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Sectoral Composition of Output and the Wage Share: a
Two-Sector Kaleckian Model

Elton Beqiraj∗, Lucrezia Fantit, Luca Zamparelli‡§

Abstract

This paper looks at structural change as one additional source of decline in the wage share. First, we provide a decomposition of changes in aggregate wage shares into changes due to variations in output composition and in sectoral wage shares for nine OECD countries between 1977 and 2010. We show that the rise in the service sector is a relevant factor in explaining the fall of the wage share, at least for some countries. Next, we develop a two-sector Kaleckian growth model consisting of the service and manufacturing sectors. We assume that structural change is exogenous as it arises from a shift in consumers’ preferences or in the saving rate. We show that, when mark-ups are relatively higher in the service sector, a shift in the sectoral composition of demand in favor of the service sector good generates a rise in the profit share.

Keywords: structural change, functional income distribution, manufacturing, service

JEL Classification: D33, E11, O14

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1 Introduction

At the onset of modern growth theory, Kaldor (1961) suggested that long-run stability of factors income shares is one of the main ‘stylized facts’ of market economies. Yet, recent contributions (Jayadev and Rodriguez, 2013; Karabarbounis and Neiman, 2014; OECD, 2015) have shown that the labor share has declined over the past three decades in both developed and developing countries. While the possibility of short- and medium-run fluctuations in factors shares has long been acknowledged (Bentolila and Saint-Paul, 2003; Young, 2004), the prolonged decline in labor share seems to point to either a long-run negative trend in the labor share or a shift to a lower steady state wage share as more plausible descriptions of the evidence.

Several explanations for such a trend have been put forward and investigated both from theoretical and empirical standpoints. Economists working within the neoclassical framework have emphasized the importance of the shape of production function and the nature of technical change in determining factors shares trends. As is well known, a unitary elasticity of substitution ($\sigma$) between capital and labor, that is a Cobb-Douglas production function, necessarily implies constant factors shares. There are two possibilities to obtain a fall in the labor share: either capital deepening (in efficiency terms) when labor and capital are substitutes ($\sigma > 1$), or a reduction in the capital-labor ratio when the elasticity of substitution is less than one. Piketty and Zucman (2014) and Karabarbounis and Neiman (2014) support the first mechanism; Acemoglu (2003) analyzes the second possibility in the context of induced technical change, though he only applies it to deviations from the stable steady state wage share.

Other researchers (Berthold et al., 2002; Bental and Demougin, 2010; Checchi and García-Peñalosa, 2010) have investigated the relation between changes in labor market institutions and the labor share trend. Checchi and García-Peñalosa (2010), in particular, show that a reduction in unions’ bargaining power might have played a role in reducing the share of income accruing to workers.

Multiple elements of globalization have also been singled out as factors behind the falling labor share. They range from trade (Brock and Dobbelaere, 2006; Doan and Wan, 2017), to offshoring (Elsby et al., 2013), to capital account openness (Jayadev, 2007).

Finally, economists working within the Post-Keynesian tradition (Dünhaupt, 2017; Stockhammer, 2017) have looked at the increasing size of the financial sector as an additional determinant of the decline in the wage share.
Relatively little attention, on the other hand, has been paid to the possible influence of structural change on functional income distribution. De Serres et al. (2001) show that changes in the sectoral structure of the economy help explaining the decline in the aggregate wage share observed in five European countries and in the US over the 1980s and 1990s. From a theoretical point of view, a recent paper by Alvarez-Cuadrado et al. (2018) explains the decline in the labor share in a two sector neoclassical growth model, where non-homothetic preferences and sectoral differences in productivities growth and factors’ elasticities of substitution produce an endogenous rise in the service sector relative to manufacturing. Their quantitative analysis shows that within-industries income shares dynamics rather than changes in the sectoral composition of output is mostly responsible for the fall in the aggregate wage share in the US. Pothier and Puy (2014) study the relation between demand composition and inequality over the business cycle; they do not focus, however, on long run changes in output composition and on the functional income distribution.

In this paper, we investigate the relation between changes in the composition of output and the aggregate wage share. First, as an empirical motivation to our theoretical contribution, we elaborate on the methodology and the evidence provided by De Serres et al. (2001) and Alvarez-Cuadrado et al. (2018) to assess the effect of changes in the sectoral composition of output on the wage share in nine OECD countries between 1977 and 2010. We show that while changes in sectoral wage shares are the main driver of the fall in the aggregate wage share, the rise in the service sector share of total value added still provides a significant contribution to the trend, particularly in UK, Italy and Germany.

Next, we build on Dutt (1988) and Dutt (1990) to develop a two-sector Kaleckian model of growth and distribution, where the economy consists of the service and manufacturing sectors. The service good is only used for consumption while the manufacturing good is used both for consumption and the accumulation of capital stock. We assume that structural change is exogenous as it arises from shifts in consumers’ preferences and in the saving rate. We study two versions of the model, with and without profit rates equalization across sector. Under both specifications we show that, when mark-ups are relatively higher in the service sector, a shift in the sectoral composition of demand in favor of the service sector generates a rise in the steady state profit share. The unique (non-trivial) steady state equilibrium is asymptotically stable. The crucial assumption that mark-ups are relatively higher in the service sector is motivated by the empirical finding that wage shares in the manufacturing sector are consistently higher than in the service sector.

While seminal contributions by Dutt (1988, 1990); Park (1995); Dutt (1997); Lavoie and
Ramirez-Gaston (1997); Franke (2000) laid the foundations of the two-sectors Keynesian-Kaleckian model of growth and distribution, recent papers have generalized the model to investigate additional issues. Nishi (2018) analyzes the effects of introducing sectoral endogenous labor productivity growth on cyclical demand, growth and distribution. Fujita (2018) explores how changes in sectoral mark-ups affect sectoral and aggregate capacity utilization and capital accumulation. Murakami (2018) studies the effect of sectoral interactions on business cycles in a Keynesian model, without focusing on income distribution. None of these recent contributions, however, consider the role that changes in consumers’ preferences and, in turn, in demand composition may produce on income distribution.

The rest of the paper is organized as follows. Section 2 illustrates the empirical motivation of the paper; Section 3 develops the model and states the theoretical results; Section 4 offers some concluding remarks while proofs of the propositions and the full description of data can be found in Section 5.

2 Data and Empirical Motivation

Starting from the empirical evidence provided by De Serres et al. (2001) and Alvarez-Cuadrado et al. (2018), we investigate the time series related to both aggregate and sectoral wage shares and to structural change, that is the change in sectoral output over the total value added, in order to assess the relevance of the sectoral shift from manufacturing to service upon the declining pattern of the wage share.

We use the 35-sectors EU-KLEMS data (O’Mahony 2009) to build time series of aggregate and sectoral wage shares and sectoral shares of total value added, from 1977 to 2010, for nine advanced economies: US, UK, Germany, Italy, Netherlands, France, Spain, Japan and Austria.\(^1\) For each country, we compute the aggregate wage share \(\omega\) as the ratio between total industries labor compensation and value added, whereas sectoral wage shares have been computed as sectoral labor compensations over sectoral value added. If we denote sectors by \(i\), aggregate and sectoral wage shares are related as follows:

\[
\omega_t = \sum_i a_{i,t} \omega_{i,t},
\]

where \(i = m, s, r\) indicates the manufacturing, service and the other sectors included in

\(^1\)See subsection 5.1 in the Appendix for a detailed description of the data.
Table 1: Changes in wage shares and service sector weights by country (1977-2010).

<table>
<thead>
<tr>
<th>Country</th>
<th>$\Delta \omega_s$</th>
<th>$\Delta \omega_m$</th>
<th>$\Delta \omega$</th>
<th>$\Delta a_s$</th>
<th>$\omega_m - \omega_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>-0.06</td>
<td>-0.20</td>
<td>-0.09</td>
<td>0.15</td>
<td>0.10</td>
</tr>
<tr>
<td>UK</td>
<td>0.01</td>
<td>0.02</td>
<td>-0.01</td>
<td>0.24</td>
<td>0.10</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.05</td>
<td>-0.03</td>
<td>-0.08</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.14</td>
<td>0.01</td>
<td>-0.1</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-0.12</td>
<td>-0.1</td>
<td>-0.10</td>
<td>0.16</td>
<td>0.01</td>
</tr>
<tr>
<td>France</td>
<td>-0.13</td>
<td>-0.1</td>
<td>-0.14</td>
<td>0.18</td>
<td>0.08</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.09</td>
<td>0.01</td>
<td>-0.09</td>
<td>0.22</td>
<td>-0.01</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.20</td>
<td>-0.02</td>
<td>-0.13</td>
<td>0.15</td>
<td>0.01</td>
</tr>
<tr>
<td>Austria</td>
<td>-0.11</td>
<td>-0.16</td>
<td>-0.12</td>
<td>0.15</td>
<td>0.10</td>
</tr>
</tbody>
</table>

For all countries, we observe a decrease in aggregate wage share and an increase in the service sector output share of total value added, that is a sectoral shift towards services. The picture regarding changes in sectoral wage shares is more nuanced. In the service sector there is a substantial decline of the wage share in all countries save for UK, where it barely rises. The manufacturing wage share, on the other hand, does not fall in Italy, Spain, and UK, while it drops in all other countries, Austria and US in particular. Additionally, we report a positive difference between the average wage share in manufacturing and in service sector for all countries except for Spain, thus providing empirical support for our hypothesis (see subsection 3.1) about higher mark-ups in the service sector.

Our goal is to quantify how changes in the sectoral composition of the economy contribute to the decline in the aggregate wage share. To the purpose, we follow De Serres et al. (2001)
and differentiate equation (3.1) to find the following decomposition:

$$
\Delta \omega_t = \sum_i \omega_{i,t} \Delta a_{i,t} + \sum_i a_{i,t} \Delta \omega_{i,t} a_{i,t}.
$$

At any point in time, the change in the aggregate wage share equals the sum of changes due to variations in the composition of output and in sectoral wage shares. Differently from De Serres et al. (2001), we do not perform a sub-sample change analysis because, given the relatively high variation of the components reported in equation (2), results would be highly sensitive to the sub-sample starting and ending values interval. Therefore, we perform the decomposition over the whole sample. Moreover, we use the trend component of both sectoral weights and wage shares to make our conclusions unaffected by the highly cyclical behavior of the time series involved. On the other hand, our computation of the influence of structural change on wage share changes differs from Alvarez-Cuadrado et al. (2018). While they calculate it by assuming sectoral wage shares fixed at their average value over the sample, the use of equation (2) let us take changes in sectoral wage shares into account.

Figure 1: Decomposition of changes in the aggregate wage share
<table>
<thead>
<tr>
<th>Country</th>
<th>Sectoral reallocation</th>
<th>Variation of sectoral wage shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.07</td>
<td>0.93</td>
</tr>
<tr>
<td>UK</td>
<td>0.62</td>
<td>0.38</td>
</tr>
<tr>
<td>Germany</td>
<td>0.29</td>
<td>0.71</td>
</tr>
<tr>
<td>Italy</td>
<td>0.31</td>
<td>0.69</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.07</td>
<td>0.93</td>
</tr>
<tr>
<td>France</td>
<td>0.13</td>
<td>0.87</td>
</tr>
<tr>
<td>Spain</td>
<td>0.03</td>
<td>0.97</td>
</tr>
<tr>
<td>Japan</td>
<td>0.20</td>
<td>0.80</td>
</tr>
<tr>
<td>Austria</td>
<td>0.12</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Table 2: Decomposition of the effect of sectoral reallocation and within sectors wage share variation upon the pattern of aggregate wage share (1977-2010).

Figure 1 and Table 2 summarize our findings. They clearly show that the fall in the aggregate wage share is mostly due to variations in sectoral wage shares. Still, at least for UK, Italy and Germany the rise in the service sector share of value added cannot be dismissed as a relevant source of change in income distribution as it explains close to or more than 30% of the overall reduction in the wage share.

3 The Model

3.1 Production and technology

The economy consists of the service good and the manufacturing good. Output in both sectors \(X_i\) is produced through a sector-specific Leontief production function:

\[
X_i = min[u_iB_iK_i, A_iL_i], \quad i = s, m
\]  

(3)

where \(B\) and \(A\) are capital and labor productivities, \(K\) is the capital stock, \(L\) is employment, and \(u\) is the degree of capacity utilization. We assume no depreciation of capital. Profit maximization ensures:

\[
X_i = u_iB_iK_i = A_iL_i.
\]  

(4)

Since we do not consider changes in capital productivities and no particular insights can be learned from their heterogeneity, without loss of generality we assume \(B_i = B = 1\) to
keep our notation more parsimonious.

3.2 Society and preferences

There are two classes in society. Workers supply labor services inelastically and receive the wage rate $w$, uniform across sectors. They consume their whole income. Capitalists earn profits on the capital stock they own. Their propensity to save is $s > 0$. Workers and capitalists share the same preferences, which are defined over the two goods. We assume that individual utility of agent $j$ is:

$$U_j(c_s, c_m) = \min[c_s, \alpha c_m],$$

where $c_i$ is consumption of good $i$, and $\alpha > 0$. The fixed coefficient structure of preferences implies $c_s = \alpha c_m$. The same fixed proportion carries over to total demand:

$$C_s \equiv \sum_j c_s = \sum_j \alpha c_m = \alpha \sum_j c_m \equiv \alpha C_m.$$

(5)

The assumption of Leontief utility function (zero elasticity of substitution between consumption goods) may appear restrictive, but we consider it appropriate in order to portray a shift in demand composition. In any case, in section (3.8) we generalize the preferences structure to any finite value of elasticity of substitution, and show that our conclusions carry over to the general framework.

3.3 Mark-up prices

In line with the original Kaleckian literature, we assume that firms set prices by charging a constant mark-up ($z_i$) over unit labor cost. Mark-ups are sector specific and our crucial hypothesis is that they are relatively higher in the service sector. The evidence reported in Table 1 justifies the assumption, as the wage share in the manufacturing sector is on average higher than the one in the service sector for all countries but Spain. If we let $p_i$ be the price of good $i$, and we choose the service sector good as the numeraire we have

$$p_s = 1 = (1 + z_s)w/A_s$$

and

$$p_m \equiv p = (1 + z_m)w/A_m,$$

with $z_s > z_m$. Accordingly

$$w = \frac{A_s}{1 + z_s},$$

(6)

We denote by $s$ both the service sector and the saving rate. Given context no ambiguity should arise.
\[ p = \frac{1 + z_m A_s}{1 + z_s A_m} \equiv \frac{1 + z_m}{1 + z_s \gamma}. \] (7)

### 3.4 Value added distribution

In each sector, value added is distributed as wages and profits to labor and capital employed in production. If we let \( r_i \) be the interest rate in sector \( i \) we have \( p_i X_i = wL_i + r_i p_m K_i \), which, after using (4), (6), (7) and rearranging, yields

\[ r_s = \frac{z_s}{1 + z_m} \frac{1}{\gamma} u_s, \] (8)

and

\[ r_m = \frac{z_m}{1 + z_m} u_m. \] (9)

### 3.5 Output uses

The service good is only used for consumption so that \( X_s = C_s \). In what follows, it will be useful to distinguish consumption depending on its income source. We denote consumption out of wages as \( C_i^w \), and consumption out of profits as \( C_i^\pi \), so that

\[ X_s = C_s = C_s^w + C_s^\pi. \] (10)

Manufacturing output, on the contrary is used both for consumption and for investment (\( I \)) in the service and in the manufacturing sectors:

\[ X_m = C_m + I. \] (11)

### 3.6 Balanced growth under alternative closures

The discussion between Park (1995) and Dutt (1997) on the risk of over-determination in the Kaleckian two sector growth model clarified that there are two possible consistent specifications of the model. In the first one, there is no sectoral capital mobility in the short run, so that \( K_s \) and \( K_m \) are given; we can specify sectoral growth rates, and profit rates will not be equalized unless by a fluke. The second version of the model assumes that the stock of capital moves between sectors to equate sectoral profit rates in the short run; in this framework, since the sectoral capital stocks are not state variables we can only specify
the aggregate growth rate, rather than the sectoral ones.\textsuperscript{6} We analyze the two specifications of the model in turn, and we show that the qualitative results on income distribution and structural change are independent of the model closure.

3.6.1 The model without profit rates equalization

Since workers do not save, the whole wage fund is spent as consumption out of wages. Using (5) and (4) we have

\[ C_m^w + C_s^w = C_s^w / \alpha + C_s^w = w(L_s + L_m) = w \left( \frac{u_s K_s}{A_s} + \frac{u_m K_m}{A_m} \right). \]

Hence, factorizing \( C_s^w \) and substituting for the wage rate from (6) yields

\[ C_s^w = \alpha \frac{A_s}{1 + \alpha} \left( \frac{u_s K_s}{A_s} + \frac{u_m K_m}{A_m} \right) = \alpha \frac{1}{1 + \alpha} \frac{1}{1 + z_s} (u_s K_s + \gamma u_m K_m). \quad (12) \]

On the other hand, capitalists’ propensity to consume out of profits is \((1 - s)\). Accordingly

\[ C_m^\pi + C_s^\pi = C_s^\pi / \alpha + C_s^\pi = (1 - s)\left( r_m p K_m + r_s p K_s \right), \]

which, using (7),(8) and (9) implies

\[ C_s^\pi = \alpha \frac{1 - s}{1 + \alpha \frac{1}{1 + z_s}} (z_s u_s K_s + z_m \gamma u_m K_m). \]

Once we know consumption out of wages and profits in the service sector, we can use equation (10) to find

\[ X_s = \alpha \frac{1 - s}{1 + \alpha \frac{1}{1 + z_s}} (z_s u_s K_s (1 + (1 - s) z_s) + \gamma u_m K_m (1 + (1 - s) z_m)). \]

Define \( \delta \equiv K_s / K \in [0,1] \) as the share of the capital stock employed in the service sector. Dividing both sides of the previous equation by \( K \) and rearranging yields

\[ \delta u_s = (1 - \delta) u_m \gamma \frac{\alpha (1 + (1 - s) z_m)}{1 + z_s (1 + \alpha s)} \equiv (1 - \delta) u_m \gamma \Gamma(\alpha, s). \quad (13) \]

It is easy to show that \( \Gamma \) is a positive function of \( \alpha \) and negative function of \( s \), when \( z_s > z_m \).

\textsuperscript{6}A variant of this version of the model assumes that profit rates equalization is a slow process. Sectoral capital stocks are given and sectoral investment depends on the profit rates differential. We explore this variant in the stability analysis.
Let us now turn to the equilibrium in the manufacturing sector. If we let $g_i$ be the growth rate of sector $i$, under the assumption of no sectoral capital mobility, equation (11) becomes

$$X_m = C_s/\alpha + g_mK_m + g_sK_s = X_s/\alpha + g_mK_m + g_sK_s,$$

where we used (5). Using factors demands found in (4), and dividing both sides by $K$, the previous condition becomes

$$u_m(1-\delta) = u_s\delta/\alpha + r(1-\delta) + g_s\delta.$$

(14)

The Kaleckian tradition posits that investment depends on utilization of capacity as a measure of aggregate demand. In our case, the actual growth rate of capital in each sector is a function of the sector's degree of capacity utilization:

$$g_m = g_m(u_m),$$

(15)

and

$$g_s = g_s(u_s).$$

(16)

Finally, balanced growth requires that sectoral growth rates be equalized

$$g_m = g_s.$$

(17)

We have a consistent system of five equations, (13),(14),(15),(16) and 17, in the five unknowns $\delta, u_m, u_s, g_m, g_s$. Our focus is on income distribution. The profit share $\pi$ is the ratio between the value of total profits and value added. We can use (4), (7), (8), (9), (13) to calculate its equilibrium value

$$\pi^* = \frac{r_s p K_s + r_m p K_m}{X_s + p X_m} = \frac{r_s \delta + r_m(1-\delta)}{\delta u_s + (1-\delta)pu_m} = \frac{p u_s \delta/\gamma + z_m u_m(1-\delta)}{1 + z_m \delta u_s + (1-\delta)pu_m} = \frac{z_s \Gamma(\alpha, s) + z_m}{(1 + z_s) \Gamma(\alpha, s) + (1 + z_m)}.$$

(18)

Inspection of (18) shows that $\pi^*$ is economically meaningful being bounded between zero and one. It is a function of the sectoral mark-ups, consumers' preferences between the two consumption goods, and the saving rate.

We are now able to state:

**Proposition 1.** *an increase in consumption demand of the service good relative to the*
manufacturing good (a rise in $\alpha$) raises the equilibrium profit share.

Proof. See the Appendix.

**Proposition 2.** a decrease in the saving rate raises the equilibrium profit share.

Proof. See the Appendix.

In both Proposition 1 and 2 the rise in the profit share follows the increase in $\Gamma$ due to shocks to $\alpha$ and $s$. In order to understand the economic meaning of an increase in $\Gamma$ we can re-write equation (13) as $X_s/(X_m\gamma) = L_s/L_m = \Gamma(\alpha, s)$. When $\Gamma$ rises, employment in the service sector rises relative to the manufacturing one. Given labor productivities and sectoral mark-ups, the change in relative employment carries over to relative sectoral value added. Therefore, the increase in the profit share depends on the change in the composition of production in favor of the sector with higher mark-up, which can be caused either by a change in consumers’ preferences or by a reduction in the saving rate. Contrary to the standard one sector Kaleckian growth model, the profit share depends on savings. The intuition for the influence of the saving rate on the composition of output is the following. The reduction in the saving rate increases capitalists’ consumption, raising both service and manufacturing consumption according to the constant proportion $\alpha$. However, since manufacturing is used both as consumption and as investment while service output is wholly consumed, the increase in consumption of both sectors weighs relatively less in manufacturing and the composition of output changes in favor of the service sector.

### 3.6.2 The model with profit rates equalization

In the second version of the model, sectoral capital stocks are not state variables since capital adjusts in the short run to ensure profit rates equalization. Accordingly, there are no sectoral growth rates and we need to replace equation (14) with

$$u_m(1 - \delta) = u_s\delta/\alpha + g.$$  \hspace{1cm} (19)

Equation (13) is not affected by the new closure, whereas we need to drop (15) and (16) and replace them with a single equation for the growth rate of capital. We assume it depends on the degree of capacity utilization in both sectors:

$$g = g(u_s, u_m).$$  \hspace{1cm} (20)
Next, we impose the equalization of profit rates across sectors, so that \( r_s = r_m \). Using (8) and (9), the equalization yields:

\[ u_s = \gamma \frac{z_m}{z_s} u_m. \tag{21} \]

We now have a consistent system of four equations, (13),(19),(20) and (21), in the four unknowns \( \delta, u_m, u_s, g \). In particular, use (21) into (13) to find

\[ \delta^* = \frac{\Gamma(\alpha, s)}{\Gamma(\alpha, s) + z_m/z_s} \in (0, 1). \tag{22} \]

Let us now turn to the profit share:

\[ \pi^* = \frac{r_s p K_s + r_m p K_m}{X_s + pX_m} = \frac{r p}{\delta u_s + (1 - \delta) p u_m} = \]

\[ = \frac{z_m}{(1 - \delta) ((1 + z_s) \Gamma(\alpha, s) + 1 + z_m)} = \frac{z_s \Gamma(\alpha, s) + z_m}{(1 + z_s) \Gamma(\alpha, s) + (1 + z_m)}, \tag{23} \]

where we used the equalization of profit rates, (4), (7), (8), (9), (13), and (22). Equations (15) and (23) show that the final expression for the profit share is the same irrespective of the model closure; therefore, a shift of consumers’ preferences in favor of the service sector and a decrease in the saving rate bring about an increase in the profit share, whether we assume profit rates equalization or not.

### 3.7 Stability

We now turn to the stability analysis of the balanced growth path. In the model with profit rate equalization, however, the adjustment to the balanced growth equilibrium is instantaneous and there is no transitional dynamics. In order to introduce a dynamic adjustment in this version of the model, we assume that sectoral profit rates are different in the short run, but changes in sectoral investment bring about profit rates equalization in the long run. This is the process known as ‘classical competition’. After this modification, the dynamics of the economy in both models is described by the slow adjustment in the allocation of capital between sectors. To the purpose, we derive a differential equation for \( \delta \), the share of capital employed in the service sector. Given the definition of \( \delta \), taking time derivative and rearranging yields
\[
\dot{\delta} = \delta(1 - \delta)(g_s - g_m). \tag{24}
\]

### 3.7.1 The model without profit rate equalization

In order to study the dynamic behavior of \(\delta\), we start by assuming explicit functional forms for sectoral growth rates. Equations (15) and (16) become

\[
g_s = \vartheta_0 + \vartheta_1 u_s \tag{25}
\]

and

\[
g_m = \beta_0 + \beta_1 u_m. \tag{26}
\]

We can use the two previous equations together with (13) and (14) to solve for utilization rates as functions of \(\delta\):

\[
u_s(\delta) = \frac{\gamma \Gamma(\alpha, s)}{\Theta} \left( \beta_0 \frac{1 - \delta}{\delta} + \vartheta_0 \right) \tag{27}
\]

and

\[
u_m(\delta) = \left( \beta_0 + \vartheta_0 \frac{\delta}{1 - \delta} \right) / \Theta, \tag{28}
\]

where \(\Theta = [1 - (\beta_1 + \gamma \Gamma(\alpha, s)(\vartheta_1 + 1/\alpha))]\). Notice that economically meaningful (positive) solutions for \(u_m\) and \(u_s\) require \(\Theta > 0\), that is \((\beta_1 + \gamma \Gamma(\alpha, s)(\vartheta_1 + 1/\alpha)) < 1\). This condition is the equivalent of the standard Keynesian 'stability' condition in one-sector Kaleckian growth models, which states that investment need be less responsive than saving to economic activity. \(\beta_1\) and \(\vartheta_1\) represent how sectoral investment reacts to capacity utilization; the role of saving is captured by \(\Gamma(\alpha, s)\), which is a negative function of the saving rate.

We can rewrite (24) as

\[
\dot{\delta} = \delta(1 - \delta)[g_s(\delta) - g_m(\delta)],
\]

and state

**Proposition 3.** The system has two locally unstable trivial steady states at \(\delta = 0\) and \(\delta = 1\). The system has one non-trivial steady state \(\delta^*\), which is asymptotically stable for \(\delta \in (0, 1)\).
Proof. see the Appendix.

Proposition 3 shows that if the initial condition of the system is such that both sector exist, the economy will converge towards the non-trivial steady state. If, on the other hand, the economy consists of only one sector at the beginning of time, the two-sector structure will never appear. Notice, however, that \( \delta = 1 \) does not have economic meaning because there cannot be accumulation of capital without production of the manufacturing good. When \( \delta = 0 \), on the other hand, we are back to the standard one-sector model, where the only output is used for both consumption and investment.

### 3.7.2 The model with profit rates equalization

In order to introduce a dynamic adjustment in this version of the model, we assume that profit rates equalization is not instantaneous. Sectoral profit rates are different in the short run, but changes in sectoral investment bring about profit rates equalization in the long run. We follow Dutt (1997) in assuming that the difference in sectoral growth rates depends on the profit rates differential

\[
g_s - g_m = \lambda(r_s - r_m), \lambda > 0.
\]

On the other hand, firms choose the total rate of investment based on the average degree of capacity utilization in the economy \( \bar{u} \). Assuming a linear form for the investment function we have:

\[
g = g(\bar{u}) = \mu_0 + \mu_1 \bar{u}, \tag{30}
\]

where

\[
\bar{u} = \frac{X_s + pX_m}{pK} = \frac{1 + z_m u_s \delta}{1 + z_s \gamma} + u_m(1 - \delta)B_m.
\]

We can now use (13), (19), (8), (9) and (30) to solve for sectoral profit rates as functions of \( \delta \):

\[
r_s(\delta) = \frac{z_s}{1 + z_m \gamma} u_s(\delta) = \frac{z_s}{1 + z_m \Psi} \frac{1}{\delta}, \tag{31}
\]

and

\[
r_m(\delta) = \frac{z_m}{1 + z_m} u_m(\delta) = \frac{z_m}{1 + z_m \Psi} \frac{1}{1 - \delta}, \tag{32}
\]

where \( \Psi = \left[1 - \gamma \Gamma(\alpha, s)/\alpha - \mu_1 \left(1 + \frac{1 + z_m}{1 + z_m} \Gamma(\alpha, s)\right)\right] \). Economically meaningful (pos-
tive) solutions for \( r_m \) and \( r_s \) require \( \Psi > 0 \). Similarly to the previous case, we can interpret it as the equivalent of the standard Keynesian 'stability' condition in one-sector Kaleckian growth models. Using the latest results and (29) in (24) we find

\[
\dot{\delta} = \lambda \delta (1 - \delta) [r_s(\delta) - r_m(\delta)] . \tag{33}
\]

We can state

**Proposition 4.** The system has two locally unstable trivial steady states at \( \delta = 0 \) and \( \delta = 1 \). The non-trivial steady state \( \delta^* = \frac{\Gamma(\alpha,s)}{\Gamma(\alpha,s) + z_m/z_s} \) is asymptotically stable over \( \delta \in (0,1) \).

**Proof.** See the Appendix.

Similarly to the comparative dynamics results found in Section 3.6, the comparison between Proposition 3 and 4 show that the stability properties of the model are independent of the model closure.

### 3.8 A generalization: the model with CES utility function

In this section we release the assumption of zero elasticity of substitution between consumption goods. To the purpose, assume that the utility function of agent \( j \) is of the constant elasticity of substitution type:

\[
U_j(c_s, c_m) = [(1 - \nu) c_s^{\rho} + \nu c_m^{\rho}]^{\frac{1}{\rho}},
\]

where \( \nu \in (0,1) \) is a parameter that measures the relative utility weight of the two goods and \( \rho \in (-\infty, 1) \) regulates their elasticity of substitution, with \( \sigma = 1/(1 - \rho) \).

From the first order conditions of the consumer utility maximization problem we have

\[
\frac{\partial U}{\partial c_m} / \frac{\partial U}{\partial c_s} = \frac{\nu}{1 - \nu} \left( \frac{c_s}{c_m} \right)^{1-\rho} = \frac{p_m}{p_s} = \gamma \frac{1 + z_m}{1 + z_s} .
\]

Hence,

\[
c_s = \left( \frac{1 - \nu}{\nu} \gamma \frac{1 + z_m}{1 + z_s} \right)^{\frac{1}{1-\rho}} c_m \equiv \left( \tilde{\alpha} \gamma \frac{1 + z_m}{1 + z_s} \right)^{\sigma} c_m
\]

By summing individual demands over all consumers, we obtain the proportion between total
the saving rate if

\[ C_s = \sum_j c_s = \sum_j \left( \frac{\tilde{\alpha} \gamma}{1 + z_s} \right)^\sigma c_m = \left( \frac{\tilde{\alpha} \gamma}{1 + z_s} \right)^\sigma \sum_j c_m = \left( \frac{\alpha \gamma}{1 + z_s} \right)^\sigma C_m. \tag{34} \]

Notice that \( \tilde{\alpha} \) measures consumers’ relative preference for the service good. Not surprisingly, an increase in \( \tilde{\alpha} \) raises the economy’s service to manufacturing goods ratio.

Using (34) and (4) we have

\[ C_m' + C_s' = C_s' / \left( \frac{\tilde{\alpha} \gamma}{1 + z_s} \right)^\sigma + C_s' = w(L_s + L_m) = w \left( \frac{u_s K_s}{A_s} + \frac{u_m K_m}{A_m} \right). \]

Hence, factorizing \( C_s' \) and substituting for the wage rate from (6) yields

\[ C_s' = \frac{(\tilde{\alpha} \gamma)^\sigma (1 + z_m)^\sigma}{(1 + z_s)^\sigma + (\tilde{\alpha} \gamma)^\sigma (1 + z_m)^\sigma} \frac{1}{1 + z_s} (u_s K_s + \gamma u_m K_m) \equiv \frac{\Lambda}{1 + z_s} (u_s K_s + \gamma u_m K_m). \tag{35} \]

Similarly, using (34) in \( C_s' + C_m' = (1 - s) \Pi \) and proceeding as in subsection 3.6.1 we can find

\[ C_s' = \frac{(1 - s) \Lambda}{1 + z_s} (z_s u_s K_s + \gamma z_m u_m K_m). \]

Plugging the new solutions for \( C_s' \) and \( C_s' \) in (10) and rearranging we obtain

\[ \delta u_s = (1 - \delta) u_m \gamma \frac{\Lambda(1 + (1 - s) z_m)}{1 + z_s - \Lambda(1 + (1 - s) z_s)} \equiv (1 - \delta) u_m \gamma \tilde{\Gamma}(\tilde{\alpha}, s). \tag{36} \]

We can now use (4), (7), (8), (9) and (36) to find the equilibrium profit share as

\[ \pi^* = \frac{z_s \tilde{\Gamma}(\tilde{\alpha}, s) + z_m}{(1 + z_s) \tilde{\Gamma}(\tilde{\alpha}, s) + (1 + z_m)}. \]

Since \( d\pi^*/d\tilde{\alpha} = \frac{(z_s - z_m) \tilde{\Gamma}'(\tilde{\alpha}, s)}{(1 + z_s) \tilde{\Gamma}(\tilde{\alpha}, s) + (1 + z_m) \gamma} \) and \( d\pi^*/d\gamma = \frac{(z_s - z_m) \tilde{\Gamma}'(\tilde{\alpha}, s)}{(1 + z_s) \tilde{\Gamma}(\tilde{\alpha}, s) + (1 + z_m) \gamma} \), the profit share rises: after a preference shift in favor of the service good if \( \tilde{\Gamma}'(\tilde{\alpha}, s) > 0 \), after a decrease in the saving rate if \( \tilde{\Gamma}'(\tilde{\alpha}, s) < 0 \). We can use the definition of \( \Lambda \) to re-write \( \tilde{\Gamma} \) as

\[ \tilde{\Gamma} = \frac{(\tilde{\alpha} \gamma)^\sigma (1 + z_m)^\sigma}{(1 + z_s)^\sigma + (\tilde{\alpha} \gamma)^\sigma (1 + z_m)^\sigma} \frac{1 + (1 - s) z_m}{1 + z_s - (1 - s) z_s} \frac{(\tilde{\alpha} \gamma)^\sigma (1 + z_m)^\sigma}{(1 + z_s)^\sigma + (\tilde{\alpha} \gamma)^\sigma (1 + z_m)^\sigma} = \frac{(\tilde{\alpha} \gamma)^\sigma (1 + z_m)^\sigma (1 + (1 - s) z_m)}{(1 + z_s)^\sigma z_s + z_m (\tilde{\alpha} \gamma)^\sigma (1 + z_m)^\sigma}. \]
Whence, $\tilde{\Gamma}'_\alpha = \frac{\gamma^\sigma (1+z_m)^\sigma (1+(1-s)z_m)\sigma \alpha^{\sigma-1}(1+z_m)^{\sigma+1}}{((1+z_m)^{\sigma+1}+sz_s(\tilde{\alpha}\gamma)^{\sigma}(1+z_m)^{\sigma})^2} > 0,$

and $\tilde{\Gamma}'_s = -\frac{(\tilde{\alpha}\gamma)^{\sigma}(1+z_m)^{\sigma}\{z_s(z_s+sz_s(\tilde{\alpha}\gamma)^{\sigma}(1+z_m)^{\sigma})\}^{\sigma+1}+sz_s(\tilde{\alpha}\gamma)^{\sigma}(1+z_m)^{\sigma}}{((1+z_m)^{\sigma+1}+sz_s(\tilde{\alpha}\gamma)^{\sigma}(1+z_m)^{\sigma})^2} < 0$. Therefore, results obtained in Proposition 1 and 2 are robust to the preferences generalization.

We have carried out the generalization within the framework of the model without profit rates equalization. If we use (36) instead of (13) to solve for the steady state income distribution in the model with profit rates equalization, it is easy to show that the equilibrium profit share is identical under both closures of the model. Therefore the results obtained in this subsection hold also for the model with profit rates equalization.

4 Conclusions

Evidence on the process of structural change shows that the service sector share in the economy tends to rise as countries become richer (Herrendorf et al., 2014). If the wage share in the service sector is relatively low, growth and structural transformation in mature economies necessarily bring about a reduction in the aggregate wage share, absent mitigating factors. As a consequence, changes in the composition of output and employment across sectors should be taken into account when investigating the ongoing negative trend in the wage share in most industrialized countries.

This paper has analyzed this mechanism from both an empirical and a theoretical standpoint. First, we have provided a quantitative assessment of how empirically relevant the rise in the service sector is in explaining the decline in the wage share in nine OECD countries. While variations in sectoral wage shares are clearly the main driver of the fall in aggregate wage shares, we have shown that the sectoral reallocation channel still matters to explain the wage share dynamics, particularly in UK, Italy and Germany. Second, we have developed a two-sector Kaleckian model of growth and distribution, where the economy consists of the service and manufacturing sectors. Shocks to consumers’ preferences or to the saving rate that change the sectoral composition of demand in favor of the service sector generate a rise in the steady state profit share. The unique (non-trivial) balanced growth path is asymptotically stable.
5 Appendix

5.1 Data

We use the EU-KLEMS 2012 release in ISIC Rev.4 (NACE 2) available at https://www.euklems.net/. Following the EU-KLEMS sectoral classification, we construct manufacturing sector (C) so to include: food products, beverages and tobacco (10-12), textiles, wearing apparel, leather and related products (13-15), wood and paper products; printing and reproduction of recorded media (16-18), coke and refined petroleum products (19), chemicals and chemical products (20-21), rubber and plastics products, and other non-metallic mineral products (22-23), basic metals and fabricated metal products, except machinery and equipment (24-25), electrical and optical equipment (26-27), machinery and equipment (28), transport equipment (29-30) and other manufacturing, repair and installation of machinery and equipment (31-33).

We include in service sectors the following: wholesale and retail trade (G), transportation and storage (H), accommodation and food service activities (I), information and communication (J), financial and insurance activities (K), real estate activities (L), professional, scientific, technical, administrative and support service activities (M-N) and arts, entertainment, recreation and other service activities (R-S).

We compute total industries value added and labor compensation by taking into account, together with manufacturing and service, also agriculture, forestry and fishing (A), mining and quarrying (B), electricity, gas and water supply (D-E) and construction (F). These sectors are also considered within the ‘residual’ component of the decomposition analysis presented in section 2. Community social and personal service (O-Q) and activities of households as employers (T) are excluded from our analysis.

5.2 Proof of Proposition 1

\[
\frac{d\pi^*}{d\alpha} = \frac{(z_s - z_m)\Gamma'(\alpha, s)}{((1 + z_s)\Gamma(\alpha, s) + (1 + z_m)\gamma)^2} > 0,
\]

since \(z_s > z\) and \(\Gamma'(\alpha, s) = \frac{\alpha(1+(1-s)z_m)/(1+z_s\gamma)}{(1+z_s\gamma)^2} > 0\).

5.3 Proof of Proposition 2

\[
\frac{d\pi^*}{ds} = \frac{(z_s - z_m)\Gamma'(\alpha, s)}{((1 + z_s)\Gamma(\alpha, s) + (1 + z_m)\gamma)^2} < 0,
\]
since \( z_s > z \) and \( \Gamma'_s(\alpha, s) = \frac{-\alpha(z_m + \alpha z_s + z_m z_s(1 + \alpha))}{(1 + z_s(1 + \alpha))} < 0 \).

### 5.4 Proof of Proposition 3

Let us start with the two trivial steady states. Inspection of (24) shows that \( \dot{\delta} = 0 \) at \( \delta = 0 \), and \( \delta = 1 \). Turning to stability, we have \( \frac{d\delta}{d\delta} = (1 - \delta) [g_s(\delta) - g_m(\delta)] - \delta [g_s(\delta) - g_m(\delta)] + \delta (1 - \delta) [g'_s(\delta) - g'_m(\delta)] \). At \( \delta = 0 \), \( g_s(0) \) is not defined but \( \lim_{\delta \to 0} \frac{d\delta}{d\delta} = \lim_{\delta \to 0} [g_s(\delta) - g_m(\delta)] \to \infty > 0 \), so that the first trivial steady state is locally unstable. At \( \delta = 1 \), \( g_m(1) \) is not defined but \( \lim_{\delta \to 1} \frac{d\delta}{d\delta} = \lim_{\delta \to 1} [g_s(\delta) - g_m(\delta)] \to \infty > 0 \), so that the second trivial steady state is locally unstable. Let us now turn to prove the existence and stability of the non-trivial steady state. Plug 27 and (28) into (24) to find

\[
\dot{\delta} = \delta (1 - \delta) \left[ \vartheta_0 + \vartheta_1 \frac{\Gamma(\alpha, s)}{\beta_0} \left( \beta_0 \frac{1 - \delta}{\delta} + \vartheta_0 \right) - \frac{\beta_0 - \beta_1}{\beta_0} \right] / \vartheta
\]

\[
= \delta (1 - \delta) \left[ \vartheta_0 - \beta_0 + \vartheta_1 \frac{\Gamma(\alpha, s)}{\beta_0} \vartheta_0 - \beta_1 \vartheta_0 \right] + (1 - \delta) \vartheta_1 \frac{\Gamma(\alpha, s)}{\beta_0} \beta_0 - \beta_1 \vartheta_0 / \vartheta
\]

\[
= \delta^2 (K_2 - K_1 - K_3) + (K_1 - 2K_2) + K_2, \text{ where } K_1 = \vartheta_0 - \beta_0 + \vartheta_1 \frac{\Gamma(\alpha, s)}{\beta_0} \vartheta_0 - \beta_1 \vartheta_0 / \vartheta,
\]

\[
K_2 = \vartheta_1 \frac{\Gamma(\alpha, s)}{\beta_0} \vartheta_0, \text{ and } K_3 = \beta_1 \vartheta_0 / \vartheta. \text{ Therefore } \dot{\delta} \text{ is a quadratic function. As a first step, notice that } \dot{\delta}(0) = K_2 > 0, \text{ and } \dot{\delta}(1) = -K_3 < 0. \text{ Since } \dot{\delta} \text{ is continuous, over the domain } \delta \in [0, 1], \text{ it must cross the horizontal axis from above at least once in order to move from positive to negative values, according to Bolzano’s theorem for continuous functions defined over a compact set. In principle, there could be a second root since the function is quadratic, but that cannot be the case or there would need to be a third real root for the function to approach a negative value as } \delta \to 1. \text{ Therefore, for } \delta \in (0, 1) \text{ there can only be one steady state } \delta^*. \text{ It is asymptotically stable as } \delta < 0 \text{ for } \delta > \delta^* \text{ and } \dot{\delta} > 0 \text{ for } \delta < \delta^*.

### 5.5 Proof of Proposition 4

The analysis of the two trivial steady states is analogous to the proof of proposition 3. Inspection of (33) shows that \( \dot{\delta} = 0 \) at \( \delta = 0 \) and \( \delta = 1 \). Turning to stability, we have \( \frac{d\delta}{d\delta} = \lambda (1 - \delta) [r_s(\delta) - r_m(\delta)] - \lambda \delta [r_s(\delta) - r_m(\delta)] + \lambda \delta (1 - \delta) [r'_s(\delta) - r'_m(\delta)] \). At \( \delta = 0 \), \( r_s(0) \) is not defined but \( \lim_{\delta \to 0} \frac{d\delta}{d\delta} = \lim_{\delta \to 0} [r_s(\delta) - r_m(\delta)] \to \infty > 0 \), so that the first trivial steady state is locally unstable. At \( \delta = 1 \), \( r_m(1) \) is not defined but \( \lim_{\delta \to 1} \frac{d\delta}{d\delta} = \lim_{\delta \to 1} [r_s(\delta) - r_m(\delta)] \to \infty > 0 \), so that the second trivial steady state is locally unstable.

Let us now turn to prove stability of the non-trivial steady state. \( \forall \delta \in (0, 1) \) we have \( d\delta / d\delta = -\frac{\lambda}{1 + z_m^2} \vartheta (1 - \delta) \left( z_s \frac{\Gamma(\alpha, s)}{\alpha} \frac{1}{\delta} - + z \frac{1}{(1 - \delta)^2} \right) < 0 \). Hence \( \delta^* = \frac{\Gamma(\alpha, s)}{\Gamma(\alpha, s) + z_m/z_s} \) is asymptotically stable over \( \delta \in (0, 1) \).
References


22


