Preference matching, Income, and Population Distribution in Urban and Adjacent Rural Regions

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by

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Abstract

We analyze the impact of preference matching and income on the distribution of the population in an aggregate economy consisting of an urban and an adjacent rural region. It costs more (less) to live in the urban (rural) region. Individuals choose freely to live either in the urban or in the rural region. They differ in their incomes. These incomes are uniformly distributed on the unit interval. Our analysis leads to four results. First, when the cost differential parameter satisfies a condition, both regions are occupied in the equilibrium. Second, when this parametric condition holds, in any equilibrium in which the mean income of individuals varies across the two regions, every resident of the rural region has a lower income than every resident of the urban region. Third, there exists an income threshold and all individuals with higher (lower) incomes choose to live in the urban (rural) region. Finally, in the equilibrium with income sorting, it is possible to make everyone better off by slightly modifying their residential choices.

Keywords: Income, Population Distribution, Preference Matching, Rural Region, Urban Region

JEL Codes: O18, R12
1. Introduction

Development economists and regional scientists have both been interested in studying rural and urban regions but typically for different reasons. Development economists have pointed out that the process of development tends to be uneven and that this unevenness leads to rapid growth in some parts of the economy while other parts are left behind to stagnate and even shrink. As such, it is no surprise to learn from a prominent development economics text---see Ray (1998, p. 345)---that by “far the most important structural feature of developing countries is the distinction between the rural and the urban sector.” Today, this dichotomy may hold in some developing nations but it is not a very useful way of thinking about regions in an increasingly urbanizing world.

Using the lens of development economics, urban regions are typically dynamic, they display relatively rapid rates of economic growth, they are industrial, and they are often technologically advanced. In contrast, rural regions are generally not as dynamic, they are frequently agricultural, they display slow economic growth rates, and they are technologically backward. This perspective explains why the early literature in development economics---see Lewis (1954), Sen (1966), and Rakshit (1982)---was preoccupied with the modeling and the analysis of the so called dual economy. Even so, it should be noted that this traditional focus on dual development has changed substantially in the past few decades.⁴

Unlike development economists, regional scientists have focused primarily on rural and urban regions in the developed world. Even though they have recognized that many rural regions in the developed world are agricultural in nature, in the main, regional scientists have not studied

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⁴ Also see Oladi and Gilbert (2011).
rural regions as the provider of resource flows to urban regions in a dual economy setting. Instead, they have pointed to rural-urban disparities in metrics such as education (Jordan et al. (2014), health (Hall et al. 2006), and income (Yamamoto 2008). This focus has led regional scientists to address questions pertaining to the viability of rural regions as independent entities in the face of ever increasing urbanization and the concomitant rise of cities.\(^5\) In turn, this concern with the viability of rural regions has now given rise to a literature on the connections between so called “leading” and “lagging” regions where, unsurprisingly, rural regions are frequently the lagging regions.\(^6\)

The historical view of rural regions as backward notwithstanding, in many of the so called Organization for Economic Cooperation and Development (OECD) countries, this view is flawed. In this regard, the work of Ward and Brown (2009), Korpela et al. (2010), and Skelhorn et al. (2014) tells us that in many OECD nations, rural regions are energetic and vibrant places because of, inter alia, a low population density, an abundance of natural landscapes, and a clean, healthy, and safe environment. Despite the energy and the vibrancy of rural regions, Ward and Brown (2009, p. 1237) rightly note that “[r]arely are rural and urban areas, and the complex flows and relationships which bind them together, considered in an integrated and holistic way.”

Given this state of affairs, we would now like to emphasize three points. First, the literature on leading and lagging regions has theoretically studied linkages between rural and urban regions but the linkages studied thus far\(^7\) are typically production or technology related

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\(^6\) Examples of recent contributions to this literature include Batabyal and Nijkamp (2014a, 2014b), Batabyal and Beladi (2015), Batabyal (2018), and the many references cited in these four papers.

\(^7\) See the references cited in footnote 6.
linkages. Second, despite the increasing salience of the interactions between the residents of urban regions and adjacent rural regions, there are no theoretical studies of these interactions. Finally, the interactions mentioned in the second point above are directly related to the preferences of the residents and they have very little to do with either production or technological factors.

Given the lacuna in the literature noted in the preceding paragraph, our basic objective in this paper is to analyze the impact of preference matching and income on the distribution of the population in an aggregate economy consisting of an urban region and an adjacent rural region. Section 2 delineates the theoretical framework. In this framework, it is more expensive to live in the urban region than in the adjacent rural region. Individuals choose freely to live either in the urban or in the rural region. However, they differ in their incomes and these (random) incomes are uniformly distributed on the interval $[0, 1]$. Section 3 shows that when a parameter in our model satisfies a particular condition (on which more below), both regions are occupied in the equilibrium. Section 4 supposes that the section 3 parametric condition holds and then demonstrates that in any equilibrium in which the mean income of individuals varies across the two regions, every resident of the rural region has a lower income than every resident of the urban region. Section 5 solves for an income threshold $I^*$ and then points out that all individuals with incomes higher (lower) than this threshold choose to live in the urban (rural) region. Section 6 notes that in the equilibrium with income sorting, it is possible to make everyone better off by slightly modifying their residential choices. Finally, section 7 concludes and then discusses two extensions of the research described in this paper.

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See Pateman (2011), Long and Reed (2013), Millsap (2017), and Kimhi and Menahem-Carmi (2017) for further details on this point.
2. The Theoretical Framework

The aggregate economy of interest is made up of a rural and an urban region. We index these two regions with the subscript \( j \) where \( j = R, U \). The subscript \( R \) denotes the rural region and the subscript \( U \) denotes the urban region. Consistent with existing evidence---see Pateman (2011) and Long and Reed (2013)---we suppose that it is more expensive to live in the urban region than in the adjoining rural region. Specifically, it costs \( c_R > 0 \) to live in the rural region and \( c_U = c_R + \theta \) to live in the urban region where \( \theta > 0 \). We suppose that \( c_R \) and \( c_U \) are constant over space and time.

Individuals in our aggregate economy differ in terms of their incomes. These incomes, which we denote by \( I \), are assumed to be uniformly distributed on the interval \( [0, 1] \). It is important to understand that individuals in our aggregate economy care about the incomes of those living in their region. The mean income in region \( j = R, U \) is a function of the average value of \( I \) in that region and we denote this mean by \( \hat{I}_j, j = R, U \).

An individual with income \( I \) who chooses to live in region \( j \) with mean income \( \hat{I}_j \) obtains gross utility denoted by \( U_j = (1 + I)(1 + \hat{I}_j) \). We know that this same individual bears a cost of living in region \( j \) that is given by \( c_j \). Therefore, putting these two pieces of information together, this individual’s net utility function is

\[
U_j = (1 + I)(1 + \hat{I}_j) - c_j. \tag{1}
\]

Inspecting equation (1), it should be clear to the reader that richer individuals place a greater value on living together with other rich individuals.\(^9\) This is the sense in which the net utility function in equation (1) displays the phenomenon of preference matching that we alluded to in

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\(^9\) See Badger (2015) for empirical evidence on this point.
the last paragraph of section 1. Our next task is to study a particular circumstance involving the parameter $\theta$ in which individuals reside in both the rural and in the urban regions in the equilibrium.

3. Individuals Reside in Both Regions

Suppose that all the individuals in our aggregate economy make their residential choices simultaneously. In addition, suppose that the difference in the cost of living between the urban region and the adjacent rural region is neither too high nor too low. We model this last feature by stipulating that the cost differential parameter $\theta \in (0.5, 1)$. We now want to study the residential choices of all the individuals in an equilibrium which has the property that no individual wishes to move given the residential choices of everyone else in the aggregate economy.

Let us begin by supposing that all individuals live in the rural region. The mean income now is $\bar{I}_R = 0.5$. The point to note is that the poorest individual will now have no incentive to move to the urban region. To see this, observe that the individual’s net utility in the rural region is

$$U_R = (1 + 0)(1 + \bar{I}_R) - c_R = 1.5 - c_R. \quad (2)$$

In contrast, if this poorest individual moves to the urban region then the mean income changes to $\bar{I}_U = 0$. Note that for the moment, we are considering the case in which all individuals live in the rural region. So, there is zero population in the urban region. Also, the poorest individual has zero income. Therefore, when this poorest individual moves to the urban region, the population of the urban region consists of one person whose income is zero. Therefore, the mean income in

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10 Note that an individual’s decision to live either in the urban region or in the adjoining rural region does not tell us anything about where this individual works. Even though Loehr (2016), Helliwell et al. (2018), and others have written about the scenario in which an individual works in the urban region but lives in the rural region, and this is one possible outcome in our model, other scenarios are also possible. Since we are interested mainly in studying how the phenomenon of preference matching influences where individuals live, we do not model the commuting decisions of individuals who live in one region but choose to work in the other.
the urban region with this single resident who has zero income is also zero and in symbols we have \( \hat{I}_U = 0 \). That said, the moving individual’s net utility in the urban region is

\[
U_U = (1 + 0)(1 + 0) - c_R - \theta = 1 - c_R - \theta, \tag{3}
\]

where the right-hand-side (RHS) follows because \( c_U = c_R + \theta \). Comparing the RHSs of equations (2) and (3), it is clear that the poorest individual benefits by staying in the rural region.

What about the \textit{richest} individual? When residing in the rural region, this individual’s net utility is

\[
U_R = (1 + 1)(1 + \hat{I}_R) - c_R = 3 - c_R \tag{4}
\]

because \( \hat{I}_R = 0.5 \). In contrast, if this individual moves to the urban region in the model, then his net utility is

\[
U_U = (1 + 1)(1 + 1) - c_R - \theta = 4 - c_R - \theta \tag{5}
\]

since \( \hat{I}_U = 1 \). When the richest individual moves from the rural region to the urban region, the population of the urban region increases from zero to one and this one person is the richest individual whose income is one. This is why we have \( \hat{I}_U = 1 \). Now, inspecting the RHSs of equations (4) and (5) we see that the net utility from moving to the urban region exceeds the net utility from staying in the rural region when \( \theta < 1 \). This finding tells us that the richest individual \textit{may} have an incentive to move to the urban region.

A similar line of reasoning tells us that if all individuals live in the urban region then the richest individual will gain nothing by moving to the adjoining rural region. In contrast, the poorest individual will gain by moving to the rural region as long as \( \theta > 0.5 \). Therefore, combining the arguments we have made thus far in this section, we conclude that the individuals populating our aggregate economy will choose to live in both the rural and in the urban regions in equilibrium as long as the cost differential parameter \( \theta \) lies in the interval \((0.5, 1)\). Note that
our reasoning thus far has two additional implications for extreme values of \( \theta \). First, when \( \theta > 1 \), all the individuals in our aggregate economy will choose to live in the rural region \textit{exclusively}. Second, when \( \theta < 0.5 \), all the individuals will live in the urban region and the rural region will have \textit{no} residents. Let us now study the properties of the equilibrium when the condition \( \theta \in (0.5, 1) \) holds and the mean income of individuals varies across the rural and the urban regions.

4. Mean Income Varies Across the Two Regions

We want to show that when the mean income varies across the two regions, every individual living in the rural region must have a \textit{lower} income than every individual living in the urban region. To demonstrate this result, we proceed with a proof by contradiction. Now, suppose that an individual with high income \( I_H \) lives in the rural region and that an individual with low income \( I_L \) lives in the urban region. Clearly, \( I_H > I_L \).

If the residential choices supposed in the preceding paragraph constitute an equilibrium then it must be the case that

\[
(1 + I_H)(1 + \hat{I}_U) - c_R > (1 + I_H)(1 + \hat{I}_U) - c_R - \theta, \quad (6)
\]

and

\[
(1 + I_L)(1 + \hat{I}_U) - c_R - \theta > (1 + I_L)(1 + \hat{I}_R) - c_R. \quad (7)
\]

After several algebraic steps, the inequalities in (6) and (7) can be simplified to

\[
\frac{\theta}{1+I_H} > \hat{I}_U - \hat{I}_R \text{ and } \hat{I}_U - \hat{I}_R > \frac{\theta}{1+I_L}. \quad (8)
\]

The two inequalities in (8) together tell us that
which is clearly false. Therefore, our initial supposition that an individual with high income \( I_H \) lives in the rural region and that an individual with low income \( I_L \) lives in the urban region cannot be an equilibrium. In turn, this tells us that when \( \theta \in (0.5, 1) \) and the mean income varies across the two regions, every individual living in the rural region must have a lower income than every individual living in the urban region. We now solve for an income threshold \( I^* \) and then show that all individuals with incomes higher (lower) than this threshold choose to live in the urban (rural) region.

5. The Income Threshold

To summarize, we have demonstrated thus far that when the cost differential parameter \( \theta \in (0.5, 1) \), the individuals in our aggregate economy choose to live in both regions and that the total population of individuals is divided by income. Now suppose that the highest income in the rural region is \( I^* \). Then, the mean income in this region is \( 0.5I^* \). In addition, the mean income in the urban region now is \( 0.5(1 + I^*) \). Since \( I^* \) is the threshold level of income, it follows that the individual with this level of income will be indifferent between residing in the rural and in the urban regions. Mathematically, we can express this indifference as

\[
(1 + I^*)(1 + 0.5I^*) - c_R = (1 + I^*)(1 + 0.5(I^* + I^*)) - c_R - \theta. \tag{10}
\]

Solving equation (10) for the income threshold \( I^* \), we get

\[
I^* = 2\theta - 1. \tag{11}
\]

Combining our results from sections 3 and 4 with equation (11), we conclude that all individuals in our aggregate economy who have incomes higher than the threshold level \( I^* \) will choose to live in the urban region and those who have incomes that are less than this same
threshold will choose to live in the adjoining rural region. Our final task in this paper is to show that in the equilibrium with income sorting, it is possible to make everyone in our aggregate economy better off by slightly modifying their residential choices.

6. Making Everyone Better Off

The reader should understand that the equilibrium with income sorting\textsuperscript{11} that we have been studying thus far cannot be improved upon by moving only one individual from one region to the other. This is because the equilibrium under study is individually rational which means that no individual wishes to move given the living choices of all the other individuals in our aggregate economy.

Therefore, to make everyone better off, it will be necessary to move a group of individuals from one region to the other. Specifically, suppose we move all individuals to the rural region. This move clearly raises the utility of all current rural region residents because the mean income rises. But what can we say about the utility of current urban region residents? Mathematically, our proposed move to the rural region benefits an arbitrary urban region resident with income \( I \) as long as the inequality

\[
(1 + I)(1.5) - c_R > (1 + I)(1.5 + 0.5I^*) - c_R - \theta
\]

is satisfied.

From equation (11), we know that \( I^* = 2\theta - 1 \). Using this value to simplify the inequality in (12), we get

\[
\frac{1}{2\theta - 1} > I.
\]

\textsuperscript{11} This “income sorting equilibrium” can also be thought of as a “location equilibrium.”
Inspecting the inequality in (13), we see that the ratio on the left-hand-side (LHS) attains a minimum of 1 when $\theta = 1$. Therefore, this inequality is clearly satisfied for all incomes $l \leq 1$.

This last result shows that situating the entire population in our aggregate economy in the rural region is, in our model, a better outcome than the outcome in which the individuals live in both regions.

The result we have just obtained is a limiting result and hence it ought not to be interpreted literally. That said, if we think of the urban region in terms of its central business district (CBD) then this result is consistent with an empirical feature of the CBDs of many North American cities. What we mean by this observation is that many of these CBDs are veritable “ghost towns” after office hours because most of the office workers who work in these CBDs do not actually live in or around these CBDs but commute in from elsewhere.$^{12}$ This completes our discussion of preference matching, income, and population distribution in urban and adjoining rural regions.

7. Conclusions

In this paper, we studied the effect of preference matching and income on the distribution of the population in an aggregate economy consisting of an urban and an adjacent rural region. It was more expensive to live in the urban region than in the rural region. Individuals chose freely to live either in the urban or in the rural region. They differed in their incomes and these incomes were uniformly distributed on the interval $[0, 1]$. Our analysis led to four findings. First, when the cost differential parameter $\theta$ satisfied a particular condition, both regions were occupied in the equilibrium. Second, once again when this same parametric condition was satisfied, in any equilibrium in which the mean income of individuals varied across the two regions, every

$^{12}$ See Polese (2014), Pham (2015), and Carmody (2016) for a more detailed corroboration of this claim.
resident of the rural region had a lower income than every resident of the urban region. Third, there existed an income threshold \( I^* \) and all individuals with higher (lower) incomes chose to live in the urban (rural) region. Finally, in the equilibrium with income sorting, it was possible to make everyone better off by slightly modifying their residential choices.

The analysis in this paper can be extended in a number of different directions. Here are two suggestions for extending the research described here. First, consistent with our observations in footnote 10, it would be useful to explicitly model the commuting decisions of individuals who live in one region but work in the other. In this case, the net utility functions of the different individuals would display the phenomenon of preference matching and aversion to long commute times. Second, following the work of Batabyal (2018), it would be helpful to study a scenario in which the decision to live in either the rural or the urban region is based on the differential valuation placed by the individuals in our aggregate economy on the provision of one or more local public goods. Studies that analyze these aspects of the underlying problem about individual living choices will increase our understanding of the connections between preference based behavior on the one hand and the residential appeal of both rural and urban regions on the other.

**Compliance with Ethical Standards**

Both authors declare that they have no funding to report for this paper. Both authors also declare that they have no conflict of interest to report with regard to the research conducted for this paper.
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