# Monitoring Intensity and Technology Choice in A Model of Unemployment

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Abstract

The interaction among a firm's choices of output, technology, and monitoring intensity is studied in a general equilibrium model. Firms engage in oligopolistic competition and unemployment is a result of the existence of efficiency wages. The following results are derived analytically. First, an increase in the cost of exerting effort leads a firm to choose a more advanced technology and a lower level of monitoring intensity. Second, an increase in the discount rate does not change a firm's choices of technology and monitoring intensity. Third, an increase in the elasticity of substitution among goods leads a firm to choose higher levels of monitoring intensity and technology. In a model in which the level of monitoring is exogenously given, there is a negative relationship between the wage rate and the monitoring intensity. In this model with endogenously chosen monitoring intensity, the wage rate and the monitoring intensity can move either in the same direction or in opposite directions.

**Keywords:** Unemployment, efficiency wages, monitoring intensity, the choice of technology, oligopoly

**JEL Classification Numbers:** E24, J64, L13

## 1. Introduction

In their seminal paper on unemployment, Shapiro and Stiglitz (1984) have demonstrated the existence of equilibrium unemployment. In their model, a worker chooses whether to exert effort or to shirk. A worker will shirk if the lifetime utility from shirking is higher than that from not shirking. One interesting result that Shapiro and Stiglitz establish is the existence of a negative relationship between the level of monitoring and the wage rate. Researchers have conducted empirical and experimental studies to test this negative relationship. First, Groshen and Krueger (1990) have demonstrated that the wages of nurses tend to fall with the extent of supervision, consistent with the efficiency wage theory. Second, in his study of contractual workers in the petrochemical industry, Rebitzer (1995) has shown that high levels of supervision are associated with lower wage levels, supporting the efficiency wage theory. Finally, in their experimental study of telephone call centers in which callers solicit donations, Nagin et al. (2002) have found that a significant ratio of workers cheat (shirk) when monitoring intensities of potential cheating decrease. However, there are empirical studies such as Neal (1993) failing to find the negative relationship between monitoring and pay. The efficiency wage approach has been challenged on this inconsistency between theory and empirical evidence.

The inconsistency between theory and some evidence motivates the incorporation of endogenous monitoring intensity into the efficiency wage approach of unemployment. In Shapiro and Stiglitz (1984), the possibility that shirking is detected is assumed to be exogenously given. In concluding their paper, they have discussed a firm's choice of monitoring intensity as a generalization of their model. In reality, as shown in time clocks and spot checks, firms choose monitoring intensities and spend significant amounts of resources in preventing shirking (Dickens et al., 1989). A higher level of monitoring by the firm can increase the probability that a worker's shirking is detected. Suppose the wage rate of a firm is exogenously given due to the existence of unions or government regulations. This exogenous wage rate may not be the optimal one to prevent shirking. Then having the choice of monitoring intensity can be valuable to this firm. A model of endogenous choice of monitoring intensity will be helpful to understand how monitoring intensities are affected by more fundamental parameters, such as the discount rate of an individual.

If a firm can choose its monitoring intensity, it can also choose other aspects of its operation, such as the levels of technology and output. Prendergast (1990) shows that it is feasible for firms to choose technologies. He also shows that a firm's choice of technology is affected by its level of output: a higher level of output induces a firm to choose a technology with a higher fixed cost but a lower marginal cost of production.

In this paper, we study a firm's choices of monitoring intensity, technology, and output in a general equilibrium model. Similar to Neary (2003, 2016), Qiu and Zhou (2007), and Liu and Wang (2010), firms engage in oligopolistic competition.[1] There is a continuum of goods. Capital and labor are the two factors of production. To produce each good, there is a continuum of technologies with different levels of fixed and marginal costs of production. A more advanced technology has a higher fixed cost but a lower marginal cost of production. Unemployment is a result of the existence of efficiency wages (Shapiro and Stiglitz, 1984).

We show that an increase in the cost of exerting effort leads a firm to choose a more advanced technology, a lower level of monitoring intensity, and the equilibrium wage rate

[1] The importance of oligopoly in a modern society is illustrated in Chandler (1990). The Second Industrial Revolution occurred in the United States near the end of the 19th century and the start of the 20th century. During that period, with increasing returns in production, management, and distribution, important industries such as the steel industry began to be dominated by oligopolistic firms.

increases. When the amount of capital increases, a firm chooses a higher level of monitoring intensity, and the equilibrium wage rate also increases. Thus, monitoring intensity and the wage rate can move either in the same direction or in opposite directions when the level of monitoring intensity is endogenously chosen. This is different from the negative relationship between monitoring intensity and the wage rate when the level of monitoring is exogenously given. That is, the incorporation of endogenous monitoring intensity helps explain empirical evidence (Goerke, 2001; Allgulin and Ellingsen, 2002).

As discussed in Neary (2003, 2016) and Ruffin (2003), there are some potential difficulties of incorporating oligopoly into a general equilibrium model. One difficulty is that a firm engaging in oligopolistic competition may have market power in both the product market and the labor market. The determination of the wage rate may not be straightforward when a firm has market power in the labor market. Neary (2003, 2016) has proposed the incorporation of a continuum of goods to eliminate a firm's market power in the labor market, which is assumed in this paper. In this paper, while a firm has market power in the product market and can influence the wage rate through its choice of monitoring intensity, this firm has to take the price index and the unemployment rate as given. Thus, the wage rate is determined through the non-shirking condition for a worker. This assumption that firms have different degrees of market power in different markets is like the "semi-small" open economy assumption discussed in Turnovsky (2000, p. 358) under which a country is assumed to be small in the international asset market and in the market for its imported goods. However, this country is assumed to have market power in the market for its export goods.

The pioneering work of Shapiro and Stiglitz (1984) has started a long line of literature. For example, Kimball (1994) has addressed out-of-the-steady-state dynamics of the efficiency wage models. Davis and Harrigan (2011) have studied a model of international trade in which firms engage in monopolistic competition and firms differ in their monitoring intensities. A firm's choices of technology and monitoring intensity are not examined in those models. Goerke (2001) and Allgulin and Ellingsen (2002) have considered a firm's choice of monitoring intensity. There are some significant differences between their models and this one. First, their models do not focus on equilibrium unemployment. Second, technology choice is not addressed in their models.

This paper is related to models on technology choice, such as Zhou (2004, 2009, 2015). Zhou (2004) provides a formal analysis of the mutual dependence between the division of labor and the extent of the market in a general equilibrium model. Zhou (2009) studies the impact of population growth on the possibility of industrialization in a dynamic model. One essential difference between this paper and Zhou (2004, 2009) is that unemployment and monitoring intensity are not modeled in Zhou (2004, 2009). Zhou (2015) shows that a higher level of capital stock may not reduce the unemployment rate. One important difference between this paper and Zhou (2015) is that monitoring intensity is endogenously chosen in this model while it is exogenously given in Zhou (2015).

The plan of the paper is as follows. Section 2 specifies the model and establishes the equilibrium conditions for a steady state. Section 3 establishes the existence of a unique equilibrium and conducts comparative statics to explore the properties of the steady state. Section 4 discusses some potential generalizations and extensions of the model and concludes. The Appendix contains alternative proofs of some results and provides some additional results not proved in the text.

## 2. The model

Time is continuous. There is a continuum of goods indexed by $\varpi \in [0,1]$. All goods have the same costs of production and they enter a consumer's utility function in a symmetric way.[2] The amount of capital is $K$, which is exogenously given and does not change over time. The interest rate is $r$. Capital is owned equally by all individuals and each individual receives $\eta$ from owning capital, which will be determined endogenously. The wage rate is $w$ and the unemployment rate is $u$. The size of the population is $L$ and it does not change over time.

### 2.1. Individual behavior

Each individual may supply one unit of labor if not shirking. An individual's income is $I$. For an employed individual, the level of income is the sum of wage income and return from owing capital: $I = w + \eta$. For an unemployed individual, the level of income is the return from owing capital: $I = \eta$.

The price of good $\varpi$ is $p(\varpi)$. An individual's consumption of good $\varpi$ is $c(\varpi)$. A consumer's budget constraint is

[2] Similar to Neary (2003, 2016), the purpose of assuming a continuum of goods rather than one good is to make sure that a firm treats the price index as given.

$$\int_0^1 p(\varpi)c(\varpi)d\varpi = I. \tag{1}$$

An individual's discount rate is $\rho$ and the cost of exerting effort is $e$. For the constant $0 < \alpha < 1$, a consumer's utility function is specified as

$$\int_0^\infty U(t)e^{-\rho t}dt,$$

$$U(t) = \left[\int_0^1 c(\varpi)^\alpha d\varpi\right]^{1/\alpha} - se. \tag{2}$$

If an individual exerts effort, $s$ is equal to one; otherwise, it is equal to zero. For $\sigma \equiv \frac{1}{1-\alpha}$, let $P$ denote the price index: $P \equiv \left[\int_0^1 p(\varpi)^{1-\sigma} d\varpi\right]^{1/(1-\sigma)}$. Then from the specification of the utility function in equation (2), $U(t) = \frac{I}{P} - se$. From the specification of the utility function, the absolute value of the elasticity of demand for a good is $\sigma$.

If an individual shirks, the probability that shirking is detected is $q$, which is chosen optimally by a firm. The cost of monitoring in terms of the amount of capital used is $\frac{\theta}{2}q^2$, where $\theta$ is a positive constant. A higher spending on monitoring will increase the probability that shirking is detected. If a worker is found shirking, this worker is fired.

The exogenous job separation rate for a worker is $b$. For a shirker, the asset equation is given by

$$\rho V_E^S = U(w+\eta) + (b+q)(V_u - V_E^S).$$

Rearrangement of this equation yields

$$V_E^S = \frac{U(w+\eta)+(b+q)V_u}{\rho+b+q}. \tag{3}$$

The asset equation for a non-shirker is given by

$$\rho V_E^N = U(w+\eta) - e + b(V_u - V_E^N).$$

Rearrangement of this equation yields

$$V_E^N = \frac{U(w+\eta)-e+bV_u}{\rho+b}. \tag{4}$$

An individual will not shirk if the lifetime utility of a shirker is smaller than that for a non-shirker. From equations (3) and (4), the non-shirking condition is

$$U(w+\eta) \geq \rho V_u + \frac{(\rho+b+q)e}{q}. \tag{5}$$

For an unemployed individual, the probability per unit time of acquiring a job is $a$, which is endogenously determined. The asset equation for an unemployed individual is

$$\rho V_u = U(\eta) + a(\max[V_E^N, V_E^S] - V_u). \tag{6}$$

In equilibrium, $\max[V_E^N, V_E^S] = V_E^N$. Plugging equation (4) into equation (6) yields

$$\rho V_u = \frac{a[U(w+\eta)-e]+(\rho+b)U(\eta)}{a+b+\rho}. \tag{7}$$

Plugging equation (7) into (5), the non-shirking condition becomes

$$U(w+\eta) \geq \frac{a[U(w+\eta)-e]+(\rho+b)U(\eta)}{a+b+\rho} + \frac{(\rho+b+q)e}{q}.$$

In equilibrium, the above relationship will hold with equality. With the utility function specified in equation (2), the non-shirking condition changes to

$$\frac{w+\eta}{P} = \frac{a\left(\frac{w+\eta}{P}-e\right)+(\rho+b)\frac{\eta}{P}}{a+b+\rho} + \frac{(\rho+b+q)e}{q}.$$

As will be discussed in detail later, each good is produced by $m$ firms and each firm produces $x$ units of output at a marginal cost of $\beta$ units of labor. Thus, total employment is $\int_0^1 m(\varpi)\beta x(\varpi)d\varpi$. With a job separation rate of $b$, the flow into the unemployment pool is $b\int_0^1 m(\varpi)\beta x(\varpi)d\varpi$. With a job acquisition rate $a$, the flow out is $a(L - \int_0^1 m(\varpi)\beta x(\varpi)d\varpi)$. Since time is continuous, the change in unemployment rate is

$$\dot{u} = \frac{1}{L}\left\{b\int_0^1 m(\varpi)\beta x(\varpi)d\varpi - a[L - \int_0^1 m(\varpi)\beta x(\varpi)d\varpi]\right\}.$$

In a steady state, there is no change in unemployment rate: $\dot{u} = 0$. Thus, $a = b\frac{\int_0^1 m(\varpi)\beta x(\varpi)d\varpi}{L-\int_0^1 m(\varpi)\beta x(\varpi)d\varpi}$. From equation (12) later, $a = b\frac{1-u}{u}$. Plugging this value of $a$ into the non-shirking condition, the non-shirking condition becomes[3]

$$\frac{w}{P} = e + \frac{e}{q}\left(\frac{b}{u} + \rho\right). \tag{8}$$

## 2.2. Firm behavior

Since the price index is determined by prices of a continuum of goods and a firm produces only one good, a firm takes the price index as given. Firms producing the same good engage in Cournot competition. The number of identical firms producing good $\varpi$ is $m(\varpi)$. Like Zhou (2004), Wen and Zhou (2012), and Gong and Zhou (2014), to produce each good, there is a continuum of technologies indexed by $n > 0$. A higher value of $n$ indicates a more advanced

[3] To emphasize the wage rate is the equilibrium value, we may add an asterisk mark over this variable. That is, we may use $w^*$ instead of $w$ in equation (8).

technology. For technology $n$, the level of fixed cost in terms of the amount of capital used is $f(n)$ and the level of marginal cost in terms of the amount of labor used is $\beta(n)$. A more advanced technology has a higher fixed cost but a lower marginal cost of production: $f'(n) > 0$ and $\beta'(n) < 0$.[4] When a firm's level of output is $x$, this firm's total revenue is $px$, cost of capital is $\frac{\theta}{2}q^2r + f(n)r$, and labor cost is $\beta(n)xw(q)$. Thus, a firm's profit is

$$px - \frac{\theta}{2}q^2r - f(n)r - \beta(n)xw(q).$$

A firm takes the unemployment rate and other firms' output as given and chooses its output, technology, and monitoring intensity optimally to maximize its profit.[5] First, the first order condition for a firm's optimal choice of output $x$ is

$$x + p\frac{\partial x}{\partial p} - \beta w = 0.$$

Since a firm takes other firms' output as given in a Cournot competition and the absolute value of the elasticity of demand of a consumer is $\sigma$, this optimal choice of output leads to marginal revenue equals marginal cost:

$$p\left(1 - \frac{1}{m\sigma}\right) = \beta w. \tag{9}$$

For equation (9), when the number of firms is one ($m = 1$), this equation degenerates to $p\left(1 - \frac{1}{\sigma}\right) = \beta w$, which is commonly seen in models of monopolistic competition.

Second, the first order condition for a firm's optimal choice of technology $n$ requires that

$$-f'(n)r - \beta'(n)xw(q) = 0. \tag{10}$$

Equation (10) shows that a firm's choice of technology is affected by its level of output. Other things equal, a higher level of output will induce a firm to choose a more advanced technology because the higher fixed cost can be spread to a higher level of output and thus the average cost is lower.

Third, the first order condition for a firm's optimal choice of monitoring intensity $q$ requires that

$$-\theta qr + \frac{\beta xe}{q^2}\left(\frac{b}{u} + \rho\right) = 0. \tag{11}$$

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[4] To make sure that the second order condition for the optimal choice of technology is satisfied, we also assume that $f''(n) \geq 0$ and $\beta''(n) \geq 0$.

[5] When a firm chooses its monitoring intensity and technology, this firm does not take strategic impact on other firms' output into consideration. This is consistent with the "open loop" approach in the R&D literature when firms engage in oligopolistic competition, as studied in Vives (2008).

The second order condition for a firm's optimal choice of monitoring intensity is always satisfied. Equation (11) shows the tradeoff faced by a firm in choosing monitoring intensity. The marginal cost from choosing a higher level of monitoring is a higher level of capital used for monitoring. The marginal benefit is that the wage rate is lower. That is, a firm takes the impact of monitoring intensity on the wage rate into consideration (as shown in equation (8)) when choosing monitoring intensity.

Equations (10) and (11) can be used to demonstrate the interaction between choices of technology and monitoring intensity. When two decisions mutually reinforce one another, they may be called "strategic complements". When two decisions mutually offset one another, they may be called "strategic substitutes" (Bulow, Geanakoplos, and Klemperer, 1985). Partial differentiation of equation (10) with respect to $q$ or partial differentiation of equation (11) with respect to $n$, the resulting cross derivatives are negative. Since the cross derivatives with respect to the profit function are negative, choices of technology and monitoring intensity are "strategic substitutes". However, with firms engaging in oligopolistic competition, there are other effects affecting the choices of technology and monitoring intensity. Thus, a firm's choice of technology and choice of monitoring intensity may not always move in opposite directions.

### 2.3. Market clearing conditions

For the labor market, each firm demands $\beta x$ units of labor and total demand for labor of this economy is $\int_0^1 m(\varpi)\beta x(\varpi)d\varpi$. Effective supply of labor of this economy is $(1-u)L$. Equilibrium in the labor market requires that

$$\int_0^1 m(\varpi)\beta x(\varpi)d\varpi = (1-u)L. \tag{12}$$

For the market for capital, each firm demands $f+\frac{\theta q^2}{2}$ units of capital and total demand for capital of this economy is $\int_0^1 m(\varpi)\left(f+\frac{\theta q^2}{2}\right)d\varpi$. Total supply of capital of this economy is $K$. The clearance of the market for capital requires that

$$\int_0^1 m(\varpi)\left(f+\frac{\theta q^2}{2}\right)d\varpi = K. \tag{13}$$

Total income for all individuals from owning capital is $\eta L$ and total return to capital in this economy is $rK$. In equilibrium, they should be equal:

$$\eta L = rK. \tag{14}$$

For the market for goods, total demand for goods from all consumers is $\eta L + (1-u)wL$ and total value of output is $\int_0^1 p\,(\varpi)m(\varpi)x(\varpi)d\varpi$. The clearance of the market for goods requires that

$$\eta L + (1-u)wL \; = \int_0^1 p\,(\varpi)m(\varpi)x(\varpi)d\varpi. \tag{15}$$

In this model, the number of firms producing a good is a real number rather than restricted to be an integer. In equilibrium, the number of firms producing a good is determined by the zero-profit condition.[6] The zero-profit condition requires that

$$px - \left(f + \frac{\theta}{2}q^2\right)r - \beta x w = 0. \tag{16}$$

We focus on a symmetric equilibrium in which all goods have the same production technology, monitoring intensity, and price. Also, the number of firms producing each good and the level of output of each firm are the same. Since the total measure of goods is one, for simplicity, we drop the integration operator in a symmetric equilibrium. In a steady state, equations (8)-(16) form a system of nine equations defining a system of nine variables $p, x, m, n, w, u, q, \eta$ and $r$ as functions of exogenous parameters.[7] A steady state is a tuple $(p, x, m, n, w, u, q, \eta, r)$ satisfying equations (8)-(16). For the rest of the paper, a representative good is used as the numeraire:

$$p \equiv 1.$$

Since the price of a representative good is normalized to one and all goods with a total measure of one have the same price, the price index will equal to one. Thus, equation (8) simplifies to

$$w = e + \frac{e}{q}\left(\frac{b}{u} + \rho\right). \tag{17}$$

## 3. Comparative statics

In this section, we study properties of the steady state. From equation (9), $m = 1/[\sigma(1-\beta w)]$. Plugging this value of $m$ into equation (13) yields

$$f + \frac{\theta}{2}q^2 = (1-\beta w)\sigma K. \tag{18}$$

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[6] For examples that firms engage in Cournot competition with free entry, see Dasgupta and Stiglitz (1980), Zhang (2007), and Liu and Wang (2010).

[7] When equations (8)-(14) and (16) are satisfied, equation (15) is automatically satisfied. That is, one equation is redundant. This redundancy is consistent with Walras's law.

The system of equations (8)-(16) defining the steady state is reduced to the following system of three equations defining three endogenous variables $u$, $n$, and $q$ as functions of exogenous parameters:[8]

$$\Gamma_1 \equiv \beta e\sigma K\left(\frac{b}{u}+\rho\right)-\theta q^3 = 0, \tag{19a}$$

$$\Gamma_2 \equiv -f'\beta-\beta'\left(\sigma K-f-\frac{\theta}{2}q^2\right)=0, \tag{19b}$$

$$\Gamma_3 \equiv \frac{1}{\beta}-e-\frac{f+\frac{3\theta q^2}{2}}{\beta\sigma K}=0. \tag{19c}$$

Partial differentiation of equations (19a)-(19c) with respect to $u, n, q, \rho, K, \theta$ and $e$ yields

$$\begin{pmatrix} \frac{\partial\Gamma_1}{\partial u} & \frac{\partial\Gamma_1}{\partial n} & \frac{\partial\Gamma_1}{\partial q} \\ 0 & \frac{\partial\Gamma_2}{\partial n} & \frac{\partial\Gamma_2}{\partial q} \\ 0 & \frac{\partial\Gamma_3}{\partial n} & \frac{\partial\Gamma_3}{\partial q} \end{pmatrix}\begin{pmatrix} du \\ dn \\ dq \end{pmatrix} = -\begin{pmatrix} \frac{\partial\Gamma_1}{\partial\rho} \\ 0 \\ 0 \end{pmatrix}d\rho-\begin{pmatrix} \frac{\partial\Gamma_1}{\partial K} \\ \frac{\partial\Gamma_2}{\partial K} \\ \frac{\partial\Gamma_3}{\partial K} \end{pmatrix}dK$$

$$-\begin{pmatrix} \frac{\partial\Gamma_1}{\partial\theta} \\ \frac{\partial\Gamma_2}{\partial\theta} \\ \frac{\partial\Gamma_3}{\partial\theta} \end{pmatrix}d\theta-\begin{pmatrix} \frac{\partial\Gamma_1}{\partial e} \\ 0 \\ \frac{\partial\Gamma_3}{\partial e} \end{pmatrix}de-\begin{pmatrix} \frac{\partial\Gamma_1}{\partial\sigma} \\ \frac{\partial\Gamma_2}{\partial\sigma} \\ \frac{\partial\Gamma_3}{\partial\sigma} \end{pmatrix}d\sigma. \tag{20}$$

Partial differentiation of equations (19a)-(19c) reveals that $\frac{\partial\Gamma_1}{\partial u}<0, \frac{\partial\Gamma_2}{\partial n}<0, \frac{\partial\Gamma_2}{\partial q}<0, \frac{\partial\Gamma_3}{\partial q}<0$, and $\frac{\partial\Gamma_3}{\partial n}>0$. Thus, the determinant of the coefficient matrix of (20) is negative: $\Delta \equiv \frac{\partial\Gamma_1}{\partial u}\left(\frac{\partial\Gamma_2}{\partial n}\frac{\partial\Gamma_3}{\partial q}-\frac{\partial\Gamma_2}{\partial q}\frac{\partial\Gamma_3}{\partial n}\right)<0$. With $\Delta$ nonsingular, a unique equilibrium exists (Turnovsky, 1977, chap. 2). We thus proceed to explore properties of the steady state. While we may expect there is no monotonic relationship between monitoring intensity and the wage rate when the level of monitoring is endogenously chosen, what are the circumstances that the two variables move in the

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[8] The derivation of equations (19a)-(19c) is as follows. First, from equation (16), $r = x(1-\beta w)/(f+\frac{\theta}{2}q^2)$. Plugging this value of $r$ into equation (16) yields $\beta e\left(\frac{b}{u}+\rho\right)\left(f+\frac{\theta}{2}q^2\right)-\theta(1-\beta w)q^3 = 0$. Plugging equation (18) into this equation yields equation (19a). Second, plugging $r = x(1-\beta w)/(f+\frac{\theta}{2}q^2)$ into equation (10) yields $f'(1-\beta w)+\beta' w\left(f+\frac{\theta}{2}q^2\right)=0$. Plugging the value of $w$ from equation (18) into this equation yields equation (19b). Third, plugging the value of $w$ from $f+\frac{\theta}{2}q^2=(1-\beta w)\sigma K$ into equation (17) yields $\frac{1}{\beta}-e-\frac{e}{q}\left(\frac{b}{u}+\rho\right)-\frac{f+\frac{\theta}{2}q^2}{\beta\sigma K}=0$. Plugging equation (18) into this equation yields equation (19c).

same or opposite directions? To answer this question, we need to study how various parameters affect the monitoring intensity and the wage rate.

Technological progress such as development of the internet can change the level of monitoring costs. An increase in $\theta$ means that the level of monitoring costs increases. The following proposition studies the impact of an increase in the level of monitoring costs on the equilibrium level of monitoring intensity.

Proposition 1: An increase in the level of monitoring costs leads a firm to choose a lower level of monitoring intensity.

Proof: Applying Cramer's rule to equation (20) yields

$$\frac{dq}{d\theta} = \frac{\partial \Gamma_1}{\partial u}\left(\frac{\partial \Gamma_2}{\partial \theta}\frac{\partial \Gamma_3}{\partial n} - \frac{\partial \Gamma_2}{\partial n}\frac{\partial \Gamma_3}{\partial \theta}\right)/\Delta < 0. \blacksquare$$

Proposition 1 is intuitive. An increase in the level of monitoring costs increases the marginal cost of choosing a higher monitoring intensity without changing the marginal benefit. Thus, the equilibrium level of monitoring decreases. From the Appendix, a change in the level of monitoring costs changes neither the equilibrium level of technology nor the wage rate. From equation (17), since the wage rate does not change and the monitoring intensity decreases, an increase in the level of monitoring costs will cause an increase in the equilibrium unemployment rate.

An increase in the discount rate indicates an individual is less concerned with future. Individuals in different countries may have quite different discount rates as shown in the different saving rates among countries. How will difference in the discount rate affect the unemployment rate and levels of monitoring intensity and technology?[9]

Proposition 2: An increase in the discount rate increases the unemployment rate and changes neither the level of technology nor the level of monitoring.

Proof: Applying Cramer's rule to equation (20) yields

$$\frac{dn}{d\rho} = 0,$$

[9] The impact of a change in the exogenous job separation rate is like that from a change in the discount rate and is not presented here.

$$\frac{dq}{d\rho} = 0,$$

$$\frac{du}{d\rho} = -\frac{\partial \Gamma_1}{\partial \rho} \Big/ \frac{\partial \Gamma_1}{\partial u} > 0. \blacksquare$$

The intuition behind Proposition 2 is as follows. From equation (10), the discount rate does not directly affect the marginal cost and marginal benefit of the choice of technology. The discount rate could affect the choice of technology indirectly through affecting variables such as the level of output and the interest rate. However, the level of output is positively related to the interest rate, and impact from those two variables on the level of technology work in opposite directions and cancel out. Overall, the equilibrium level of technology is not affected by a change in the discount rate. Similarly, neither the equilibrium monitoring intensity nor the equilibrium wage rate is affected by a change in the discount rate.

The following proposition studies the impact of an increase in the amount of capital on the levels of monitoring intensity and the equilibrium level of technology.

Proposition 3: An increase in the amount of capital leads a firm to choose higher levels of monitoring intensity and technology.

Proof: From equation (19c), $\frac{f+\frac{3}{2}\theta q^2}{\sigma K} < 1$. Partial differentiation of equations (19b) and (19c) yields

$$\frac{\partial \Gamma_2}{\partial q}\frac{\partial \Gamma_3}{\partial K} - \frac{\partial \Gamma_2}{\partial K}\frac{\partial \Gamma_3}{\partial q} = \frac{\theta q \beta'}{\beta K}\left(\frac{f+\frac{3}{2}\theta q^2}{\sigma K} - 3\right) > 0.$$

Applying Cramer's rule to equation (20) yields

$$\frac{dq}{dK} = \frac{\partial \Gamma_1}{\partial u}\left(\frac{\partial \Gamma_2}{\partial K}\frac{\partial \Gamma_3}{\partial n} - \frac{\partial \Gamma_2}{\partial n}\frac{\partial \Gamma_3}{\partial K}\right)/\Delta > 0,$$

$$\frac{dn}{dK} = \frac{\partial \Gamma_1}{\partial u}\left(\frac{\partial \Gamma_2}{\partial q}\frac{\partial \Gamma_3}{\partial K} - \frac{\partial \Gamma_2}{\partial K}\frac{\partial \Gamma_3}{\partial q}\right)/\Delta > 0. \blacksquare$$

When the amount of capital increases, some of the increased capital is absorbed in choosing a higher level of technology and some is absorbed in choosing a higher level of monitoring intensity. In equilibrium, both the level of technology and the level of monitoring intensity increase. From the Appendix, the equilibrium wage rate increases when the amount of capital increases. From equation (17), since both the wage rate and the monitoring intensity increase, the equilibrium unemployment rate decreases when the amount of capital increases.

Proposition 3 can be used to illustrate some events in the process of the Industrial Revolution. Initially, merchants provided inputs and purchased outputs from independent producers. Later workers produced at factories. Workers were monitored more intensively than independent producers and factories use more advanced technologies. Using Proposition 3, those increases in the level of monitoring intensity and technology could be viewed as a result of capital accumulation at that time.

The following proposition studies the impact of an increase in the cost of exerting effort on the level of technology, monitoring intensity, and the wage rate.

Proposition 4: An increase in the cost of exerting effort leads a firm to choose a lower level of monitoring intensity and a more advanced technology. In addition, the equilibrium wage rate increases.

Proof: Applying Cramer's rule to equation (20) yields

$$\frac{dq}{de} = -\frac{\partial \Gamma_1}{\partial u}\frac{\partial \Gamma_2}{\partial n}\frac{\partial \Gamma_3}{\partial e}/\Delta < 0,$$

$$\frac{dn}{de} = \frac{\partial \Gamma_1}{\partial u}\frac{\partial \Gamma_2}{\partial q}\frac{\partial \Gamma_3}{\partial e}/\Delta > 0.$$

From equation (18), the wage rate can be expressed as

$$w = \frac{1}{\beta}\left(1 - \frac{f + \frac{\theta}{2}q^2}{\sigma K}\right).$$

While $e$ does not show up in the above equation for the wage rate, $e$ can affect the wage rate through $n$ and $q$. Differentiation of this equation with respect to $e$ yields

$$\frac{dw}{de} = -\left[\frac{f'}{\beta\sigma K} + \frac{\beta'}{\beta^2}\left(1 - \frac{f + \frac{\theta}{2}q^2}{\sigma K}\right)\right]\frac{dn}{de} - \frac{\theta q}{\beta\sigma K}\frac{dq}{de}.$$

From equation (19b), $\frac{f'}{\beta\sigma K} + \frac{\beta'}{\beta^2}\left(1 - \frac{f + \frac{\theta}{2}q^2}{\sigma K}\right) = 0$. Thus, $\frac{dw}{de} = -\frac{\theta q}{\beta\sigma K}\frac{dq}{de} > 0$. ∎

The intuition behind Proposition 4 is as follows. When the cost of exerting effort increases, monitoring intensity needs to decrease to make sure that the non-shirking condition remains valid. When the level of monitoring decreases, the wage rate increases. When a firm chooses a more advanced technology, the marginal benefit comes from saving of wage costs. A higher wage rate increases a firm's incentive to choose a more advanced technology because the saving from wage costs increases. Thus, equilibrium level of technology increases. Like the proof of Proposition 4, it can be shown that impact of a change in the cost of exerting effort on the unemployment rate is ambiguous.

When $\sigma$ increases, the elasticity of substitution among goods increases. The elasticity of substitution may be used to measure the degree of competition in an industry. Will a more competitive industry have a higher level of monitoring? The following proposition studies the impact of a change in the elasticity of substitution among goods on the levels of monitoring intensity and the equilibrium level of technology.

Proposition 5: An increase in the elasticity of substitution among goods leads a firm to choose higher levels of monitoring intensity and technology.

Proof: From equation (19c), $f + \frac{3}{2}\theta q^2 < \sigma K$. Partial differentiation of equations (19b) and (19c) yields

$$\frac{\partial \Gamma_2}{\partial q}\frac{\partial \Gamma_3}{\partial \sigma} - \frac{\partial \Gamma_2}{\partial \sigma}\frac{\partial \Gamma_3}{\partial q} = \frac{\theta q \beta'}{\beta K}\left(\frac{f+\frac{3}{2}\theta q^2}{\sigma K} - 3\right) > 0.$$

Applying Cramer's rule to equation (20) yields

$$\frac{dq}{d\sigma} = \frac{\partial \Gamma_1}{\partial u}\left(\frac{\partial \Gamma_2}{\partial \sigma}\frac{\partial \Gamma_3}{\partial n} - \frac{\partial \Gamma_2}{\partial n}\frac{\partial \Gamma_3}{\partial \sigma}\right)/\Delta > 0,$$

$$\frac{dn}{d\sigma} = \frac{\partial \Gamma_1}{\partial u}\left(\frac{\partial \Gamma_2}{\partial q}\frac{\partial \Gamma_3}{\partial \sigma} - \frac{\partial \Gamma_2}{\partial \sigma}\frac{\partial \Gamma_3}{\partial q}\right)/\Delta > 0. \blacksquare$$

To understand Proposition 5, a firm's choices of technology and monitoring intensity increase with the level of output. A higher elasticity of substitution among goods tends to increase a firm's output, and equilibrium technology and monitoring intensity increase. From the Appendix, the equilibrium wage rate increases when the elasticity of substitution among goods increases. From equation (17), because both the wage rate and the monitoring rate increase, the equilibrium unemployment rate decreases when the elasticity of substitution among goods increases.

It is interesting that the level of monitoring intensity and the level of technology are not affected by the population size. The reasoning is as follows. Population size does not affect the marginal cost and benefit of choices of technology and monitoring intensity directly. It could affect the choices of technology and monitoring intensity indirectly through affecting the level of output and the interest rate. Because impact from those two variables cancel out, equilibrium levels of technology and monitoring intensity are not affected by population size.

From the discussions after Propositions 1-5, when the monitoring intensity is endogenously chosen, the level of monitoring intensity and the wage rate can move either in the same direction or in opposite directions. Also, they can be unrelated. That is, there is no monotonic relationship between the monitoring intensity and the wage rate. This is different from the case that the wage rate decreases with the level of monitoring intensity when monitoring intensity is exogenously given.

## 4. Conclusion

In this paper, we have studied the interaction among a firm's choices of output, technology, and monitoring intensity in a general equilibrium model. In this model, firms engage in oligopolistic competition and the existence of efficiency wages leads to unemployment. The model is tractable, and we have established the following results analytically. First, an increase in the amount of capital induces a firm to choose higher levels of monitoring intensity and technology, and the equilibrium wage rate increases. Second, an increase in the cost of exerting effort for a worker leads a firm to choose a more advanced technology and a lower level of monitoring intensity. Third, an increase in the discount rate of a consumer increases the unemployment rate and does not change the levels of technology and monitoring intensity. Finally, an increase in the level of monitoring cost or a decrease in the elasticity of substitution among goods leads a firm to choose a lower level of monitoring intensity and the equilibrium unemployment rate increases.

There are some interesting generalizations and extensions of the model. First, in this model the amount of capital is exogenously given. To address how time preference affects unemployment rate, the model can be generalized to the case that the amount of capital is endogenously determined by saving. Second, in this model firms are homogenous in terms of monitoring intensity and marginal cost of production. To fit reality better, firm heterogeneity through difference in either monitoring intensity or marginal cost may be introduced into the model. Finally, this paper studies a closed economy. As shown in Brecher, Chen, and Yu (2013), the impact of trade on unemployment is an interesting issue. With the introduction of one additional sector of production, the model may be extended to study how a country's comparative advantage is affected by monitoring intensity and how the opening of international trade affects the monitoring intensity and the unemployment rate.

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**Appendix**

The system of equations (8)-(16) defining the steady state can be reduced alternatively into the following system of three equations defining three variables $w$, $n$, and $q$ as functions of exogenous parameters:

$$\Omega_1 \equiv f + \frac{\theta}{2}q^2 - (1-\beta w)\sigma K = 0, \quad \text{(A1)}$$

$$\Omega_2 \equiv -f' - \beta' w\sigma K = 0, \quad \text{(A2)}$$

$$\Omega_3 \equiv \theta q^2 - \beta\sigma K(w-e) = 0. \quad \text{(A3)}$$

The derivation of equations (A1)-(A3) is as follows. First, equation (A1) is the same as equation (18). Second, equation (A2) is derived by plugging the value of $r$ from equation (16) and equation (18) into equation (10). Third, equation (A3) is derived by plugging the value of $u$ from equation (17) into equation (19a).

Partial differentiation of equations (A1)-(A3) yields

$$\begin{pmatrix} \frac{\partial\Omega_1}{\partial w} & 0 & \frac{\partial\Omega_1}{\partial q} \\ \frac{\partial\Omega_2}{\partial w} & \frac{\partial\Omega_2}{\partial n} & 0 \\ \frac{\partial\Omega_3}{\partial w} & \frac{\partial\Omega_3}{\partial n} & \frac{\partial\Omega_3}{\partial q} \end{pmatrix} \begin{pmatrix} dw \\ dn \\ dq \end{pmatrix} = -\begin{pmatrix} \frac{\partial\Omega_1}{\partial K} \\ \frac{\partial\Omega_2}{\partial K} \\ \frac{\partial\Omega_3}{\partial K} \end{pmatrix} dK - \begin{pmatrix} \frac{\partial\Omega_1}{\partial\theta} \\ 0 \\ \frac{\partial\Omega_3}{\partial\theta} \end{pmatrix} d\theta - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial\Omega_3}{\partial e} \end{pmatrix} de - \begin{pmatrix} \frac{\partial\Omega_1}{\partial\sigma} \\ \frac{\partial\Omega_2}{\partial\sigma} \\ \frac{\partial\Omega_3}{\partial\sigma} \end{pmatrix} d\sigma. \quad \text{(A4)}$$

Let $\Delta_\Omega$ denote the determinant of the coefficient matrix of (A4). For stability, it is assumed that $\Delta_\Omega < 0$. Comparative statics results from (A4) are the same as those from (20). For example, applying Cramer's rule to (A4) yields $\frac{dw}{de} = \frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial e}/\Delta_\Omega > 0$. This result is the same as that in Proposition 4. However, the system (A4) can be used to derive the following additional results. Those results are not available from (20).

Partial differentiation of (A1) and (A3) reveals that $\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_3}{\partial K} - \frac{\partial\Omega_1}{\partial K}\frac{\partial\Omega_3}{\partial q} = \frac{2qf\theta}{K} > 0$. With $\frac{\partial\Omega_2}{\partial n} < 0$ and $\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial K}\frac{\partial\Omega_3}{\partial n} > 0$, it is clear that $\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial K} - \frac{\partial\Omega_1}{\partial K}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial q} - \frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial K}\frac{\partial\Omega_3}{\partial n} < 0$. Partial differentiation of equations (A1) and (A3) reveals that $\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_3}{\partial\sigma} - \frac{\partial\Omega_1}{\partial\sigma}\frac{\partial\Omega_3}{\partial q} = \frac{2fq\theta}{\sigma} > 0$. With $\frac{\partial\Omega_2}{\partial n} < 0$ and $\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial\sigma}\frac{\partial\Omega_3}{\partial n} > 0$, $\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial\sigma} - \frac{\partial\Omega_1}{\partial\sigma}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial q} - \frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial\sigma}\frac{\partial\Omega_3}{\partial n} < 0$. Applying Cramer's rule to (A4) yields

$$\frac{dw}{dK} = \left(\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial K} - \frac{\partial\Omega_1}{\partial K}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial q} - \frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial K}\frac{\partial\Omega_3}{\partial n}\right)/\Delta_\Omega > 0,$$

$$\frac{dw}{d\sigma} = \left(\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial\sigma} - \frac{\partial\Omega_1}{\partial\sigma}\frac{\partial\Omega_2}{\partial n}\frac{\partial\Omega_3}{\partial q} - \frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_2}{\partial\sigma}\frac{\partial\Omega_3}{\partial n}\right)/\Delta_\Omega > 0.$$

Partial differentiation of equations (A1) and (A3) reveals that $\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_3}{\partial\theta} - \frac{\partial\Omega_1}{\partial\theta}\frac{\partial\Omega_3}{\partial q} = \theta q^3 - \theta q^3 = 0$. Applying Cramer's rule to (A4) yields

$$\frac{dw}{d\theta} = \frac{\partial\Omega_2}{\partial n}\left(\frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_3}{\partial\theta} - \frac{\partial\Omega_1}{\partial\theta}\frac{\partial\Omega_3}{\partial q}\right)/\Delta_\Omega = 0,$$

$$\frac{dn}{d\theta} = \frac{\partial\Omega_2}{\partial w}\left(\frac{\partial\Omega_1}{\partial\theta}\frac{\partial\Omega_3}{\partial q} - \frac{\partial\Omega_1}{\partial q}\frac{\partial\Omega_3}{\partial\theta}\right)/\Delta_\Omega = 0.$$

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