# What stunts economic growth and causes the poverty trap?

Mizushima, Atsue

Otaru University of Commerce

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Atsue Mizushima†

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**Abstract**

In spite of an identical initial condition, why are some parts of the world so rich and others so poor? To address this question, this paper constructs a simple theoretical model that incorporates human infrastructure and child labor The first part of the paper shows that the condition of bifurcation from an identical initial condition depends on the technology level. We also show that current dynamic trends highly depend on initial endowments and productivity. The second part of the paper examines the effect of development assistance in recipient countries. By analyzing two types of programs; the elimination of child labor and support to strengthen human infrastructure, we show that the former (latter) program is effective for middle- (low-) income countries.

**Keywords:** Child labor, human infrastructure, human capital, divergence, poverty trap, development aid

**Classification Numbers:** I00; J01; O11; O41

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†Correspondence to: Atsue Mizushima, Department of Economics, Otaru University of Commerce, 3-5-21 Midori, Otaru, 047-8501, JAPAN, E-mail: mizushima@res.otaru-uc.ac.jp

# 1 Introduction

As noted by Clark (2008), the average person in the year 1800 was no better off than the average person in 100,000 BC. However, the Industrial Revolution of two hundred years ago irreversibly changed the possibilities for material consumption. Incomes per person began to exhibit sustained growth in a favored group of countries, and the richest economies are now 10 to 20 times wealthier than the average economy in 1800. Prosperity, however, is not the case in all societies. Material consumption in some countries, mainly in sub-Saharan Africa, is now well below pre-industrial levels.

The Industrial Revolution increased mechanization and the associated labor, and many unskilled workers including children played a role in economic activities. Mechanization improved productivity and fostered economic development. The highly successful countries of today increased their stock of human capital and infrastructure in the early stages of industrial development, which increased household income and decreased the ratio of child labor (see Figure 1). A lag delayed some countries from developing and setting up manufacturing, but the fruits of prosperity were gradually realized (see Figure2). The countries with a significant lag in prosperity are still suffering economically, and many children are involved in economic activities (see Figure 3)[1]. Why has industrialization failed to make all countries rich, and why do large parts of the world still suffer from poverty? Because an increasing presence of emerging country is expected to be a driving force in the global economy, identifying the cause of stagnation and resolving it in less-developed countries has gained much attention. Many studies have addressed the importance of infrastructure such as roads, bridges, public transportation, and energy facilities on economic development (see for example, Calderon and Chong (2004), Easterly and Rebelo (1993),Gramlich (1994), and Garcia-Mila, McGuire, and Porter (1996) ) and show the positive correlation between the stock of infrastructure and development (The World Bank (1994)). These types of infrastructure are known as physical infrastructure or public capital, and they improve the efficiency of economic activities. Because economic growth can also shape the demand and supply of physical infrastructure services, the underdevelopment of infrastructure is considered the cause of a lag in development

[1]The proportion of children aged between five and 17 years who work amounts to 19.6 % in Africa, 7.4 % in Asia and the Pacific countries, 5.3 % in the Americas, 4.1 % in Europe and Central Asia, and 2.9 % in Arab States (ILO (2017)).

(Esfahani and Ramirez-Giraldo (2003)). There is no doubt that the development of physical infrastructure is central to economic development; however, other aspects of infrastructure, such as the level of health care, education, and nutrition, also contribute to economic development. For example, better public health improves the overall well-being of society because healthier individuals are less myopic and tend to place greater value in the future[2]; healthy individuals prefer to acquire human capital (Jimenez (1995) and Bedi and Edwards (2002)). This type of infrastructure classified apart from physical infrastructure and known as human infrastructure (Jimenez (1995)). Human infrastructure increases the level of human capital and productivity, which leads to higher wages, reduced child labor, and enhances investment in human capital. The mechanism is expected to create a virtuous cycle endogenously, but the analysis of the effect of human infrastructure on economic development has not received much attention in existing literature. The analytical research on how the accumulation of human infrastructure affects economic development is extremely important to clarify the cause of poverty and to find its solution. For this purpose, we set up a simple theoretical model incorporating child labor and human infrastructure and investigate the causes of bifurcation from an identical initial condition. By assuming that an increase in human infrastructure improves the marginal productivity of labor, the first part of this paper analyzes the transition dynamics to determine the reason why one economy can develop and another cannot despite having the same initial condition. The second part of this paper investigates the possibility of sustainable growth in developing countries. Because development aid is the main source of support for less developed countries, and the amount of net official development assistance is increasing year by year[3], we examine the effect of development aid on poverty-stricken countries.

The remainder of this paper is organized as follows. Section 2 sets up the model. Section 3 characterizes the dynamic equilibrium path and analyzes the condition of divergence and poverty. Section 4 analyzes the effect of development aid, and Section 5 concludes the paper.

[2] Agenor (2010).

[3] US$6.817 billion (1970), US$34.475 (1980), US$58.242 (1990), US$49.804 (2000), US$130.685 (2010), and US$157.676 (2016) (See World Bank (2018)).

## 2 The Model

Consider an infinite-horizon economy composed of households, perfectly competitive firms, and a government. A new generation of a unit mass, referred to as generation $t$, is born in period $t-1$. Time is indexed by $t = 1, 2, \cdots$. Generation $t \geq 1$ is composed of a continuum of $N_t$ units who live for three periods: childhood, adulthood, and oldhood.

**Human Infrastructure**

As noted by Jimenez (1995), human infrastructure can be classified separately from physical infrastructure (transport, energy, water). It is defined as public capital that improves societal levels of health status, education, nutrition, and labor productivity. Human infrastructure also includes sanitation, environment, and legal and political institutional systems. When the level of human infrastructure is considered as an index $g_t$, the level of human infrastructure in period $t+1$ $g_{t+1}$ accumulates through investment in human infrastructure, $i_t^G$, and the level of human infrastructure in the preceding period $t$ $g_t$:

$$g_{t+1} = i_t^G + (1-\delta)g_t, \tag{1}$$

where $\delta \in (0,1)$ is the depreciation rate of human infrastructure.

**Firms**

In each period, there exists two potential sectors that are employed by Hazan and Berdugo (2002). Production will take place in either one of them or in both. The existence of one sector is independent of the existence of the other, and the existence of each sector is determined by the optimal choices of individuals. Each sector produces consumption goods but employs different factors: one sector employs only raw labor while the other employs efficiency units of labor. We refer to the former as the "traditional sector" and to the later as the "modern sector." The production function of the traditional sector is $y_t^{tra} = \bar{y} l_t$, where $y^{tra}$, $\bar{y}$, and $l_t$ represent per capita output in the traditional sector, the productivity in the traditional sector, and the quantities of raw labor, respectively. The wage rate in the traditional sector is $w^{tra} = \bar{y}$. The modern production function is given by $y_t^{mo} = y(g_t, h_t) = \eta\phi(g_t)h_t$, where $y_t^{mo}$, $\eta > 0$, and $h_t$, show per capita output in the modern sector, the level of technology, and the level of human capital at period $t$, respectively. $\phi(g_t)$ represents labor productivity, which is augmented by the level of per capita human infrastructure where $\phi'(g_t) > 0$, $\phi''(g_t) < 0$, and $\phi(0) = 0$. The wage rate in the modern sector is set as $w_t^{mo} = \eta\phi(g_t)$.

### Government

We assume that government monitors child labor and human infrastructure investment. The government levies income tax on adult agents[4] and allocates it between two policy programs: child labor inspection and investment in human infrastructure. By assuming that whether a firm employs children is private information, the government must inspect each firm to reveal the extent of child labor by investing the cost $\gamma$[5]. Inspection is not always successful, and the revealed child labor probability depends on the inspection cost $\pi(\gamma)$ $(0 \leq \pi(\gamma) < 1)$ and also has the properties: $\pi'(\gamma) > 0, \pi''(\gamma) < 0$, $\pi(0) = 0, \lim_{\gamma\to\infty} \pi'(\gamma) = 0$, and $\lim_{\gamma\to 0} \pi'(\gamma) = \infty$. If the government can expose child labor, the company must pay a fine depending on the amount that its child workers produce. Noting that the detection probability is $\pi(\gamma)$, the income from child labor is $w^c = (1-\pi(\gamma))\bar{y}$, and aggregate fines are $\pi(\gamma)\bar{y}l_t^c$ where $l_t^c$ shows the amount of child labor. The fines are redistributed to non-returnable government investments such as self-defense.[6] The residual amount of tax income is allocated to human infrastructure investment: $i_t^G = T_t - \gamma$[7] where $T_t$ shows aggregate tax revenue.

### Individuals

The individuals of generation $t$ are identical in their preferences. They live for three periods: child, adult, and old period, with an endowed one unit of time in each period. The adult agent (generation $t$) has a child and allocates their child (generation $t-1$) to either work $l_t^c$ or school $e_t$ in period $t$. When parents send their children to work, children can earn wage income $w_t^c$. The sum of the time spent by children at work or in school must be unity, and each value for time spent must lie between zero and unity in each period:

$$l_t^c + e_t = 1, \quad 0 \leq l_t^c \leq 1, \quad 0 \leq e_t \leq 1. \tag{2}$$

[4]Because child labor is a hidden action, the government cannot levy tax on child labor.

[5]For example, the Ministry of Labor and Social Security in the Republic of Turkey introduced a program enforcing the supervision of child labor from 1994. As a result, the number of working children who were 15 years old or less decreased between 1997 and 1999 in Turkey (OECD (2003)) .

[6]When the fines are redistributed to investment in human infrastructure, we obtain the same results.

[7]If we set $\gamma = \mu T_t$ where $\mu \in (0, 1)$, we obtain the same results.

As in Galor and Weil (1996, 2000), and Holtz-Eakin, Lovely, and Tosun (2004), we assume that the level of human capital is an increasing function of the time devoted to schooling $e_t$. Additionally, the level of human capital is also an increasing and strictly concave function of human infrastructure $g_t$. For analytical tractability, we assume that individuals born in period $t$ obtain their human capital as follows:

$$h_{t+1} = h(e_t, h_t) \equiv \theta e_t g_t^{\sigma}, \quad \theta > 0,\ 0 < \sigma < 1, \tag{3}$$

where $\theta$ and $\sigma$ express the parameters of education efficiency and the adjusted elasticity of human capital with respect to educational level, respectively.

When an individual becomes an adult, that individual has children and decides whether to donate all of their time to the traditional or modern sector. The decision is made according to aggregate income: $(1-\tau)w^{tra}l_t$ and $(1-\tau)w_t^{mo}h_t l_t$. Noting that $l_t = 1$, potential income $Y_t$ is decided as follows,

$$Y_t = \max\{(1-\tau)w^{tra},\ (1-\tau)w_t^{mo}\}. \tag{4}$$

When parents consume all of their family income; that is, their own income and that of their children, we have a budget constraint for that adult as follows:

$$c_t = Y_t + (1-\pi(\gamma))w_t^c l_t^c. \tag{5}$$

Figure 4 shows a sketch of the relationship between each generation.

An individual from generation $t$ has preference over their consumption $c_t$ and the level of human capital of their children $h_{t+1}$. We assume the following utility function:

$$U_t = \log c_t + \beta \log h_{t+1}, \tag{6}$$

where $\beta \in (0,1)$ indicates the degree of altruism by parents toward their children.

At each time point, adult individuals first decide their occupation by comparing the income of both sectors and then allocating their children's time by maximizing (6) subjects from (4) to (5). Hence, for given levels of $w^{tra}$, $w_t^{mo}$, and $w^c$, the optimization problem of each individual of generation $t$ can be expressed as follows:

$$\begin{aligned} \max_{l_t} U_t &= \log\{Y_t + (1-\pi(\gamma))w^c l_t^c\} + \beta \log\{\theta(1-l_t)g_t^{\sigma}\}, \\ \text{s.t.} \quad 0 &\le l_t^c < 1. \end{aligned}$$

The first-order condition with respect to $l_t$ is given by

$$\frac{\partial U_t}{\partial l_t^c} = \frac{(1-\pi(\gamma))w_t^c}{Y_t + (1-\pi(\gamma))w_t^c l_t^c} - \frac{\beta}{1-l_t^c} \lesseqgtr 0, \ (with\ equality\ if\ 0 < l_t^c < 1). \tag{7}$$

The first and second terms on the right-hand side of (7) represent the marginal benefit and the marginal cost of child labor.

# 3 Equilibrium

In this section, we examine the equilibrium and dynamic system of the economy.

## 3.1 Equilibrium

At first, the adult individual makes the occupational choice by comparing an expected income. Substituting the wage rate into (4), the individual who satisfies $h_t \leq \bar{y}/\eta\phi(g_t) \equiv \underline{h}$ chooses an occupation in the traditional sector and $\bar{h} \leq h_t$ chooses an occupation in the modern sector. Next, the individual maximizes their utility by choosing the working time of their children. Noting the individual who has $\underline{h} \leq h_t$ work in the modern sector, we obtain the following equilibrium relationship between child labor and human capital:

$$l_t^c(h_t, g_t) \equiv l_t^c = \begin{cases} \dfrac{1-\pi(\gamma)-\beta(1-\tau)}{(1+\beta)(1-\pi(\gamma))} & \text{if} \quad h_t \leq \underline{h}, \\ \dfrac{(1-\pi(\gamma))\bar{y} - \beta(1-\tau)\eta\phi(g_t)h_t}{(1+\beta)(1-\pi(\gamma))\bar{y}}, & \text{if} \quad \underline{h} \leq h_t \leq \bar{h}, \\ 0, & \text{if} \quad \bar{h} \leq h_t, \end{cases} \tag{8}$$

where $\underline{h} \equiv \frac{\bar{y}}{\eta\phi(g_t)}$ and $\bar{h} \equiv \frac{(1-\pi(\gamma))\bar{y}}{\beta(1-\tau)\eta\phi(g_t)}$. (8) show that the time spent working by children depends on the level of human capital of the child's parents. When the level of human capital is sufficiently low, parents cannot earn sufficient income to survive and have to rely on their children for income.

## 3.2 The Dynamic Equilibrium Path

From (2), (3), and (8), we obtain the dynamics of human capital as follows:

$$h_{t+1} = \begin{cases} \dfrac{\theta\beta(2-\tau-\pi(\gamma))}{(1+\beta)(1-\pi(\gamma))} g_t^\sigma, & \text{if} \quad h_t \leq \underline{h}, \\ \dfrac{\theta\beta((1-\pi(\gamma))\bar{y} + (1-\tau)\eta\phi(g_t)h_t)}{(1+\beta)(1-\pi(\gamma))\bar{y}} g_t^\sigma, & \text{if} \quad \underline{h} \leq h_t \leq \bar{h}, \\ \theta g_t^\sigma, & \text{if} \quad \bar{h} \leq h_t. \end{cases} \tag{9}$$

Accordingly, investment in human infrastructure is decided as

$$i_t^G = \begin{cases} \tau\bar{y} - \gamma, & \text{if } \ h_t \leq \underline{h}, \\ \tau\eta\phi(g_t)h_t - \gamma, & \text{if } \ \underline{h} \leq h_t, \end{cases} \tag{10}$$

where $\gamma$ is the inspection cost of child labor. By using (1), (8), and (10), we obtain the following:

$$g_{t+1} = \begin{cases} (1-\delta)g_t + \tau\bar{y} - \gamma, & \text{if } \ h_t \leq \underline{h}, \\ (1-\delta)g_t + \tau\eta\phi(g_t)h_t - \gamma, & \text{if } \ \underline{h} \leq h_t. \end{cases} \tag{11}$$

(9) and (11) show the system of this economy and indicate that the dynamics systems of the economy are completely described by these difference equations in $h_t$ and $g_t$. Before analyzing the equilibrium, we consider the borderline between the regimes. For analytical tractability, we impose[8]

$$\phi(g_t) = g_t. \tag{12}$$

This leads to the borderline relations: $\underline{h}$ and $\bar{h}$ are as follows:

$$\underline{h} \equiv h_t = \frac{\bar{y}}{\eta g_t}, \tag{13}$$

$$\bar{h} \equiv h_t = \frac{(1-\pi(\gamma))\bar{y}}{\beta(1-\tau)\eta g_t}. \tag{14}$$

By using (9), (11), (13), and (14), we can draw the phase diagram on the $(h_t, g_t)$ plane. We refer to the locus on the $(h_t, g_t)$ plane, representing $h_{t+1} = h_t$ as $HH$ locus and that representing $g_{t+1} = g_t$ as $GG$ locus. The $HH$ and $GG$ loci represent equal parts of (9) and (11), respectively. By applying (12), we have a complete dynamic systems as follows:

$$h_{t+1} \geq h_t \Leftrightarrow$$
$$h_t \leq \begin{cases} \dfrac{\theta\beta(2-\tau-\pi(\gamma))}{(1+\beta)(1-\pi(\gamma))} g_t^\sigma \equiv H_0(g_t), & \text{if } h_t \leq \underline{h}, \\ \dfrac{\theta\beta(1-\pi(\gamma))\bar{y}g_t^\sigma}{(1+\beta)(1-\pi(\gamma))\bar{y} - \theta\beta(1-\tau)\eta g_t^{1+\sigma}} \equiv H_1(g_t), & \text{if } \underline{h} \leq h_t \leq \bar{h}, \\ \theta g_t^\sigma \equiv H_2(g_t) & \text{if } h_t \geq \bar{h}, \end{cases} \tag{15}$$

$$g_{t+1} \geq g_t \Leftrightarrow \quad \begin{array}{ll} g_t \leq \dfrac{\tau\bar{y} - \gamma}{\delta} \equiv G_0, & \text{if } \ h_t \leq \underline{h} \\ h_t \geq \dfrac{\delta g_t + \gamma}{\tau\eta g_t} \equiv G_1(g_t), & \text{if } \ \underline{h} \leq h_t, \end{array} \tag{16}$$

[8]If we impose $\phi(g_t) = g^\epsilon, \ 0 < \epsilon < 1$, we have the same dynamic system.

$H_1(g_t)$ and $H_2(g_t)$ in (15) satisfies $dH_1/dg_t > 0$, $\lim_{g \to \underline{g}} \partial H_1/\partial g_t = 0$, $\lim_{g_t \to \bar{g}} \partial H_1/\partial g_t = \infty$[9] and $\partial H_2/\partial g_t > 0$, $\partial^2 H_2/\partial g_t^2 < 0$. Additionally, $G_1(g_t)$ in (16) also satisfies $\partial G_1/\partial g_t < 0$ and $\partial^2 G_1/\partial g_t^2 > 0$. The phase diagrams in the economy are shown in Figure 5 and Figure 6. For simplicity, we denote the saddle path described in Figure 5 and 6 as the $SS$ curve. Because $h_t$ and $g_t$ are state variables, their initial values $h_0$ and $g_0$ are historically given, and the economies that are initially on the $SS$ curve converge to the steady-state equilibrium. The steady-state equilibrium $(h^*, g^*)$ is at the intersection of the $HH$ and $GG$ locus. There exist two possible equilibria, and the next lemma summarizes the differences of these equilibria.

**Lemma 1** *Suppose that $\eta > (<)\bar{\eta}$, the high steady-state equilibrium is in the regime without (with) child labor; that is, $\bar{h} \leq h_t(\underline{h} \leq h_t \leq \bar{h})$. where $\bar{\eta} = \frac{(1-\pi(\gamma))\bar{y}}{\theta\beta(1-\tau)\eta}(\frac{\tau(1-\pi(\gamma))\bar{y}-\gamma\beta(1-\tau)}{\delta\beta(1-\tau)})^{-(1+\sigma)}$.*

**Proof.** When the equilibrium is in the regime $\bar{h} < h_t$, the intersect point of $H_1(g_t)$ (equation (15) and $\bar{h}$ is larger than the intersect point $G_1(g_t)$ (equation (16)) and $\bar{h}$. Now, we define the previous intersect point as $g^h$ and the latter as $g^g$. Thus, we have

$$g^h \equiv \Big(\frac{(1-\pi(\gamma))\bar{y}}{\theta\beta(1-\tau)\eta}\Big)^{\frac{1}{1+\sigma}} > \frac{\tau(1-\pi(\gamma))\bar{y}-\gamma\beta(1-\tau)}{\delta\beta(1-\tau)} \equiv g^g.$$

By arranging the equation, it becomes $\eta > \bar{\eta} \equiv \frac{(1-\pi(\gamma)\bar{y}}{\theta(1-\tau)}\Big(\frac{\tau(1-\pi(\gamma)\bar{y}-\gamma\beta(1-\tau)}{\delta\beta(1-\tau)}\Big)^{-(1+\sigma)}$. When productivity is high, the economy has low steady-state equilibrium and vice versa. ■

High productivity decreases the marginal benefit from child labor and decreases child labor in the long run. As a result, the economy with higher productivity experiences the steady-state equilibrium without child labor. The next proposition shows the stability of the equilibrium.

**Proposition 1** *There exists multiple equilibrium ($E_1$ and $E_2$). The higher equilibrium $E_2$ is a saddle point stability and satisfies the following:*

$$\eta < \bar{\eta} \;\; : \;\; (g^*)^{1+\sigma}\tau\eta\theta\beta(1-\pi(\gamma))\bar{y} = (\delta g^* + \gamma)((1+\beta)(1-\pi(\gamma)\bar{y}) - \theta\beta\eta(1-\tau)(g^*)^{1+\sigma}),$$
$$\eta > \bar{\eta} \;\; : \;\; \tau\eta\theta(g^*)^{1+\sigma} = \delta g^* + \gamma.$$

**Proof.** See Appendix. ■

A case where the economies are initially on the $SS$ curve is rare. In general, an economy

[9] $\underline{g} \to \bar{g} \equiv (1+\beta)(1-\pi(\gamma))\bar{y}/\gamma\beta\eta(2-\tau-\pi(\gamma)))^{1/(1+\sigma)}$ shows the level of $g$ that satisfies $H_1(g_t) = \underline{h}$, and $\bar{g} \equiv (\gamma\beta(2-\tau-\pi(\gamma))/(1+\beta)(1-\pi(\gamma)))^{\sigma}$ shows the level of $g$ that satisfies $H_1(g_t) = \bar{h}$.

is endowed with $(h_0, g_0)$ above or below the $SS$ curve. If $(h_0, g_0)$ is at the point below the $SS$ curve, such as the point $B$ (low level of human capital and human infrastructure) in Figure 5 and Figure 6, the output is too low for workers to earn sufficient wages and to invest in human infrastructure. Because wage incomes are low, parents must rely on their children to supplement family income, which results in diminished accumulation of human capital. Lower human capital prevents workers from entering the modern sector and results in fewer resources invested in human infrastructure and lower productivity levels and wage rates. These mechanisms create a vicious cycle, and the economy finally converges to the poverty trap equilibrium. On the other hand, if $(h_0, g_0)$ is at a point above the $SS$ curve, such as point $G$ (a high level of human capital and human infrastructure) in Figure 5 and Figure 6, output and human capital are sufficient for workers to earn sufficient wages. A high wage released children from the need to work and enhances the government's ability to invest in human infrastructure. A high wage also improves the productivity of human capital and wage rates. These mechanisms create a virtuous cycle, and the economy can perform sustainable development.

Next, we examine the case where initial endowments is $D$ in Figure 5 and Figure 6. Note that the economy that has the same initial endowments has completely different dynamics. One economy in Figure 6 experiences sustainable growth, the other economy in Figure 5 falls into the poverty trap. The next proposition summarizes the results.

**Proposition 2** *Even if initial conditions are the same, whether the economy continues to achieve economic growth depends on productivity. When productivity is high enough, the economy can sustain long-run growth.*

As noted, when productivity is high, individuals receive higher wages, and they do not rely on their children for family income. Additionally, higher wages increase the tax income of government and results in greater investment in human infrastructure. This creates a virtuous cycle, and the economy experiences sustainable growth.

## 3.3 Numerical Simulation

The purpose of the paper is to construct a theoretical model that explains the historically observed changes in child labor and substantial divergence. To understand the properties of these dynamic equilibrium paths, we used numerical calibrated version simulations of the model. For illustrative purpose, we parameterized the function $\pi(\gamma)$ as $\pi(\gamma) = \alpha\gamma/(1 + \alpha\gamma)$ where $\alpha \in (0, 1)$. The parameters used in the baseline simulation are given in Table 1. Treating each period as 25 years, we calculated the equilibrium values for the 10 periods from 1750 to 2000. Figure 7 to Figure 11 show the numerically analyzed transition paths of human capital $h$, human infrastructure $g$, and child labor $l^c$, respectively, where the initial level of stocks are the points $A, B, G$, and $D$ in Figure 5 and 6. These paths show that when the initial point starts below (above) the $SS$ curve, the transition dynamics of $g$ and $h$ fall into the vicious cycle (diverge). Figure 7 shows the transition of the vicious cycle. In the economy, low accumulation of human capital and social infrastructure increases the level of child labor. These trends indicate the economy cited in Figure 3. Moreover, the economy in Figure 1 and 2 can be adapted in the economy of Figure 9 and Figure 8, respectively. For these paths, the accumulation of human infrastructure creates a vicious cycle and increases the level of child labor. Figure 10 and 11 show the transition path with the same initial condition. As proposition 2 showed when productivity is low, the economy converges to a lower level of the steady-state equilibrium.

# 4 The Effect of Foreign Development Assistance

This section investigates the effect of development assistance on recipient countries. Currently, aid is the main source of support to less-developed countries, and international organizations such as the United Nations and The World Bank support these countries through multiple schemes (The World Bank (2009))[10]. One of the main schemes is a child protection program. Because child labor prevents children from obtaining an education and hinders their physical and mental development, the eradication of child labor is one of the goals of child protection. The other scheme is development aid whereby rich countries support poor countries by investing in infrastructure.

We examine how these tied aids influence individual decision making and steady-state equilibrium in recipient countries. First, we examine the effect of the child protection program. To eradicate child labor, this aid strengthens the inspection of child labor by increasing the

[10]Particularly, the International Labor Organization and The World Bank jointly provide poverty eradication programs to poor countries.

inspection cost from $\gamma$ to $\gamma' = \gamma + a$ where $a$ shows the total amount of aid distributed to each developing country. Because the program decreases the marginal utility of child labor, it decreases the incidence of child labor and contributes to increased human capital. The aid also shifts $HH$ loci upwards as follows:

$$h_{t+1} \geq h_t \Leftrightarrow$$
$$h_t \leq \begin{cases} \dfrac{\theta\beta(2-\tau-\pi(\gamma'))}{(1+\beta)(1-\pi(\gamma'))} g_t^{\sigma} \equiv H_0'(g_t), & \text{if } h_t \leq \underline{h}, \\ \dfrac{\theta\beta(1-\pi(\gamma'))\bar{y}g_t^{\sigma}}{(1+\beta)(1-\pi(\gamma'))\bar{y} - \theta\beta(1-\tau)\eta g_t^{1+\sigma}} \equiv H_1'(g_t), & \text{if } \underline{h} \leq h_t \leq \bar{h}, \\ \theta g_t^{\sigma} \equiv H_2'(g_t) & \text{if } h_t \geq \bar{h}, \end{cases}$$

Aid from the child protection program also shifts the $\bar{h}$ line (equation (14)) downwards. Figure 12 shows the effect of aid on the dynamics, and the dotted line shows the dynamics with aid. The aid increases (decreases) the steady-state level of human capital of $E_1(E_2)$, decreases the level of human infrastructure of $E_2$, and shifts the $SS$ line downwards. Therefore, the economy that initially occurs on the regime between $SS$ and $SS'$ curve (see point $J$ in Figure 12) has the potential for long-run growth. The child protection program has two effects on the steady-state welfare level. One is an income effect because the child protection program decreases the child labor in terms of time. However, this leads to lower levels of family income and consumption. Thus, the income effect has a negative effect on the steady-state welfare level. The other effect is an education effect, the child protection program encourages education and increases the stock of human capital. Thus, the education effect has a positive effect on the steady-state welfare level. The overall effect on welfare depends on the level of human infrastructure. When the level of human infrastructure is high, the positive effect dominates the negative effect and vice versa.

Next, we examine the effect of development aid. Development aid supports the accumulation of human infrastructure and changes investment as $i_t^G = T_t - \gamma$ to $i_t^{G'} = T_t - \gamma + a$. The $GG$ loci will be amended as follows:

$$g_{t+1} \geq g_t \Leftrightarrow \quad \begin{array}{ll} g_t \leq \dfrac{\tau\bar{y} - \gamma + a}{\delta} \equiv G_0', & \text{if } \; h_t \leq \underline{h} \\ h_t \geq \dfrac{\delta g_t + \gamma - a}{\tau\eta g_t} \equiv G_1'(g_t), & \text{if } \; \bar{h} \leq h_t, \end{array}$$

The aid does not have a direct effect on the accumulation of human capital, but it indirectly increases human capital. With the new dynamics, the $G_0'$ line shifts to the right, and the steady-state level of human capital and human infrastructure increase. The shift in line $G_1'$ depends on the level of aid $a$. When $\gamma > a$, $G_1'$ shifts downwards (see Figure 13), this results in a low steady-state equilibrium for human capital and human infrastructure. The $SS$ line also shifts downwards; however, the economy that initially occurs on the regime between $SS$ and $SS'$ curve (see point $J$ in Figure 13) also has the potential for long-run growth. On the other hand, when $\gamma < a$, $G_1'$ shifts upwards (see Figure 14), this results in a higher steady-state equilibrium for human capital and human infrastructure. The $SS$ line also shifts upwards, and the economy that initially occurs on the regime between the $SS$ and $SS'$ curve (see point $J$ in Figure 14) misses the potential for development. Development aid also has two effects; consumption effect and education effect. For the lower steady-state equilibrium ($E_1$) where parents work in the traditional sector, the aid does not have a direct effect on family income. The aid indirectly increases the level of human capital, which positively affects the steady-state welfare level. For the higher steady-state equilibrium ($E_2$) where the parents work in the modern sector and employ child labor, the effect of aid on welfare depends on the level of aid. When the level of aid is sufficiently large, a decrease in the marginal utility of child labor reduces the level of child labor. However, the increased stock of human infrastructure enhances the adult income and results in a positive income effect on the steady-state welfare level. The more the level of human infrastructure enhances the stock of human capital, the more positive the effect of human capital on the steady-state welfare level. When the level of aid is sufficiently low, a decrease in the marginal utility from child labor reduces the level of child labor. Because the aid increases the level of human infrastructure and the income of parents, the aggregate income effect on the steady-state welfare level is ambiguous. The aid also has a negative human capital effect on the steady-state welfare level. Therefore, the overall effect on welfare is ambiguous. The sufficiently large level of aid is generous for development aid. However, as we noted before, the economy that initially occurs on the $SS$ line is rare. The majority of economy are initially endowed with $(h_0, g_0)$ above or below the $SS$ curve. Particularly for economies initially endowed between the $SS$ and $SS'$ line, aid is detrimental to the economy. Therefore, when the income of the recipient country is not extremely low but there is child labor, the aid to prevent child labor is more effective than development assistance aid.

When child labor is conspicuous, the aid that reduces child labor is considered more effective than aid to enhance infrastructure. Because the direct reason for child labor is poverty, and families rely on their children for income, aid designed to prevent child labor is detrimental to welfare. Although development aid may not seem directly related to the eradication of child labor, it has the effect of enhancing adult income and results in low levels of child labor.

# 5 Conclusion

The average person in year 1800 was no better off than the average person in 100,000 BC. However, the Industrial Revolution that occurred two hundred years ago irrevocably changed the possibilities for material consumption. Incomes per person reached a phase of sustained growth in a favored group of countries. The richest modern economies are now 10 to 20 times wealthier than the average economy in the year 1800. Prosperity, however, is not enjoyed by all societies. Material consumption in some countries, mainly in sub-Saharan Africa, is now well below pre-industrial levels. Why did industrialization fail to benefit the whole world? Why did it make substantial areas of the world even poorer? To address these questions, this paper constructs a simple theoretical model, which incorporates human infrastructure and child labor. In the first part of the paper, we show that when the economy has the same initial condition, the economy with a high level of technology exhibits sustainable growth. However, the economy with a low level of technology falls into the poverty trap. In addition, we show the historically observed trends in child labor and development. The second part of the paper examines the effect of development assistance in recipient countries. By comparing development assistance programs that call for the elimination of child labor and the strengthening of human infrastructure, we find that aid for the elimination of child labor is effective for middle-income countries, and aid to strengthen human infrastructure is effective for low-income countries.

# Appendix

## Appendix A

We examine the stability of the equilibriums $E_1$ and $E_2$. To examine the local dynamics of equilibrium $E_1$ (or $E_2$), we take a first-order Taylor expansion of the system around the $E_1 = (h^*, g^*)$. Noting that $\check{h}_t \equiv h_t - h^*$ and $\check{g}_t \equiv g_t - g^*$, the linearizing of equations (9) and (11) in the equilibrium $E_1$ is expressed as:

$$\begin{pmatrix} \check{h}_{t+1} \\ \check{g}_{t+1} \end{pmatrix} = \begin{pmatrix} \Omega(g^*)^{1+\sigma} & \Omega(X(g^*)^{\sigma-1} + (1+\sigma)\eta h^*(g^*)^{\sigma}) \\ \tau\eta g^* & 1-\delta+\tau\eta h^* \end{pmatrix} \begin{pmatrix} \check{h}_t \\ \check{g}_t, \end{pmatrix}$$

where $g^*$ and $h^*$ are shown in proposition 1; and $\Omega \equiv \frac{\theta\beta(1-\tau)\eta}{(1+\beta)(1-\pi(\gamma))\bar{y}}$ and $X \equiv \frac{\sigma(1-\pi(\gamma)\bar{y}}{(1-\tau)\eta}$. The characteristic polynomial becomes:

$$P(\kappa) = \kappa^2 - T\kappa + D,$$

$$\begin{aligned} T &= \Omega(g^*)^{1+\sigma} + 1 - \delta + \tau\eta h^* > 0, \\ D &= \Omega(g^*)^{1+\sigma}(1-\delta+\tau\eta h^*) - \Omega\tau\eta g^*(X(g^*)^{\sigma-1} + (1+\sigma)h^*(g^*)^{\sigma}). \end{aligned}$$

Azariadis (1993) checks that the steady state is a saddle point, when $P(1) < 0$ and $P(-1) > 0$ hold simultaneously.

$$\begin{aligned} P(1) &= 1 - \Omega(g^*)^{1+\sigma} - (1-\delta+\tau\eta h^*) + \Omega(g^*)^{1+\sigma}(1-\delta+\tau\eta h^*) - \tau\eta\Omega(g^*)(X(g^*)^{\sigma-1} + (1+\sigma)h^*(g^*)^{\sigma}) \\ &= (1-\Omega(g^*)^{1+\sigma})(\delta-\tau\eta h^*) - \tau\eta\Omega(g^*)(X(g^*)^{\sigma-1} + (1+\sigma)h^*(g^*)^{\sigma}). \end{aligned}$$

Now, we suppose that $1 - \Omega(g^*)^{1+\sigma} = 1 - \frac{\theta\beta(1-\tau)}{(1+\beta)(1-\pi(\gamma))\bar{y}}(g^*)^{1+\sigma} < 0$. To hold this assumption, $(g^*)^{1+\sigma}$ should be larger than $\frac{(1+\beta)(1-\pi(\gamma))\bar{y}}{\theta\beta\eta(1-\tau)}$. Although Lemma 1 shows the steady-state equilibrium is in the regime $(g^*)^{1+\sigma} < \frac{(1-\pi(\gamma))\bar{y}}{\theta\beta\eta(1-\tau)} \equiv g^h$, because $\frac{(1-\pi(\gamma))\bar{y}}{\theta\beta\eta(1-\tau)} < \frac{(1+\beta)(1-\pi(\gamma))\bar{y}}{\theta\beta\eta(1-\tau)}$, the assumption is contradicted.

Next, we examine the sign of $\delta - \tau\eta h^*$. Substituting the equilibrium value $h^* = (\delta g^* + \gamma)/\tau\eta g^*$ (see equation (16) in the previous equation, $\delta - \tau\eta h^* = -\tau\eta\gamma < 0$, we have $P(1) < 0$.

$$\begin{aligned} p(-1) &= 1 + \Omega(g^*)^{1+\sigma} + 1 - \delta + \tau\eta h^* + \Omega(g^*)^{1+\sigma}(1-\delta+\tau\eta h^*) - \tau\eta\Omega X((g^*)^{\sigma} - (1+\sigma)\tau\eta\Omega h^*(g^*)^{1+\sigma}), \\ &= 1 + \Omega(g^*)^{1+\sigma} + 1 - \delta + \Omega(g^*)^{1+\sigma}(1-\delta) + \tau\eta h^* - \tau\eta\Omega X((g^*)^{\sigma} - \sigma\tau\eta\Omega h^*(g^*)^{1+\sigma}). \end{aligned}$$

Substituting the equilibrium value of $h^*$ (see 15), we have

$$\begin{aligned}&\tau\eta h^* - \tau\eta\Omega X((g^*)^\sigma - \sigma\tau\eta\Omega h^*(g^*)^{1+\sigma})\\ &= \frac{\tau\eta\theta\beta(g^*)^\sigma}{(1+\beta)(1-\pi(\gamma)\bar{y} - \theta\beta(1-\tau)\eta(g^*)^{1+\sigma}}(1-\sigma)(1-\pi(\gamma)\bar{y}(1+\beta)^2 > 0.\end{aligned}$$

Therefore, we have $P(1) < 0$ and $P(-1) < 0$.

Next, we examine the local dynamics of equilibrium $E_2 = (h^*, g^*)$. Using the same procedure, the linearization of equations (9) and (11) in the equilibrium $E_2$ is expressed as:

$$\begin{pmatrix}\check{h}_{t+1}\\ \check{g}_{t+1}\end{pmatrix} = \begin{pmatrix}0 & \sigma\theta(g^*)^{\sigma-1}\\ \tau\eta g^* & 1-\delta+\tau\eta h^*\end{pmatrix}\begin{pmatrix}\check{h}_t\\ \check{g}_t\end{pmatrix}.$$

$$T = 1-\delta+\tau\eta h^* > 0, \qquad D = \sigma\theta(g^*)^{\sigma-1}\tau\eta g^* > 0.$$

Since $P(-1) = 1 + T + D > 0$ and $P(1) = \delta - \tau\eta h^* - \sigma\theta\tau\eta(g^*)^\sigma$. Because $\delta - \tau\eta h^a st = -gamma/g^* < 0$, $P(1) < 0$. Therefore, $E_2$ is a saddle stable equilibrium.

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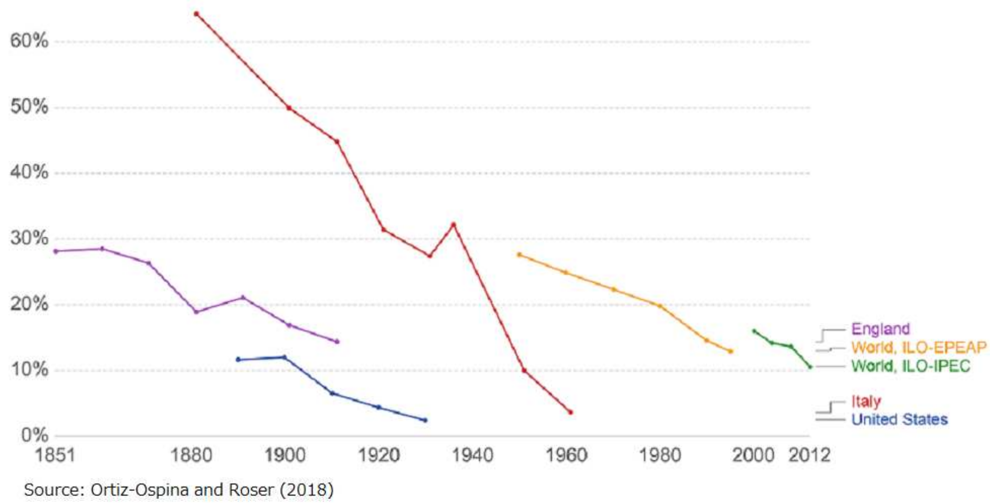


Figure 1: Child Labor Trends in Developed Countries

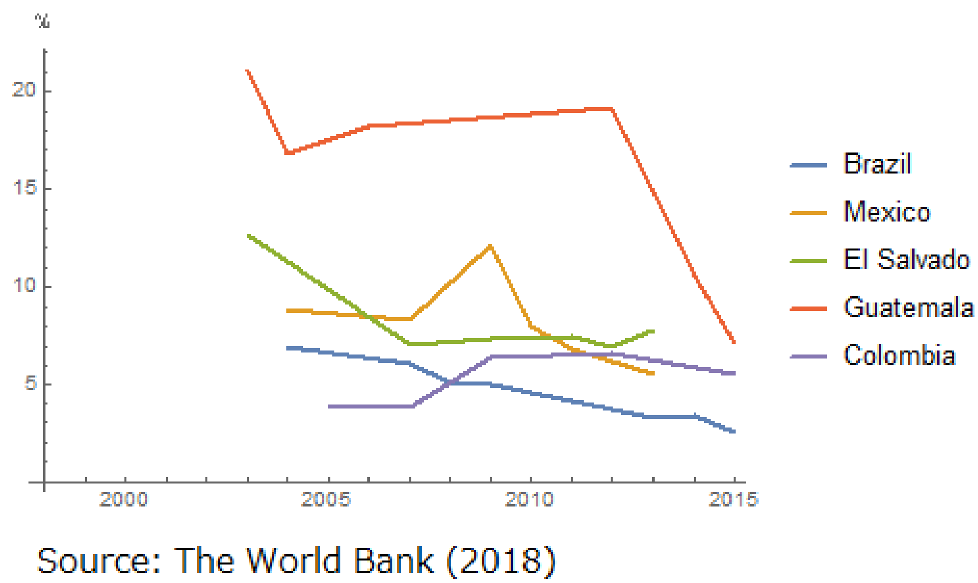


Figure 2: Child Labor Trends in Lagging Countries

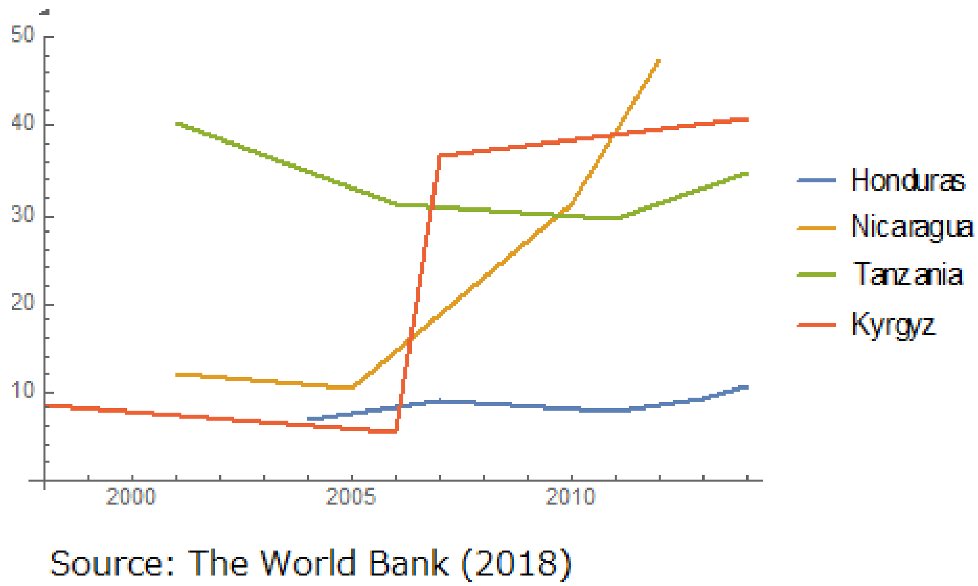


Figure 3: Child Labor Trends in less-developed Countries

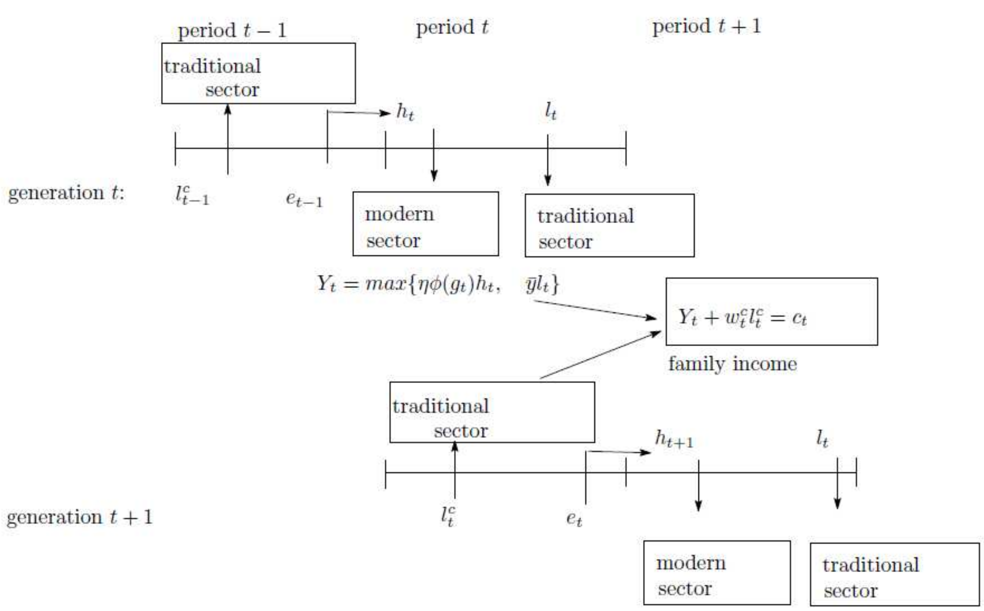


Figure 4: Generation Linkage

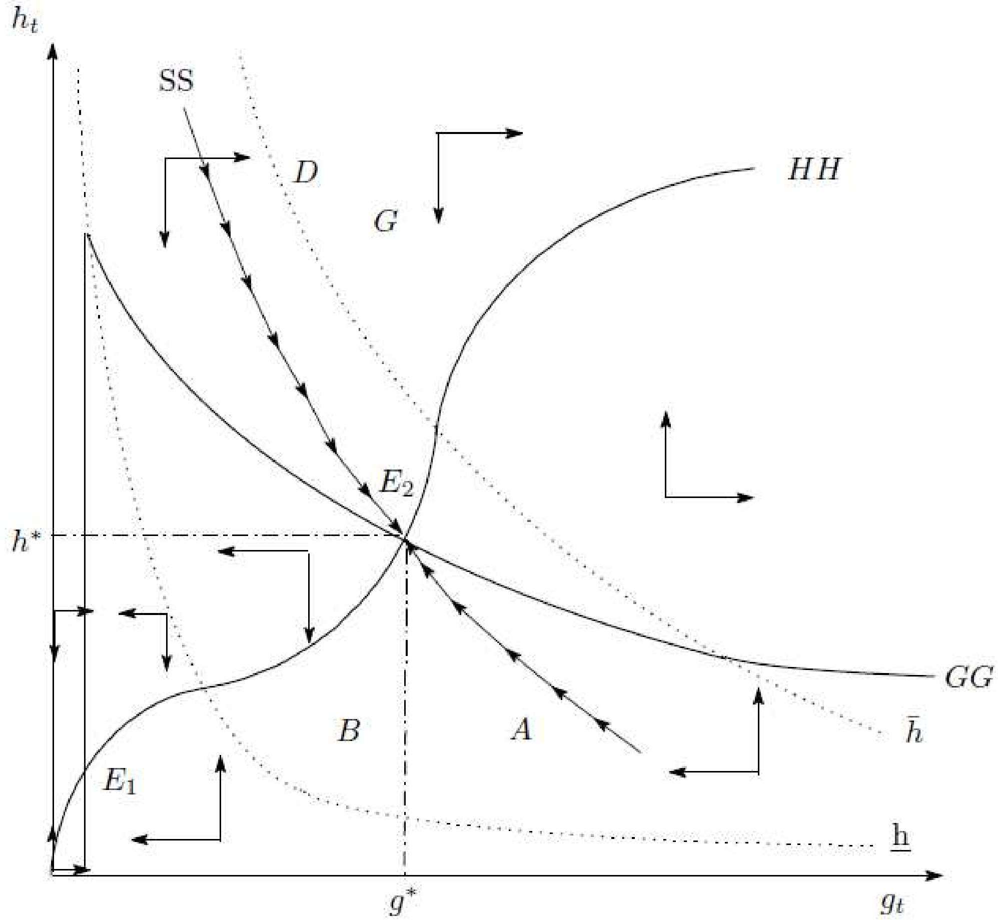


Figure 5: Phase Dynamics When $\eta > \bar{\eta}$

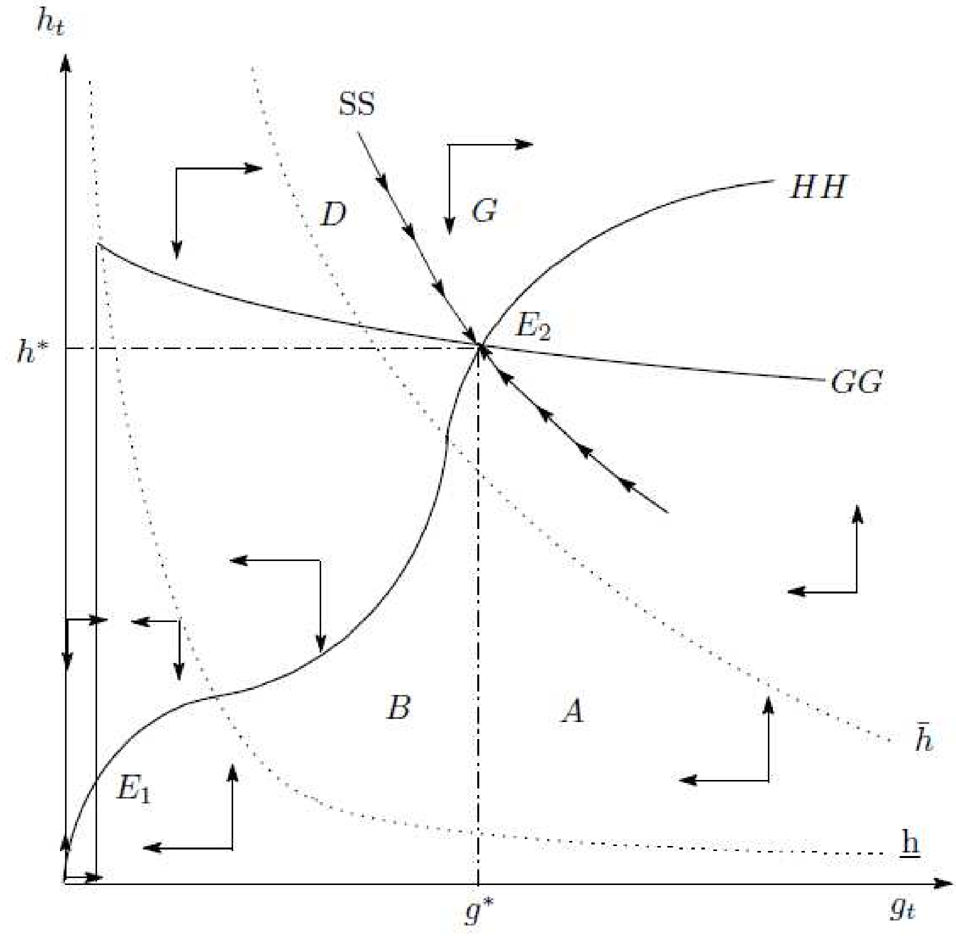


Figure 6: Phase Dynamics When $\eta < \bar{\eta}$

| Description | Parameter | Value |
|---|---|---|
| The utility weight on offspring | $\beta$ | 0.5 |
| Productivity in the modern sector | $\eta$ | 12 |
| Productivity in the traditional sector | $y$ | 12 |
| Parameters for human capital accumulation | $\theta$ | 1.2 |
| | $\sigma$ | 0.3 |
| Parameters for the revealed probability of child labor | $\gamma$ | 0.6 |
| | $\alpha$ | 0.7 |
| | $\pi$ | 0.2 |
| Depreciation rate of human infrastructure | $\delta$ | 0.6 |
| Labor income tax rate | $\tau$ | 0.05 |
| Initial points $(h_0, g_0)$ | $A$ | (0.8, 2.0) |
| | $B$ | (0.8, 1.0) |
| | $G$ | (2.2, 2.0) |
| Initial point $(h_0, g_0)$ | $D$ | (2.2, 1.8) |
| Productivity | $\eta$ | 15 |
| | | 5 |

Table 1: The Simulation Parameters

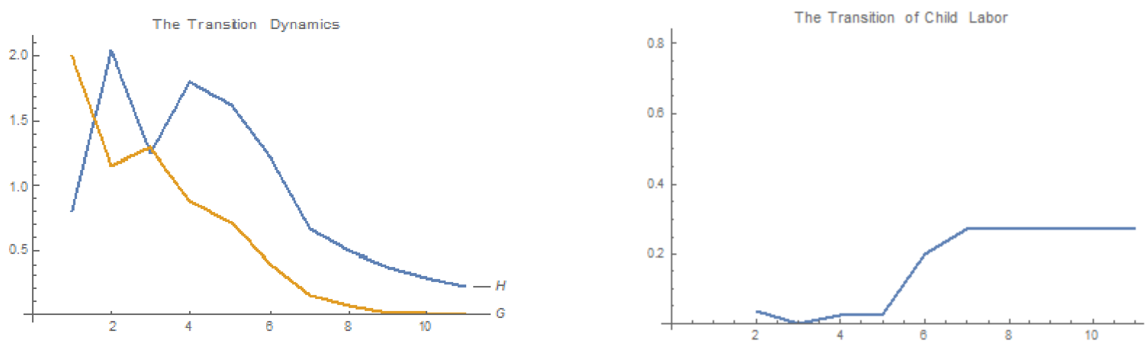


Figure 7: The Transition Dynamics (A)

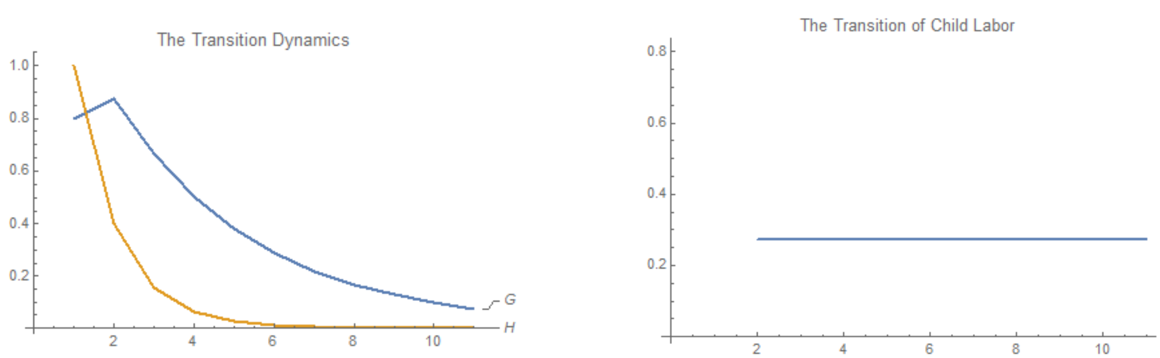


Figure 8: The Transition Dynamics (B)

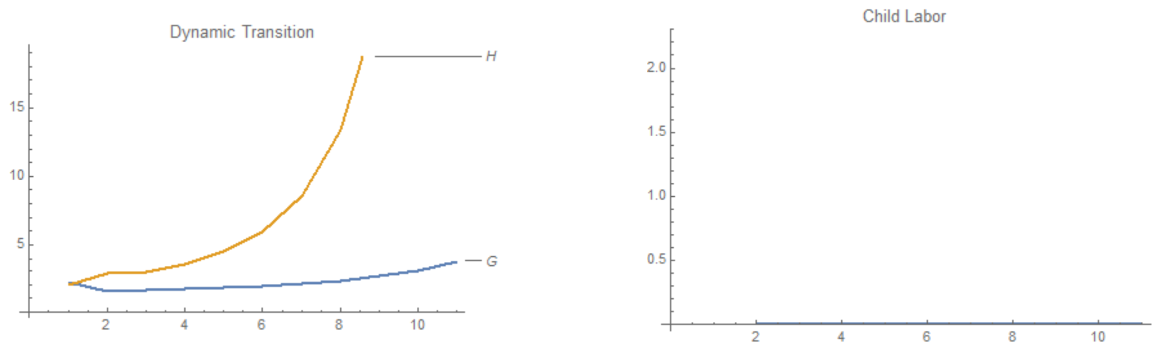


Figure 9: The Transition Dynamics (G)

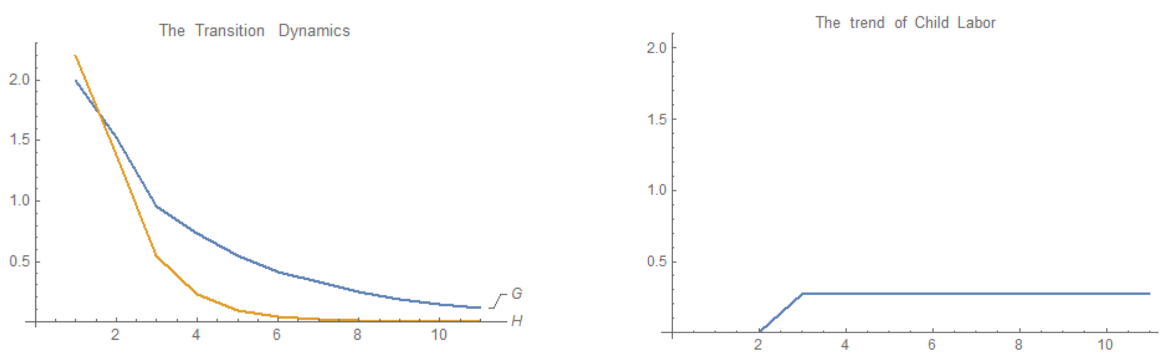


Figure 10: The Transition Dynamics (D) $\eta$=7

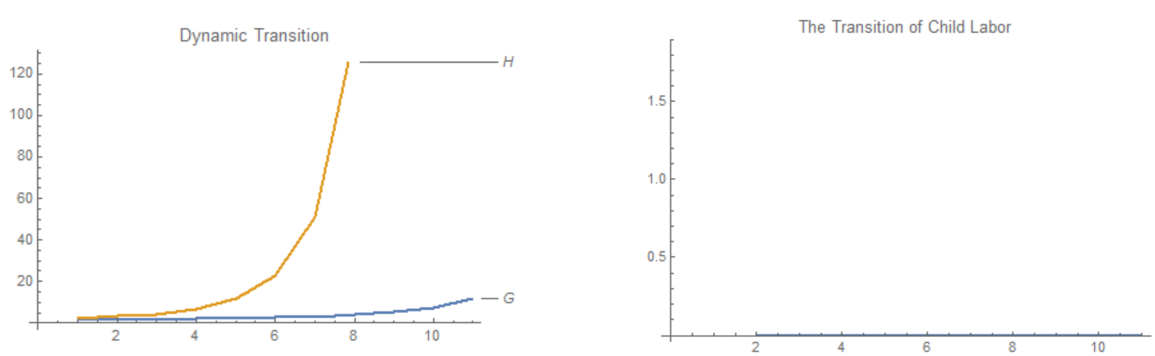


Figure 11: The Transition Dynamics (D) $\eta$=15

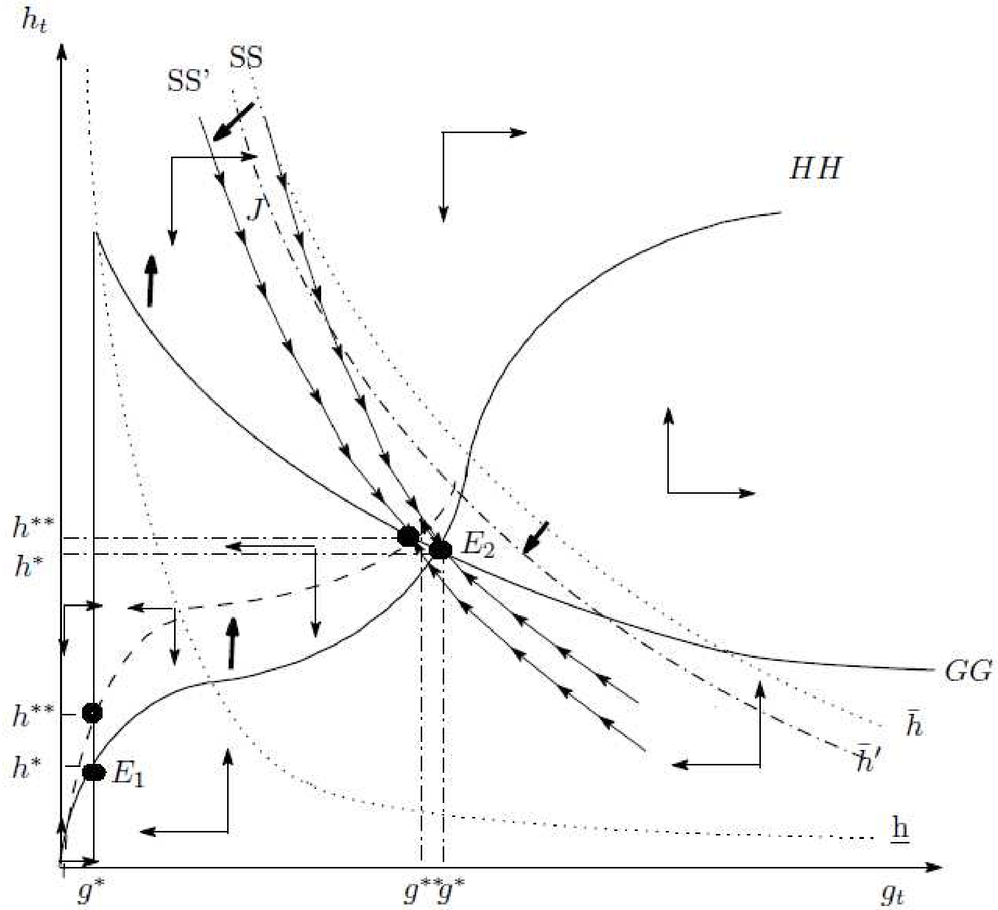


Figure 12: The Effect of Aid on Child Labor Protection

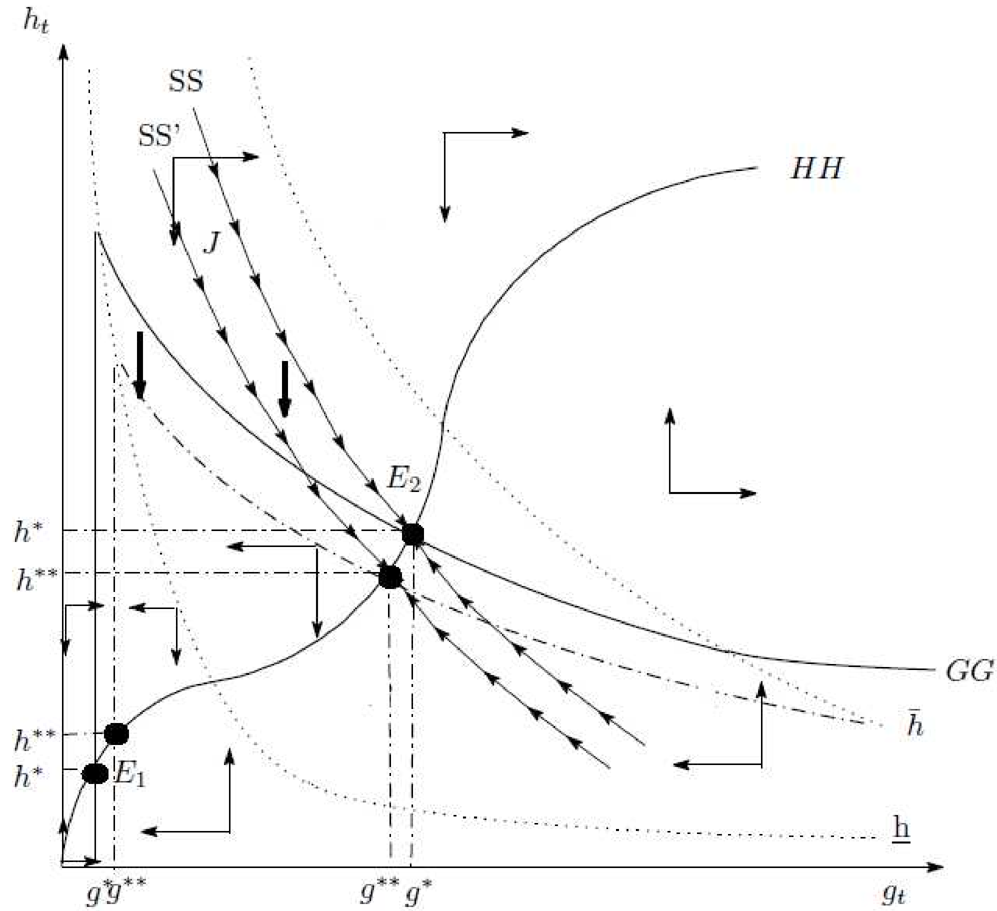


Figure 13: The Effect of Aid on Human Infrastructure $a < \gamma$

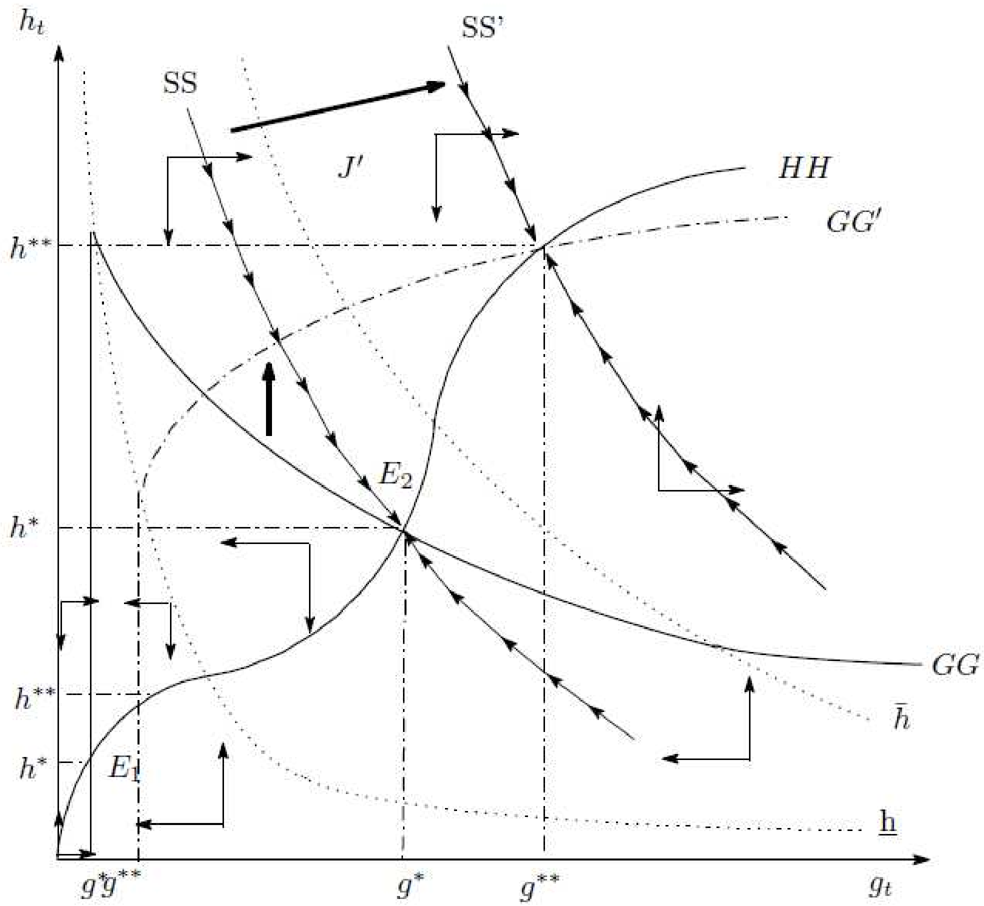


Figure 14: The Effect of Aid on Human Infrastructure $a > \gamma$