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Abstract

Mortality forecasting become a big challenge not only for demographers but also for actuaries. Different modeling were proposed for this issue while insuring efficiency and simplicity. These models has been based on time and age dimensions. The analysis of mortality reductions schemes by age shows some inequalities related to age. Generally, the difference is well apparent between lower and higher ages. This can't be only explained by aging phenomenon but also by the year of birth. Considering the cohort effect in morality forecasting has to improve the fitting and by the way the forecasting quality. In the present paper, we propose to forecast specific-age mortality in Algeria with considering cohort effect by comparison between a set of proposed models.

Key-words: Mortality forecasting, Cohort, fitting, Algeria, life annuities

1 Introduction

During the past half-century, the Algerian population has earned about 30 years in life expectancy at birth and more than 6 years in life expectancy at 60 (ONS, 2012). this improvement make a challenge not only for demographers but also for actuaries.We now wonder about the future trend of mortality reduction. the recent changes in the Algerian insurance market impose to life insurers to adopt more accurate estimation methods. Life-insurance calculations in Algerian are still based on static life table constructed on old mortality data (CNA, 2004).

In a previous works (Flici, 2013. Flici, 2014-a), we have tried to construct prospective life-tables for the Algerian population applying Lee carter (1992) and Renshaw and Haberman (2003). Since we are looking for more accuracy, the objective of the present paper is to propose a dynamic life table trying to consider more element which can affect or explain the future evolution of the mortality pattern of the Algerian population aged 60 and over. the effect of the year of birth in the mortality profile should be considered. the mortality profile is greatly affected by the infant life conditions (Elo and Preston, 1992; Bengtsson and Broström, 2009).

this work is based on the global population mortality data published by the Algerian Office of National Statistics (ONS) covering the period 1977- 2013. This data allows to construct incomplete mortality surface. For some years the ONS has not published life tables. Also, some of the published life tables were closed-out before the open age group [80 and +]. We have dedicated a previous work to estimate the missing data in the Algerian mortality surface (Flici, 2014-b).

To estimate the parameters of the Age-Period-Cohort model we use the process proposed by Renshaw and Haberman (2006) and Haberman and Renshaw (2009) which represent an extension of Lee-Carter model (Lee and Carter, 1992). Some modifications were proposed by the presents work. We use XL-Solver for the residual matrix decomposition based on the LSE minimisation. Suppression of the Constraints allows to reduce the fitting errors (Wilmouth, 1993). The old age mortality is estimated using the model proposed by Coale & Kisker (1990) with single age formulation.

The mortality trend in Algeria has marked a bump during the 90 because of the terrorism events. During this period, known also as the black decade, the curve of life expectancy evolution has marked a recession by about 2 years (Flici and Hammouda, 2014) without considering the interactive effects which should affect the actual life expectancy. this bump should be corrected before extrapolation of the mortality trends in the future. Bell (1997) showed the necessity to correct such a bump observed erstwhile in USA in 1918.

In final, the obtained result are used to resume the future evolution of life expectancy at 60 and also on the life annuities pricing.

2 Methodology

2.1 Background on Cohort effect in mortality forecasting

Almost of the models that have been proposed for mortality forecasting during the recent years are based on the decomposition principal proposed by Roland lee and Lawrence Carter in 1992 (Lee and Carter, 1992). The principal of the method aims to decompose the logarithm of the central death rate observed for the age (x) during the year (t) u_{xt}in three components as following:

$$ln(\mu_{xt}) = \alpha_x + \beta_x * \kappa_t + \xi_{xt} \tag{1}$$

For each age $x \in [X_1, ..., X_n]$, α_x is estimated by the average of the observed $ln(\mu_{xt})$ throughout the observation period $(T_1 - T_p)$:

$$\alpha_x = ln(\prod_{t=T_1}^{T_n} (\mu_{xt}))^{\frac{1}{(Tn - T_1)}}$$

a simple arithmetique mean formula can also be used: ()

$$\alpha_x = \frac{1}{p} \sum_{t=1}^{p} ln(\mu_{xt})$$

Thereafter, the residual matrix is decomposed into two vectors representing the time mortality index (κ_t) and the sensitivity of the different ages to the time mortality variation (β_x):

$$ln(\mu_{xt}) - \alpha_x \approx \hat{\beta}_x * \hat{\kappa}_t$$

with as a constraints: $\sum_{x=X_1}^{X_n} \beta_x = 1$ and $\sum_{t=T_1}^{T_p} \kappa_t = 0$.

in their original paper ((Lee and Carter, 1992), the authors proposed a two stages decomposition process. In the first stage we decompose the residual matrix by SVD techniques by minimising the sum of the squared errors between the residual matrix $ln(\mu_{xt})-\alpha_x$and the product of the two vectors $\hat{\beta}_x$and $\hat{\kappa}_t$obtained by the optimisation prosses. The optimization problem is written by:

$$minS = \sum_{x=0,t=1}^{n,p} [ln(\mu_{xt}) - \alpha_x\text{-}\hat{\beta}_x * \hat{\kappa}_t]^2$$

This process leads to some differences between the expected and the observed number of deaths. For this, in the second estimation stage, $\hat{\beta}_x$and $\hat{\kappa}_t$are adjusted to fit the observed number of deaths at every year (t). the optimization problem is then:

$$minS = \sum_{t=1} \sum_{x=0}^{pn} [exp(\alpha_x + \hat{\beta}_x * \hat{\kappa}_t) L_{xt} - D_{xt}]^2$$

where $D_{xt}$is the observed number of deaths at age x and time t, and $L_{xt}$is the death exposure to the death risque at age x and time t.

Wilmoth (1993) proposed a one stage decomposition process based on the Weighed Least Squared Errors.

$$minS = \sum_{x=0,t=1}^{n,p} W_{xt}[ln(\mu_{xt}) - \alpha_x\text{-}\hat{\beta}_x * \hat{\kappa}_t]^2$$

The weight W_{xt} can be the observed number of death at each point x and t D_{xt}.

The same author proposed also to re-estimate $\alpha_x$by the same estimation process as $\hat{\beta}_x$and $\hat{\kappa}_t$ and to use the original values obtained by mean over time of the $ln(\mu_{xt})$ as a starting value.

Renshaw and Haberman (2006) extended the LC model by adding a cohort Component. That gave the following formula:

$$ln(\mu_{xt}) = \alpha_x + \beta_x^{(1)} * \kappa_t + \beta_x^{(2)} * \gamma_{t-x} + \xi_{xt} \quad (2)$$

Currie (2006) simplified the formula (2) by imposing as a constraints: $\beta_0^{(1)} = \beta_1^{(1)} = ... = \beta_{X_n}^{(1)} = \frac{1}{n}$ and $\beta_0^{(2)} = \beta_1^{(2)} = ... = \beta_{X_n}^{(2)} = \frac{1}{n}$. That leads to the formula:

$$ln(\mu_{xt}) = \alpha_x + \frac{1}{n}\kappa_t + \frac{1}{n}\gamma_{t-x} + \xi_{xt} \quad (3)$$

This model is well known as the Age-Period-Cohort model (APC).

We remind that the transition from the model (2) to model (3) was by introducing about the sensitivity coefficients:

$$\beta_x^{(1)} = \beta_x^{(2)} = \frac{1}{n} \quad (4)$$

which implies to modify somehow two other constraints:

$$\sum_{x=X_1}^{X_n} \beta_x^{(1)} = \sum_{x=X_1}^{X_n} \beta_x^{(2)} = 1 \quad (5)$$

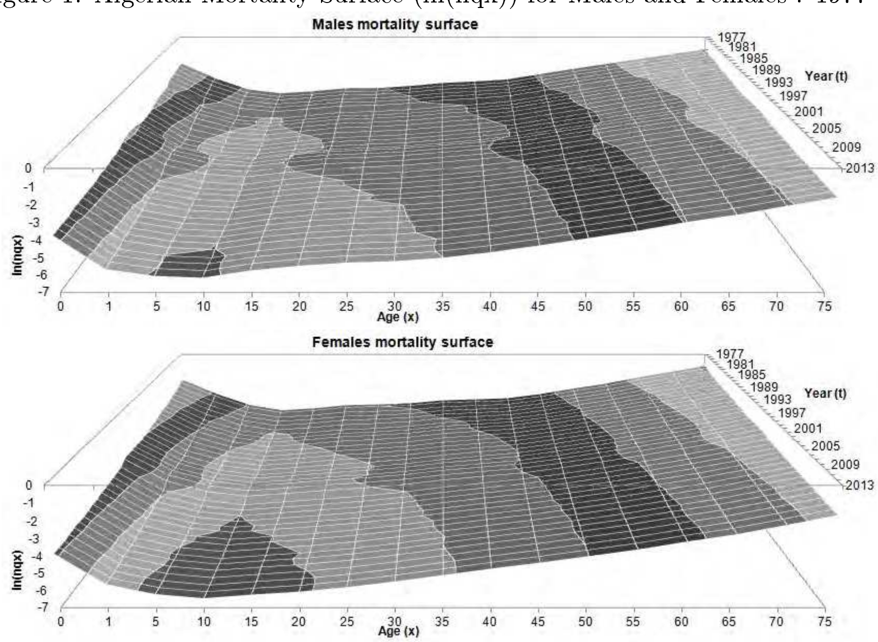
In first, we apply the four model on the mortality surface (1977 - 2010). we will project the results for the period 2011-2012-2013 and we compare : to select the best model.

3 Data

The first Algerian life-table based on the civil registration data has been constructed in 1977 by The Algerian office for National Statistics (ONS). In the beginning, it was difficult to insure an annual publication of such a statistics. It become possible since 1998. So, there are some calendar years missing life-tables for the period 1977 - 1997. The missing years were: 1979, 1984, 1986, 1988, 1990, 1992 and 1997. Also, Some life tables were closed-out before the open age group [80 and +[. For the period 1983-1987, the closing age was [70 and +[. for the period 1993-1996, the published life-tables were closed-out at the age group [75 and +[. For the rest, it was [80 and +[or higher.

Mortality forecasting needs continuous and complete series of historical mortality data expressed as a mortality surface. Since that, it was necessary to estimate the missing data in the Algerian mortality surface. In a previous work (Flici, 2014-b), we proposed to do this by using a modified Lee Carter model. Since some changes are observed in the mortality trend between 1977-1998, we segmented the global surface on three sub-surfaces. To improve again the quality of the fitting, we applied LC model by segmented age. The missing data was estimated by interpolating the estimated mortality trend index for the missing years. the completed mortality surface obtained with this proceedings is shown in the following figure:

Figure 1: Algerian Mortality Surface (ln(nqx)) for Males and Females : 1977-2013



Source : Life tables published by the ONS. The missing data was estimated by FLICI (2014-b)

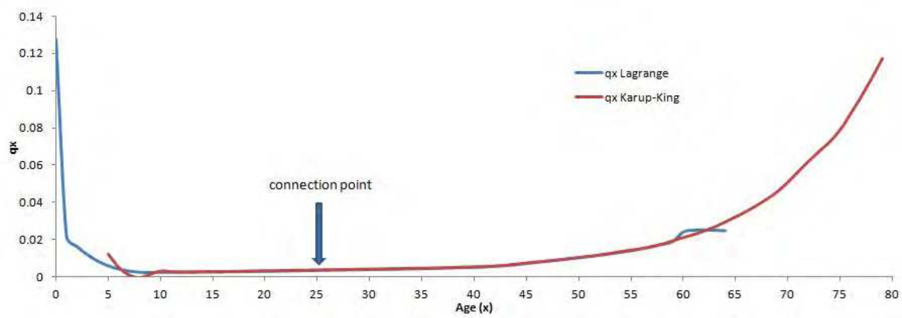
To consider the cohort effect in mortality forecasting, the age and time description of the mortality rates must be compatible. Until now, we have a matrix (X=17,T=36). X which represents the age length is defined as following: 0-1, 1-5, and after by 5-age groups until 80

years. For the time dimension (t), the data is presented by one-year description from 1977 to 2013. To insure the same description for the data, we should make the life tables in one-years description. So, the five years mortality rates denoted ($_5q_x$) must be broken out to single-age mortality rate (q_x). we note that in some works, the data was arranged by 5-years range for age and time (Richards and al., 2006).

4 Single-ages mortality surface

In the literature of interpolation methods, Several methods were proposed to interpolate single-age values (Exposure, Observed deaths, Deaths rates) from five-age description. (king, 1908; Sprague, 1880; Beers, 1944 and 1945). Generally, some complications can be found because of the structure of the abridged life-table. The length of the age groups is 5 years except the two first groups [0-1[and [1-5[. Some interpolation methods were proposed to deal with unequal age-intervals principally based on the Karup-King and Lagrange interpolation methods (Crofts, 1998; Srivastava and Srivastava, 2012). For the present work, we preferred to use both Lagrangian and Karup-King methods (Shrock and al., 1993). The first method is more adequate with the lower age groups while the second is more adapted with the higher age groups. The connection point is defined in the way to minimize the gap between the two curves around this points.

Figure 2: Single age mortality rates interpolated by junction of Lagrange and Karup-King formulas



Data: Algerian Males life table - 1977

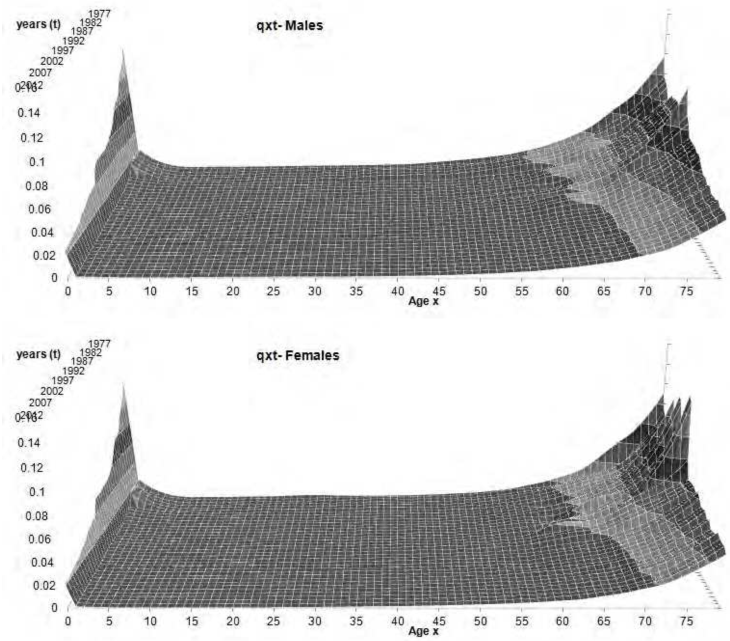
As shown in Figure 2, the connection point is 25 years. if we consider $q_x^*$the mortality rate at age (x) obtained with the Karup-king Method and $q_x^+$is the rate obtained with Lagrange for the same age (x). The connection point (k) is defined in the order to minimize the Mean Absolute Errors (MAE) between the two curves around this point. This implies :

$$MinA = \frac{1}{5}\sum_{x=k-2}^{k+2} \left\| q_k^* - q_k^+ \right\|, ...k = 20 - 50,$$

The connection point is different by period and sex.

The obtained single age mortality surface for males and females is shown by figure 3:

Figure 3: Single ages mortality surface for males and females qxt (x=0-80, t=1977-2013)



5 Best model selection based on the data 1977 - 2010

To choose the best model to use for mortality forecasting, we apply the methods presented above on the data of the period 1977 - 2011 for males and females. The best model should gave the best forecasting quality if the obtained results are compared to the observed data of the years 2011, 2012 and 2013.

In the way to improve the fitting quality for all the models, we propose some modifications related to the estimation process:

In the original LC model, the alpha parameter $\alpha_x$is defined to be the mean over time of the ln of the central death rate. Some authors accepted that this relation can be partially respected. In the way to improve the quality of the model, Renshaw and Haberman (2006) used the original values of $\alpha_x$as a starting values which was re-estimated by the same optimization process with all the parameters in RH model. To insure the same fitting quality for all models, we will use the same process by using XL-Solver.

5.1 Models implementation

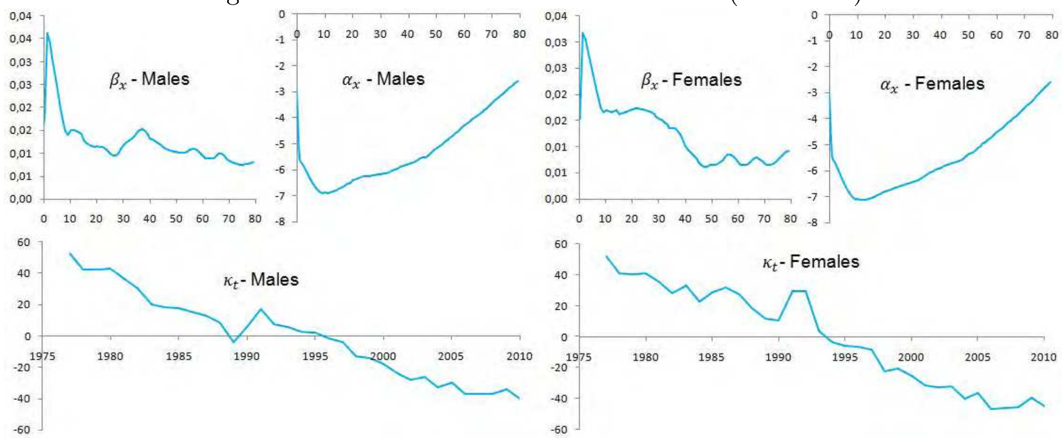
5.1.1 Lee-Carter Model (LC)

In first, we estimate the parameters by the classical way. $\alpha_x = ln(\prod_{t=T_1}^{T_n}(\mu_{xt}))^{\frac{1}{(T_n-T_1)}}$. and then we decompose the residual matrix by Weighed Least Squared Errors Method. :

$$MinA = \sum D_{xt}[ln(q_{xt}) - \alpha_x - \hat{\beta}_x \hat{\kappa}_t]^2$$

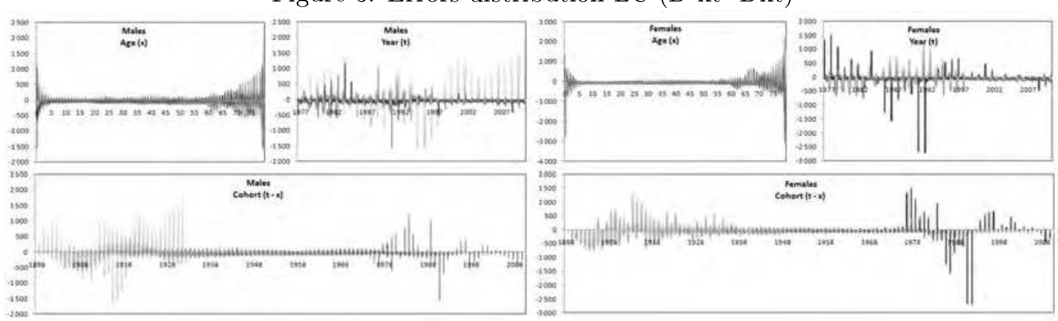
with as a constraints: $\sum_{x=X_1}^{X_n} \beta_x = 1$, $\sum_{t=T_1}^{T_p} \kappa_t = 0$ and $\beta_x \geqslant 1$. To improve the fitting quality, we re-estimate $\alpha_x$by including it solving the optimization problem. The obtained result are shown in the figure below (LC).

Figure 4: LC model - Parameters estimation (1977-2010)



Considering the difference in age mortality patterns by sex, the bump around 20th is more apparent for males compared with females whoes marks a small bump between [28 - 45] which is respectively related to the motor car and maternal mortality. Starting from the age of 45, the two curves follow a linear trend. For the mortality trend indexes, the two populations marks a high mortality level by the beginning of the black decade (1990th). in terms of age sensitivity of mortality reduction index, some differences are observed by sex in the age range [10 - 35]. At that age, females are more sensitive to the mortality reduction compared with males.

Figure 5: Errors distribution LC (D"xt -Dxt)

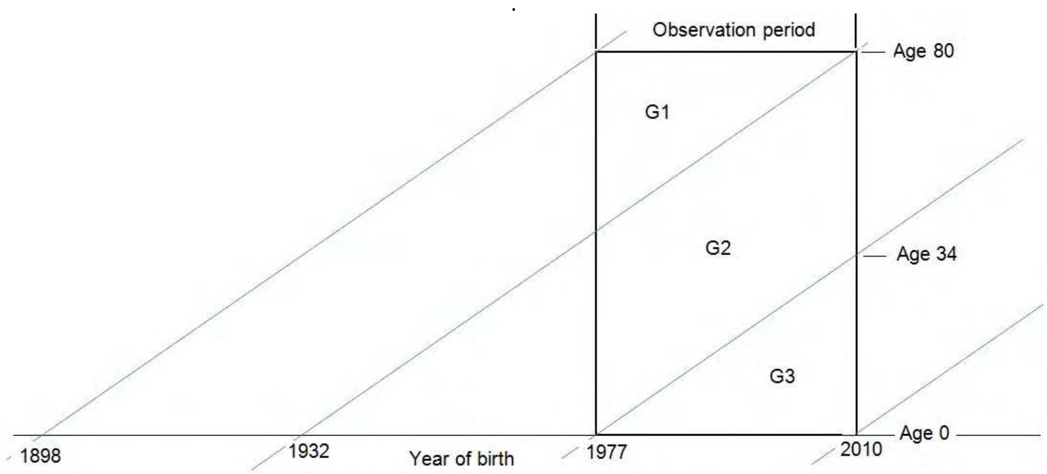


The errors distribution expressed by the difference between the expected and the observed number of deaths at every square x and t allows to get more clear idea about the quality of fitting. This difference noted $e_{xt} = D_{xt} - l_{xt} * exp(\hat{\alpha_x} + \hat{\beta_x} * \hat{\kappa_t})$

We remind that in the present application, the LC model was fitted on the mortality rate q_{xt}which represents the observed number of deaths at age x and time t D_{xt}divided by the survival number at the exact age x at time t l_{xt}. This latest replaces the exposure to death in the formula above.

To better shows the errors distribution, we crossed $e_{xt}$with age, time and year of birth. this last will serves as a comparison when cohort effect is considered. According to age, we observe that the error is more important at the extreme age groups where an important number of deaths is observed. The use of the relative errors in deaths allows to eliminate this imperfection in errors distribution. the time distribution is more stable. The errors distribution by years of birth shows an important irregularities by the extremes of the series: before 1932 and after 1976 for the two sexes. This is purely statistical effect related to the age range considered for each cohort. with a series of 34 annual life tables (1977 - 2010), we can not observe the trajectory until complete fall of any generation. The maximum number of observations by generation can't exceed 34. The following scheme shows the observation period compared to the years of birth by a simplified Lexis Diagram:

Figure 6: Observation period compared to the years of birth

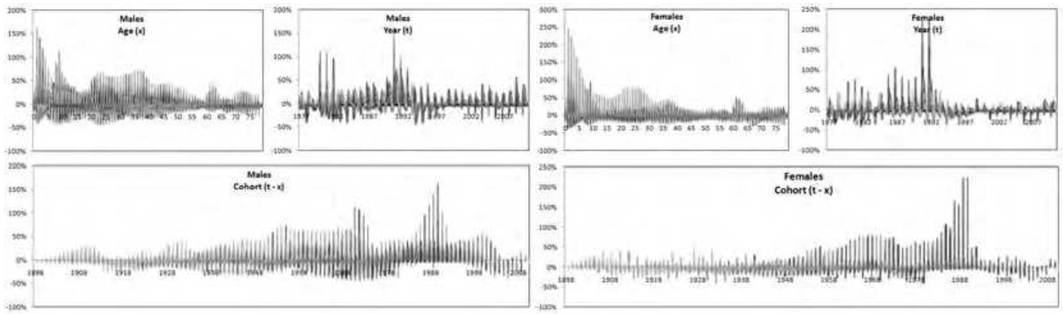


As is shown in the figure above, for the considered observation period (1977-2010) and age range (0-80) the years of birth go from 1898 to 2010. According to the year of birth and the observation period, we can distinguish three groups of large generations (G1, G2, G3). The medium group G2 has the maximum number of observation (34) corresponding to the age range [46-80] for the 1932's cohort and to the age range [0-34] for the 1977's cohort. This group is not significantly affected by the effect of big number of birth at the extreme ages (0, 1 and 79). The first group G1, has a croissant number of observation going from 1 for the cohort 1898 to 33 for the cohort 1931. The weight of the age group 79 is decreasing with time (t). The adverse is observed for the group G3 which has decreasing weight of the deaths observed at lower ages (0 and 1). For this that we obtained such a cohort-errors distribution shown above.

The idea is to do the same errors representation in relative terms. $e_{xt}(\%) = \frac{l_{xt}*exp(\hat{\alpha_x}+\hat{\beta_x}*\hat{\kappa_t})-D_{xt}}{D_{xt}}$. The obtained results are shown in the figure below:

The relative errors between observed and expected deaths is very important. The absolute relative errors is 14% for males and 12% for females. This errors is greatest at the lower ages than at higher ones especially for females. for males, we observe a reduction starting from the age of 55. for females, it's starting from 45 years that errors are relatively reduced. in terms of

Figure 7: Relative Errors distribution ($\hat{D}_{xt}$-D_{xt})/D_{xt}



time, errors are stables and we observe some bumps around 1980 and 1990 for the two sexes. by cohort, the errors are biggest for the cohorts born between 1950 and 1970.

5.1.2 Renshaw and Haberman (APC model)

We use the parameters estimated in LC model as a starting values to estimate the APC model. In first, we will try to reduce errors by estimating the additional component related to cohort effect $\beta_x^{(2)} * \gamma_{t-x}$. in a second stage, we re-estimate all the parameters by the same solving process in XL-solver including α_x. The model to estimate is:

$$ln(q_{xt}) = \alpha_x + \beta_x^{(1)} * \kappa_t + \beta_x^{(2)} * \gamma_{t-x} + \xi_{xt}$$

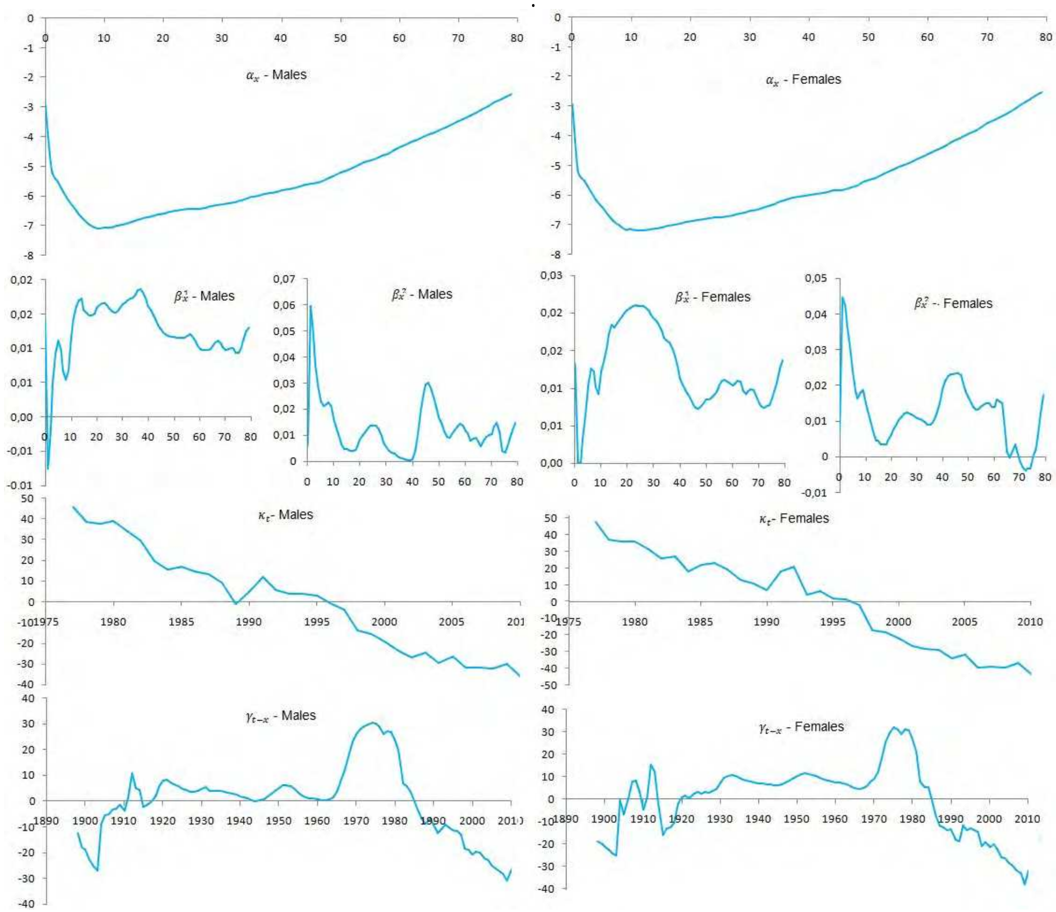
with the constraints: $\sum_{x=X_1}^{X_n} \beta_x^{(1)} = \sum_{x=X_1}^{X_n} \beta_x^{(2)} = 1$**,** $\sum_{t-x=T_1-X_n}^{T_p-X_n} \gamma_{t-x} = 0$ **and** $\sum_{t=T_1}^{T_p} \kappa_t = 0$**.**

The optimization problem aims to minimize the weighted least squared errors:

$$MinA = \sum D_{xt}[ln(q_{xt}) - \alpha_x - \hat{\beta_x^{(1)}} \hat{\kappa_t} - \hat{\beta_x^{(2)}} \gamma_{t-x}]^2$$

The obtained results are shown in the figure bellow.

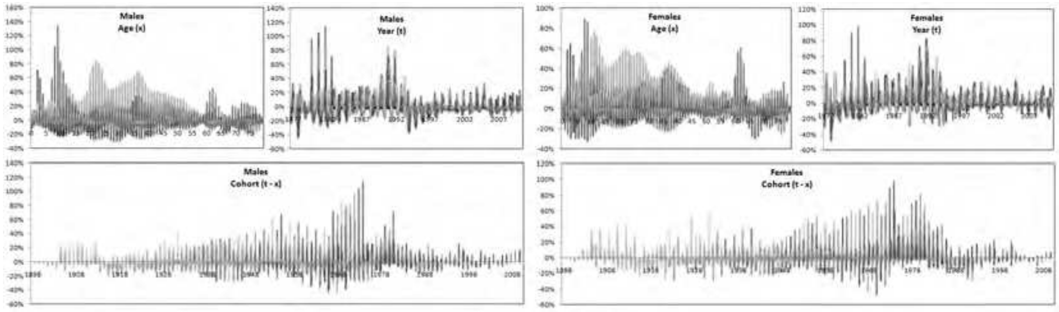
Figure 8: RH model estimation parameters (1977-2010)



The estimated parameters have undergone some changes. The parameters $\beta_x^{(1)}$marked some apparent change at the lower ages compared with the results obtained with LC model. This modification is related to the introducing of the cohort effect. This parameters has a negative sign at the age 1 for the two sexes. That means that the mortality at this age decrease less than the global trend in mortality reduction. The change in the mortality trend index is fewer. The $\beta_x^{(2)}$follows generally an adverse trend compared with $\beta_x^{(1)}$. The cohort effect marks a stable trend between 1920 to 1975 followed by an increase until 1977. After, it decreases until 2010. This bump can be related to the events of the black decade during the 1990th. The born whose were born between 1968 and 1977 have had 18 and 25 years in 1995. This age category was the most concerned by deaths during the black decade.

The introducing of the cohort effect may to reduce errors.

Figure 9: Relative errors distribution (RH - 1977-2010)



The Average relative error in $q_{xt}$is 10,7% for males varying between 0,01% to 139 %. For females, the average is 10,7% with 0,01% and 103 % as minimum and maximum. According to the age distribution of errors, it's only beyond the age of 55 that errors start decreasing . The time distribution of errors shows that starting from 1998, errors become relatively stable. The same is observed for the cohort distribution errors starting from 1988.

5.1.3 Simpler APC model (APC*)

As we said above the model proposed by Currie (2006) is a special case of the initial version of APC model, where the parameters $\beta^{(1)}$and $\beta^{(2)}$are supposed to be constant over age and equal to $\frac{1}{n}$. The advantage of thise process is to reduce the number of parameters to estimate. Compared to the APC model where the total number to estimate is equal to n+p+(p+n-1)+2n=2p+4n-1, the simpler APC model allows to reduce this number by 2n. in our case, we pass from 487 to 327 parameters to estimate.

To estimate the parameters of this model, we use the results obtained with APC model as a starting values but with imposing $\beta_x^{(1)} = \beta_x^{(2)} = \frac{1}{n}$. we re-estimate all the other parameters by the same solving process including α_x. The function to estimate is:

$$ln(q_{xt}) = \alpha_x + \frac{1}{n} * \hat{\kappa}_t + \frac{1}{n} * \hat{\gamma}_{t-x}$$

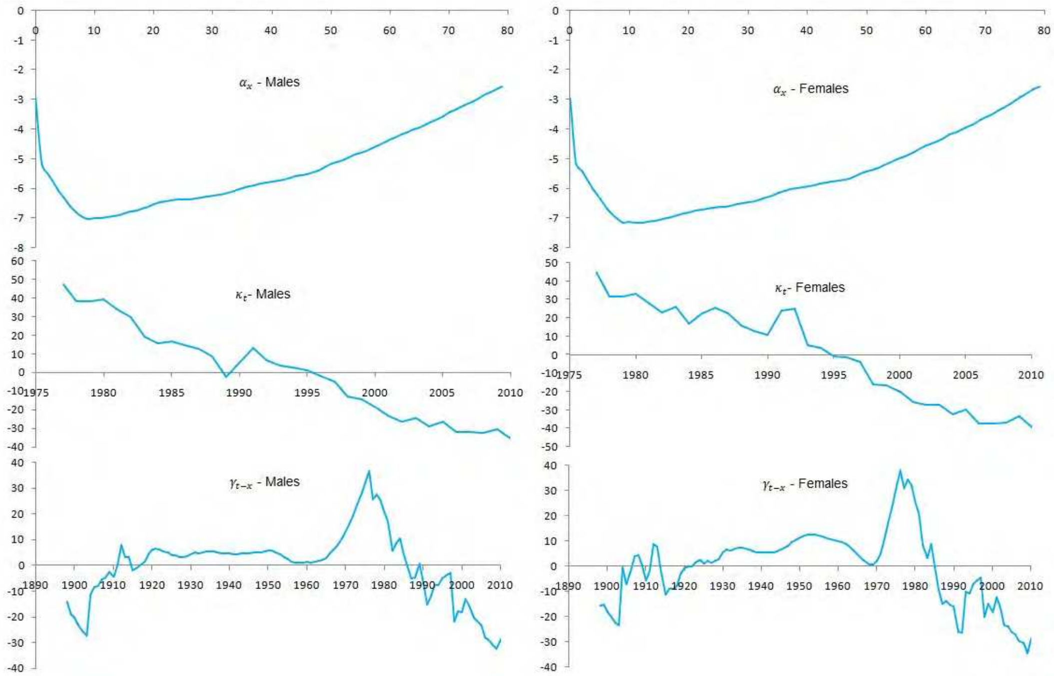
The quantity to minimize is:

$$MinA = \sum D_{xt}[ln(q_{xt}) - \alpha_x - \frac{1}{n}\hat{\kappa_t} - \frac{1}{n}\hat{\gamma}_{t-x}]^2$$

with the constraints: $\sum_{t-x=T_1-X_n}^{T_p-X_n} \gamma_{t-x} = 0$ and $\sum_{t=T_1}^{T_p} \kappa_t = 0$.

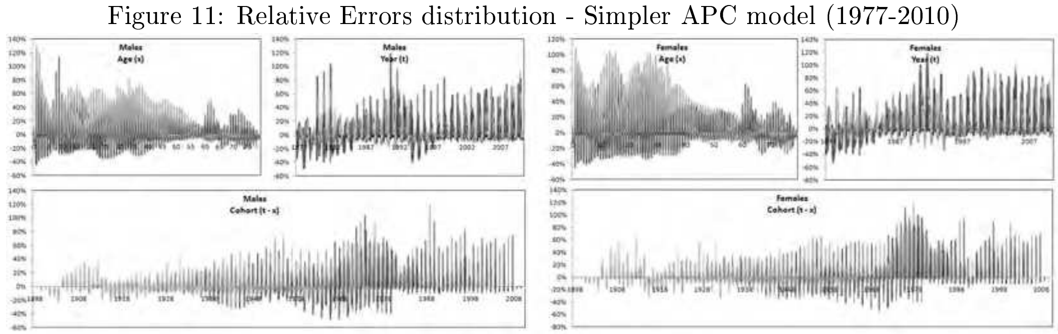
The obtained readjusted parameters are shown in the following figure.

Figure 10: Simpler APC model - Parameters estimation results (1977 - 2010)



Supposing the constancy of the age sensitivity to the time and cohort effect has to decrease the quality of fitting. But the re-estimation of the obtained parameters is done in order to recover a part of the lost information.

Figure 11: Relative Errors distribution - Simpler APC model (1977-2010)



The Average relative error in $q_{xt}$is 17% for males varying between 0% to 130 %. For females, the average is 20% with 0 and 120 % as minimum and maximum. According to the age distribution of errors, it's only beyond the age of 55 that errors start decreasing . The time distribution of errors shows that starting from 1998, errors have almost positive sign. that implies that the estimated mortality trend index has trend to upper estimate the real trend of mortality decreasing. The same is observed for the cohort distribution, where the errors become positive starting from 1980.

5.2 Comparison of the fitting quality of the three model

5.3 Mortality time index forecasting

In the literature of the mortality forecasting, many techniques have been used and specified to project the mortality time index in the future. In the original Lee-Carter application, a random walk with drift model was used (ARIMA (0,1,0)).

$$k_t - k_{t-1} = d_1 + \delta_t$$

With the drift d_1 represents the mean annual change in κ_t, and $\delta_t$represents the errors term. This model was also used in several works (Dowd et al., 2011; Zhou et al., 2013; Cairns et al., 2011).

In other works , an first order auto-regressive model ARIMA (1, 0, 0) model was be used:

$$k_t = d_1 + d_2 k_{t-1} + \delta_t$$

the coefficient $d_2$represents the slope of decreasing of k_t.

The use of such a models is based on the stationnarity over time of the series k_t. If this series is not stationary itself, it can be it by differentiation. The first difference (or more) can be foretasted using ARIMA (1,1, 0) model.

$$k_t - k_{t-1} = d_1 + d_2(k_{t-1} - k_{t-2}) + \delta_t$$

Yang and Wang (2013) used an ARIMA(2,1,0)

$$k_t - k_{t-1} = d_1 + d_2 k_{t-1} + d_3(k_{t-1} - k_{t-2}) + \delta_t$$

When a change in mortality trend index is observed, Structural change model can also be used (Nunes and Coehlo, 2009). In case of irregularity of the mortality trend, we use only the recent trend. Also, some projections take in account the projected life expectancy, so the $\kappa_t$is adjusted to fit the projected life expectancy (Li and Gerland, 2011).

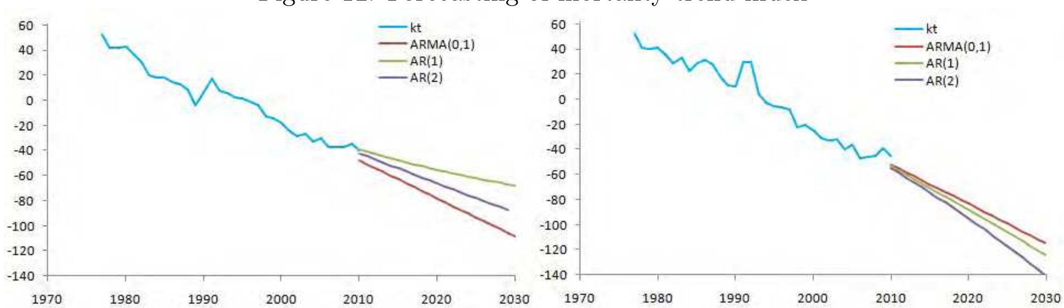
For our application we compare between three models: ARIMA(0,1,0), ARIMA(1,0,0) and ARIMA (2,0,0). The results of the estimation of the parameters of the three models are given by the following table:

Table 1: $k_t$forecasting models

			d_1	d_2	d_3	$\sum \delta_t^2$	p	$\frac{1}{p}\sum \delta_t^2$
ARMA(0,1)	$k_t = d_1 + k_{t-1} + \delta_t$	Males	-3,04			1695,5	33	51,4
		Females	-3,15			1920,7	33	58,2
AR(1)	$k_t = d_1 + d_2 k_{t-1} + \delta_t$	Males	-2,74	0,976		1096,7	33	33,2
		Females	-3,21	1,004		1900,1	33	57,6
AR(2)	$k_t = d_1 + d_2 k_{t-1} + d_3 k_{t-2} + \delta_t$	Males	-1,93	1,247	-0,249	917,8	32	28,7
		Females	-5,59	0,214	0,809	1850,0	32	57,8

AR(2) give the lowest normalized squared errors for male's series compared with the other models. For females, the same model give approximatively the same result as AR (1). To ensure some coherence in terms of sex differential mortality forecasting, we use AR(2) to forecast mortality time index for the two populations. The obtained results are shown in the figure below:

Figure 12: Forecasting of mortality trend index



All the time series forecasting models lead to an increasing gap between males and females, which have to increase in long terms projection the difference in life expectancy between the two sexes. The small difference in the trends of $k_t$observed along the observation period can explain such a divergence. The two series of $k_t$have marked in summary identical structural change points: 1992 which coincide with the beginning of the terrorism events in Algeria, and 1998 which coincide with the end of the black decade and also which the change in the methodology of constructing life tables. Theses structural changes points was studied in a previous work (Flici et Hammouda, 2014). Considering the recent break point in mortality reduction trend, The use of the time range (1998 - 2010) may lead to more consistent results.

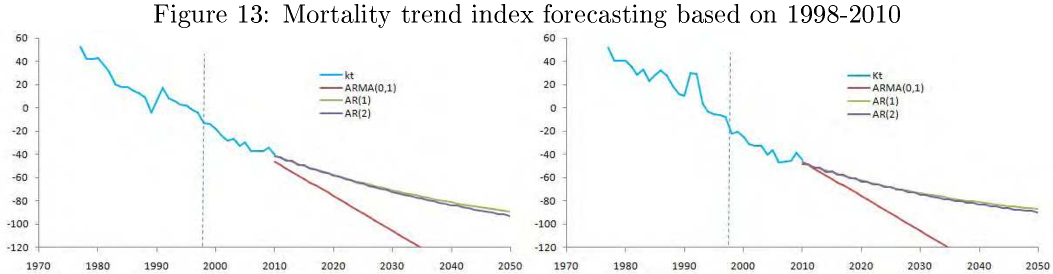
To do this we re-estimate the three models presented above to forecast mortality trend for males and females. The obtained results and the estimated parameters of these models are shown in the table and the table below:

Table 2: Mortality trend index forecasting (1998 - 2010)

			d_1	d_2	d_3	$\sum \delta_t^2$	p	$\frac{1}{p}\sum \delta_t^2$
ARMA(0,1)	$k_t = d_1 + k_{t-1} + \delta_t$	Males	-2,98			225,2	13	17,3
		Females	-3,23			408,7	13	31,4
AR(1)	$k_t = d_1 + d_2 k_{t-1} + \delta_t$	Males	-2,80	0,98		93,7	13	7,2
		Females	-3,16	0,969		160,7	13	12,4
AR(2)	$k_t = d_1 + d_2 k_{t-1} + d_3 k_{t-2} + \delta_t$	Males	-5,33	0,022	0,937	73,7	12	6,1
		Females	-6,00	0,010	0,935	141,7	12	11,8

The sum of the squared errors is very important in model (1) compared with models (2) and (3), and more important for females than for males. The difference between model (2) and model (3) is fewer. This difference may lead to some differences in tong term forecasting.

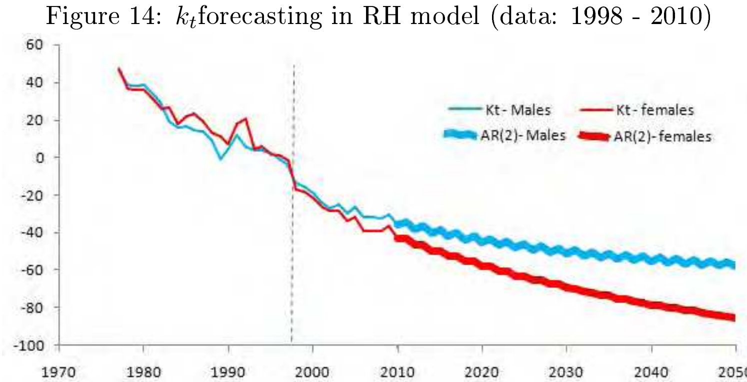
Figure 13: Mortality trend index forecasting based on 1998-2010



The gap between the results obtained with model (1) and the models (2-3) is important. Model (2) and (3) lead to closer forecasted series with a very slow divergence at the long term. For all the forecasting Period, model (3) leads to $k_t^{females} < k_t^{males}$. For some years, model (2) leads to the adverse. That allows to confirm that AR(2) has a better forecasting quality compared with the two other models. this model respects some constraints in long term mortality evolution between males and females.

The mortality trend index in RH or Simpler APC has some difference compared with the trend in LC model. these trend must be also be forcasted using the sames model AR(2) estimated on the period 1998-2010.

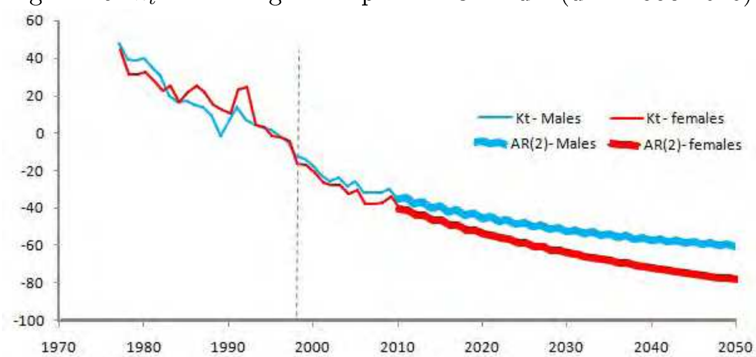
Figure 14: $k_t$forecasting in RH model (data: 1998 - 2010)



We observe some divergence over time of the two forecasted series of $k_t$between males and females and may lead to an important difference in life expectancy in long term forecast. If we observe the historical reduction of the mortality trend index for males and females, a crossover point can be detected during the mid 90th. The mortality reduction slope is more important for the females population.

By the same way, the mortality index obtained by the simpler APC model is also forecasted using a second order Auto-regressive model AR(2). The obtained results are shown in the figure below:

Figure 15: $k_t$forecasting in simpler APC model (data:1998-2010)



Compared with the forecast in the previous case, here the divergence is relatively less importante, but the two forecasted series follow globally the same evolution trend.

5.4 Cohort effect forecasting

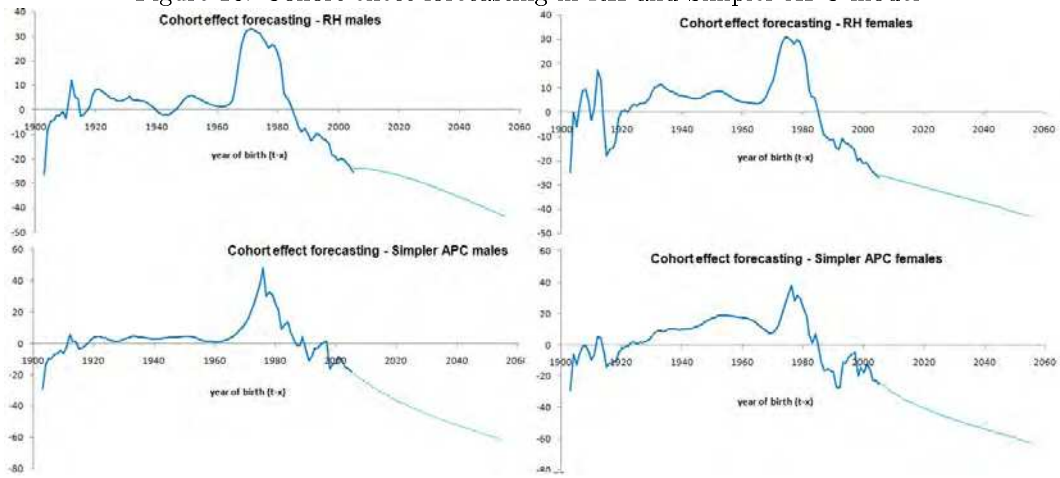
The age specific mortality forecasting need to forecast all the parameters related to time (mortality index and cohort effect). The cohort effect, as the mortality index, must be forecasted using the time series forecasting techniques with respecting some specifities. In first and to insure the homogeneity of the serie γ_{t-x}based on the same number of observation. The other observation corresponding to the beginning and the end of the series are neglected. The cohort to include in the forecasting process are 1930 - 1977. For these cohort, 34 successive mortality rate were considered. the other cohorts (before 1930 and after 1977) have less than 34 observation.

To forecast the cohort effect, Chan et al.(2014) used a first order Auto-regressive model ARIMA(1,0,0). If we make $s = t - x$ as a cohort index. The cohort effect corresponding to the s^{th}cohort can be forecasted by: $\gamma_s = c_0 + c_1\gamma_{s-1} + \delta_s$.

Dowd et al. (2011) used a ARIMA (1,1,0) : $\gamma_s - \gamma_{s-1} = c_0 + c_1(\gamma_{s-1} - \gamma_{s-2}) + \delta_s$. This leads to: $\gamma_s = c_0 + \gamma_{s-1} + c_1(\gamma_{s-1} - \gamma_{s-2}) + \delta_s$.

Cairns et al. (2011) used a second order auto-regressive model AR(2) which is same as ARIMA(2,0,0): $\gamma_s = c_0 + c_1\gamma_{s-1} + c_2\gamma_{s-2} + \delta_s$.

Figure 16: Cohort effect forecasting in RH and Simpler APC model



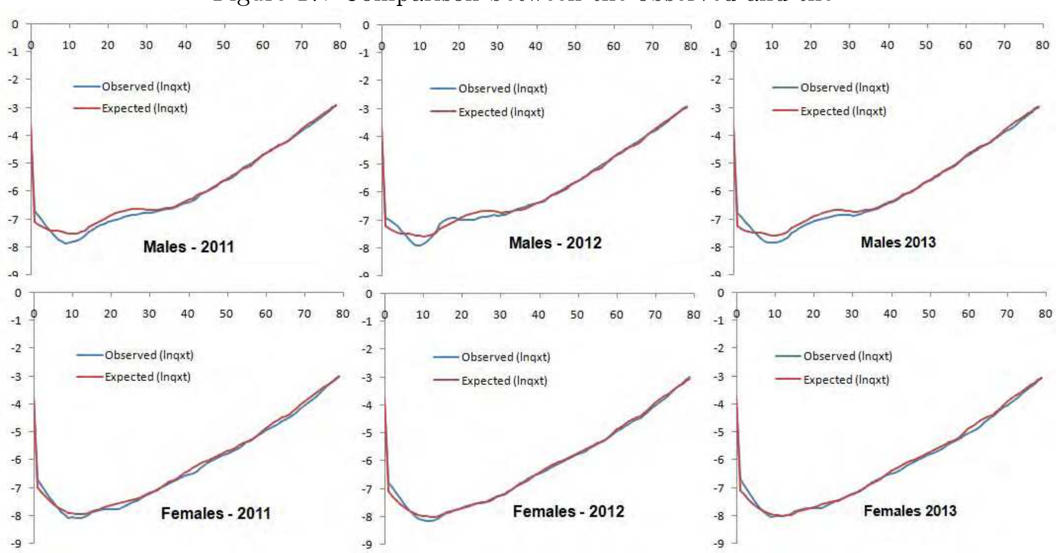
Here, we compare these three model on the series of γ_{t-x} obtained with RH model and the simpler APC model.

The best model to forecast the bcohort effect in our application is the model ARIMA (1, 1, 0).

5.5 Comparison of the forecasting capacity of the three model

The forecasting capacity of a model is not necessary related to its fitting quality. a good fitting quality doesn't garantee a good forecasting capacity. for this, we use some recent data to verify in which estimation condition, each model give a better result.

Figure 17: Comparison between the observed and the



For males, errors are important at the lower ages because of the bump due to the motor deaths at younger ages, and starting from the age of 32, the expected and observed $ln(q_{xt})$are well fitted. The errors in females life tables are more regular.

Figure 18: comparison of the observed and expected ln qxt RH model (2011 - 2013)

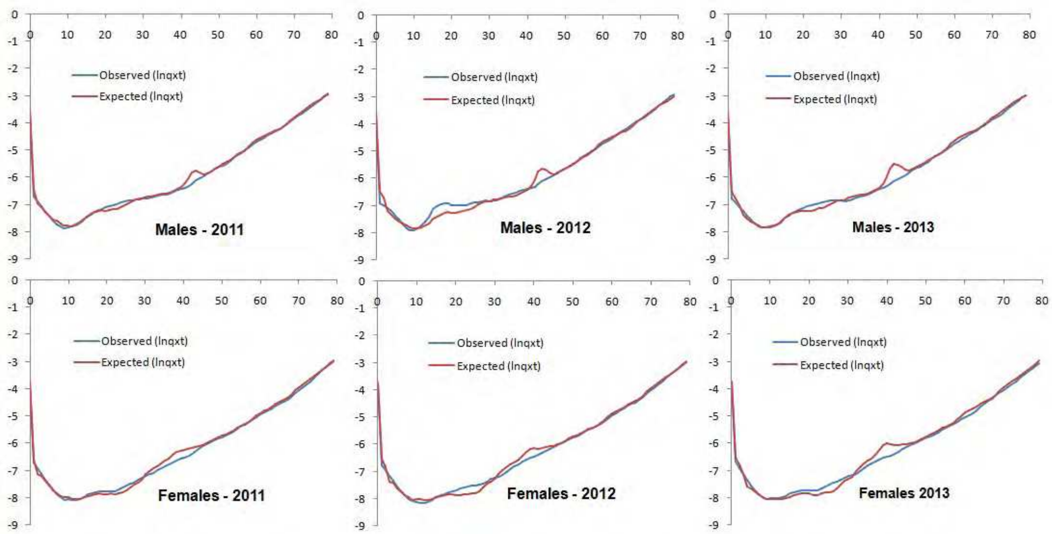
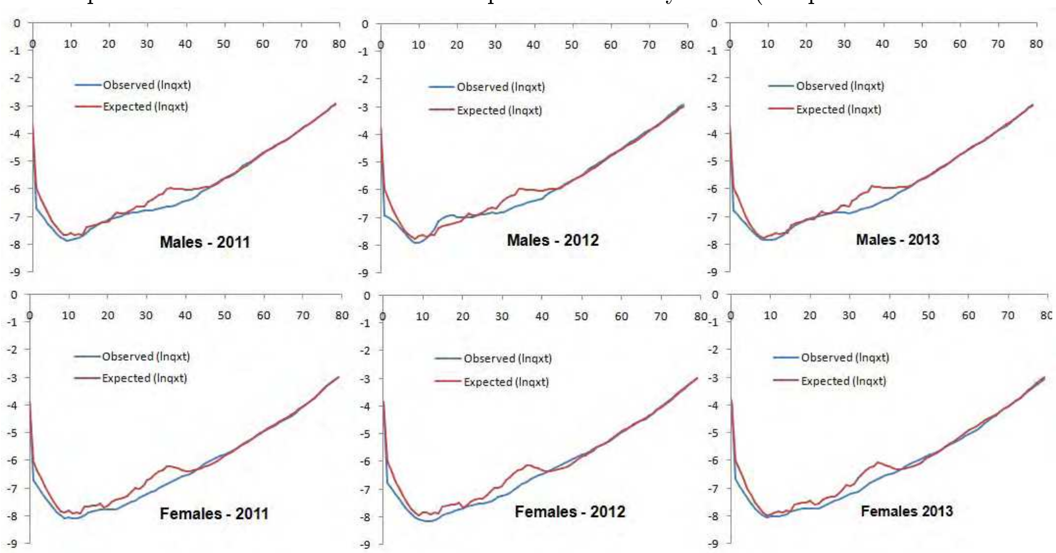
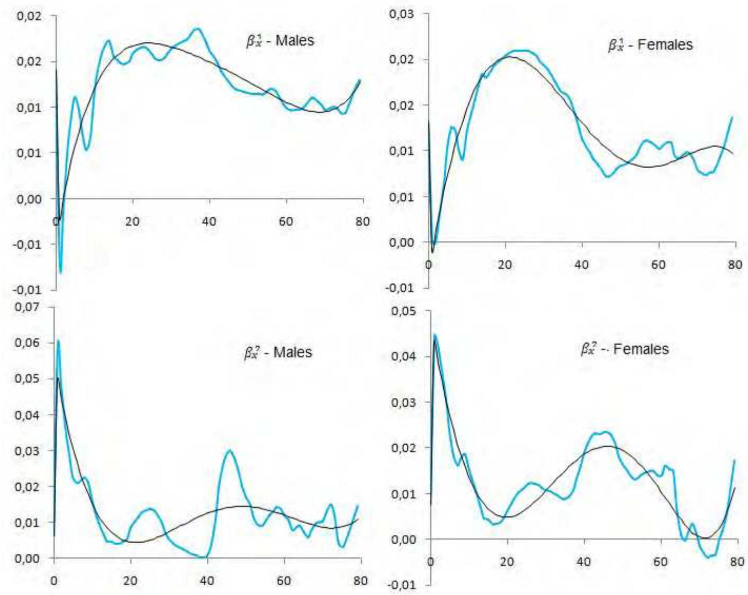


Figure 19: Comparison of the observed and the expected mortality rates (Simpler APC model : 2010-2013)



Both in RH and APC*, we observe that the curves of $lnq_{xt}$are less well fitted before the age of 45 compared with the results obtained with LC model. That can be due to the bump observed for cohorts that were born between 1966 and 1977. These cohorts were aged 18 - 30 during the black decade. this age category is supposed to be the most exposed ages to the terrorism events during the black decade. as in LC model were the bump related to the terrorism events

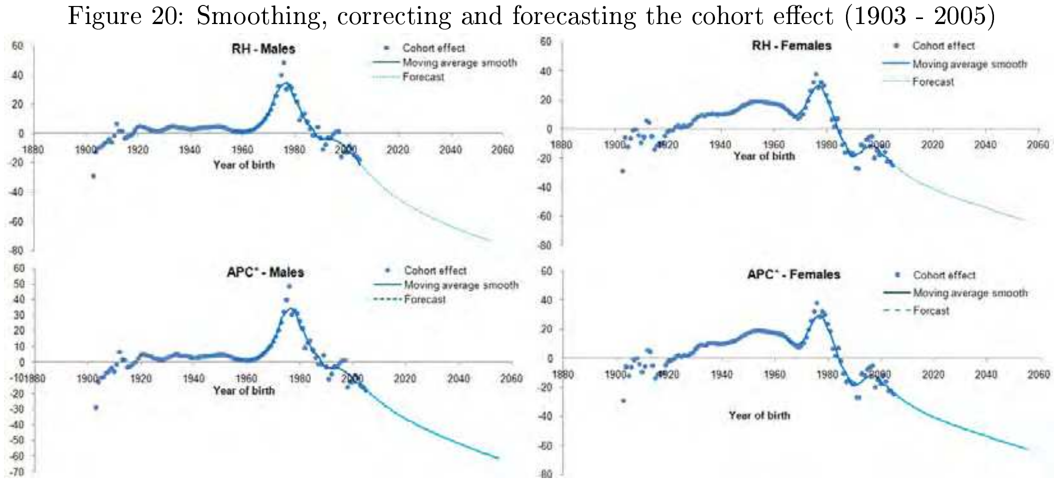
Figure 21: Smoothing of $B_x^{(1)}$and $B_x^{(2)}$



was excluded in the forecasting process, the same correction should be done with the cohort effect. Also, since the observed cohort effect during the observed period will be reported in the future life for each cohort, the effect cohort must be corrected and smoothed to get more regular trend.

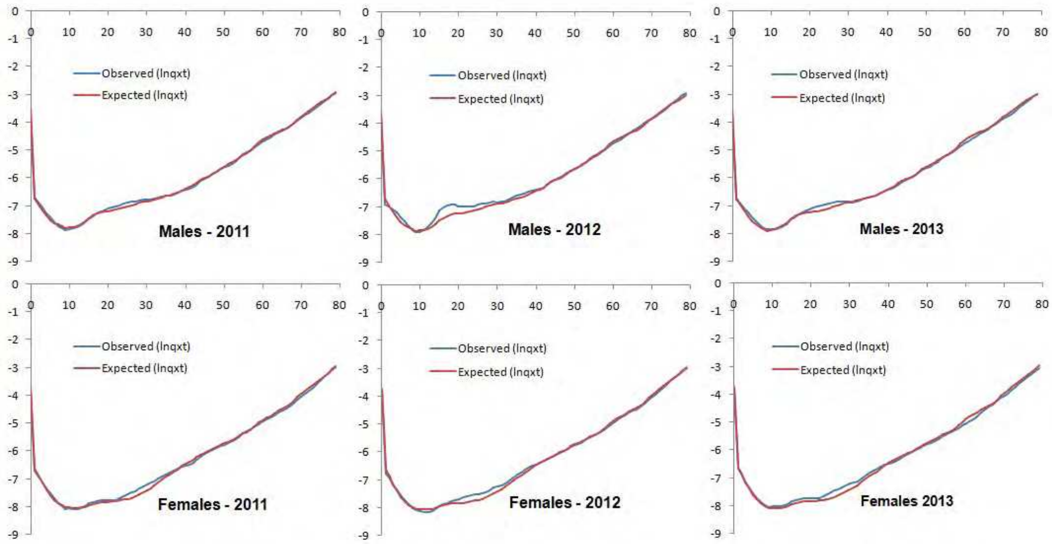
Our idea is to eliminate the bump in γ_{t-x}series for the period 1966 - 1981, and to smooth all the series with an 4th order polynome in the order to give some regularity for the the historical series of the cohort effect and also for the annual expected life table obtained under the RH and APC* models. In the following figures, we shows the results of the smoothing and correcting process applied to the models RH and APC* for the male and the female population. The forecast has been done with the same model used before:

Figure 20: Smoothing, correcting and forecasting the cohort effect (1903 - 2005)



To reduce the information lack due to the correcting process, and as suggested by some authors (Qiu and al., 2010), we readjust the age parameters for the two considered models.

Figure 22: Comparison of RH model after correcting cohort effect trend



We observe that when the bumps of 1963-1986 for males and 1967-1987 for females were corrected and the series of the following years (until 2005) were smoothed, we got more regular mortality curves expected with the RH model. For The two sexes, RH forecasting leads to a slight upper estimation of the death probabilities for the age range [18 - 30]. That can not be explained by a upper estimation of the time mortality reduction slope since it's not observed for all ages. It can be related to the age parameters. The errors are less than the original RH model with uncorrected cohort effect and less than in LC model.

Figure 23: Comparison APC* model with corrected cohort effect trend

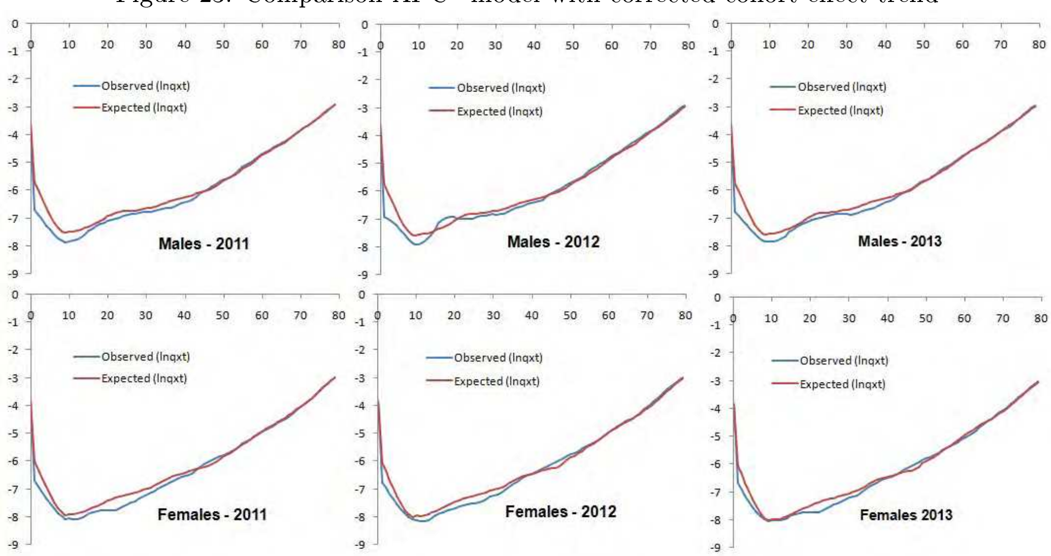


Figure 25: Comparaison of the predictive capacity of the three models - time error evolution

	LC		RH		RH corrected		APC*		APC* Corrected	
	Males	Females	Males	Females	Males	Females	Males	Females	Males	Females
2011	1,68%	1,67%	1,22%	1,53%	1,12%	0,82%	2,34%	2,70%	2,33%	2,15%
2012	1,48%	1,04%	1,89%	1,79%	1,10%	1,45%	2,72%	2,67%	2,45%	2,07%
2013	1,70%	1,43%	1,84%	2,51%	1,57%	1,01%	2,27%	2,69%	2,20%	1,79%

It's starting from the age of 45 for males and 55 for females that the expected and the observed mortality curves become well juxtaposed. for the two populations (males and females), the death probability for ages between 0 and 40 is under-estimated by the model. It can be explained by an imperfection in cohort effect trend correcting, since the age parameter was re-adjusted.

To better show the difference between the results of forecasting with the different models compared to the observed life tables for the years 2011, 2012 and to 2013, and to show the contribution of the correctiong of the cohort effect trend in the quality of the obtained result, we summarise in the following table some indicators. For this we use as a comparison indicator the Mean Percentage Absolute Deviation (MPAD) between the observed and the expected $ln(q_{xt})$.

Figure 24: Comparaison of the predictive capacity of the three models

	LC		RH		RH corrected		APC*		APC* Corrected	
	Males	Females	Males	Females	Males	Females	Males	Females	Males	Females
Global	1,62%	1,38%	1,65%	1,94%	1,09%	1,27%	2,44%	2,69%	2,33%	2,00%
0-14	3,13%	1,89%	1,38%	2,62%	1,19%	0,80%	3,49%	3,83%	5,99%	3,37%
15-39	1,95%	0,59%	1,52%	2,08%	1,43%	1,74%	3,83%	4,71%	2,27%	2,72%
40-79	0,85%	1,68%	1,83%	1,60%	0,85%	1,14%	1,19%	1,00%	0,99%	1,04%

All the models give their best fitting quality at the age range (40 - 80) because ofh the relative regularity of the mortality curve at that ages. Globally, the corrected RH model and the LC model have better predictive capacity compared with the APC* (corrected or not). The same mean errors is obtained with LC and corrected RH models for males at ages (40 - 80). For females, the best results for the same age range is obtained with the two APC* models. We can conclude that APC* model can be more adapted to the old population mortality forecasting compared with the global population forcasting.

The predective quality of the forecasting aims also a close look on the behaviour of the errors with the lenght of the projection. A comparaison for just three years is certainly not sufficient for such a evaluation but allows to detect some differences between the different used models for the short term. Here we compare the Mean Annual errors obtained with the different model for the years 2011, 2012 and 2013:

In term of the time evolution of the errors, we can observe that APC* model provides a relative stability of errors with time compared with the RH model however if the cohort trend was corrected or not. LC model provides too a similar quality. The errors for males are around 1, 68% in 2011 and 1,70% in 2013. for females, we passed from 1,67% to 1,43 in 2013.

6 Mortality forecasting based on the data (1977-2013)

We use the results obtained on the period (1977-2010) and those expected for the years from 2011 to 2013 as a starting value to estimate the new parameters for the period (1977 - 2013). and we use the same methodology used before. In addition, we will not extend mortality to the old age, we will use simply the temporary life expectancy (Arriaga, 1984) to compare results. In final, we would like to compare the two forecasting with and without cohort effect in terms of life expectancy evolution, before we will try to compare the impact in some life insurance reserving cases.

The mortality time index is forecasting by using a second order Auto Regressive model for the three model. The cohort effect is forecasted by using an ARIMA (1, 1, 0) for the RH and APC models. This cohort effect is smoothed by 5th odrer Moving average. The age parameters in RH models are smoothed using a 6th order polynomial function, while keeping the nitial hypothesis. The results are presented bellow for each model.

Age-Period approach (Lee-Carter)

Figure 26: Parameters estimation LC model (data: 1977-2013)

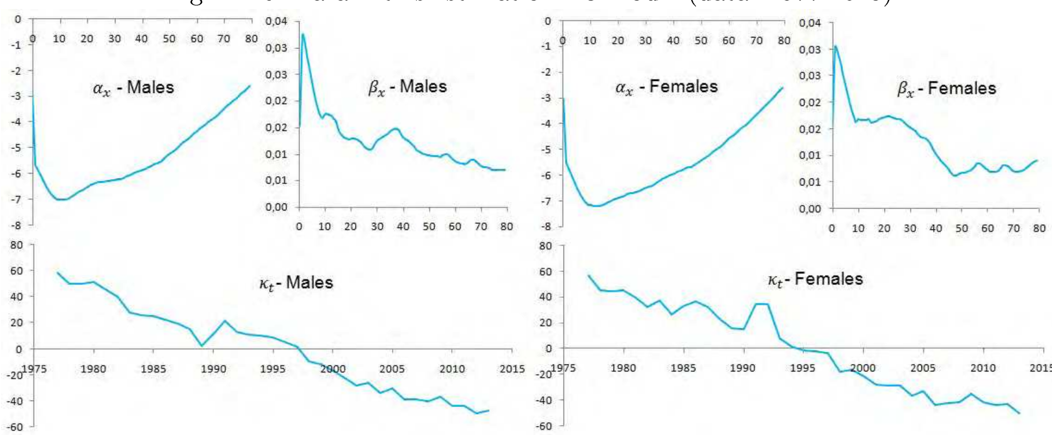
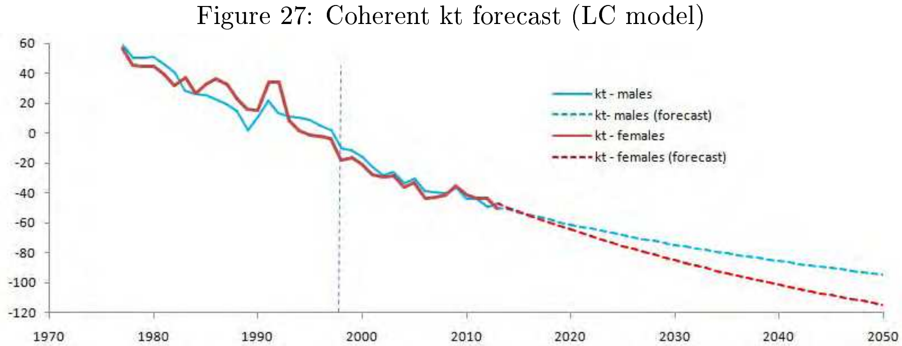


Figure 27: Coherent kt forecast (LC model)



Age-Period-Cohort approach (Renshaw-Haberman)

Figure 28: estimated and smoothed age related parameters (RH : 1977-2013)

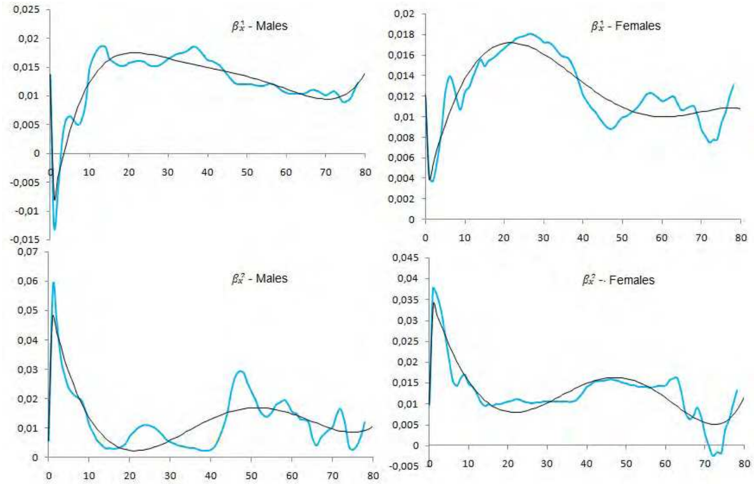
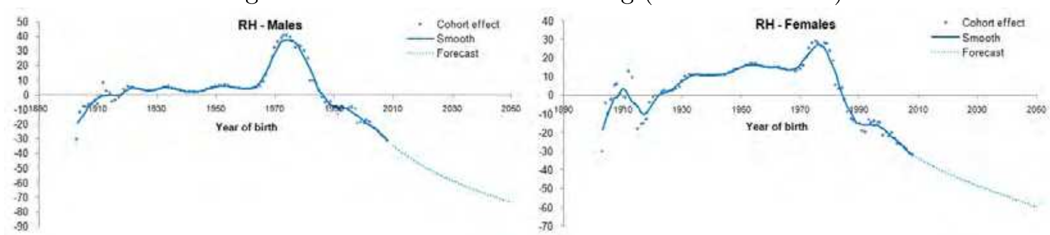


Figure 29: Cohort effect forecasting (RH - 1977-2013)



Age-Period-Cohort approach (Simpler APC)

Figure 30: mortality time index forecast (APC * - 1977 - 2013)

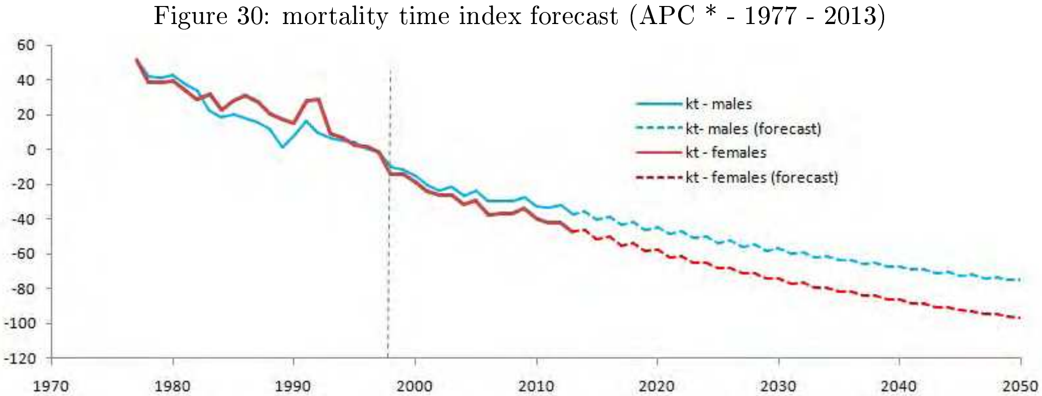
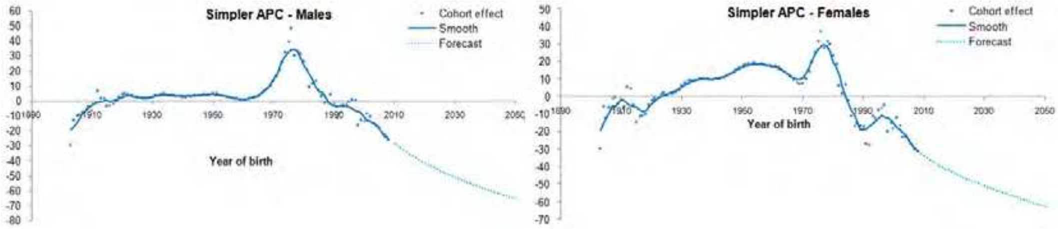


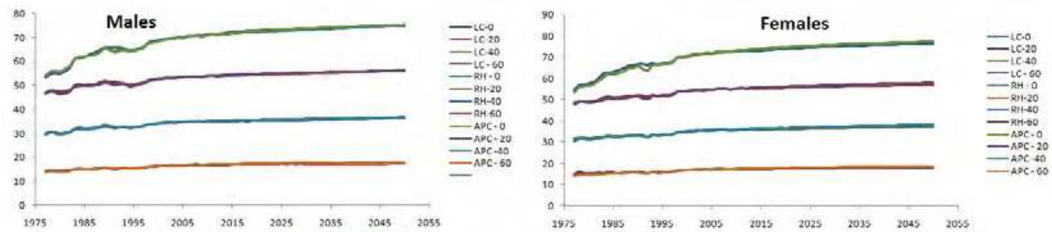
Figure 31: Cohort effect forecast with simpler APC model (1903- 2008)



6.1 Comparaison

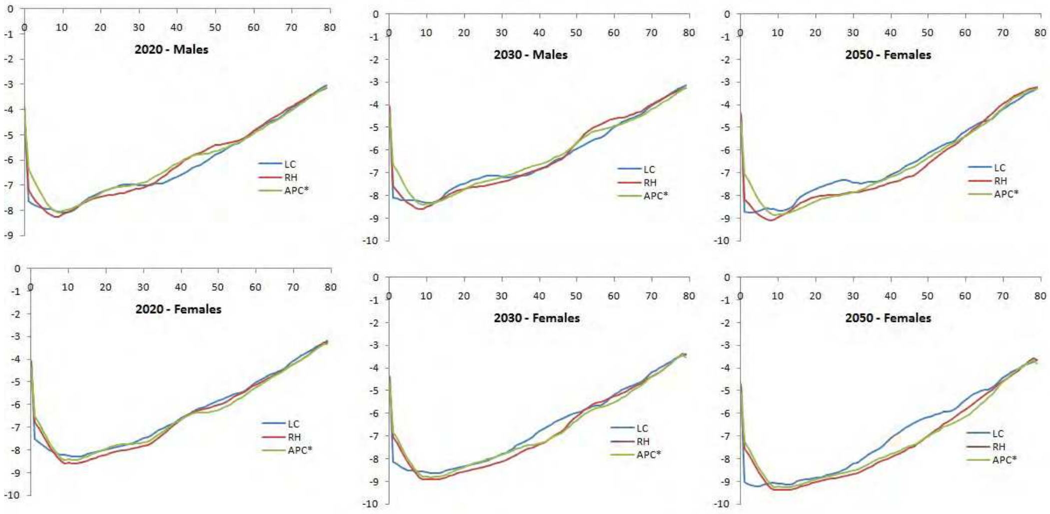
To compare between the results obtained with the three models, we use the temporary life expectancy (see. Arriaga, 1984) until age of 80 as a basis for comparaison. Here, we compare the temporary life expectancy at different age (0, 20, 40, and 60) obtained with the three models. Globally, the three model lead to the very near values over time.In 2050, we expecte that Temporary life expectancy between 0 and 80 will be about 75,2 for males and 76,8 for males. LC model gave the lowest value, RH the Medium and the APC*model gave the higher values. The life expectancy at 60 will be about 17,8 years for males and 18,5 for females.

Figure 32: Projected temporary life expectancy at different ages (LC, RH and APC* models)



Since Life expectancy is an average value for the age at death of a set of individual suppozed born in the same year, a comparison based on such a parameters doesn't lead to better distinguish the differences in the specific age mortality schemes especially between a model to another while they gave an approximative life expecyancues. The following figure compare the mortality curves obtained on the three model by 2050:

Figure 33: projected specific age mortality petterns at different horizons



The specificity of the Age-period-Cohort models (RH or APC* or others) is that capture the particularity of the mortality surface according to the year of birth, and it (APC model) allows to reproduce this effect in the future not according to time but also to year of birth. we observe some difference at the lower ages more apparent for females than for males.

7 Some simulations with life insurance reserving

To show the impact of considering cohort effects in mortality projections, we use an example of temporary life- annuity of (1 usd) payable from January 1st, 2015 until death or reashing the age of 80 year. We compare the annuity price at different ages, 55, 60, 65 and 70 years, for males and females. The interest rate is supposed to be constant all over the covered period.

Table 3: Annuity pricing at different ages (50, 55, 60, ... 75)

	Males			Females		
	LC	RH	APC*	LC	RH	APC*
50	18,44	18,43	18,46	18,47	18,49	18,51
55	16,65	16,65	16,68	16,69	16,71	16,72
60	14,19	14,18	14,22	14,23	14,25	14,25
65	11,42	11,42	11,44	11,46	11,48	11,48
70	8,31	8,32	8,33	8,34	8,36	8,37
75	4,84	4,85	4,85	4,86	4,87	4,88

The prices of the temporary life annuity calculated with LC and RH and APC* models are approximatively same, with changing difference between males and females from a model to another. The females annuities at 50 year are 0,05 in average expensive than the males annuities and however the males model is.

8 Conclusion

The objective of the present paper was to show and to estimate the impact of considering of cohort effect in mortality forecasting for the algerian population. For this, we have tried to compare between the Age-Period and the Age-Period-Cohort approaches. The LC model is very easy to emplement and to forecast and leads to regular results. The RH model by adding a supplimentary component related to the year of birth, leads , in first, to reduce the error terms, and by the same to improve the quality of the fitting. a high fitting quality doesn't garantee a high forecasting capacity. The complixity of the forecasting process of the cohort effect reduce the quality of the forecasting. The first estimated cohort parameters should be smoothed to ensure the regularity of the final obtained series. Also, the estiseparate mation process doesn't garantee to better the time from the cohort effect. if we start for exemple from the results obtained with LC model to estimate the Cohort effect in RH model, in such a case, the cohort effect vector that we will try to estimate is more related to the LC initial distribution. the best way is to estimate the time and the cohort parameters by the same process.

For our case, The three models lead approximatively to the same results with some differences in the age specific mortality schemes. The bump observed for the cohorts borns between 1965 - 1975, which represents princippally the important deaths number occured during the black decade among the age range (20-30) years. This effect is reproduced for the futurs years. This leads to a difference in mortality schemes that we signaled above. For mortality time index forecasting, the bump of the black decade was corrected, This bump corresponds to the bump of 1970'th. To ensure more accurate calculation, the two bumps should be corrected by the same maner.

The complexity of the cohort effect estimation and forecasting is almost relation to the weight of the observations corresponding to each generation. we should define an estimation criterion which take in account the observation distribution according to the year of birth.

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