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Staking plans in sports betting under unknown true probabilities of the event

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Abstract

Kelly staking method has been shown to maximize long-term growth of bankroll. However, it demands for the estimation of the true probabilities for each event. As a result many sport tipsters have abandoned this staking method and opted for a flat staking plan ('unit loss') or, less frequently, an 'unit win' plan. We analyze under which assumptions these methods correspond to the Kelly staking method and propose a different staking plan: 'unit impact,' under the hypothesis that this plan fits better with the Kelly staking method. We test our predictions using a betting database from Pyckio, one of the most popular tipster platforms in the world. Results show empirical support for our hypothesis.

Keywords: sports betting; Kelly criterion; staking methods; tipster;
JEL CODES: L83; G11; D81

1 Introduction

Sports betting markets have grown in recent years, from 46.5 billion dollars in 2005 to an estimation of 90.9 billion dollars in 2017 (Collignon and Sultan, 2014) and process more transfers per day than stock markets (Croxson and

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James Reade, 2014). This has raised the attention of academics on sport betting markets, analyzing several topics such as market biases (Levitt, 2004; Snowberg and Wolfers, 2010; Feddersen et al., 2017), how fast new information is incorporated into prices (Croxson and James Reade, 2014), the difference between bookmakers and exchange markets (Smith et al., 2009) or the optimal betting strategy (Ethier, 2004; Baker and McHale, 2013).

In this last line of research, the Kelly staking criterion has been shown to maximize long term growth (Kelly, 1956) and also median fortune (Ethier, 2004), so that it has been used in practice in a number of areas, such as sports betting, blackjack or stock markets (Thorp, 2008)¹.

However, an important requirement is needed to apply Kelly criterion in practice: to be able to estimate the 'true' probabilities of an event. Many sport tipsters acknowledge that they are not able to do it². As a consequence, they abandon the optimal Kelly criterion and embrace a flat staking plan ('unit loss') or, less frequently, an 'unit win' plan .

The main contributions of this paper are: (i) analyzing under which assumptions the 'unit loss' and the 'unit win' staking methods correspond to the Kelly staking method, (ii) proposing a different staking method: 'unit impact', (iii) empirically test which of the staking plans fits better with the Kelly staking method and (iv) analyze bankroll evolution under each staking method. Results show that the 'unit impact' plan fits well with the Kelly staking method while the 'unit loss' and 'unit win' do not and that, consequently, bankroll growth under the 'unit impact' plan is much higher than under the other methods.

The structure of the paper is as follows: Section 2 describes the three staking plans and their Kelly equivalence. Section 3 presents the database and methodology used. Section 4 reports the results achieved. Section 5 analyzes the evolution of bankroll and Section 6 provides some concluding remarks.

2 Staking plans and their Kelly equivalence

In this section we explain the three staking plans ('unit loss', 'unit win' and 'unit impact') and develop the equivalence between each of them and the Kelly staking

¹For a review of other (good and bad properties) of Kelly staking criterion, see (Maclean et al., 2010). For a extension of Kelly criterion to pairs of binary wagers, see (Eisenberg and Diao, 2017)

²(Baker and McHale, 2013) highlight that Kelly betting criterion assumes no uncertainty in estimation of 'true' probabilities and show that, under uncertainty in the estimation, a shrinkage factor should be applied

criteria according to the price at which the bet is wagered. The idea is the following: to be equivalent to the Kelly staking method, each of the three plans implicitly require an assumption regarding the specific function representing the relationship between the expected yield of the bet and the odds at which the bet has been placed.

To ease the exposition, we assume no fees from the bookmaker (or, alternatively, we think about price after these fees are discounted). According to the Kelly criterion, the percentage of bankroll to be used in every bet is $\frac{1}{c-1}(pc-1)$ where c is the decimal price provided by the bookmaker and p is the 'true' probability for the event to happen.

This formula can be written as $\frac{1}{c-1}(\frac{p}{p_c}-1) = \frac{EY}{c-1}$ where p_c is the probability implicit in the price provided by the bookmaker so that $(\frac{p}{p_c}-1)$ is the expected yield (EY) of the bet. Obviously p should be greater than p_c for the bet to show a positive expected value.

Many tipsters know about the Kelly staking criterion and are able to estimate if p is greater than p_c . However, they acknowledge that they are not able to estimate the EY from each bet with some degree of precision (that is, they are not able to accurately estimate p), which refrains them to apply the Kelly criterion in practice. So, what do they do?

Many of them have turned to a 'unit loss' strategy risking a constant percentage of their bankroll in every bet³. This strategy parallels the 'naive diversification' strategy analyzed in the financial literature (Benartzi and Thaler, 2001). Obviously, the 'unit loss' strategy has the consequence that the bets placed at higher price will have a much higher impact on final bankroll than bets placed at short price in case of a win (for example, a similar increase in bankroll is caused by 1 win at price 5 than by 8 straight wins at price 1.5).

A somewhat less used alternative is the 'unit win' staking plan. Instead of holding constant the loss in case of a failure, this method holds constant the amount won in case of a win. Tipsters think that, as they are not able to estimate the expected yield of each bet, they can just 'forget' about this part of the formula and bet a percentage of bankroll equal to a constant divided by $(c-1)$. Obviously, the 'unit win' strategy may risk a large loss if a bet is placed at short prices (for example, to win a 1% of the bankroll at decimal price 1.1, 10% of the bankroll should be risked. However, if the decimal price is 2 only 1% of the bankroll should be risked)

³For example, around 60 percent of PRO tipsters in Pyckio (www.pyckio.com) do use flat stakes

We propose an alternative staking plan that, to our knowledge, has not been proposed before. We may label this plan 'unit impact', meaning that what is held constant is not the absolute win or loss but the difference between winning and losing. That is, every bet has exactly the same impact in the bankroll, no matter how long or short the price of the bet is.

Due to the fact that Kelly staking criterion has been shown to maximize long term growth and median fortune, a crucial question is:

How do these three staking plans relate to the Kelly staking criterion?

The main idea is that, although, we obviously do not know the true probabilities from every bet, each of the different criteria imply a different relationship between expected yield and observed price according to the Kelly criterion.

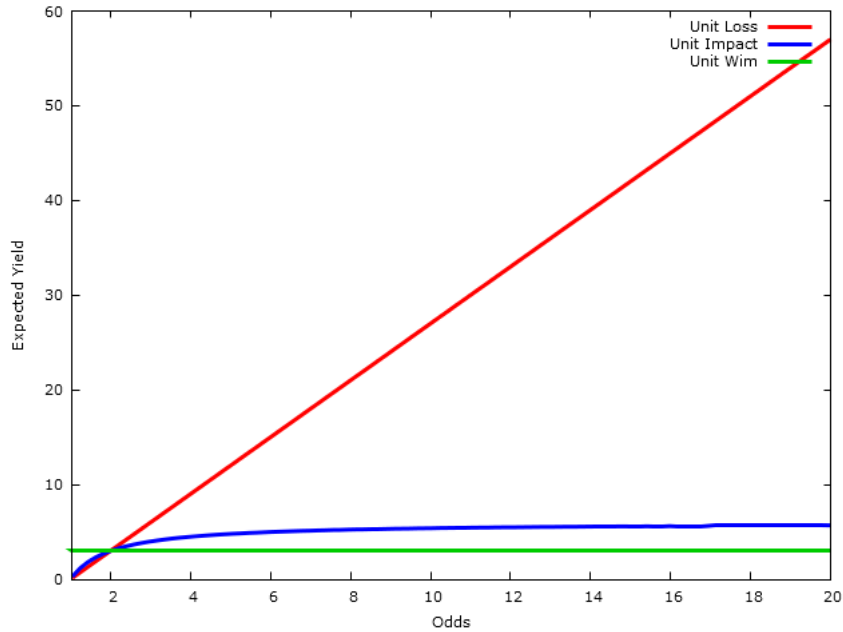
If we call s_i the percentage of bankroll riskied in every bet:

First, from the 'unit loss' strategy, we have that the percentage of bankroll staked s is constant so that $s = \frac{1}{c-1} EY$. Therefore $EY = s(c-1)$ and $\frac{\partial EY}{\partial c} = s$. That is, if a tipster expect that her yield increases linearly with the price, then the 'unit loss' strategy is equivalent to the Kelly criterion. In other words, if a tipster were able to correctly estimate the true probabilities for each event and the expected yield from these true probabilities increases linearly with the price, then the utilization of the Kelly criterion will actually result in a 'unit loss' strategy.

Second, the 'unit win' strategy means that the stake placed is $s_i = \frac{a}{c-1}$ where a is a constant that can be used to normalize so that the total amount wagered is the same following each method. That is, the 'unit win' strategy matches the Kelly criterion if there is no relationship between EY and price. In other words, if a tipster were able to correctly estimate the true probabilities for each event and the expected yield from these true probabilities is unrelated to the price, then the utilization of the Kelly criterion will actually result in a 'unit win' strategy.

Third, the 'unit impact' strategy means that the amount placed in each bet is $s = \frac{b}{c}$, where b is a constant that can be used to normalize so that the total amount wagered is the same following each method. For this strategy to match the Kelly criterion, it should happen that $EY = b\frac{c-1}{c}$. That is, EY increases linearly with $\frac{c-1}{c}$ (not with c). In fact, it means that $\frac{\partial EY}{\partial c} = b\frac{1}{c^2}$. In other words, if a tipster were able to correctly estimate the true probabilities for each event and the expected yield from these true probabilities increases linearly with $\frac{c-1}{c}$, then the utilization of the Kelly criterion will actually result in a 'unit impact'

Figure 1: Relationship between expected yield and price according to the different staking methods

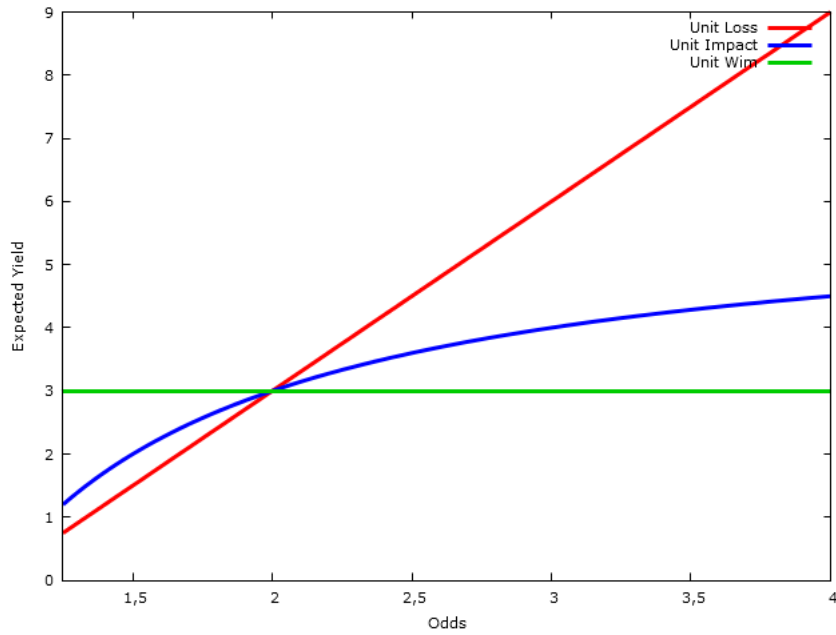


strategy.

To summarize, each of the different staking methods assume a specific relationship between the expected yield and the price at which the bet is wagered. Figure 1 plots these relationships for prices between 1.05 and 20 while Figure 2 provides a zoom for prices between 1.25 and 4 (93.97% of bets from Pyckio professional tipsters belong to this range). These figures are drawn departing from a 3% expected yield in a bet with price equal to 2.

Table 1 shows how the different strategies would potentially perform at different prices. We take 9 prices corresponding to the following probabilities: 95%, 90%, 75%, 66.6% and 50%, 33.3%, 25%, 10% and 5%. We depart from the unit loss strategies and adjust the other strategies so that the total amount staked in every strategy is equal to 9% of bankroll. We can clearly see the difference between the three strategies. First, unit loss plan holds constant the amount potentially lost but it gives very different potential earnings depending on the price of the bet. As a consequence, bets at higher prices would greatly determine the final bankroll. Second, unit win holds constant the amount potentially won but it gives very different potential losses depending on the price of the bet. As a consequence, bets at lower prices would greatly determine the

Figure 2: Relationship between expected yield and price according to the different staking methods (price between 1.25 and 4)



final bankroll. Finally, unit impact holds constant the difference between potential wins and potential losses meaning that each bet has exactly the same impact on determining the final bankroll.

Table 1: Illustration of the different stake plans

Price	Unit Loss			Unit Win			Unit impact		
	Stake	Potential Win	Potential Loss	Stake	Potential Win	Potential Loss	Stake	Potential Win	Potential Loss
1.05	1	0.05	-1	4.74	0.24	-4.74	1.88	0.09	-1.88
1.1	1	0.1	-1	2.37	0.24	-2.37	1.79	0.18	-1.79
1.25	1	0.25	-1	0.95	0.24	-0.95	1.58	0.39	-1.58
1.5	1	0.5	-1	0.47	0.24	-0.47	1.32	0.66	-1.32
2	1	1	-1	0.24	0.24	-0.24	0.99	0.99	-0.99
3	1	2	-1	0.12	0.24	-0.12	0.66	1.32	-0.66
4	1	3	-1	0.08	0.24	-0.08	0.49	1.48	-0.49
10	1	9	-1	0.03	0.24	-0.03	0.20	1.78	-0.20
20	1	19	-1	0.01	0.24	-0.01	0.10	1.87	-0.10

3 Data and methodology

We use a betting database from Pyckio, which is one the more popular worldwide tipster platforms. This database include all bets made until 22nd May 2018. We will test the hypotheses from previous section using two different methods.

First, we will use the data from professional tipsters only. To avoid survivorship bias we just use the bets placed by those professional tipsters only since they were considered grandmaster tipsters⁴.

The final database is composed of 55,891 bets in different sports, such as soccer, basketball, tennis, baseball or volleyball. Figure 3 plots the relationship between observed yield and price for these bets. To build the plot, we ordered the 55,891 bets and grouped them in 191 groups⁵. The yield of each specific bet runs from -100% to $(c - 1) * 100\%$. Average price is 2.03 and average yield is 3.64%. We will use these groups to test for the relationship between yield and price⁶

Second, we use an alternative method to test the relationship between yield and price. This method is based on the idea that the closing prices (those at the start of the event) are those giving a best guess on the true probabilities of the event. For each price at which a bet has been placed we compute the yield against the closing price⁷ and we take the percentile 90 of the distribution of closing prices for each odd⁸ in order to compute the expected yield by 'pro bets'. Figure 4 plots this relationship.

4 Results

Results using data from professional tipsters are shown in table 2. Regressions use the 191 groups previously defined. The variable unitloss is just $(c - 1)$ while the variable unitimpact is $\frac{c-1}{c}$ as these are the linear relationships implied by the unit loss and the unit impact respectively (unit win implies no relationship between yield and price). From the first column it can be seen that the lin-

⁴The category of grandmaster tipster is achieved with a rating of 3.5, while professional tipsters need a rating of 4.25

⁵Actually, we used a command to tell Stata to cut it in 500 groups but, as price only show two decimal points, that means that for some price there are more bets than the 112 required for the division in 500 groups. The mode of the price distribution is 1.93 with 1,597 bets

⁶Results are robust to the consideration of a lower number of groups

⁷We take account of a 2.5% fee from the bookmaker. Results are robust to the consideration of alternative fees

⁸Results are robust to taking different percentiles like 95 or 85

Figure 3: Relationship between yield and price for professional tipsters

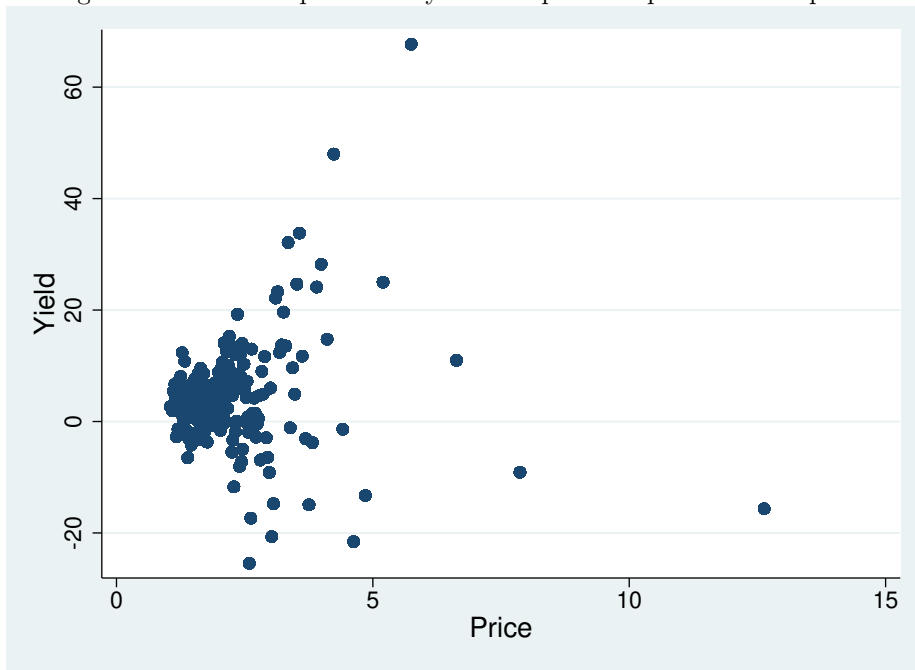
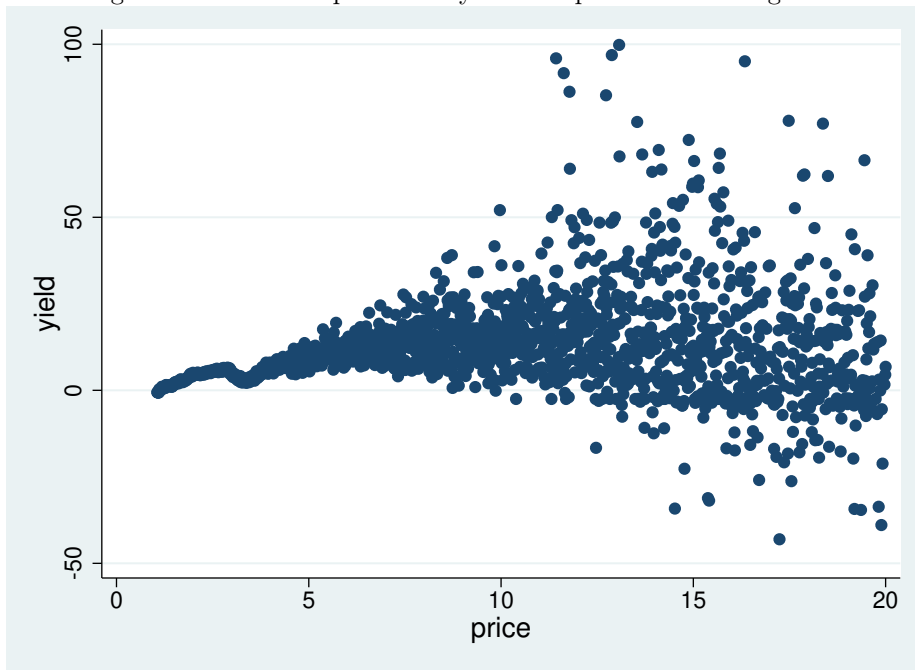


Figure 4: Relationship between yield and price from closing odds



ear relationship between expected yield and price is not statistically significant. This evidence goes against the unit loss strategy and may support the unit win strategy. However, the second column clearly shows that there exist a linear relationship between expected yield and $\frac{c-1}{c}$, being the coefficient positive and clearly significant from a statistical point of view. The R-squared in this second column is almost three times higher than in the first column, supporting the better fit of the unit impact plan with the Kelly staking method. Finally, the third column includes both variables at the same time. We see that unitimpact remains slightly significant while unitloss stills shows a non significant coefficient.

Table 2: OLS using bets from professional tipsters

	(1)	(2)	(3)
	yield	yield	yield
unitloss	0.794		-0.572
	[0.598]		[0.954]
unitimpact		9.043**	12.163*
		[4.132]	[6.647]
N	191	191	191
F	1.762	4.789	2.566
r^2	0.009	0.025	0.027

Unitloss= $c - 1$; Unitimpact= $\frac{c-1}{c}$

Robust standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Results using the observed yield against expecting odds by the 10% of better bets are shown in Table 3. From the first column it can be seen that in this case the linear relationship between expected yield and price is positive and statistically significant. This evidence goes against the unit win strategy and may favor the unit loss strategy. However, the second column clearly shows that the relationship between expected yield and price fits much better with that implied by the unit impact strategy. Not only is the coefficient positive and clearly significant from a statistical point of view, but also the R-squared increases by 66% and the F-statistic is approximately ten times higher. Last column provides further evidence that the unit impact strategy fits data much better than the unit loss strategy.

Table 3: OLS using yield against closing odds

	(1)	(2)	(3)
	yieldco	yieldco	yieldco
unitloss	0.655*** [0.171]		0.120 [0.281]
unitimpact		27.008*** [2.520]	23.890*** [5.089]
N	1784	1784	1784
F	14.658	114.886	352.244
r2	0.020	0.030	0.030

Unitloss= $c - 1$; Unitimpact= $\frac{c-1}{c}$

Robust standard errors in brackets

* $p < 0.10$, ** $p < 0.05$, $p < 0.01$

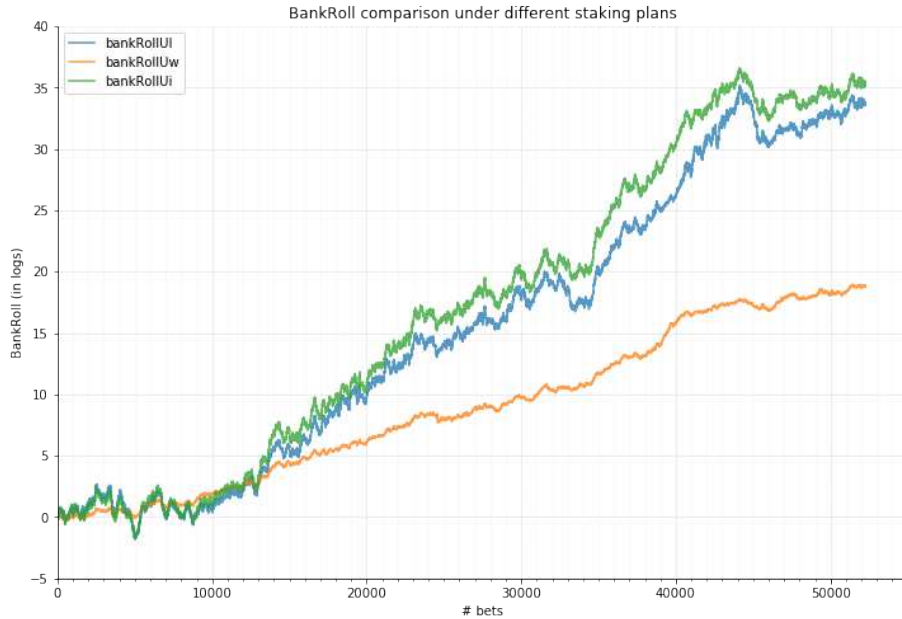
5 Bankroll evolution

In this section we compare the evolution of bankroll under the different staking strategies. One way to compare them would be to stake exactly the same amount of money after all bets have been placed. However this is not possible because the utilization of a variable bankroll together with the superiority of the unit impact strategy makes the growth of the bankroll using this strategy so much higher that it is impossible to match the amount staked. We would rely instead on a different alternative: comparing the three strategies under an optimal choice of the constants s , a and b . That is, we select the constant that achieves the larger possible growth of bankroll under each specific strategy.

We use the database composed of 55,891 bets from PRO tipsters. The optimal value of s is .03557. That is, under a unit loss strategy the best (ex-post) choice of stake would be 3.557% of bankroll in each bet. Regarding the optimal value for a in the unit win strategy we found a corner solution because the maximum possible value should be 0.01 so that the percentage of bankroll to be staked in a single bet remains under the 100% of the bankroll.⁹ This means that for a bet at price 2, a 1% of bankroll is risked. Finally, the optimal value of b is 0.7615. That is, under a unit impact strategy the best (ex-post) choice of stake for a bet priced at 2 is 3.807% of the bankroll. . Figure 5 plots bankroll evolution when the optimal stakes are used using a log scale. We can see that the unit impact strategy provides the larger growth of the bankroll. More precisely, the final bankroll is 5 times larger than final bankroll when

⁹The bet with minimum price is 1.01 so that values of a greater than 0.01 would be to stake more than 100% of the bankroll in this bet.

Figure 5: Evolution of bankroll using optimal stakes for each strategy



using the unit loss strategy and 16 million times larger than the final bankroll using the unit win strategy.

6 Conclusions

In this paper we have dealt with the important issue of the staking method to be used in sports betting and other financial investments. The Kelly staking criterion has been shown to maximize long-term growth. However, this criterion demands for exact estimation of the true probabilities of the events so that in the real world many tipsters opt by a naive diversification strategy, labelled in this context as the 'unit loss' strategy. In addition, some tipsters have relied on an alternative strategy ('unit win') that holds constant the potential win instead of the potential loss.

One main contribution of this paper is to propose an alternative staking method (the 'unit impact') that holds constant the difference between potential wins and losses and analyze under which assumptions each of these staking methods correspond to the Kelly staking criterion.

We use a betting database from Pyckio, which is one of the more popular worldwide tipster platforms, with the aim to analyze if these staking methods are compatible with the Kelly criterion according to the relationship between expected yield and the price of the bet. Results show that 'unit impact' staking method fits much better with the Kelly criterion than 'unit win' or 'unit loss' staking methods. We simulate the evolution of the bankroll under the different methods and we find that the final bankroll is remarkably higher under the unit impact strategy than under the unit loss strategy, being the unit win strategy the worst performing one.

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