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Troug, Haytem

University of Exeter

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Haytem Troug*

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Abstract

To show how fiscal policy affects the transmission mechanism of monetary policy, we extend a standard new Keynesian model for a small open economy to allow for the presence of non-separable government consumption in the utility function. We show how monetary policy should optimally respond to demand and supply shocks when the government sector is incorporated into the model. The introduction of government consumption affects the transmission of monetary policy. When government consumption has a crowding in effect on private consumption, it will dampen the transmission mechanism of monetary policy, and vice versa. Nevertheless, the degree of openness will minimise the effect of the introduction of government consumption in a non-separable form. Data for 35 OECD countries empirically support these findings, and the empirical results are robust to the zero lower bound period. The theoretical model also shows that, once we model the rest of the world economy, domestic government consumption and foreign government consumption will have opposing effects on private consumption, which contradicts with the existing literature.

Keywords: New Keynesian models, Business Cycle, Monetary Policy, Open Economy Macroeconomics, Joint Analysis of Fiscal and Monetary Policy.

JEL classification: E12, E32, E52, E63, F41.

*University of Exeter Business School, Department of Economics, Streatham Court, Rennes Drive, Exeter, EX4 4PU, UK. E-mail: ht321@exeter.ac.uk

1 Introduction

The substantial role of government consumption in influencing economic activity raises the necessity for monetary policy to take into account the behaviour of fiscal policy and to also take into account how the presence of the fiscal sector affects the transmission mechanism of monetary policy. Despite being a flexible tool that can address several macroeconomic issues, DSGE models have been rarely used to analyse the interaction between monetary and fiscal policy until the post-financial crisis. The recent literature (see, e.g., Christiano et al. 2011; Davig and Leeper 2011) focused on the impact of fiscal policy only when monetary policy is constrained by the zero-lower-bound, leaving a gap in the analysis of how governemnt consumption affects the transmission mechanism of monetary policy[1]. That motivates this paper to analyse how government consumption affects the dynamics of a small open economy, once the former is included in a non-separable form to the utility function. To the best of our knowledge, this issue has not been addressed by the literature, and we aim to do so in this paper.

The standard hypothesis of DSGE models introduces government consumption as either complete waste (Obstfeld and Rogoff 1995; 1996) or included in preferences in a non-separable form. While the former became an obsolete assumption in the recent literature, the inclusion of government consumption to preferences in a separable form was adopted both in New Keynesian models (Smets and Wouters 2007 and Gali and Monacelli 2008) and in RBC models (Baxter and King 1993). Despite being understudied, as noted by Cantore et al. 2014, the inclusion of government consumption seems appealing, given that agents gain utility from government consumption and the purpose of the government delivering services to households supports this claim. However, assuming prior separability in preferences between private consumption and government consumption can produce biased estimates of the response of private consumption, labour supply, and, hence, of output to a government consumption shock, as recently highlighted by Ercolani and Azevedo 2014.

Gali, Lopez-Salido, et al. 2007 challenged the adverse effect of government consumption on private consumption produced by these DSGE models. They highlighted the discrepancy between the

[1]Following the work of Troug 2019, we use the term "government consumption" in this paper for non-fixed capital formation government expenditure. For instance, it represents government provisions of goods and services, excluding compensations of state employees.

estimates of these DSGE models and the ones produced by some empirical models which illustrate a positive effect of government consumption on private consumption. Bouakez and Rebei 2007 and Pieschacon 2012, among others, have also supported the complementarity assumption between private consumption and government consumption. The two papers employ an RBC model and use the complementarity assumption to analyse the effect of a government consumption shock on the economy. Nevertheless, Ercolani and Azevedo 2014 showed results that indicate that government consumption is a substitute for private consumption in a New Keynesian framework. Ganelli 2003 also introduced government consumption as a substitute for private consumption. The elasticity of substitution between government consumption and private consumption in Ganelli's model governs how much private consumption needs to decline in response to an increase in government consumption in order to hold the utility on the same indifference curve. The discrepancy in the literature goes beyond the theoretical models, where even empirical models illustrate different estimates for the effect of government consumption, which depend on the modelled time frame and the adopted estimation method[2].

The above complications raise a crucial question regarding the reaction of monetary policy to different shocks once government consumption is included to the utility function in a non-separable form. Once government consumption is included in a non-separable form, it will affect the marginal utility of consumption and consequently the labour supply condition and the consumption smoothing condition. As a result, this inclusion will affect the structure of the whole model, and the transmission mechanism of monetary policy. As noted above, the existing literature has not addressed this issue, and we aim to fill in this gap in this paper. The mechanism of this model applies to both the complementarity case and the substitutability case, and we will focus on changes in the reaction and the transmission mechanism of monetary policy once government consumption is incorporated to a standard New Keynesian model in a non-separable form.

This paper employs a New Keynesian model to study the optimal response of monetary policy to supply and demand shocks in the presence of fiscal policy for a small open economy. To do this, we extend an otherwise standard New Keynesian model for a small open economy (Galí and Monacelli 2005) and build from the model used in Troug 2019 to allow for meaningful non-

[2] For instance, while Aschauer 1985 and Ahmed 1986 find substitutability between government consumption and private consumption, Karras 1994, Fiorito and Kollintzas 2004, and Coenen, Straub, and Trabandt 2013 find complementarity between the two variables.

separable government consumption, financed by means of lump-sum taxes, in the utility function. Extending the model to a small open economy case complicates the problem for monetary policy to the extent that the authorities must additionally take into account how the exchange rate affects other macroeconomic variables. Similar to the closed-economy case, we are able to derive an optimal monetary policy rule which takes into account developments in government consumption, in addition to developments in the rest of the world economy. The other extension that this paper adds to the canonical small open economy model is modelling the rest of the world economy as an aggregate of identical small open economies, and each of them has a size of zero, following the work of Unalmis et al. 2008. This will allow us to trace the spillover effects of supply and demand shocks in the foreign economy on the domestic economy. I choose this framework over the two-country model, adopted in (Obstfeld and Rogoff 1995; 1996) and Ganelli 2003, to prevent spillovers from the domestic economy to the rest of the world economy that might complicate the analysis.

In the open-economy case, the degree of openness will minimise the deviation of the slope of the IS curve from the standard case both in the complementarity and the substitutability case. Moreover, the degree of openness minimises the crowding-out (-in) effect of government consumption on monetary policy when the former is a complement (substitute) to private consumption. We also find that the fiscal multiplier is also minimised by the degree of openness of the economy, in comparison to the closed-economy version of the model. We additionally find that the size of the fiscal multiplier is negatively affected by the response of monetary policy and the flexibility of the exchange rate, which is in line with the theoretical findings of, among others, Woodford 2011, and the empirical results of Koh 2017 and contradicts with the findings of Troug 2019.

Moreover, in the case of the spillover effect of external shocks, the amount of exchange rate volatility will determine how much the domestic economy will be affected by external shocks. In this regard, we find that the main difference in the dynamics between this model and the one used in Ganelli 2003 is the exchange rate channel in the two models. In this model, the exchange rate is a product of the interest rates differentials, and the purchasing power of the domestic consumers will be affected by any changes in the exchange rate. On the other hand, in Ganelli's model, the exchange rate is a function of the money demand, which is, in return, a function of private consumption and government consumption. This difference in the dynamics between the two models produces a conflicting effect between domestic and foreign government consumption on domestic private consumption in our model, while the two have the same adverse effect on domestic consumption in Ganelli's model.

The empirical results of this paper support the findings of the theoretical model. First, the results show that government consumption in 35 OECD countries has a crowding in effect on private consumption. Second, this positive effect of government consumption dampens the negative effect of the policy rule on private consumption. Nevertheless, the degree of openness in the economy, measured as net exports as a ratio of output, minimises the crowding out effect of fiscal policy on monetary policy. These results are robust to the zero lower bound period.

The remainder of the paper is organised as follows. We will first demonstrate the structure of our model in the second section. In the third section, we show the parametrisation of the model. The equilibrium dynamics of the model will be discussed in the fourth section. The analysis of the impulse response functions is presented in the fifth section. In the sixth section, we show the results of the welfare loss calculations and the second moments of the primary variables of the small open economy. Lastly, the concluding remarks will take place in the seventh section.

2 Small open economy model[3]

2.1 Households in the domestic economy

Our economy is populated by a representative household that derives utility from aggregate consumption and leisure. The household is assumed to live infinitely and in each period is endowed with one unit of time that is divided between work and leisure: $N_t + L_t = 1$. The representative consumer seeks to maximise the following discounted lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\bar{C}_t, N_t) \tag{1}$$

The utility function is assumed to be continuous and twice differentiable. Where N_t is the number of hours worked; β is the discount factor; $\bar{C}_t$ is the aggregate consumption bundle. The aggregate consumption bundle is a constant elasticity of substitution that consists of private con-

[3] We will only display the model for the small open economy in the main body of this paper. We show the log-linearised version of the rest of the world economy in **??**, where we adopted the same model used by Kripfganz et al. 2017 for the closed economy.

sumption C_t and government consumption G_t[4]:

$$\bar{C}_t = \left[\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{2}$$

Where δ is the share of private consumption in the aggregate consumption bundle and χ is the inverse elasticity of substitution between private consumption and government consumption. Equations (1) and (2) show that the utility function is non-decreasing in government consumption G_t. The above utility function is subject to the following budget constraint:

$$\int_0^1 P_{H,t}(j) C_{H_t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i_t}(j) dj di + E_t Q_{t,t+1} D_{t+1} \leq D_t + W_t N_t + T_t \tag{3}$$

Where D_t is the nominal payoff at period t+1 of bonds held at the end of period t including shares in firms, government bonds, and different types of deposits. $Q_{t,t+1}$ is a stochastic discount factor of nominal payoffs and it is equal to $\frac{1}{R_t}$; W_t is the wage; T_t is lump-sum transfers to the households net of lump-sum taxes. All units are expressed in terms of domestic currency.

The utility function that we use assumes two separabilities. The first one is the separation between consumption and the amount of hours worked, and the second one is time separability. The household's problem is also analysed in two stages here: we first deal with the expenditure minimisation problem faced by the representative household to derive the demand functions for domestic and foreign goods. In the second stage, the households choose the level of C_t and N_t, given the optimally chosen combination of goods. C_t is our basic private consumption bundle, and it is a CES composite of home and foreign goods defined as follows:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{4}$$

The above equation is the same household consumption bundle used by Galí and Monacelli 2005, which is the workhorse for small open economies. α here is the degree of openness in the

[4]Government consumption in this framework can be thought of as a public good that households consume at a free cost. It can also be thought of as government expenditure on security and defence which stimulates private consumption and increases the utility of households. Basically, the government is assumed to consume non-tradable goods only.

economy which represents the share of imported goods $C_{F,t}$ in the household's consumption bundle. Conversely, the home bias parameter $(1-\alpha)$ produces the possibility of a different consumption bundle in each economy. This is a consequence of having different consumption baskets in each country, despite the law of one price holding for each individual good. $\eta > 0$ is the elasticity of substitution between domestically produced goods and imported goods in the household's consumption bundle. Consumption goods that are produced either at home or in any foreign country are represented by the unit interval: $C_{j,t} = \int_0^1 C_{j,t}(i)^{\frac{\epsilon-1}{\epsilon}} di$ for $i \in [0,1]$ and $j = [H, F, i \in [0,1]]$. Now as a first step, the households must minimise their expenditure by optimally choosing the share of each good in the aggregate consumption bundle. Doing so will yield the following demand functions:

$$C_{i,t}(j) = \Big(\frac{P_{i,t}(j)}{P_{i,t}}\Big)^{-\epsilon} C_{i,t}\,, C_{H,t}(j) = \Big(\frac{P_{H,t}(j)}{P_{H,t}}\Big)^{-\epsilon} C_{H,t}\,, C_{i,t} = \Big(\frac{P_{i,t}}{P_{F,t}}\Big)^{-\gamma} C_{F,t} \tag{5}$$

Where $P_{i,t} \equiv (\int_0^1 P_{i,t}(j)^{1-\epsilon} dj)^{1-\epsilon}$ is the aggregate price index for imported goods from country (i), $P_{H,t} \equiv (\int_0^1 P_{H,t}(j)^{1-\epsilon} dj)^{1-\epsilon}$ is the aggregate price index for domestic goods, and $P_{F,t}$ is the aggregate price index for imported goods. The first two terms show domestic demand for good (j) in one of the foreign economies and in the home economy, respectively. The parameter ϵ represents how much the demand for good (j) will decline if the relative price of that good increased by 1 unit. A lower elasticity of substitution indicates higher consumption of the good of interest. This shows that the goods in the consumption bundle are not perfect substitutes. The third equality is domestic demand for goods produced in country (i) as a function of total domestic demand for foreign goods, and $\gamma > 0$ is the elasticity of substitution between goods from different origins. We finally show the demand functions of domestic and foreign goods from their expenditure minimisation given total consumption:

$$C_{H,t} = (1-\alpha)\Big(\frac{P_{H,t}}{P_t}\Big)^{-\eta} C_t\,; \qquad C_{F,t} = \alpha\Big(\frac{P_{F,t}}{P_t}\Big)^{-\eta} C_t \tag{6}$$

Now we turn our attention to the per-period utility function in the following form[5]:

[5] We replaced private consumption in the utility function with the aggregate consumption bundle. As noted above, this is one of the deviations that we make from the standard New Keynesian models.

$$U(C_t, N_t; G_t) = \frac{\bar{C}_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}, \tag{7}$$

Where σ is the inverse elasticity of intertemporal substitution. Setting σ equal to 1 implies that the household has a log-utility in consumption; φ is the inverse Frisch labour supply coefficient, and $\varphi > 0$ also measures the curvature of the marginal disutility of labour. The above equation is subject to the aggregate budget constraint, which we get by plugging the above demand bundles and price indices in equation (3):

$$P_t C_t + E_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + T_t \tag{8}$$

Where E_t is the conditional expectations operator. The household's total expenditure basket is equal to: $P_t C_t = \int_0^1 P_{H,t}(j) C_{H_t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i_t}(j) dj di$. P_t is the consumer price index (CPI) and it is equal to: $P_t = \Big[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\Big]^{\frac{1}{1-\eta}}$. From Equation (7) and (8) we can write the standard optimality condition for households as follows:

$$\frac{W_t}{P_t} = N_t^{\varphi} \bar{C}_t^{\sigma} \Big(\frac{C_t}{\bar{C}_t}\Big)^{\chi} \delta^{-\chi} \tag{9}$$

The intertemporal optimality condition is:

$$\beta \Big(\frac{\bar{C}_{t+1}}{\bar{C}_t}\Big)^{\chi-\sigma} \Big(\frac{P_t}{P_{t+1}}\Big) \Big(\frac{C_t}{C_{t+1}}\Big)^{\chi} = Q_{t,t+1} \tag{10}$$

Taking the conditional expectation of equation (10) and rearranging the terms we get:

$$\beta R_t E_t \Big[\Big(\frac{\bar{C}_{t+1}}{\bar{C}_t}\Big)^{\chi-\sigma} \Big(\frac{P_t}{P_{t+1}}\Big) \Big(\frac{C_t}{C_{t+1}}\Big)^{\chi}\Big] = 1 \tag{11}$$

Where $R_t = \frac{1}{E_t(Q_{t,t+1})}$ is the one-period return from a riskless bond and $Q_{t,t+1}$ is the price of that bond. Equations (9) and (11) deviate from the standard open economy literature. This deviation is due to the fact that we have included government consumption in the aggregate CES basket with private consumption in a non-separable form. The first equation depicts the labour supply dynamics. It shows labour supply as a function of the real wage given the aggregate consumption bundle and private consumption, and it shows how the effect of the aggregate consumption bundle on labour supply depends on the value of χ. Noting that in the Cobb-Douglas case, when $\chi = \sigma$, the labour supply equation collapses to its canonical form as government consumption would have no effect on private consumption. When $\chi > \sigma$, government consumption will have a negative effect on real wages given its positive effect on labour supply. On the other hand when $\chi < \sigma$, government consumption will have a positive effect on real wages, resulting from its negative effect on labour supply. The second equation is the Euler equation which characterises consumption smoothing. The Euler equation in this model also deviates from the standard form found in the literature. In this case, the smoothing of the aggregate consumption bundle $\bar{C}$ is included in the above Euler equation. In the Cobb-Douglas case, when $\chi = \sigma$, the above equation would also collapse to the canonical version of the Euler equation. When $\chi > \sigma$, changes in the current value of the aggregate consumption bundle will have a positive effect on private consumption. However, the aggregate consumption bundle will have an adverse effect on private consumption when $\chi < \sigma$.

2.2 Firms

2.2.1 Price Setting Behaviour

The firms in this model set their prices in a staggered way following Calvo 1983[6]. Under Calvo contracts, we have a random fraction $1 - \theta$ of firms that can reset their prices at period t, while the remaining firms of size θ keep their prices fixed at the previous period's price level. Therefore, we can say that θ^k is the probability that a price set at period t will still be valid at period $t + k$. Thus, the probability of a firm re-optimising its prices will be independent of the time elapsed since it last re-optimised its prices, and the average duration for prices not to change is $\frac{1}{1-\theta}$. Given the above information, the aggregate domestic price level will have the following form:

$$P_{H,t} = \left[\theta(P_{H,t-1})^{1-\epsilon} + (1-\theta)(\bar{P}_{H,t}^{1-\epsilon})\right]^{\frac{1}{1-\epsilon}} \tag{12}$$

Where $\bar{P}_{H,t}$ is the new price set by the optimising firms. From the derivations shown in Appendix 2, we get the following form for inflation:

[6]The Calvo model makes aggregation easier because it gets rid of the heterogeneity in the economy. The alternative pricing scheme is the quadratic cost of price adjustment by Rotemberg 1982. The two dynamics are equivalent up to a first-order approximation.

$$\Pi_{H,t}^{1-\epsilon} = \theta + (1-\theta)\Big(\frac{\bar{P}_{H,t}}{P_{t-1}}\Big)^{1-\epsilon} \tag{13}$$

The above equation shows that the inflation rate at any given period will be solely determined by the fraction of firms that reset their prices at that period. When a given firm in the economy sets its prices, it seeks to maximise the expected discounted value of its stream of profits, conditional that the price it sets remains valid:

$$max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}[c_{jt+k|t}(\bar{P}_{H,t} - \Psi_{t+k})]\} \tag{14}$$

The above equation is subject to a sequence of demand constraints: $c_{jt+k} = \Big(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\Big)^{-\epsilon} C_t$. Solving this problem (also shown in Appendix 2) yields the following optimal decision rule:

$$\sum_{k=0}^{\infty} \theta^k E_t\Big\{Q_{t,t+k}C_{t+k}\Big[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \mathcal{M}MC_{t+k|t}\Pi^H_{t-1,t+k}\Big]\Big\} = 0 \tag{15}$$

Where $\mathcal{M}$ is the firm's price markup at the steady state and MC_t is the real marginal cost that firms face in the domestic economy. As we can see from equation (15), in the sticky price scheme producers, given their forward-looking behaviour, adjust their prices at a random period to maximise the expected discounted value of their profits at that period and in the future. Thus, firms in this model will set their prices to equal a price markup plus the present value of the future expected stream of their marginal costs. This is done because firms know that the price they set at period t will remain valid for a random period of time in the future. We also assume that all firms in the economy face the same marginal cost, given the constant return to scale assumption imposed on the model and the subsidy that the government pays to firms, as we will see in the following section. The firms also use the same discount factor β as the one used by households, and this is attributed to the fact that the households are the shareholders of these firms. Also, all the firms that optimise their prices in any given period will choose the same price, and this is also a consequence of the firms facing the same marginal cost. Equation (15) also shows that the inflation rate is proportional to the discounted sum of the future real marginal costs additional to a markup resulting from the monopolistic power of the firms.

2.2.2 Production

Firm (j) in the domestic economy produces a differentiated good following a linear production function:

$$Y_t(j) = A_t N_t(j) \tag{16}$$

Where $Y_t(j)$ is the output of final good (j) in the domestic economy. A_t is the level of technology in the production function, and it is assumed to be common across all firms in the economy and exogenously evolves. $N_t(j)$ is the labour force employed by firm (j). The log form of total factor productivity $a_t = log(A_t)$ is assumed to follows an AR(1) process: $a_t = \rho_a a_{t-1} + \epsilon_{a,t}$. $0 < \rho_a > 1$ is the autocorrelation of the shock, and the innovation to technology $\epsilon_{a,t}$ has a zero mean and a finite variance σ_a. We exclude capital from production in this model for the sake of tractability. Aggregate output and aggregate employment in the domestic economy are defined by the Dixit and Stiglitz 1977 aggregator:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} ; N_t = \left(\int_0^1 N_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{17}$$

Given the common technology assumption across all firms of the economy, the total cost function for firm (j) is defined as follows:

$$TC_t(j) = \frac{(1-\tau)W_t Y_t(j)}{A_t} \tag{18}$$

In the above equation, we left W_t without any firm specification, which is attributed to the fact that we have a competitive labour market. Also, τ is the subsidy that the government gives to firms in order to eliminate the markup distortion created by the firms' monopolistic power. Taking the first order condition of equation (18), with respect to production, yields the following marginal cost equation:

$$MC_t(j) = \frac{(1-\tau)W_t}{A_t} \tag{19}$$

From the above equation, it is clear that the constant return to scale assumption makes the marginal cost independent of the firm's production level and this will make marginal cost common across all firms: $MC_t(j) = MC_t$. Now the common real marginal cost will look like:

$$MC_t^r = \frac{(1-\tau)}{A_t}\frac{W_t}{P_{H,t}} \tag{20}$$

The marginal cost equation is expressed in terms of the domestic prices level $P_{H,t}$, wages W_t, total factor productivity A_t and τ, which is the subsidy that the government gives to firms, in order to eliminate the markup distortion created by the firms' monopolistic power.

Lastly, after the aggregation of output and employment, we get the following aggregate production function:

$$Y_t = A_t N_t \tag{21}$$

2.3 International Linkages

We first start by the defining the terms of trade as the ratio of imported prices to domestic prices. The bilateral terms of trade between the domestic economy and another small economy (country i) is defined as: $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$. Thus, the aggregate terms of trade is defined as: $S_t = \left(\int_0^1 S_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$. Defining $P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$ allows us to define the aggregate effective terms of trade as:

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{22}$$

If we plug in the log-linearised representation of the imported prices index from the above equation ($p_{F,t} = s_t + p_{H,t}$) in the log-linearised form of the CPI price index equation, we will be able to derive the CPI index as a function of the domestic prices index and the terms of trade:

$$p_t = p_{H,t} + \alpha s_t \tag{23}$$

The above function shows that a gap exists between the CPI index and the domestic price index which is filled with the terms of trade. The gap is parametrised by the degree of openness of the domestic economy. Before progressing to further derivations, we first define the bilateral exchange rate $\mathcal{E}_{i,t}$ as the value of country i's currency in terms of the domestic currency. Assuming that the law of one price holds, the price of any good in country (i) will be equal to:

$$P_{i,t}(j) = \mathcal{E}_{i,t} P^i_{i,t}(j) \tag{24}$$

Integrating the above equation yields the price index for country (i). Solving this integrate for the imported prices index in the domestic economy yields:

$$P_{F,t} = \mathcal{E}_t P^*_t \tag{25}$$

The nominal effective exchange rate is equal to $\mathcal{E}_t \equiv \int_0^1 \mathcal{E}_{i,t} di$, and the world price index is defined as $P^*_t \equiv \int_0^1 P_{i,t} di$. Plugging the value of the imported prices index from the above equation in the definition of the terms of trade yields:

$$S_t = \frac{\mathcal{E}_t P^*_t}{P_{H,t}} \tag{26}$$

We now define the bilateral real exchange rate as the ratio of the price index in country (i) to the CPI index in the domestic economy: $REER_{i,t} = \frac{\mathcal{E}_{i,t} P^i_t}{P_t}$. Integrating the bilateral real exchange rate equation yields the real effective exchange rate equation for the domestic economy: $REER_t = \frac{\mathcal{E}_t P^*_t}{P_t}$. From the definitions of the terms of trade and the real effective exchange rate, we can define the equation that links the two variables in a log-linearised form as follows:

$$q_t = (1 - \alpha) s_t \tag{27}$$

Under the assumption of complete international financial markets, the price of a one-period riskless bond from country (i) dominated in the domestic economy's currency is equal to: $\mathcal{E}_{i,t} Q^i_t = E[\mathcal{E}_{i,t+1} Q_{t,t+1}]$. If we add this equation to the domestic bond's price equation ($Q_t = E[Q_{t,t+1}]$), we get the uncovered interest rate parity condition:

$$\frac{Q^i_t}{Q_t} = E_t\Big(\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}\Big) \tag{28}$$

The uncovered interest parity condition is crucial for the no-arbitrage condition to hold in the international bonds market. Under the uncovered interest parity we assume that foreign bonds are perfect substitutes for domestic bonds once both of them are expressed in the same currency. The uncovered interest parity equation also implies that higher foreign interest rates or depreciation in the exchange rate will put upward pressure on domestic interest rates.

The last thing that we need do in this section is to derive the international risk condition. Under the assumptions of complete international markets and the identical preferences assumption, the foreign consumer's Euler equation can be transformed to:

$$\beta\Big(\frac{\bar{C}^*_{t+1}}{\bar{C}^*_t}\Big)^{\chi-\sigma}\Big(\frac{P^*_t}{P^*_{t+1}}\Big)\Big(\frac{C^*_t}{C^*_{t+1}}\Big)^{\chi}\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big) = Q_{t,t+1} \tag{29}$$

We then divide the domestic inter-temporal optimality condition (eq. 10) by the foreign economy's inter-temporal optimality condition (eq. 29) to get:

$$1 = E_t\left(\frac{\Big(\frac{\bar{C}_{t+1}}{\bar{C}_t}\Big)^{\chi-\sigma}\Big(\frac{P_t}{P_{t+1}}\Big)\Big(\frac{C_t}{C_{t+1}}\Big)^{\chi}}{\Big(\frac{\bar{C}^*_{t+1}}{\bar{C}^*_t}\Big)^{\chi-\sigma}\Big(\frac{P^*_t}{P^*_{t+1}}\Big)\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big)\Big(\frac{C^*_t}{C^*_{t+1}}\Big)^{\chi}}\right) \tag{30}$$

Plugging the definition of the real effective exchange rate in the above equation yields:

$$C_t = \mathcal{V}_t C^*_t (REER_t)^{\frac{1}{\chi}}\Big(\frac{\bar{C}_t}{\bar{C}^*_t}\Big)^{\frac{\chi-\sigma}{\chi}} \tag{31}$$

Where $\mathcal{V}_t = \frac{C_{t+1}\bar{C}_t^{*\frac{\chi-\sigma}{\chi}}}{C^*_{t+1}\bar{C}_{t+1}^{\frac{\chi-\sigma}{\chi}}REER_{t+1}^{\frac{1}{\chi}}}$ is a constant which depends on the initial relative wealth position. We assume that we have a symmetric initial condition and set $\mathcal{V}_t = 1$, meaning that the net position of foreign assets is equal to zero. Thus, the international risk sharing condition simplifies to:

$$C_t = C^*_t (REER_t)^{\frac{1}{\chi}}\Big(\frac{\bar{C}_t}{\bar{C}^*_t}\Big)^{\frac{\chi-\sigma}{\chi}} \tag{32}$$

Complete security markets ensure that risk-averse consumers can trade away the risks and the shocks they encounter. Under this setting, consumers can purchase contingent claims for realisations of all idiosyncratic shocks, and this will enable them to diversify all idiosyncratic risk through the capital markets. Also, the above international risk sharing condition depicts how a depreciation in the real effective exchange rate would boost domestic consumption relative to the foreign economy's consumption. The log-linearised form of the above international risk sharing condition is:

$$c_t = c_t^* + \frac{(\sigma - \sigma_\delta)}{\sigma_\delta}(g_t^* - g_t) + \frac{1}{\sigma_\delta} q_t. \tag{33}$$

Where $\sigma_\delta = \delta\sigma + (1-\delta)\chi$ is a weighted average of the intertemporal elasticity of substitution σ and the inverse elasticity of substitution between government consumption and private consumption χ. The above equation illustrates how the effect of domestic and foreign government consumption is governed by χ. In the Cobb-Douglas case when $\chi = \sigma$, the above international risk sharing condition collapses back to its standard representation as $\sigma_\delta = \sigma$. In this case, government consumption both in the domestic and foreign economy will not affect private domestic consumption. When $\chi > \sigma$, domestic government consumption has a positive effect on private consumption, while foreign government consumption has a negative effect on domestic private consumption. On the other hand, when $\chi > \sigma$ domestic government consumption will have a negative effect on private consumption and foreign government consumption will have a positive effect on private consumption. This last point makes a clear distinction between our model and the one used in Ganelli 2003. Where in the latter's both domestic and foreign government consumption have the same negative effect on private consumption. In this setting, as will be made clear in the simulations below, monetary policy will react to changes in government consumption, conditional on how government consumption affects the economy. The exchange rate will react to the movement in the interest rates differential affecting the purchasing power of domestic private consumers.

2.4 Market Clearing Conditions

We start by identifying the market clearing condition for the domestically produced products in the small open economy. Where domestic output of good (j) is absorbed both by domestic demand and foreign demand:

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j)di \tag{34}$$

In the above equation, $C_{H,t}(j)$ is domestic demand for good (j) and $C_{H,t}^i$ is country (i)'s demand for good (j) in the domestic economy. We plug the domestic demand function for good (j) (eq.5). As for foreign demand for domestic good (j), we use the assumption of symmetric preferences across all the countries of the world economy to get:

$$C^i_{H,t}(j) = \Big(\frac{P_{H,t}(j)}{P_{H,t}}\Big)^{-\epsilon}\Big(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\Big)^{-\gamma}\Big(\frac{P^i_{F,t}}{P^i_t}\Big)^{-\eta} \tag{35}$$

Plugging in the respective demand bundles transforms the market clearing condition for domestic production of good (j) to:

$$Y_t(j) = \Big(\frac{P_{H,t}(j)}{P_{H,t}}\Big)^{-\epsilon}\Big((1-\alpha)\Big(\frac{P_{H,t}}{P^i_t}\Big)^{-\eta}C_t + \alpha\int_0^1 \Big(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\Big)^{-\gamma}\Big(\frac{P^i_{F,t}}{P^i_t}\Big)^{-\eta}C^i_t(j)di\Big) \tag{36}$$

Using the Dixit-Stiglitz aggregater of domestic output, we can write the above equation in aggregate terms:

$$Y_t = \Big(\frac{P_{H,t}}{P^i_t}\Big)^{-\eta}\Big((1-\alpha)C_t + \alpha\int_0^1 \Big(\frac{\mathcal{E}_{i,t}P^i_{F,t}}{P_{H,t}}\Big)^{\gamma-\eta}Q^{\eta}_{i,t}C^i_t di\Big) \tag{37}$$

In the above equation, we took $\Big(\frac{P_{H,t}}{P^i_t}\Big)^{-\eta}$ as common factor. We have also used the definition of the bilateral real exchange rate. If we divide and multiply the term $\Big(\frac{\mathcal{E}_{i,t}P^i_{F,t}}{P_{H,t}}\Big)^{\gamma-\eta}$ by $P_{i,t}$ we get: $\Big(\frac{P_{i,t}}{P_{H,t}}\frac{\mathcal{E}_{i,t}P^i_{F,t}}{P_{i,t}}\Big)^{\gamma-\eta}$. The two terms that we get are essentially the effective terms of trade for country (i) and the bilateral terms of trade between the domestic economy and country (i), and equation (37) simplifies to:

$$Y_t = \Big(\frac{P_{H,t}}{P^i_t}\Big)^{-\eta}\Big((1-\alpha)C_t + \alpha\int_0^1 \Big(S^i_t Si,t\Big)^{\gamma-\eta}Q^{\eta}_{i,t}C^i_t di\Big) \tag{38}$$

Taking the first order log-linearisation of the above equation around a symmetric steady state yields:

$$y_t = (1-\alpha)c_t + \alpha c^*_t + \alpha[\gamma + \eta(1-\alpha)]s_t \tag{39}$$

Adding the international risk sharing condition to the above equation yields:

$$y_t = y^*_t + \frac{(1-\alpha)(\sigma-\sigma_\delta)}{\sigma_\delta}(g^*_t - g_t) + \frac{\omega_\alpha}{\sigma_\delta}s_t \tag{40}$$

where $\omega = \sigma_\delta\gamma + (1-\alpha)(\eta\sigma_\delta - 1)$ and $\omega_\alpha = (1-\alpha) + \alpha\omega$. The above equation links actual output to foreign and domestic government consumption, the rest of the world's output, and the terms of trade. From the above equation, we notice that the terms of trade variable is the only channel through which monetary policy could have an effect on the actual rate of output. In this regard, the monetary policy rule will be adjusted to achieve the required movement in the terms of trade to close the output gap.

2.4.1 The Supply Side of the Economy

The equation of the natural rate of output (derived in Appendix 1) takes the following form:

$$\bar{y}_t = \Big(\frac{\omega_\alpha(1+\varphi)}{\omega_\alpha\varphi+\sigma_\delta}\Big)a_t + \Big(\frac{\sigma_\delta(\omega_\alpha-1)}{\omega_\alpha\varphi+\sigma_\delta}\Big)y_t^* - \Big(\frac{\alpha\omega(\sigma-\sigma_\delta)}{\omega_\alpha\varphi+\sigma_\delta}\Big)g_t^* - \Big(\frac{(1-\alpha)(\sigma-\sigma_\delta)}{\omega_\alpha\varphi+\sigma_\delta}\Big)g_t \tag{41}$$

In the above equation, the effect of technology on the natural rate of output is positive and this positive effect is robust against different values of χ. Nevertheless, changes in the value of χ will determine the magnitude of the effect of technology on the natural rate of output. In the Cobb-Douglas case, when $\chi = 1$, the reaction of the natural rate of output to a TFP shock is identical to its reaction in the canonical version of the model. In the complementarity case, the reaction of the natural rate of output will be less than its reaction in the canonical version of the model. In the substitutability case, the reaction of the natural rate of output is magnified, and it is higher than its reaction in the canonical version of the mode. These findings are consistent with the findings of the closed-economy version of the model found in Troug 2019. However, the degree openness in the economy will minimise the deviation of the response of the natural rate of output to a TFP shock?under both the complementarity and the substitutability assumptions?from the reaction of the natural rate of output in the canonical version of the model.

Moreover, once we deviate from the Cobb-Douglas case, the foreign economy's output will have an effect on the natural rate of output. In the complementarity case, the foreign economy's output will have a positive effect on the natural rate of output. While in the substitutability case, the foreign economy's output will have an adverse effect on the natural rate of output. Also, both domestic and foreign government consumption will have a positive effect on the natural rate of output in the complementarity case, and an adverse effect in the substitutability case. To construct a relationship between the marginal cost variable and the output gap in the domestic economy, we subtract the above equation from equation (58) in Appendix 1 to get:

$$\hat{mc}_t = \frac{\varphi\omega_\alpha + \sigma_\delta}{\omega_\alpha} x_t \tag{42}$$

Adding the above equation to the derived Phillips curve derived in Appendix 2 enables us to write domestic inflation as a function of the output gap:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa \frac{\varphi\omega_\alpha + \sigma_\delta}{\omega_\alpha} x_t \tag{43}$$

In the above Phillips curve equation, the effect of the output gap on domestic inflation will be higher in the complementarity case than in the standard case. While in the substitutability case, it will be relatively lower.

2.4.2 The Demand Side of the Economy

In the open economy version of the model, adding the domestic economy's market clearing condition (eq. 39) to the log form of the Euler equation (eq. 10) yields:

$$\begin{aligned} y_t =& E_t\{y_{t+1}\} - \frac{(1-\alpha)}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\}) - \alpha[\gamma + \eta(1-\alpha)]\Delta E_t\{s_{t+1}\} - \alpha \Delta E_t\{y^*_{t+1}\} \\ &+ \frac{(1-\alpha)(\sigma - \sigma_\delta)}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{(1-\alpha)}{\sigma_\delta}(r_t - E_t\{\pi_{H,t+1}\}) - \frac{\alpha\omega}{\sigma_\delta}\Delta E_t\{s_{t+1}\} - \alpha\Delta E_t\{y^*_{t+1}\} \\ &+ \frac{(1-\alpha)(\sigma - \sigma_\delta)}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{H,t+1}\}) - \alpha(\omega - 1)\Delta E_t\{y^*_{t+1}\} + \frac{(1-\alpha)(\sigma - \sigma_\delta)}{\sigma_\delta}\Delta E_t\{g_{t+1}\} \\ &+ \frac{\alpha(\sigma - \sigma_\delta)}{\sigma_\delta}\Delta E_t\{g^*_{t+1}\} \end{aligned} \tag{44}$$

In the above system of equations, we made use of the CPI index equation in the domestic economy (eq. 23) and replaced the value of the terms of trade in equation (40). Moreover, we show that the effects of the domestic variables (government consumption and domestic real interest rate) on output are parametrised by the home-bias parameter $(1-\alpha)$, while the effects of the external variables are parametrised by the degree of openness in the economy α. This is inherited from the market clearing condition of the domestic economy.

The slope of the IS curve changes, similar to the closed-economy version, according the degree of substitutability between government consumption and private consumption. Nevertheless, the degree of openness in the economy minimises the effect of introducing non-separable government consumption on the slope of the IS curve. Therefore, as the degree of openness in the economy tends to zero, the slope of the IS curve starts to converge to the slope of the closed-economy case both in the substitutability and the complementarity case[7]. Solving the above IS curve for the output gap yields:

$$x_t = E_t\{x_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\} - \bar{rr}_t) \tag{45}$$

Where:

$$\begin{aligned} \bar{rr}_t = &- \frac{\sigma_\delta(1+\varphi)(1-\rho_a)}{\varphi\omega_\alpha}a_t + \frac{\alpha(\omega-1)\varphi\sigma_\alpha}{\varphi\omega_\alpha}\Delta E_t\{y^*_{t+1}\} - \frac{\alpha\omega(\sigma-\sigma_\delta)\varphi(1-\rho_{g^*})}{\varphi\omega_\alpha}g^*_t \\ &- \frac{(1-\alpha)\omega(\sigma-\sigma_\delta)\varphi(1-\rho_g)}{\varphi\omega_\alpha}g_t \end{aligned} \tag{46}$$

The reaction of the natural rate of interest to a percentage change in productivity includes σ_δ to account for the presence of government consumption, as in the closed-economy version of this model. Nevertheless, unlike the closed-economy version of the model, the open economy's natural rate of interest takes into account the degree of openness in the economy, represented by ω_α in the denominator.

In addition, the responses of the natural rate of interest to changes in the foreign economy's variables are governed by the degree of openness, and also take into account the inclusion of government consumption in the model. Also, the reaction to domestic government consumption, on the other hand, is governed by the home-bias parameter, and is also affected by the degree of openness in the domestic economy. Moreover, the effect of the foreign economy's output is positive in the complementarity case and negative in the substitutability case.

[7]In the closed-economy case, the slope of the IS curve in the complementarity case becomes flatter, making output less responsive to changes in interest rates. In the substitutability case, on the other hand, the IS curve becomes steeper making output more responsive to changes in interest rates.

2.5 Fiscal and Monetary Policy

2.5.1 Fiscal Policy

The government budget constraint in the economy is:

$$P_tG_t + (1 + R_{t-1})B_{t-1} = B_t + P_tT_t \tag{47}$$

B_t is the quantity of a riskless one-period bond maturing in the current period, paying one unit of the domestic currency. R_t denotes the gross nominal return on bonds purchased in period t. The government levies a non-distortionary lump-sum tax T_t to finance its expenditure. Following the logic of , G_t is government consumption, and is assumed to evolve exogenously according to a first order autoregressive process:

$$\frac{G_t}{G} = \left\{\frac{G_{t-1}}{G}\right\}^{\rho_g} exp(\zeta_{G,t}) \tag{48}$$

Where $0 < \rho_g < 1$ is the autocorrelation parameter of government consumption, and $\zeta_{G,t}$ represents an i.i.d government consumption shock with constant variance σ^2. Another important feature that we add to this model is assuming that the government consumes from a different market than the one occupied by the private agents[8]. For this instance, we assume that while the private agents only consume tradable goods, the government consumes non-tradable goods. Also, government consumption is assumed to be produced costlessly.

2.5.2 Monetary Policy

The monetary authorities in this model use short-term interest rates as their policy tool. As the model uses a cashless economy, money supply is implicitly determined to achieve the interest rate target. It is also assumed that the central bank will meet all the money demanded under the policy rate it sets.

The first rule in the model will be the optimal rule:

[8] In Appendix 3 we show how the effect of government consumption on private consumption changes under different values of the elasticity of substitution between the two.

$$\frac{R_t}{R} = \left\{\frac{RR_t}{RR}\right\}\left\{\frac{\Pi_{H,t}}{\Pi_H}\right\}^{\phi_\pi}\left\{\frac{Y_t}{Y}\right\}^{\phi_x} \tag{49}$$

The optimal rule tells us that by setting domestic inflation and the output gap to zero, the policy rate will equal the natural rate of interest, and it will be able to follow the developments in the natural rate of output. Thus, the optimal policy reproduces the flexible price equilibrium output, given that the government will pay a subsidy τ to offset the monopolistic distortion in the economy. The above policy rule will provide a useful benchmark to evaluate the performance of different monetary policy rates on the basis of a welfare-loss function. The first policy rule we set to evaluate in the small open economy is a naive CPI-targeting Taylor rule equation:

$$\frac{R_t}{R} = \left\{\frac{\Pi_t}{\Pi}\right\}^{\phi_\pi} \tag{50}$$

The second rule is a Taylor rule domestic-inflation targeting equation:

$$\frac{R_t}{R} = \left\{\frac{\Pi_{H,t}}{\Pi_H}\right\}^{\phi_\pi} \tag{51}$$

The third rule is the exchange rate peg equation. Where the domestic economy will follow the policy rule implemented by the foreign economy:

$$\mathcal{E}_t = 0 \tag{52}$$

The parameters of the above equations (ϕ_π, ϕ_x) describe the strength of the response of the policy rate to deviations in the variables on the right-hand side. These parameters are also assumed to be non-negative. Also, the last three rules are referred to as naive interest rules, resulting from the fact that they only make use of observable variables. Finally, the inflation response parameter ϕ_π in the above policy rules must be strictly higher than one in order for the solution of the model to be unique, as shown by Bullard and Mitra 2002 and further explained in Appendix 4.

3 Parametrisation

β	Discount factor	0.99
σ	inverse elasticity of intertemporal substitution	1
φ	inverse Frisch labour supply elasticity	3
α	share of foreign goods in core consumption	0.4
χ	inverse elasticity of substitution between $C_t \& G_t$	20 & 0.01
η	elasticity of substitution between domestic and foreign goods	1
ϵ	elasticity of substitution	6
δ	share of private consumption in the aggregate consumption bundle	0.95
θ	Calvo probability	0.75
ρ_g	AR(1) coefficient of domestic government expenditure	0.9
ρ_{g^*}	AR(1) coefficient of foreign government expenditure	0.9
ϕ_π	inflation elasticity of the nominal interest rate	1.5
ϕ_x	output gap elasticity of the nominal interest rate	0.5
ρ_a	AR(1) coefficient of domestic productivity shock	0.9
ρ_{a^*}	AR(1) coefficient of foreign productivity shock	0.9
γ	elasticity of substitution between goods in the world economy	1

The values of the parameters of the model are listed in the above table. We Set θ equal to 0.75, implying that firms only change their prices once a year. Our discount factor β is equal to 0.99, which implies?given that $\beta = 1/r$ at the steady state?that annual return is approximately equal to 4 %. We set φ equal to 3, under the assumption that the labour supply elasticity is $\frac{1}{3}$. We set ϕ_π & ϕ_x equal to 1.5 and 0.5 following Taylor 1993. We also set the inverse elasticity of substitution between government expenditure and private consumption χ equal to 20 following Bouakez and Rebei 2007 and Pieschacon 2012, and we use 0.01 for the substitutability case. The size of private consumption in the aggregate consumption bundle δ equals to 0.95. In this regard, and as mentioned in Troug 2019, the weight of δ hast to be strictly less than 1 for government consumption to influence the dynamics of the model, and different values of δ between 0 and 1 only affect the model quantitatively. Moreover, change in the value of χ do not qualitatively affect the behaviour of the model.

The inverse elasticity of intertemporal substitution of private consumption σ is set equal to 1, implying log utility in consumption. We set the elasticity of substitution between domestic and foreign produced goods η to 1. This elasticity describes the change in consumption of imported goods in response to changes in the prices of foreign goods relative to domestic prices. The value of the parameter implies that demand of imported goods increases precisely by 1 % when the relative price of foreign goods declines by 1 %. The share of foreign consumption goods in the private consumption basket is set to 40 %, while the elasticity of substitution between the domestically produced goods ϵ equals 6, corresponding to a steady state markup of 1.2. Also, we adopt the persistence parameter of government consumption in the two economies ($\rho_g \& \rho_{g^*}$) from Gali, Lopez-Salido, et al. 2007. As for the standard deviations of the two shock processes, we use the standard deviation of the TFP shock in Galí and Monacelli 2005 $\sigma_a = 0.0071$, and for the government consumption shock we use the one in Coenen and Straub 2005 $\sigma_g = 0.323$.

4 Equilibrium Dynamics

The key equations that we use to analyse the model's equilibrium implications are the non-policy block equations (IS demand curve, NKPC) and the interest rate rule conducted by the monetary authority:

$$\begin{aligned} x_t &= E_t\{x_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{H,t+1}\} - \bar{rr}_t) \\ \pi_{H,t} &= \beta E_t\{\pi_{H,t+1}\} + \kappa_g x_t \\ r_t &= \overline{rr_t} + \phi_\pi \pi_{H,t} + \phi_x x_t \end{aligned} \tag{53}$$

Where $\kappa_g = \kappa \frac{\varphi\omega_\alpha + \sigma_\delta}{\omega_\alpha}$. combining the first equation with the third equation allows us to simplify the above system of equations to only two equations. Solving for the output gap and domestic inflation as a function of their respective expectations yields:

$$\begin{aligned} x_t &= \frac{\sigma_\delta}{\sigma_\delta + \omega_\alpha\phi_x + \kappa_g\omega_\alpha\phi_\pi} E_t\{x_{t+1}\} + \frac{\omega_\alpha(1-\beta\phi_\pi)}{\sigma_\delta + \omega_\alpha\phi_x + \kappa_g\omega_\alpha\phi_\pi} E_t\{\pi_{H,t+1}\} \\ \pi_{H,t} &= \frac{\kappa_g\sigma_\delta}{\sigma_\delta + \omega_\alpha\phi_x + \kappa_g\omega_\alpha\phi_\pi} E_t\{x_{t+1}\} + \frac{\beta(\sigma_\delta + \omega_\alpha\phi_x) + \kappa_g\omega_\alpha}{\sigma_\delta + \omega_\alpha\phi_x + \kappa_g\omega_\alpha\phi_\pi} E_t\{\pi_{H,t+1}\} \end{aligned} \tag{54}$$

The above system of equations could be easily presented in a matrix form:

$$\begin{bmatrix} x_t \\ \pi_{H,t} \end{bmatrix} = A \begin{bmatrix} x_{t+1} \\ \pi_{H,t+1} \end{bmatrix}$$

Where $A = \Omega \begin{bmatrix} \sigma_\delta & \omega_\alpha(1-\beta\phi_\pi) \\ \kappa_g\sigma_\delta & \beta(\sigma_\delta+\omega_\alpha\phi_x)+\kappa_g\omega_\alpha \end{bmatrix}$ and $\Omega = \frac{1}{\sigma_\delta+\omega_\alpha\phi_x+\kappa_g\omega_\alpha\phi_\pi}$.

Under the assumed values of the policy parameters and the non-policy parameters of the model shown in the parametrisation section, we find that both eigenvalues of matrix A lie inside the unit circle, making the solution of the model unique. The results also apply to the domestic inflation targeting rule, the CPI targeting rule, and the exchange rate peg rule as well. Where the equilibrium in each model is unique, and it satisfies the following condition[9]:

$$\kappa_g\omega_\alpha(\phi_\pi - 1) + (1-\beta)\omega_\alpha\phi_x > 0 \tag{55}$$

From the above equation it is clear that the inflation parameter has to be strictly greater than one for this rule to be determined, along with a trivial condition which requires $0 < \beta < 1$.

5 IRF

5.1 Open Economy

5.1.1 A Technology Shock in the Open Economy:

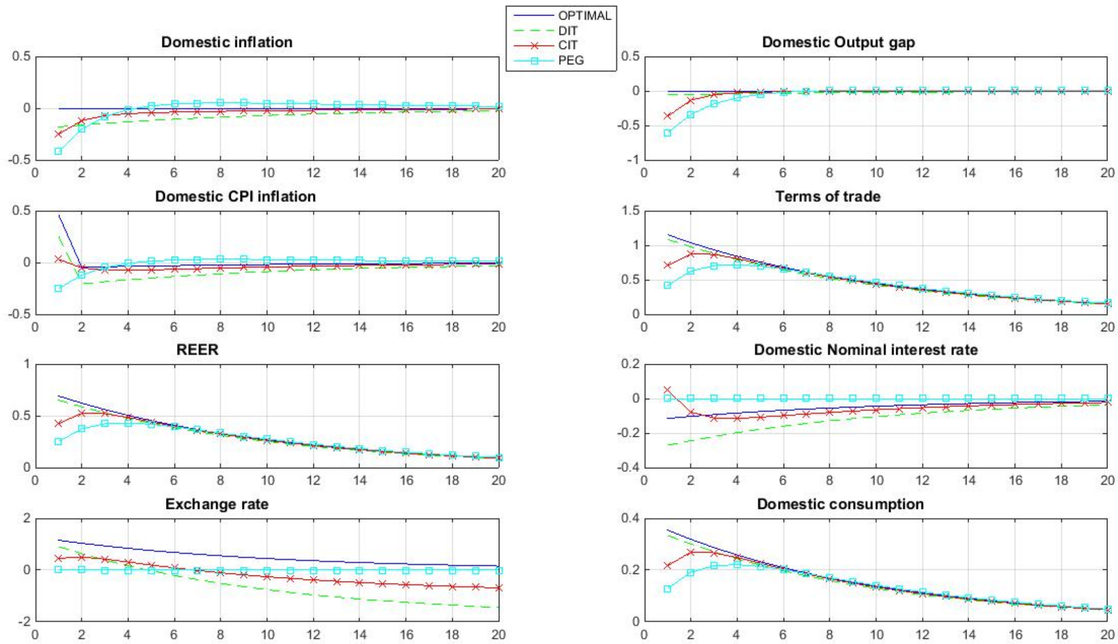
The analysis of this section focuses on the effect of a TFP shock in the small open economy. The shock is similar to the one found in Galí and Monacelli 2005, which gives us a good anchor, along with the closed-economy version of this model, on the effect of introducing government consumption to a New Keynesian model for a small open economy.

The response of the natural rate of interest in this version of the model is still consistent with the closed-economy version, as the natural rate of interest is reduced to accommodate the transitory

[9]See Appendix 4.

expansion in the natural rate of output. The natural rate of interest still takes into account the introduction of government consumption to the model, similar to the closed-economy version case, and this makes the reduction of the interest rate higher than the standard case. Nevertheless, the degree of openness in the economy partly offsets the effect of introducing government consumption on the behaviour of the model. Conversely, the positive effect of the technology shock on the natural rate of output is less in this version of the model than in a standard New Keynesian one. However, the effect here is still higher than in the closed-economy version one, mainly due to the offsetting effect of the degree of openness in the economy on the introduction of government consumption to the model.

Figure 1: Response to a Domestic TFP shock



The responses of the output gap and domestic inflation are zero under the optimal policy rule. Once we set domestic inflation and the output gap to zero, the policy rate will be lowered to follow the path of the natural rate of interest. The exchange rate depreciates after the reduction of the domestic policy rule, given that the foreign economy's policy rate is constant in this case. This depreciation will fully reflect on the terms of trade since world inflation, and domestic inflation are both fixed at zero. The depreciation of the terms of trade will boost the growth of the actual rate of output until it equals its natural level.

Consumption is lower in the open economy version than in the closed economy version. This is attributed to the difference in the market clearing conditions between the two economies, give that domestic output is not just absorbed by domestic consumption in the open economy, but by foreign consumption as well. Also, under a technology shock, the economy will still suffer from a decline in employment, similar to the closed-economy version of the model. Nevertheless, this decline in employment is minimised in the open economy case also due to the offsetting effect of the degree of openness of the economy.

The behaviour of the domestic inflation targeting rule closely resembles the behaviour of the optimal policy. Despite having a nominal value less than the optimal policy rate, the domestic inflation rule still has a contractionary policy stance as its real value $(i_{H,t} - \pi_{H,t+1})$ is above the natural rate of interest. Also, the policy's inability to guide inflation expectation to zero will lead the terms of trade to depreciate by less than the required amount to close the output gap to zero. The lack of depreciation in the terms of trade will lead to a negative output gap and negative domestic inflation rates.

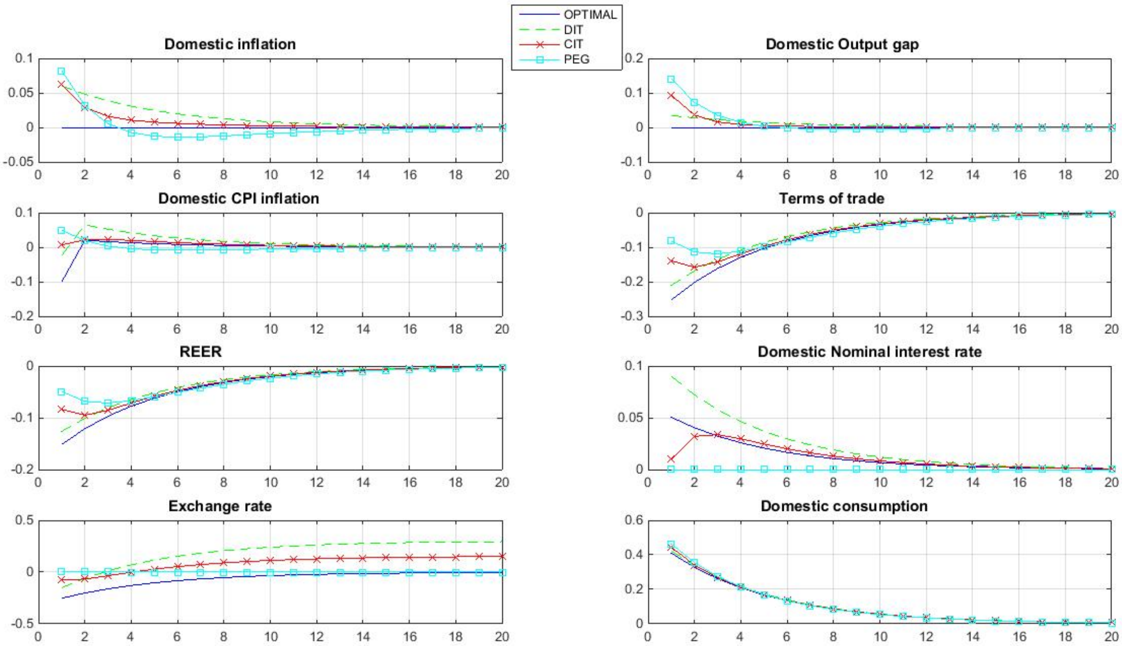
Under the exchange rate-peg regime, smoothing the terms of trade by keeping the exchange rate fixed will result in higher volatility in the domestic variables. On the other hand, the main difference between the CPI-targeting rule and the domestic inflation-targeting rule lies in the behaviour of the terms of trade. The CPI-rule targets both domestic inflation and the terms of trade. For this reason, the CPI-targeting rule will not allow the terms to depreciate enough to boost output until it reaches its potential level, and this explains the hump-shaped in the terms of trade and the exchange rate under the CPI-targeting rule. As a result, the output gap under the CPI-targeting rule will be higher than the one under the domestic inflation-targeting rule.

5.1.2 A Domestic Government Shock in the Open Economy:

The effect of a government consumption shock in the small open-economy follows the dynamics of the same shock in the closed-economy version of this model: An increase in government consumption will have a positive effect on the natural rate of output, and a positive response by the natural rate of interest to limit the inflationary pressure of this demand shock. Nevertheless, the effect of this shock is minimised by the degree of openness in the economy when compared with the same shock in the closed-economy version of this model.

Given that domestic inflation and the output gap are set to zero under the optimal policy rate, the increase in the natural rate of interest causes an appreciation in the nominal exchange rate. This appreciation will sufficiently transmit to the terms of trade. The appreciation of the terms of trade will, in return, dampen the growth of domestic output and keep it at an equivalent value to its natural level.

Figure 2: Response to a Domestic Government Consumption Shock



The dynamics of the other policy rates follow their same behaviour under the technology shock, but in an opposite manner. We notice that under a domestic inflation-targeting rule, the nominal value of the policy rate increases above the neutral level of interest, but its real value is below it, indicating an expansionary monetary policy stance. Moreover, this increasein interest rates is not fully reflected on the exchange rate because of the inability of this policy rule to guide expectations of inflation to zero levels. The less than required appreciation in the exchange rate will cause the actual rate of output to grow above its natural level, causing a positive output gap.

The overheating in domestic output will cause positive rates of domestic inflation, consequently.

Under the CPI-targeting rule, we also notice a hump-shaped response in the policy rate. This, as explained above, is due to the fact that under this rule the terms of trade are also targeted by the monetary authorities, in addition to domestic inflation. As a result, the policy rate starts to increase until the point when the necessity of stabilising the terms of trade arises. While under the exchange rate-peg regime, smoothing the terms of trade by allowing them to only appreciate through domestic inflation causes more volatility in the output gap and domestic inflation.

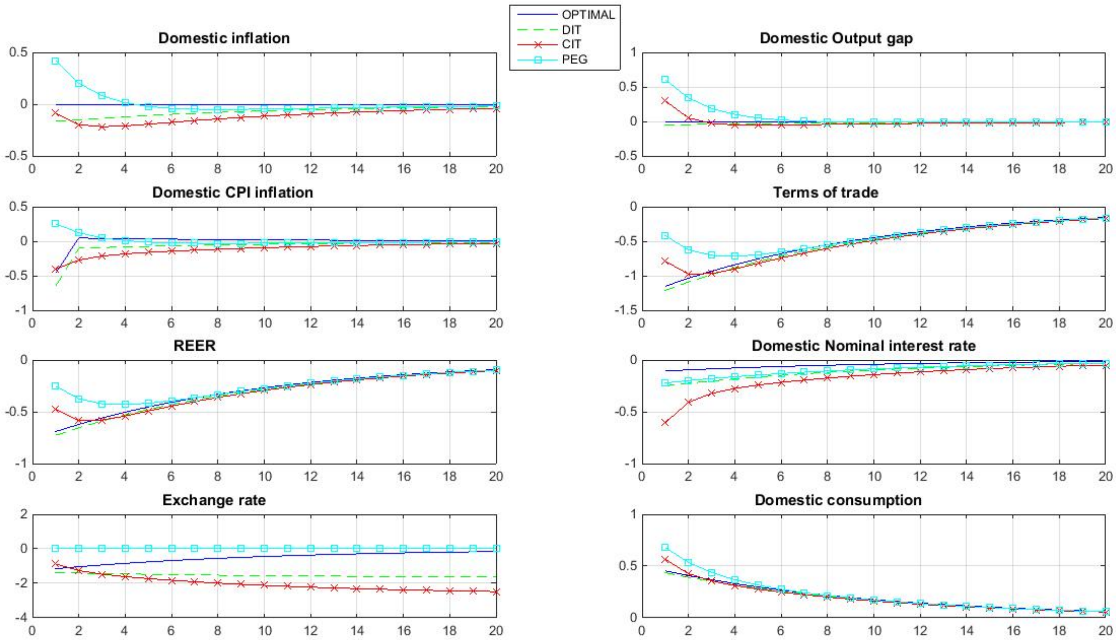
Lastly, we notice that consumption in this open version of the model is at higher levels than consumption in the closed version of the model under the same government consumption shock. This is because the appreciation of the exchange rate boosts the purchasing power of domestic households and in return, that increases domestic consumption of imported goods to the point that domestic consumption exceeds domestic output. We observe the opposite under a technology shock in the open economy, and this explains how the degree of openness minimises the crowding out effect of fiscal policy on monetary policy.

5.2 Spillover Effect on the Domestic Economy

The structure of the model enables us to construct further analysis on the domestic small open economy. In this regard, by modelling the foreign economy, we can capture the effect of shocks in the foreign economy on domestic variables via the three important channels. The first is changes in world demand for domestic goods, which is illustrated in the domestic market clearing condition. The second is the price differential in the two economies which affects the competitiveness between the two economies. The last, which is the vital one, is the interest rates differential between the two economies. We only limit this analysis to one policy rule in the foreign economy, and that rule is the optimal policy in the foreign (closed) economy.

5.2.1 The Effect of a Technology Shock in the Foreign Economy on the Domestic Economy:

Figure 3: Response to a TFP Shock in the Foreign Economy



A shock in the foreign economy's TFP causes growth in world demand for domestically produced goods. The main channel in these dynamics is, as noted above, the interest rates differential between the two economies. In this regard, given that the monetary authority in the foreign economy will lower its policy rate to accommodate the expansion in the natural rate of output, the domestic authorities should try to manage how its exchange rate should behave in order to achieve internal stability in the domestic economy.

Under the optimal policy rule, the domestic policy rule aims to achieve a positive interest rate differential against the foreign economy's interest rate. This positive gap in the interest rates will cause the terms of trade to appreciate to the extent that it keeps actual output at its natural level. Thus, under the optimal policy rule, inflationary external demand for domestic products is reduced by making domestic products more expensive for foreign households, and by increasing the purchasing power of domestic households to enable them to consume more from abroad.

Under the domestic inflation targeting rule however, the policy rate is lowered at a level equivalent to the foreign economy's policy rate. Nevertheless, its inability to manage expectation of zero-domestic inflation expectations makes the terms of trade appreciate more than required, causing a negative output gap. The double dimensional structure of the CPI-targeting rule will aim at lowering the policy rate to close the negative gap of domestic inflation (its first target), and this causes the actual output to grow above its natural level. Nevertheless, as the terms of trade start reaching undesirable negative values, the policy rule is reversed to stabilise the terms of trade (its second target).

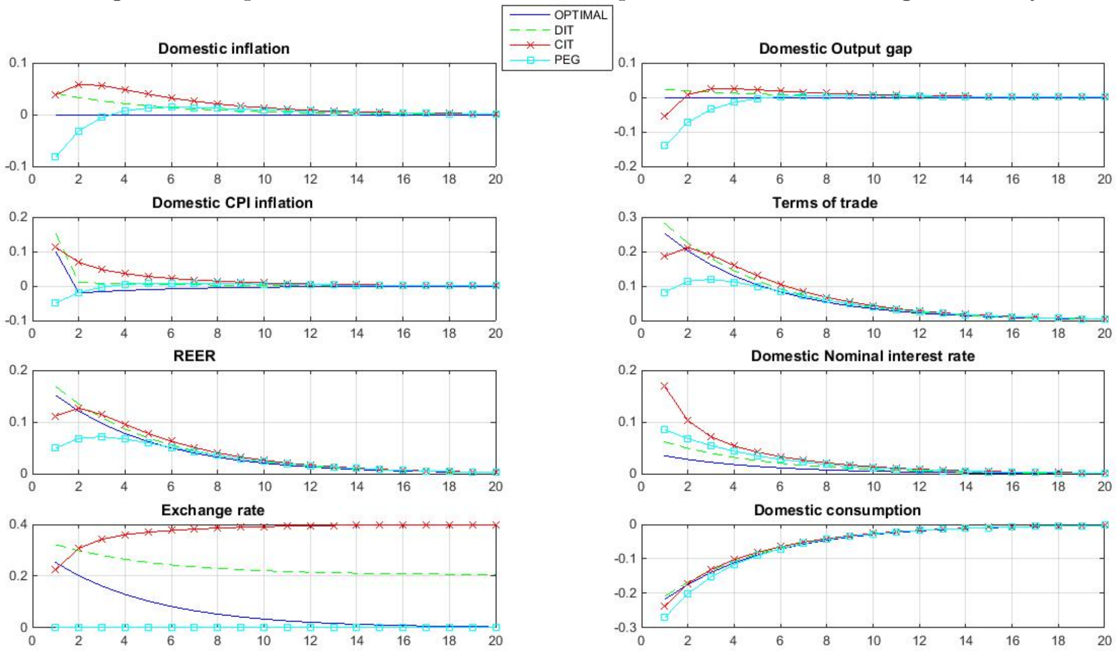
Under the exchange rate-peg regime, the domestic authorities follow the rule conducted by the foreign economy's authorities. As a result, the domestic policy rule will be lower than the neutral rate of interest. This expansionary stance will boost the actual rate of output to grow above its natural level, causing inflationary pressure. Consistent with the analysis of shocks in the small open economy, pegging the exchange rate causes smooth behaviour in the terms of trade, given that they are only affected by the sticky prices of the two economies only. As a result, adopting this rule will cause more volatility in the domestic variables (domestic inflation, the output gap).

5.2.2 The Effect of a Government Shock in the Foreign Economy on the Domestic Economy:

Under a government consumption shock in the foreign economy, the foreign economy's policy rate will increase to limit the inflationary pressure arising from the increase in demand. From the uncovered interest rate parity condition, the relative increase in foreign interest rates will cause downward pressure on the value of the domestic economy's currency, and foreign demand for domestic goods will also increase in this case. Thus, leaving the domestic policy rule unchanged in this case will boost the actual rate of output to grow far beyond its natural level leading to high levels of inflation.

Under the optimal policy rule, the gap between the interest rates is reduced to limit the depreciation in the domestic currency. This depreciation will fully reflect on the terms of trade, leading the actual rate of output to grow at a rate equivalent to its natural level, and this will cause zero levels of domestic inflation. Limiting the depreciation of the domestic currency will also minimise the adverse effect of an increase in the foreign economy's policy rate on domestic consumption.

Figure 4: Response to a Government Consumption Shock in the Foreign Economy



We also notice that the domestic inflation-targeting rule fails, in this case as well, to guide expectation of inflation to the zero-levels, despite the fact that the nominal value of the policy rate, in this case, was above the value of the optimal policy rate. Under this rule, the gap between the interest rates is minimised more than the optimal policy rule. Nevertheless, this interest rate differential will not fully reflect in the exchange rate, due to the presence of non-zero domestic inflation levels. The high level of depreciation in the domestic currency, in this case, will cause the actual rate of domestic output to grow above its natural level leading to positive rates of inflation in return.

Under the exchange rate-peg regime, the domestic authorities will follow the rate adopted by the foreign economy's monetary authorities in this case as well. This rate will cause the terms of trade to only grow via the relative sticky prices in the two economies. The actual rate of output will grow less than its potential level, causing a negative output gap and negative inflation rates as a result. Thus, adopting the exchange rate-peg regime will lead the economy into recession because of the domestic authorities' inability to adjust the external balances (the exchange rate, the terms of trade), in order to achieve internal stability (the output gap, domestic inflation). Under the CPI-targeting rule, the policy rate is first increased to limit the high levels of domestic inflation causing a less than needed depreciation in the terms of trade. However, the policy rule reverts once domestic inflation starts going down. The terms of trade are never a concerning issue in this case since the depreciation in the exchange rate is offset by the positive levels of domestic inflation.

This section highlights the main differences between the dynamics of this model and the one used in Ganelli 2003. In this model, the interest rates differential will affect the purchasing power of the domestic consumers, due to the reaction of monetary policy to changes in aggregate demand. This will result in a contradictory effect between domestic government consumption and foreign government consumption on domestic private consumption, both in the complementarity and substitutability case.

5.3 The Substitutability Case

In this section we illustrate how the dynamics of the model change, once we incorporate government consumption as a substitute to private consumption[10]:

5.3.1 A Technology Shock in the Open Economy:

The response of the natural rate of interest to a technology shock, in this case, is consistent with its response under the other cases. The natural rate of interest is reduced in this case as well to accommodate the expansion in the natural rate of output. Nevertheless, the reduction of the natural rate of interest, in this case, will be less than all of the other cases. This is attributed to the fact that output is more sensitive to changes in interest rates under the substitutability assumption, as noted earlier. Also, the positive effect of a technology shock on the natural rate of output is higher than all of the other cases.

Similar to all the other cases, under the exchange rate-peg rule and the CPI-targeting rule, the relative smoothing of the terms of trade will cause more volatility in domestic inflation and the output gap.

[10]The impulse response function graphs, are reported in Appendix 5.

5.3.2 A Government Shock in the Open Economy:

Government consumption will have an adverse effect on output in this case. The role of monetary policy is to reduce interest rates to alleviate the adverse effect of this shock on the economy. The decline in the domestic rates will cause depreciation in the exchange rate, and this, in return, will direct private consumption towards domestically produced goods. The exchange rate-peg role is still the worst performing role among the evaluated rules.

5.3.3 The Effect of a Technology Shock in the Foreign Economy on the Domestic Economy:

The spillover effect of a technology shock in the foreign economy on the domestic economy in the substitutability case is similar to the effect in the complementarity case. Given that foreign interest rates are reduced to accommodate the expansion in output, the domestic interest rates must keep the appropriate positive interest rates differential to reduce foreign demand for domestically produced goods. This positive gap will make domestic goods more expensive than foreign goods, through the appreciation of the exchange rate. Failure to do so, similar to the CPI-targeting rule and the exchange rate-peg rule, will cause more welfare losses in the form of more fluctuations in the output gap and domestic inflation.

5.3.4 The Effect of a Government Shock in the Foreign Economy on the Domestic Economy:

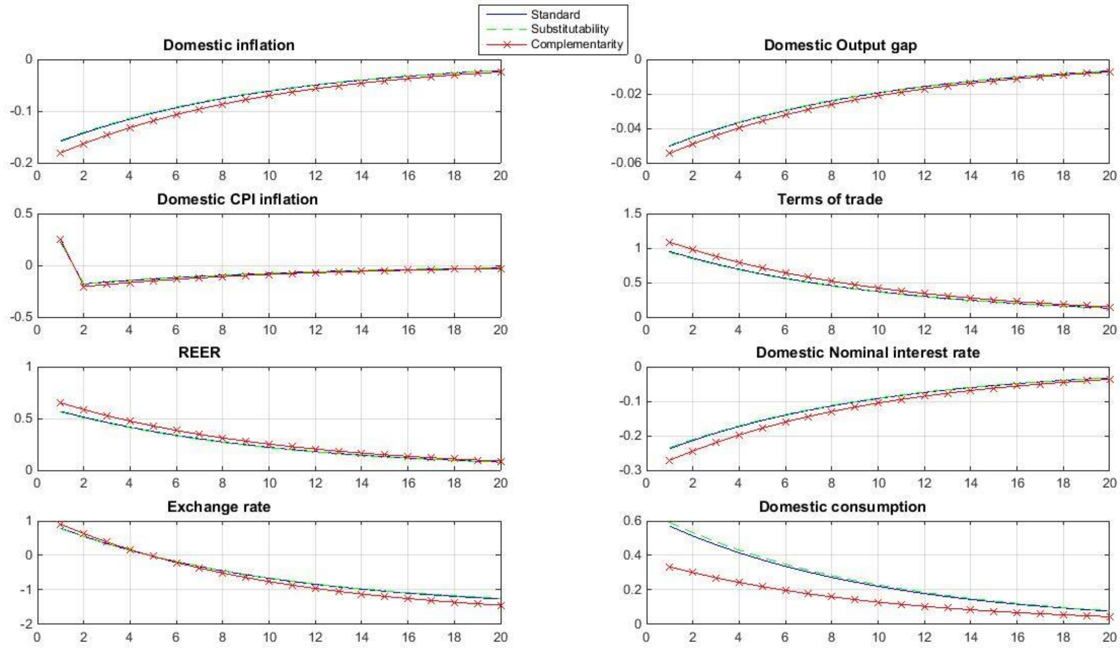
The spillover effect of a government shock in the foreign economy will take place through two channels. The first is the direct positive effect of the foreign economy's government consumption on domestic private consumption in the substitutability case. Similar to the above technology shock, foreign interest rates will be reduced to limit the adverse effect of government consumption on the foreign economy, which creates the second channel (interest rates differential channel). The role of domestic monetary policy is to keep a positive gap between domestic interest rates and foreign interest rates to direct household consumption towards consumption of foreign goods. This, as a result, will reduce the inflationary effect of a shock in the foreign economy's government consumption.

This section also illustrates the main difference between our model and the one used by Ganelli. We find that, even in the substitutability case, domestic government consumption and foreign government consumption have contradicting effects on domestic private consumption.

5.4 The Effect of Introducing Government Consumption on a Standard Model

In this section, I explicitly demonstrate how the behaviour of a standard New Keynesian model changes once government consumption is included in a non-separable form. In the below graph, I nest the effect of the same technology shock on the domestic economy under the standard case, the complementarity case and the substitutability case. I choose the domestic inflation targeting rule as the monetary policy rule for all three scenarios to show how the output gap and domestic inflation are affected by the technology shock across all three scenarios.

Figure 5: Response to a Technology Shock



A TFP shock requires a reduction in interest rates to accommodate growth in the economy and increase employment. The highest reduction in interest rates occurs in the complementarity case, as domestic variables become less sensitive to changes in interest rates. The reduction under the substitutability assumption, on the other hand, resembles the standard case. The reduction of interest rates transmits to depreciation in the exchange rate and the terms of trade, which, understandably, show the highest depreciation under the complementarity scenario.

Private consumption in the complementarity case increases the least, despite the fact that interest rates are reduced the most in this case, reflecting the crowding out effect of government consumption to monetary policy under the complementarity assumption. Under the substitutability case, as domestic variables become more sensitive to changes in interest rates, private consumption increases more than the other two scenarios, in spite of the fact that the reduction of interest rates in the complementarity case is equivalent to the reduction in the standard case. The simulations also show that the negative values of domestic inflation and the output gap are also greater in the complementarity case than the standard and substitutability case, also reflecting the effect of the introduction of government consumption in a non-separable form.

6 Empirical evidence: panel fixed effects estimates:

The above theoretical model clearly illustrates the crowding out effect of government consumption to monetary policy when the former has a crowding in effect on private consumption and vice versa. Moreover, the paper shows that the degree of open dampens this crowding in (out) effect of fiscal policy on monetary policy. This section empirically tests the hypothesis mentioned above by adopting the panel fixed effect model used in Troug 2019.

The panel regression start by estimating the effect of government consumption and the policy rate on private consumption, as shown in the first two specifications. In the third specification, the interaction term, which shows the interaction between the policy rate and government consumption, is added to the estimation. This paper shows how the degree of openness dampens the crowding out effect of government consumption on monetary policy by adding net exports as a proxy for openness.

Table 1: The crowding out effect of monetary policy to monetary policy

Dependent variable: Private Consumption	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Government Consumption	0.114***	0.123**	0.126***	0.114***	0.124***	0.126***	0.117***
	(0.016)	(0.016)	(0.016)	(0.016)	(0.016)	(0.016)	(0.016)
Changes in Policy Rate		-0.0004**	-0.0004**		-0.0004**	-0.0004**	-0.0004**
		(0.0002)	(0.0002)		(0.0002)	(0.0002)	(0.0002)
Net Exports							-0.23**
							(0.109)
$\Delta(\Delta r_t * G_c)$			0.0001**			0.0001**	0.0001
			(0.00006)			(0.00007)	(0.0001)
$\Delta(\Delta r_t * G_c * \frac{NX}{Y})$							0.0001
							(0.0001)
Output	0.741***	0.744***	0.747***	0.741***	0.744***	0.746***	0.743
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)
Trend Consumption	0.0284***	0.0297***	0.0304***	0.0284**	0.0298***	0.0305***	0.0178**
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.008)
Trend Output	-0.0264***	-0.0294***	-0.0301***	-0.0263***	-0.0291***	-0.0301***	-0.0172**
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.008)
ZLB Dummy				-0.008	-0.009	-0.006	0.005
				(0.060)	(0.059)	(0.059)	(0.060)
Constant	-0.890	1.061	1.511	-0.983	0.956	1.439	0.345
	(1.692)	(1.701)	(1.739)	(1.820)	(1.828)	(1.865)	(1.93)
Country Fixed Effects	yes	yes	yes	yes	yes	yes	yes
Observations	3,220	3,150	3,185	3,220	3,185	3,150	3,150
Countries	35	35	35	35	35	35	35
R-squared	0.47	0.51	0.51	0.47	0.50	0.50	0.51

Standard errors are in parentheses.

Significance levels: *** p <0.01, ** p <0.05, * p <0.1.

The specification of the model takes the following form:

$$C_c = \alpha + (\beta_1 + \beta_2 G_c + \beta_3 NX_t * G_c)\Delta i^p_{it} + \beta_4 G_c + \beta_5 Y_c + \beta_6 Y_{HP} + \beta_7 C_{HP} + \beta_8 NX_t + \beta_9 D_1 \quad (56)$$

In the above equation, C_c is cyclical private consumption, G_c is cyclical government consumption and Y_c is cyclical output. Y_{HP} and C_{HP} are the trend components of output and private consumption, respectively. $D1$ is a dummy variable that accounts for the zero lower bound period (2008q1-2013q4). These variables are similar to the ones used in . This paper adds net exports as a proxy variable and an additional interaction term between the policy rate, government consumption and net exports.

The analysis is conducted on 35 OECD countries (excluding Turkey). As mentioned above, private consumption, government consumption and output are detrended. We also control for trend private consumption and output. The data for these three variables come from the OECD database. All variables are expressed in real terms. Also, data on interest rates come from the IMF's IFS database, and it is expressed in nominal terms. The data is in quarterly frequency, and it covers the period 1995q1 - 2017q4.

The results of all the panel regressions are shown in the above table. The first specification shows how government consumption has a crowding in effect on private consumption. A 10 % increase in government consumption causes a contemporaneous and statistically significant increase of 1.14 % in private consumption. The size of this effect is robust through all specifications of the above table. Starting from the second specification, we add the policy interest rate, which represents the monetary policy instrument. The results show an adverse, significant effect of the policy rate on private consumption in the second and third specifications.

In the third specification, we add an interaction variable between government consumption and the policy rate. Even though the coefficient of the policy rate (β_1) remains unchanged, (β_2) shows the crowding out effect of government consumption on monetary policy. After adding the interaction term, the combined effect ($\beta_1 + \beta_2$) shows a decline in the effect of the policy rate on consumption from -0.0004 to -0.0003, supporting the predictions of the above theoretical model.

In the last specification, net exports and an additional interaction term are added to the model. The interaction term is a product of the policy rate, government consumption and net exports as a ratio of output. The results show that the additional interaction term, although insignificant, dampens the crowding out effect of the first interaction term, leading to an increase in the effect of the policy rate from -0.0003($\beta_1 + \beta_2$) in the third specification to -0.0004 ($\beta_1 + \beta_2\beta_3$) in the seventh specification. The last four specifications show that the estimations of the model are robust to the zero lower bound period.

7 Conclusion

In this paper, we developed a small open economy model in a dynamic general equilibrium framework which incorporates meaningful government consumption into the utility function in a non-separable form. We perform this exercise to show the effect of this introduction on the structure and dynamics of a standard New Keynesian model. The analysis of the model is twofold. First, we add a government sector to a standard open-economy New Keynesian model. Doing so allows us to make the analysis against two benchmark models. The first one is the closed-economy version of the model, while the second one is the Galí and Monacelli 2005 model, which is the workhorse model for small open economies. The second part of the analysis is the spillover effect of shocks in the foreign (closed) economy on the small open economy, and we make our analysis for this part against the analysis of Ganelli 2003.

In the open economy case, the effect of introducing government consumption in a non-separable form on monetary policy is reduced by the degree of openness in the economy. Monetary policy affects the internal balances in the domestic economy through its influence on the movement of the exchange rate and the terms of trade. Monetary policy behaves differently under the two imposed shocks on the model. Under a technology (supply) shock, the policy rate should be lowered to accommodate the expansion in output, and failure to do so will cause a high reduction in employment, resulting in higher welfare losses. Once interest rates are lowered, the purchasing power of the domestic currency will weaken, and that will make domestic output more competitive. The decline in the purchasing power of the domestic currency will, as a result, direct domestic consumers to consume domestically produced goods. Under the government consumption (demand) shock, the policy rate is increased to offset the inflationary demand increase in the complementarity case. This increase in the interest rates will cause the domestic currency to appreciate, and that will increase the purchasing power of the domestic households. This increase, in return, will make domestic production less competitive and will direct private consumption towards imported goods. In the substitutability case, given the adverse effect of government consumption on private consumption, monetary policy will lower interest rates to minimise the decline in private consumption. This will negatively affect the purchasing power of the domestic consumers and direct their consumption towards domestically produced goods.

The empirical results of this paper support the findings of the theoretical model. First, the results show that government consumption in 35 OECD countries has a crowding in effect on private consumption. Second, this positive effect of government consumption dampens the negative effect of the policy rule on private consumption. Nevertheless, the degree of openness in the economy, measured as net exports as a ratio of output, minimises the crowding out effect of fiscal policy on monetary policy. These results are robust to the zero lower bound period.

In the second part of this exercise, we evaluate the response of the domestic economy's authorities to shocks in the foreign economy. We find that the most significant transmission channel of these shocks is the interest rates differential channel. Given that the foreign economy's authorities will change the policy rates in response to any shock in the foreign economy, the exchange rate will immediately be affected by these changes. The role of the domestic economy's authorities, in this case, is to minimise the resulting depreciation or appreciation in the domestic currency to ensure internal stability. The dynamics of the model also highlight significant differences with the model used by in regards to the spillover effect of foreign government consumption on domestic consumption. In our model, the international risk sharing condition shows how domestic government consumption and foreign government consumption have conflicting effects on domestic private consumption. Also, the interest rates differential affects the purchasing power of the domestic consumers due to the reaction of monetary policy to changes in aggregate demand. This further contributes to the contradicting effects of domestic government consumption and foreign government consumption on domestic consumption both in the complementarity and substitutability case. While in Ganelli's model the two will have the same adverse effect on domestic private consumption.

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Appendices

Appendix 1

- Derivation of the natural rate of output:

We start the derivation of the natural rate of output from the log form of the marginal cost equation:

$$\begin{aligned} mc &= -v + w_t - p_{H,t} - a_t \\ &= -v + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -v + \varphi n_t + \sigma_\delta c_t + (\sigma - \sigma_\delta) g_t + \alpha s_t - a_t \\ &= -v + \varphi y_t + \sigma_\delta y_t^* + (\sigma - \sigma_\delta) g_t^* + s_t - (1+\varphi) a_t \end{aligned} \tag{57}$$

In the above system of equations we made use of the log form of the domestic economy's labour supply equation (eq.9), the CPI index equation (eq.23), and the international risk sharing condition (eq.33). Plugging in the value of the terms of trade from the market clearing condition of the domestic economy (eq.40) yields:

$$mc_t = \Big(\frac{\varphi\omega_\alpha + \sigma_\delta}{\omega_\alpha}\Big) y_t + \Big(\frac{\sigma_\delta(\omega_\alpha - 1)}{\omega_\alpha}\Big) y_t^* + \Big(\frac{\alpha\omega(\sigma - \sigma_\delta)}{\omega_\alpha}\Big) g_t^* + \Big(\frac{(1-\alpha)(\sigma - \sigma_\delta)}{\omega_\alpha}\Big) g_t - (1+\varphi) a_t \tag{58}$$

To solve for the natural rate of output, we first set $mc_t = -\mu$. Where μ is the markup under flexible prices. Solving the above equation for output yields:

$$\bar{y}_t = \Big(\frac{\omega_\alpha(1+\varphi)}{\omega_\alpha\varphi + \sigma_\delta}\Big) a_t + \Big(\frac{\sigma_\delta(\omega_\alpha - 1)}{\omega_\alpha\varphi + \sigma_\delta}\Big) y_t - \Big(\frac{\alpha\omega(\sigma - \sigma_\delta)}{\omega_\alpha\varphi + \sigma_\delta}\Big) g_t^* - \Big(\frac{(1-\alpha)(\sigma - \sigma_\delta)}{\omega_\alpha\varphi + \sigma_\delta}\Big) g_t \tag{59}$$

- Log-linearised model:

Domestic economy

- Aggregate consumption bundle:

$$\bar{c}_t = \delta c_t + (1-\delta) g_t \tag{60}$$

- IS curve:

$$x_t = E_t\{x_{t+1}\} - \frac{\omega_\alpha}{\sigma_\delta}(r_t - E_t\{\pi_{t+1}\} - \bar{r}r_t) \tag{61}$$

- Natural rate of interest:

$$\begin{aligned} \bar{r}r_t = &- \frac{\sigma_\delta(1+\varphi)(1-\rho_a)}{\varphi\omega_\alpha} a_t + \frac{\alpha(\omega-1)\varphi\sigma_\alpha}{\varphi\omega_\alpha}\Delta E_t\{y^*_{t+1}\} - \frac{\alpha\omega(\sigma-\sigma_\delta)\varphi(1-\rho_{g^*})}{\varphi\omega_\alpha} g^*_t \\ &- \frac{(1-\alpha)\omega(\sigma-\sigma_\delta)\varphi(1-\rho_g)}{\varphi\omega_\alpha} g_t \end{aligned} \tag{62}$$

- Phillips curve:

$$\hat{\pi}_{H,t} = \beta E_t[\hat{\pi}_{H,t+1}] + \frac{\kappa(\varphi\omega_\alpha + \sigma_\alpha)}{\omega_\alpha}\hat{x}_t, \tag{63}$$

- Flexible-price output:

$$\bar{y}_t = \Big(\frac{\omega_\alpha(1+\varphi)}{\omega_\alpha\varphi+\sigma_\delta}\Big)a_t - \Big(\frac{\sigma_\delta(\omega_\alpha-1)}{\omega_\alpha\varphi+\sigma_\delta}\Big)y^*_t - \Big(\frac{\alpha\omega(\sigma-\sigma_\delta)}{\omega_\alpha\varphi+\sigma_\delta}\Big)g^*_t - \Big(\frac{(1-\alpha)(\sigma-\sigma_\delta)}{\omega_\alpha\varphi+\sigma_\delta}\Big)g_t \tag{64}$$

- Output gap:

$$x_t = y_t - \bar{y}_t. \tag{65}$$

- CPI inflation:

$$\hat{\pi}_t = \hat{\pi}_{H,t} + \alpha\left(\hat{s}_t - \hat{s}_{t-1}\right). \tag{66}$$

- Monetary policy:

$$\begin{cases} r_t = \overline{rr_t} + \phi_\pi\pi_t + \phi_x x_t, & \text{Optimal policy,} \\ r_t = \phi_\pi\pi_{H,t}, & \text{Domestic inflation targeting,} \\ r_t = \phi_\pi\pi_t, & \text{CPI inflation targeting,} \\ e_t = 0, & \text{Exchange rate peg,} \end{cases} \tag{67}$$

- Exogenous processes:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t} \tag{68}$$

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \tag{69}$$

- Labour Supply:

$$w_t - p_t = \phi n_t + \sigma_\delta c_t + (\sigma - \sigma_\delta) g_t \tag{70}$$

- Production:

$$y_t = a_t + n_t \tag{71}$$

Rest of the world

- Aggregate consumption bundle in the rest of the world:

$$\bar{c}_t^* = \delta c_t^* + (1 - \delta) g_t^* \tag{72}$$

- IS curve in the rest of the world:

$$x_t^* = E_t\{x_{t+1}^*\} - \frac{1}{\sigma_\delta}[r_t^* - E_t\{\pi_{t+1}^*\} - \bar{r}r_t^*] \tag{73}$$

- Natural rate of interest:

$$\bar{r}r_t^* = -\frac{\sigma_\delta(1 - \rho_{a^*})(1 + \varphi)}{\varphi + \sigma_\delta} a_t^* + -\frac{((\sigma - \sigma_\delta)(1 - \rho_{g^*})\varphi}{\varphi + \sigma_\delta} g_t^* \tag{74}$$

- Phillips curve:

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \kappa(\varphi + \sigma_\delta) x_t^* \tag{75}$$

- Flexible-price output:

$$\bar{y}_t^* = -\Big(\frac{\sigma - \sigma_\delta}{\varphi + \sigma_\delta}\Big) g_t^* + \Big(\frac{1 + \varphi}{\varphi + \sigma_\delta}\Big) a_t^* \tag{76}$$

- Output gap:

$$x_t^* = y_t^* - \bar{y}_t^*. \tag{77}$$

- Monetary policy:

$$r_t^* = \overline{rr_t}^* + \phi_\pi^* \pi_t{}^* + \phi_x^* x_t^* \tag{78}$$

- Rest of the world market clearing condition:

$$y_t^* = c_t^* \tag{79}$$

- Exogenous process:

$$a_t^* = \rho_a^* a_{t-1}^* + \epsilon_{a,t}^*, \tag{80}$$

$$g_t^* = \rho_g^* g_{t-1}^* + \epsilon_{g,t}^*. \tag{81}$$

- Labour Supply:

$$w_t^* - p_t^* = \phi n_t^* + \sigma_\delta c_t^* + (\sigma - \sigma_\delta) g_t^* \tag{82}$$

- Production:

$$y_t^* = a_t^* + n_t^* \tag{83}$$

International linkages

- Goods market clearing:

$$y_t = (1-\alpha)c_t + \alpha c_t^* + \alpha[\gamma + \eta(1-\alpha)]s_t \tag{84}$$

- Terms of trade:

$$s_t - s_{t-1} = e_t - e_{t-1} + \pi_t^* - \pi_{H,t}. \tag{85}$$

- Real exchange rate:

$$q_t = (1-\alpha)s_t. \tag{86}$$

- International risk sharing:

$$c_t = c_t^* + \frac{(\sigma - \sigma_\delta)}{\sigma_\delta}(g_t^* - g_t) + \frac{1}{\sigma_\delta} q_t. \tag{87}$$

Appendix 2

To understand the inflation dynamics in the model, we start by analysing the price-setting behaviour of firms. We follow the steps of Galí and Monacelli 2005, and the 3rd chapter of Gali 2008 to derive the price-setting behaviour of firms in the model under a sticky prices framework. The aggregate domestic price index in the model is a weighted average of prices that have been adjusted at period t and prices that have not been adjusted:

$$P_{H,t} = \left[\theta (P_{H,t-1})^{1-\epsilon} + (1-\theta)(\bar{P}_{H,t})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{88}$$

$\bar{P}_{H,t}$ is the re-optimised price that a fraction of the firms $(1-\theta)$ choose at period t, and this is normally higher than the prevailing price during the last period before. P_{t-1} is the price imposed by the other fraction of firms who have not been able to adjust their prices, and this is why we keep last period's prices as the prevailing prices for those firms. We divide the above equation by $P_{H,t-1}$ to get:

$$\Pi_{H,t}^{1-\epsilon} = \theta + (1-\theta)\Big(\frac{\bar{P}_{H,t}}{P_{H,t-1}}\Big)^{1-\epsilon} \tag{89}$$

Log-linearising the above equation around a steady state with zero inflation yields[11]

$$\pi_{H,t} = (1-\theta)(\bar{p}_{H,t} - p_{H,t-1}) \tag{90}$$

In the above equation, inflation at the current period is affected by the price adjustment that a fraction of the firms in the economy makes to their prices. Therefore, as mentioned above, we start deriving the price-setting behaviour of firms to capture the dynamics of prices in the economy. When firms set their prices according to Calvo 1983 contract scheme, they aim to maximise the expected discounted value of their profits under the assumption that the newly set price will still

[11] Log-linearising around a steady state of zero inflation allows us to get rid of the price dispersion created by the nominal friction in the model.

be effective:

$$max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}[c_{jt+k|t}(\bar{P}_{H,t} - \Psi_{t+k})]\} \quad (91)$$

Ψ is the cost function, θ^k is the probability that the re-optimised price at period t will remain effective at period t+k, and $Q_{t,t+k}$ is a the discount factor of nominal pay off and it is defined in equation (3). $c_{jt+k|t}$ is the Expected demand/production for period t+k at period t. The equation is subject to the following demand constraint: $c_{jt+k} = \left(\frac{\bar{P}_t}{P_{t+k}}\right)^{-\epsilon} C_t$. Plugging in the demand function into the firm's maximisation problem yields:

$$max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}[\Big(\frac{\bar{P}_{H,t}}{P_{t+k}}\Big)^{-\epsilon} C_{t+k}(\bar{P}_t - \Psi_{t+k})]\} \quad (92)$$

Taking the first order condition of the above equation yields:

$$\sum_{k=0}^{\infty} \theta^k E_t\{Q_{t,t+k}C_{t+k}[\bar{P}_{H,t} - \mathcal{M}\psi_{t+k|t}]\} = 0 \quad (93)$$

ψ is the nominal marginal cost, and $\mathcal{M}$ is the gross mark-up and its equal to $\frac{\epsilon}{\epsilon-1}$. Now, we divide the above equation by $P_{H,t-1}$ and divide and multiply the second term by $P_{H,t+k}$:

$$\sum_{k=0}^{\infty} \theta^k E_t\Big\{Q_{t,t+k}C_{t+k}\Big[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \mathcal{M}MC_{t+k|t}\Pi^H_{t-1,t+k}\Big]\Big\} = 0 \quad (94)$$

Where $\Pi^H_{t-1,t+k} = \frac{P_{H,t+k}}{P_{H,t-1}}$, and $MC_{t+k|t} = \frac{\psi_{t+k|t}}{P_{H,t+k}}$. We log-linearise the above equation around a zero-inflation steady state. Noting that $Q_{t,t+k}$ in the steady state will equal β^k:

$$\bar{p}_{H,t} - p_{H,t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k|t} + \mu + (p_{H,t+k} - p_{H,t-1})\} \quad (95)$$

We notice from the above equation that the firms discount the expected stream of their future profits using the household's discount factor. This is simply attributed to the fact that the households are the share holders of those firms. Rearranging the above equation gives:

$$\bar{p}_{H,t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k|t} + p_{H,t+k}\} \quad (96)$$

The above equation is describing how firm set their prices with a certain mark-up and the discounted present value of the stream of marginal costs. In the case when $\theta = 0$, all firms will be able to adjust their prices in each period (flexible prices scheme), and the above equation will simplify to:

$$\bar{p}_{H,t} = \mu + mc_t \tag{97}$$

The price the firms set in this case is equal to their markup over the nominal marginal cost. Of course, this shows that the price set by the firms is above their marginal cost since the markup is greater than 1. As a result, output will be lower than its level under perfect competition. It will be shown how the government can offset this distortion by giving the firms a certain employment subsidy. Now going back to equation (95), if we rewrite down the equation in a compact form we get:

$$\bar{p}_{H,t} - p_{H,t-1} = \beta\theta E_t\{p_{H,t+1} - p_{H,t}\} + \pi_t + (1-\beta\theta)\hat{mc}_t \tag{98}$$

Where $\hat{mc}_t = mc_t - p_{H,t} + \mu$. Adding the above equation to the price setting equation gives us:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa\hat{mc}_t \tag{99}$$

Where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. The above equation is the core New Keynesian Phillips Curve. We develop it in the text to link inflation to the output gap through the relationship between the $\hat{mc}$ and the output gap x_t. κ in the Phillips curve equation is strictly decreasing in the stickiness parameter θ. From the above equation, we see that inflation in this type of models is a result of aggregate price-setting of the firms who adjust their prices based on current and future stream of their marginal costs.

Appendix 3

In this section we aim to exploit how the effect of government consumption on private consumption changes under different values of the elasticity of substitution between the two. We show this under three different values for the inverse elasticity of substitution between government expenditure and private consumption χ: a) The Cobb-Douglas scenario when the the elasticity between government consumption and private consumption is equal to the inverse elasticity of intertemporal substitution

$\chi = \sigma$, b) The complementarity case when $\chi > \sigma$, and c) and the last case when $\chi < \sigma$ when the two items are substitutes. Maximising the utility function with respect to the budget constraint:

$$\ell = \frac{\left[\left(\delta^{\chi} C_t^{1-\chi} + (1-\delta)^{\chi} G_t^{1-\chi}\right)^{\frac{1}{1-\chi}}\right]^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t (D_t + W_t N_t + P_t T_t - P_t C_t - E_t[Q_{t,t+1} D_{t+1}]) \tag{100}$$

We take the F.O.C with respect to consumption to show how the marginal utility of consumption reacts to changes in government spending under different elasticities of substitution:

$$\frac{\partial \ell}{\partial C_t} = \delta^{\chi} \bar{C}_t^{-\sigma} \left(\frac{C_t}{\bar{C}_t}\right)^{-\chi} - P_t \lambda_t = 0 \tag{101}$$

Now we check the response of the marginal utility of consumption to changes in G_t:

$$\frac{\partial \lambda_t}{\partial G_t} = \chi - \sigma \bar{C}_t^{\chi-\sigma-1} \left(\frac{C_t G_t}{\bar{C}_t}\right)^{-\chi} \left(\frac{\delta^{\chi}(1-\delta)^{\chi}}{P_t}\right) \tag{102}$$

In the above equation, it is obvious that the reaction of the marginal utility of consumption for a given level of consumption will depend on $\chi - \sigma$:

a) $\chi = \sigma$: In the Cobb-Douglas case when $\chi = 1$, the above ratio will collapse to 0, regardless of the size of δ in the utility function.

b) $\chi > \sigma$: In this case, the effect of government expenditure will be positive, and as $\chi \to \infty$ the two items will be perfect complements.

c) $\chi < \sigma$, In this case, the sign of the term above will turn into negative, and any changes in government expenditure will have an adverse effect on consumption. Also, as $\chi \to 0$ the two items will be perfect substitutes.

It is easy to see from above that once we change the size of χ, the dynamics of the whole model will follow. In the separable case when $\chi = \sigma$, the whole model collapses to the basic model of Gali 2008 since the government consumes different goods than the ones consumed by the representative consumer.

When $\chi < \sigma$ an increase in government consumption will cause a drop in private consumption, and this will cause a negative output gap which will push domestic prices down. The response of monetary policy, in this case, will be a reduction of the policy rate to minimise the decline in private consumption. In the substitutability case, government expenditure will have an adverse effect on the natural rate of output, and this will cause a reduction in the natural rate of interest. Also, when $\chi < \sigma$, the slope of the IS curve will be steeper than the other two case. This translates to a higher response of consumption to changes in the real interest rate.

Appendix 4

Following Bullard and Mitra 2002, we try to explain how determinacy of monetary policy rules is derived from the below matrix:
$A = \Omega \begin{bmatrix} \sigma_\delta & \omega_\alpha(1-\beta\phi_\pi) \\ \kappa_g\sigma_\delta & \beta(\sigma_\delta + \omega_\alpha\phi_x) + \kappa_g\omega_\alpha \end{bmatrix}$ and $\Omega = \frac{1}{\sigma_\delta + \omega_\alpha\phi_x + \kappa_g\omega_\alpha\phi_\pi}$.

The characteristic polynomial of the above matrix is:

$$p(\lambda) = \lambda^2 - \lambda a_1 + a_2 \tag{103}$$

For the eigenvalues of matrix A to be inside the unit circle, the following conditions have to be met:

$$|a_2| < 1 \tag{104}$$

$$|a_1| < 1 + |a_2| \tag{105}$$

Where a_1 is the trace of matrix A and a_2 is the determinant of the same matrix:

$$a_2 = \frac{\beta\sigma_\delta}{\sigma_\delta + \omega_\alpha\phi_x + \kappa_g\omega_\alpha\phi_\pi} \tag{106}$$

The determinant of this matrix should satisfy the first condition: $\beta\sigma_\delta < \sigma_\delta + \omega_\alpha\phi_x + \kappa_g\omega_\alpha\phi_\pi$. This rule is easily satisfied since $0 < \beta < 1$ and given the fact that all of the parameters have positive values. The trace of the matrix a_1 is:

$$a_1 = \frac{-(\sigma_\delta + \beta\sigma_\delta + \beta\omega_\alpha\phi_x + \kappa_g\omega_\alpha)}{\sigma_\delta + \omega_\alpha\phi_x + \kappa_g\omega_\alpha\phi_\pi} \tag{107}$$

From the second condition, the following inequality must hold for the eigenvalues of the matrix to lie inside the unit circle: $\kappa_g \omega_\alpha (\phi_\pi - 1) + (1 - \beta)\omega_\alpha \phi_x > 0$. The inflation parameter has to be greater than 1 for this rule to hold along with the first condition.

Appendix 5

Figure 6: Response to a Domestic TFP shock (Substitutability)

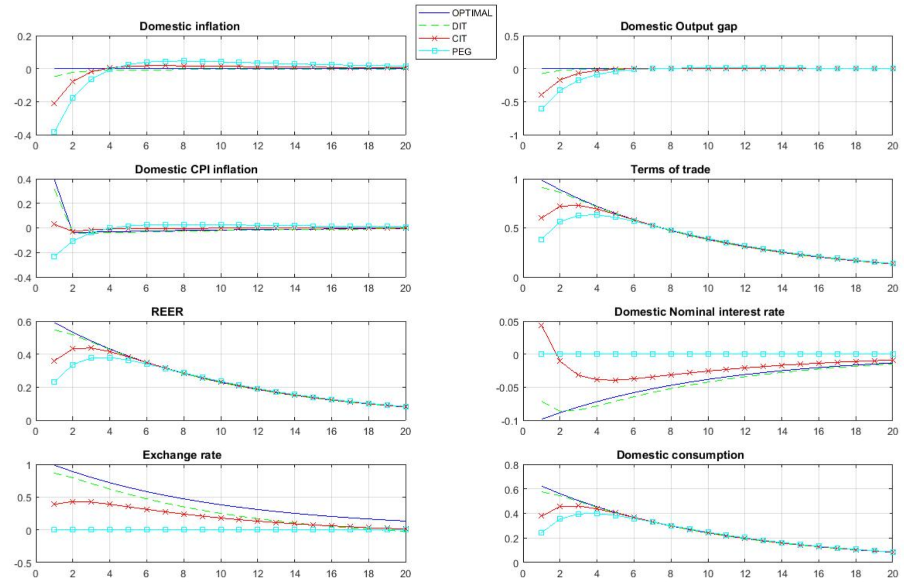

Figure 7: Response to a Domestic Government Consumption Shock (Substitutability)

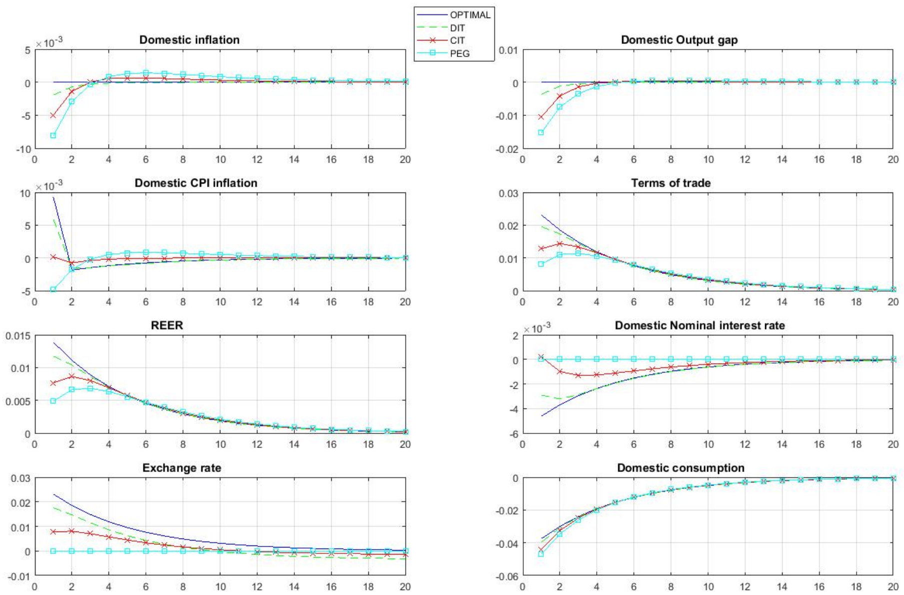

Figure 8: Response to a TFP Shock in the Foreign Economy (Substitutability)

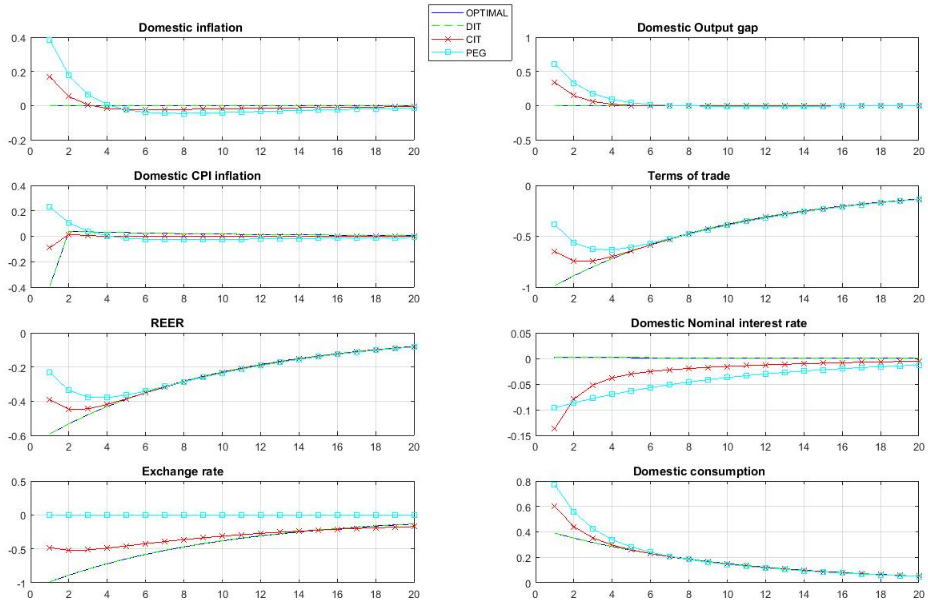

Figure 9: Response to a Government Consumption Shock in the Foreign Economy (Substitutability)