotheses. This research method is targeted at finding regularities and relations between the obtained responses. Another goal is to draw conclusions on the practice of managerial decision-making in enterprises operating in Poland. Testing of hypotheses is counted among primary quantitative research methods.

In the presented research, the quantitative research phase should be treated only as an introductory phase aiming at bringing to the readers' attention the particularities exiting in the managerial decision-making in Poland. The author's intention was far from providing a comprehensive overview of this subject in the Polish companies.

For the purposes of the introductory quantitative research, the following hypotheses have been adopted:

H1: The application of multicriteria decision-making enhancement methods increases the probability of taking an integral managerial decision.

H2: The application of multicriteria decision-making enhancement methods does not increase the probability of taking an integral managerial decision.

It should be observed that hypotheses H1 and H2 be are used only in the quantitative research phase. As such, they are relevant exclusively to the introductory (quantitative) phase of the presented research task. A distinction should also be made between the hypotheses of the quantitative research phase (H1 and H2) and general research thesis together with its general sub-theses T1, T2 and T3, which are valid for the entire research task treated in this monograph.

3.2. Results of the quantitative research stage

Verma & Burnett (1996) and Henry (2009) point at various ways of limiting the sample size, out of which the following apply to the presented study:

- stratification of the research target group – employed mixed-sampling technique, namely the purposive expert sampling narrows down the group of respondents to decision-makers: owing to stratification the sample size can be reduced up to 25%;
- reduction of the non-response rate through an e-mail or telephone announcement of the survey prior to its distribution – this technique can further reduce the sample size up to 50%;
- reduction of the alpha level (Type I error) to 10% – instead of the initially planned 5%;
- reduction of statistical power of the test up to 70% (or increase of the beta level – Type II error). Mann, Crouse & Prentice (1991, p. 11) state that a reasonable statistical power of the test that provides still meaningful results is assured by the beta level between 0.7 and 0.9.

Basing on the above, some assumptions were made for the calculation of a representative sample size. These include: according to the Polish Government's Central Registration and Information on Business database, the number of active companies that are subject to the Polish commercial law is 313 000 (CEIDG, 2018). Therefore, at an assumed 8% error margin, a 0.5 fraction size and 90% confidence level, the required sample size should amount to 106 respondents[14]. However, as already mentioned, the quantitative research is merely an introductory element of a larger research task. It aims only at providing a general idea on the decision-making pattern of the Polish managers, with a special regard to the issue of application or non-application of tools for decision-making enhancement. Neither does it have ambitions to provide grounds for generalisation of research results over the general population, or analysing this issue in detail.

Due to the introductory nature of the quantitative research stage, the application of mixed-type sampling method and the possibilities of reducing alpha and beta levels (presented earlier) a reduction of the initial sample composed of 106 respondents seems possible. The adopted error margin could be raised to 10%, as the results were not generalised on the entire population. As the composition of research sample covered only active managers of tactical and strategic level, the fraction size could be adopted at a very high level of 90% – this is the fraction of the research sample that presumably possesses the characteristic of being an expert in management. The remaining 10% are perceived as part of respondents, who although active managers, lack the competence in managerial decision-making (e.g. due to little managerial experience). The power of the test could then rise to 95%. Therefore, at the assumed

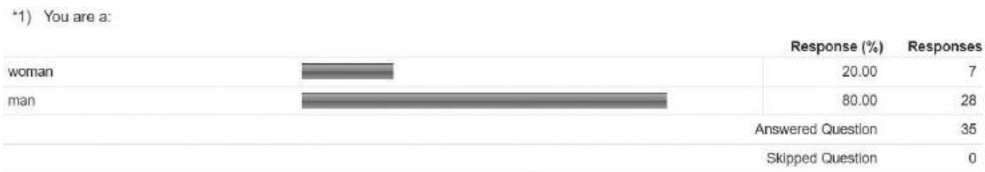
[14] Calculated online with Naukowiec.org (Accessed 11 March, 2017 at: <www.naukowiec.org/dobor.html>).

10% error margin, a 0.9 fraction size and 95% confidence level, the final research sample of 35 respondents turned out to be satisfying enough to draw conclusions from the introductory quantitative research phase (but giving grounds for generalisations of these observations onto the general population).

After an initial telephone announcement and having obtained the respondents' verbal agreement, the questionnaires were sent to over 100 management practitioners in the companies operating in Poland. The result was thirty-five completely answered questionnaires, which constitutes an acceptable survey return rate of around 35%.

The practical application of mixed-type sampling consisted in an initial deliberate limitation of the group of respondents to management practitioners, who were defined as managerial decision-making experts (purposive sampling – expert type). Managers who agreed to answer the questionnaire were asked to forward the link to the questionnaire to other decision-makers. This proceeded horizontally, that is to lower and higher level managers inside the same department of the company, and vertically, that is, to similar level managers in other departments of the company. Additionally, same respondents agreed to send links to the questionnaire to managerial level executives among their business partners. This part of the sampling procedure accounted for the snowball sampling part of the mix. The elements of complementary sampling techniques were ensured by recruiting the respondents from among the managerial stratum of company employees only, who were requested to state the occupied managerial level (stratified sampling). The random sampling modus operandi consisted in stratification (active managers) with no other differentiator employed to collect the research sample. The respondents who agreed to forward the link to the questionnaire to other managers were specifically instructed not to limit their choice of snowball respondents in any way.

The recruited respondents were management practitioners – executives of local and international companies operating in Poland, including 20% (seven persons) female and 80% (twenty-eight persons) male (Figure 3.1). The unequal gender composition of the research sample did not affect the results, and is a challenging field to explore in separate research.


*1) You are a:

	Response (%)	Responses
woman	20.00	7
man	80.00	28
	Answered Question	35
	Skipped Question	0

Figure 3.1. Gender composition of the quantitative research sample
Source: Compiled on the basis of quantitative research results.

The largest part of the respondents (65.71% - twenty-three persons) were between 36 and 45 years of age, 17.14% (six persons) were between 46 and 55 years old, 8.57% (three persons) were over 55 and the same amount of respondents were aged between 26 and 35. Such results (Figure 3.2) suggest that the answers were obtained from active professionals, which strengthens their representativeness for the needs of the analysis of managerial decision-making preferences towards the use of tools for decision-making enhancement.

*2) How old are you?

	Response (%)	Responses
younger than 25	0.00	0
26-35	8.57	3
36-45	65.71	23
46-55	17.14	6
older than 55	8.57	3
	Answered Question	35
	Skipped Question	0

Figure 3.2 Age span of the respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

As Figure 3.3 indicates, all the respondents had a university degree, including 5.71% (two persons) with fist-cycle university degree, Bachelor or Engineer, 62.86% (twenty-two persons) second-cycle university degree, Master's, and quite a large number of 31.43% (eleven persons) third-cycle university education with the scientific title of doctor (Ph.D.) or higher. Such a high percentage of practitioners who appreciate academic self-development could be an interesting field of future research.

*3) What is your education?

	Response (%)	Responses
primary	0.00	0
basic vocational	0.00	0
technical secondary	0.00	0
upper secondary	0.00	0
University first cycle (Bachelor, Engineer)	5.71	2
University second cycle (Master)	62.86	22
University third cycle (Ph.D. or higher)	31.43	11
	Answered Question	35
	Skipped Question	0

Figure 3.3. Education level of the respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

Most respondents (74.29%, twenty-six persons) lived in a metropolis (a city with over 500.000 inhabitants), 8.57% (three persons) in a big city (between 150.000 and 500.000 inhabitants), 5.71% (two persons) in a small city (up to 50.000 inhabitants) and 11.43% (four persons) declared themselves as countryside residents (Figure 3.4). The last category can be misleading, as an important number of Polish citizens are leaving big cities for their more rural entourage, nevertheless the neighbouring metropolis or a big city remains the centre of their living and professional activities.

*4) You live:

	Response (%)	Responses
countryside	11.43	4
small city (up to 50.000 inhabitants)	5.71	2
medium city (50.000 to 150.000 inhabitants)	0.00	0
big city (150.000 to 500.000 inhabitants)	8.57	3
metropolis (above 500.000 inhabitants)	74.29	26
	Answered Question	35
	Skipped Question	0

Figure 3.4. Place of residence of the respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

The analysis of the size of the company employing decision-makers who agreed to participate in the quantitative research stage indicates (Figure 3.5) that most of them (62.86%, twenty-two persons) worked for a big enterprise (defined as the average yearly number of employees in past two fiscal years above 250 persons, yearly net turnover at the end of one of these two years above 50m € – or equivalent in PLN – or total assets at the end of one of these two years above 43m € – or equivalent in PLN. 17.14% (six persons) of the respondents worked in a middle-sized enterprise (average yearly number of employees in past two fiscal years below 250 persons, yearly net turnover at the end of one of these two years below 50m €– or equivalent in PLN – or total assets at the end of one of these two years below 43m € – or equivalent in PLN). 8.57% (three respondents) worked in small enterprises (average yearly number of employees in past two fiscal years below 50 persons, yearly net turnover or total assets at the end of one of these two years below 10m € – or equivalent in PLN) and 11.43% (four persons) in micro-enterprises (average yearly number of employees in past two fiscal years below 10 persons, yearly net turnover or total assets at the end of one of these two years below 2m € – or equivalent in PLN). The definitions of types of enterprises were taken from the European Commission Recommendation of 6 May 2003 concerning the definition of micro, small and medium-sized enterprises (European Commission, 2003).

*5) Your workplace is: (according to the definitions below) microenterprise – average yearly number of employees in past two fiscal years below 10persons, yearly net turnover or total assets at the end of one of these two years below 2 m € (or equivalent in PLN); small enterprise – average yearly number of employees in past two fiscal years below 50 persons, yearly net turnover or total assets at the end of one of these two years below 10 m € (or equivalent in PLN); medium enterprise – average yearly number of employees in past two fiscal years below 250persons, yearly net turnover at the end of one of these two years below 50 m € (or equivalent in PLN) or total assets at the end of one of these two years below 43 m € (or equivalent in PLN); big enterprise - other.

	Response (%)	Responses
microenterprise	11.43	4
small enterprise	8.57	3
medium enterprise	17.14	6
big enterprise	62.86	22
	Answered Question	35
	Skipped Question	0

Figure 3.5. Size of companies managed by the respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

62.86% of the participants in the study project (twenty-two persons) were strategic, higher level managers (e.g. Chief Executive Officer, Chairman of the Board, General Director, etc.), whereas 20% (seven persons) were tactical, medium level managers (e.g. Head of Department) and 17.14% (six persons) were operational, lower level managers (e.g. Team Manager). The graphic illustration is presented in Figure 3.6 below.

*6) At which managerial level in your organization is your work post situated

	Response (%)	Responses
functional (lower) – e.g. Team Manager	17.14	6
operational (medium) – e.g. Head of Department	20.00	7
strategic (higher) – e.g. Chief Executive Officer, Chairman of the Board, General Director	62.86	22
	Answered Question	35
	Skipped Question	0

Figure 3.6. Managerial level occupied by the respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

Quite interesting results have been obtained from the inquiry on the driving factors of professional decision-making habits of the interviewed managers. An overwhelming majority of 77.14% agreed that their decisions are made most of all on the basis of their own analytical assessment (1st rank attributed). The 2nd most popular decision-making pattern was decision-makers' own intuition (with 20% of indications), although in the general picture the majority of respondents (28.57%) gave this factor 4th rank. Even though the opinions are divided here, the 2nd or 4th ranks of intuition's importance in managerial decision-making is a strong indication of the self-confidence of the interviewed decision-makers. The question arises to what extent this self-confidence is substantiated.

22.86% of the respondents pointed at available reports from external analysts or institutions as the 3rd most important driver of their managerial decision-making, but 28.57% declared that their own intuition came into the scope right after. There was no consensus on the 5th driving factor. An equal number of respondents (20% in each case) pointed at available reports of external analysts or institutions (ruled out, as a higher number, 22.86% of respondents pointed at it as 3rd) or again own intuition (declared 4th by 28.57%) and suggestions given by subordinates. As this last category did not appear in higher ranked positions, it can be attributed 5th place among the driving factors of managerial decision-making in the eyes of the interviewed managers. When analysed from the majority of votes in this category only, this rank falls to 6th position. A distant before-last position (6th rank) was attributed to solutions suggested by any tool for the enhancement of decision-making processes. Nevertheless, an even bigger number of respondents (37.14%) judged this option as least important, with rank 7 held ex aequo with the superior's opinion (25.71%).

*7) What are the driving factors of your decision-making at work? Grade the answers from most to least influential (drag & drop).

	1	2	3	4	5	6	7	Responses	Total
available reports from external analysts or institutions	2.86%	14.29%	22.86%	17.14%	20.00%	17.14%	5.71%	35	14%
a democratically reached common position of my team	5.71%	17.14%	20.00%	8.57%	17.14%	17.14%	14.29%	35	14%
my own analytical assessment	77.14%	11.43%	5.71%	2.86%	2.86%	0%	0%	35	14%
a solution suggested by a tool for enhancement of decision-making processes	8.57%	14.29%	5.71%	5.71%	8.57%	20.00%	37.14%	35	14%
the opinion of my superior	5.71%	14.29%	14.29%	14.29%	11.43%	14.29%	25.71%	35	14%
my own intuition	0%	20.00%	20.00%	28.57%	20.00%	5.71%	5.71%	35	14%
suggestions of my employees	0%	8.57%	11.43%	22.86%	20.00%	25.71%	11.43%	35	14%

Figure 3.7. Driving factors of managerial decision-making of the respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

The above results show not a very great trust of decision-makers employed by companies operating in Poland in the opinions of external analysts or other decision-support bodies. The tendency to employ scientific tools for the enhancement of decision-making in a manager's daily routine is even lower. This last opinion is further strengthened by the willingness to apply tools for the enhancement of decision-making processes by the questioned managers, cf. Figure 3.8. Actually, the questioned managers mostly tend to rely on their own assessments, perceptions and intuition, rather than on professional opinions of their employees or superiors. Democratic decision-making is not a common and respected practice either, even though it was perceived as 3rd in the ranking, but only by 20% of respondents.

*8) How often do you employ tools for enhancement of decision-making processes?

	Response (%)	Responses
never	11.43	4
hardly ever	22.86	8
sometimes	34.29	12
in half of the cases	5.71	2
often	20.00	7
almost always	5.71	2
always	0.00	0
	Answered Question	35
	Skipped Question	0

Figure 3.8. Willingness to apply tools for enhancement of decision-making processes among the respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

When asked about their willingness to apply professional research-grounded tools for the enhancement of decision-making processes, most respondents (together 68.58%, which corresponds to twenty-four persons) declared that they seldom profited from such a decision-making aid (Figure 3.8). 11.43% (four persons) did not use any tools at all, 22.86% (8 persons) hardly ever employed them and 34.29% (twelve persons) used them sometimes. Two managers (5.71%) declared the use in half of the cases, whereas 20% (seven persons) confessed using them often. Only two managers (5.71%) declared the application of decision-making support tools in most of the cases – almost always, however, no manager resorts to decision-making support in every decision. The fact that the majority of respondents hardly ever employ support tools in their professional decision-making tasks supports the ranks of particular driving factors of managerial decision-making presented in Figure 3.7.

Figure 3.9 explores the issue further by showing what types of decision-making tools are favoured by managers who declare using them, even if not often. Eight respondents (17.78%) confirmed that they did not use any tools supporting decision-making processes, which proves that half of the 22.86% (eight persons) who declared hardly ever applying them (Figure 3.8) did not use them at all. This, in turn, additionally confirms the statement about 68.58% of managers practically not referring to any decision-making support tools at all.

*9) What decision-making enhancement tools do you use? (you can check more than one answer)

	Response (%)	Responses
I don't use any tools supporting decision-making processes	17.78	8
I employ quantitative tools, based on statistical data (e.g. Statistica, MatLab, SPSS, Big Data Analysis, prognostic models, etc.)	35.56	16
I employ qualitative tools, based on qualitative criteria (e.g. SWOT, SPACE, Force Field Analysis, scenario methods, etc.)	37.78	17
I employ mixed qualitative-quantitative multicriteria tools (e.g. Analytic Hierarchy Process, Analytic Network Process, Fuzzy Sets, Fuzzy Logic, Artificial Neural Networks, ELECTRE, PROMETHEE, TOPSIS, BIPOLAR, verbal methods, etc.)	8.89	4
	Answered Question	35
	Skipped Question	0

Figure 3.9. Types of decision-making enhancement tools used by the respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

Those who do employ support in their decision-making almost equally apply qualitative and quantitative tools. The first group, which includes qualitative determinants of the decision problem, with such tools as SWOT, SPACE, Force Field Analysis, scenario methods and others gained the recognition of 37.78% of managers (seventeen persons). Quantitative tools, e.g. Statistica, MatLab, Minitab, SPSS, Big Data Analysis, prognostic models, etc. found the recognition among 35.56% of managers (sixteen persons). Only 8.89% (four persons) declared benefiting from mixed qualitative-quantitative multicriteria tools, such as Analytic Hierarchy Process, Analytic Network Process, fuzzy sets, fuzzy logic, Artificial Neural Networks, ELECTRE, PROMETHEE, TOPSIS, BIPOLAR, verbal and other complex decision-making methods.

In question 10 of the survey the respondents were asked to explain their preference from Figure 3.9. The obtained statements can be grouped into three sub-questions:

I. What specific instruments do you employ in your decision-making?
II. For what precisely do you employ decision-making support?
III. General explanatory remarks.

The responses to sub-question I were: analysis of data from the internet and other sources; objective data (e.g. financial); excel files that describe revenue, costs, profitability, revenue per employee, costs and revenue in the entire project; planned costs and revenue in the entire project, resource occupancy files and demand for necessary resources, forecasts of costs, revenue and profitability; mainly SWOT analysis and statistical tools, sometimes forecasting models; specialist market reports, MS Excel calculations or SWOT analysis for decisions on alternative variants for commercialisation of technologies; statistical assessment tools and comparative analysis at tactical managerial level; statistical analysis of data for creation of sale strategies; I base my problem analysis on quantitative indexes and qualitative data and use them as supplementary material for my team when working on a common decision, or analyse them myself, with regard to my experience; I use mostly Key Performance Indicators, such as Return on Investment, Click Through Rate, Conversion Rate, statistical cohorts, Customer Lifetime Value; benchmarks; mostly trends in big data analysis; SWOT and scenario methods at strategic planning; mostly SWOT.

The responses to sub-question II: analysis of complex issues and decision aid in technological and investment fields.

The responses to sub-question III (general remarks): only a multicriteria analysis fully reflects the environment's complexity; decision-making based on qualitative and quantitative data helps state one's own faulty opinions and correct them; more advanced decision-making tools are employed at higher managerial levels than mine; at strategic managerial level the analysis is a complex process that cannot be described and limited to typical decision-making support tools; the decision problem determines the question that needs to be asked, which in turn suggests the decision-making method – as in scientific research; decision-making tools are only a suggestion and are complementary to more comprehensive analyses.

The above additional comments point at several issues: first of all, SWOT analysis and scenario methods are quite recognised by active managers, which can be due to the simplicity and low time-consuming application of the method, but also to the fact that they are covered by economic education programmes. Secondly, managers' awareness of the methods seems to grow with increasing specificity of managerial tasks. In other terms, simple managerial tasks do not force decision-makers to employ specific tools. On the other hand, with growing complexity or complication of a decision problem, the necessity of employing decision-making support methods seems to be growing. Thirdly, managers seem to see the value in combining quantitative and qualitative analysis of decision problems, which points at the need of creation of appropriate mixed tools that support complex decision-making processes.

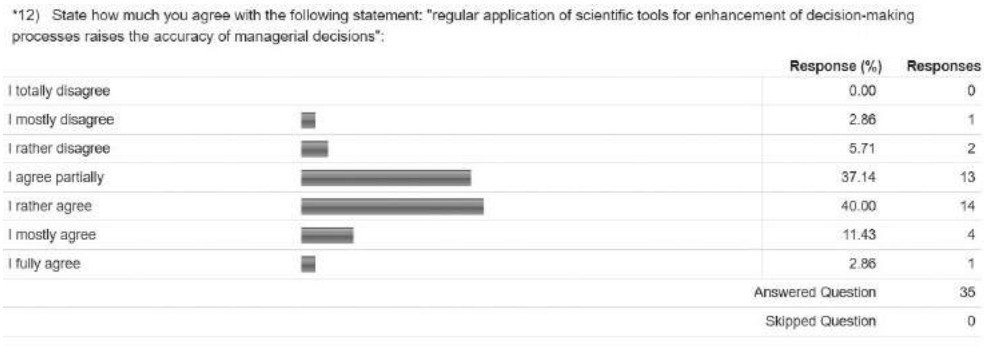
The data on the managers' assessment of their decision-making patterns are a cause for concern. Figure 3.10 shows that a vast majority declares they are satisfied with the accuracy of their decision-making (altogether 91.43%, which corresponds to thirty-two persons). Only two managers (5.71%) declared being sometimes happy about the results and one (2.86%) stated satisfaction in half of the cases.

*11) How often are you satisfied with the accuracy of your managerial decisions?

	Response (%)	Responses
never	0.00	0
hardly ever	0.00	0
sometimes	5.71	2
in half of the cases	2.86	1
often	62.86	22
almost always	28.57	10
always	0.00	0
	Answered Question	35
	Skipped Question	0

Figure 3.10. Satisfaction of the respondents to the quantitative research stage from taken managerial decisions
Source: Compiled on the basis of quantitative research results.

At the same time a slight majority (54.29%, which corresponds to nineteen persons), but still a majority of respondents agrees that a regular application of scientific tools for the enhancement of decision-making processes increases the accuracy of managerial decisions (Figure 3.11). 40% (fourteen persons) rather agree, 11.43% (four



*12) State how much you agree with the following statement: "regular application of scientific tools for enhancement of decision-making processes raises the accuracy of managerial decisions":

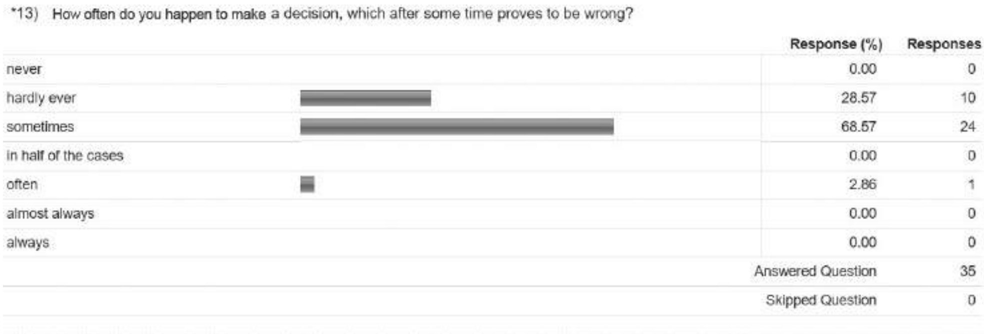
	Response (%)	Responses
I totally disagree	0.00	0
I mostly disagree	2.86	1
I rather disagree	5.71	2
I agree partially	37.14	13
I rather agree	40.00	14
I mostly agree	11.43	4
I fully agree	2.86	1
	Answered Question	35
	Skipped Question	0

Figure 3.11. Opinion of the questioned managers on the relation between the application of decision-making tools and accuracy of final decisions
Source: Compiled on the basis of quantitative research results.

persons) mostly agree and 2.86% (one person) fully agree with this statement. It is important to notice that question No. 12 (Figure 3.11) together with question No. 14 (Figure 3.14) of the survey are key questions of the quantitative research stage. This is because they both are directly associated with hypothesis H1 of the quantitative research stage (the application of multicriteria decision-making enhancement methods increases the probability of taking an integral managerial decision), which follows Stachak's (2006, p. 91) condition of research hypotheses formulation.

The post factum perception of the results of managerial decision-making with regard to decisions that finally proved to be completely wrong in the eyes of the respondents to the quantitative research stage shows that only one manager (2.86%) confessed to often taking wrong decisions (Figure 3.12). The rest declared taking them sometimes (68.57%, twenty-four persons) and hardly ever (28.57%, ten persons).

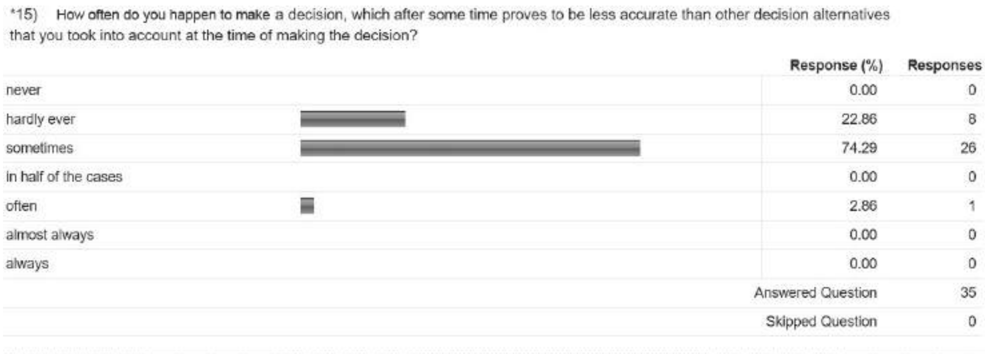
A similar number declared post factum satisfaction with the results of decision-making with regard to non-optimal decisions (Figure 3.13). 74.29% (twenty-six



*13) How often do you happen to make a decision, which after some time proves to be wrong?

	Response (%)	Responses
never	0.00	0
hardly ever	28.57	10
sometimes	68.57	24
in half of the cases	0.00	0
often	2.86	1
almost always	0.00	0
always	0.00	0
	Answered Question	35
	Skipped Question	0

Figure 3.12. Post factum perception of the results of decision-making by the respondents to the quantitative research stage – completely wrong decisions
Source: Compiled on the basis of quantitative research results.



*15) How often do you happen to make a decision, which after some time proves to be less accurate than other decision alternatives that you took into account at the time of making the decision?

	Response (%)	Responses
never	0.00	0
hardly ever	22.86	8
sometimes	74.29	26
in half of the cases	0.00	0
often	2.86	1
almost always	0.00	0
always	0.00	0
	Answered Question	35
	Skipped Question	0

Figure 3.13. Post factum perception of the results of decision-making by the respondents to the quantitative research stage – non-optimal decisions
Source: Compiled on the basis of quantitative research results.

managers) declared that sometimes a better decision could have been made and 22.86% (eight persons) saw this option as hardly ever possible. Only one person (2.86%) confessed that often his or her decisions were not optimal. These answers are very similar to those from Figure 3.10, which proves high consistency of the respondents' answers and a high level of confidence of the managers in their decision-making patterns, at least in their declarations.

High satisfaction with their decision-making results declared by the managers is followed by their declaration about the need of incorporating a large number of decision criteria into managerial decision-making (Figure 3.14). Only one person (2.86%) negated the benefits of multicriteria decision analysis, whereas the rest supported this statement – 22.86% (eight persons) partly, 34.29% (twelve persons) to a larger extent (rather agree), 25.71% (nine persons) mostly agree and 14.29% (five persons) fully agree. Altogether the support rate for multicriteria decision making reaches an outstanding 97.14% (thirty-four out of thirty-five respondents).

*14) * State how much you agree with the following statement: "incorporation of a larger number of decision criteria into the decision-making process increases the probability of making a more accurate managerial decision":

	Response (%)	Responses
I totally disagree	0.00	0
I mostly disagree	2.86	1
I rather disagree	0.00	0
I agree partially	22.86	8
I rather agree	34.29	12
I mostly agree	25.71	9
I fully agree	14.29	5
	Answered Question	35
	Skipped Question	0

Figure 3.14. Multiple criteria analysis and the accuracy of managerial decisions in the eyes of respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

*16) How relevant for the quality of the decision-making process is in your oppinion the inclusion into decision-making praxis of both quantitative (e.g. level of sales, income, etc.) and qualitative (e.g. organizational maturity, motivation of employees , etc.) decision criteria?

	Response (%)	Responses
totally irrelevant	0.00	0
irrelevant	2.86	1
rather irrelevant	2.86	1
partially relevant	8.57	3
rather relevant	28.57	10
relevant	34.29	12
crucial	22.86	8
	Answered Question	35
	Skipped Question	0

Figure 3.15. Qualitative-quantitative criteria analysis and the accuracy of managerial decisions in the eyes of respondents to the quantitative research stage
Source: Compiled on the basis of quantitative research results.

Most managers (94.28%, which corresponds to thirty-three persons) perceive the qualitative-quantitative criteria analysis as relevant for the accuracy of taken managerial decisions (Figure 3.15). One person (2.86%) sees it as irrelevant, one as rather irrelevant, three (8.57%) as partly relevant, ten (28.57%) as rather relevant, twelve (34.29%) as relevant and eight (22.86%) as crucial.

Figures 3.14 and 3.15 show a discrepancy between the willingness to use decision support tools, expected results of their application and decision-makers' satisfaction with the accuracy of taken decisions. One of the reasons, pointed out in one of the explanatory remarks above, is limited knowledge about the availability of decision-making tools or their difficult applicability.

An additional inquiry about decision-makers' expectations about decision support tools was incorporated into the questionnaire. In the managers' opinion, the key features that would increase their interest in decision-making enhancement tools include the ease and speed of application, affordable price, correlation with decision-maker's industry, simplicity, high competence of experts involved in the decision-making and a data analysis module, rooting in the business environment of the company, tailor-made, clear and easy interface, results in tables and graphs, short presentation of results, efficiency, high track record proving that the tool works in practice, inclusion of qualitative and quantitative criteria, actual data, multiple comparison options, fast data interpretation, compatible with other software used in the company (SAP, MS Office, others), multicriteriality (including cross-cultural issues), inclusion of a database of criteria based on former case studies, automation of data collection, possibility of verification of results, prognostic module, elasticity, transparency.

Meeting all the above expectations comprised within one tool for managerial decision-making enhancement is difficult. Nevertheless, the methodological framework of the proposed Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes is such an attempt owing to its joint qualitative-quantitative character.

An analysis of the answers obtained in the quantitative research stage is given in 3.3 below.

3.3. Analysis and interpretation of quantitative research results

Dietrich & Schulze (2000, pp. 18-131) define descriptive statistics as a research field that aims at providing generalisations valid for the entire population. Inferential (analytical) statistics covers the research performed on specific groups extracted from the general population – the random samples. Inferential statistical methods are required whenever results cannot be often and precisely replicated. Problems with research repeatability can result from uncontrolled and uncontrollable external effects. Such impacts cause dispersion of analysed statistical attributes (e.g. measurement results).

The statistical inference of a random sample over general population is called indirect inference. For each measured index a confidence interval needs to be determined. This means that the real value of the estimated magnitude belongs to the confidence interval, at a given probability level.

The analysis of statistical samples is targeted at generalisation of the observed behaviour of a researched feature onto the general population to which the sample belongs. Such generalisation is possible only when the behaviour of the discussed feature conforms to some credible assumptions and the analysed states of observation can be accepted as roughly to reality. This leads to various statistical distributions:

- probability distributions (distributions of random variables) – which describe the population containing the observed magnitudes;
- parametric distributions (distributions of probability, described in mathematical notation through probabilistic parameters) – which are employed for the calculation of statistical indexes (e.g. intervals of dispersion of results, confidence intervals) or in statistical tests; most commonly applied parametric distributions are:
    - normal distribution;
    - Student's t-distribution;
    - χ^2(chi-squared) distribution;
    - F-Snedecor test.

Depending on the features of analysed population, probability distributions can be divided into discrete or continuous. Normal distribution, most commonly met in statistical analysis, is an example of continuous probability distribution. Normal distribution is popular because:

- in experiments and observations an important amount of statistical features show normal distribution of their values; especially the distribution of a sum of many independent random variables with various distributions approaches the normal distribution – the closer the higher the number of totalled values;
- the distribution of features which do not fall under normal distribution can often be approximated by normal distribution; an assumption that they can be described via normal distribution relatively often leads to logical and practically useful results;
- some features that do not conform to normal distribution can be transformed so that the transformed variable will be subject to normal distribution;
- a number of complicated distributions can in boundary conditions be substituted by normal distribution in a practically useful way;
- mathematical operations on normal distributions are easy to perform, e.g. with the use of statistical tables.

The probability density function (random variable density) is a non-negative real function that describes the probability distribution so that its integral, calculated within proper limits, equals to the likelihood of the occurrence of a given random event. It can be defined for one-dimensional and multi-dimensional probability distributions and is characteristic of continuous probability distributions. The probability density function in mathematical notation is assigned the symbol $g(x)$, whereas its corresponding distribution function is noted as $G(x)$.

The normal distribution curve results from the value stream or the histogram of a finite random sample, under the assumption that the general population being subject to normal distribution contains an infinite amount of the measured values.

The distribution function $G(x)$ results from empirical data. The value stream shows the frequency of occurrence of particular results. Its graphic illustration shows how often the given x value occurs. Values grouped in classes form a frequency diagram (Figure 3.16.a) or a histogram (Figure 3.16.b) on which a theoretical value of the density function is imposed.

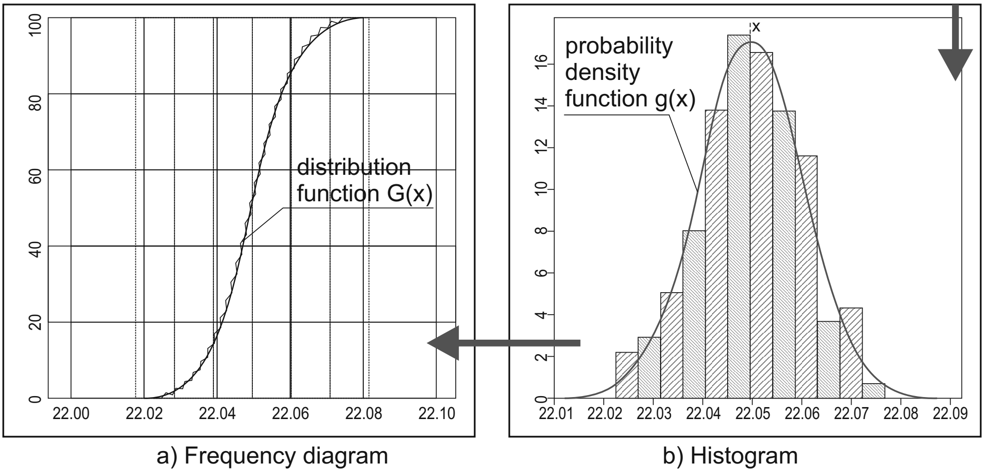


Figure 3.16. Normal distribution example – a) distribution function $G(x)$;
b) probability density function $g(x)$
Source: Dietrich & Schulze (2000, p. 46).

The shape of distribution function $G(x)$ derives from the calculated average value and standard deviation. The frequencies of occurrence of the values totalled from the histogram (Figure 3.16.b) form a step line of the sums of empirical values. The total of all probability density functions $g_i(x)$ from the histogram forms a theoretical curve – the distribution function $G(x)$ (Figure 3.16.a), to which the step curve tends.

The arithmetic average of a random sample is denoted by $\overline{x}$, whereas the expected value in the general population by μ (this parameter is usually unknown). The $\overline{x}$ value calculated from the random sample is only an approximated estimation of real value μ.

In a general population the index corresponding to the standard deviation s of a random sample is σ – the standard deviation of general population. The value of s parameter calculated from the random sample is only an approximated estimation of real value σ.

Equation 1 gives the mathematical formula of probability density of normal distribution $g(x)$:

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \text{(Eq. 1)}$$

In practical calculations a standardised form of normal distribution is employed for which the expected value $\mu = 0$ and variance $\sigma^2 = 1$. The distance of the result from the average value is given in units of standard deviation and denoted by u, defined in Equation 2:

$$u = \frac{x-\mu}{\sigma} \qquad \text{(Eq. 2)}$$

Then the probability density is given by the formula (Equation 3):

$$g(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} \qquad \text{(Eq. 3)}$$

The area below the probability density curve $g(u)$, i.e. the sum of probabilities from $u = -\infty$ to $u = \infty$, equals $G(u) = 1$ and is mathematically described as an integral of probability densities.

A number of statistical methods assume the provenience of values from a normal general population (falling under normal distribution). Proving this assumption (or rather the probability of its occurrence) requires the verification of a hypothesis on the normality of sample distribution. This can be with using a relevant statistical test. The choice of test type depends on the research sample size and is based on empirical research. The most commonly applied statistical tests for sample distribution normality are:

- χ^2 (chi-squared) test – for $n \geq 50$;
- d'Agostino K^2 (K-squared) test – for $50 \leq n \leq 1000$;
- Epps-Pulley test – for $8 \leq n \leq 200$;
- Shapiro-Wilk test – for $3 \leq n \leq 50$;
- Shapiro-Wilk Expanded test – for very small samples.

The practical application of tests for normality of distribution requires the use of statistical tables or their electronic versions in statistical computer software (e.g. Statistica, Matlab, Minitab, SPSS).

However one could search for some features of a normal distribution among some of the answers to the questionnaire (e.g. Figures 3.11, 3.14, 3.15), a question arise on the sense of such tests in the case of the discussed introductory quantitative research stage arises. Which feature of the research problem would the tested normality of distribution account for? Tests for normality of distribution are applied to easily measurable characteristics that can be indisputably expressed with a number. In the discussed case such a feature does not exist. The transposition of the respondents' statements into numerical criteria is possible (e.g. with use of AHP), but irrelevant for the purpose of this research.

The goal of the quantitative research stage was to find out whether in the eyes of management practitioners the application of multicriteria decision-making enhancement methods increases the probability of taking an integral managerial decision (H1). However, as proven in the above reasoning, testing the quantitative research stage research results for normality of distribution will not provide answers to such main research hypothesis. The concordance of the respondents' answers to the key survey questions will be checked instead.

There are two questions that cover to the largest extent the hypothesis H1 of the quantitative stage of discussed research task. One of these is No. 12: State how much you agree with the statement: "regular application of scientific tools for enhancement of decision-making processes raises the accuracy of managerial decisions". The other one is No. 14: State how much you agree with the statement: "incorporation of a larger number of decision criteria into the decision-making process increases the probability of making a more accurate managerial decision". The answers were provided on a 7-point Likert-type scale, with possible answers: I totally disagree, I mostly disagree, I rather disagree, I agree partially, I rather agree, I mostly agree, I fully agree. The answers can be divided into two groups:

- answers supporting hypothesis H1, stating that the application of multicriteria decision-making enhancement methods increases the probability of taking an integral managerial decision – answers 4-7 (I agree partially, I rather agree, I mostly agree, I fully agree), which point at the purposefulness of the application of tools for managerial decision-making enhancement;
- answers supporting hypothesis H2, stating that the application of multicriteria decision-making enhancement methods does not increase the probability of taking an integral managerial decision – answers 1-3 (I totally disagree, I mostly disagree, I rather disagree), which discourage the use of decision-making support methods for managerial decision problems.

The proportion of answers to each group of answers additionally validates this division:

- answers supporting H1 (answers 4-7): 91.43% – 97.14%;
- answers supporting H2 (answers 1-3): 2.86% – 8.57%.

For the assessment of respondents' answers concordance with those two key questions Kendall-Smith coefficient (also called Kendall's W coefficient of concordance) was applied.

Głuszak & Leśniak (2015, pp. 90-92) mention that Kendall's W coefficient of concordance shows the degree of association of ordinal assessments provided by multiple judges who assess the same samples. It adopts values from 0 to 1. The higher the *W* value the closer this association. This means that high or relatively significant W values are a proof that the judges keep up to the same assessment standards in their answers (Cabała 2012, pp. 191-196). Equation 4 gives a mathematical notation of Kendall's *W* coefficient of concordance for strong ordering of values:

$$W = \frac{12S}{m^2\left(n^3 - n\right)} \quad \text{(Eq. 4)}$$

where:
m = number of judgements;
n = number of factors evaluated by judges;
S = the sum of squared deviations, given by the formula from Equation 5:

$$S = \sum_{i=1}^{n}\left(R_i - \overline{R}\right)^2 = \sum_{i=1}^{n}\left(R_i - \frac{m(n+1)}{2}\right)^2 \quad \text{(Eq. 5)}$$

where:
R_i = total rank given to factor *i* (the sum of all ranks of factor *i*) (Eq. 6);
$\overline{R}$ = mean value of all total ranks (Equation 7).

R_i is calculated from Equation 6:

$$R_i = \sum_{j=1}^{m} r_{ij} \quad \text{(Eq. 6)}$$

where r_{ij} = evaluation (rank or points) attributed to the i^{th} factor in j^{th} evaluation;

where $\overline{R}$ as = mean value of all total ranks is denoted in Equation 7:

$$\overline{R} = \frac{1}{n}\sum_{i=1}^{n} R_i \quad \text{(Eq. 7)}$$

Then, the formula for Kendall's *W* coefficient of concordance that can be applied for the analysis of respondents' judgements consistency takes the form (Equation 8):

$$W = \frac{12\sum_{i=1}^{n}\left(\sum_{j=1}^{m} r_{ij} - \frac{1}{n}\sum_{i=1}^{n} R_i\right)^2}{m^2\left(n^3 - n\right)} \quad \text{(Eq. 8)}$$

In practical applications for the assessments with no ranks tied values a simplified version of Equation 5 can be used (Equation 9):

$$S = \sum_{i=1}^{n}\left(R_i - \frac{m(n+1)}{2}\right)^2 \quad \text{(Eq. 9)}$$

Following Cabała's (2010, pp. 43-45) reasoning, Kendall's test can be interpreted as if $W = 1$, then all survey respondents provided unanimous evaluations and each judge assigned the same order to the list of factors. If $W = 0$, then no trend in the responses or any agreement among the respondents can be perceived – the obtained judgements are random. Intermediate values of W indicate a greater or lesser degree of unanimity among the respondents or concordance between their assessment standards. Stabryła (2005, p. 106) provides the intervals of Kendall's W coefficient of concordance:

- sufficient: $0.20 \leq W \leq 0.40$;
- good: $0.41 \leq W \leq 0.60$;
- plus good: $0.61 \leq W \leq 0.80$;
- very good: $0.81 \leq W \leq 0.95$;
- ideal: $0.96 \leq W \leq 1$.

When judges provide evaluations of n factors on a 1 to k scale, where k is a natural number and $k < n$, some of the factors will be attributed the same rank (tied values). Such a rank order is called weak ordering. In such a case, the coefficient of concordance W needs to include a correction for ties and is defined as in Equation 10:

$$W = \frac{S}{mG} \quad \text{(Eq. 10)}$$

where G denotes the sum of squares of deviations of all ranks from series' average and can be formulated as in Equation 11:

$$G = \sum_{i,j=1}^{m,n} \left(a_{ij} - \frac{(n+1)}{2} \right)^2 \quad \text{(Eq. 11)}$$

with the relation noted in Equation 12:

$$mG = \frac{m^2 (n^3 - n)}{12} = S_{\max} \quad \text{(Eq. 12)}$$

In the case of weak ordering (ranks with ties), the ties between values tend to reduce the W value. Therefore the following relationship holds (Equation 13):

$$mG = S_{\max} - mT_j \quad \text{(Eq. 13)}$$

In such a case the correction factors T_j need to be computed. T_j is the correction factor required for the set of ranks for judge j, i.e. the j^{th} set of ranks and is noted as in Equation 14:

$$T_j = \sum_{i=1}^{g_j} \left(t_i^3 - t_i \right) \quad \text{(Eq. 14)}$$

where:
t_i – the number of tied ranks in the i^{th} group of tied ranks (a group is a set of values having constant, i.e. tied rank);
g_j – the number of groups of ties in the set of ranks (from 1 to n) for judge j.

The formula for Kendall's W coefficient of concordance with correction for tied values takes the form as in Equation 15):

$$W = \frac{12\sum_{i=1}^{n} R_i^2 - 3m^2 n(n+1)^2}{m^2 n(n^2-1) - m\sum_{j=1}^{m} T_j} \qquad \text{(Eq. 15)}$$

where:
R_i – the sum of the ranks for object i;
$\sum_{j=1}^{m} T_j$ – the sum of the values of T_j over all m sets of ranks (Siegel & Castellan, 1988, p. 266).

Kendall's W applications with correction for tied values can be additionally backed by an assessment of the significance of the coefficient of concordance via the χ_r^2 (chi-squared) characteristic with k-1 degrees of freedom. Its formula is expressed by Equation 16:

$$\chi_r^2 = \frac{S}{\frac{1}{12} mn(n+1) - \frac{1}{n-1} T} \qquad \text{(Eq. 16)}$$

The application of Kendall's W test allows a statistical assessment of respondents' consensus as regards two hypotheses:

- $H(K)_1$ – the concordance of respondents' opinions is not random, therefore judges are competent and their evaluations credible;
- $H(K)_0$ – the concordance of respondents' opinions is random.

In the discussed research task a relatively strong ordering was observed, which means that there were no ties between the obtained ranks. This is why the appropriate formulas for Kendall's W coefficient of concordance are those for ranks with no ties (from Equation 4 to Equation 9). In order to clarify the calculations, their simplified versions from Equations 4 and 9 were applied. The necessary calculations for questions No. 12 and No. 14 are given below (Example 3.1).

Example 3.1 – assumptions:

- $n = 7$ – number of possible answers to questions No. 12 and No. 14;
- m_i – number of answers to the i^{th} question
- n_i – number of questions in the analysed area.

Table 3.1 demonstrates all 7-point Likert-type scale answers, with the number of respondents' choices of each possible answer, shown separately for questions No. 12 and No. 14. For computational reasons, a column with the sum of all j evaluations attributed to the i^{th} answer is added. In order to check the validity of hypothesis H1 questions 4-7 were taken into consideration in the calculation of calculating Kendall's W coefficient of concordance. In Table 3.1 they are represented by the bolded square.

Table 3.1.

Kendall's W assessment of concordance of respondents' answers to questions No. 12 and No. 14 of the quantitative research stage questionnaire

7-point Likert-type scale answer No.	**Sum of evaluations of the ith factor (R_i)**	**No of judgements to question 12(m_i)**	**No of judgements to question 14 (m_i)**
1	0	0	0
2	2	1	1
3	3	2	0
4	5	13	8
5	7	14	12
6	9	4	9
7	10	1	5
	Kendall's W coefficient =	**0.917** (very good)	**0.56** (good)

*m_i is the number of answers to the i^{th} question
Source: Compiled on the basis of quantitative research results.

The calculations for the sum of squared deviations formula from Equation 9 and question No. 12 are expressed by Equation 17:

$$S_{12} = \left(5 - \frac{13(7+1)}{2}\right)^2 + \left(7 - \frac{14(7+1)}{2}\right)^2 + \left(9 - \frac{4(7+1)}{2}\right)^2 + \qquad \text{(Eq. 17)}$$

$$+\left(10 - \frac{1(7+1)}{2}\right)^2 = 4695$$

The calculation pattern of the value of Kendall's W coefficient of concordance for ranks with no ties, with the use of formula from Equation 4 and question No. 12 is expressed in Equation 18:

$$W_{12} = \frac{12 \cdot 4695}{32^2\left(4^3 - 4\right)} = \frac{56340}{61440} = 0.917 \qquad \text{(Eq. 18)}$$

Analogous calculations for question No. 14 can be found in Equations 19 and 20:

$$S_{14} = \left(5 - \frac{8(7+1)}{2}\right)^2 + \left(7 - \frac{12(7+1)}{2}\right)^2 + \left(9 - \frac{9(7+1)}{2}\right)^2 + \qquad \text{(Eq. 19)}$$

$$+\left(10 - \frac{5(7+1)}{2}\right)^2 = 3239$$

$$W_{14} = \frac{12 \cdot 3239}{34^2\left(4^3 - 4\right)} = \frac{38868}{69360} = 0.560 \qquad \text{(Eq. 20)}$$

The interpretation of the calculated Kendall's *W* coefficient of concordance values leads to a statement that the respondents to two key questions of the questionnaire provided their evaluations in a deliberate manner, basing on their best knowledge and experience in the field of managerial decision-making. The obtained results allow a statement that enhancement of managerial decision-making tasks with the use of multicriteria support tools is justified. Therefore hypothesis H1 of the introductory quantitative research stage, stating that the application of multicriteria decision-making enhancement methods increases the probability of taking an integral managerial decision can be confirmed. At the same time hypothesis H2 of the introductory quantitative research stage, stating that the application of multicriteria decision-making enhancement methods does not increase the probability of taking an integral managerial decision should be rejected.

On the basis of the research results obtained in the quantitative research stage the general sub-thesis T1, stating that in a dynamically changing environment of companies managerial decision-making processes require the inclusion of a growing number of decision criteria can be confirmed. The results also provide ground for positive verification of the general sub-thesis T2, stating that the incorporation of quantitative (measurable) and qualitative (immeasurable) criteria into such analyses can improve the integrity of the final decision.

The confirmation of the main research hypothesis of the quantitative research stage H1 and general sub-theses T1 and T2 provides justification for the construction of the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes. A proposal of the model construction pattern, illustrated using its application procedure is presented in Chapter 4.

4. CONSTRUCTION OF MODULAR MULTICRITERIA MANAGERIAL DECISION-MAKING MODEL (MMUMADEMM)

4.1. The Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) method was developed by Saaty (1980). Kou et al. (2017, p. 1209) state that AHP was initially developed making the most use of field data, laboratory results, and experts' experience, for the applications in multicriteria decision-making, planning, resource allocation and conflict resolution. The core of the AHP method is the Pairwise Comparison Matrix composed of values obtained through expert judgements. Goossens et al. (2008, p. 236) state that expert judgement has always played a large role in science [and] is recognised as another type of scientific data.

The core of AHP are pairwise comparisons of decision alternatives, parent criteria, sub-criteria and their indicators between each other, on an "each-with-each" basis. For this purpose a hierarchy of factors influencing the decision problem is formed. Their gradation constitutes the most important step of AHP method. Next these factors are analysed in pairs on each hierarchical level. As a result, the dominant factor from the pair below is being linked with the dominant factor from the pair straight above, which results in a ranking of the importance of all criteria.

An important advantage of the AHP method is that it allows including the immeasurable qualitative criteria into the decision-making process. While the pairwise comparisons are effectuated, these qualitative determinants of the decision environment can be transposed into quantitative ones. The numerical data obtained in this way can be used for statistical analysis, comparisons, or provide a numerical hierarchy that clearly shows the gradation of the significance of particular factor for the decision-making process in question. Additionally, mathematical notation constitutes a quantitative proof of the superiority of alternative A over alternative B, in terms of the adopted hierarchy of criteria. The above features make AHP the basic tool for combining qualitative and quantitative research approaches within one model. This in turn corresponds with general sub-thesis T3, which states that joint qualitative-quantitative research method is appropriate for the construction of the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes.

Figure 4.1 illustrates a theoretical example of a hierarchical composition of a decision-making model constructed in the AHP framework.

The main goal level represents the overriding purpose of the decision-making process. Cyfert & Krzakiewicz (2009, pp. 30-31) provide a set of characteristics that

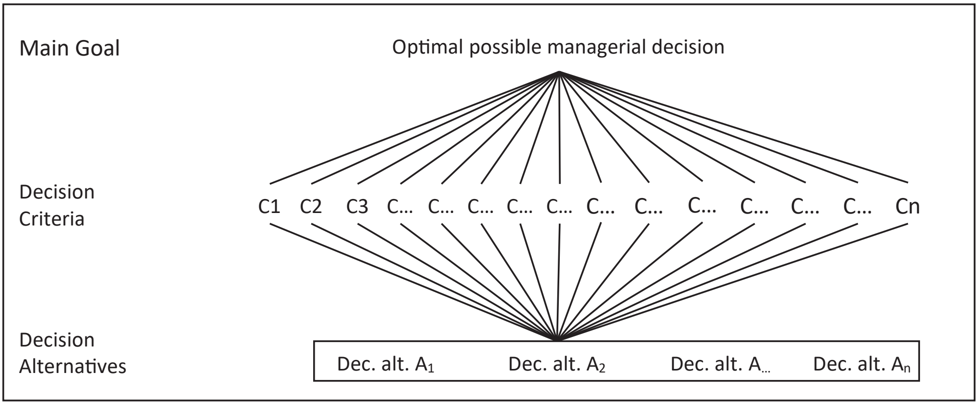


Figure 4.1. Theoretical example of an AHP hierarchical decision-making model
Source: Compiled on the basis of Saaty, 1996, pp. 5-6.

make a goal, in this case the main goal of the decision-making process, a properly formulated one:

- a goal needs to be extensively described;
- the denotation and formulation of goals has to conform to the standards and practices of the scientific discipline;
- the expected level and time span of achievement of the goal need to be instantiated;
- no alternative contradictory goals exist in the system;
- elements belonging to one group (here: the decision criteria or decision alternatives) have to be comparable and evaluable;
- the hierarchy of goals has to be flexible and allow the introduction of changes – from entire system's perspective – in the frame of one decision-making process, the main goal of the decision-making remains unchangeable.

The decision criteria level incorporates parent criteria, sub-criteria and their indicators, which are used for the evaluation of the significance of particular decision-making factors. The level of decision alternatives shows the set of available decisions from which the optimal solution should arise (Saaty, 1996, pp. 5-6). What is important, parent criteria and sub-criteria, as well as decision alternatives need to be independent of each other so that they do not bias the effects of the model application.

AHP can be applied to decision problems when the optimal solution has to be chosen from a set of alternatives on a subjective basis (i.e. a managerial decision). Another case is when the problem structure can be presented as a hierarchy, where the upper elements do not interact nor affect those lower on the hierarchical ladder (Saaty, 2001, pp. 23-24). Gawlik & Motyka (2006, p. 415) observe that AHP is a method for modelling decision-making problems when there is a necessity of multicriteria evaluation of decision variants.

To perform an analysis with the application of the Analytic Hierarchy Process the following steps have to be performed:

I. Formulation of a decision-making hierarchy;
II. Selection of experts;
III. Expert evaluations stage 1 (decision criteria);
IV. Expert evaluations stage 2 (decision alternatives);
V. Consistency checks of expert opinions;
VI. Evaluation of decision alternatives;
VII. Sensitivity analysis.

The first step of AHP-based decision-making is the development of a decision-making hierarchy. It is composed of:

- identification of the decision problem;
- formulation of goals that need to be achieved by the decision-making;
- determination of which groups of independent parent criteria and sub-criteria have an impact on the decision problem.

In the second step, a team of evaluators needs to be invited. There is no universal expert selection procedure, as the composition of group of experts is decision problem-specific. Therefore a separate expert selection task needs to be considered for each decision-making process. A common feature is that all experts need to possess a high level of expertise in the field of the decision problem in question.

The third step, expert evaluations stage 1, is meant to collect expert judgements on the significance of all parent criteria and their sub-criteria. This happens via the mentioned pairwise comparison process. The experts' task is to state which of two objectives from each pair of the compared criteria is more significant with respect to the higher node of decision hierarchy. Pairwise comparisons are repeated until all parent criteria and sub-criteria have been compared. As a result a Pairwise Comparison Matrix is constructed. Its values represent the significance of particular decision criteria with respect to the main goal of the decision-making.

The same procedure is repeated in step four – expert evaluations stage 2; however, this time decision alternatives are assessed. Pairwise comparisons between each pair of available decision alternatives, with respect to every parent criterion and particular sub-criteria are performed. Similarly, pairwise comparisons are repeated until all decision alternatives have been compared and a Pairwise Comparison Matrix is obtained. However, this time it represents the significance of decision alternatives with respect to all decision criteria simultaneously.

Szarucki (2014, p. 110) defines evaluation as a statement assessing the material value of something, which happens in the process of estimation of evaluation's subject or an opinion on something or someone, formed through an analysis. In the AHP applications evaluation is a strictly defined process of attribution of preference statements in pairwise comparisons between two decision parent criteria (or sub-criteria or decision alternatives), in relation to the higher node of the decision hierarchy (parent criterion or main goal of the decision-making process). Similarly to above definition, it results in a measure (material value) of the significance of each evaluated element for the achievement of main goal of the decision-making. A completed evaluation process results in a hierarchical ranking of significance of decision criteria and alternatives.

The qualitative-quantitative transition of significance of decision-making criteria within an AHP decision-making model is effectuated through three instruments:

- the Fundamental Comparison Scale;
- the Pairwise Comparison Matrix;
- the Consistency Check (Bhushan & Rai, 2004, pp. 15-17)[15].

The Fundamental Comparison Scale is applied at the pairwise comparison level. It allows the experts to express their preferences in terms of showing how strong the dominance of one factor above the other (from the same pair) is. The expert chooses the most suitable descriptive term to state one factor's dominance over the other from the following set: equal, weak, strong, very strong or absolute. Numbers 1, 3, 5, 7, 9 are attributed respectively. Numbers 2, 4, 6, 8 describe intermediary situations, when a strict choice cannot be made.

The construction of Saaty's Fundamental Comparison Scale has been subject to a wave of criticism (Belton & Gear, 1983; Dyer, 1990; Holder, 1990). As a result, other designs of judgement scales together with appropriate Consistency Check procedure have been admitted as well (Saaty, 2001, pp. 70-92); they can be found in Franek & Kresta (2014). An interesting discussion on the shortcomings of different types of scales within AHP, as well as a 9-point scale rationalisation proposal, can be found in Goepel (2017).

The Pairwise Comparison Matrix is a rectangular array of numbers derived from expert ratings. Saaty (1996, pp. 17-25) provides its mathematical construction, as expressed by Equations 21 to 23 below. Equation 21 shows the notation of an *i* verse j row a_{ij} element of the Pairwise Comparison Matrix:

$$\left[a_{ij}\right] = \left(n \cdot n\right) A \qquad \text{(Eq. 21)}$$

where:
$i = \{1,2,\ldots,n\}, j = \{1,2,\ldots,n\}$;
$a_{ij} = \frac{1}{a_{ij}}$, $\rightarrow$ if $i = j \leftrightarrow a_{ij} = 1$.

Equation 22 shows the Pairwise Comparison Matrix A:

15 A discussion of this topic can also be found in (Gawlik, 2012).

$$A = \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \dots & 1 \end{bmatrix} \quad \text{(Eq. 22)}$$

For the calculation of matrix's eigenvector, the above matrix needs to be normalised using Equation 23:

$$\overline{a_{ij}} = \frac{a_{ij}}{\sum_{j=1}^{n} a_{ij}} \quad \text{(Eq. 23)}$$

The sum of normalised verses of the matrix allows the calculation of matrix eigenvector, which is a representation of the priorities obtained through expert evaluations through achieving all pairwise comparisons, according to the "each with each" principle (Saaty, 2003).

Step five, i.e. Consistency Checks of expert opinions, has to be performed after each round of expert evaluations in order to assure the credibility of their evaluations. The Consistency Check targets at excluding inconsistent expert opinions. The Consistency Ratio (CR) formula is expressed by Equation 24:

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}} \quad \text{(Eq. 24)}$$

where:
CR = Consistency Ratio;
CI = Consequence Index;
RI = Random Index.

The Consequence Index (CI) can be calculated from the formula expressed by Equation 25:

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n - 1} \quad \text{(Eq. 25)}$$

where:
λ_{max} = matrix eigenvalue;
n = dimension of the matrix.

The Random Index values (RI) have been empirically verified (Saaty, 1996, p. 21). They are also dependent on the dimension of the matrix (**N**) and can be found in Table 4.1:

Table 4.1.

Values of the Random Index with regard to matrix dimension

N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Source: Saaty (1996, p. 21).

Saaty (1996, 21) states that expert opinions are inconsistent when the Consistency Ratio (CR) value exceeds 0.1 (or 10%). There are several ways of dealing with inconsistent expert opinions. Most AHP software (e.g. Expert Choice, Comparion Suite, Super Decisions, AHP Online System) provides partial Consistency Checks for each obtained local priority, after each parent criterion has been assessed. In such a case, when an expert realises that the inconsistency of his/ her evaluations exceeds 10%, they can reassess one or all of their preference statements. It has to be done very consciously, though there is as a risk of self-suggestion or willingness to be more consistent than truthful.

An absolute 100% consistency of expert evaluations is impossible due to:

- the construction of comparison scales (Dyer, 1990; Franek & Kresta, 2014; Goepel, 2017);
- the character of the evaluation process, which limits the possibility of assistance to experts at the time of providing their judgements; however, precise definitions of all parent criteria, sub-criteria, objectives and alternatives have to be presented to experts at times of evaluation, the possibility of obtaining further explanations should be rather restricted, in order not to influence experts' judgements;
- imperfect rationality of human judgements;
- approximate nature of scientific research, where absolute statements based on 100% probability should automatically cause misbelief and doubt.

For the reasons above, an acceptable level of consistency of expert evaluations has to be adopted, bearing in mind that it constitutes a self-imposed limitation of accuracy research results. In most AHP applications the $CR_{max} < 10\%$ suggested by Saaty (1996, p. 21) is adopted. It reminds one of a similar case of quantitative research when statistical methods are used, where pre-adopted α and β levels anticipate a reasonable and acceptable level of Type I and Type II errors.

When employing AHP as a research method, corrections to formerly given statements are a very delicate matter and should be avoided in principle. Therefore, in cases when evaluations of one expert significantly exceed the accepted 10% inconsistency level, they should not be taken into account in making the final decision. From the methodological perspective, all assessments of the highly inconsistent expert should be removed from the Pairwise Comparison Matrix. Such a step can be applied only in special cases, as the scientific truthfulness always takes the priority over researcher's or decision-maker's expectations. What absolutely cannot be accepted is any manipulation of data by the researcher after the expert evaluations have been received.

Numerous improvements of the quantitative and qualitative Consistency Check procedure (including AHP-related) can be found in the literature on the subject, e.g. Barzilai (1998a), Duszak & Koczkodaj (1994), Giove et al. (2002), Błaszczyński et al. (2009), Błaszczyński, Słowiński & Stefanowski (2010), Zhai & Janicki (2010), Brunelli, Critch, & Fedrizzi (2013), Szeląg, Greco & Słowiński (2014), Kułakowski (2014, 2015a, 2015b) or Koczkodaj, Kułakowski & Ligęza (2014).

Another way of indirectly improving the general consistency of decision-making could effected by the fuzzification of the decision-making results of the Hierarchical Module of the proposed decision-making model. For this reason a Fuzzy Module and mixed AHP-Fuzzy expansions will be considered.

The sixth step is the evaluation of decision alternatives, this phase leads to the final result of model application which points at the final decision and is the last step of AHP application originally proposed by Saaty (1980). After the obtained matrices have been normalised, criteria eigenvector and decision alternatives eigenvector can be drawn. Vector values signify approximated weights of particular criteria with respect to their contribution in the accomplishment of the adopted goal of decision-making. As a result, each decision alternative is attributed weights that reflect the extent to which it contributes to the accomplishment of the main goal of the decision-making process, with simultaneous fulfilment of all the criteria. The decision alternative that achieves this to the largest extent is the optimal possible one.

The seventh step, i.e. the sensitivity analysis, was not initially part of the AHP method. However, it provides important information on the susceptibility of the chosen decision alternative to change of prioritisation of any decision criterion. Sensitivity analysis aims at finding which criterion and which performance measure could cause a change in the ranking between a pair of alternatives (rank reversal), even if there are relatively small changes in the preference statements.

The first to mention the need for sensitivity analysis in decision-making processes was French (1986). It was almost immediately implemented into operations research through linear programming and inventory models (Wendell, 1992). In management science, sensitivity analysis became a part of investment analysis models. The methodology for sensitivity analysis for multicriteria decision-making models was proposed by Rios Insua (1990) and Masuda (1990), and further developed by Triantaphyllou & Sanchez (1997) and Triantaphyllou (2000).

Triantaphyllou & Sanchez (1997, pp. 156-159) propose to approach sensitivity analysis by determining the most critical criterion, which is verbally defined as the criterion C_k with the smallest change of the current weight w_k by the amount of $\delta_{k,i,j}$ changing the ranking between the alternatives A_i and A_j. The mathematical notation of this definition follows. First, the changes in the current weights of decision criteria need to be defined:

Let $\delta_{k,1,j}$ (for $1 \leq i < j \leq m$ and $1 \leq k \leq n$) denote the minimum change in the current weight w_k of criterion C_k such that the ranking of alternatives A_i and A_j will be reversed (Equation 26):

$$\delta'_{k,i,j} = \delta_{k,i,j} \frac{100}{w_k} \qquad \text{(Eq. 26)}$$

for any $1 \leq i < j \leq m$ and $1 \leq k \leq n$, where $\delta'_{k,i,j}$ expresses changes in relative terms.

Basing on Equation 26, the most critical criterion can be defined. There are two possibilities:

- the Percent-Top[16] (PT) critical criterion, which is the most critical criterion with the smallest change $\delta_{k,1,j}$ changing the ranking of the best (top) alternative; it is calculated when the decision-maker is interested only in changes in the best decision alternative (usually noted as A_1); it corresponds to the smallest $\left|\delta'_{k,i,j}\right|$ for $1 \leq j \leq m$ and $1 \leq k \leq n$ value;
- the Percent-Any[17] (PA) critical criterion, which is the most critical criterion with the smallest change $\delta_{k,1,j}$ changing the ranking of any alternative; it is calculated when the decision-maker is interested in changes in the ranking of any of the decision alternatives; it corresponds to the smallest $\left|\delta'_{k,i,j}\right|$ for $1 \leq i < j \leq m$ and $1 \leq k \leq n$ value.

To create the sensitivity matrix for each pair of alternatives A_i and A_j, for $1 \leq i < j \leq m$ and $1 \leq k \leq n$ the following calculations need to be performed (Equation 27):

$$\delta_{k,i,j}\left(w_k, A_i, A_j\right) = \frac{P_j - P_i}{a_{jk} - a_{ik}} \qquad \text{(Eq. 27)}$$

$$\text{where } \left|\delta'_{k,i,j}\left(w_k, A_i, A_j\right)\right| \leq w_k$$

In Equation 27 P_i stands for the preference for alternative A_i in terms of the AHP method and is calculated by the weighted sum model (WSM) formula expressed by Equation 28:

$$P_i = \sum_{j=1}^{n} a_{ij} w_j \qquad \text{(Eq. 28)}$$

where:
w_j – the weight of criterion C_j;
a_{ij} – the performance measure of alternative A_i with respect to criterion C_j.

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16 In the literature the Percent-Top (PT) critical criterion is sometimes referred to as the Absolute-Top (AT) critical criterion.

17 In the literature the Percent-Any (PA) critical criterion is sometimes referred to as the Absolute-Any (AA) critical criterion.

In AHP applications performance values are usually normalised via formula expressed by Equation 29:

$$\sum_{i=1}^{m} a_{ij} = 1 \qquad \text{(Eq. 29)}$$

for any $j = 1,2,3,...,n$.

As a result a sensitivity matrix is obtained, out of which the most critical criterion C_j can be extracted. In the case of Percentage-Top (PT) critical criterion this will be the one with the smallest decision alternative change (in %) representing the best (top) decision alternative A_1. In the case of Percentage-Any (PA) critical criterion this will be the one with the smallest decision alternative change (in %) representing any of the decision alternatives A_i, A_j.

In the search for the most critical measure of performance, Triantaphyllou & Sanchez (1997, p. 176) state that in AHP applications the threshold value by which the measure of performance of alternative A_i in terms of criterion C_j needs to be modified so that the ranking of alternatives A_i and A_k will change is given by formula from Equation 30:

$$\tau'_{i,j,k} = \frac{P_i - P_k}{\left[P_i - P_k + w_j\left(a_{kj} - a_{ij} + 1\right)\right]} \frac{100}{a_{ij}} \qquad \text{(Eq. 30)}$$

where $\tau'_{i,j,k} \leq 100$

The Percentage-Any (PA) critical performance measure for the decision alternative A_i is the criterion C_j, because it is the smallest value X (in %), that will change its ranking in relation to alternative A_j drastically (by X %).

The most recent applications of the AHP method cover multiple areas of management. These include business (Oudah, Jabeen & Dixon, 2018); development of cities (Wang et al., 2018; Neisani Samani, Karimi & Alesheikh, 2018); financial supervision (Adamus & Łasak, 2010); governance of energy supply (Udie, Bhattacharyya & Ozawa-Meida, 2018); human resources (Gawlik & Jacobsen, 2016); production engineering (Heydaryan, Suaza Bedolla & Belingardi, 2018; Gajdzik & Gawlik, 2018); quality management (Gręda, 2010); real estate management (Marona & Wilk, 2016; Małkowska, Głuszak & Gawlik, 2017); work-life balance (Gawlik, 2017) and others mentioned by Saaty (2008).

Some authors question the outcome from AHP-based decision-making models. Belton & Gear (1983) question the validity of empirical justifications of the correctness of AHP applications. Another issue arises from the semantic problems with the transposition of the Fundamental Scale of expert judgements into numbers, which is the core of the method in question. The same authors argue that the 1-9 judgement scale (Fundamental Comparison Scale) is incomplete, due to its linearity and difficulties of transposing some linguistic nuances into integer numbers.

The rank reversal phenomenon mentioned earlier was discussed by Holder (1990). He argued that the additive hierarchical construction of sets of decision criteria in the AHP method, which is the basis for pairwise comparisons, is at the same time its serious weakness. An additional irrelevant decision alternative could reverse the final ranking of alternatives. Dyer (1990) argued that rankings obtained via the AHP applications are arbitrary due to the principle of independence of AHP hierarchy higher levels elements from its lower levels. In his analysis of the mathematical decomposition of value functions Barzilai (1998b) observed that AHP generated non-equivalent value functions and rankings from equivalent decompositions. Pérez, Jimeno & Mokotoff (2006) added another issue – adding indifferent criteria (which should not influence the final decision in any way) causes significant alterations of the final aggregated priorities of decision alternatives. This again could lead to rank reversal, which questions the precision of AHP-based decision-making.

Answers to most of the critical remarks of AHP point at limited, but still present rationality of decision-makers that will construct hierarchies of decision criteria according to their best knowledge and sense. Preliminary reduction of criteria should exclude those irrelevant, or interfering with each other[18] (which is a frequently neglected necessity when building AHP sets of criteria). A similar argument can be used towards sets of potential decision alternatives (Saaty, Vargas & Whitaker, 2009; Whitaker, 2007). The construction of judgement scales became an almost separate field of research in the frame of the AHP method. Franek & Kresta (2014) discuss recent developments, which answer most of the earlier criticism. The mentioned authors also provide additional restraints to Saaty's Consistency Check, which additionally raises the accuracy of decisions made with the use of AHP. Dyer himself provided a solution to his earlier criticism of AHP in terms of the multiattribute utility concept (Dyer, 1990). Goepel (2017) states that the original AHP scale seems to present a kind of compromise with respect to the maximum number of criteria, weight dispersion and weight uncertainty. Nevertheless, he proposes an improvement – an adaptive-balanced scale, which keeps the weight ratio at nine for any number of criteria [and] results in evenly distributed weights across the judgement range, and is with respect to weight uncertainty still preferable to the original AHP scale (Goepel, 2017).

As the mentioned limitations could potentially cause some imperfections of AHP-based modelling of managerial decisions, a development of the proposed solution has been considered. Following the recommendations of the prescriptive approach (making the optimal possible decisions, but under real environmental circumstances and restrictions, i.e. integral decisions), fuzzy expansion of the proposed decision-making

[18] A proposal of a method for the number of AHP decision criteria reduction can be found in (Gawlik, 2008).

model will be envisaged. A combination of AHP and Fuzzy sets theory was already employed by Woropay & Muślewski (2005), but with a reversed order. AHP's role was then limited to the evaluation of system functioning quality criteria.

4.2. Fuzzy expert systems in managerial decision-making

The increasing complexity of scientific methods is their immanent feature, which happens in a sequence (Ćwiklicki, 2011, pp. 28-29). First, the initial fields of application of a method expand into new ones. This creates the need for the inclusion into the initial toolbox of new modules that facilitate the solution of new categories of problems. As a response to these needs, the method is complemented by auxiliary methods and techniques. Consequently, the structure of the method evolves and expands through the inclusion of new modes of operation, but also through the application of additional methods and research tools. The author derives these findings from a literature search, but also realises the limitations of the use of such methodology, i.e. deficiencies of management science's legacy. Recalling Martyniak's (1999, p. 38) observations, several factors that preclude a complete literature review on the evolution of managerial methods can be enumerated:

- the lack of papers describing some of managerial methods used in the past;
- the state of the economy;
- the existence and coverage of scientific journals;
- the extent of researcher's cooperation with practitioners of management;
- the fact that most research projects are conducted in big organisations, which can distort observation.

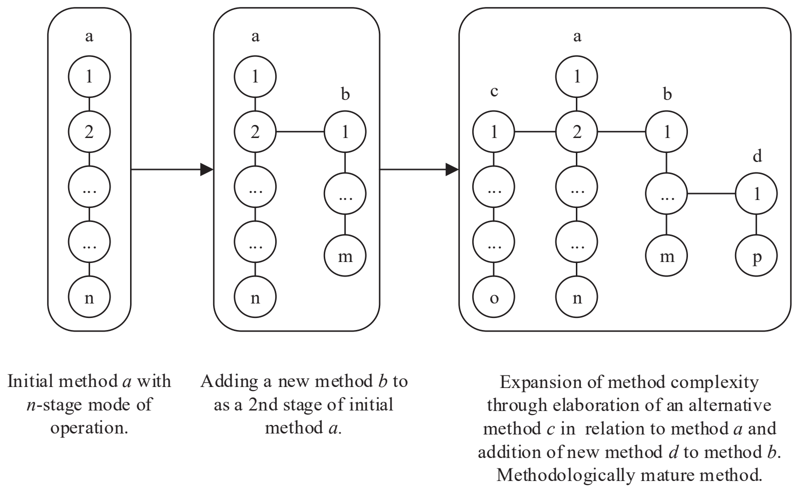

Figure 4.2. Evolutionary increase of the research method complexity
Source: Ćwiklicki (2011, p. 28).

Ćwiklicki continues by stating that management methods are vulnerable to evolutionary changes (2011, p. 29) and backs this statement with Mikołajczyk's (1994, p. 40) opinion that one of the features of methods of organisation and management is the openness to new modes of operation and complementary techniques. A scheme of evolutionary increase of research method's complexity is presented in Figure 4.2.

As the enhancement of managerial decisions is a sub-discipline of the field of study management in the academic discipline of economics, it can be assumed that the presented views on the evolution of managerial methods apply also to the scientific problem covered by this monograph.

Rębiasz & Macioł (2015) provide arguments for and against the substitution of classical multicriteria decision-making methods by fuzzy rule-based methods. Prakash & Barua (2015, p. 4, after Mostert, Niemann & Kotzé, 2017, p. 6) and Mardani et al. (2015, after Marais, Du Plessis & Saayman, 2017, p. 6) and especially Peng et al. (2011) suggest an expansion of AHP applications by other decision-making methods rather than by replacing AHP with them. This position has been adopted in this monograph for the construction of the decision-making model.

The polyvalent fuzzy logic methodology, together with the resulting inference was proposed by Zadech in 1965, although as early as 1911 Łukasiewicz proposed his many-valued logic theory, which is perceived by some as the foundations of fuzzy logic. It soon proved to be useful for expert decision support systems. Therefore it is considered as an evolutionary development of the complexity of the proposed decision support method, aiming at increasing the accuracy of the entire managerial decision-making process.

Fuzzy logic uses linguistic information based on descriptive notions of qualitative character (e.g. small, big, very small, very big, etc.). Each such linguistic descriptor represents a blurred (fuzzy) notion. Fuzzy logic application aims at establishing rules of possibly rigorous systems characterising the relations between descriptors, together with system's learning and testing algorithms. For this purpose linguistic descriptors are attributed numerical values, which become numerical variables. These in turn can be attributed adequate functions (called membership functions) which describe the range of variability of their parameters. The resulting sets of values are called fuzzy sets. Therefore in fuzzy logic numerical variables coexist with linguistic descriptors with attributed numerical values (Zadech, 1965). This constitutes a premise that, similarly to AHP, fuzzy logic can be applied as a mixed, qualitative-quantitative method for the support and enhancement of managerial decision-making processes.

Before proceeding to the presentation of the decision-making enhancement model construction, the main concepts and notions characterizing the Fuzzy sets theory need to be explained.

A linguistic variable is an input or output measure or a state variable that is assessed via descriptive notions, called linguistic values. In practice, the assessment of linguistic values is mixed, both through linguistic descriptors and fuzzy numbers.

A linguistic value is a verbal assessment of the linguistic magnitude (e.g. very, big, small, medium, negative, positive, old, new, tall, short, etc.).

Out of eight definitions of fuzzy numbers presented by Dijkman, van Haeringen & de Lange, the following seem to summarise all their features: a fuzzy number is a generalisation of a regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1 (Dijkman, van Haeringen & de Lange, 1983, p. 303). Fuzzy numbers can be applied for modelling of systems with a known, unequivocal projection of input into output, $y = f(X)$, when input signals cannot be precisely measured, but only approximated (e.g. expected cost around EUR 1000, time-span of investment not longer than twelve months, profitability around 3%, etc.).

Variables characterised by fuzzy numbers can be measured with specialised instruments only, whereas linguistic values can be measured via expert evaluations (which will be the case of decision-making model in construction).

An interesting issue arises with the definition of the true value (positive logical/truth value) of a linguistic determinant. Lootsma (1993, p. 12) states that since it is not easily acceptable to define a concept on the basis of subjective feelings, attempts have been made to introduce a more objective, frequentist definition of the true value. Therefore the true value is sometimes interpreted as the fraction of a sufficiently large number of referees agreeing with the fact that a given statement belongs to a particular set, e.g. the room temperature of 21°C is comfortable, as a relative majority of the population thinks so. Nevertheless, the "comfortable" statement does not have to apply precisely to 21°C, as the subjective feeling of comfort depends also on other factors, e.g. air humidity, wind, etc. Following the fuzzy logic concept a true comfort temperature would be a set of values of all descriptive parameters that in the eyes of a representative sample of population or in experts' assessments instigate the feeling of comfort.

In his introduction to the Fuzzy sets theory Wołoszyn (1990, pp. 11-40) provides further definitions and equations. The linguistic term-set is a set containing all linguistic values that are used for the assessment of a given linguistic variable. This space is marked with capital Latin letters. The linguistic term-set is a finite-dimensional space. Some examples of linguistic term-set notation are presented in Equations 31 and 32:

$$X_L = \{\text{small, medium, big}\} = \{x_{L1}, x_{L2}, x_{L3}\} \quad \text{(Eq. 31)}$$

$$Y_L = \{\text{high, low}\} = \{y_{L1}, y_{L2}\} \quad \text{(Eq. 32)}$$

The membership function and grade of membership reflect the arrangement of linguistic term-set elements in space. This arrangement results from associating a given feature with the elements of the set. The value of the membership function of a given element describes the grade of its membership to this set (Equation 33):

$$\mu(x): X \to [0,1], \forall x \in X \qquad \text{(Eq. 33)}$$

The membership function can be presented in following forms:

- graphic, in the form of a continuous or discrete diagram (Figure 4.3);
- tabular – in a table (Table 4.2);
- in mathematical notation – a sum (Equation 34) or an integral (Equation 35);
- via a membership vector (Equation 36).

Figure 4.3 is a graphic presentation of the membership function, with its continuous and discrete types:

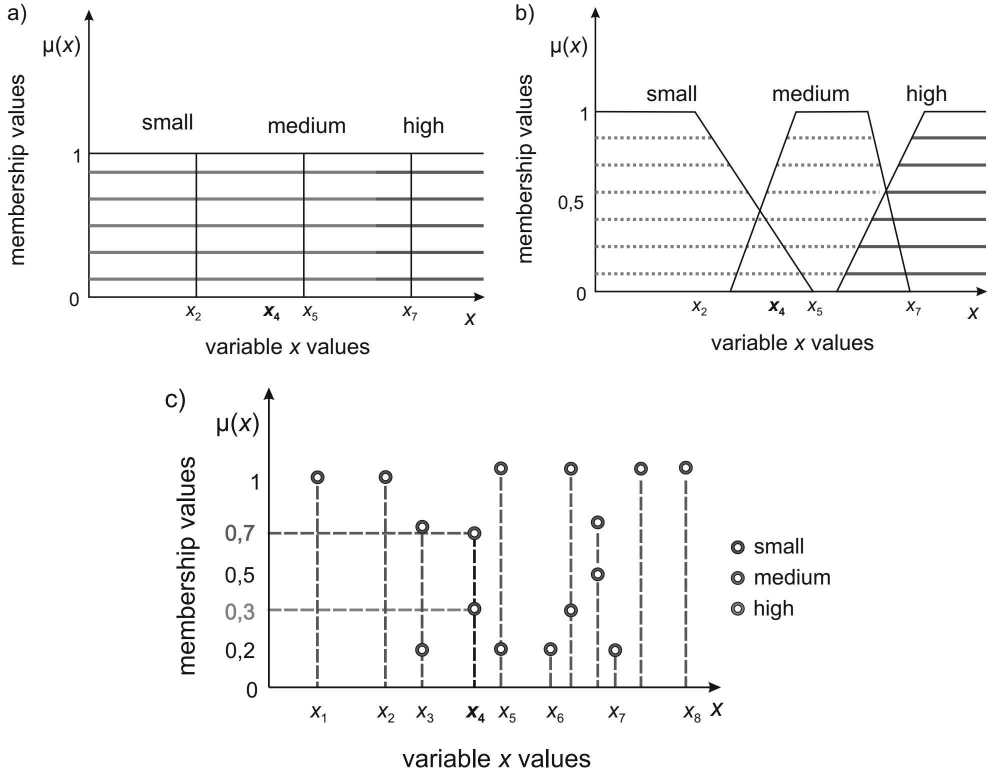

Figure 4.3. Graphic presentation of the membership function – continuous: a) & b); discrete: c)
Source: Compiled on the basis of Grzesik (2016, p. 12).

Table 4.2 shows the tabular notation of the membership function. For presentation purposes the data have been taken from Figure 4.3.c):

Table 4.2.

Tabular notation of the membership function

$x \in X$	$x_1 = 80$	$x_2 = 140$	$x_3 = 310$	$x_4 = 370$
$\mu_{low}(x)$	1,0	1,0	0,37	0.18

Source: Compiled on the basis of example c) from Figure 4.3.

Equation 34 expresses the mathematical notation of the membership function as a sum:

$$A = \frac{\mu A(x_1)}{x_1} + \frac{\mu A(x_2)}{x_2} + \ldots + \frac{\mu A(x_n)}{x_n} = \sum_{i=1}^{n} \frac{\mu A(x_i)}{x_i} \quad \text{(Eq. 34)}$$

The integral-based notation of the membership function is shown by Equation 35:

$$A = \int \frac{\mu A(x)}{x} \quad \text{(Eq. 35)}$$

The membership vector describing the membership is expressed in Equation 36. The order of all n elements x_i in the contemplated space X has been strictly determined.

$$V_A = \{\mu_A(x_1), \mu_A(x_2), \ldots, \mu_A(x_n)\} \quad \text{(Eq. 36)}$$

Some examples of most commonly applied membership functions are shown in Figures 4.4 to 4.8 and cover the following types (Rutkowska, Piliński & Rutkowski, 1997):

- singleton membership function (Figure 4.4);
- triangular membership function (Figure 4.5);
- trapezoidal membership function (Figure 4.6);
- simple Gaussian membership function (Figure 4.7)
- composite Gaussian membership function (Figure 4.8).

An example of a singleton membership function shown in Figure 4.4 is defined by the mathematical formula expressed by Equation 37:

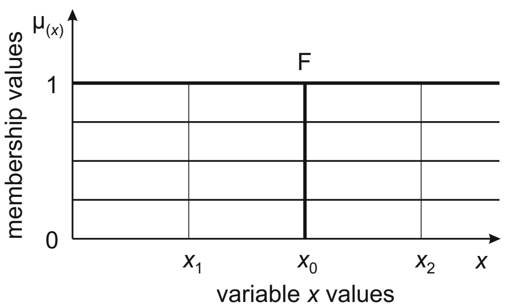

Figure 4.4. A singleton membership function
Source: Compiled on the basis of Grzesik (2016, p. 14).

$$\mu_A(x) = 1 \text{ for } x = x_0;\ \mu_A = 0 \text{ for } x \neq x_0 \qquad \text{(Eq. 37)}$$

An example of a symmetric triangular membership function defined by a peak c, a lower limit $c - d$ and an upper limit $c + d$ is shown in Figure 4.5.

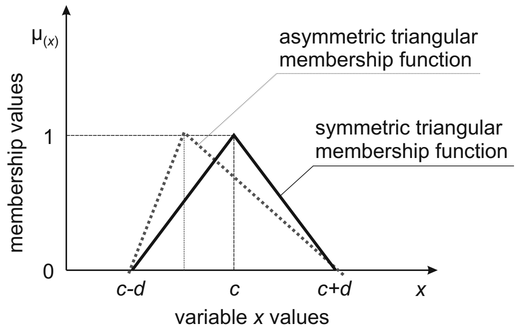

Figure 4.5. Triangular membership functions: symmetric and asymmetric
Source: Compiled on the basis of Grzesik (2016, p. 14).

The mathematical notation of a triangular membership function is given in Equation 38:

$$\mu_A(x) = \left\{1 - \frac{|x - c|}{d}\right\};\ \text{for } x \in [c - d, c + d] \qquad \text{(Eq. 38)}$$

where:
$\mu_A(x) = 0$ lies outside this interval;
c = peak of the symmetric triangular function ($\mu_A(c) = 1$);
$c - d$ = lower limit of x values;
$c + d$ = upper limit of x values.

Examples of trapezoidal membership functions defined by an upper limit z, lower support limit y, support range t, central point c, and slope s are shown in Figure 4.6.

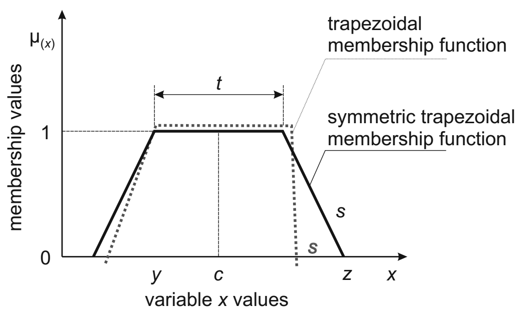

Figure 4.6. Trapezoidal membership functions
Source: Compiled on the basis of Grzesik (2016, p. 15).

In mathematical notation the trapezoidal membership function has to comply with the assumptions presented in the equations below (Equations 39-42):

$$\mu_A(x_i) = 0 \quad \text{for} \quad x > z \quad or \quad x < y \qquad \text{(Eq. 39)}$$

$$\mu_A(x_i) = 1 \quad \text{for} \quad c - \frac{t}{2} \leq x \leq c + \frac{t}{2} \qquad \text{(Eq. 40)}$$

$$\mu_A(x_i) = s(z - x) \quad \text{for} \quad c + \frac{t}{2} \leq x \leq z \qquad \text{(Eq. 41)}$$

$$\mu_A(x_i) = s(x - y) \quad \text{for} \quad y \leq x \leq c - \frac{t}{2} \qquad \text{(Eq. 42)}$$

where:

$y = c - \frac{t}{2} - \frac{1}{s}$;

$z = c + \frac{t}{2} + \frac{1}{s}$;

c = central value;
y = lower support limit;
i = upper limit;
t = support limit range;
s = value of the slope.

Please note, that if $t = 0$, then the trapezoidal membership function becomes a triangular membership function.

A Gaussian membership function defined by a central value m and a standard deviation $k > 0$ is mathematically noted in Equation 43. The smaller the k parameter, the narrower the "bell" of the graphic illustration of function becomes.

$$\mu_A(x) = e^{-\frac{(x-m)^2}{2k^2}} \qquad \text{(Eq. 43)}$$

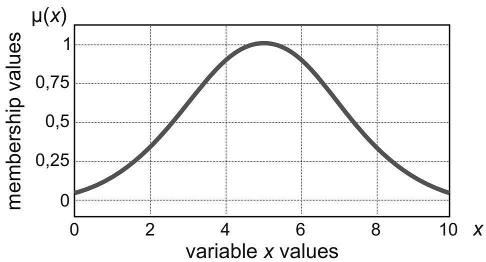

Figure 4.7 An example of a simple Gaussian membership function
Source: Compiled on the basis of Dietrich & Schulze (2000, pp. 49-50).

There are two basic types of Gaussian membership functions: symmetric and asymmetric. A symmetric Gaussian membership function is based on simple Gaussian distribution curve (Figure 4.7).

An asymmetric Gaussian membership function is based on a two-sided composite of two different Gaussian curves (Figure 4.8).

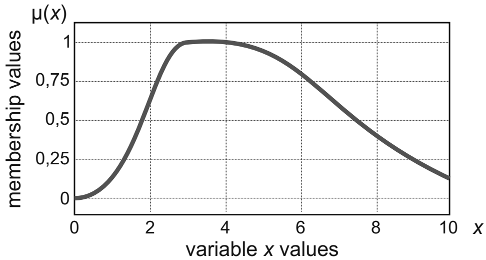

Figure 4.8. An example of a composite Gaussian membership function
Source: Compiled on the basis of Dietrich & Schulze (2000, pp. 49-50).

Although the Gaussian type membership function seems to offer the most precise fuzzy projection of a linguistic term-set, in practice the type of the membership function to be applied for research purposes is usually context dependent. Moreover, it will be chosen arbitrarily, basing on the experience of the decision-maker or involved experts (Mendel, 1995, p. 350). In need of close modelling of managerial decisions, the complex, asymmetric character of decision environment, together with its uncertainty suggests that polygonal membership functions should be considered.

The advantages of polygonal membership functions include:

- a small amount of data necessary to define the membership function;
- they fulfil the condition of summing up to 1, which means that the sum of grades of membership for each x value equals 1;
- they allow a fast mapping of system's input → output model resulting in a hypersurface built of line segments;
- they allow a fast modification of parameters (modal values) of membership function, basing on model's input → output measurements data.

There are two main disadvantages of polygonal membership functions: their discontinuity and non-differentiability. In one's choice of a membership function, one should observe the following procedure: at low availability of information about the system, the simplest functions, composed of straight line segments should be used, as for the identification of their parameters small amounts of data are needed. When there is a lot of information about the system, complex functions defined by a bigger number of parameters are advised (e.g. Gaussian or polygonal functions), as they offer a higher precision of the model. However, in most managerial decision-making applications, due to incomplete information available, symmetric membership functions need to be applied.

Fuzzy numbers and fuzzy sets can be subject to basic mathematical operations, i.e. summation, subtraction, multiplication, division (Piegat, 1999, pp. 112-133).

Taking into account the substantive aspects of managerial decision-making and the scope of research presented earlier in this monograph, triangular membership functions were employed for further processing of decision-making results obtained via the Hierarchical Module of the constructed multicriteria model. Two separate cases were analysed:

- case I – decisions alternatives A_1 and A_2 are disjoint sets;
- case II – decisions alternatives A_1 and A_2 intersect (they are not disjoint sets).

Case I – decision alternatives A_1 and A_2 are disjoint sets and are defined as in Equation 44:

$$A_1 = \left(m_{A_1}, x_{A_1 1}, x_{A_1 2}\right); A_2 = \left(m_{A_2}, x_{A_2 1}, x_{A_2 2}\right) \quad \text{(Eq. 44)}$$

Final decision D is a sum of decisions alternatives A_1 and A_2 and is defined as in Equation 45:

$$D = \left(A_1 + A_2\right) = \left(m_{A_1} + m_{A_2}, x_{A_1 1} + x_{A_2 1}, x_{A_1 2} + x_{A_2 2}\right) \quad \text{(Eq. 45)}$$

A graphic illustration of parameters of two fuzzy numbers A_1 and A_2 and their sum D is shown in Figure 4.9.

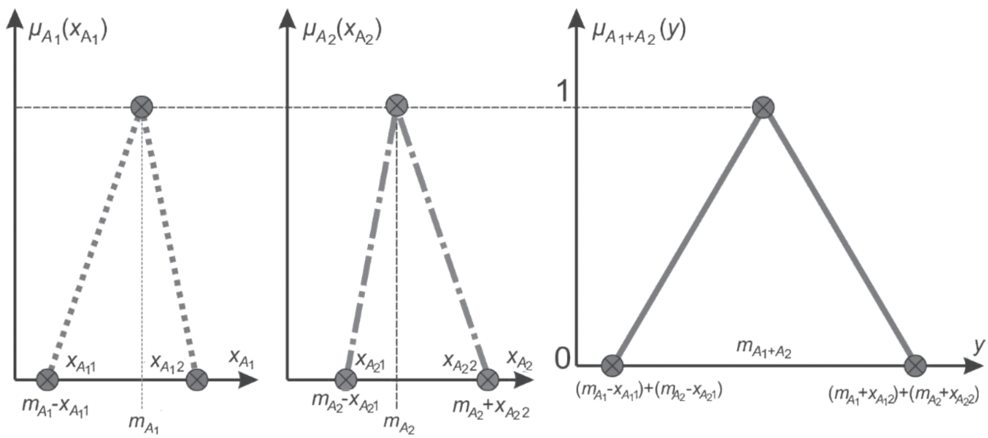

Figure 4.9. Parameters of sum of two disjoint fuzzy sets (A_1 and A_2)
Source: Compiled on the basis of Grzesik (2016, p. 24).

In the discussed case fuzzy numbers A_1 and A_2 correspond to best (A_1) and second-best (A_2) alternative decisions resulting from the Hierarchical Module of the developed multicriteria decision-making model. A calculation example is given below (Example 4.1).

Example 4.1: $D = \begin{Bmatrix} m_{A_1} = 500 & m_{A_2} = 450 \\ x_{A_1 1} = 15 & x_{A_2 1} = 10 \\ x_{A_1 2} = 25 & x_{A_2 2} = 5 \end{Bmatrix}$

Following the relations from Equations 44 and 45 $D = (A_1 + A_2) = (500,15,25) + (450,10,5) \rightarrow 950$, which means that the final decision $D = 950$, with lower limit value $D_{low} = 950 - (15+10) = 925$ and upper limit value $D_{high} = 950 + (25+5) = 980$. The graphic illustration of resulting final decision D with partial, AHP-resulting decision alternatives A_1 and A_2 is shown on Figure 4.10.

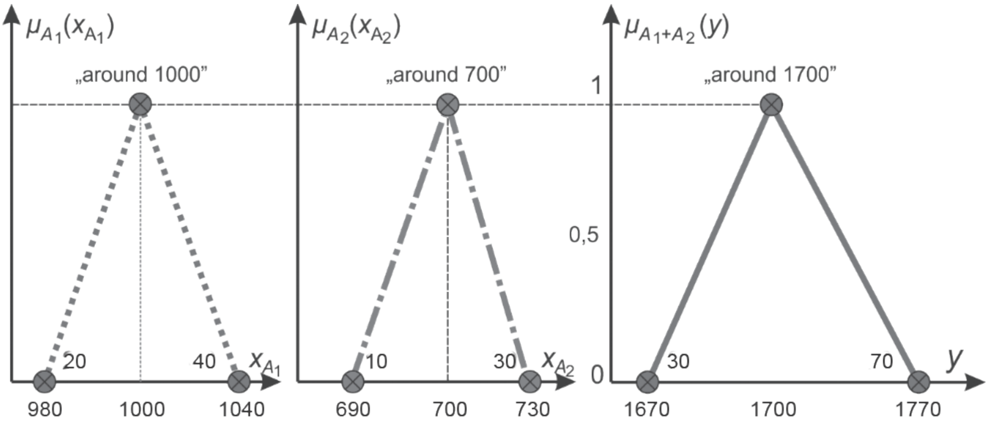

Figure 4.10. Parameters of decision D (Example 4.1), resulting from a sum of two disjoint fuzzy sets – decision alternatives A_1 and A_2
Source: Compiled on the basis of Grzesik (2016, p. 25).

Case II – decision alternatives A_1 and A_2 intersect (they are not disjoint sets).

In such a case the notion of intersection (conjunction) of two fuzzy sets A_1 and A_2, belonging to decision set D is used. The conjunction in question is defined as in Equation 46:

$$A_1 \cap A_2 = \{x : x \in A_1 \text{ i } x \in A_2\} \quad \text{(Eq. 46)}$$

The set defined by Equation 46 is also a fuzzy set, with a membership function defined for all values $x \in D$ as shown in Equation 47 (as a minimal value) and Equation 48 (as a product):

$$\mu_{A_1 \cap A_2}(x) = \text{MIN}\{\mu_{A_1}(x), \mu_{A_2}(x)\}, \forall x \in D \quad \text{(Eq. 47)}$$

$$\mu_{A_1 \cap A_2}(x) = \mu_{A_1}(x) \cdot \mu_{A_2}(x), \forall x \in D \tag{Eq. 48}$$

A graphic illustration of a conjunction of two fuzzy sets A_1 and A_2 is presented in Figure 4.11.

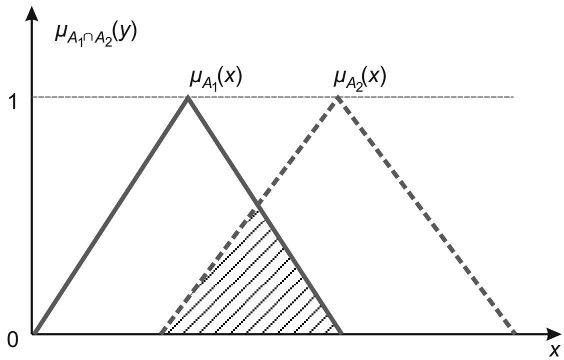

Figure 4.11. Parameters of intersecting fuzzy sets (A_1 and A_2)
Source: Compiled on the basis of Grzesik (2016, p. 46).

A mathematical operation of set intersection, denoted by the symbol "$\cap$", has the following features (valid for fuzzy sets, too):

- commutativity – the order of sets in intersection operations does not influence its final result – Equation 49:

$$A_1 \cap A_2 = A_2 \cap A_1 \tag{Eq. 49}$$

- associativity – a product of many sets can be calculated gradually as a product of pairs of sets; the order of association of sets into pairs does not influence the final result – Equation 50:

$$(A_1 \cap A_2) \cap A_3 = A_2 \cap (A_1 \cap A_3) \tag{Eq. 50}$$

- idempotence – multiple conjunctions of the fuzzy set do not change its initial result – Equation 51:

$$A_1 \cap A_1 = A_1 \tag{Eq. 51}$$

- absorption by an empty set Ø – the result of an intersection of any set with an empty set Ø results in an empty set Ø – Equation 52:

$$A_1 \cap \varnothing = \varnothing \tag{Eq. 52}$$

- identity – the result of an intersection of any set A_1 with an universe set D results in the universe set D – Equation 53:

$$A_1 \cap D = D \tag{Eq. 53}$$

where D is the universe set of all possible decisions, containing all decision alternatives;

- complementarity contradiction – the result of an intersection of any set A_1 with its complementary set A_1' results in an empty set Ø – Equation 54:

$$A_1 \cap A_1' = \varnothing \tag{Eq. 54}$$

where:
A_1' – the complement of A_1 – a set of elements not being part of A_1.

A mathematical operation of a union of two fuzzy sets A_1 and A_2, belonging to the universal set of all decision alternatives D, denoted by the symbol "$\cup$", is defined by Equation 55:

$$A_1 \cup A_2 = \{x : x \in A_1 \text{ i } x \in A_2\} \tag{Eq. 55}$$

The membership function defined for all values $x \in D$ is shown on Equation 56 (as a maximal value) and Equation 57 (as an algebraic sum):

$$\mu_{A_1 \cup A_2}(x) = \text{MAX}\{\mu_{A_1}(x), \mu_{A_2}(x)\}, \forall x \in D \tag{Eq. 56}$$

$$\mu_{A_1 \cup A_2}(x) = \mu_{A_1}(x) + \mu_{A_2}(x) - \mu_{A_1}(x) \cdot \mu_{A_2}(x), \forall x \in D \tag{Eq. 57}$$

Set union operations have the following features (valid for fuzzy sets, too):

- commutativity – the order of sets in union operations does not influence its final result – Equation 58:

$$A_1 \cup A_2 = A_2 \cup A_1 \tag{Eq. 58}$$

- associativity – a sum of many sets can be calculated gradually as a sum of pairs of sets; the order of association of sets into pairs does not influence the final result – Equation 59:

$$(A_1 \cup A_2) \cup A_3 = A_2 \cup (A_1 \cup A_3) = A_1 \cup A_2 \cup A_3 \tag{Eq. 59}$$

- idempotence – multiple unions of the fuzzy set do not change its initial result – Equation 60:

$$A_1 \cup A_1 = A_1 \tag{Eq. 60}$$

- absorption of an empty set Ø – the result of a union of any set A_1 with an empty set Ø results in the initial set A_1 – Equation 61:

$$A_1 \cup \emptyset = A_1 \tag{Eq. 61}$$

- absorption by the universe set D – the result of a union of any set A_1 with a universe set D results in the universe set – Equation 62:

$$A_1 \cup D = D \tag{Eq. 62}$$

where D is the universe set of all possible decisions, containing all decision alternatives;

- complementarity – the result of a union of any set A_1 with its complement A_1' results in a universe set D – Equation 63:

$$A_1 \cup A_1^{'} = D \tag{Eq. 63}$$

where:
A_1' – the complement of A – a set of elements not being part of A.

Figure 4.12 is a graphic illustration of a union of two fuzzy sets A_1 and A_2.

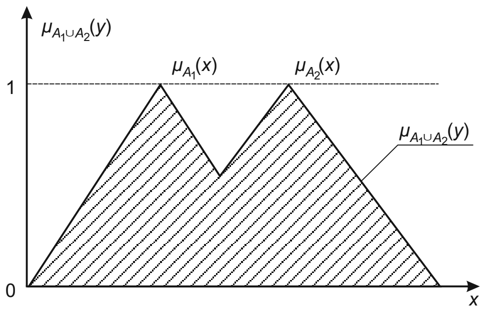

Figure 4.12. Parameters of a union of two fuzzy sets (A_1 and A_2)
Source: Compiled on the basis of Grzesik (2016, p. 46).

A fuzzy expert system can be built with the use of fuzzy sets, based on the idea of fuzzy coding of information. Fuzzy expert systems operate on fuzzy sets instead of numbers, which provides ground for generalisation of outputs. Figure 4.13 shows a fuzzy reasoning scheme.

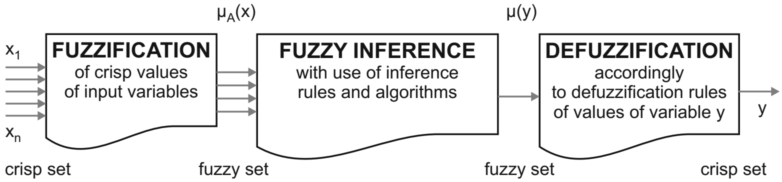

Figure 4.13. Fuzzy reasoning scheme
Source: Compiled on the basis of Grzesik (2016, p. 47).

Fuzzy modelling is composed of three steps: fuzzification, inference and defuzzification. A scheme of fuzzy modelling is shown in Figure 4.14.

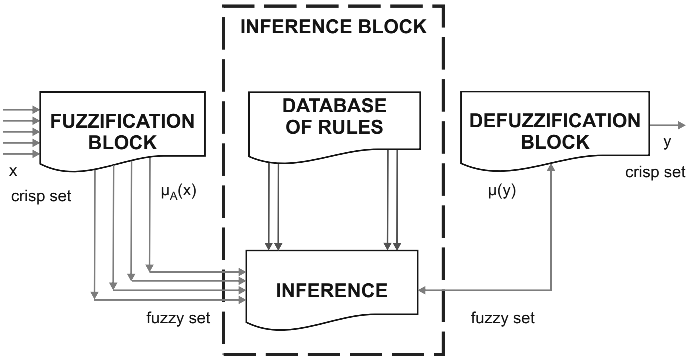

Figure 4.14. Fuzzy modeling scheme, with crisp inputs *x*, fuzzification $\boldsymbol{\mu(x_A)}$, inference $\boldsymbol{\mu(y)}$, defuzzification and crisp outputs *y*.
Source: Compiled on the basis of Grzesik (2016, p. 48).

Fuzzification block procedure is performed in the following way: determined crisp values are given at the input to fuzzification block (crisp values are exact inputs: $x_1 \div x_n$). In the fuzzification process they are transposed into fuzzy variables through the calculation of their membership function $\mu(x_A)$, $A \in \{1,N\}$, for $x_A \in X_A$. At the output of fuzzification block values of grades of membership are calculated. They provide information about the level of membership of *x* input values in particular input fuzzy set *X*.

The inference block contains:

- a database of rules;
- inference algorithms;
- $\mu(y)$ membership functions of variable *y*.

The inference block generates a fuzzy set of variable *y*. The resulting membership function often has a complicated form and its calculation is performed through the so-called inference. There are several ways of mathematical execution of the inference procedure. In actual application the inference block is perceived in terms of the black-box approach, which leaves the precise occurrences within it, which reaches beyond the scope of this research. Only the interrelations between input and output data are analysed.

In the defuzzification block this membership function is transposed into certain crisp values *y*.

Management literature overview reveals an important number of publications dealing with uncertainty in AHP applications, connected with the Fuzzy sets theory. These publications include Abdullah & Najib (2014), Dede, Kamalakis & Sphicopoulos (2015), Fu, Xu & Yang (2016), Karatas (2017), Krejci, Pavlacka & Talasova (2017), Kubler et al. (2016), Li, Wang & Tong (2016), Liu, Zhang & Zhang (2014), Mosadeghi et al. (2015), Pamucar et al. (2017), Shams, Mohamed & Fayek (2014), Shidpour, Da Cunha & Bernard (2016), Tuysuz (2018), Zamani-Sabzi et al. (2016), Zhang et al. (2017). However, it is difficult to find directly applicable solutions that employ AHP and fuzzification as a sequence rather than an immediate combination. Therefore an example-based proposal of the construction pattern of the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes is offered.

4.3. Proposal of a multicriteria model for enhancement of managerial decision-making

Managerial decisions are in most cases made under uncertainty, which instigates the risk of wrong or not optimal results of the decision-making process. In view of the basic features of fuzzy sets presented above and the characteristics of the AHP method presented earlier, it is fully justified to create a managerial decision-making enhancement system merging both methods. Therefore the construction of a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes, which is the main goal of this monograph, was carried in three phases:

- construction of the Hierarchical Module;
- creation of the Fuzzy Module;
- combining AHP and fuzzy sets methods into a mixed AHP-Fuzzy Module.

The Hierarchical Module provides the first optimal possible decision, resulting from prioritisation of decision criteria, whereas the Fuzzy and mixed AHP-Fuzzy Modules expand the spectrum of decision parameters by those most relevant from the second-best decision alternative. As the theoretical framework has already been provided in previous chapters, this part of the monograph is devoted to the proposal of a universal form of the model. In practical applications general variables will be substituted with real characteristics.

A scheme of the model is visualized in Figure 4.15.

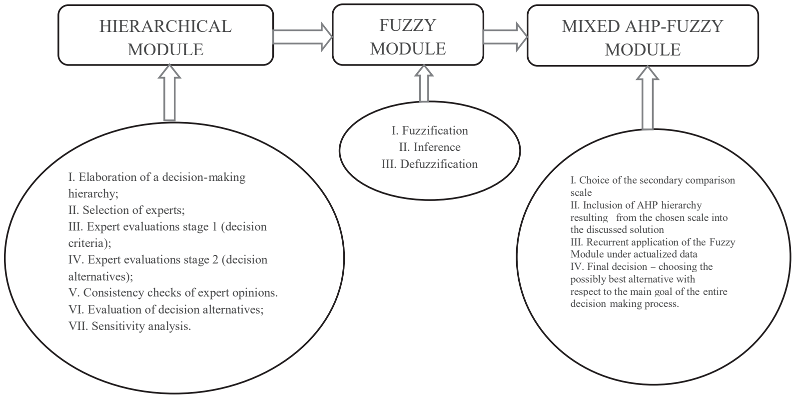

Figure 4.15. Structure of the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for enhancement of managerial decision-making processes.
Source: Compiled on the basis of the methodological framework for construction of decision-making models.

Construction of the Hierarchical Module

The Hierarchical Module, based on the Analytic Hierarchy Process method was formulated. As mentioned before, an AHP decision-making model is constructed in a series of consecutive steps: (i) establishing a decision-making hierarchy; (ii) selection of experts; (iii) expert evaluations stage 1 (decision criteria); (iv) expert evaluations stage 2 (decision alternatives); (v) Consistency Checks of expert opinions; (vi) evaluation of decision alternatives; (vii) sensitivity analysis.

To establish a decision-making hierarchy the decision problem needs first to be identified. In the discussed case the decision problem is to support any managerial decision, taken at the tactical or strategic level in an enterprise. The decision problem is operationalised through searching for answers to the questions:

1. What is the main goal of the decision to be made?
2. What criteria are relevant for the analysed decision problem?
3. Are both quantifiable (measurable) and non-quantifiable (immeasurable) decision criteria included?
4. As multicriteria managerial decisions hardly ever have a deterministic character, is it possible to include in the developed model the decision alternatives that vary slightly from the optimal possible solution (e.g. the second-best variant)?

For the needs of the constructed Hierarchical Module the main goal of decision-making is to make an optimal possible managerial decision. This is a general statement, resulting from the universality assumption of model construction. In practical applications the main goal needs to be reformulated into a more precise statement (e.g. choosing the most appropriate construction method; investing in the most profitable company; buying the most efficient machinery).

Once the main goal has been defined, the decision hierarchy needs to be built. Here the parent criteria are: Parent criterion 1, Parent criterion 2, Parent criterion 3. Each group of criteria has a set of belonging sub-criteria. Parent criterion 1 will encompass Sub-criterion 1.1, Sub-criterion 1.2, Sub-criterion 1.3, Sub-criterion 1.4 and Sub-criterion 1.5. Parent criterion 2 will stand in the hierarchy over Sub-criterion 2.1, Sub-criterion 2.2, Sub-criterion 2.3 and Sub-criterion 2.4. Parent criterion 3 will be superior to Sub-criterion 3.1, Sub-criterion 3.2 and Sub-criterion 3.3.

In practical applications these general criteria need to be defined for each decision problem separately and should not be interdependent. The need for a clear separation of the criteria in the hierarchy together with a precise definition of decision alternatives is an immanent feature of the method. An invalid hierarchy of criteria, resulting from the inclusion of interrelated criteria into the hierarchy will cause unacceptable bias in prioritisation results, which could make the entire decision-making process doubtful. An example of parent criteria for buying a car could include the price, design, performance and safety. An example of sub-criteria under the parent criterion design could include the body type, interior design, exterior design, the quality of finishing materials.

Additionally, as seen in the quantitative research presented earlier, a significant share of managers believe in incorporating into the criteria hierarchy those quantifiable, of measurable character, together with those non-quantifiable, immeasurable ones. A graphic illustration of a hierarchy of criteria is shown in Figure 4.16.

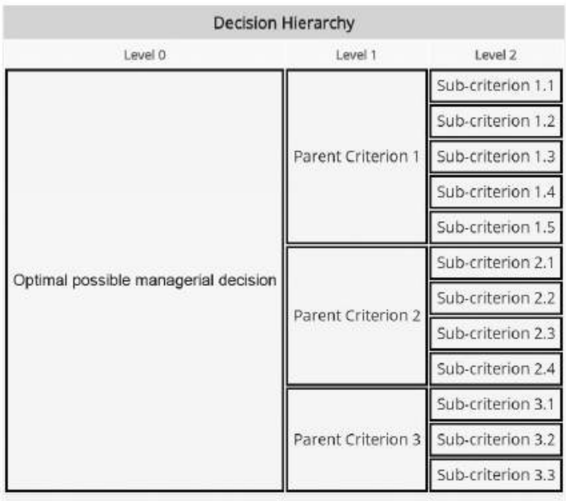

Figure 4.16. The hierarchy of criteria of the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for enhancement of managerial decision-making processes.
Source: Compiled on the basis of the AHP Online System software.

Similarly, decision alternatives need to be defined. In the universal version of the model they will be called Decision alternative A_1, Decision alternative A_2, Decision alternative A_3, Decision alternative A_4. Again, in real applications these will be clearly differentiable solutions (e.g. MAN, Scania, Mercedes, Mack – when buying a truck; block of flats, office building or shopping mall – when searching for investment opportunities in real estate, etc.).

The entire Hierarchical Module, including a complete decision-making hierarchy, with the main goal level, direct and indirect stakeholders of the decision-making process, parent criteria, their sub-criteria and available decision alternatives, is presented in Figure 4.17.

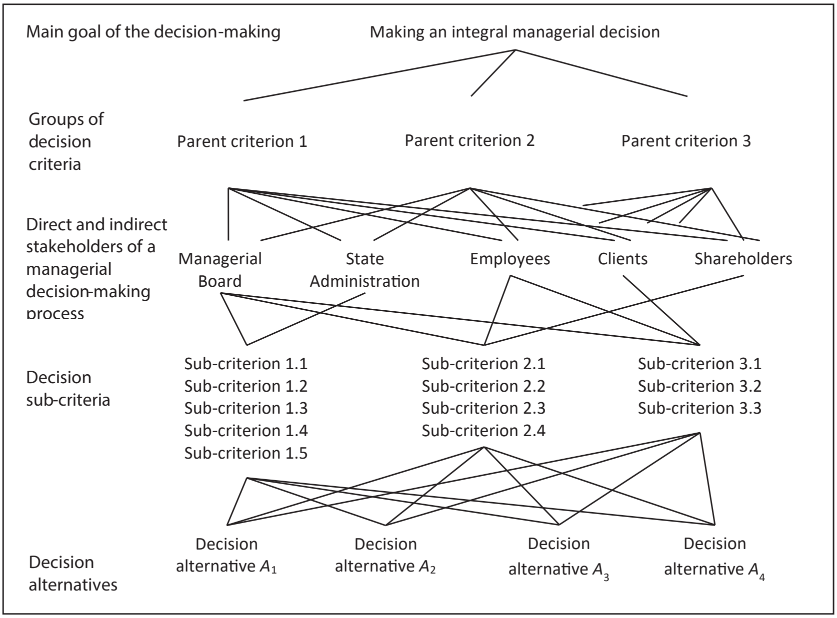

Figure 4.17. The Hierarchical Module of the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for enhancement of managerial decision-making processes
Source: Compiled on the basis of Saaty (1996, p. 13).

The second step of the construction of the model's Hierarchical Module is the selection of evaluators who will perform the pairwise comparisons. The most sensitive issue is the selection of experts who will follow strictly the criteria definitions within the hierarchy and other methodological constraints of the AHP method. The reason is the threat of bias in expert evaluations caused by an insufficient level of expertise in the field of decision-making, an individual interpretation of given decision criteria or a lack of concentration at later stages of pairwise comparison process.

In order to minimise the risk of such occurrences, the author suggests composing the respondents' sample through the arbitral choice method. The arbitral choice method should be employed for a non-random sampling of the composition of experts for the direct semi-structured interviews. Frankfort-Nachmias & Nachmias (2001, s. 199) state that the goal of arbitral choice method is the creation of a nearly representative research sample. Experts for this stage of the construction of the Hierarchical Module of the decision-making model should be recruited from theoreticians of management science and practitioners of management. When choosing theoreticians of management science, the leading sampling criterion should be the relevance of their publications to the research area (managerial decision-making). The sampling criteria for practitioners should be the branch opinions on their efficiency as managers and the development trends of their enterprises since in service. In this monograph arbitral choice is seen as the primary qualitative research method.

Direct semi-structured interviews can also be used to additionally test the level of expertise on the decision-making problem of preselected evaluators. The application of this method is highly recommended – a special focus should be put on the expert's knowledge on the particularities of the field in which the decision-making happens. For instance, a practitioner of management can be perceived as an expert in this field, although different levels and fields of expertise will be expected from a hospital director and different from the financial manager of a biotechnological company. In this research task direct semi-structured interviews are counted among the secondary qualitative research methods.

One should notice that the limitation of the number of experts involved in the decision-making process constitutes one of the huge advantages of the AHP method. Scientific practice allows ascertaining that a higher degree of expert knowledge allows the limitation of the number of experts. Saaty discusses their number in some of his works (1980, 1996, 2001). Finally, 5-9 respondents are a rational amount, on the assumption of their high level of expertise in the subject. Therefore, in the constructed model a group of five fictional highly qualified experts (Expert A, Expert B, Expert C, Expert D and Expert E) was employed, which falls into Saaty's constraint, assuming a high level of their professional expertise in the field of the decision problem under examination.

It needs to be underlined that every managerial task requires a separate hierarchy construction process and a different set of experts. Therefore the results of the evaluation of expert evaluations stage 1 (decision criteria) and expert evaluations stage 2 (decision alternatives), as well as the results of Consistency Checks and finally the evaluation of decision alternatives are case-dependent. They have the characteristics of a dynamic and interactive procedure and will be presented here with the use of example numerical data. For this reason the calculations for all the presented modules are used as examples and cannot be perceived as a unique or ultimate form of the elaborated managerial decision-making model. This flexibility, incorporated in the model's construct constitutes one of its most important cognitive and applicatory values.

Example results from the Expert evaluations stage 1 (decision criteria) are presented in Figure 4.18. The numbers in squares in levels 1 and 2 of the decision hierarchy represent the local priorities of decision criteria and sub-criteria. They are the ratio-scale weights of a sub-criteria node with respect to the parent criterion. In ideal conditions they should add up to 1 within one parent criterion, however, due to the imperfect consistency of expert answers and tolerated inconsistency level of less than 10%, very slight deviations can be observed.

Decision Hierarchy			
Level 0	Level 1	Level 2	Glb Prio.
Possibly optimal managerial decision	Parent Criterion 1 0.600	Sub-criterion 1.1 0.415	24.9%
		Sub-criterion 1.2 0.296	17.8%
		Sub-criterion 1.3 0.162	9.7%
		Sub-criterion 1.4 0.088	5.3%
		Sub-criterion 1.5 0.040	2.4%
	Parent Criterion 2 0.287	Sub-criterion 2.1 0.616	17.7%
		Sub-criterion 2.2 0.174	5.0%
		Sub-criterion 2.3 0.136	3.9%
		Sub-criterion 2.4 0.074	2.1%
	Parent Criterion 3 0.113	Sub-criterion 3.1 0.631	7.1%
		Sub-criterion 3.2 0.263	3.0%
		Sub-criterion 3.3 0.107	1.2%
			1.0

Figure 4.18. Expert evaluations stage 2 – decision criteria
*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
Source: Compiled on the basis of the AHP Online System software.

"Glb Prio." stands for global priorities, which are the ratio-scale weights of any parent criterion with respect to the main goal. The shades of grey denote the significance of a given sub-criterion to the accomplishment of the main goal of the decision-making, i.e. making an optimal possible managerial decision. The higher the grey colour intensity the higher the significance of the respective criterion. In ideal conditions, global priorities of all the lowest level sub-criteria sum up to 100%, although similarly to local priorities very slight computational deviations can be observed. Global priorities for each sub-criterion, with relation to each expert's evaluations are presented in Table 4.3, with the shades of grey used as a graphic additional representation of normalised preference vector values (in %).

Table 4.3.

Global priorities of decision sub-criteria – expert-sensitive

Partici-pants	Sub-criterion 1.1	Sub-criterion 1.2	Sub-criterion 1.3	Sub-criterion 1.4	Sub-criterion 1.5	Sub-criterion 2.1	Sub-criterion 2.2	Sub-criterion 2.3	Sub-criterion 2.4	Sub-criterion 3.1	Sub-criterion 3.2	Sub-criterion 3.3	CR_{max}
Group result	**24.9%**	**17.8%**	**9.7%**	**5.3%**	**2.4%**	**17.7%**	**5.0%**	**3.9%**	**2.1%**	**7.1%**	**3.0%**	**1.2%**	**5.0%**
Expert E	18.9%	10.9%	8.5%	3.3%	2.7%	22.5%	7.1%	6.2%	2.9%	11.0%	3.9%	2.1%	7.2%
Expert D	22.3%	27.8%	17.2%	6.5%	3.2%	7.2%	1.6%	1.0%	0.6%	8.5%	2.9%	1.3%	9.0%
Expert C	27.9%	21.2%	12.2%	4.8%	2.1%	13.4%	3.0%	3.1%	2.0%	6.0%	3.3%	0.9%	4.9%
Expert B	22.8%	19.2%	7.9%	5.5%	2.2%	17.8%	10.1%	4.3%	2.0%	5.5%	1.8%	0.8%	9.0%
Expert A	23.6%	9.8%	4.1%	4.6%	1.3%	30.8%	6.5%	7.3%	4.2%	4.3%	2.5%	0.9%	8.0%

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
Source: Compiled on the basis of the AHP Online System software.

Figure 4.19 clarifies the significance of expert evaluations with relation to criteria objectives. Normalised results of pairwise comparisons of sub-criteria with relation to their parent criterion and of parent criteria with relation to the main goal of the decision-making process are clearly indicated. The comparisons were performed on an each-with-each basis, with Sub-criterion 1 being the one that contributes the most to the achievement of an optimal possible managerial decision, with 24.9% significance.

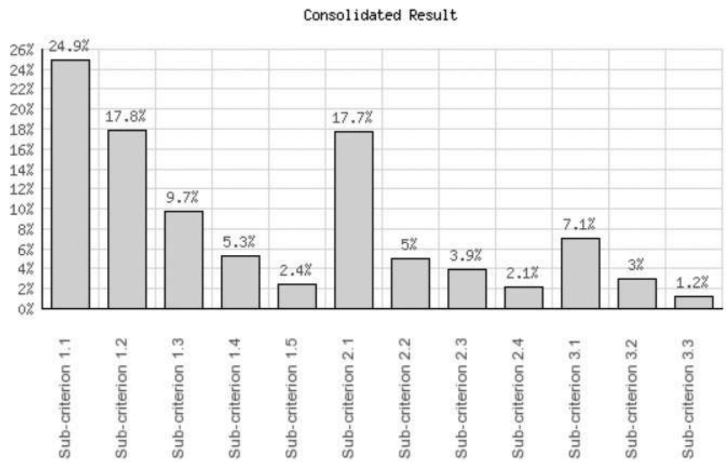

Figure 4.19. Hierarchy of decision criteria – weights of sub-criteria.
*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
Source: Compiled on the basis of the AHP Online System software.

Expert evaluations stage 2 (decision alternatives) resulted in decision alternatives ranking, under former evaluations of relevance of all the decision parent criteria, together with their sub-criteria. The extent to which a given decision alternative (A_1, A_2, A_3 or A_4) contributes to the achievement of the main goal under a given sub-criterion is shown in squares under Decision Alternative A_i column (Figure 4.20).

Decision Hierarchy							
Level 0	Level 1	Level 2	Glb Prio.	Decision Alternative A1	Decision Alternative A2	Decision Alternative A3	Decision Alternative A4
Optimal possible managerial decision	Parent Criterion 1 0.600	Sub-criterion 1.1 0.415	24.9%	0.398	0.396	0.123	0.083
		Sub-criterion 1.2 0.296	17.8%	0.399	0.378	0.154	0.069
		Sub-criterion 1.3 0.162	9.7%	0.434	0.363	0.146	0.057
		Sub-criterion 1.4 0.088	5.3%	0.425	0.327	0.160	0.088
		Sub-criterion 1.5 0.040	2.4%	0.386	0.372	0.139	0.102
	Parent Criterion 2 0.287	Sub-criterion 2.1 0.616	17.7%	0.319	0.427	0.161	0.094
		Sub-criterion 2.2 0.174	5.0%	0.367	0.438	0.122	0.073
		Sub-criterion 2.3 0.136	3.9%	0.384	0.362	0.147	0.107
		Sub-criterion 2.4 0.074	2.1%	0.425	0.304	0.160	0.110
	Parent Criterion 3 0.113	Sub-criterion 3.1 0.631	7.1%	0.426	0.330	0.152	0.092
		Sub-criterion 3.2 0.263	3.0%	0.351	0.397	0.158	0.094
		Sub-criterion 3.3 0.107	1.2%	0.332	0.396	0.162	0.110
			1.0	38.7%	38.5%	14.5%	8.3%

Figure 4.20. Expert evaluations stage 2 – decision alternatives
*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
Source: Compiled on the basis of the AHP Online System software.

The Consistency Checks of expert opinions were performed at each stage of criteria evaluation. In the presented examples the Consistency Ratio (CR) of each set of evaluations oscillates between 0.4% and 5.0%, which is a very good result. Both values are lower than 10%, which means that expert opinions are relatively consistent and therefore reliable. In the case of CR value exceeding 10%, experts should perform a reassessment of preference statements. Most of AHP decision-making software offers such an opportunity after each round of evaluations. Should the CR value exceed 10% on a more regular basis, the expert should be replaced and his evaluations rejected as not reliable.

The resulting ranking of alternatives was reached at a very high consensus of 91.5% among all the five experts. The aggregated consensus value was calculated from all the consensus percentages of pairwise comparisons of decision parent criteria and sub-criteria.

Table 4.4 presents the Consistency Ratios (CR's) and consensus levels for each parent criterion and the main goal level.

Table 4.4.

Consistency ratio and the consensus level of expert evaluations stage 1 (decision criteria)

Criterion	Consistency Ratio (CR)	Consensus (%)	Consensus level
Main goal level	0.4%	87.1%	very high
Parent criterion 1	5.0%	95.1%	very high
Parent criterion 2	4.5%	96.4%	very high
Parent criterion 3	3.2%	98.3%	very high

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
Source: Compiled on the basis of the AHP Online System software.

The resulting ranking of alternatives was reached at a moderate, but acceptable consensus of 74.26% among all the five experts, at a very satisfactory Consistency Ratio (CR) oscillating between 0.9% and 5.7%. The aggregated consensus value was calculated from all the consensus percentages, including those on decision alternatives. Table 4.5 presents Consistency Ratios (CR's) and consensus levels for each sub-criterion, with regard to decision alternatives.

Table 4.5.

Consistency ratio (CR) and the consensus level of expert evaluations stage 2 (decision criteria)

Criterion	Consistency Ratio (CR)	Consensus [%]	Consensus level
Sub-criterion 1.1	0.9%	80.5%	high
Sub-criterion 1.2	5.7%	70.2%	moderate
Sub-criterion 1.3	1.6%	78.3%	high
Sub-criterion 1.4	3.0%	67.3%	moderate
Sub-criterion 1.5	1.3%	66.9%	moderate
Sub-criterion 2.1	4.1%	74.8%	moderate
Sub-criterion 2.2	4.4%	76.1%	high
Sub-criterion 2.3	3.0%	68.8%	moderate
Sub-criterion 2.4	4.4%	63.3%	low
Sub-criterion 3.1	3.5%	67.9%	moderate
Sub-criterion 3.2	5.0%	76.7%	high
Sub-criterion 3.3	1.8%	70.1%	moderate

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
Source: Compiled on the basis of the AHP Online System software.

The final result of decision-making with the Hierarchical Module of the developed model arises at the evaluation of decision alternatives phase. The weights of each alternative decision in relation to the main goal of decision-making – an optimal possible managerial decision – are obtained. Figure 4.21 presents an example of a final result of AHP-enhanced decision-making via the Hierarchical Module of the developed decision-making model. The obtained vector values were normalised, which yielded the percentage values presented in Figure 4.21. It is clear that the decision alternative that contributes to the highest extent to the accomplishment of the main goal of the decision-making process, which was making an optimal possible managerial decision, is the alternative A_1. The 38.7% value means that this alternative contributes in 38.7% to the fulfilment of all the decision criteria simultaneously, with respect to the achievement of the main goal and under given circumstances and does it to the highest extent from all the available decision alternatives. The distant place of decision alternatives A_3 (14.5%) and A_4 (8.3%) leaves them out of the scope of further examination, however, a very close position of the second-best alternative A_2 (38.5%) provides grounds for further discussion.

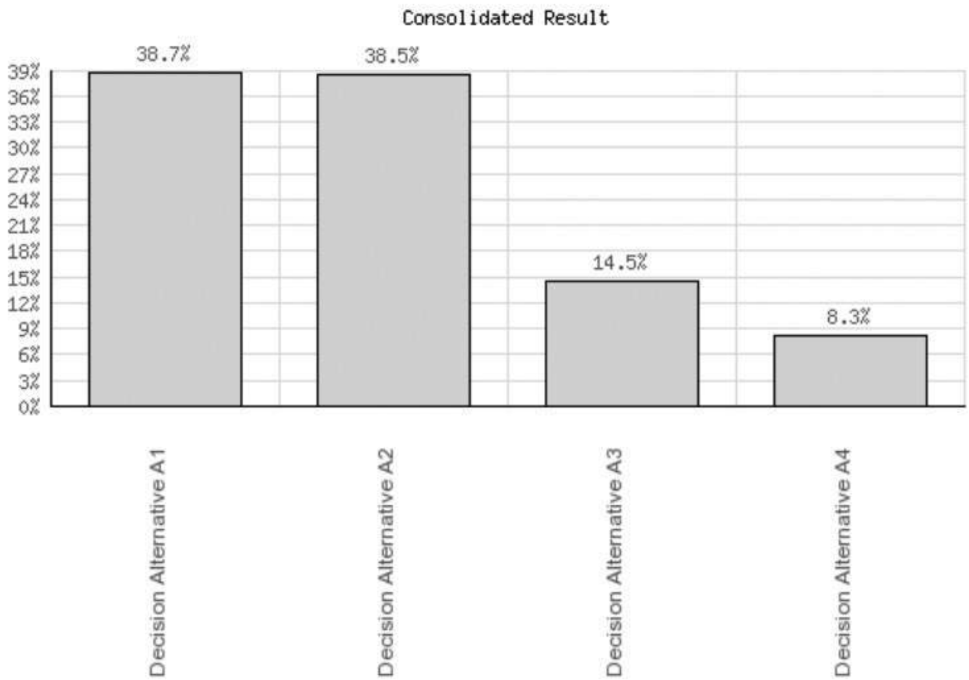

Figure 4.21. Optimal possible managerial decision - weights of alternative decisions $A_1 – A_4$
*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
Source: Compiled on the basis of the AHP Online System software.

As already mentioned, the above results were reached at a reasonable 74.26% consensus among all the participating experts. This imperfect consensus comes from different prioritisation of criteria relevance by individual experts, which can be seen in Table 4.6. Reasonable variations in expert evaluations are a normal occurrence, however such close ranking results as between decision alternatives A_1 and A_2 suggest that a sensitivity analysis is necessary. Table 4.6 shows expert-sensitive results of the decision-making process obtained via the application of Saaty's 9-point linear Fundamental Comparison Scale. The grey colour intensity inside a cell denotes the level of significance of a particular expert statement toward the accomplishment of the main goal of decision-making, when the respective decision alternative is chosen. Even if the group consensus is moderate (74.3%), individual preference statements of all the experts are below 10% threshold (CR_{max} = 9.7%), also their group judgements are still consistent (CR_{max} = 5.7%). The shades of grey denote the normalised values of the preference vector with regard to decision alternatives. The results obtained with the use of alternative scales are shown as well.

Table 4.6.

Expert-sensitive weights of alternative decisions $A_1 – A_4$ – Fundamental Comparison Scale (linear)

Participants	**Decision Alternative A_1**	**Decision Alternative A_2**	**Decision Alternative A_3**	**Decision Alternative A_4**	**CR_{max}**
Group result	**38.7%**	**38.5%**	**14.5%**	**8.3%**	5.7%
Expert E	8.3%	72.7%	9.7%	9.3%	*9.6%*
Expert D	45.4%	32.3%	15.9%	6.5%	*8.1%*
Expert C	52.7%	23.5%	14.8%	9.0%	*8.7%*
Expert B	50.3%	29.9%	13.7%	6.1%	*9.4%*
Expert A	51.0%	28.0%	13.5%	7.4%	*9.7%*

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
**Group consensus: 74.3% (moderate);
Source: Compiled on the basis of the AHP Online System software.

As discussed in the theoretical introduction, the sensitivity analysis in the frame of the AHP method aims at finding which criterion and which performance measure could cause rank reversal in the obtained ranking of decision alternatives. Table 4.7 provides data for the identification of the most critical criterion, whereas Table 4.8 those of the most critical measure of performance.

Table 4.7.

Results of application of the Hierarchical Module of constructed decision-making model – the most critical criterion

Criteria	Sub-criterion 1.1	Sub-criterion 1.2	Sub-criterion 1.3	Sub-criterion 1.4	Sub-criterion 1.5	Sub-criterion 2.1	Sub-criterion 2.2	Sub-criterion 2.3	Sub-criterion 2.4	Sub-criterion 3.1	Sub-criterion 3.2	Sub-criterion 3.3
weights	24.90%	17.75%	9.72%	5.27%	2.38%	17.67%	4.99%	3.89%	2.12%	7.13%	2.97%	1.21%
A_1 - A_2	n/a	8.96%	2.62%	1.91%	n/a	-1.72%	-2.62%	n/a	**1.54%**	1.93%	n/a	n/a
A_1 - A_3	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
A_1 - A_4	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
A_2 - A_3	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
A_2 - A_4	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
A_3 - A_4	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a

Source: Compiled on the basis of the AHP Online System software.

The sensitivity analysis presented in Table 4.7 shows that the Percentage-Top (PT) critical criterion is Sub-criterion 2.4, as a change from 2.12% by -1.54% will change the ranking between the top decision alternative A_1 and decision alternative A_2. The Percentage-Any (PA) critical criterion is the same as above, as 1.54% is the lowest value in the table.

Table 4.8.

Results of application of the Hierarchical Module of constructed decision-making model – the most critical measure of performance

A_i	Sub-criterion 1.1	Sub-criterion 1.2	Sub-criterion 1.3	Sub-criterion 1.4	Sub-criterion 1.5	Sub-criterion 2.1	Sub-criterion 2.2	Sub-criterion 2.3	Sub-criterion 2.4	Sub-criterion 3.1	Sub-criterion 3.2	Sub-criterion 3.3	A_k
A_1	**0.74%**	1.05%	2.01%	3.75%	7.30%	0.94%	3.35%	4.65%	9.05%	2.79%	5.63%	12.61%	A_2
A_1	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_3
A_1	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_4
A_2	-0.75%	-1.03%	-1.81%	-3.31%	-8.30%	-1.19%	-4.16%	-4.89%	-8.47%	-2.43%	-7.00%	-19.61 %	A_1
A_2	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_3
A_2	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_4
A_3	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_1
A_3	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_2
A_3	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_4
A_4	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_1
A_4	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_2
A_4	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	A_3

Source: Compiled on the basis of the AHP Online System software.

The sensitivity analysis presented in Table 4.8 shows that the Percentage-Any (PA) critical measure of performance can be found for decision alternative A_1 under the Sub-criterion 1.1. A change by -0.74% will change its ranking with decision alternative A_2. A change from 39.76% (local weight value of Sub-criterion 1.1 for decision alternative A_1 calculated with the weighted sum method – WSM) by -0.74% to 39.02% could cause rank reversal, with decision alternative A_2 becoming the best (top) prioritised one.

As shown in Table 4.6, the second-best decision alternative A_2 obtained a very close ranking of 38.5% to decision alternative A_1, with 38.7% of relevance. Such a close relevance of two top alternatives for the achievement of the main goal of the decision-making process suggests a possibility of rank reversal resulting from the application of a different comparison scale. This issue is visualised in Tables 4.9 to 4.17. The meaning of shades of grey and percentages in these tables are the same as in Table 4.6.

Table 4.9.

Expert-sensitive weights of alternative decisions $A_1 – A_4$ – Logarithmic Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**34.5%**	**33.6%**	**18.7%**	**13.3%**	2.4%
Expert E	15.1%	51.4%	17.0%	16.5%	*3.9%*
Expert D	37.9%	30.4%	19.9%	11.9%	*3.0%*
Expert C	41.4%	25.7%	19.0%	13.9%	*3.6%*
Expert B	40.5%	29.5%	18.4%	11.6%	*3.9%*
Expert A	40.8%	28.2%	18.4%	12.6%	*3.4%*

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
**Group consensus: 73.9% (moderate);
Source: Compiled on the basis of the AHP Online System software.

Table 4.9. shows the expert-sensitive results of the decision-making process obtained via the application of the Logarithmic Comparison Scale. Although no rank reversal happens in this case and the hierarchy of alternatives becomes slightly strengthened by a marginally larger distance between alternatives A_1 and A_2, the group consensus ratio is lower. Individual and group expert opinions are consistent.

Table 4.10.

Expert-sensitive weights of alternative decisions $A_1 - A_4$ – Square-Root Scale.

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**32.5%**	**32.5%**	**19.9%**	**15.0%**	1.4%
Expert E	15.9%	49.0%	17.7%	17.3%	*2.3%*
Expert D	35.8%	30.0%	20.8%	13.4%	*2.0%*
Expert C	38.7%	25.8%	20.0%	15.5%	*2.1%*
Expert B	38.0%	29.2%	19.6%	13.2%	*2.3%*
Expert A	38.3%	28.0%	19.5%	14.2%	*2.3%*

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
**Group consensus: 75.2% (high);
Source: Compiled on the basis of the AHP Online System software.

Table 4.10 shows the expert-sensitive results of the decision-making process obtained via the application of the Square-Root Comparison Scale. What can be seen is that with the application of the Square-Root Comparison Scale alternatives A_1 and A_2 become equal in the hierarchy of decision alternatives. The group consensus ratio of such prioritisation becomes higher than in the two preceding Comparison Scales (75.2%). Individual and group expert opinions are consistent.

Table 4.11.

Expert-sensitive weights of alternative decisions $A_1 - A_4$ – Inverse Linear Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**28.6%**	**30.6%**	**22.3%**	**18.6%**	0.8%
Expert E	9.0%	71.6%	9.9%	9.5%	*7.7%*
Expert D	34.6%	29.9%	21.0%	14.4%	*8.2%*
Expert C	36.7%	25.4%	20.6%	17.3%	*8.1%*
Expert B	36.7%	28.5%	20.9%	13.9%	*11.4%*
Expert A	37.8%	27.1%	20.2%	15.0%	*14.2%*

*Overall Consistency Ratio CR < 0.1 => group expert opinions are consistent, but individual expert opinions of Expert A and Expert B surpass the CR margin of 10%;
**Group consensus: 80.3% (high);
Source: Compiled on the basis of the AHP Online System software.

Table 4.11 shows the expert-sensitive results of the decision-making process obtained via the application of the Inverse Linear Comparison Scale. In the case of the expert evaluations obtained via the Inverse Linear Comparison Scale a rank reversal between alternatives A_1 and A_2 happens at the group consensus ratio of 80.3% (high). The new prioritisation points at decision alternative A_2 as the one contributing to the highest extent (30.6%) to making an optimal possible managerial decision. Even though the group consistency of expert opinions is still acceptable, Expert A and Expert B exceed the margin of 10%, which makes their preference statements unreliable.

Table 4.12.

Expert-sensitive weights of alternative decisions $A_1 – A_4$ – Balanced Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**30.5%**	**33.0%**	**20.7%**	**15.9%**	1.5%
Expert E	8.7%	72.3%	9.7%	9.4%	*3.9%*
Expert D	36.9%	31.3%	19.8%	12.0%	*4.7%*
Expert C	40.4%	25.1%	19.2%	15.3%	*4.6%*
Expert B	39.9%	29.3%	19.3%	11.4%	*7.0%*
Expert A	40.8%	27.8%	18.6%	12.8%	*10.4%*

*Overall Consistency Ratio CR < 0.1 => group expert opinions are consistent, but individual expert opinions of Expert A surpass the CR margin of 10%;
**Group consensus: 79.0% (high);
Source: Compiled on the basis of the AHP Online System software.

Table 4.12 shows the expert-sensitive results of the decision-making process obtained via the application of the Balanced Comparison Scale. It maintains the rank reversal between alternatives A_1 and A_2 at the priority of alternatives from Table 4.11. The group consensus ratio of 79.0% is slightly lower than in the case of the Inverse Linear Scale, however, still high and superior to the Fundamental Comparison Scale. The group consistency of expert opinions is below the 10% threshold, but Expert A breaches the margin of 10%, which makes his statements unreliable.

Table 4.13 shows the expert-sensitive results of the decision-making process obtained via the application of the Balanced-*n* Comparison Scale. Rank reversal happens again, with decision alternative A_2 being the top-prioritised one. The group consensus ratio of 77.6% is still high and superior to the Fundamental Comparison Scale. The individual and group consistencies of expert opinions are below the 10% threshold, which allows the approval of their statements as consistent and therefore reliable.

Table 4.13.

Expert-sensitive weights of alternative decisions $A_1 – A_4$ – Balanced-*n* Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**32.6%**	**35.3%**	**18.8%**	**13.3%**	2.2%
Expert E	8.4%	72.7%	9.6%	9.3%	*2.7%*
Expert D	39.3%	32.2%	18.6%	9.9%	*5.1%*
Expert C	44.2%	24.7%	17.8%	13.2%	*3.2%*
Expert B	43.1%	29.9%	17.6%	9.4%	*3.9%*
Expert A	43.9%	28.3%	17.0%	10.9%	*7.2%*

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
**Group consensus: 77.6% (high);
Source: Compiled on the basis of the AHP Online System software.

Table 4.14.

Expert-sensitive weights of alternative decisions $A_1 – A_4$ – Adaptive-Bal Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**34.3%**	**37.8%**	**17.0%**	**10.9%**	3.6%
Expert E	3.8%	87.5%	4.5%	4.2%	*7.0%*
Expert D	43.3%	33.2%	16.3%	7.2%	*10.4%*
Expert C	49.9%	23.5%	15.8%	10.9%	*8.7%*
Expert B	48.6%	29.8%	15.0%	6.7%	*12.5%*
Expert A	49.3%	28.2%	14.3%	8.3%	*21.3%*

*Overall Consistency Ratio CR < 0.1 => group expert opinions are consistent, but individual expert opinions of Expert A and Expert B surpass the CR margin of 10% significantly;
**Group consensus: 74.5% (moderate);
Source: Compiled on the basis of the AHP Online System software.

Table 4.14 shows expert-sensitive results of the decision-making process obtained via application of the Adaptive-Bal Comparison Scale. Rank reversal between alternatives A_1 and A_2 happens here as well, however, doubt is raised by the high level of inconsistency of the preference statements of Expert A (21.3%) and Expert B (12.5%). The group consensus is moderate as well, which altogether makes the prioritisation obtained by the Adaptive-Bal Comparison Scale untrustworthy.

Table 4.15.

Expert-sensitive weights of alternative decisions $A_1 - A_4$ – Adaptive Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**43.1%**	**42.7%**	**9.8%**	**4.3%**	13.0%
Expert E	3.7%	87.6%	4.5%	4.2%	*22.6%*
Expert D	52.7%	32.5%	11.8%	3.0%	*18.9%*
Expert C	64.5%	19.6%	10.8%	5.1%	*20.2%*
Expert B	60.2%	28.3%	8.9%	2.5%	*21.9%*
Expert A	61.0%	26.3%	8.8%	3.9%	*22.8%*

*Overall Consistency Ratio CR > 0.1 => group and individual expert opinions are inconsistent;
**Group consensus: 70.5% (moderate);
Source: Compiled on the basis of the AHP Online System software.

Table 4.15 shows the expert-sensitive results of the decision-making process obtained via the application of the Adaptive Comparison Scale. The initial hierarchy of decision alternatives A_1 to A_4 is maintained, however, neither individual, nor group expert evaluations are consistent, with CR coefficient significantly exceeding 10% threshold in each case. Also the group consensus level is moderate (70.5%). All the above make this hierarchy inapplicable.

Table 4.16.

Expert-sensitive weights of alternative decisions $A_1 - A_4$ – Power Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**46.2%**	**45.3%**	**6.4%**	**2.2%**	23.5%
Expert E	1.6%	94.8%	1.9%	1.8%	*41.9%*
Expert D	57.9%	31.7%	9.0%	1.4%	*34.9%*
Expert C	73.5%	15.5%	8.1%	2.9%	*37.1%*
Expert B	67.6%	25.8%	5.6%	1.0%	*40.7%*
Expert A	68.0%	24.0%	5.8%	2.1%	*42.9%*

*Overall Consistency Ratio CR > 0.1 => group and individual expert opinions are inconsistent;
**Group consensus: 66.5% (moderate);
Source: Compiled on the basis of the AHP Online System software.

Table 4.16 shows the expert-sensitive results of the decision-making process obtained via the application of the Power Comparison Scale. The case from Table 4.15 is repeated in Table 4.16 with even higher inconsistency (reaching almost 43%) and lower group consensus (66.5%). Prioritisation results obtained with the Power Scale need to be rejected.

Table 4.17.

Expert-sensitive weights of alternative decisions $A_1 - A_4$ – Geometric Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**38.0%**	**47.5%**	**10.0%**	**4.5%**	15.3%
Expert E	0.5%	98.2%	0.7%	0.6%	*16.8%*
Expert D	51.0%	36.3%	10.6%	2.1%	*36.3%*
Expert C	67.1%	17.3%	10.2%	5.4%	*21.4%*
Expert B	62.9%	27.6%	7.8%	1.6%	*29.5%*
Expert A	60.8%	28.2%	7.5%	3.5%	*49.4%*

*Overall Consistency Ratio CR > 0.1 => group and individual expert opinions are inconsistent;
**Group consensus: 66.0% (moderate);
Source: Compiled on the basis of the AHP Online System software.

Table 4.17 shows the expert-sensitive results of the decision-making process obtained via the application of the Geometric Comparison Scale. The rank reversal between decision alternatives A_1 and A_2 happens again, but the inconsistency is unacceptable (up to 49.4%) and group consensus of 66% is not satisfactory. Therefore prioritisation results obtained with the Geometric Scale need to be rejected.

In conclusion, a very close position of alternatives A_1 and A_2 in the decision hierarchy, the results of sensitivity analysis and possibility of rank reversal due to the application of different comparison scales points at the need for the application of an additional method for the enhancement of decision-making precision. Therefore a fuzzy expansion of the model in form of a Fuzzy Module was proposed, which corresponds with the formerly discussed evolutionary methodological approach in management science by Ćwiklicki.

Elaboration of the Fuzzy Module

The second phase encompasses the construction of the Fuzzy Module of the Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes. It is in the Fuzzy sets theory, the theoretical framework of which has been provided earlier in this chapter.

Due to the nature of fuzzy sets applications, its creation has to be based on a real decision-making problem. Therefore the main aim of the decision-making process becomes taking an integral managerial decision (previously defined). The input data for the Fuzzy Module result from preceding application of the Hierarchical Module. It means that the output of the Hierarchical Module – the hierarchy of decision alternatives obtained via AHP – will constitute the input to the Fuzzy Module.

Tóth-Laufer & Takács (2012, p. 133) state that when a crisp value instead of a fuzzy set is required at the system's output (as in the discussed decision problem – at the output of the Fuzzy Module), the defuzzification method has to be selected depending on the application. Therefore in the practical applications of fuzzy sets each the decision alternative needs to be attributed a numerical value. The membership vector of each input value of the Fuzzy Module $[\mu_1 \ldots \mu_k]$ is transformed in the defuzzification block into a singular crisp numerical value.

There are a considerable number of defuzzification methods discussed in the scientific literature. Filo (2017, pp. 15-16) enumerates some of the most popular ones: Centre of Gravity method (CoG, also called Centre of Area method – CoA), Centre of Sums method (CoS), Bisector Method (BM, also called the Bisector of Area method – BoA), Maxima methods – First of Maxima (FoM), Last of Maxima (LoM) and Mean of Maxima (MoM, also called the Mean-Max method), and finally the Weighted Average Method (WAM). A more comprehensive discussion about the choice of the appropriate defuzzification method, according to the application of a fuzzy system (here a fuzzy decision-making module) can be found in Tóth-Laufer & Takács (2012, pp. 133-134).

The Centre of Gravity / Centre of Area (CoG / CoA) method results in providing a centre point of the area under the aggregated membership functions curve. This defuzzified output value y is calculated by Equation 64:

$$y_{\mathrm{CoG}} = \frac{\int x\mu_A(x)dx}{\int \mu_A(x)dx} \qquad \text{(Eq. 64)}$$

where:
x = input linguistic variables (carrier of linguistic area for output variable);
$\mu_A(x)$ = resulting membership function, composed of activated membership functions to particular fuzzy intervals.

The main disadvantage of CoG / CoA method is its calculation complexity, especially the calculation of crisp value for the complex-shaped partial conclusions.

In the Centre of Sums (CoS) method the defuzzified output value y is calculated in two steps: first, the geometric Centre of Area for each membership function is calculated; second, the calculation of the weighted average of the geometric centre of area for all membership functions. The mathematical notation of the Centre of Sums is given in Equation 65:

$$y_{\mathrm{CoS}} = \frac{\sum_{i=1}^{k} A_i \overline{x_i}}{\sum_{i=1}^{k} A_i} \qquad \text{(Eq. 65)}$$

where:
A_i = the area of i^{th} variable;
k = total number of fuzzy variables;
$\bar{x}_1$ = centre of area A_i.

The Bisector of Area (BoA) method draws a vertical line dividing the area into two sub-areas of equal size – as in Equation 66:

$$\int_{\alpha}^{y_{\mathrm{BoA}}} B^*(y)\,dy = \int_{y_{\mathrm{BoA}}}^{\beta} B^*(y)\,dy \qquad \text{(Eq. 66)}$$

where:
$\alpha = \min\{y; y \in \mathrm{B}^*\}$;
$\beta = \max\{y; y \in \mathrm{B}^*\}$;
y = BoA is a vertical line that partitions the area determined by $y = \alpha$, $y = \beta$, $z = 0$ and $z = \mathrm{B}^*(y)$ into two equal sub-areas.

The Maxima Methods focus on the values with maximum membership. There are three main Maxima methods:

- First of Maxima (FoM), which determines the smallest value of the fuzzy set with the maximum membership value;
- Last of Maxima (LoM) which determines the largest value of the fuzzy set with the maximum membership value;
- Mean of Maxima (MoM), in which the defuzzified value is the mean value of maximas of all elements with the highest membership values – Equation 67:

$$y_{\mathrm{MoM}} = \frac{\sum x_i \in M x_i}{|M|} \qquad \text{(Eq. 67)}$$

where:
$x \in X$, $M = \{x_i\}$;
$\mu_A(x_i)$ – equal to the height of the fuzzy set A_i;
$|M|$ – the cardinality of the set M.

In the Weighted Average Method (WAM) each membership function obtains a weight equal to its maximum membership value. It can be applied for fuzzy sets with symmetrical output membership functions. Although similar to CoG method, it is easier to compute. The defuzzified crisp value is calculated with Equation 68:

$$y_{\mathrm{WAM}} = \frac{\sum_{i=1}^{N}(\mu_i Y_i)}{\sum_{i=1}^{N}(\mu_i)} \qquad \text{(Eq. 68)}$$

where:
N = number of fuzzy sets with a membership output value $\neq 0$;
μ_i= the value of grades of membership of particular fuzzy sets;
Y_i= output value for particular fuzzy sets.

As already mentioned, for precise modelling of the decision-making environment polygonal membership functions are more adequate, however they can be applied only when a full possible full set of information about the system (here the decision environment and decision problem) is available. As this is not the case, symmetric membership functions will be expected at Fuzzy Module's output. An additional argument in favour of symmetric membership functions is the discontinuity and non-differentiability of their polygonal versions. Moreover, an approximation of the final result of decision-making is performed in the Hierarchical Module of the built model, which provides an extra opportunity for the simplification of the initial complexity of decision environment. Taking all of the above into account, for decision-making enhancement with the use of the Fuzzy Module of the proposed decision-making model, triangular symmetric membership functions were applied. Next, defuzzification was performed with the use of the Weighted Average Method (WAM).

Each decision problem requires a separate setup of both hierarchical and Fuzzy Modules. In each of its applications the mode of operation of the Fuzzy Module shows the characteristics of a dynamic and interactive procedure and was presented with example numerical data.

Example 4.2 provides a practical insight into the application of the Fuzzy Module of the developed model. As already mentioned in the theoretical introduction, the Fuzzy Module consists of three blocks:

- fuzzification block;
- interference block;
- defuzzification block.

<u>Example 4.2</u> – assumptions:

- crisp input values – AHP-resulting decision alternatives A_1 = 38.7% and A_2 = 38.5%;

- triangular membership functions were applied for the calculation of grades of membership at the exit of the fuzzification block – denoted by Equation 69 (same as Equation 38):

$$\mu_A(x) = \left\{1 - \frac{|x-c|}{d}\right\}; \text{ for } x \in [c-d, c+d] \quad \text{(Eq. 69);}$$

where:
$\mu_A(x) = 0$ lies outside this interval;
c = peak of the symmetric triangular function ($\mu_A(c) = 1$);
$c - d$ = lower limit of x values;
$c + d$ = upper limit of x values.

- $\mu_A(y)$ membership functions at the output of the inference block are continuous and symmetric;
- Weighted Average Method (WAM) will be used for defuzzification, calculated from Equation 70 (same as Equation 68):

$$y_{\mathrm{WAM}} = \frac{\sum_{i=1}^{N}(\mu_i Y_i)}{\sum_{i=1}^{N}(\mu_i)} \quad \text{(Eq. 70)}$$

where:
N = number of fuzzy sets with a membership output value $\neq 0$;
μ_i = the value of grades of membership of particular fuzzy sets;
Y_i = output value for particular fuzzy sets.

Table 4.18 summarises the results of the application of the Hierarchical Module of the model, out of which crisp values of decision alternatives A_1 = 38.7% and A_2 = 38.5% derived. Alternatives A_3 and A_4 were rejected as much lower in the decision hierarchy and fell out of the scope of further analysis, however, together with the evaluations of decision making criteria they remain part of the calculations of the maximum Consistency Ratio CR_{max} = 5.7%.

Table 4.18.

Prioritization of decision alternatives resulting from expert-sensitive weights of alternative decisions $A_1 - A_4$ – Fundamental Comparison Scale (linear)

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**38.7%**	**38.5%**	**14.5%**	**8.3%**	5.7%

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
**Group consensus: 74,3% (moderate);
Source: Compiled on the basis of Table 4.6.

A_1 = 38.7% and A_2 = 38.5% values form the crisp inputs into the fuzzification block of the Fuzzy Module of the model. The fuzzification procedure results in obtaining two fuzzy sets of potential decisions $A_1 \in$ (36.49%, 40.91%); $A_2 \in$ (36.31%, 40.69%) . These intervals result from the fuzzification of initial A_1 and A_2 crisp values (38.7% and 38.5% respectively) with the use of Equation 69, where d value is equal to the CR_{max} = 5.7%. Justification: as the Consistency Ratio of experts' answers is imperfect ($CR_{max} \neq 0$), the AHP decision-making result is imperfect, too – to the extent described by the maximal Consistency Ratio (CR_{max}), calculated for all the evaluators. Therefore $\mu_{A_1}(x)$ triangular membership function at the output of the fuzzification block was determined by A_1 – d = 38.7% – 0.057 · 38.7% = 36.49% lower limit value and $A_1 + d$ = 38.7 + 0.057 · 38.7% = 40.91% upper limit value. Therefore the grades of membership of alternative A_1 belong to the interval $\mu_{A_1} \in$ (36.49%, 40.91%). Per analogiam, the grades of membership of alternative A_2 belong to the interval $\mu_{A_2} \in$ (36.31%, 40.69%), as the lower limit value of the membership function $\mu_{A_2}(x)$ equals $A_2 - d$ = 38.5% – 0.057 · 38.5% = 36.31% and its upper limit value equals $A_2 + d$ = 38.5 + 0.057 · 38.5% = 40.69%.

Once fuzzification has been accomplished, the calculation of $\mu_{A_1}(y)$ membership functions via the inference block rules and algorithms follows. As the membership functions at the exit of the fuzzification block are triangular, the output values of A_1 and A_2 are at their maximum. Therefore their grades of membership $\mu_{A_1} = \mu_{A_2} = 1$, as the peak of the symmetric triangular function ($\mu_A(c) = 1$). Because the $\mu_{A_1}(y)$ membership functions at the output of the inference block are continuous and symmetric, the Weighted Average Method (WAM) can be applied for defuzzification, basing on Equation 70. The calculation with numerical data expressed by Equation 71:

$$y_{A_{1,2}} = \frac{1 \cdot 38.7\% + 1 \cdot 38.5\%}{2} = 38.6\% \qquad \text{(Eq. 71)}$$

The interpretation of result from Equation 71 is as follows: the value of fuzzy decision for both decision alternatives $y_{A_{1,2}}$ = 38.6%, which equals the arithmetic average of both decision alternatives A_1 and A_2. However, such a result has a low applicatory value, as in their search for the enhancement of their decision-making the majority of managers require a more precise indication of the optimal possible solution, which is an integral one.

As explained before, such an ideal situation is unlikely to happen, because of the limitations of individual and group consistency of experts' evaluations, denoted in AHP by the Consistency Ratio index. It can then be assumed that if CR_{max} = 5.7% (using the numerical data from the same calculation example as earlier in the text), the fuzzy grades of membership of both decisions A_1 and A_2, resulting from the Hierarchical Module, are equal and conform to the following relation: $\mu_{A_1} = \mu_{A_2} = 1 - 0.057 = 0.943$. In such a case the defuzzified values of decisions A_1 and A_2 can be calculated by Equations 72 and 73, respectively:

$$A_1^{'} = \mu_{A_1} \cdot A_1 = 0.943 \cdot 38.7\% = 36.49\% \qquad \text{(Eq. 72)}$$

$$A_2^{'} = \mu_{A_2} \cdot A_2 = 0.943 \cdot 38.5\% = 36.31\% \qquad \text{(Eq. 73)}$$

In this way, after the application of the Fuzzy Module of the developed model real rather than ideal priorities of AHP decision alternatives are obtained. The difference lies in the fact that real hierarchization of decision alternatives covers the inconsistency of experts' evaluations and lowers the significance rankings of decision alternatives in question respectively.

It is noticeable that such a result could be obtained without the application of the Fuzzy Module, by simply deducting the "inconsistent" percentage of the significance ranking of a given decision alternative from its initial magnitude. However, such an interpretation is valid only in a unique case, when the membership functions are symmetric and their grades of membership can be calculated with the use of WAM. In a universal case, the shapes of membership functions do not have to be triangular and their values can vary. This could force the decision-maker to use methods other than WAM defuzzification. Such a solution takes into account neither the discrepancies nor even rank reversal, caused by the application of a different comparison scale in the Hierarchical Module (the issue discussed in sub-chapter 4.1, Table 4.6 and Tables 4.9 to 4.17). For this reason a mixed AHP-Fuzzy development of the traditional fuzzy logic procedure was proposed.

Construction of mixed AHP-Fuzzy Module

The aim of the mixed AHP-Fuzzy expansion of the Hierarchical and Fuzzy Modules of the developed multicriteria model for the enhancement of managerial decision-making is to include into the scope of analysis scales other than the Fundamental Comparison Scale. The advantages and disadvantages of the application of various comparison scales were discussed in sub-chapter 4.1.

The modus operandi of the mixed AHP-Fuzzy Module also shows the characteristics of a dynamic and interactive procedure and will be explained on example numerical data (Example 4.3).

The mixed AHP-Fuzzy Module consists of the following phases:

- choice of the secondary comparison scale;
- inclusion of AHP hierarchy resulting from the chosen scale into the discussed solution;
- recurrent application of the Fuzzy Module under actualized data;
- final decision – choosing integral alternative with respect to the main goal of the entire decision-making process.

Example 4.3 – the application of mixed AHP-Fuzzy Module.

First, a secondary comparison scale needs to be chosen. Several comparison scales, used for explanatory purposes can be found in Table 4.6 and Tables 4.9 to 4.17. The following steps need to be performed in order to use the most accurate secondary comparison scale:

- rejection of inconsistent comparison scales – following Saaty's assumption of acceptable inconsistency of expert evaluations (CR < 10%, Saaty, 1996, p. 21) individual CR_{max} of each expert and group CR_{max} needs to be verified – all scales with individual or group CR_{max} > 10% need to be rejected; continuing with formerly used data only the following tables observe this rule and can be processed further:

Table 4.19.

Prioritization of decision alternatives resulting from expert-sensitive weights of alternative decisions $A_1 – A_4$ – Logarithmic Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**34.5%**	**33.6%**	~~**18.7%**~~	~~**13.3%**~~	2.4%

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
**Group consensus: 73.9% (moderate);
Source: Compiled on the basis of Table 4.9.

Table 4.20.

Prioritization of decision alternatives resulting from expert-sensitive weights of alternative decisions $A_1 – A_4$ – Square-Root Scale

Participants	Decision Alternative A_1	Decision Alternative A_2	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**32.5%**	**32.5%**	~~**19.9%**~~	~~**15.0%**~~	1.4%

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
**Group consensus: 75.2% (high);
Source: Compiled on the basis of Table 4.10.

Table 4.21.

Prioritization of decision alternatives resulting from expert-sensitive weights of alternative decisions $A_1 – A_4$ – Balanced-*n* Scale

Participants	Decision Alternative A_1^*	Decision Alternative A_2^*	Decision Alternative A_3	Decision Alternative A_4	CR_{max}
Group result	**32.6%**	**35.3%**	~~**18.8%**~~	~~**13.3%**~~	2.2%

*Overall Consistency Ratio CR < 0.1 => expert opinions are consistent;
Group consensus: **77.6% (high);
Source: Compiled on the basis of Table 4.13.

- choice of the table with the highest group consensus – which coincides with the rationale for the application of Kendall's W coefficient of concordance in the Fuzzy Module; in our case this will be the Balanced-*n* Scale (Table 4.21), as the group consensus of expert evaluations is the highest (77.6%, ranked as high);
- in the case of a very close group consensus (with differences below 1%) an additional measure is suggested – choosing the table with the lowest group CR_{max} – in the discussed case no such need exists, as the differences between group consensus of all available scales are significant.

AHP hierarchy resulting from the chosen scale is included into the discussed solution by the calculation of average values of decision alternatives A_1^{mix} and A_2^{mix} resulting from the Hierarchical Module, from Equation 74:

$$A_i^{\text{mix}} = \frac{A_i + A_i^*}{2} \tag{Eq. 74}$$

where:
A_i – the value of the decision alternative resulting from the application of the Hierarchical Module;
A_i^* – the value of the decision alternative resulting from the secondary comparison scale.

Equations 75 and 76 provide calculation examples of A_1^{mix} and A_2^{mix} based on the same data as before:

$$A_1^{\text{mix}} = \frac{38.7\% + 32.6\%}{2} = 35.65\% \tag{Eq. 75}$$

$$A_2^{\text{mix}} = \frac{38,5\% + 35,3\%}{2} = 36.90\% \tag{Eq. 76}$$

The obtained average values of decision alternatives, after the inclusion of those from the secondary comparison scale are used for the recurrent application of the Fuzzy Module under actualized data.

$A_1^{mix} = 35.65\%$ and $A_2^{mix} = 36.90\%$ form the crisp inputs into the fuzzification block of the mixed AHP-Fuzzy Module of the proposed model. The fuzzy set of potential decisions is calculated with the same $\mu_{A_i^{mix}}(x)$ triangular membership functions, however this time the grades of membership for each membership function are calculated with the use of their appurtenant CR_{max}. Explanation: as A_1 has been ranked in the Hierarchical Module as the decision alternative that contributes the most to taking an optimal possible managerial decision, its $CR_{max} = 5.7\%$ is taken from the Fundamental Comparison Scale from the Hierarchical Module (Table 4.18); but when using the secondary comparison scale, the Balanced-*n* Scale, this priority was attributed to decision alternative A_2^{*} [19], with a CR_{max} = 2.2% – taken from the Balanced-*n* Scale (Table 4.21). Therefore d value from Equation 69 for A_1^{mix} remains unchanged ($d_1 = 0.057$), but changes for A_2^{mix} into $d_2 = 0.022$. In case with no rank reversal between decision alternatives A_1^{*} and A_2^{*}, the same d value could be employed ($d_1 = d_2 = 0.057$).

The $\mu_{A_i^{mix}}(x)$ membership function at the output of the fuzzification block was determined by $A_1^{mix} - d_1 = 35.65\% - 0.057 \cdot 35.65\% = 33.62\%$ lower limit value and $A_1^{mix} + d_1 = 35.65\% + 0.057 \cdot 35.65\% = 37.68\%$ upper limit value. Therefore the grades of membership of decision alternative A_1^{mix} belong to the interval $\mu_{A_1^{mix}} \in (33.62\%, 37.68\%)$. Per analogiam, the grades of membership of alternative A_2^{mix} belong to the interval $\mu_{A_2^{mix}} \in (36.09\%, 37.71\%)$, as the lower limit value 9of the membership function $\mu_{A_2^{mix}}(x)$ equals $A_2^{mix} - d_2 = 36.90\% - 0.022 \cdot 36.90\% = 36.09\%$ and its upper limit value equals $A_2^{mix} + d_2 = 36.90\% + 0.022 \cdot 36.90\% = 37.71\%$.

The calculation of $\mu_{A_i^{mix}}(y)$ membership functions in the inference block falls under the same conditions as $\mu_{A_i^{mix}}(y)$ membership functions. They are also symmetric and triangular and their output values A_1^{mix} and A_2^{mix} are at their maximum. Therefore their grades of membership $\mu_{A_1^{mix}} = \mu_{A_2^{mix}} = 1$. As the $\mu_{A_i^{mix}}(y)$ membership functions at the output of the inference block are continuous and symmetric, the Weighted Average Method (WAM) can again be applied for their defuzzification, basing on Equation 77 (similar to Equation 70, with modified notation).

$$y_{\mathrm{WAM}} = \frac{\sum_{i=1}^{N}\left(\mu_{A_i^{mix}} Y_{A_i^{mix}}\right)}{\sum_{i=1}^{N}\left(\mu_{A_i^{mix}}\right)} \qquad \text{(Eq. 77)}$$

where:
N = number of fuzzy sets with a membership output value $\neq 0$;
$\mu_{A_i^{mix}}$ = the value of grades of membership of particular average fuzzy sets;
$Y_{A_i^{mix}}$ = output value for particular average fuzzy sets.

The calculation using numerical data is expressed in Equation 78.

$$y_{A_{1,2}}^{\mathrm{mix}} = \frac{1 \cdot 35.65\% + 1 \cdot 36.90\%}{2} = 36.28\% \qquad \text{(Eq. 78)}$$

Equation 78 represents the value of the fuzzy decision for both decision alternatives $y_{A_{1,2}^{mix}} = 36.28\%$, calculated as the arithmetic average of both decision alternatives A_1^{mix} and A_2^{mix}.

In order to proceed to the last step of the application of the entire developed decision-making model – to choose the decision alternative that would most likely result in making an integral managerial decision – the membership grades for the

[19] Decision alternative is the same decision alternative as A_2, but with a different value – therefore a separate notation with “*” has been employed.

$\mu_{A_i^{\mathrm{mix}}}(y)$ membership function need to be calculated. Equation 79 provides a general mathematical notation, and Equations 80 and 81 are the calculation examples with appropriate $\mathrm{CR}_{\max}$ values:

$$\mu_{A_i^{\mathrm{mix}}} = 1 - d_i \qquad \text{(Eq. 79)}$$

where $d_i = \mathrm{CR}_{\max}$ for the i^{th} average (mixed) decision alternative.

$$\mu_{A_1^{\mathrm{mix}}} = 1 - 0.057 = 0.943 \qquad \text{(Eq. 80)}$$

$$\mu_{A_2^{\mathrm{mix}}} = 1 - 0.022 = 0.978 \qquad \text{(Eq. 81)}$$

Using the above grades of membership of $\mu_{A_i^{\mathrm{mix}}}(y)$ membership functions, the defuzzified values of decisions A_1^{mix} and A_2^{mix} can be calculated with use of Equation 82, with real calculation values in Equations 83 and 84:

$$A_i'' = \mu_{A_i^{\mathrm{mix}}} A_i^{\mathrm{mix}} \qquad \text{(Eq. 82)}$$

$$A_1'' = \mu_{A_1^{\mathrm{mix}}} \cdot A_1^{\mathrm{mix}} = 0.943 \cdot 35.65\% = 33.62\% \qquad \text{(Eq. 83)}$$

$$A_2'' = \mu_{A_2^{\mathrm{mix}}} \cdot A_2^{\mathrm{mix}} = 0.978 \cdot 36.90\% = 36.09\% \qquad \text{(Eq. 84)}$$

The final decision, which is the last step of the application of the mixed AHP-Fuzzy Module consists of the measurement of the distance of A_1'' and A_2'' values from the average $y_{A_{1,2}^{\mathrm{mix}}}$ value, which equals 36.28%. It is clear that $A_2'' = 36.09\%$ is closer to $y_{A_{1,2}^{\mathrm{mix}}} = 36.28\%$ as $A_1'' = 33.62\%$. This constitutes the final suggestion for the decision-maker that the final decision A_2 is an integral one.

The application of all three modules of the proposed decision-making model indicates choosing the alternative decision A_2 as the one that contributes to the highest extent to the main goal of the decision-making, i.e. making an integral managerial decision. This confirms the general sub-thesis T3, stating that the joint qualitative-quantitative research method is appropriate to address the scientific purpose of this monograph – the complementation of the decision theory methodology in the field of complex managerial problems by developing a method of the enhancement of managerial decision-making processes through a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM).

As a result performing the decision-making process using the proposed model, a hierarchy of decisions is obtained. However, the result of the application of the model points at one certain solution. And even though the Fuzzy and mixed AHP-Fuzzy Modules enrich the study by incorporating the second-best decision alternative into the analysis, it is important to remember that all decision-making models aim at the enhancement of the decision-making process only. The final decision always has to be made by the manager. The role of the decision-making enhancement tools is to expand the manager's awareness of the available choices and his or her understanding of their potential implications after the decision has finally been made. However, the expectation that any decision-making model will completely replace the manager in the decision-making process is purely irrational.

CONCLUDING REMARKS

The presented monograph embraces a congruous research area of scientific modelling and enhancement of the decision-making process. The author's input into the development of scientific discipline of management science will be outlined below.

Following the science development characteristics and valuation of research problems proposed by Niemczyk (2015a, pp. 18-19; 2015b, pp. 112-116), the proposed research task can be attributed an evolutionary cumulative influence on the development of science. This is due to the evolution brought to the methodological level of the sub-discipline enhancement of managerial decisions, with consequences for the management field of study. A deeper understanding of the character of the interrelations between decision criteria and accuracy of managerial decision-making processes has been achieved as well. An additional outcome of the presented research is the development of a model for the enhancement of decision-making that allows the optimisation of final decisions with regard to the changeability of decision environment. This in turn is a premise for adaptive changes in the Polish economy, this issue, however, requires further research.

Moreover, the presented research problem constitutes a new solution or a development of an existing one, so far not applied to the context in question. Therefore it can be perceived as innovative and innovatory (Niemczyk, 2015b, p. 113). This is mainly due to:

- integration of measurable and immeasurable determinants of decision-making processes in one complex model, in the frame of one consistent research task;
- sequential application of the qualitative and quantitative methods for the enhancement of managerial decision-making processes;
- modelling of the abovementioned process using the research methods applied mainly in technical sciences – multicriteria analysis, Analytic Hierarchy Process and fuzzy logic;
- applicatory character of the research task, which opens a vast field for the use of the proposed solution in managerial practice.

Considering Niemczyk's comments on the innovative and innovatory developments in the scientific research provokes an interesting observation on the direction of these trends. On the one hand, a growing number of applications of research methods traditionally belonging to the technical or medical sciences can be observed in management science. On the other hand, the research methods typical of economics, such as econometric models, are increasingly employed for the analysis of phenomena of sociological or even psychological nature (e.g. Żyra, 2013).

The originality of the proposed research task can be derived from a successful attempt at answering questions such as "how?", "why?" and "in what way?". At the same time, the research concept of the proposed research satisfies all four key conditions of scientific originality:

- identification of relations;
- projection of relations;
- determining the associations of elements with entity;
- matching a method.

The above then leads to the statement that the presented research conforms to the process of filling the research gap formulated by Strużyna (2015, pp. 61-64).

The scientific purpose of the presented monograph was the complementation of the decision theory methodology in the field of complex managerial problems. This purpose has been fully achieved by the application of scientific methodology to the development of a method for the enhancement of managerial decision-making processes through a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM). Also the main research problem of the monograph, the development of a sequential, qualitative-quantitative method for the enhancement of managerial decision-making, appropriate for multicriteria management problems has been solved. Additionally, the procedure of the application of the formulated decision support solution has been illustrated.

All the research tasks have been conducted. Decision-making related issues have been identified in the scientific literature. Literature studies encompassed mainly management sources, however, other fields of science have been explored as well. This was due to a wider popularity and availability of scientific publications of MCDM in other branches of science (e.g. engineering, IT or medicine). An inquiry on the practice of decision-making in enterprises has been performed on a sample of management professionals, belonging to the tactical and strategic management levels. The quantitative research methods have been employed for this task. The testing of hypotheses at the quantitative research stage resulted in positive validation of the H1 research hypothesis, stating that the application of multicriteria decision-making enhancement methods increases the probability of taking an integral managerial decision. The research hypothesis H2, stating that the application of multicriteria decision-making enhancement methods does not increase the probability of taking an integral managerial decision has been rejected. The qualitative methods have been employed to accomplish the third research step, which resulted in the creation of a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes. The formulated solution is composed of three modules:

- the AHP-based Hierarchical Module, incorporating measurable and immeasurable determinants of decision environment on the input side and a hierarchy of decision alternatives on the output side;
- a Fuzzy Module, which allows reconsideration of the second-best decision alternative, which is very often close to the optimal possible solution from the Hierarchical Module; as a result, the decision field is widened by a discussion of the rejected, but still important decision alternatives and corresponding rankings of decision criteria;
- a mixed AHP-Fuzzy Module, an expansion aiming at counterbalancing ranking inconsistencies derived from the calculation of comparison scales other than the Fundamental Comparison Scale of AHP; providing an integral managerial decision.

An innovative input into the development of management science results from the application of a joint (AHP + fuzzy logic) and mixed AHP-Fuzzy research methodology (AHP and Fuzzy sets theory) for solving managerial decision problems. As an effect, a new quality in the modelling of multicriteria decision-making processes has been achieved, by joining the choice of one precise decision alternative, incorporating the arguments in favour of the second-best solution. Also, the author's original definition of an integral managerial decision has been proposed.

All the research questions posed in order to stimulate in-depth research have been investigated. Re 1) The decision-making practice of tactical and strategic managers in companies has been unveiled by means of a questionnaire. Re 2) The questioned managers pointed at their need to include both measurable and immeasurable decision determinants as relevant for their decision-making practices. Re 3) An overview of multicriteria methods for enhancing managerial decision-making present in the management science literature has been performed, however, due to the dynamic development of the discipline a separate meta-analysis of this topic in scientific publications could prove useful. Re 4) Combining the qualitative and quantitative methods of enhancement of managerial decision-making processes within one model turned out not only to be possible, but also promising for the increase of precision and efficiency of managerial decision-making. Re 5) Fuzzification of the obtained hierarchy of decision alternatives in terms of the proposed solution allowed the second-best decision variant to be included into the analysis.

The thinking process incited by the research questions allowed the formulation of the main goal of the monograph, which was the presentation of the developed Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes.

It has been achieved through a series of detailed goals of the monograph, all of which have been accomplished as well. Re 1) Definition of an integral managerial decision in the frame of the prescriptive approach has been proposed. Re 2) The practice of managerial decision-making in enterprises operating in Poland with regard to the readiness to use the decision-making processes enhancement tools has been researched. Re 3) An analysis of the multicriteria methods enhancing managerial decision-making in scientific literature has been performed. Re 4) The methodological bases for the development of a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes have been presented. Re 5) As an immediate result of the accomplishment of all the previous research goals a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) for the enhancement of managerial decision-making processes that includes the second-best decision alternative into the decision-making outcome has been built.

The performed research allows all the three general sub-theses to be positively verified:

- T1. In a dynamically changing environment of companies managerial decision-making processes require the inclusion of a growing number of decision criteria.
- T2. The inclusion of the quantitative (measurable) and qualitative (immeasurable) criteria into this analysis can improve the integrity of the final decision.
- T3. The joint qualitative-quantitative research method is appropriate to address the scientific purpose of this monograph – the complementation of the decision theory methodology in the field of complex managerial problems by developing a method of the enhancement of managerial decision-making processes through a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM).

Therefore **the general research thesis** of this monograph, stating that the application of a sequential qualitative-quantitative model for the enhancement of multicriteria managerial decision-making processes allows integral managerial decisions to be made **can be confirmed**.

The confirmation of the general research thesis and the study presented in this monograph implies some outcomes for the practice of enterprise management:

1. Offering the decision-makers a Modular Multicriteria Managerial Decision-Making Model (MMUMADEMM) will equip them with a tool for a more frequent making of integral decisions in their business practice.
2. This in turn will positively affect the competitiveness of the company, branch of industry and in broader consequence – the entire economy.
3. A common usage of the devised tool will allow reducing the frequency of faulty decision-making, which could minimise lost opportunity costs.
4. Practical implementation of the research results will disengage a part of managers' attention from routine decisions and will allow them to shift

from the tactical to strategic managerial level, e.g. search for innovative development directions, potential business niches or evolve towards socially responsible companies.

The construction of the accomplished study project imposed some limitations. One of these is the territorial span of the quantitative research, which covered only enterprises operating in Poland. It could be interesting to carry out a separate research project expanding the territorial coverage of the questioned enterprises by European or global companies. The other limitation originated in the composition of experts' sample – only tactical and strategic level managers were addressed. An expansion of the evaluators' group with operational level managers could provide interesting data for future comparative research, as well as the expansion of the territorial span of the analysed enterprises.

Moreover, the qualitative research stage provides basis for further research. One of the possible directions could be the construction of scenarios for the analysis of potential future developments of the decision environment. Kononiuk & Nazarko (2014, pp. 19-21) trace the scenario analysis method back to ancient times, when these were applied in the military and philosophical fields. The same authors point at Kahn (1962) who was the first to combine the scenario analysis with decision theory, and a dynamic development of the method ever since. Therefore a multitude of the developed model's possible applications for scientific anticipation of potential developments of conceivable business scenarios arises.

A separate research should focus on devising a classification that would combine a specific type of managerial decision problems with appropriate membership functions (similar to the one on MCDM application appropriateness to given decision problems proposed by Piotrowski, 2009, pp. 150-151). Such a classification would enrich the toolbox of managers who employ scientific models and procedures for the enhancement of their decision-making processes.

Last, but not least – another desirable field for future research derives from the main limitation of this study, which was the relatively uncomplicated employed mathematical toolbox. Even if such an approach was appropriate for the analysed decision problem, and also justified by the explanatory character of the discussed examples, further research should concentrate on the applications of more advanced types of membership functions for the calculation of their membership grades (including those in the fuzzy inference block). A combination of the proposed decision-making enhancement framework with sophisticated mathematical apparatus has all the chances to evolve into an even more powerful tool for the enhancement of complex decision-making processes at all levels of management. If combined with the innovative scenario analysis postulated by Kononiuk & Nazarko (2014), an anticipation of various possible environmental development scenarios at early stages of managerial decision-making could become possible as well.

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