Should the Government Subsidize Innovation or Automation?

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Abstract

This study introduces automation into a Schumpeterian growth model to explore the effects of R&D and automation subsidies. R&D subsidy increases innovation and growth but decreases the share of automated industries and the degree of capital intensity in the aggregate production function. Automation subsidy has the opposite effects on these macroeconomic variables. Calibrating the model to US data, we find that raising R&D subsidy increases the welfare of high-skill workers but decreases the welfare of low-skill workers and capital owners, whereas increasing automation subsidy increases the welfare of high-skill workers and capital owners but decreases the welfare of low-skill workers.

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1 Introduction

Automation allows machines to perform tasks that are previously performed by workers. On the one hand, automation may be a threat to the employment of workers. For example, a recent study by Frey and Osborne (2017) examines 702 occupations and finds that almost half of them could be automated within the next two decades. On the other hand, automation reduces the cost of production and frees up resources for more productive activities. Given the rising importance of automation,[1] we develop a growth model with automation to explore its effects on the macroeconomy.

Specifically, we introduce automation in the form of capital-labor substitution into a Schumpeterian growth model. Then, we apply the model to explore the effects of R&D subsidy versus automation subsidy on innovation, economic growth and the welfare of different agents in the economy. In our model, an industry uses labor as the factor input before automation occurs. When the industry becomes automated, it then uses capital as the factor input. Innovation in the form of a quality improvement can arrive at an automated or unautomated industry. When an innovation arrives at an automated industry, the industry becomes unautomated and once again uses labor as the factor input.[2] Therefore, the share of automated industries, which is also the degree of capital intensity in the aggregate production function, is endogenously determined by automation and innovation.

In this growth-theoretic framework, we obtain the following results. An increase in R&D subsidy leads to a higher level of innovation and a higher rate of economic growth. However, the increase in skilled labor for innovation crowds out skilled labor for automation and leads to a lower share of automated industries as well as a lower degree of capital intensity in the aggregate production function. This effect is absent in previous studies with exogenous capital intensity in production. Capital intensity affects output and welfare because it determines the returns to scale of capital, which is a reproducible factor that can be accumulated. An increase in automation subsidy has a negative effect on innovation and economic growth but a positive effect on the share of automated industries and capital intensity in production.

Calibrating the model to aggregate US data, we find that increasing R&D subsidy increases the welfare of high-skill workers but decreases the welfare of low-skill workers and capital owners. Intuitively, high-skill workers engage in innovative activities and benefit from R&D subsidies, which however hurt low-skill workers and capital owners due to the tax burden from increasing subsidies and the lower capital share of income. Increasing automation subsidy increases the welfare of high-skill workers and capital owners but decreases the welfare of low-skill workers. Intuitively, high-skill workers also engage in automation and benefit from the subsidies, whereas capital owners benefit from the higher capital share of income. However, low-skill workers are worse off due to the tax burden from increasing subsidies and the lower labor share of income. Simulating transition dynamics, we find that increasing the automation subsidy rate by 5 percentage points leads to a welfare gain equivalent to a permanent increase in consumption of 3.13% for capital owners and 2.35% for high-skill workers as well as a welfare loss of 1.47% for low-skill workers.

[1] See for example Agrawal *et al.* (2018) for a comprehensive discussion on artificial intelligence, which is the latest form of automation.

[2] See Acemoglu and Restrepo (2018) for empirical evidence that "humans have a comparative advantage in new and more complex tasks."

This study relates to the literature on innovation and economic growth. Romer (1990) develops the seminal R&D-based growth model in which innovation is driven by the invention of new products. Then, Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) develop the Schumpeterian quality-ladder model in which innovation is driven by the development of higher-quality products. Many subsequent studies in this literature use variants of the R&D-based growth model to explore the effects of R&D subsidies; see for example, Peretto (1998), Segerstrom (2000), Zeng and Zhang (2007), Impullitti (2010), Chu *et al.* (2016) and Chu and Cozzi (2018). These studies do not feature automation, and hence, the degree of capital intensity in the aggregate production function is exogenous or simply zero.

This study also relates to the literature on automation and innovation; see Aghion *et al.* (2017) for a comprehensive discussion of this literature. An early study by Zeira (1998) develops a growth model with capital-labor substitution, which forms the basis of automation in subsequent studies. Zeira (2006) contributes to the literature by introducing endogenous invention of technologies into Zeira (1998). Peretto and Seater (2013) propose a growth model with factor-eliminating technical change in which R&D serves to increase capital intensity in the production process. Our study relates to Peretto and Seater (2013) by considering both factor-eliminating technical change (i.e., automation) and factor-augmenting technical change (i.e., innovation) and exploring their relative importance on growth and welfare. Recent studies by Acemoglu and Restrepo (2018) and Hemous and Olson (2018) generalize the model in Zeira (1998) and introduce directed technological change between automation and variety expansion in order to explore the effects of automation on the labor market and income inequality.[3] Our study complements these interesting studies by embedding endogenous automation into the Schumpeterian quality-ladder model.[4] While Acemoglu and Restrepo (2018) assume in their variety-expanding model that when a new unautomated product arrives, a previous automated product becomes obsolete, our Schumpeterian model features an endogenous cycle of innovation and automation on a fixed variety of products.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 compares the effects of the two subsidies. The final section concludes.

2 A Schumpeterian growth model with automation

We introduce automation in the form of capital-labor substitution as in Zeira (1998) into a canonical Schumpeterian growth model. We consider a cycle of automation and innovation. An unautomated industry that currently uses labor as the factor input can become automated and then use capital as the factor input. Innovation in the form of a quality improvement can arrive at an automated or unautomated industry. When an innovation arrives at an automated industry, the industry becomes unautomated and once again uses labor as the factor input until the next automation arrives.[5] We will derive the equilibrium condition that supports this cycle of automation and innovation.

[3] See also Prettner and Strulik (2017) for a variety-expanding model with automation and education.

[4] See also Aghion *et al.* (2017) who develop a Schumpeterian model with *exogenous* automation.

[5] Acemoglu and Restrepo (2018) make a similar assumption that all new inventions are first produced by labor until they are automated.

2.1 Agents

There are three types of agents in the model. Their lifetime utility functions are given by

$$U^j = \int_0^\infty e^{-\rho t} \ln c_t^j dt, \tag{1}$$

where $j \in \{k, l, h\}$. c_t^k is the consumption of a representative capital owner. c_t^l is the consumption of a representative low-skill worker. c_t^h is the consumption of a representative high-skill worker. For simplicity, we assume that they all have the same discount rate $\rho > 0$.[6]

Only the capital owner accumulates (tangible and intangible) capital. He/she maximizes utility subject to the following asset-accumulation equation:

$$\dot{a}_t + \dot{k}_t = r_t a_t + (R_t - \delta) k_t - c_t^k. \tag{2}$$

a_t is the real value of assets (i.e., the share of monopolistic firms), and r_t is the real interest rate. k_t is physical capital, and $R_t - \delta$ is the real rental price net of capital depreciation. From standard dynamic optimization, the Euler equation is

$$\frac{\dot{c}_t^k}{c_t^k} = r_t - \rho. \tag{3}$$

Also, the no-arbitrage condition $r_t = R_t - \delta$ holds.

The representative low-skill worker supplies l units of low-skill labor, whereas the representative high-skill worker supplies one unit of high-skill labor. $w_{l,t}$ and $w_{h,t}$ are respectively the real wage rates of low-skill labor and high-skill labor. Workers simply consume their after-tax wage income such that $c_t^l = (1 - \tau_t) w_{l,t} l$ and $c_t^h = (1 - \tau_t) w_{h,t}$, where τ_t is the rate of labor-income tax (or transfer).[7]

2.2 Final good

Competitive firms produce final good y_t using the following Cobb-Douglas aggregator over a unit continuum of differentiated intermediate goods:

$$y_t = \exp\left(\int_0^1 \ln x_t(i) di \right). \tag{4}$$

$x_t(i)$ denotes intermediate good $i \in [0, 1]$,[8] and the conditional demand function for $x_t(i)$ is

$$x_t(i) = \frac{y_t}{p_t(i)}, \tag{5}$$

where $p_t(i)$ is the price of $x_t(i)$.

[6] In our model, only the capital owner's discount rate affects the equilibrium allocations.

[7] We assume that taxes are levied on workers instead of capital owners for two reasons. First, labor income tends to be more heavily taxed than capital income. According to the classical Chemley-Judd result, the optimal capital tax rate is zero. In an R&D-based growth model, Chen *et al.* (2019) find that the optimal tax rate on labor income is much higher than that on capital income. Second, although our analysis of increasing subsidies is biased against workers, we still find positive welfare effects on high-skill workers.

[8] We follow Zeira (1998) to interpret $x_t(i)$ as intermediate goods. Alternatively, one could follow Acemoglu and Restrepo (2018) to interpret $x_t(i)$ as tasks.

2.3 Intermediate goods

There is a unit continuum of industries, which are also indexed by $i \in [0,1]$, producing differentiated intermediate goods. If an industry is not automated, then the production process uses low-skill labor and the production function is

$$x_t(i) = z^{n_t(i)} l_t(i), \tag{6}$$

where the parameter $z > 1$ is the step size of each quality improvement, $n_t(i)$ is the number of quality improvements that have occurred in industry i as of time t, and $l_t(i)$ is the amount of low-skill labor employed in industry i. Given the productivity level $z^{n_t(i)}$, the marginal cost function of the leader in an unautomated industry i is $w_{l,t}/z^{n_t(i)}$.

The monopolistic price $p_t(i)$ is a markup over the marginal cost $w_{l,t}/z^{n_t(i)}$. Grossman and Helpman (1991) and Aghion and Howitt (1992) assume that the markup is equal to the quality step size z, due to limit pricing between current and previous quality leaders. Here we follow Howitt (1999) and Dinopoulos and Segerstrom (2010) to consider an alternative scenario in which new quality leaders do not engage in limit pricing with previous quality leaders because after the implementation of the newest innovations, previous quality leaders exit the market and need to pay a cost before reentering. Given the Cobb-Douglas aggregator in (4), the unconstrained monopolistic price would be infinite. We follow Evans *et al.* (2003) to consider price regulation under which the regulated markup ratio cannot be greater than $\mu > 1$ such that[9]

$$p_t(i) = \mu \frac{w_{l,t}}{z^{n_t(i)}}. \tag{7}$$

In this case, the wage payment in an unautomated industry is

$$w_{l,t} l_t(i) = \frac{1}{\mu} p_t(i) x_t(i) = \frac{1}{\mu} y_t, \tag{8}$$

and the amount of monopolistic profit in an unautomated industry is

$$\pi_t^l(i) = p_t(i) x_t(i) - w_{l,t} l_t(i) = \frac{\mu - 1}{\mu} y_t. \tag{9}$$

If an industry is automated, then we follow Zeira (1998) to assume that the production process uses capital. The production function is[10]

$$x_t(i) = \frac{A}{Z_t} z^{n_t(i)} k_t(i), \tag{10}$$

where $A > 0$ is a parameter that captures an exogenous productivity difference between automated and unautomated industries. Z_t denotes aggregate technology capturing an erosion effect of new technologies that reduce the adaptability of existing physical capital.[11] Given

[9] This additional markup parameter enables us to perform a more realistic quantitative analysis.

[10] If we consider a more general specification $x_t(i) = A z^{n_t(i)} k_t(i)/Z_t^{\xi}$ where $\xi \in [0,1)$, then automation subsidy would have an additional positive effect on economic growth and give rise to an overall inverted-U effect on growth; see an earlier version of this study in Chu *et al.* (2018). However, the equilibrium condition for the automation-innovaton cycle would not hold for $\xi \in [0,1)$.

[11] See Galor and Moav (2002) for a similar erosion effect of technology on human capital.

the productivity level $z^{n_t(i)}$, the marginal cost function of the leader in an automated industry i is $Z_t R_t/[Az^{n_t(i)}]$. The monopolistic price $p_t(i)$ is also a markup μ over the marginal cost $Z_t R_t/[Az^{n_t(i)}]$ such that

$$p_t(i) = \mu \frac{Z_t R_t}{Az^{n_t(i)}}. \tag{11}$$

The capital rental payment in an automated industry is

$$R_t k_t(i) = \frac{1}{\mu} p_t(i) x_t(i) = \frac{1}{\mu} y_t, \tag{12}$$

and the amount of monopolistic profit in an automated industry is

$$\pi_t^k(i) = p_t(i) x_t(i) - R_t k_t(i) = \frac{\mu - 1}{\mu} y_t. \tag{13}$$

2.4 Automation-innovation cycle

In this section, we derive the equilibrium condition that supports a cycle of automation and innovation. In order for automation to yield a lower marginal cost of production than an existing innovation, we need the following condition to hold: $Z_t R_t/A < w_{l,t}$. In order for the next innovation to yield a lower marginal cost of production than automation, we need the following condition to hold: $w_{l,t}/z < Z_t R_t/A$. Combining these two conditions yields $w_{l,t}/z < Z_t R_t/A < w_{l,t}$. In Lemma 1, we derive the steady-state equilibrium expression for this condition, in which $g_y \equiv \dot{y}_t/y_t$ is the steady-state growth rate of output.

Lemma 1 *The steady-state equilibrium condition for the automation-innovation cycle is*

$$\frac{1}{z} < \left[\frac{\mu}{A}(g_y + \rho + \delta)\right]^{\frac{1}{1-\theta}} < 1.$$

Proof. See the Appendix A. ■

2.5 R&D and automation

Equations (9) and (13) show that $\pi_t^l(i) = \pi_t^l$ and $\pi_t^k(i) = \pi_t^k$ for each type of industries. Therefore, the value of inventions is also the same within each type of industries such that $v_t^l(i) = v_t^l$ and $v_t^k(i) = v_t^k$.[12] The no-arbitrage condition that determines the value v_t^l of an unautomated invention is

$$r_t = \frac{\pi_t^l + \dot{v}_t^l - (\alpha_t + \lambda_t) v_t^l}{v_t^l}, \tag{14}$$

[12] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian model.

which states that the rate of return on v_t^l is equal to the interest rate. The return on v_t^l is the sum of monopolistic profit π_t^l, capital gain $\dot{v}_t^l$ and expected capital loss $(\alpha_t + \lambda_t)v_t^l$, where α_t is the arrival rate of automation and λ_t is the arrival rate of innovation.[13]

Similarly, the no-arbitrage condition that determines the value v_t^k of an automation is

$$r_t = \frac{\pi_t^k + \dot{v}_t^k - \lambda_t v_t^k}{v_t^k}, \tag{15}$$

which states that the rate of return on v_t^k is also equal to the interest rate. The return on v_t^k is the sum of monopolistic profit π_t^k, capital gain $\dot{v}_t^k$ and expected capital loss $\lambda_t v_t^k$, where λ_t is the arrival rate of innovation. The condition in Lemma 1 ensures that the previous automation becomes obsolete when the next innovation arrives.

Competitive entrepreneurs recruit high-skill labor to perform innovation. The arrival rate of innovation in industry i is given by

$$\lambda_t(i) = \varphi_t h_{r,t}(i), \tag{16}$$

where $\varphi_t \equiv \varphi h_{r,t}^{\epsilon-1}$. The aggregate arrival rate of innovation is $\lambda_t = \varphi h_{r,t}^{\epsilon}$, where $h_{r,t}$ denotes aggregate R&D labor. Here the parameter $\epsilon \in (0,1)$ captures an intratemporal duplication externality as in Jones and Williams (2000) and determines the degree of decreasing returns to scale in R&D at the aggregate level. In a symmetric equilibrium, the free-entry condition of R&D becomes

$$\lambda_t v_t^l = (1-s)w_{h,t}h_{r,t} \Leftrightarrow \varphi v_t^l = (1-s)w_{h,t}h_{r,t}^{1-\epsilon}, \tag{17}$$

where $s < 1$ is the R&D subsidy rate.[14]

There are also competitive entrepreneurs who recruit high-skill labor to perform automation. The arrival rate of automation in industry i is given by

$$\alpha_t(i) = \phi_t h_{a,t}(i), \tag{18}$$

where $\phi_t \equiv \phi(1-\theta_t)h_{a,t}^{\epsilon-1}$. The endogenous variable $\theta_t \in (0,1)$ is the fraction of industries that are automated at time t. In other words, $1-\theta_t$ captures the following effect: a larger mass of currently unautomated industries that can be automated makes automation easier to complete.[15] The aggregate arrival rate of automation is $\alpha_t = \phi h_{a,t}^{\epsilon}$, where $h_{a,t}$ denotes aggregate automation labor and we have used the condition that $h_{a,t}(i) = h_{a,t}/(1-\theta_t)$. In a symmetric equilibrium, the free-entry condition of automation becomes

$$\alpha_t v_t^k = (1-\sigma)w_{h,t}h_{a,t}/(1-\theta_t) \Leftrightarrow \phi(1-\theta_t)v_t^k = (1-\sigma)w_{h,t}h_{a,t}^{1-\epsilon}, \tag{19}$$

where $\sigma < 1$ is the automation subsidy rate.[16]

[13] When the next innovation occurs, the previous technology becomes obsolete. See Cozzi (2007) for a discussion on the Arrow replacement effect.

[14] If $s < 0$, then it acts as a tax on R&D.

[15] Otherwise, if $\theta_t \to 1$, then $h_{a,t}(i) = h_{a,t}/(1-\theta_t)$ would become unbounded and have an infinite probability of automating an industry. Recall that automation is only directed to currently unautomated industries, which have a mass of $1-\theta_t$.

[16] If $\sigma < 0$, then it acts as a tax on automation.

2.6 Government

The government collects tax revenue to finance the subsidies on R&D and automation. The balanced-budget condition is

$$\tau_t(w_{l,t}l + w_{h,t}) = sw_{h,t}h_{r,t} + \sigma w_{h,t}h_{a,t}. \tag{20}$$

2.7 Aggregate economy

Aggregate technology Z_t is defined as[17]

$$Z_t \equiv \exp\left(\int_0^1 n_t(i)di \ln z\right) = \exp\left(\int_0^t \lambda_\omega d\omega \ln z\right), \tag{21}$$

where $\int_0^1 n_t(i)di \equiv \overline{n}_t$ is the aggregate number of innovations that have occurred in the economy and the last equality in (21) uses the law of large numbers. Differentiating the log of Z_t in (21) with respect to time yields the growth rate of technology given by

$$g_{z,t} \equiv \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z. \tag{22}$$

Substituting (6) and (10) into (4) yields[18]

$$y_t = \left(\frac{Ak_t}{\theta_t}\right)^{\theta_t}\left(\frac{Z_t l}{1-\theta_t}\right)^{1-\theta_t}, \tag{23}$$

where the share θ_t of automated industries also determines the degree of capital intensity in the aggregate production function. The evolution of θ_t is determined by

$$\dot{\theta}_t = \alpha_t(1-\theta_t) - \lambda_t\theta_t, \tag{24}$$

where $\alpha_t = \phi h_{a,t}^{\epsilon}$ and $\lambda_t = \varphi h_{r,t}^{\epsilon}$ are respectively the arrival rates of automation and innovation. Using (2), one can derive the familiar law of motion for capital as follows:[19]

$$\dot{k}_t = y_t - c_t - \delta k_t, \tag{25}$$

where $c_t \equiv c_t^k + c_t^l + c_t^h$. From (8) and (12), the capital and labor shares of income are

$$R_t k_t = \frac{\theta_t}{\mu} y_t, \tag{26}$$

$$w_{l,t} l = \frac{1-\theta_t}{\mu} y_t. \tag{27}$$

[17] Recall that automation does not improve quality but only allows for capital-labor substitution.
[18] Recall that $k_t(i) = k_t/\theta_t$ and $l_t(i) = l/(1-\theta_t)$.
[19] Derivations are available upon request.

2.8 Decentralized equilibrium

The equilibrium is a time path of allocations $\{a_t, k_t, c_t^k, c_t^l, c_t^h, y_t, x_t(i), l_t(i), k_t(i), h_{r,t}(i), h_{a,t}(i)\}$ and a time path of prices $\{r_t, R_t, w_{l,t}, w_{h,t}, p_t(i), v_t^l(i), v_t^k(i)\}$ such that the following conditions hold in each instance:

- agents maximize utility taking $\{r_t, R_t, w_{l,t}, w_{h,t}\}$ as given;
- competitive final-good firms produce $\{y_t\}$ to maximize profit taking $\{p_t(i)\}$ as given;
- each monopolistic intermediate-good firm i produces $\{x_t(i)\}$ and chooses $\{l_t(i), k_t(i), p_t(i)\}$ to maximize profit taking $\{w_{l,t}, R_t\}$ as given;
- competitive entrepreneurs choose $\{h_{r,t}(i), h_{a,t}(i)\}$ to maximize expected profit taking $\{w_{h,t}, v_t^l(i), v_t^k(i)\}$ as given;
- the market-clearing condition for capital holds such that $\int_0^{\theta_t} k_t(i)di = k_t$;
- the market-clearing condition for low-skill labor holds such that $\int_{\theta_t}^1 l_t(i)di = l$;
- the market-clearing condition for high-skill labor holds such that $\int_0^1 h_{r,t}(i)di + \int_{\theta_t}^1 h_{a,t}(i)di = 1$;
- the market-clearing condition for final good holds such that $y_t = \dot{k}_t + \delta k_t + c_t^k + c_t^l + c_t^h$;
- the value of inventions is equal to the value of the household's assets such that $\int_0^{\theta_t} v_t^k(i)di + \int_{\theta_t}^1 v_t^l(i)di = a_t$; and
- the government balances the fiscal budget.

3 Growth and welfare effects of R&D and automation

From (9) and (13), the amount of monopolistic profits in both automated and unautomated industries is

$$\pi_t^l = \pi_t^k = \frac{\mu - 1}{\mu} y_t. \tag{28}$$

The balanced-growth values of an innovation and an automation are respectively[20]

$$v_t^l = \frac{\pi_t^l}{\rho + \alpha + \lambda} = \frac{\pi_t^l}{\rho + \phi h_a^\epsilon + \varphi h_r^\epsilon}, \tag{29}$$

$$v_t^k = \frac{\pi_t^k}{\rho + \lambda} = \frac{\pi_t^k}{\rho + \varphi h_r^\epsilon}. \tag{30}$$

[20] It is useful to note that $r - g_\pi = \rho$, where g_π is the growth rate of π_t^l and π_t^k and equal to the growth rate of output and consumption.

Substituting (29) and (30) into the free-entry conditions in (17) and (19) yields

$$\frac{\varphi(1-\sigma)h_a^{1-\epsilon}}{\phi(1-\theta)(1-s)h_r^{1-\epsilon}} = \frac{\rho+\phi h_a^\epsilon + \varphi h_r^\epsilon}{\rho+\varphi h_r^\epsilon},$$

which can be reexpressed as

$$\frac{1-\sigma}{1-s}\left[\frac{\varphi}{\phi}+\left(\frac{1-h_r}{h_r}\right)^\epsilon\right] = \left(\frac{h_r}{1-h_r}\right)^{1-\epsilon} + \left(\frac{h_r}{1-h_r}\right)^{1-2\epsilon}\frac{\phi}{\varphi+\rho/h_r^\epsilon}. \tag{31}$$

If we assume $\epsilon \leq 1/2$,[21] then the right-hand side of (31) is increasing in h_r, whereas the left-hand side is always decreasing in h_r. Therefore, there exists a unique steady-state equilibrium value of R&D labor h_r and automation labor h_a. R&D labor $h_r(s,\sigma)$ is increasing in R&D subsidy s but decreasing in automation subsidy σ, whereas automation labor $h_a(s,\sigma)$ is increasing in automation subsidy σ but decreasing in R&D subsidy s.

From (24), the steady-state share of automated industries is

$$\theta(\underset{-}{s},\underset{+}{\sigma}) = \frac{\alpha}{\alpha+\lambda} = \frac{\phi h_a^\epsilon}{\phi h_a^\epsilon+\varphi h_r^\epsilon}, \tag{32}$$

which is increasing in automation subsidy σ but decreasing in R&D subsidy s. The steady-state equilibrium growth rate of technology is

$$g_z(\underset{+}{s},\underset{-}{\sigma}) = \lambda \ln z = \varphi h_r^\epsilon \ln z, \tag{33}$$

which is increasing in R&D subsidy s but decreasing in automation subsidy σ. Given that y_t and k_t grow at the same rate on the balanced growth path, the aggregate production function in (23) implies that the steady-state equilibrium growth rate of output y_t is

$$g_y(\underset{+}{s},\underset{-}{\sigma}) = g_z = \lambda \ln z = \varphi h_r^\epsilon \ln z, \tag{34}$$

which is increasing in R&D subsidy s but decreasing in automation subsidy σ. Proposition 1 summarizes these results

Proposition 1 *An increase in the R&D subsidy rate s has a positive effect on the technology growth rate g_z, a negative effect on the share θ of automated industries and a positive effect on the output growth rate g_y. An increase in the automation subsidy rate σ has a negative effect on the technology growth rate g_z, a positive effect on the share θ of automated industries and a negative effect on the output growth rate g_y.*

Proof. See Appendix A. ■

[21] In the appendix, we derive a weaker parameter condition.

We now examine the effects of R&D/automation subsidies on the welfare of capital owners, high-skill workers and low-skill workers. Given that the balanced growth level of consumption is $c_t^j = c_0^j \exp(g_c^j t)$, the steady-state level of welfare U^j can be expressed as $U^j = \int_0^\infty e^{-\rho t}(\ln c_0^j + g_c^j t)dt = (\ln c_0^j)/\rho + g_c^j/\rho^2$, which in turn can be re-expressed as

$$\rho U^j = \ln c_0^j + \frac{g_c^j}{\rho} \tag{35}$$

for $j \in \{k, l, h\}$.

From $c_t^l = (1-\tau)w_{l,t}l$, the steady-state welfare of low-skill workers is given by

$$\rho U^l = \ln(1-\tau) + \ln w_{l,0}l + \frac{g_y}{\rho} = \ln(1-\tau) + \ln\left(\frac{1-\theta}{\mu}y_0\right) + \frac{g_y}{\rho}, \tag{36}$$

where the second equality uses (27). U^l depends on the after-tax wage income of production labor. On the balanced growth path, the wage rate $w_{l,t}$ grows at the same rate as output y_t, which in turn determines the growth rate of low-skill workers' consumption. Therefore, R&D/automation subsidies affect the welfare of low-skill workers through the tax rate τ, the wage income of production labor and the growth rate of output.

From $c_t^h = (1-\tau)w_{h,t}$, the steady-state welfare of high-skill workers is given by

$$\rho U^h = \ln(1-\tau) + \ln w_{h,0} + \frac{g_y}{\rho}, \tag{37}$$

where the wage income of high-skill workers can be expressed as

$$w_{h,0} = \frac{\varphi/h_r^{1-\epsilon}}{1-s}\frac{\pi_0^l}{\rho + \phi h_a^\epsilon + \varphi h_r^\epsilon} = \frac{\phi(1-\theta)/h_a^{1-\epsilon}}{1-\sigma}\frac{\pi_0^k}{\rho + \varphi h_r^\epsilon}, \tag{38}$$

which uses (17), (19), (29) and (30). Therefore, R&D/automation subsidies affect the welfare of high-skill workers through the tax rate τ, the wage income of research/automation labor and the growth rate of output.

The welfare of capital owners can be expressed as

$$\rho U^k = \ln c_0^k + \frac{g_y}{\rho}, \tag{39}$$

where the initial level of their consumption is given by

$$c_0^k = \rho(a_0 + k_0), \tag{40}$$

which is obtained by imposing balanced growth on (2). Therefore, R&D/automation subsidies affect the welfare of capital owners through the value of intangible/tangible capital and the growth rate of output.

3.1 Quantitative analysis

In this section, we calibrate the model to aggregate US data in order to perform a quantitative analysis on the growth and welfare effects of the two subsidies. The model features the following set of parameters $\{\rho, \delta, \mu, z, \varphi, \phi, \epsilon, s, \sigma, A\}$.[22] We choose a conventional value of 0.05 for the discount rate ρ. As for the capital depreciation rate δ, we calibrate its value using an investment-capital ratio of 0.0765 in the US. We use the estimate in Laitner and Stolyarov (2004) to consider a value of 1.10 for the markup ratio μ. We calibrate the quality-step size z using a long-run technology growth rate of 0.0125 in the US. We calibrate the R&D productivity parameter φ using an innovation arrival rate of one-third as in Acemoglu and Akcigit (2012). We calibrate the automation productivity parameter ϕ using a labor-income share of 0.60 in the US. As for the intratemporal externality parameter ϵ, we follow Jones and Williams (2000) to set ϵ to 0.5. Given that the US currently does not apply different rates of subsidies to innovation and automation, we consider a natural benchmark of symmetric subsidies $s = \sigma$.[23] Then, we follow Impullitti (2010) to set the rate of subsidies in the US to 0.188. Finally, we pick a value of A that satisfies the condition for the automation-innovation cycle in Lemma 1 for the range of $\{s, \sigma\}$ that we consider. Table 1 summarizes the calibrated parameter values.

Table 1: Calibration

ρ	δ	μ	z	φ	ϕ	ϵ	s	σ	A
0.050	0.064	1.100	1.039	0.403	0.296	0.500	0.188	0.188	0.141

In the rest of this section, we simulate the separate effects of R&D subsidy s and automation subsidy σ on the technology growth rate g_z, the share θ of automated industries, the output growth rate g_y and the steady-state welfare U^j for the three types of agents.[24] Figure 1 simulates the effects of R&D subsidy s. Figure 1a shows that R&D subsidy s has a positive effect on the technology growth rate. Figure 1b shows that R&D subsidy s has a negative effect on the share of automated industries. Figure 1c shows that R&D subsidy s has a positive effect on the growth rate of output. Figure 1d-1f shows that R&D subsidy s increases the welfare of high-skill workers but decreases the welfare of low-skill workers and capital owners. Intuitively, high-skill workers engage in R&D and benefit from the subsidies, whereas low-skill workers are hurt by the higher tax burden despite the higher share of production wage income and capital owners are worse off due to the lower capital share of income.[25]

[22] Our calibration does not require us to assign a value to low-skill production labor l. Although the welfare function in (36) features the level of low-skill production labor l, it only affects the level of social welfare but not the change in welfare.

[23] In our simulation, we will change the individual values of s and σ separately.

[24] We focus on the steady state in this section and consider transition dynamics in the next section.

[25] If we were to assume that taxes are levied on high-skill workers but not low-skill workers, then R&D subsidies would hurt high-skill workers due to the higher tax burden and benefit low-skill workers due to the higher share of production wage income.

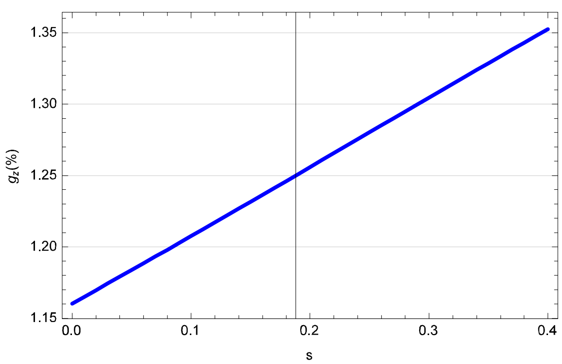

Figure 1a: Effect of s on g_z

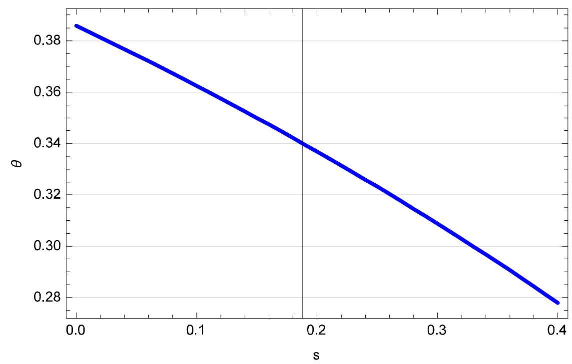

Figure 1b: Effect of s on θ

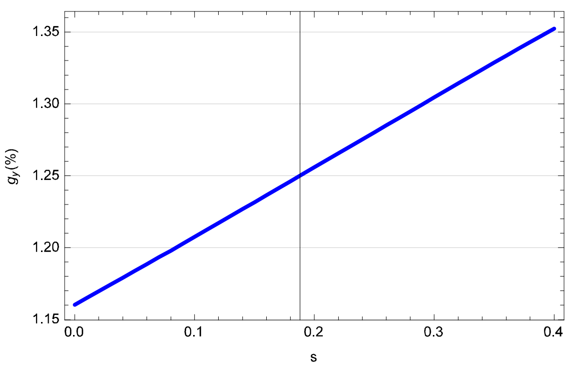

Figure 1c: Effect of s on g_y

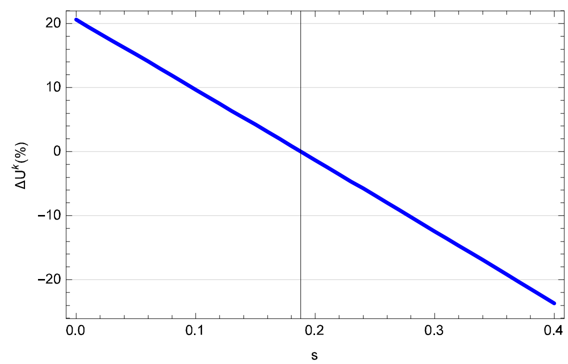

Figure 1d: Effect of s on steady-state U^k

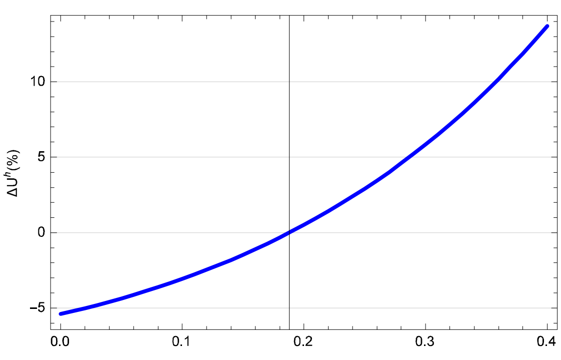

Figure 1e: Effect of s on steady-state U^h

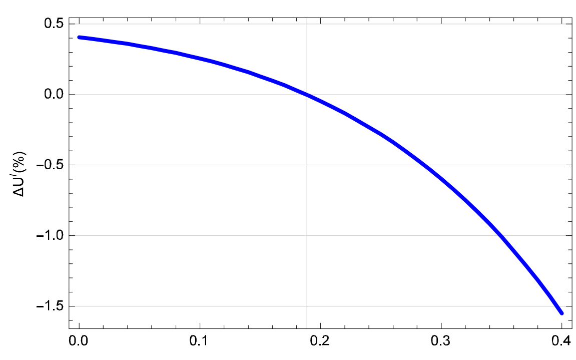

Figure 1f: Effect of s on steady-state U^l

Figure 2 simulates the effects of automation subsidy σ. Figure 2a shows that automation subsidy σ has a negative effect on the technology growth rate. Figure 2b shows that automation subsidy σ has a positive effect on the share of automated industries. Figure 2c shows that automation subsidy σ has a negative effect on the growth rate of output. Figure 2d-2f shows that automation subsidy σ increases the welfare of high-skill workers and capital owners but decreases the welfare of low-skill workers. Intuitively, high-skill workers engage in automation and benefit from the subsidies, whereas capital owners benefit from the higher capital share of income; however, low-skill workers are hurt by the higher tax burden and the lower share of production wage income.[26]

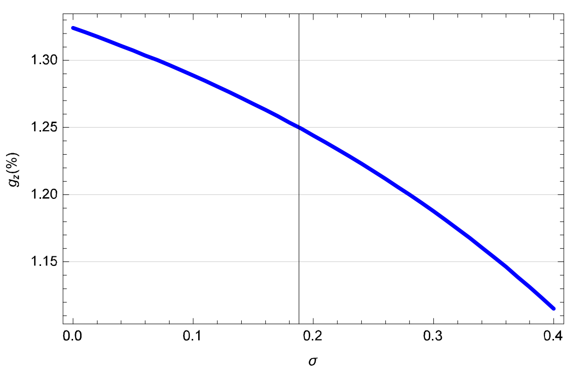

Figure 2a: Effect of σ on g_z

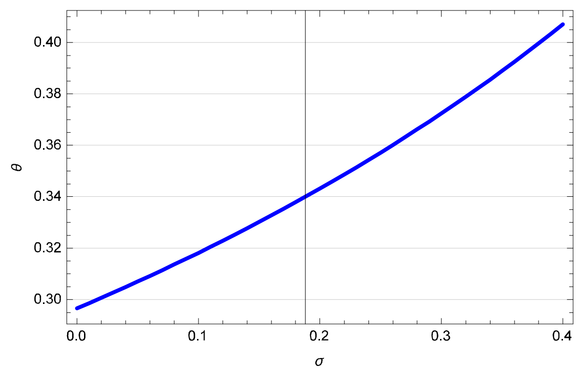

Figure 2b: Effect of σ on θ

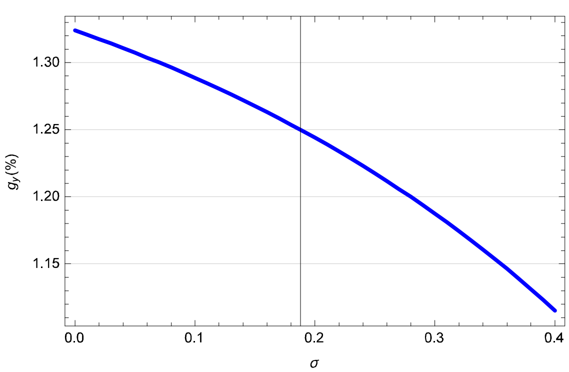

Figure 2c: Effect of σ on g_y

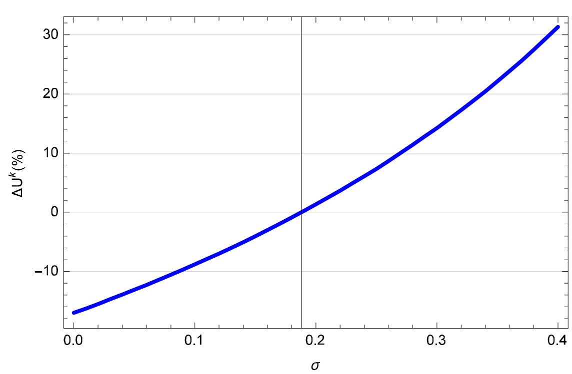

Figure 2d: Effect of σ on steady-state U^k

[26] These results would be qualitatively the same if we were to assume that taxes are levied on high-skill workers but not low-skill workers.

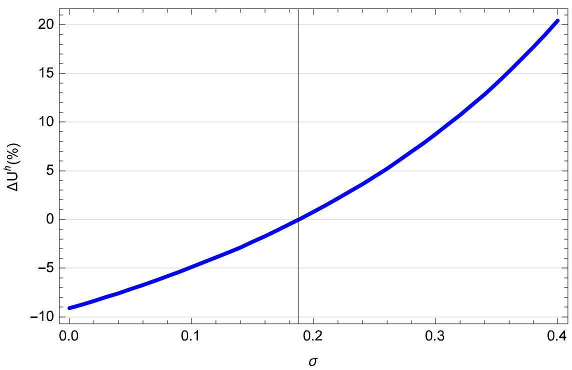

Figure 2e: Effect of σ on steady-state U^h

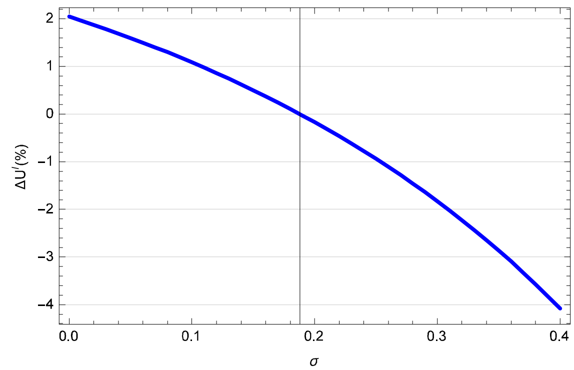

Figure 2f: Effect of σ on steady-state U^l

3.2 Transition dynamics

We use the relaxation algorithm in Trimborn *et al.* (2008) to simulate the transitional dynamic effects of raising automation subsidy σ from 0.188 to 0.238.[27] Figure 3a shows that an increase in automation subsidy leads to a lower technology growth rate $g_{z,t}$. The initial drop in $g_{z,t}$ is larger than the decrease in the long run. As shown in Figure 3b, capital intensity θ_t increases towards a higher level that requires a large amount of automation labor $h_{a,t}$, which crowds out R&D labor $h_{r,t}$. Figure 3c shows that despite the fall in technology growth $g_{z,t}$, the output growth rate $g_{y,t}$ increases after one year before gradually falling towards the new steady state, which is below the initial steady state. The drastic initial increase in output growth $g_{y,t}$ is due to the high initial growth in capital intensity θ_t.

Figure 3d and 3e show that the (log) level of consumption of capital owners and high-skill workers gradually converges to a higher balanced growth path (BGP), which however has a lower growth rate than the initial BGP. Given that the transitional path of consumption is below the new BGP, the transitional welfare gains are likely to be smaller than the steady-state welfare gains computed in the previous section. Figure 3f shows that the level of consumption of low-skill workers falls below the new BGP and gradually converges to it from below. Therefore, the transitional welfare loss on low-skill workers is likely to be larger than the steady-state welfare loss in the previous section. Comparing the new transitional path of consumption and its initial BGP, we compute a welfare gain equivalent to a permanent increase in consumption of 3.13% for capital owners and 2.35% for high-skill workers as well as a welfare loss of 1.47% for low-skill workers. Finally, Figure 4a and 4b show that the transitional welfare effects of automation subsidy σ on capital owners and high-skill workers are about one-half to two-thirds of the steady-state welfare effects of σ in Figure 2d-2e, whereas Figure 4c shows that the transitional welfare effects of automation subsidy σ on low-skill workers are about twice the steady-state welfare effects in Figure 2f. Therefore, focusing on the steady state may overstate the welfare effects on some groups but understate the welfare effects on others.

[27] See Appendix B for a summary of the dynamic equations.

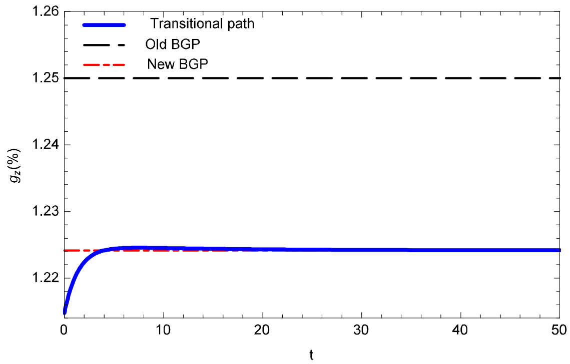

Figure 3a: Dynamic effect of σ on g_z

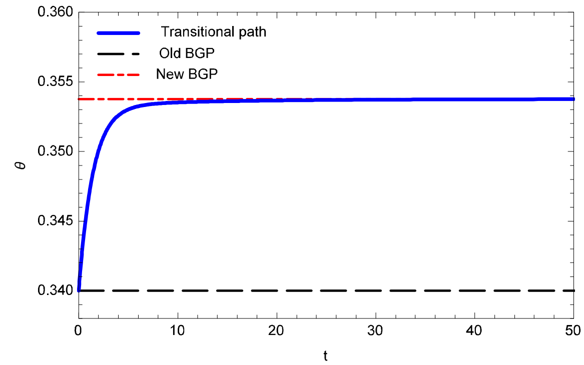

Figure 3b: Dynamic effect of σ on θ

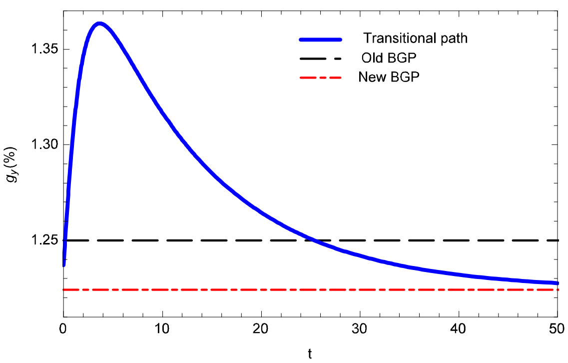

Figure 3c: Dynamic effect of σ on g_y

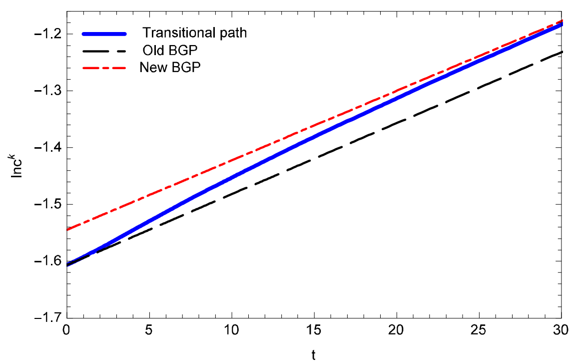

Figure 3d: Dynamic effect of σ on c^k

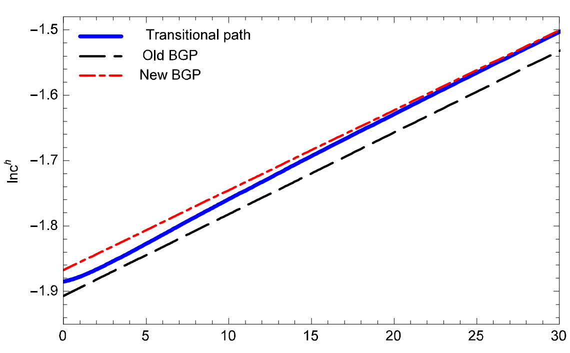

Figure 3e: Dynamic effect of σ on c^h

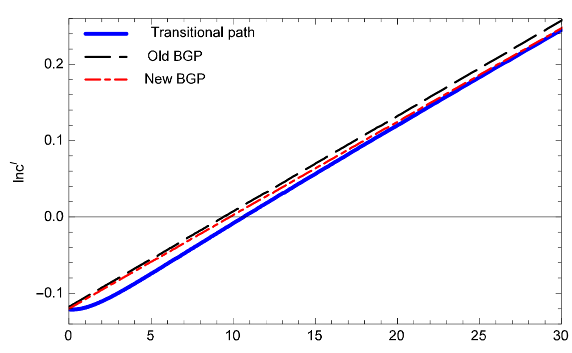

Figure 3f: Dynamic effect of σ on c^l

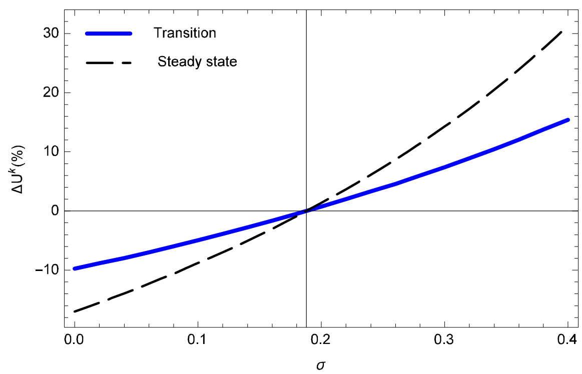

Figure 4a: Effect of σ on transitional U^k

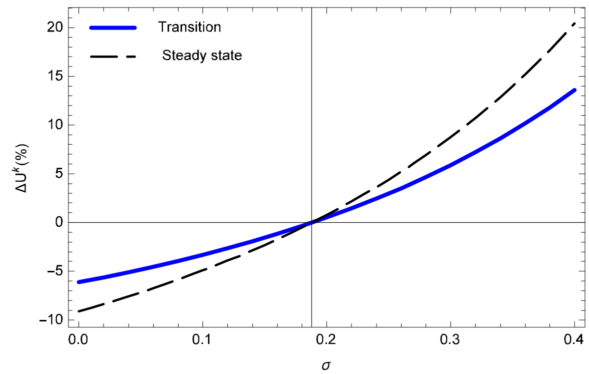

Figure 4b: Effect of σ on transitional U^h

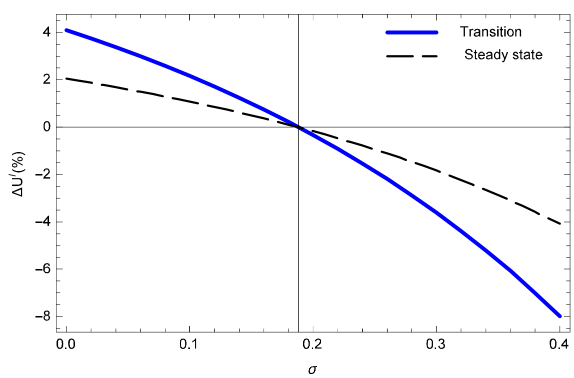

Figure 4c: Effect of σ on transitional U^l

4 Conclusion

In this study, we have developed a simple Schumpeterian growth model with automation. Our model features innovation in the form of quality improvement and also automation in the form of capital-labor substitution. Innovation gives rise to technological progress whereas automation increases the returns to scale of capital in production. R&D subsidy increases innovation but crowds out automation, whereas automation subsidy has the opposite effects. As a result, increasing R&D subsidy has a positive effect on innovation and growth but a negative effect on capital intensity in aggregate production. In contrast, increasing automation subsidy has a negative effect on innovation and growth but a positive effect on capital intensity in aggregate production. Our quantitative analysis shows that increasing R&D subsidy improves the welfare of high-skill workers but hurts the welfare of low-skill workers and capital owners, whereas increasing automation subsidy improves the welfare of high-skill workers and capital owners but hurts the welfare of low-skill workers. In other words, subsidizing automation has different welfare implications on different groups in the economy. Therefore, a redistribution of income is a better approach than Universal Basic Income to mitigate the negative effects of automation.

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Appendix A: Proofs

Proof of Lemma 1. Using the no-arbitrage condition $r = R - \delta$ and the Euler equation $r = g_y + \rho$, we can reexpress the equilibrium condition that supports a cycle of automation and innovation as

$$\frac{1}{z} < \frac{Z}{A}\left(\frac{g_y + \rho + \delta}{w_l}\right) < 1. \tag{A1}$$

We substitute production labor income $w_l l = (1-\theta)\, y/\mu$ and the aggregate production function $y = (Ak/\theta)^{\theta}\left[Zl/(1-\theta)\right]^{1-\theta}$ into (A1) to derive

$$\frac{1}{z} < \left(\frac{1}{A}\right)^{\frac{1}{1-\theta}}\left(\frac{\theta y}{k}\right)^{\frac{\theta}{1-\theta}}\left[\mu\left(g_y + \rho + \delta\right)\right] < 1. \tag{A2}$$

From capital income $Rk = \theta y/\mu$, the steady-state capital-output ratio is given by

$$\frac{k}{y} = \frac{\theta}{\mu R} = \frac{\theta}{\mu\,(r+\delta)} = \frac{\theta}{\mu\left(g_y + \rho + \delta\right)}. \tag{A3}$$

Substituting (A3) into (A2) yields the steady-state equilibrium condition for the automation-innovation cycle. ■

Proof of Proposition 1. We first establish the following sufficient parameter condition for the uniqueness of the equilibrium:

$$\epsilon < \frac{\phi + \rho}{2\phi + \rho} \in (1/2, 1). \tag{A4}$$

The left-hand side (LHS) of (31) is decreasing in h_r, whereas the derivative of the right-hand side (RHS) of (31) is given by

$$\frac{d}{dh_r}RHS = \frac{1}{(1-h_r)^2}\left(\frac{1-h_r}{h_r}\right)^{\epsilon}\underbrace{\left[\frac{\epsilon\rho\phi\,(1-h_r)^{1+\epsilon}}{(\varphi h_r^{\epsilon} + \rho)^2} + (1-\epsilon) - (2\epsilon - 1)\frac{\phi\,(1-h_r)^{\epsilon}}{\varphi h_r^{\epsilon} + \rho}\right]}_{\equiv\Phi}. \tag{A5}$$

Equation (A5) shows that when $\epsilon < 1/2$, RHS of (31) is monotonically increasing in h_r. As for $\epsilon > 1/2$, we consider the following lower bound of Φ:

$$\Phi > (1-\epsilon) - (2\epsilon - 1)\frac{\phi\,(1-h_r)^{\epsilon}}{\varphi h_r^{\epsilon} + \rho} > (1-\epsilon) - \frac{\phi\,(2\epsilon - 1)}{\rho}. \tag{A6}$$

Equation (A6) shows that $\epsilon < (\phi + \rho)/(2\phi + \rho)$ in (A1) is a sufficient condition for $\Phi > 0$; in this case, RHS of (31) is monotonically increasing in h_r. Therefore, we have established that the equilibrium h_r is uniquely determined by (31) as shown in Figure 5.

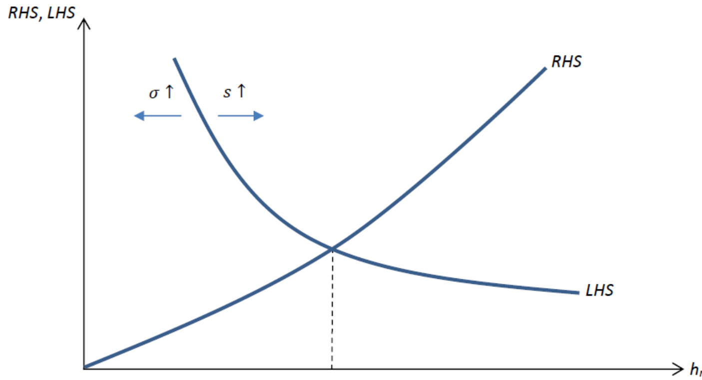

Figure 5: Equilibrium uniqueness

LHS of (31) being increasing in s (decreasing in σ) implies that h_r is monotonically increasing from 0 to 1 as $s < 1$ increases on its domain (decreasing from 1 to 0 as $\sigma < 1$ increases on its domain).[28] For the effects of $\{s, \sigma\}$ on θ, we use (32) to derive that θ is increasing in σ but decreasing in s. As for the effects of $\{s, \sigma\}$ on $\{g_z, g_y\}$, we use (33) and (34) to establish that both g_z and g_y are increasing in s but decreasing in σ. ■

[28]Recall that s and σ can be negative, in which case they act as taxes.

Appendix B: Dynamic equations

This appendix describes the dynamics of the economy. Using (23) and (26), we obtain

$$r_t = R_t - \delta = \frac{\theta_t y_t}{\mu k_t} - \delta = \frac{A^{\theta_t} Z_t^{1-\theta_t}}{\mu} \left(\frac{\theta_t}{1-\theta_t} \frac{l}{k_t} \right)^{1-\theta_t} - \delta. \tag{B1}$$

Based on $c_t^l = (1-\tau_t) w_{l,t} l$ and $c_t^h = (1-\tau_t) w_{h,t}$, we make use of (17), (19), (20) and (27) to obtain

$$\frac{c_t^l}{k_t} + \frac{c_t^h}{k_t} = \frac{(1-\tau_t)(w_{l,t} l + w_{h,t})}{k_t} = \left(\frac{1-\theta_t}{\mu} \right) \frac{y_t}{k_t} + \frac{\lambda_t v_t^l}{k_t} + \frac{\alpha_t (1-\theta_t) v_t^k}{k_t}. \tag{B2}$$

Substituting (B1) into (3) yields the growth rate of consumption as

$$\frac{\dot{c}_t^k}{c_t^k} = \frac{A^{\theta_t} Z_t^{1-\theta_t}}{\mu} \left(\frac{\theta_t}{1-\theta_t} \frac{l}{k_t} \right)^{1-\theta_t} - \delta - \rho. \tag{B3}$$

Using (9), (13), (23), (B1), $\lambda_t = \varphi h_{r,t}^\epsilon$ and $\alpha_t = \phi h_{a,t}^\epsilon$, we reexpress (14) and (15) as

$$\frac{\dot{v}_t^l}{v_t^l} = \frac{A^{\theta_t} Z_t^{1-\theta_t}}{\mu} \left(\frac{\theta_t}{1-\theta_t} \frac{l}{k_t} \right)^{1-\theta_t} - \delta + \phi h_{a,t}^\epsilon + \varphi h_{r,t}^\epsilon - \frac{A^{\theta_t} (\mu-1)/\mu}{(\theta_t)^{\theta_t} (1-\theta_t)^{1-\theta_t}} \frac{[k_t/(l Z_t)]^{\theta_t}}{v_t^l/(l Z_t)}, \tag{B4}$$

$$\frac{\dot{v}_t^k}{v_t^k} = \frac{A^{\theta_t} Z_t^{1-\theta_t}}{\mu} \left(\frac{\theta_t}{1-\theta_t} \frac{l}{k_t} \right)^{1-\theta_t} - \delta + \varphi h_{r,t}^\epsilon - \frac{A^{\theta_t} (\mu-1)/\mu}{(\theta_t)^{\theta_t} (1-\theta_t)^{1-\theta_t}} \frac{[k_t/(l Z_t)]^{\theta_t}}{v_t^k/(l Z_t)}. \tag{B5}$$

From (23), (25) and (B2), we derive the growth rate of capital k_t as

$$\frac{\dot{k}_t}{k_t} = \frac{[1-(1-\theta_t)/\mu] A^{\theta_t} Z_t^{1-\theta_t}}{(\theta_t)^{\theta_t} (1-\theta_t)^{1-\theta_t}} \left(\frac{l}{k_t} \right)^{1-\theta_t} - \frac{c_t^k}{k_t} - \left(\varphi h_{r,t}^\epsilon \right) \frac{v_t^l}{k_t} - \left(\phi h_{a,t}^\epsilon \right) (1-\theta_t) \frac{v_t^k}{k_t} - \delta, \tag{B6}$$

where we have used $\lambda_t = \varphi h_{r,t}^\epsilon$ and $\alpha_t = \phi h_{a,t}^\epsilon$. The dynamics of θ_t and Z_t are given by

$$\dot{\theta}_t = \left(\phi h_{a,t}^\epsilon \right) (1-\theta_t) - \left(\varphi h_{r,t}^\epsilon \right) \theta_t, \tag{B7}$$

$$\frac{\dot{Z}_t}{Z_t} = \varphi h_{r,t}^\epsilon \ln z. \tag{B8}$$

Differential equations in (B3)-(B8) describe the autonomous dynamics of $\{c_t^k, v_t^l, v_t^k, k_t, \theta_t, Z_t\}$ along with the following two static conditions:

$$h_{r,t} = \frac{\left[\varphi (1-\sigma) v_t^l \right]^{1/(1-\epsilon)}}{\left[\phi (1-s)(1-\theta_t) v_t^k \right]^{1/(1-\epsilon)} + \left[\varphi (1-\sigma) v_t^l \right]^{1/(1-\epsilon)}}, \tag{B9a}$$

$$h_{a,t} = \frac{\left[\phi (1-s)(1-\theta_t) v_t^k \right]^{1/(1-\epsilon)}}{\left[\phi (1-s)(1-\theta_t) v_t^k \right]^{1/(1-\epsilon)} + \left[\varphi (1-\sigma) v_t^l \right]^{1/(1-\epsilon)}}, \tag{B9b}$$

which are obtained by eliminating $w_{h,t}$ from (17) and (19) to derive

$$\frac{h_{r,t}}{h_{a,t}} = \left[\frac{\varphi (1-\sigma)}{\phi (1-s)(1-\theta_t)} \frac{v_t^l}{v_t^k} \right]^{1/(1-\epsilon)} \tag{B10}$$

and by substituting (B10) into $h_{a,t} + h_{r,t} = 1$. Finally, one can divide $\{c_t^k, v_t^l, v_t^k, k_t\}$ by $l Z_t$ to define stationarized variables and also eliminate l from the dynamic system.