Bank Capital Regulation and Endogenous Shadow Banking Crises

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Abstract

We study the macroeconomic effects of bank capital requirements in an economy with two banking sectors. Banks are connected through a wholesale funding market. Anticipated banking crises occur endogenously in the form of self-fulfilling wholesale funding rollover crises. Retail bank capital requirements can reduce the frequency and severity of banking crises. Tightening retail bank capital requirements increases the size and leverage of the shadow banking sector through a novel channel that works through the anticipation of banking crises. A policy which corrects this spillover is more than twice as effective in reducing the frequency and severity of banking crises.

Keywords: Bank capital regulation, shadow banking, anticipated bank runs.

JEL Classification: E440; G240; G280.

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1 Introduction

The optimal design of bank capital regulation has always been the subject of an extensive debate among policymakers and researchers.[1] However, regulators now face several new challenges that came only into the spotlight after the financial crisis of 2008: For example, bank capital requirements should be designed in a way to ensure that systemic banking crises can be avoided and fears about financial crises mitigated.[2] Moreover, the emergence of financial intermediaries outside the traditional retail banking sector - the so-called shadow banks - means that regulating the traditional banks can have unintended consequences like regulatory arbitrage.[3] This latter point is a special concern, since financial instability during the financial crisis of 2008 originated to a large extent in the shadow banking sector, e.g. in the form of shadow bank runs.[4] There are, however, several unresolved research questions: How costly are shadow banking crises, as well as fears about such crises? Through which channels do they affect the real economy? How can bank capital regulation contribute to mitigating or even eliminating such crises? Do financial crises lead to new costs of bank capital regulation?

In this paper, we study the macroeconomic effects of imposing capital requirements on retail banks or shadow banks in an economy in which shadow banking crises arise occasionally and endogenously. They take the form of rollover crises on the wholesale funding market, on which retail banks lend to shadow banks. We present several novel findings: First, shadow bank runs are costly in welfare terms: Eliminating shadow bank runs increases welfare in consumption equivalent terms by about 1.7 percent. Most of this welfare loss arises, because fears about future banking crises substantially reduce the leverage capacity of banks and hence financial intermediation. Second, retail bank capital requirements are an effective policy to reduce the frequency of shadow

[1] See, e.g. Admati and Hellwig (2014) and Gorton and Winton (2017) for an overview.

[2] See e.g. Angeloni and Faia (2013), Gertler et al. (2016), Begenau (2016), Begenau and Landvoigt (2018).

[3] See e.g. Plantin (2015), Ordoñez (2018).

[4] See e.g. Gorton and Metrick (2012) or Covitz et al. (2013). For a recent narrative of the financial crisis supporting this view, see Gertler and Gilchrist (2018).

bank runs by allowing retail banks to more easily absorb the liquidated assets of shadow banks in a run. Third, retail bank capital requirements create a novel spillover effect that increases the size and leverage of the shadow banking sector as retail bank capital requirements tighten. This spillover works through shadow banks having a higher leverage capacity as the fears about future banking crises are reduced. Fourth, because of this additional spillover effect, bank run risk strengthens the motive to jointly regulate retail and shadow banks.

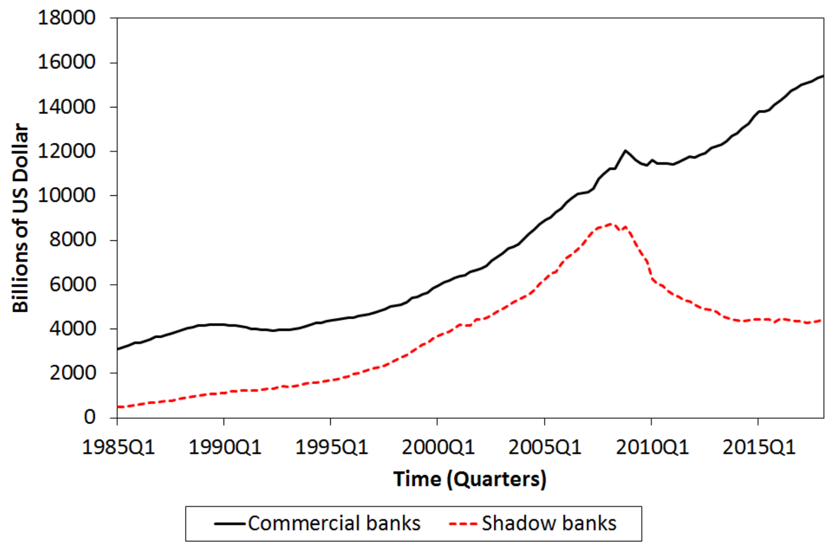

Figure 1: Total financial assets of commercial banks and shadow banks in the United States. The data are from the flow of funds and constructed as in Adrian and Shin (2011). Commercial banks are Total financial assets of U.S.-chartered depository institutions, including IBFs (FL764090005). Shadow banks are Total financial assets of funding corporations (FL504090005), finance companies (FL614090005) and issuers of asset-backed securities (FL674090005).

We define shadow banks as financial institutions that first, borrow from other financial institutions on the wholesale funding market, second, are highly leveraged, and third, are more efficient than retail banks in lending to the real economy.[5] Examples of shadow banks by our definition include finance companies, funding corporations, and issuers of asset-backed securities. Taking

[5]The last characteristic can be thought of as arising due to either benefits of specialization or due to fewer regulatory restrictions for shadow banks.

shadow banks into consideration is crucial for three reasons: First, as we can see in Figure 1, the shadow banking sector has grown tremendously over the last decades into an essential part of the modern financial system in the United States.[6] Second, it was the collapse of the shadow banking sector that led to the financial turmoil which eventually turned into a global financial crisis. Third, there are spillover effects from regulating retail banks on the shadow banking sector.

Specifically, we consider an economy in the spirit of Gertler, Kiyotaki, and Prestipino (2016) (henceforth GKP) populated with households, retail banks, shadow banks, and firms. Households, retail banks and shadow banks make retail loans to firms. Households can make deposits at banks. Banks can borrow and lend on a wholesale funding market. Banks differ in their retail lending efficiency, and payout policy. Retail and shadow banks face an endogenous leverage constraint due to a moral hazard problem. In addition, banks face a capital requirement, which is set by the regulator. An important part of the shadow bank business model is the securitization of assets.[7] To capture the effects of securitization in a simple and stylized way, we assume that retail loans financed with wholesale loans as well as wholesale loans themselves are subject to the moral hazard problem to a lesser degree than deposit financed retail loans. In equilibrium, households make deposits to retail banks, which in turn lend on a wholesale funding market to shadow banks.

Depending on the state of the economy, which includes the net worth of retail and shadow banks, a second shadow bank run equilibrium with low asset prices and insolvent shadow banks may exist. This shadow bank run equilibrium resembles a rollover crisis in Cole and Kehoe (2000) or a bank run in Gertler and Kiyotaki (2015). The intuition is that a fall in the endogenous price of assets can reduce the value of the banks' assets below the value of their liabilities, making them insolvent. This in turn reduces the demand for assets, leading to a low asset price. Importantly, bank runs are anticipated, and fears

[6] According to the Global Shadow Banking Monitoring Report 2016 by the Financial Stability Board, in 2015, shadow banking accounts for 13% of the total financial system, and the shadow banking to GDP ratio is around 70%.

[7] See e.g. Adrian and Ashcraft (2012b), Acharya et al. (2013).

about bank runs affect the economy also outside a bank run by pushing up the spread between the return on wholesale loans and the return on deposits, reducing the leverage capacity of shadow banks, and hence investment and output. As in the models of Cole and Kehoe (2000) and Gertler and Kiyotaki (2015), the state space of the economy can be divided into three zones: a safe zone, where only the no-run equilibrium exists, a crisis zone with both equilibria and a zone where only the run equilibrium exists. In the crisis zone, which equilibrium actually occurs will be determined by a sunspot shock.

In comparative statics exercises, we show that our model captures the following trade-off for retail bank capital requirements: On the positive side, a higher capital requirement reduces the frequency and severity of banking crises. Under a higher retail bank capital requirement, retail banks can use the capital buffers they build during expansions to absorb liquidated assets of shadow banks during the fire sales that occur after a run on the shadow banking sector. This leads to an investment boom, which increases the liquidation price of assets, which in turn reduces the liquidation loss of the retail banks in a run. As the likelihood of a shadow bank run is positively related to this liquidation loss, the probability of bank runs is reduced.

On the negative side, higher retail bank capital requirements lead to less financial intermediation, an increase in the leverage of the shadow banking sector and a reallocation of assets from the retail to the shadow banking sector: Intuitively, a higher retail bank capital requirement pushes up the financing cost for retail banks, which is further passed through wholesale funding to the shadow banks. This results in less financial intermediation and a higher required return on capital, a lower aggregate capital stock, and eventually a lower output of the economy. A reallocation of assets occurs whenever wholesale loans have a lower weight in the capital requirement of the retail banks than retail loans.[8] Consequently, the relative share of financial intermediation conducted in the shadow banking sector will increase as the capital require-

[8] This is the empirically relevant case, since wholesale lending receives a lower risk weight in the risk-weighted capital ratio of banks. An alternative interpretation would be that retail banks can hide a fraction of their investments into shadow banks from the regulator and the public.

ment on retail banks increases, which in turn makes bank runs on the shadow banking sector more costly.

As a first numerical exercise, we compute the welfare cost of bank runs. We do this by eliminating the sunspot shock, such that the economy always ends up in the no-run equilibrium, even when it is in the crisis zone. The welfare cost of bank runs is large: households are willing to pay 1.7 percent in permanent consumption equivalent units to eliminate bank runs. To understand better why bank runs are costly, we decompose the total cost of runs in the cost of bank run fears and the cost of realized bank runs. We do this by comparing an economy, in which bank runs are anticipated, but never realize, to the baseline economy with anticipated and realized bank runs. We find that most of the welfare cost of bank runs stems from bank run fears: Even without realized bank runs, the welfare gain from eliminating bank run fears is 1.4 percent in consumption equivalent terms or about 80 percent of the total welfare gain.

The welfare cost of bank run fears stems from two sources: First, bank run fears reduce the leverage capacity of shadow banks by reducing the value of operating a shadow bank relative to the value of diverting assets. Moreover, a higher fear of bank runs increases the cost of wholesale financing. Together, these two effects reduce financial intermediation, hence the capital stock and therefore output and consumption.

We second investigate the effects of retail and shadow bank capital requirements on financial stability and welfare. We investigate the effect of increasing retail bank capital requirements from 10 to 12.5 percent. If this capital requirement is lifted during a bank run, it reduces the frequency of bank runs from 3 runs per 100 years to 2.8 runs per 100 years. The effectiveness of higher retail bank capital requirements is so limited, since they lead to a spillover on shadow banks: The leverage of shadow banks increases by 10.4 percent. This spillover effect comes from two sources: First, retail banks shift from lending on the retail lending market to the wholesale lending market, since the amount of equity required per unit of lending on the retail market is higher than that required on the wholesale market. Second, a lower risk of bank runs due to tighter retail bank regulation increases the leverage capacity of shadow banks.

This latter channel is unique to the model with anticipated banking crises. We decompose the spillover effect into these two channels by comparing the effect of imposing retail bank capital requirements in the model with and without bank runs. We find that more than 60 percent of the spillover effect on shadow banks is due to the latter, novel bank run effect.

Finally, we consider the quantitative importance of this spillover effect by comparing the effects of a policy that only increases retail bank capital requirements to another policy that imposes the same retail bank capital requirement, but additionally corrects for the spillover by imposing a capital requirement on shadow banks such that shadow banks have the same leverage as in the model without regulation. We find that the costs of the spillover effect are quantitatively important: The policy that corrects for the spillover is more than twice as effective at reducing the frequency and severity of bank runs than the policy which does not do so.

Related Literature This paper is closely related to three strands of literature. The first literature incorporates shadow banks into macroeconomic models. We build directly on the work of GKP, who develop a canonical macroeconomic framework of financial crises in the form of shadow bank runs. They extend Gertler and Kiyotaki (2015) by including a wholesale or shadow banking sector, which played an important part in the onset of the 2007-09 financial crisis. We embed their model into a real business cycle model with endogenous capital accumulation and productivity shocks to analyze the welfare and financial stabilization effects of bank capital regulation.

Another paper which studies a similar research question to this paper is Begenau and Landvoigt (2018). The authors study retail bank capital requirements in an economy with an unregulated external shadow banking sector and endogenous capital accumulation. The key difference to their framework is the flow of funds in the economy. In our model, households have direct access to capital markets and there is a wholesale funding market that links the retail and the shadow banking sector. In their model, households hold both debt and equity of retail and shadow banks but have no access to the capital market, and there is no interbank market between the two banking sectors. Consequently, the spillover effects of regulating retail bank capital on shadow bank decisions are small in their model, but quite large in this model. They also model bank runs, but the probability of a bank run is determined exogenously and independently of the liquidation loss in their model.

There is also a set of papers that provides microfoundations for the role of shadow banks in more stylized, theoretical models, e.g. Gennaioli et al. (2013), Luck and Schempp (2016), Moreira and Savov (2017) and Chretien and Lyonnet (2017). Farhi and Tirole (2017) provide a theoretical model of optimal macroprudential regulation in the presence of shadow banks. Relatedly, there is a theoretical literature that emphasizes the role of the shadow banking system for regulatory arbitrage, for example Plantin (2015), Huang (2018) and Ordoñez (2018).

Meeks et al. (2017) study the effects of the existence of a shadow banking sector on the propagation of various macroeconomic shocks, including financial shocks, but do not consider endogenous financial risk in the form of banking crises. Ferrante (2018) presents a new channel through which shadow banks endogenously affect the asset quality of the economy, which leads to business cycle and banking crisis amplification. He does not consider bank capital regulation.

There is a second literature which studies the causes and non-linear propagation of severe financial crises in models with financial intermediation. The closest paper to this paper is Gertler et al. (2017), which also introduces banking crises through into a macroeconomic model. A key distinction is that we include both a retail and a shadow banking sector, which allows us to study spillover effects of bank regulation. Moreover, we discuss the motive for and welfare effects of bank capital regulation and compute the welfare cost of bank runs. Other papers, which model banking crises as rollover crises driven by sunspots, are Martin et al. (2014) and Paul (2018). There are other paper which introduce financial crises in a different way, e.g. due to occasionally binding borrowing constraints (Mendoza (2010), Bianchi (2011), He and Krishnamurthy (2012), Brunnermeier and Sannikov (2014)) or due to market freezes (Boissay et al. (2016)).

The paper is also related to a third literature, which discusses macroprudential regulation. The results of our paper are related to Angeloni and Faia (2013), who study the effectiveness of dynamic capital requirements in the presence of bank runs. There are several differences: First, there are multiple banking sectors in our model. This gives us a spillover effect of regulation of retail banks on the shadow banking sector. Second, we consider runs on the interbank market as opposed to depositor runs. Financial instability in our model arises therefore for a different reason, and gives a different motive to regulate retail banks. Third, banks in our model invest into long-lived assets, which gives rise to *systemic* self-fulfilling crises through changes in asset prices. In their model, runs are instead *bank-specific* runs of the Diamond and Dybvig (1983)-type. Therefore, macroprudential policy can have additional effects through affecting asset prices. Other papers have considered optimal regulation in the presence of externalities that arise in general equilibrium, e.g. Lorenzoni (2008), Bianchi and Mendoza (2018), Di Tella (2016) and Dávila and Korinek (2017). Relative to this literature, we study the effects of regulation in the presence of heterogeneous intermediaries and an interbank market.

More generally, this paper builds on the literature that studies the role of financial frictions as a driving force of financial crises. Early models include Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In these papers, financial frictions are embedded in the financial structure of firms rather than financial intermediaries.

Outline We proceed as follows: In section 2, we outline the model. We characterize the equilibrium of the model in section 3. We present comparative statics which describe the intended effects of retail bank capital regulation and the unintended spillover effect on shadow bank leverage in section 4. We calibrate the model and compare it to the data in section 5. Finally, we show simulation results for the welfare cost of bank runs as well as the welfare effects of bank capital regulation in section 6. Section 7 concludes.

2 Model

In this section, we present a model with a detailed financial sector which is based on GKP. The crucial feature of the model is a wholesale funding market, where retail and shadow banks can make risky loans to each other. Moreover, households and banks make retail loans to consumption goods producers. The consumption goods producers use these loans to purchase capital from capital producers. Households also lend on the deposit market to banks.

Time is discrete, with $t = 0, 1, \ldots, \infty$. We follow the convention that lower case letters for variables denote individual variables, while upper case letters denote aggregate variables.

2.1 Banks

We begin with an exposition of the problem of a bank. The rest of the model is quite standard.

2.1.1 Objective Function and Balance Sheet

Objective Function There is a unit measure of both retail banks (R-banks) and shadow banks (S-banks) in the economy. All banks are owned by households. J-banks, $J \in \{R, S\}$ maximize expected discounted payouts to households, which are given by

$$\mathbb{E}_t \left[\sum_{s=t}^{T} \Lambda_{t,t+s} (1-\sigma^J)^{s-t-1} \sigma^J n_s^J \right], \tag{2.1}$$

where $\mathbb{E}_t$ denotes the expectation conditional on time t information. σ^J is an exogenous exit probability and n_s^J is the net worth of the bank in period s. $\Lambda_{t,t+s}$ is the stochastic discount factor of the household between period t and period $t+s$, defined in the household problem. T is the period in which the bank defaults due to insolvency, with possibly $T \to \infty$. Intuitively, this payoff function implies that banks accumulate net worth until they exit, in which case they pay out their net worth to the households.

Balance Sheet At time t, banks use deposit funding from households, d_{t+1}^J, and their own net worth, n_t^J, to finance retail loans to the non-financial firms, a_{t+1}^J. These retail loans are state-contingent and long-term and can be interpreted as direct claims to the capital stock of the non-financial firms.[9] Hence, these loans are valued at the market price of capital Q_t. Banks incur a bank-specific, linear loan servicing cost f_t^J for outstanding retail loans at the end of period t. In addition, banks can borrow or lend on the wholesale funding market, b_{t+1}^J. $b_{t+1}^J > 0$ means that bank J lends on the wholesale funding market, while $b_{t+1}^J < 0$ denotes that bank J borrows on the wholesale funding market. Hence, the balance sheet of a bank is given by

$$(Q_t + f_t^J)a_{t+1}^J + b_{t+1}^J = d_{t+1}^J + n_t^J \tag{2.2}$$

Net Worth In period t, incumbent banks obtain a gross return on retail loans issued in period $t-1$, $R_t^A a_t^J$. Banks also pay a gross return from borrowing or receive a gross return from lending on the wholesale funding market, $x_t R_t^B b_t^J$. $x_t \leq 1$ denotes the fraction of the promised repayment on wholesale funding that wholesale lenders receive. In a bank run, wholesale lenders receive only a fraction of their return on wholesale lending, i.e. $x_t < 1$. Otherwise $x_t = 1$. Banks repay $R_t^D d_t^J$ to households for their deposits. A bank's net worth in period t is given by

$$n_t^J = R_t^A a_t^J + x_t R_t^B b_t^J - R_t^D d_t^J. \tag{2.3}$$

Loan Servicing Fee For the retail loans on their balance sheet at the end of the period, banks and households have to pay a loan servicing cost. This loan servicing cost can be interpreted as the cost of monitoring outstanding loans. Households, retail banks and shadow banks have access to different loan

[9] In practice, banks' lending to the non-financial sector is largely in the form of debt rather than equity. In the context of our model, banks' investment in the non-financial sector takes the form of equity investment rather than debt. This is a common assumption in the literature with financial intermediation for simplicity - otherwise another layer of liability of the non-financial sector has to be added. Under the current assumption, default on bank loans can be related to bankruptcy of the non-financial firms.

servicing technologies. Loan servicing is provided by specialized firms which operate in a competitive industry.[10] These firms have a cost function that is quadratically increasing in the total amount of loans serviced, $\tilde{A}^J_{t+1}$. It is given by

$$\eta^J \left(\frac{\tilde{A}^J_{t+1}}{A_t} \right)^2 A_t \tag{2.4}$$

Loan servicing firms charge the banks a linear fee f^J_t for their service.[11] Regarding the cost of screening, we make the following assumption:

Assumption 1. *Shadow banks have lower loan servicing costs than retail banks. Households have the highest loan servicing costs:* $\eta^H > \eta^R > \eta^S = 0$.

Adrian and Ashcraft (2012a) discuss reasons for the existence of shadow bank credit intermediation in addition to retail bank credit intermediation. They argue that securitization allowed shadow banks to reduce informational frictions in credit markets, thereby being able to offer loans to high-risk creditors which yield a superior return. Assuming that shadow banks have a lower loan servicing cost than retail banks allows us to capture this salient fact in a simple and tractable way.

Entry and Exit With probability σ^J, a bank of type J receives an exit shock. In the case of such a shock, the bank sells its assets, repays its liabilities and exits the economy. To keep the measure of banks constant over time, new banks with mass σ^J enter the economy and receive an exogenous endowment $\tilde{n}^J_t = \upsilon K_t / \sigma^J$ from households.[12] We make the following assumption regarding the exit probability of retail and shadow banks:

[10] Such companies are for example appraisal management companies, which determine the value of a property, or credit bureaus, which determine the credit worthiness of a household.

[11] This assumption is important for technical reasons. It ensures that the decision problem of the retail banks is linear in their net worth. Therefore it is sufficient to characterize the decision problem of a representative retail bank.

[12] We scale the endowment of newly entering banks by the capital stock to ensure that the arguably stylized assumptions on entry do not affect the comparative statics through changes in the relative size of the endowment.

Assumption 2. *The exit probability of retail banks is lower than the exit probability of shadow banks, i.e.* $\sigma^R < \sigma^S$*.*

This assumption is necessary to ensure that operating a R-bank is not strictly worse than operating a S-bank: R-banks have higher lending costs, but they can accumulate net worth for a longer period of time.[13] Together, Assumptions 1 and 2 capture in a simple way the idea that shadow banks are opened to exploit specific investment opportunities and closed down again when those investment opportunities vanish.

2.1.2 Financial Friction and Default

Moral Hazard Problem Banks can divert a fraction of their assets after they have made their borrowing and lending decisions. How much they can divert depends both on the type of assets and the financing of the assets. Following GKP, we make the following assumptions regarding the diversion of assets:

Assumption 3. *A fraction*

- ψ, $0 < \psi < 1$, *of equity or deposit financed retail loans,*
- $\gamma\psi$, $0 < \gamma < 1$, *of equity or deposit financed wholesale loans, and*
- $\omega\psi$, $0 < \omega < 1$, *of wholesale funding financed retail loans*

can be diverted.

Intuitively, retail loans are easier to divert than wholesale loans, and assets financed through wholesale funding are harder to divert compared to those financed by deposits or bank equity.[14]

[13] While we do not model the choice between operating a R-bank and a S-bank, this assumption ensures that the relative size of the R-banking sector, which is endogenous, is not too small.

[14] Diversion entails the liquidation of the banks' assets and a subsequent default on creditors. One way to justify why equity financed assets cannot be diverted fully is that diversion creates a loss to the diverting bank which is equal to $1 - \psi$ times the diverted assets. Since banks utility is linear in consumption, such a cost may either be a pecuniary cost in the form of a penalty or a stigmatic utility cost.

We motivate $\omega < 1$ by wholesale lenders having stronger incentives to screen the investments of borrowers than depositors or shareholders.[15] For $\gamma < 1$, the intuition lies in the higher standardization of wholesale loans compared to retail loans.[16]

Incentive Constraint If a bank diverts assets, it will not repay its liabilities. Its creditors will subsequently force the bank to exit the economy. Because diversion occurs at the end of the period before the uncertainty about the next period is resolved, an incentive constraint on the bank can ensure that diversion will never occur in equilibrium. This incentive constraint states that the benefit of diversion must be smaller or equal to the franchise value of continuing to operate the bank. Denote this franchise value by V_t^J. If the bank lends on the wholesale funding market, i.e. $b_{t+1}^J > 0$, the incentive constraint is

$$\psi \left[(Q_t + f_t^J) a_{t+1}^J + \gamma b_{t+1}^J \right] \leq V_t^J. \tag{2.5}$$

If the bank borrows on the wholesale funding market, i.e. $b_{t+1}^J \leq 0$, the incentive constraint is instead

$$\psi \left[(Q_t + f_t^J) a_{t+1}^J + (1 - \omega) b_{t+1}^J \right] \leq V_t^J, \tag{2.6}$$

i.e. a fraction $(1 - \omega)$ of wholesale financed retail loans is excluded from diversion.

[15] Adrian and Ashcraft (2012a) argue that due to deposit insurance, depositors have a lower incentive to screen investments of the borrower than wholesale lenders who lend against securitized assets. For the former, the implicit government guarantee provided by the Federal Deposit Insurance is enough to ensure depositors that their lending is risk-free, whereas for the latter, the riskiness of their lending depends on the diversification of the borrower.

[16] The collateral underlying for example a repo contract, which is a typical wholesale lending instrument, was often a high quality government bond or a collateralized debt obligation. Those assets were considered safe and their market value was easy to verify for creditors (see e.g. Gorton and Metrick (2012)). The collateral underlying a retail loan can for example be commercial real estate or the physical capital stock of a firm, for which only a rough estimate of the market value exists. Hence, the potential for diversion is much higher for retail loans compared to wholesale loans.

Bank Default Equations 2.5 and 2.6 imply that a bank with zero or negative net worth cannot lend or borrow. By the definition of bank net worth this also means that the assets of the bank are insufficient to cover its liabilities: $n_t^J \leq 0 \iff R_t^A a_t^J + R_t^B b_t^J \leq R_t^D d_t^J$. Hence, a bank with a negative net worth is insolvent and has to default on its liabilities. The creditors of the bank recover a fraction ξ of the bank's assets. If the bank borrows exclusively from the wholesale funding market, i.e. $d_{t+1}^J = 0$ and $b_{t+1}^J < 0$, the recovery rate is

$$x_t = \xi \frac{R_t^A a_t^J}{|R_t^B b_t^J|}. \tag{2.7}$$

If the bank is insolvent, the creditors do not recover their claim in full: $n_t^J < 0 \Rightarrow x_t < 1$. The fraction $1 - \xi$ that is not recovered is rebated lump sum to households.

2.1.3 Bank Leverage

Define the leverage ratio of a bank as

$$\phi_t^J \equiv \frac{(Q_t + f_t^J)a_{t+1}^J + \gamma b_{t+1}^J \mathbb{1}(b_{t+1}^J > 0)}{n_t^J}, \tag{2.8}$$

i.e. the fraction of bank assets that require some equity financing divided by the net worth, or equity, of the bank. Remember that a fraction $1 - \gamma$ of wholesale loans is non-divertable and hence does not require equity financing.

Define further $\Omega_t^J \equiv \frac{V_t^J}{n_t^J}$ as the average market value of a unit of net worth. Since the problem of the bank is linear in net worth, this corresponds to the marginal value of an additional unit of net worth to a bank of type J.

With these definitions, we can rewrite equation 2.5 as

$$\psi \phi_t^J \leq \Omega_t^J. \tag{2.9}$$

Similarly, equation 2.6 can be rewritten as

$$\psi \left[\omega \phi_t^J + 1 - \omega\right] \leq \Omega_t^J. \tag{2.10}$$

Hence, the incentive constraint of a bank can be interpreted as a market-imposed leverage constraint. It implies that the leverage of a bank that is at the incentive constraint is linearly increasing in the value of an additional unit of net worth.

2.1.4 Bank Capital Requirements

The regulator can impose a capital requirement, which is equivalent to an upper bound on leverage:

$$\phi_t^J \leq \bar{\phi}_t^J. \tag{2.11}$$

We consider a regulator that sets leverage constraints for banks of type J, $\bar{\phi}_t^J$, following a modified leverage constraint:

$$\psi \bar{\phi}_t^J (1 + \tau_t^J) = \Omega_t^J \tag{2.12}$$

for lenders on the wholesale funding market, and

$$\psi \left[\omega \bar{\phi}_t^J (1 + \tau_t^J) + 1 - \omega \right] \leq \Omega_t^J \tag{2.13}$$

for borrowers on the wholesale funding market.

The interpretation of this constraint is that the regulator internalizes that the social cost of an additional unit of bank leverage might be higher, by a factor τ_t^J, than its private cost. This is the case, since an increase in leverage leads to higher debt in the economy, which increases the likelihood and severity of a systemic shadow bank run.

2.2 Households

Preferences Households maximize utility from consumption. Their utility function is given by

$$\mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} U(c_s^H) \right], \tag{2.14}$$

where β is the discount factor of the household. c_t^H denotes household consumption in period t. $U(c)$ is the current utility function of the household, with $U'(c) > 0$, $U''(c) < 0$, and satisfying the Inada conditions.

The stochastic discount factor of the household between period t and $t+s$ is given by

$$\Lambda_{t,t+s} = \beta^{s-t} \frac{U'(c_{t+s}^H)}{U'(c_t^H)}.$$

Household Budget Constraint Households consume, invest in retail loans a_{t+1}^H and make deposits d_{t+1}^H at banks. They supply one unit of labor inelastically and receive W_t as labor income. In addition, they own the banks, capital producers and loan servicing firms and receive their profits Π_t.[17] Deposits yield a safe gross return R_{t+1}^D in the subsequent period. The net worth of the household at the beginning of period t is given by

$$n_t^H = R_t^A a_t^H + R_t^D d_t^H + W_t + \Pi_t. \tag{2.15}$$

The budget constraint of the household is

$$c_t^H + d_{t+1}^H + (Q_t + f_t^H) a_{t+1}^H = n_t^H. \tag{2.16}$$

Loan Servicing Firms Banks and households pay a linear fee f_t^J to the loan servicing companies. The profit of these firms is given by:

$$\Pi_t^{L,J} = f_t^J \tilde{A}_{t+1}^J - \frac{\eta^J}{2} \left(\frac{\tilde{A}_{t+1}^J}{A_t} \right)^2 A_t$$

where f_t^J is the loan servicing fee per unit of the loan. These firms are owned by the households. They operate in a competitive market, which means the equilibrium fee f_t^J is taken as given by households and banks, and is determined in equilibrium such that the loan servicing firms are willing to service all loans of the banks and households.

[17] Profits are 0 in steady state, but may arise outside of the steady state due to a quadratic capital adjustment cost.

2.3 Production

2.3.1 Consumption Goods Producers

Consumption goods producers use a Cobb-Douglas production technology that takes labor L_t and capital K_t as input:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}. \tag{2.17}$$

The price of the consumption good is normalized to one. Productivity Z_t follows an AR(1) process with mean μ^Z, persistence ρ^Z and volatility σ^Z:

$$\ln(Z_t) = (1-\rho^Z)\mu^Z + \rho^Z \ln(Z_{t-1}) + \sigma^Z \epsilon_t, \tag{2.18}$$

where $|\rho^Z| < 1$ and $\epsilon_t \sim N(0,1)$.

The consumption goods producers own the capital stock. Capital accumulation follows the standard law of motion

$$K_{t+1} = (1-\delta)K_t + I_t,$$

with depreciation rate δ and investment I_t.

The consumption goods producers use labor from households and their own capital to produce final goods. They finance their capital K_{t+1} exclusively using retail loans A_{t+1}, which yields their balance sheet condition:

$$K_{t+1} = A_{t+1}. \tag{2.19}$$

They maximize profits taking the aggregate wage W_t, the return on loans R_t^A and the price of capital Q_t as given:

$$\max_{\{L_s, K_{s+1}, A_{s+1}\}_{s=t}^{\infty}} \mathbb{E}_t \left[\sum_{s=t}^{\infty} \Lambda_{t,t+s} \Pi_s \right] \tag{2.20}$$

subject to

$$\Pi_t = Z_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - Q_t(K_{t+1} - (1-\delta)K_t) + Q_t A_{t+1} - R_t^A A_t,$$
$$K_{t+1} = A_{t+1}.$$

The first order conditions of the final goods producers' problem determine the wage and the state-contingent return on retail loans in equilibrium:

$$W_t = (1-\alpha) Z_t K_t^{\alpha} L_t^{-\alpha}, \tag{2.21}$$

$$R_{t+1}^A = \alpha Z_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + (1-\delta) Q_{t+1}. \tag{2.22}$$

2.3.2 Capital Goods Producers

Capital producers use a technology which transforms $\tilde{I}_t$ units of consumption goods into I_t^{supply} units of capital goods. They face a convex cost function:

$$\tilde{I}_t = I_t^{supply} + \frac{\theta}{2}\left(\frac{I_t^{supply}}{K_t} - \delta\right)^2 K_t, \tag{2.23}$$

Therefore, the relative price of capital goods Q_t is endogenous. Importantly, the cost function is scaled by the aggregate capital stock K_t, which the capital producers take as given.

Due to the convex cost function, the capital producers may earn non-zero profits outside the steady state. They are owned by the households and any profits or losses are transferred to the households each period.

The capital producers' problem can be summarized as:

$$\max_{\{I_s^{supply}\}_{s=t}^{\infty}} \mathbb{E}_t \left[\sum_{s=t}^{\infty} \Lambda_{t,t+s} \Pi_s^Q\right] \tag{2.24}$$

subject to

$$\Pi_t^Q = Q_t I_t^{supply} - I_t^{supply} - \frac{\theta}{2}\left(\frac{I_t^{supply}}{K_t} - \delta\right)^2 K_t.$$

The first order condition of the capital producer yields an expression for the capital price:

$$Q_t = 1 + \theta \left(\frac{I_t^{supply}}{K_t} - \delta \right). \tag{2.25}$$

2.4 Aggregation and Market Clearing

2.4.1 Aggregation

There is no idiosyncratic uncertainty for households, such that we can consider the problem of a representative household. Moreover, since the policy functions of an individual bank are linear in net worth, we will characterize the equilibrium in terms of the aggregate decisions of the banking sectors. Aggregate variables are denoted by capital letters. The aggregate net worth of the retail and shadow banking sector is given by the sum of the net worth of incumbent and newly entering banks:

$$N_t^J = \left(R_t^A A_t^J + R_t^B B_t^J - R_t^D D_t^J \right) (1 - \sigma^J) + \upsilon K_t$$

Aggregate profits are given by the profits of screening firms and capital producers, plus the sum of net worth of exiting retail banks and shadow banks minus the net worth of entering banks:

$$\Pi_t = \Pi_t^Q + \sigma^R n_t^R + \sigma^S n_t^S + \Pi_t^{L,H} + \Pi_t^{L,R} - 2\upsilon K_t \tag{2.26}$$

Aggregate output is given by production net of the capital holding costs:

$$Y_t = Z_t K_t^\alpha - \frac{\eta^H}{2} \left(\frac{K_{t+1}^H}{K_t} \right)^2 K_t - \frac{\eta^R}{2} \left(\frac{K_{t+1}^R}{K_t} \right)^2 K_t. \tag{2.27}$$

2.4.2 Market Clearing

The markets for retail loans, labor, deposits, wholesale loans, investment and loan services have to clear:

$$A_{t+1} = A^H_{t+1} + A^R_{t+1} + A^S_{t+1} \tag{2.28}$$

$$L_t = 1 \tag{2.29}$$

$$D^H_{t+1} = D^R_{t+1} + D^S_{t+1} \tag{2.30}$$

$$0 = B^R_{t+1} + B^S_{t+1} \tag{2.31}$$

$$I_t = I^{supply}_t \tag{2.32}$$

$$A^J_{t+1} = \tilde{A}^J_{t+1}, \quad J \in \{H, R\} \tag{2.33}$$

Since there is a representative household, the individual consumption and aggregate consumption are equal, $c^H_t = C^H_t$. Household consumption can be inferred from the aggregate resource constraint:

$$C^H_t = Y_t - \tilde{I}_t \tag{2.34}$$

3 Equilibrium and Bank Runs

In this section, we define the recursive competitive equilibrium. We characterize the flow of funds in the no-bank-run equilibrium and show how it depends on the financial constraints of retail and shadow banks. We illustrate the potential for multiple equilibria, namely one equilibrium with solvent banks and high capital prices and one with insolvent shadow banks and low capital prices.

3.1 Recursive Competitive Equilibrium

We consider a recursive competitive equilibrium. The aggregate state of the economy is given by $\mathcal{S}_t = (N^R_t, N^S_t, K_t, Z_t, \Xi_t)$. Ξ_t is a sunspot shock which can take on two values, 0 and 1. It coordinates agents on the bank run equilibrium if it takes the value 1 and on the no-bank run equilibrium otherwise. The equilibrium is a set of price functions $\mathbf{Q}(\mathcal{S})$, $\mathbf{R^A}(\mathcal{S})$, $\mathbf{R^D}(\mathcal{S})$, $\mathbf{R^B}(\mathcal{S})$, $\mathbf{W}(\mathcal{S})$, $\mathbf{f^H}(\mathcal{S})$ and $\mathbf{f^R}(\mathcal{S})$, value functions and policy functions for

- retail banks, $\mathbf{\Omega^R}(\mathcal{S}), \mathbf{A^R}(\mathcal{S}), \mathbf{D^R}(\mathcal{S}), \mathbf{B^R}(\mathcal{S})$, which maximize 2.1 subject to 2.2, 2.3 and 2.5,
- shadow banks, $\mathbf{\Omega^S}(\mathcal{S}), \mathbf{A^S}(\mathcal{S}), \mathbf{D^S}(\mathcal{S}), \mathbf{B^S}(\mathcal{S})$, which maximize 2.1 subject to 2.2, 2.3 and 2.6,
- households, $\mathbf{V^H}(\mathcal{S}), \mathbf{C^H}(\mathcal{S}), \mathbf{A^H}(\mathcal{S}), \mathbf{D^H}(\mathcal{S})$, which maximize 2.14 subject to 2.15 and 2.16,

policy functions for

- final goods producers, $\mathbf{K}(\mathcal{S}), \mathbf{A}(\mathcal{S}), \mathbf{L}(\mathcal{S})$, that solve 2.20
- capital producers, $\mathbf{I^{supply}}(\mathcal{S})$, that solves 2.24 and
- loan service providers, $\mathbf{\tilde{A}^H}(\mathcal{S})$ and $\mathbf{\tilde{A}^R}(\mathcal{S})$,

and perceived laws of motion for the aggregate state $\mathbf{G}(\mathcal{S})$ that ensure that the perceived law of motion for the aggregate state corresponds to its actual law of motion:

$$\mathcal{S}_{t+1} = \mathbf{G}(\mathcal{S}_t), \tag{3.1}$$

clear the markets specified in equations 2.28 to 2.33 satisfy the aggregate resource constraint 2.34.

3.2 The Equilibrium with Solvent Shadow Banks

Figure 2 shows an overview of the equilibrium balance sheet of the economy. Households are the ultimate lenders and have only equity on the liability side of their balance sheets. They own all banks and firms. Not displayed is their ownership of capital producers and loan servicing firms. Households lend to retail banks and to consumption goods producers. Retail banks use deposits and equity to lend to both shadow banks and consumption goods producers. Shadow banks in turn use wholesale funding and their own equity to lend to

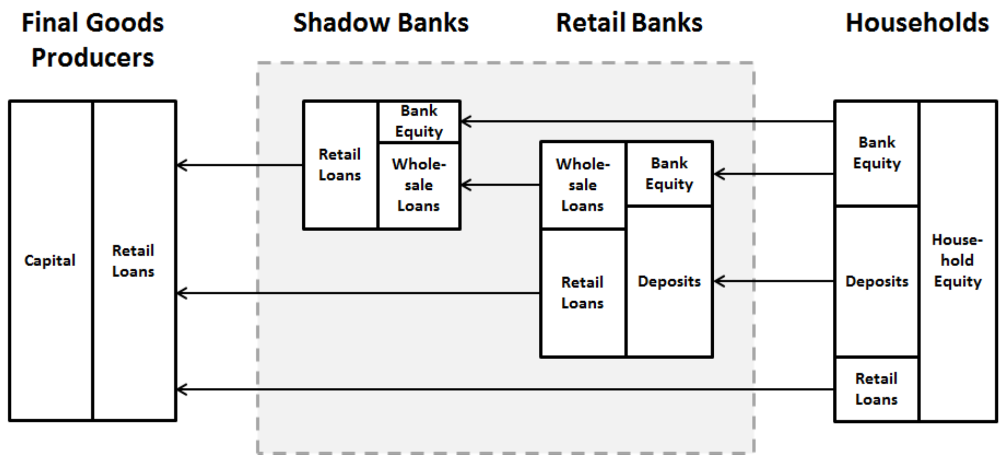

Figure 2: The equilibrium flow of funds of the economy in the no-bank run state.

consumption goods producers, which are the ultimate borrowers. They fund capital by using state-contingent claims to the capital stock which we call retail loans.

To understand how this equilibrium depends on model assumptions, we consider in Appendix C three instructive cases: A situation in which neither retail nor shadow banks are financially constrained, a situation in which shadow banks are financially constrained, but retail banks are not, and a situation in which both types of banks are financially constrained.

3.3 The Equilibrium with Insolvent Shadow Banks

Assumptions We consider only runs on the shadow banking sector, and only on the shadow banking sector as a whole. If such a run happens, the assets of the shadow banks are liquidated at the liquidation price

$$Q_t^* \equiv \mathbf{Q}(N^R(Q_t^*), N_t^S(Q_t^*), K_t, Z_t, \Xi_t = 1).$$

The retail banks recover the assets of the shadow banks instead of their lending at the recovery rate x_t^*. Incumbent shadow banks exit once their assets are liquidated. There is no entry of new shadow banks for the duration of the bank run.

Effect on the Capital Price If shadow banks are insolvent, their demand for assets is zero. Moreover, the reduction of net worth due to the losses on their wholesale loans reduces the asset demand of retail banks. This, together with the fact that they are less efficient lenders than shadow banks anyway implies that not all liquidated assets of shadow banks will be absorbed by retail banks. This reduces the overall asset and capital demand and hence the market price of capital Q_t^*.

3.4 Equilibrium Multiplicity and Sunspots

Bank Run Condition There can be multiple equilibria in the model. In other words, bank runs can be self-fulfilling. In that case, the market price of capital deteriorates in anticipation of a bank run. This weakens balance sheets of shadow banks so much that they cannot repay their liabilities. As a consequence, it is optimal for the retail banks to run on shadow banks. However, the bad equilibrium will only occur if the assets of the shadow banks, valued at the liquidation price of capital, are insufficient to cover the liabilities of shadow banks. Define

$$\mathbf{x}(\mathcal{S}_t) \equiv \xi \frac{\mathbf{R^A}(\mathcal{S}_t) a_t^S}{R_t^B b_t^S}.$$

Define further

$$x_t \equiv \mathbf{x}(N^R(Q_t), N^S(Q_t), K_t, Z_t, \Xi_t = 0),$$

and

$$x_t^* \equiv \mathbf{x}(N^R(Q_t^*), N^S(Q_t^*), K_t, Z_t, \Xi_t = 1).$$

Then, depending on the state of the economy, there can be three situations: First, if $x_t > 1$ and $x_t^* > 1$, the economy is in the safe zone, where even conditional on a sunspot shock, the bank run equilibrium will not arise. If $x_t > 1$ and $x_t^* \leq 1$, the economy is in the crisis zone, where it is susceptible to bank runs. If $x_t \leq 1$ and $x_t^* \leq 1$, shadow banks will default for sure, independently of the sunspot.

Sunspots The probability of a shadow bank run is given by

$$p_t \equiv \Pr(\Xi_t = 1)\mathbb{1}(x_t^* \le 1), \tag{3.2}$$

where $\mathbb{1}(x_t^* \le 1)$ is an indicator function that takes the value 1 if $x_t^* \le 1$ and 0 otherwise. Ξ_t is a sunspot that determines how likely a bank run is, conditional on the bank run condition being fulfilled. In the baseline model, we assume that the bank run probability is i.i.d. and inversely related to the recovery value of bank creditors in default. Formally,

$$\Pr(\Xi_t = 1) = \eta(1 - x_t^*).$$

Note that the bank run probability is endogenous for two reasons. First, the probability of the sunspot shock depends on an endogenous object, x_t^*. Second, the crisis zone where both equilibria exist depends also on x_t^*. As a consequence, bank runs in this model are driven by fundamentals and not pure sunspot events.[18]

3.5 The Effects of Bank Run Risk on Leverage

Figure 3 shows that the existence of the bank run equilibrium substantially changes the optimal policies of shadow banks in the no-run equilibrium. It depicts the leverage policy of shadow banks in the no-run equilibrium as a function of shadow bank net worth, holding all other state variables at their steady state value. We show the policy functions for two situations: The blue, solid line depicts the policy function if bank runs are anticipated, the red, dashed line if they are unanticipated.

If bank runs are anticipated, the leverage capacity of shadow banks is much lower. To see why this is the case, consider the incentive constraint of

[18] Gorton (1988) presents evidence that historically, bank runs in the United States were indeed related to an increased fundamental riskiness of deposits, that is, during times when expected losses on deposits were high. Further, the large number of retail banks in an economy and the high competition in the retail banking business reduces the ability of banks to coordinate absent, for example, a credible lender of last resort (Rochet and Vives (2004)).

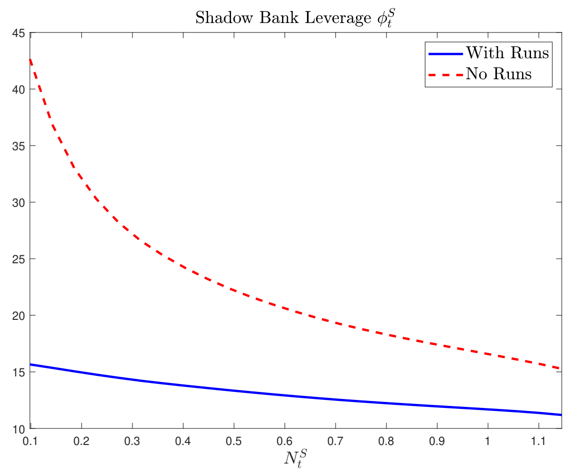

Figure 3: The policy function for shadow bank leverage ϕ_t^S in the no-run equilibrium in the baseline model as a function of shadow bank net worth. The other state variables are held constant at the steady state level. *With Runs* denotes the policy function in the case of anticipated bank runs. *No Runs* denotes the policy function if bank runs are unanticipated or if the crisis equilibrium is ruled out.

the shadow bank:

$$\begin{aligned}\psi(\omega\phi_t^S + 1 - \omega) &= \Omega_t^S \\ &= \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\sigma^S + (1-\sigma^S)\underbrace{\Omega_{t+1}^S}_{\substack{\text{Effect on Future}\\ \text{Continuation}\\ \text{Value}}}\right)\underbrace{\frac{n_{t+1}^S}{n_t^S}}_{\substack{\text{Effect on}\\ \text{Net Worth}\\ \text{Growth}}}\underbrace{(1-p_{t+1})}_{\substack{\text{Direct}\\ \text{Effect}}}\right].\end{aligned}$$

An increase in bank run risk (i.e., in p_{t+1}) can affect the leverage capacity of shadow banks through three channels: First, it directly lowers the continuation value. Second, it can lower the leverage capacity of shadow banks indirectly by lowering future capital prices and hence net worth growth. Third, it can affect the current continuation value by lowering lower future continuation values through lowering future leverage capacity. Overall, the incentive constraint delivers some form of "market prudence": Higher bank run risk will lead to lower leverage capacity of shadow banks.

4 The Intended and Unintended Effects of Regulating Retail Banks

In the last section, we showed how bank runs in the model work and demonstrated that they are costly events. In this section, we introduce regulation in the form of leverage constraints for retail and shadow banks into the model and investigate their effectiveness in reducing the frequency of shadow bank runs.

4.1 Retail Bank Capital Requirements

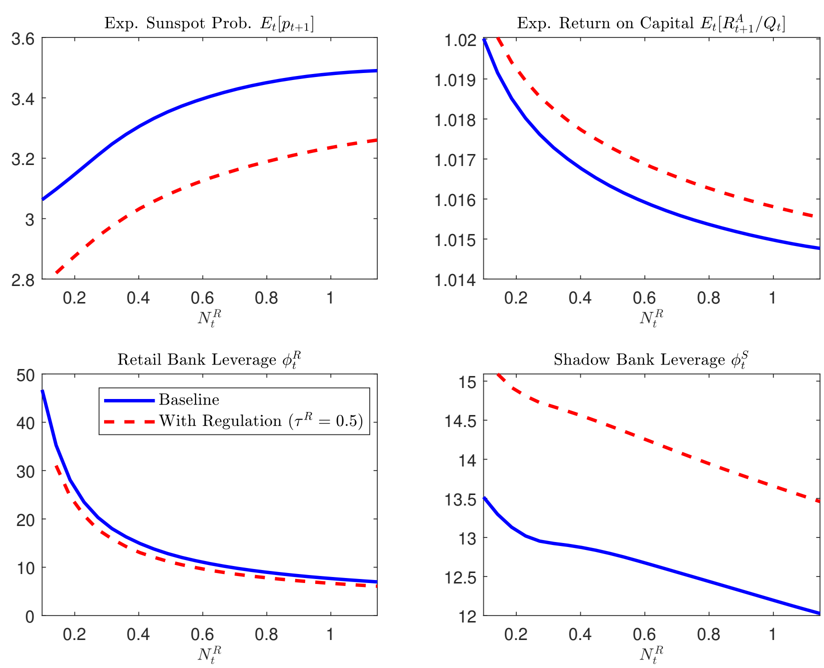

Figure 4: Policy functions for the expected bank run probability (top left), the expected return on capital (top right), retail bank leverage (bottom left) and shadow bank leverage (bottom right) as a function of retail bank net worth N_t^R. All other states are held constant at their steady state value. *Baseline* denotes the policy functions for the model without regulation, *Regulation* those for the model with a retail bank capital requirement such that $\tau_{NoRun}^R = 0.5$.

We show the effects of higher retail bank capital requirements in Figure 4. In this figure, we plot the next period bank run probability $\mathbb{E}_t[p_{t+1}]$, the expected future return on capital $\mathbb{E}_t\left[\frac{R^A_{t+1}}{Q_t}\right]$, and both retail bank leverage ϕ^R_t and shadow bank leverage ϕ^S_t as a function of the net worth of retail banks N^R_t. We keep all other state variables at their steady state value. We show two situations: The baseline model, in which there is no regulation, is the blue, solid line. The red, dashed line depicts the policy functions in a model in which the regulator increases capital requirements by imposing $\tau^R_{NoRun} = 0.5$. We set $\tau^R_{Run} = 0$, so that in the run equilibrium, the market imposed leverage constraint and the regulatory leverage constraint coincide.

We see that the introduction of the retail bank capital requirement reduces the expected future probability of bank runs substantially. To understand why, consider the existence condition for the bank run equilibrium. After substituting $Q_t a^S_{t+1} = \phi^S_t n^S_t$ and $b^S_{t+1} = (\phi^S_t - 1)n^S_t$, this becomes:

$$x^*_t = \xi \frac{\frac{R^{A*}_{t+1}}{Q_t}}{R^B_{t+1}} \frac{\phi^S_t}{\phi^S_t - 1} \leq 1. \tag{4.1}$$

There are two terms: The first one is the spread between the return on shadow bank assets in the case of a shadow bank run, the second is decreasing in the leverage of shadow banks. Inspecting the last three panels of Figure 4, we can see that the effect of higher retail bank capital requirements on the expected future bank run probability stems from an increase in the expected future return on capital: By constraining the leverage capacity of retail banks in the no-run-equilibrium, the regulator increases the leverage capacity of retail banks in the run-equilibrium, which leads them to invest more and pushes up the liquidation price of capital in the run equilibrium.

However, we can see in the bottom right panel that imposing retail bank capital requirements has a strong spillover on shadow bank leverage. This spillover occurs for two reasons: First, as the policy reduces the probability of a shadow bank run, it relaxes the incentive constraint of shadow banks, which increases their leverage capacity. Second, as $\gamma < 1$, that is, as wholesale loans enter with a lower weight in the leverage constraint of retail banks than retail loans, a tighter leverage constraint induces retail banks to shift their relative portfolio shares away from retail lending and towards wholesale lending. Looking at the bank run existence condition 4.1, we can see that a higher shadow bank run leverage counteracts the increase in the return on capital and hence reduces the effectiveness of the retail bank capital requirement policy.

These spillover effects are consistent with the empirical evidence: For example, Duca (2016) provides evidence that the share of assets funded by shadow banks is higher if event risks on the interbank market are lower and the regulation of banks relative to nonbanks was tighter.

4.2 Correcting the Spillover Effect

To understand the importance of the spillover effect better, we show in Figure 5 a policy that corrects for the spillover effect of retail bank capital requirements on shadow bank leverage. This policy imposes both a retail and a shadow bank capital requirement. We choose $\tau^R_{NoRun} = 0.5$ and $\tau^R_{Run} = 0$ as before, but now additionally set $\tau^S \approx 0.11$. We choose τ^S such that the average level of shadow bank leverage in model simulations is equal in the model with and without regulation. The dotted, black lines depict the policy functions in the case of this retail bank capital requirement with spillover correction as a function of retail bank net worth. As before, the solid blue line is the baseline case without regulation, the dashed red line the case with retail bank regulation, but without spillover correction.

We can see in the top left panel of Figure 5 that, due to the spillover correction, the retail bank capital requirement is almost twice as effective at reducing the expected future probability of shadow bank runs as without spillover correction. Moreover, there is no substantial feedback from the spillover correction to either retail bank leverage or the expected return on capital, as we see in the top right and bottom left panel. Taken together, this shows that the negative spillover lowers the effectiveness of retail bank capital requirements in reducing the frequency of shadow bank runs dramatically.

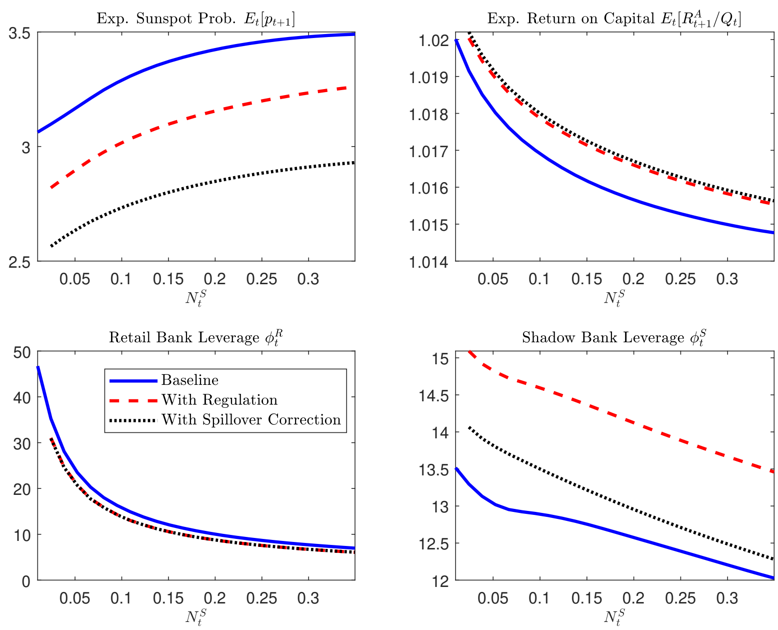

Figure 5: Policy functions for the expected bank run probability (top left), the expected return on capital (top right), retail bank leverage (bottom left) and shadow bank leverage (bottom right) as a function of retail bank net worth N_t^R. All other states are held constant at their steady state value. *Baseline* denotes the policy functions for the model without regulation, *With Regulation* denotes those for the model with a retail bank capital requirement such that $\tau^R_{NoRun} = 0.5$. *With Spillover Correction* denotes the policy functions for the model where there are a both retail and shadow bank capital requirement, with $\tau^R_{NoRun} = 0.5$ and $\tau^S \approx 0.11$.

5 Calibration

We solve the model numerically to illustrate the effects of bank run risk on the distribution of output in model simulations, as well as to illustrate the effectiveness of retail bank capital regulation and the quantitative importance of the novel spillover effect.

5.1 Calibration Strategy

Overall, there are 18 parameters to calibrate. Since we solve a complicated non-linear model, estimating all parameters is infeasible. We therefore divide the parameters into three blocks. Parameters in the first block include the technology and preference parameters. We use conventional values for those parameters. The second block of parameters are those for the financial sector. We set them to match steady state moments of the model to the data. We take the targets which we calibrate to from GKP. These targets are credit spreads as well as data from the financial accounts of the US. The third block of parameters are specific to bank runs or specify the exogenous stochastic processes. We internally calibrate those parameters using moment matching.

Role	Name	Value	Target or Source
(a) Technology and Preferences			
Capital share in production	α	0.36	Standard value
Depreciation Rate	δ	0.025	Standard value
Risk Aversion	σ	2	Standard value
Household discount factor	β	0.9902	$R^D - 1 = 4\%$ p.a.
Capital adjustment cost	θ	10	$\left.\frac{\partial \ln(Q_t)}{\partial \ln(I_t)}\right\vert = 0.25$
(b) Financial Sector			
Banks' initial equity	υ	0.001	Planning horizons of banks
Diversion benefit of wholesale lending	γ	0.6676	$R^B - R^D = 0.8\%$ p.a.
Household capital holding cost	η^H	0.0286	$R^K - R^D = 2.4\%$ p.a.
Retail bank capital holding cost	η^R	0.0071	$R^{K,R} - R^D = 1.2\%$ p.a.
Retail bank exit rate	σ^R	0.0521	$K^R/K = 0.4$
Shadow bank exit rate	σ^S	0.1273	$K^S/K = 0.4$
Asset diversion share	ψ	0.2154	$\phi^R = 10$
Diversion benefit of wholesale funding	ω	0.5130	$\phi^S = 20$
(c) Bank Runs and Stochastic Processes			
Autocorrelation, productivity	ρ^Z	0.9	$\rho(Y_t, Y_{t-1}) = 0.9$
Standard Deviation, productivity shock	σ^Z	0.01	$\sigma(Y_t) = 0.03$
Loss in Default	ξ	0.9	Retail bank net worth in run -30 %
Sunspot probability shifter	η	0.25	Crisis freq. of $\approx 0.75\%$ per quarter
Reentry probability after bank run	π	12/13	Runs last 3.25 yrs on avg

Table 1: Calibration.

Technology and Preferences. Parameters in Panel (a) of Table 1 are set following the literature. The capital share of consumption good production α and the quarterly depreciation rate of capital δ are set to 0.36 and 0.025, respectively. We assume a risk aversion σ of the households of 2. These are standard values in the literature. We set the discount rate of households β to target an annual steady state return on deposits of 4 percent. We set the adjustment cost parameter θ to match an elasticity of the capital price to the investment-to-capital ratio of 0.25, which is the target of Bernanke et al. (1999). This implies a parameter of $\theta = 10$.[19]

Financial Sector. Parameters in Panel (b) of Table 1 are specific to the financial sector. We use the same targets for these parameters as GKP. We set the banks endowments υ to yield a planning horizon of shadow banks of about two years and retail banks of about five years, similar to Gertler et al. (2016).[20] We target leverage ratios of 10 and 20 for retail banks and shadow banks, respectively, to calibrate the diversion parameters ψ and ω. This corresponds to the leverage ratios of depository institutions and broker dealers before the crisis.[21] We choose the remaining diversion parameter γ to match an average annualized spread between the return on wholesale loans and the return on deposits of 0.8 percent per year. We set the exit shock probabilities σ^R and σ^S such that the share of assets intermediated by retail banks and shadow banks in steady state are respectively 40 percent. These values correspond to the respective share of intermediated assets in the data between 2003 and 2007.[22] For the capital holding cost parameters η^H and η^R, we target spreads between the return on retail loans intermediated by shadow banks and the deposit rate of 2.4 % per year and between the return on retail loans intermediated by retail banks and the deposit rate of 1.2 percent per year.

[19] The elasticity of the capital price to investment is given by $\frac{\partial Q_t}{\partial I_t}\frac{I_t}{Q_t} = \theta \frac{1}{K_t}\frac{I_t}{1+\theta\left(\frac{I_t}{K_t}-\delta\right)}$. Evaluated in Steady State, this expression becomes $\theta\delta$.

[20] The planning horizons are simply $1/\sigma^R$ and $1/\sigma^S$. These are the same targets for the banks endowment as in Gertler and Karadi (2011).

[21] See e.g. Gertler et al. (2016) or Ferrante (2018).

[22] According to GKP, assets intermediated by retail banks comprise equity of non-financial firms, bonds, commercial paper, household and non-financial firm loans, mortgages and consumer credit. For shadow banks, intermediated assets comprise equity of non-financial firms, mortgages and consumer credit.

Bank Runs and Stochastic Processes. We calibrate the parameters in Panel (c) of Table 1 using moment matching. We choose ρ^Z and σ^Z to match roughly the conditional volatility and the autocorrelation of detrended GDP for the United States. Three key parameters for the cost of bank runs are the loss on the recovery value in a run ξ, the sunspot probability shifter η and the persistence of the run π. We choose the persistence of financial crises such that the average length of a financial crisis is 3.25 years, which matches the length of a typical financial crisis in the database of Laeven and Valencia (2012). We calibrate the sunspot probability shifter η to match an annual frequency of bank runs of 3 percent or one bank run every 33 years, also in line with Laeven and Valencia (2012). We choose the loss on the recovery value in a run to match an average decrease of retail bank net worth conditional on a bank run of 30 percent. This corresponds to the cutoff used in Baron et al. (2018) to define a bank equity crash.

5.2 Solution Method

We solve the model nonlinearly with a projection method on a sparse grid. To construct the sparse grid, we use the toolbox of Judd et al. (2014). Solving the model using global methods has two key advantages: First, and most importantly, it allows us to accurately characterize the dynamics of the economy very far away from steady state. This is important, since a financial crisis will wipe out the net worth of the shadow banking sector and substantially reduce the net worth of the retail banking sector below its steady state level. Second, the non-linear solution allows us to accurately compute risk premia in the model. This is crucial, since asset price dynamics are key for generating financial crises in the model. Details of the solution algorithm are in Appendix D.

6 Quantitative Results

We conduct three counterfactual experiments. First, we compare how the existence of the bank run equilibrium affects the economy. This experiment gives us an idea of the cost of bank runs and hence an upper bound on the positive effect of a policy designed to reduce bank runs. We find that the welfare cost of bank runs is large. We decompose the total effect of shadow bank runs into the effect that is due to realized bank runs, and the effect that is due to financial crisis fears. We show that it is the latter channel which is the more important one. Second, we consider the effects of retail bank capital requirements. We show that there is a substantial spillover effect from retail bank capital requirements on shadow bank leverage. We decompose the effect into an effect that is due to a relaxation of the shadow bank incentive constraint and one that is due to the portfolio reallocation of retail banks. Third, we consider a policy that corrects for the spillover effect by imposing a shadow bank capital requirement that offsets the increase in leverage in addition to the retail bank capital requirement. We show that the retail bank capital requirement would be more than twice as effective in reducing the frequency of shadow bank runs without the spillover effect.

6.1 The Effects of Bank Run (Fears) on the Level and Volatility of Output

Before investigating the welfare effects of bank capital requirements, we want to know how costly bank runs are in our calibrated model. For this purpose, we conduct the following experiment: We first simulate 10000 model economies for 2000 periods and discard the first 1000 periods in the model with bank runs. Moments from this simulation are reported in column 1 of Table 2. We compute welfare in consumption equivalent units using the realized consumption of households. Next, we simulate the model without bank runs. For this simulation, we re-solve the model and set the expected probability of a sunspot shock to 0. The results are reported in column 2 of Table 2.

Comparing the models with and without bank runs, we can see that bank

	With Runs	No Runs	Only Exp.
Macroeconomic Aggregates			
Mean, Output (Y)	1.088	1.114	1.093
Mean, Consumption (C^H)	0.850	0.865	0.853
Mean, Investment ($\tilde{I}$)	0.238	0.248	0.240
St. Dev., Output (Y)	3.181	3.275	3.192
St. Dev., Consumption (C^H)	2.169	2.197	2.162
St. Dev., Investment ($\tilde{I}$)	7.396	7.614	7.421
Financial Sector			
Mean, Retail Bank Leverage (ϕ^R)	10.291	10.019	10.239
Mean, Shadow Bank Leverage (ϕ^S)	13.444	19.995	13.244
Asset Prices			
Mean, Spread, Wholesale ($R^{B'} - R^{D'}$)	1.119	0.757	1.083
Mean, Spread, Retail ($E[R^{K'}/Q] - R^{D'}$)	2.990	2.505	2.862
Bank Runs			
Runs per 100 Years	3.100	0.000	0.000
Recovery Rate ($x_t \| Run_t$)	78.214	-	-
Welfare	0.850	0.865	0.853

Table 2: The macroeconomic effects of bank runs. Results are from a simulation of 10000 economies for 2000 periods, discarding the first 1000 periods. *With Runs* is the baseline model with anticipated bank runs. *No Runs* is a version of the model in which bank runs neither happen nor are expected to happen. *Only Exp.* shows the results for a model in which bank runs are expected to happen, but never occur.

runs have severe effects on the first and second moments of macroeconomic aggregates: With bank runs, output, consumption and investment are substantially lower. The effect is sizeable: Even though bank runs occur only once every 33 years on average, output is on average by 2.3 percent lower *in every period.* Consumption is by 1.7 percent lower, investment even by 4 percent.

Surprisingly, the unconditional volatility of the economy is *lower* if there are bank runs. This is because in the presence of bank run risk, the leverage of shadow banks is much lower, which reduces the strength of the feedback between banks balance sheets and the real economy. The reason that shadow bank leverage is much lower is that with anticipated bank runs, the continuation value of operating a shadow bank is lower, which makes the incentive constraint tighter. We can see that the decline in shadow bank leverage is substantial - from 20 to about 13.4. In contrast, retail banks see a relaxation of their incentive constraint due to a higher excess return on retail and wholesale lending, which increases their leverage capacity slightly. The welfare gain from eliminating bank runs is a substantial 1.7 percent in permanent consumption equivalent units.

We also want to investigate to what extent the effects of bank runs are driven by pure anticipation effects as opposed to by actual runs occurring. For that purpose, we simulate the model with anticipated bank runs, but without any realized bank runs. For this simulation, we use the solution of the model with anticipated bank runs, but set the probability that a sunspot occurs in the simulation to 0. The results are reported in column 3 of Table 2.

Comparing columns 1 and 3, we can see that most of the effects of bank runs seem to stem from bank run fears, since the economies look very similar with or without realized bank runs. In particular, shadow bank leverage is even lower without realized bank runs. Eliminating bank run fears leads to a welfare gain of 1.4 percent in permanent consumption equivalent units or 80 percent of the total welfare gain from eliminating bank runs.

6.2 Retail Bank Capital Requirements with Spillover Effects

We consider a retail bank capital requirement that is only imposed if the economy is in the no-run state, i.e. $\tau^R_{NoRun} = 0.5$, $\tau^R_{Run} = 0$. Such a requirement has the advantage that the regulator can impose higher equity buffers of retail banks during normal times, which can be used to absorb the liquidated capital from shadow banks during a run, thereby pushing up the liquidation price of capital. In this sense, the more access to deposits retail banks have during a banking crisis, the higher the fire sale price of the capital will be ex post, and the less likely bank runs would happen ex ante. Therefore, the optimal capital requirement in face of a bank run is its lower bound, i.e. zero. In what follows we focus on this specific run-contingent capital requirement.

	With Runs		No Runs	
	Baseline	$\tau^R = 0.5$	Baseline	$\tau^R = 0.5$
Macroeconomic Aggregates				
Mean, Output (Y)	1.088	1.082	1.114	1.101
Mean, Consumption (C^H)	0.850	0.848	0.865	0.860
Mean, Investment ($\tilde{I}$)	0.238	0.234	0.248	0.240
St. Dev., Output (Y)	3.185	3.204	3.279	3.302
St. Dev., Consumption (C^H)	2.174	2.245	2.203	2.249
St. Dev., Investment ($\tilde{I}$)	7.395	7.387	7.613	7.652
Financial Sector				
Mean, Retail Bank Leverage (ϕ^R)	10.291	8.057	10.019	7.571
Mean, Shadow Bank Leverage (ϕ^S)	13.444	14.847	19.993	20.820
Asset Prices				
Mean, Spread, Wholesale ($R^{B'} - R^{D'}$)	1.119	1.335	0.756	1.146
Mean, Spread, Retail ($E[R^{K'}/Q] - R^{D'}$)	2.990	3.195	2.504	2.877
Bank Runs				
Runs per 100 Years	3.096	2.899	0.000	0.000
Recovery Rate ($x_t \vert Run_t$)	78.212	78.725	-	-
Welfare	0.850	0.848	0.865	0.860

Table 3: The macroeconomic effects of tighter retail bank capital requirements. Results are from a simulation of 10000 economies for 2000 periods, discarding the first 1000 periods. *With Runs* is the baseline model with anticipated bank runs. *No Runs* is a version of the model in which bank runs neither happen nor are expected to happen.

Columns 1 and 2 of Table 3 report the results. For $\tau^R = 0.5$, retail bank leverage decreases on average by about 21 percent from 10 to about 8. This is because the increase in τ^R_{NoRun} is partially offset by an increase in asset returns, which increases the leverage capacity of retail banks. In terms of looking at the macroeconomic aggregates, a retail bank capital requirement is not a good policy: It reduces output on average by tightening the constraints of the financial intermediaries and it increases the volatility of output.

This is because while retail bank leverage is lower with the regulatory policy, shadow bank leverage increases substantially. This counteracts the effectiveness of the retail bank capital requirement in reducing the frequency and severity of financial crises: While we can see that the frequency of bank runs decreases somewhat and the recovery rate of creditors increases, the overall effect is not very large.

To understand how much of the spillover is due to the relaxation of the shadow bank incentive constraint due to the lower bank run risk, and how much is due to the portfolio reallocation of retail banks, we consider the effects of the same bank capital requirement in an economy in which there are no bank runs. Hence, in that economy, the first spillover channel does not exist. Columns 3 and 4 of Table 3 report the results.

In welfare terms, higher retail bank capital requirements reduce welfare despite reducing the frequency of bank runs. This is since capital requirements reduce the steady state capital stock, which lowers the steady state level of consumption.

6.3 Retail Bank Capital Requirements without Spillover Effects

In the last section, we established that retail bank capital requirements lead to a spillover effect on shadow bank leverage, which reduces the effectiveness of the policy. In this section, we aim to quantify by how much this spillover reduces the effect of the policy quantitatively. For this purpose, we compare two counterfactual policies: The first one corresponds to the retail bank capital requirement in the last section. We call this counterfactual *regulation with spillover*. We report the results from simulations of this model in Table 4, column 2. In the second counterfactual, the regulator imposes the same capital requirement on retail banks and levies an additional capital requirement on shadow banks such that their leverage corresponds on average to their leverage in the baseline model. We call this the *regulation without spillover*. The results are reported in Table 4, column 3. For comparison, we report results for the baseline model without regulation in Table 4, column 1.

We can see that the regulation without spillover is more than twice as effective in reducing the frequency and severity of bank runs than the regulation with spillover: The frequency of bank runs decreases by about 0.5 bank runs

	With Runs		
	Baseline	Regulation W Spillover	Regulation W/O Spillover
Macroeconomic Aggregates			
Mean, Output (Y)	1.088	1.082	1.079
Mean, Consumption (C^H)	0.850	0.848	0.846
Mean, Investment ($\tilde{I}$)	0.238	0.234	0.232
St. Dev., Output (Y)	3.184	3.202	3.179
St. Dev., Consumption (C^H)	2.172	2.243	2.227
St. Dev., Investment ($\tilde{I}$)	7.398	7.387	7.351
Financial Sector			
Mean, Retail Bank Leverage (ϕ^R)	10.291	8.057	8.033
Mean, Shadow Bank Leverage (ϕ^S)	13.444	14.847	13.436
Asset Prices			
Mean, Spread, Wholesale ($R^{B'} - R^{D'}$)	1.119	1.335	1.280
Mean, Spread, Retail ($E[R^{K'}/Q] - R^{D'}$)	2.990	3.196	3.270
Bank Runs			
Runs per 100 Years	3.105	2.909	2.630
Recovery Rate ($x_t\|Run_t$)	78.213	78.728	79.427
Welfare	0.850	0.848	0.846

Table 4: Correcting for the spillover effect. Results are from a simulation of 10000 economies for 2000 periods, discarding the first 1000 periods. *Baseline* is the baseline model without regulation and anticipated bank runs. *Regulation with spillover* is a version of the model in which retail banks are regulated, with $\tau^R_{NoRun} = 0.5$, but shadow banks are not. *Regulation without spillover* is a version of the model, in which both retail banks and shadow banks are regulated, with $\tau^R_{NoRun} = 0.5$ and $\tau^S \approx 0.11$

per 100 years in the case without the spillover effect as opposed to by only 0.2 runs per 100 years in the case with the spillover effect. Recovery rates increase by over 1.3 percentage points as opposed to only 0.5 percentage points. This shows that the spillover effect, which arises mostly due to the relaxation of the leverage constraint of shadow banks in response to safer retail banks, has substantial implications for the effectiveness of macroprudential retail bank capital requirements.

Looking at macroeconomic aggregates, we can see that a retail bank capital requirement without the spillover effect reduces output volatility somewhat, as opposed to increasing it. It does however also reduce average output, consumption and investment more than the policy which allows for the spillover effect. This also explains why the policy overall leads to an additional welfare loss.

7 Conclusion

We study the macroeconomic effects of bank capital regulation in a quantitative model with retail banks and shadow banks. In our model, financial crises occur in the form of runs on shadow banks. There is a role for regulation in the model because banks do not internalize that their decisions affect the likelihood of financial crises, which leads to over-borrowing during normal times.

Overall, we have established that the welfare gains from policies which eliminate shadow bank runs are potentially very large. Most of the welfare gains stem from eliminating fears about future shadow bank runs, which relaxes the leverage constraint of shadow banks. Hence, they can invest more, which leads to a higher level of output, consumption and investment.

Retail bank capital requirements can reduce the frequency and severity of shadow bank runs by allowing retail banks to better absorb the liquidated assets of shadow banks in a bank run. However, retail bank capital requirements lead to substantial spillover effects, especially if fears about future constraints are present. Eliminating these spillover effects has considerable scope for increasing the effectiveness of retail bank capital regulation.

An interesting extension of our model would be to include sticky prices and nominal debt. A bank run could then result in a Fisherian debt deflation spiral: The initial effects of the run depresses goods prices, which worsens the real debt burden of banks, which in term depresses investment, and so on. Bank runs can then lead to episodes that cause the economy to be at the lower bound of the nominal policy interest rate. In this case, the possibility of bank runs will also affect how monetary policy should be conducted.

References

Acharya, V. V., P. Schnabl, and G. Suarez (2013, mar). Securitization Without Risk Transfer. *Journal of Financial Economics 107*(3), 515–536.

Admati, A. and M. Hellwig (2014). *The Bankers' New Clothes: What's Wrong with Banking and What to Do about It.* Princeton University Press.

Adrian, T. and A. B. Ashcraft (2012a). Shadow Banking: A Review of the Literature. *Palgrave Dictionary of Economics 580.*

Adrian, T. and A. B. Ashcraft (2012b). Shadow Banking Regulation. *Annual Review of Financial Economics 4*, 99–140.

Adrian, T. and H. S. Shin (2011). Chapter 12 - Financial Intermediaries and Monetary Economics. In *Handbook of Monetary Economics*, Volume 3, Chapter 12, pp. 601–650. Elsevier Ltd.

Angeloni, I. and E. Faia (2013, apr). Capital Regulation and Monetary Policy with Fragile Banks. *Journal of Monetary Economics 60*(3), 311–324.

Baron, M., E. Verner, and W. Xiong (2018). Bank Equity and Banking Crises. *Unpublished Manuscript.*

Begenau, J. (2016). Capital Requirements, Risk Choice, and Liquidity Provision in a Business Cycle Model. *Working Paper, Harvard.*

Begenau, J. and T. Landvoigt (2018). Financial Regulation in a Quantitative Model of The Modern Banking System. *SSRN Electronic Journal.*

Bernanke, B. S. and M. Gertler (1989). Agency Costs, Net Worth, and Business Fluctuations. *American Economic Review 79*(1), 14–31.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The Financial Accelerator in a Quantitative Business Cycle Framework. In *Handbook of Macroeconomics*, Volume 1.

Bianchi, J. (2011). Overborrowing and Systemic Externalities in the Business Cycle. *American Economic Review 101*(7), 3400–3426.

Bianchi, J. and E. G. Mendoza (2018). Optimal Time-Consistent Macroprudential Policy. *Journal of Political Economy 126*(2), 588–634.

Boissay, F., F. Collard, and F. Smets (2016). Booms and Banking Crises. *Journal of Political Economy 124*(2), 489–538.

Brunnermeier, M. K. and Y. Sannikov (2014). A Macroeconomic Model with a Financial Sector. *American Economic Review 104*(2), 379–421.

Chretien, E. and V. Lyonnet (2017). Traditional and Shadow Banks During the Crisis. *SSRN Electronic Journal*.

Cole, H. L. and T. J. Kehoe (2000). Self-Fulfilling Debt Crises. *Review of Economic Studies 67*(1), 91–116.

Covitz, D., N. Liang, and G. A. Suarez (2013, jun). The Evolution of a Financial Crisis: Collapse of the Asset-Backed Commercial Paper Market. *The Journal of Finance 68*(3), 815–848.

Dávila, E. and A. Korinek (2017). Pecuniary Externalities in Economies with Financial Frictions. *Review of Economic Studies 84*(4), 1869.

Di Tella, S. (2016). Optimal Regulation of Financial Intermediaries. *American Economic Review (Forthcoming)*.

Diamond, D. W. and P. H. Dybvig (1983). Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy 91*(3), 401–419.

Duca, J. V. (2016, aug). How Capital Regulation and Other Factors Drive the Role of Shadow Banking in Funding Short-term Business Credit. *Journal of Banking & Finance 69*, S10–S24.

Farhi, E. and J. Tirole (2017). Shadow Banking and the Four Pillars of Traditional Financial Intermediation. *Unpublished Manuscript*.

Ferrante, F. (2018). A Model of Endogenous Loan Quality and the Collapse of the Shadow Banking System. *American Economic Journal: Macroeconomics 10*(4), 152–201.

Gennaioli, N., A. Shleifer, and R. W. Vishny (2013, aug). A Model of Shadow Banking. *The Journal of Finance 68*(4), 1331–1363.

Gertler, M. and S. Gilchrist (2018). What Happened: Financial Factors in the Great Recession. *Journal of Economic Perspectives 32*(3), 3–30.

Gertler, M. and P. Karadi (2011). A Model of Unconventional Monetary Policy. *Journal of Monetary Economics 58*, 17–34.

Gertler, M. and N. Kiyotaki (2015). Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy. *American Economic Review 105*(7), 2011–2043.

Gertler, M., N. Kiyotaki, and A. Prestipino (2016). Wholesale Banking and Bank Runs in Macroeconomic Modeling of Financial Crises. In J. B. Taylor and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Chapter 16, pp. 1345–1425.

Gertler, M., N. Kiyotaki, and A. Prestipino (2017). A Macroeconomic Model with Financial Panics. *Unpublished Manuscript*.

Gorton, G. (1988). Banking Panics and Business Cycles. *Oxford Economic Papers 40*(40), 751–781.

Gorton, G. and A. Metrick (2012). Securitized Banking and the Run on Repo. *Journal of Financial Economics 104*(3), 425–451.

Gorton, G. and A. Winton (2017, feb). Liquidity Provision, Bank Capital, and the Macroeconomy. *Journal of Money, Credit and Banking 49*(1), 5–37.

He, Z. and A. Krishnamurthy (2012). A Model of Capital and Crises. *Review of Economic Studies 79*(2), 735–777.

Huang, J. (2018). Banking and Shadow Banking. *Journal of Economic Theory 178*, 124–152.

Judd, K. L., L. Maliar, S. Maliar, and R. Valero (2014). Smolyak method for solving dynamic economic models: Lagrange interpolation, anisotropic grid and adaptive domain. *Journal of Economic Dynamics and Control 44*, 92–123.

Kiyotaki, N. and J. Moore (1997). Credit Cycles. *Journal of Political Economy 105*(2), 211–248.

Laeven, L. and F. Valencia (2012). Systemic Banking Crises Database. *International Monetary Fund 61*(2), 225–270.

Lorenzoni, G. (2008, jul). Inefficient Credit Booms. *Review of Economic Studies 75*(3), 809–833.

Luck, S. and P. Schempp (2016). Banks, Shadow Banking, and Fragility. *Unpublished Manuscript*.

Martin, A., D. Skeie, and E.-L. von Thadden (2014). Repo Runs. *Review of Financial Studies 27*(4), 957–989.

Meeks, R., B. Nelson, and P. Alessandri (2017). Shadow Banks and Macroeconomic Instability. *Journal of Money, Credit and Banking 49*(7), 1483–1516.

Mendoza, E. G. (2010, dec). Sudden Stops, Financial Crises, and Leverage. *American Economic Review 100*(5), 1941–1966.

Moreira, A. and A. Savov (2017). The Macroeconomics of Shadow Banking. *The Journal of Finance 72*(6), 2381–2432.

Ordoñez, G. (2018). Sustainable Shadow Banking. *American Economic Journal: Macroeconomics 10*(1), 33–56.

Paul, P. (2018). A Macroeconomic Model with Occasional Financial Crises. *Federal Reserve Bank of San Francisco Working Paper*.

Plantin, G. (2015). Shadow Banking and Bank Capital Regulation. *The Review of Financial Studies 28*(1), 146–175.

Rochet, J.-C. and X. Vives (2004). Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All? *Journal of the European Economic Association 128*, 1116–1147.

Appendix

For Online Publication

A Steady State

We focus on the case of the model under a binding capital requirement. Our approach is to first characterize the steady state allocation of capital for a given aggregate capital stock K. We then explain how the aggregate capital stock is determined. Given K, we can compute the gross return on capital and the wage as

$$R^K = \alpha K^{\alpha-1} + 1 - \delta \tag{A.1}$$

$$W = (1-\alpha) Z K^{\alpha} \tag{A.2}$$

Steady state interest rates are determined by the first order conditions with respect to D^H_{t+1}, and B^R_{t+1}:

$$R^D = \frac{1}{\beta} \tag{A.3}$$

$$R^B = \gamma \frac{R^K}{1+f^R} + (1-\gamma) R^D \tag{A.4}$$

Given R^K, K^H is determined by the euler equation of the household with respect to K^H

$$R^K = \frac{1}{\beta}\left(1 + \eta^H \frac{K^H}{K}\right) \tag{A.5}$$

$$\frac{K^H}{K} = \frac{1}{\eta^H}\left(\beta R^K - 1\right) \tag{A.6}$$

We can now characterize the steady state allocation for the shadow banks: First, from the balance sheet constraint of shadow banks follows

$$B = K^S - N^S \tag{A.7}$$

Plugging this into the law of motion for aggregate net worth, we can write net worth as

$$N^S = \frac{\upsilon^S K}{1 - \left[\left(R^K - R^B\right) \frac{K^S}{N^S} + R^B\right] (1 - \sigma^S)}. \tag{A.8}$$

From the incentive constraint, we then get a quadratic condition for K^S/N^S:

$$\begin{aligned} \psi \left[\omega \frac{K^S}{N^S} + (1-\omega)\right] &= \beta \left[\sigma^S + (1-\sigma^S)\psi \left[\omega \frac{K^S}{N^S} + (1-\omega)\right]\right] \frac{1}{1-\sigma^S} \left(1 - \upsilon^S \frac{K}{N^S}\right) \\ &= \beta \left[\sigma^S + (1-\sigma^S)\psi \left[\omega \frac{K^S}{N^S} + (1-\omega)\right]\right] \left(\left(R^K - R^B\right) \frac{K^S}{N^S} + R^B\right) \end{aligned} \tag{A.9}$$

We then can infer $K^S = K^S/N^S N^S$. The fraction of capital holdings of retail banks are given by the market clearing condition for capital goods:

$$\frac{K^R}{K} = 1 - \frac{K^H}{K} - \frac{K^S}{K}. \tag{A.10}$$

From the balance sheet of the retail banking sector, we get

$$D = \left(1 + \eta^R \frac{K^R}{K}\right) K^R + B - N^R. \tag{A.11}$$

This allows us to substitute out D in the law of motion for aggregate net worth:

$$\begin{aligned} N^R &= (R^K K^R + R^B B - R^D D)(1-\sigma^R) + \upsilon^R K \\ &= \left(R^K K^R + R^B B - R^D \left(\left(1 + \eta^R \frac{K^R}{K}\right) K^R + B - N^R\right)\right)(1-\sigma^R) + \upsilon^R K \\ &= \left(\left(R^K - \left(1 + \eta^R \frac{K^R}{K}\right) R^D\right) \frac{K^R}{N^R} + \left(R^B - R^D\right) \frac{B}{N^R} + R^D\right) N^R (1-\sigma^R) + \upsilon^R K \end{aligned}$$

Hence,

$$N^R = \frac{\upsilon^R K}{1-(1-\sigma^R)\left(\left(R^K-\left(1+\eta^R\frac{K^R}{K}\right)R^D\right)\frac{K^R}{N^R}+\left(R^B-R^D\right)\frac{B}{N^R}+R^D\right)}. \tag{A.12}$$

Finally, from the capital requirement, we get

$$\left(1+\eta^R\frac{K^R}{K}\right)K^R+\gamma B = \bar{\phi}\frac{\upsilon^R K}{1-(1-\sigma^R)\left(\left(R^K-\left(1+\eta^R\frac{K^R}{K}\right)R^D\right)\frac{K^R}{N^R}+\left(R^B-R^D\right)\frac{B}{N^R}+R^D\right)} \tag{A.13}$$

Substituting in the solutions for B from equation A.7, N^R from equation A.12 and K^R/K, from equation A.10 this is a complicated nonlinear equation in K only.

Some additional variables of interest can be calculated residually. Total output is given by:

$$Y = ZK^\alpha + \upsilon^R K + \upsilon^S K - \eta^H\left(\frac{K^H}{K}\right)^2 K - \eta^R\left(\frac{K^R}{K}\right)^2 K. \tag{A.14}$$

Then, household consumption is characterized by the aggregate budget constraint:

$$C^H = Y - \delta K - \sigma^R\frac{N^R-\upsilon^R K}{1-\sigma^R} - \sigma^S\frac{N^S-\upsilon^S K}{1-\sigma^S}. \tag{A.15}$$

A.1 Comparative Statics in Steady State

In this section, we explore the steady state implications of retail bank capital requirements. We characterize the non-stochastic steady state equilibrium absent of bank runs in Appendix A.

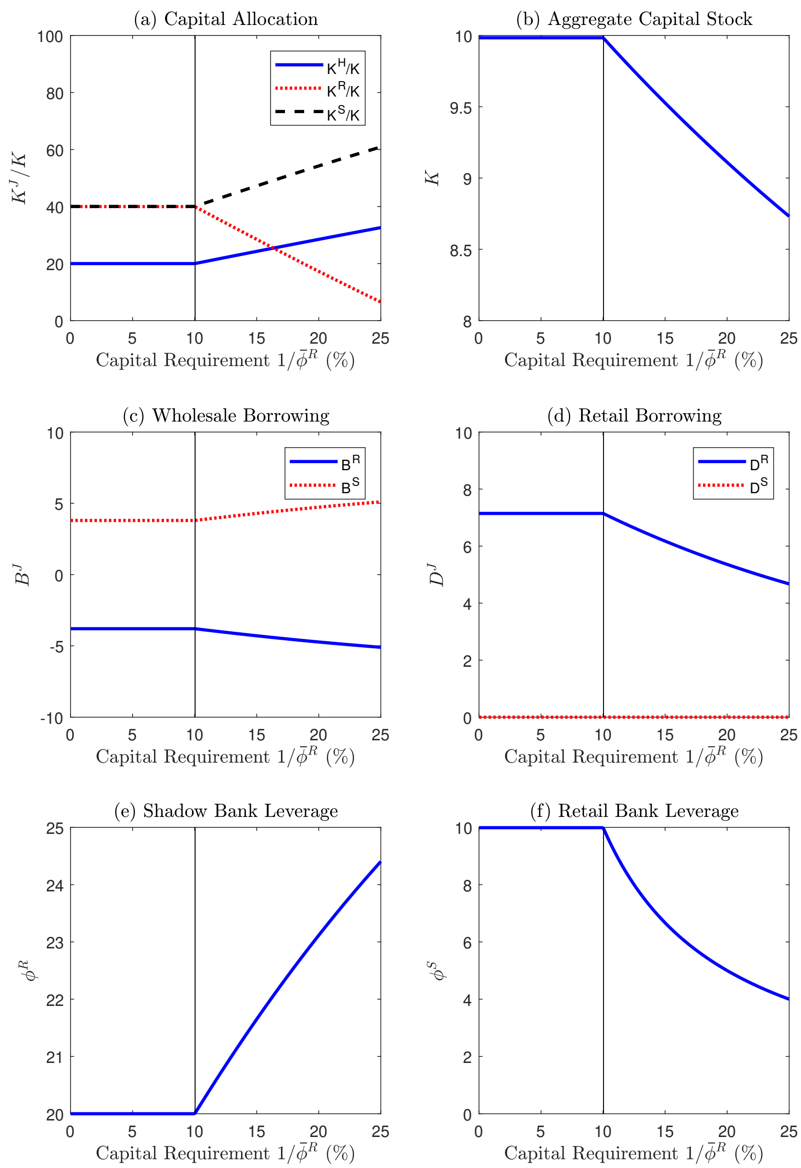

Figure 6: Steady state effect of a retail bank capital requirement on the capital allocation and the aggregate capital stock.

A.1.1 Retail Bank Capital Requirements

Figure 6 shows the comparative statics for varying the retail bank capital requirement in steady state. We vary the minimum capital requirement between 0 and 25 percent.

In panel (a), we show how increasing retail bank capital requirements changes the capital allocation among households and banks. The solid line is the share of capital held by households, the dotted line is the share of retail bank capital holdings and the dashed line is the share of shadow bank capital holdings. For a retail bank capital requirement below 10 percent, the capital requirement is not binding and the retail bank leverage ratio is determined by its incentive constraint.

For a retail bank capital requirement between 10 and 25 percent, the capital requirement is binding and retail banks will invest in both capital and wholesale lending. As the capital requirement increases in this range, retail banks substitute away from capital lending to wholesale lending. This is because an additional unit of capital lending requires $1/\bar{\phi}$ of equity finance, while an additional unit of wholesale lending only requires $\gamma/\bar{\phi}$ of equity finance. If the regulator tightens the retail bank capital requirement, wholesale lending will therefore become relatively more attractive for retail banks. Hence, direct capital holdings by retail banks decrease, and capital holdings by shadow banks increase in this range.

For a capital requirement above 25 percent, retail banks will only invest through wholesale lending. If the regulator increases the capital requirement in this range, retail banks can no longer substitute away from capital holding and therefore can only reduce wholesale lending. Consequently, both retail and shadow banks will reduce their assets in this range.

In panel (b), we show the aggregate capital stock as a function of the capital requirement. Increasing retail bank capital requirements decreases the steady state capital stock substantially. The reason for this strong effect is that banks in this economy cannot raise outside equity from households. Hence, a higher capital requirement forces retail banks to sharply cut the asset side of their balance sheet, which in turn forces the shadow banks to reduce their assets as well. If banks could issue equity to households, their required return on equity would not increase monotonically with a higher capital requirement, which would imply a lower bound on the aggregate capital stock.

In panels (c) and (d), we consider the effects of capital requirements on the deposit and wholesale funding markets. Higher retail bank capital requirements increase the wholesale funding markets, as retail banks substitute away from direct lending to the non-financial sector towards wholesale lending. Deposit borrowing decreases, as higher capital requirements imply that retail banks need to reduce leverage.

In panels (e) and (f), we show the effects of capital requirements on shadow and retail bank leverage. There is a spill-over effect from higher retail bank capital requirements on shadow bank leverage: As retail banks reduce their assets, returns on assets increase, which increases shadow bank profitability and allows them to take on more leverage. Retail bank leverage decreases with higher retail bank capital requirements by construction.

A.1.2 Shadow Bank Capital Requirements

In Figure 7, we show the comparative statics for the steady state for different levels of the shadow bank capital requirement. We again vary the shadow bank capital requirement between zero and 25 percent.

As we show in panel (a), an increase in shadow bank capital requirements reduces the fraction of capital held by shadow banks in steady state, and increases the fraction of capital held by the retail banks. Households also hold a larger fraction of the capital stock.

Nevertheless, the aggregate capital stock decreases, as we see in panel (b). The reason is that shadow banks have to reduce leverage as they have to comply with the higher capital requirement. This in turn forces them to sell capital, which will only partially be absorbed by households and retail banks, who are less efficient at holding capital.

Panels (c) and (d) show that in stark contrast to retail bank capital requirements, shadow bank capital requirements reduce the size of the wholesale funding market and the retail funding market. The former market decreases

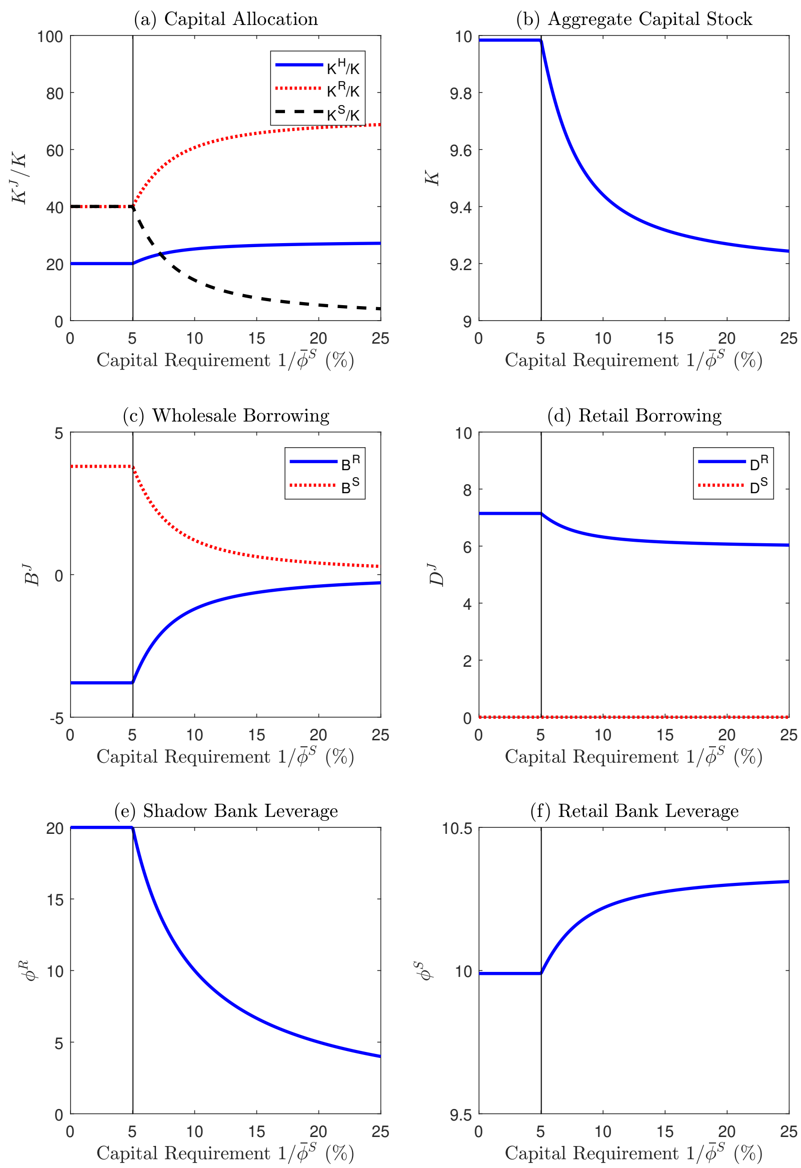

Figure 7: Steady state effect of a shadow bank capital requirement on the capital allocation and the aggregate capital stock.

directly, because shadow banks have to reduce leverage, The latter market becomes smaller, since retail banks substitute away from wholesale lending to direct lending, which can be more easily diverted and hence requires more equity financing.

By construction, shadow bank leverage decreases with a higher capital requirement, as we show in panel (e). In panel (f), we can see that retail bank leverage increases with a higher capital requirement. There are two effects at work: First, as the capital stock decreases, the return on capital goes up, which increases the profitability and hence the leverage capacity of retail banks. Second, the substitution away from wholesale lending towards direct lending increases retail bank leverage directly, since direct lending has a higher weight in the calculation of retail bank leverage.

B Equilibrium Conditions in the Dynamic Model

B.1 Households

The first-order conditions of the households' problem with respect to capital holding K^H_{t+1} and deposit D^H_{t+1} are given by:

$$FOC(K^H_{t+1}): \quad \frac{1}{C^H_t}(Q_t + \eta^H \frac{K^H_{t+1}}{K_t}) = \beta \mathbb{E}_t \left(\frac{1}{C^H_{t+1}} R^K_{t+1} \right) \tag{B.1}$$

$$FOC(D^H_{t+1}): \quad \frac{1}{C^H_t} = \beta \mathbb{E}_t \left(\frac{1}{C^H_{t+1}} R^D_{t+1} \right) \tag{B.2}$$

The interpretation of these first-order conditions is standard. In the first expression, the left-hand side and the right-hand side are the marginal cost and marginal benefit of capital holding, respectively. The marginal cost of capital holding has two components. One is the price the households have to pay for purchasing the capital goods, and the second is the capital holding cost due to households' low investment skills.

In addition, the households decide how much capital to hold through the retail banking sector. The first order condition with respect to K^R_{t+1} yields a first order condition which pins down f^R_t:

$$f^R_t = \eta^R \left(\frac{K^R_{t+1}}{K_t} \right).$$

Aggregate consumption of the household sector can be inferred from the resource constraint of the economy. Therefore, we do not have to track the net worth of households as a state variable.

$$\begin{aligned} C^H_t &= Z_t K^\alpha_t + \upsilon^R K_t + \upsilon^S K_t - \frac{\eta^H}{2} \left(\frac{K^H_{t+1}}{K_t} \right)^2 K_t - \frac{\eta^R}{2} \left(\frac{K^R_{t+1}}{K_t} \right)^2 K_t \\ &- I_t - \frac{\theta}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t - \sigma^R \frac{N^R_t - \upsilon^R K_t}{1 - \sigma^R} - \sigma^S \frac{N^S_t - \upsilon^S K_t}{1 - \sigma^S} \end{aligned}$$

B.2 Banks

B.2.1 Shadow banks

The incentive constraint of the shadow bank is given by

$$\psi(n^S_t + \omega b^S_{t+1}) = \beta \mathbb{E}_t \left[V^S_{t+1} \right]. \tag{B.3}$$

The balance sheet constraint of the shadow bank reads

$$Q_t k^S_{t+1} = n^S_t + b^S_{t+1} \tag{B.4}$$

The net worth of an incumbent shadow bank is

$$n^S_t = R^K_t k^S_t - R^B_t b^S_t \tag{B.5}$$

The value of the shadow bank before the realization of the exit shock is given by

$$\begin{aligned} V^S_t &= \sigma^S n^S_t + (1 - \sigma^S) \beta \mathbb{E}_t \left[V^S_{t+1} \right] \\ &= \sigma^S n^S_t + (1 - \sigma^S) \psi(n^S_t + \omega b^S_{t+1}), \end{aligned}$$

where the second line uses the binding incentive constraint to substitute out the continuation value. Plugging this expression into B.3 yields the following characterization for the shadow banks choices for k^S_{t+1} and b^S_{t+1}:

$$\psi(n^S_t + \omega b^S_{t+1}) = \beta \mathbb{E}_t \left[(\sigma^S + (1-\sigma^S)\psi) n^S_{t+1} + \psi\omega(1-\sigma^S) b^S_{t+2} \right] \tag{B.6}$$

$$Q_t k^S_{t+1} = n^S_t + b^S_{t+1} \tag{B.7}$$

$$n^S_t = R^K_t k^S_t - R^B_t b^S_t \tag{B.8}$$

We now conjecture and verify that the policy functions for b^S_{t+1} and k^S_{t+1} are linear in net worth, such that it is sufficient to characterize the optimal choices of the shadow banking sector as a whole in equilibrium.

Theorem B.1 (Linearity of Policy Functions)**.** *The policy functions for b^S_{t+1} and k^S_{t+1} which solve the problem of the shadow bank given by equations B.6 to B.8 are linear in net worth.*

Proof. Suppose that the policy functions are given by $b^S_{t+1} = A^S_b n^S_t$ and $k^S_{t+1} = A^S_k n^S_t$, respectively. Then, it follows from equation B.8 that

$$\begin{aligned} n^S_{t+1} &= R^K_{t+1} k^S_{t+1} - R^B_{t+1} b^S_{t+1} \\ &= (R^K_{t+1} A^S_k - R^B_{t+1} A^S_b) n^S_t \\ &= A^S_n n^S_t. \end{aligned}$$

From equation B.7 follows that

$$\begin{aligned} Q_t A^S_k n^S_t &= n^S_t + A^S_b n^S_t \\ A^S_k &= \frac{1 + A^S_b}{Q_t}. \end{aligned}$$

Finally, from B.6 follows that

$$\begin{aligned}\psi(1+\omega A_b^S)n_t^S &= \beta\mathbb{E}_t\left[(\sigma^S+(1-\sigma^S)\psi(1+\omega A_b^S))n_{t+1}^S\right]\\ &= \beta\mathbb{E}_t\left[(\sigma^S+(1-\sigma^S)\psi(1+\omega A_b^S))A_n^S\right]n_t^S\\ &= \beta\mathbb{E}_t\left[(\sigma^S+(1-\sigma^S)\psi(1+\omega A_b^S))(R_{t+1}^K\frac{1+A_b^S}{Q_t}-R_{t+1}^B A_b^S)\right]n_t^S\end{aligned}$$

This equation yields a solution for A_b^S that is independent of n_t^S.[23] Consequently, A_k^S and A_n^S are also independent of n_t^S. □

Given the linearity of policy functions, it is sufficient to characterize the policies K_{t+1}^S and B_{t+1}^S of the aggregate shadow banking sector. These choices are the solutions to

$$\begin{aligned}\psi(N_t^S+\omega B_{t+1}^S) &= \beta\mathbb{E}_t\left[(\sigma^S+(1-\sigma^S)\psi)\frac{N_{t+1}^S-\upsilon^S K_{t+1}^S}{1-\sigma^S}+\psi\omega(1-\sigma^S)B_{t+2}^S\right]\\ Q_t K_{t+1}^S &= N_t^S+B_{t+1}^S\\ N_t^S &= (R_t^K K_t^S-R_t^B B_t^S)(1-\sigma^S)+\upsilon^R K_t^S.\end{aligned}$$

B.2.2 Retail banks, No Regulation

We characterize the problem of a retail banks under a non-binding and a binding capital requirement. First, we consider the problem of a retail bank where the incentive constraint is binding. The incentive constraint is given by

$$\psi((Q_t+f_t^R)k_{t+1}^R+\gamma b_{t+1}^R)=\beta\mathbb{E}_t\left[V_{t+1}^R\right]. \tag{B.9}$$

The balance sheet constraint reads:

$$(Q_t+f_t^R)k_{t+1}^R+b_{t+1}^R=n_t^R+d_t^R. \tag{B.10}$$

---

[23] Specifically, the solution is given by $A_b^S = -p+\sqrt{p^2+q}$, with $p = -\frac{1}{2}\frac{(1/\beta-\mathbb{E}_t[R_{t+1}^K/Q_t])-\mathbb{E}_t[R_{t+1}^K/Q_t-R_{t+1}^B]}{\omega\mathbb{E}_t[R_{t+1}^K/Q_t-R_{t+1}^B]}$ and $q = \frac{1/\beta-\mathbb{E}_t[(\sigma^S+(1-\sigma^S)\psi)R_{t+1}^K/Q_t]}{\omega\mathbb{E}_t[R_{t+1}^K/Q_t-R_{t+1}^B]}$. When $\mathbb{E}_t\left[R_{t+1}^K/Q_t-R_{t+1}^B\right]>0$ and $\mathbb{E}_t\left[R_{t+1}^B-1/\beta\right]>0$, this solution is unique.

Net worth is determined according to

$$n_t^R = R_t^K k_t^R + R_t^B b_t^R - R_t^D d_t^R. \tag{B.11}$$

These three equations pin down k_{t+1}^R, d_{t+1}^R and n_t^R. b_{t+1}^R is determined by a first order condition of the retail banks problem:

$$\begin{aligned}
&\max_{\left\{k_{s+1}^R, b_{s+1}^R, d_{s+1}^R\right\}_{s=t}^{\infty}} \beta \mathbb{E}_t \left[V_{t+1}^R\right] \\
&\text{s.t.} \\
&V_t^R = \sigma^R n_t^R + (1-\sigma^R)\psi((Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R) \\
&n_t^R + d_{t+1}^R = (Q_t + f_t^R)k_{t+1}^R + b_{t+1}^R \\
&\psi((Q_t + f_t^R)k_{t+1}^R + \gamma b_{t+1}^R) = \beta \mathbb{E}_t \left[V_{t+1}^R\right] \\
&n_t^R = R_t^K k_t^R + R_t^B b_t^R - R_t^D d_t^R \\
&k_{t+1}^R, d_{t+1}^R, b_{t+1}^R \geq 0.
\end{aligned}$$

We conjecture, as in the shadow banking problem, that the policy functions for k_{t+1}^R, b_{t+1}^R and d_{t+1}^R are linear in net worth n_t^R:

$$\begin{aligned}
k_{t+1}^R &= A_k^R n_t^R \\
b_{t+1}^R &= A_b^R n_t^R \\
d_{t+1}^R &= A_d^R n_t^R
\end{aligned}$$

Plugging in the conjectured policy functions, we can rewrite the maximization problem as

$$
\begin{aligned}
&\max_{\left\{k^R_{s+1}, b^R_{s+1}\right\}_{s=t}^{\infty}} \beta \mathbb{E}_t \left[V^R_{t+1}\right] \\
&\text{s.t.} \\
&V^R_{t+1} = (\sigma^R + (1-\sigma^R)\psi((Q_{t+1} + f^R_{t+1})A^R_k + \gamma A^R_b))n^R_{t+1} \\
&\psi((Q_t + f^R_t)k^R_{t+1} + \gamma b^R_{t+1}) = \beta \mathbb{E}_t \left[V^R_{t+1}\right] \\
&n^R_{t+1} = R^K_{t+1}k^R_{t+1} + R^B_{t+1}b^R_{t+1} - R^D_{t+1}((Q_t + f^R_t)k^R_{t+1} + b^R_{t+1} - n^R_t) \\
&k^R_{t+1}, d^R_{t+1}, b^R_{t+1} \geq 0.
\end{aligned}
$$

Defining $\Omega^R_{t+1} \equiv V^R_{t+1}/n^R_{t+1} = (\sigma^R + (1-\sigma^R)\psi((Q_{t+1} + f^R_{t+1})A^R_k + \gamma A^R_b))$, the Lagrangian for this problem is given by

$$
\begin{aligned}
\mathcal{L} = {} & \beta \mathbb{E}_t \left[\Omega^R_{t+1}(R^K_{t+1}k^R_{t+1} + R^B_{t+1}b^R_{t+1} - R^D_{t+1}((Q_t + f^R_t)k^R_{t+1} + b^R_{t+1} - n^R_t))\right] \\
& + \lambda \left[\psi((Q_t + f^R_t)k^R_{t+1} + \gamma b^R_{t+1}) \right. \\
& \left. - \beta \mathbb{E}_t \left[\Omega^R_{t+1}(R^K_{t+1}k^R_{t+1} + R^B_{t+1}b^R_{t+1} - R^D_{t+1}((Q_t + f^R_t)k^R_{t+1} + b^R_{t+1} - n^R_t))\right]\right]
\end{aligned}
$$

This yields the following first order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial k^R_{t+1}} &= \beta \mathbb{E}_t \left[\Omega^R_{t+1}(R^K_{t+1} - R^D_{t+1}(Q_t + f^R_t))\right](1-\lambda) + \lambda \psi (Q_t + f^R_t) = 0 \\
\frac{\partial \mathcal{L}}{\partial b^R_{t+1}} &= \beta \mathbb{E}_t \left[\Omega^R_{t+1}(R^B_{t+1} - R^D_{t+1})\right](1-\lambda) + \lambda \psi \gamma = 0
\end{aligned}
$$

Combining these two equations and rearranging, we arrive at the condition

$$
\mathbb{E}_t \left[\Omega^R_{t+1}\left(\frac{R^K_{t+1}}{Q_t + f^R_t} - R^D_{t+1}\right)\right] = \frac{1}{\gamma}\mathbb{E}_t \left[\Omega^R_{t+1}(R^B_{t+1} - R^D_{t+1})\right].
$$

This is basically a condition that ensures that the retail bank is indifferent between lending a marginal unit of funds to final goods producers or on the wholesale funding market.

Showing the linearity of policy functions works in the same way as in the shadow bank problem. Then, in equilibrium, it is sufficient to characterize the choices K_{t+1}^R, B_{t+1}^R and D_{t+1}^R of the retail banking sector as a whole. These choices are characterized by the following system of equations:

$$\psi((Q_t + f_t^R)K_{t+1}^R + \gamma B_{t+1}^R) = \beta \mathbb{E}_t \left[\Omega_{t+1}^R \frac{N_{t+1}^R - \upsilon^R K_{t+1}^R}{1 - \sigma^R} \right]$$

$$(Q_t + f_t^R)K_{t+1}^R + B_{t+1}^R = N_t^R + D_t^R$$

$$\mathbb{E}_t \left[\Omega_{t+1}^R \left(\frac{R_{t+1}^K}{Q_t + f_t^R} - R_{t+1}^D \right) \right] = \frac{1}{\gamma} \mathbb{E}_t \left[\Omega_{t+1}^R (R_{t+1}^B - R_{t+1}^D) \right]$$

$$N_t^R = (R_t^K K_t^R + R_t^B B_t^R - R_t^D D_t^R)(1 - \sigma^R) + \upsilon^R K_t^R$$

B.3 Production

From the problem of the capital producer follows

$$Q_t = 1 + \theta \left(\frac{I_t}{K_t} - \delta \right).$$

The first order conditions of the final goods producer yield

$$r_t^K = \alpha Z_t K_t^{\alpha - 1}$$

$$W_t = (1 - \alpha) Z_t K_t^{\alpha}.$$

B.4 Full Statement of the Equilibrium Conditions

B.4.1 No Run Equilibrium

- Household:

$$
\begin{aligned}
\frac{1}{C_t^H}\left(Q_t+\eta^H\frac{K_{t+1}^H}{K_t}\right) &= \beta\mathbb{E}_t\left(\frac{1}{C_{t+1}^H}R_{t+1}^K\right)\\
\frac{1}{C_t^H} &= \beta\mathbb{E}_t\left(\frac{1}{C_{t+1}^H}R_{t+1}^D\right)\\
f_t^R &= \eta^R\frac{K_{t+1}^R}{K_t}\\
C_t^H &= Z_tK_t^\alpha+\upsilon^RK_t+\upsilon^SK_t-\frac{\eta^H}{2}\left(\frac{K_{t+1}^H}{K_t}\right)^2K_t-\frac{\eta^R}{2}\left(\frac{K_{t+1}^R}{K_t}\right)^2K_t\\
&\quad -I_t-\frac{\theta}{2}\left(\frac{I_t}{K_t}-\delta\right)^2K_t-\sigma^R\frac{N_t^R-\upsilon^RK_t}{1-\sigma^R}-\sigma^S\frac{N_t^S-\upsilon^SK_t}{1-\sigma^S}
\end{aligned}
$$

- Shadow Bank:

$$
\begin{aligned}
\psi(N_t^S+\omega B_{t+1}^S) &= \beta\mathbb{E}_t\left[(\sigma^S+(1-\sigma^S)\psi)\frac{N_{t+1}^S-\upsilon^SK_{t+1}^S}{1-\sigma^S}+\psi\omega(1-\sigma^S)B_{t+2}^S\right]\\
Q_tK_{t+1}^S &= N_t^S+B_{t+1}^S\\
N_t^S &= (R_t^KK_t^S-R_t^BB_t^S)(1-\sigma^S)+\upsilon^RK_t^S.
\end{aligned}
$$

- Retail Bank:

$$
\begin{aligned}
\bar{\phi}_tN_t^R &= (Q_t+f_t^R)K_{t+1}^R+\gamma B_{t+1}^R\\
(Q_t+f_t^R)K_{t+1}^R+B_{t+1}^R &= N_t^R+D_t^R\\
\mathbb{E}_t\left[\Omega_{t+1}^R\left(\frac{R_{t+1}^K}{Q_t+f_t^R}-R_{t+1}^D\right)\right] &= \frac{1}{\gamma}\mathbb{E}_t\left[\Omega_{t+1}^R(R_{t+1}^B-R_{t+1}^D)\right]\\
\Omega_t^R &= \left(\sigma^R+(1-\sigma^R)\psi\left((Q_t+f_t^R)\frac{K_{t+1}^R}{\tilde{N}_t^R}+\frac{B_{t+1}^R}{\tilde{N}_t^R}\right)\right)\\
N_t^R &= (R_t^KK_t^R+R_t^BB_t^R-R_t^DD_t^R)(1-\sigma^R)+\upsilon^RK_t^R\\
\tilde{N}_t^R &= \frac{N_t^R-\upsilon^RK_t^R}{1-\sigma^R}
\end{aligned}
$$

- Firms:

$$
\begin{aligned}
Q_t &= 1 + \theta \left(\frac{I_t}{K_t} - \delta \right) \\
R_t^K &= \alpha Z_t K_t^{\alpha-1} + (1-\delta) Q_t \\
W_t &= (1-\alpha) Z_t K_t^{\alpha}
\end{aligned}
$$

- Laws of Motion:

$$
\begin{aligned}
K_{t+1}^H + K_{t+1}^R + K_{t+1}^S &= (1-\delta) K_t + I_t \\
\ln(Z_t) &= (1-\rho^Z)\mu^Z + \rho_Z \ln(Z_{t-1}) + \epsilon_t
\end{aligned}
$$

B.4.2 Run Equilibrium

- Household:

$$
\begin{aligned}
\frac{1}{C_t^{H,*}} \left(Q_t^* + \eta^H \frac{K_{t+1}^{H,*}}{K_t} \right) &= \beta \mathbb{E}_t \left[(1-\pi) \frac{1}{C_{t+1}^H} R_{t+1}^K + \pi \frac{1}{C_{t+1}^{H,*}} R_{t+1}^{K,*} \right] \\
\frac{1}{C_t^{H,*}} &= \beta \mathbb{E}_t \left[(1-\pi) \frac{1}{C_{t+1}^H} R_{t+1}^D + \pi \frac{1}{C_{t+1}^{H,*}} R_{t+1}^D \right] \\
f_t^{R,*} &= \eta^R \frac{K_{t+1}^{R,*}}{K_t} \\
C_t^{H,*} &= Z_t K_t^{\alpha} + \upsilon^R K_t - \frac{\eta^H}{2} \left(\frac{K_{t+1}^{H,*}}{K_t} \right)^2 K_t - \frac{\eta^R}{2} \left(\frac{K_{t+1}^{R,*}}{K_t} \right)^2 K_t \\
&\quad - I_t^* - \frac{\theta}{2} \left(\frac{I_t^*}{K_t} - \delta \right)^2 K_t - \sigma^R \frac{N_t^{R,*} - \upsilon^R K_t}{1 - \sigma^R}
\end{aligned}
$$

- Shadow Bank:

$$
\begin{aligned}
C_t^{S,*} &= 0 \\
N_t^{S,*} &= 0 \\
B_{t+1}^{*} &= 0 \\
K_{t+1}^{S,*} &= 0
\end{aligned}
$$

- Retail Bank:

$$
\begin{aligned}
\bar{\phi}_t N_t^{R,*} &= (Q_t + f_t^R) K_{t+1}^{R,*} \\
(Q_t^* + f_t^{R,*}) K_{t+1}^{R,*} &= N_t^{R,*} + D_{t+1}^{R,*} \\
\Omega_t^R &= \left(\sigma^R + (1-\sigma^R) \psi \left((Q_t^* + f_t^{R,*}) \frac{K_{t+1}^{R,*}}{\tilde{N}_t^{R,*}} \right) \right) \\
N_t^{R,*} &= (R_t^{K,*} K_t^R + \xi_t R_t^{K,*} K_t^S - R_t^D D_t^R)(1-\sigma^R) + \upsilon^R K_t^R \\
\tilde{N}_t^{R,*} &= \frac{N_t^{R,*} - \upsilon^R K_t}{1-\sigma^R}
\end{aligned}
$$

- Firms:

$$
\begin{aligned}
Q_t^* &= 1 + \theta \left(\frac{I_t^*}{K_t} - \delta \right) \\
R_t^{K,*} &= \alpha Z_t K_t^{\alpha-1} + (1-\delta) Q_t^* \\
W_t &= (1-\alpha) Z_t K_t^{\alpha}
\end{aligned}
$$

- Laws of Motion:

$$
\begin{aligned}
K_{t+1}^{H,*} + K_{t+1}^{R,*} &= (1-\delta) K_t + I_t^* \\
\ln(Z_t) &= (1-\rho^Z)\mu^Z + \rho_Z \ln(Z_{t-1}) + \epsilon_t
\end{aligned}
$$

C Effects of Financial Frictions on the No-Bank-Run Equilibrium

Case I: Financially Unconstrained Retail and Shadow Banks Consider a situation in which neither retail banks nor shadow banks are subject to the moral hazard problem described above and have unlimited liability. In this case, both types of banks can finance themselves exclusively with debt. Moreover, from the perspective of the shadow banks, there is no distinction between deposits and wholesale loans. This is because, as spelled out in Assumption 3, liabilities are different only in the way in which they affect the incentive constraint of the banks. Hence, in equilibrium, shadow banks must be indifferent between deposits and wholesale loans, which implies that their interest rates must be the same:

$$R^B_{t+1} = R^D_{t+1}. \tag{C.1}$$

Moreover, banks will lend to consumption goods producers up to the point at which the marginal return on retail loans is equal to the marginal cost of borrowing:

$$\mathbb{E}_t\left[\Lambda_{t,t+1}\frac{R^A_{t+1}}{Q_t}\right] = \mathbb{E}_t\left[\Lambda_{t,t+1}\right]R^B_{t+1} = \mathbb{E}_t\left[\Lambda_{t,t+1}\right]R^D_{t+1}.$$

This further implies that retail banks or households should not make retail loans, due to Assumption 1. The model collapses to a model in which only fully deposit financed shadow banks make retail loans. It is observationally equivalent to a real business cycle model.

Case II: Financially Unconstrained Retail Banks, Financially Constrained Shadow Banks If retail banks are not subject to the moral hazard problem, but shadow banks are, shadow banks are no longer indifferent between deposit financing and wholesale financing. This is because wholesale financing requires shadow banks to use less (costly) net worth to finance one unit of retail loans. Further, shadow banks are now subject to default risk, which implies that x_t can be smaller than 1. Since shadow banks are constrained, they cannot lend up to the point where the return on their assets equals the return on their liabilities. Hence,

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \frac{R^A_{t+1}}{Q_t} \right] > \mathbb{E}_t \left[\Lambda_{t,t+1} x_{t+1} R^B_{t+1} \right].$$

This spread implies that it will be optimal for retail banks and households to make some retail loans. Moreover, retail banks must be indifferent between making retail loans and making wholesale loans. Hence

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \frac{R^A_{t+1}}{Q_t + f^R_t} \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} x_{t+1} R^B_{t+1} \right].$$

Retail banks will make loans up to the point where the return on deposits equals the return on retail or wholesale lending:

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \frac{R^A_{t+1}}{Q_t + f^R_t} \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} x_{t+1} R^B_{t+1} \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} \right] R^D_{t+1}.$$

In this model, both shadow banks and retail banks will make retail loans, and there will be an active wholesale lending market, where retail banks lend to shadow banks. There is a spread between the return on deposits and the return on wholesale loans which reflects the risk-adjusted probability of a shadow bank run:

$$R^B_{t+1} = \frac{\mathbb{E}_t \left[\Lambda_{t,t+1} \right]}{\underbrace{\mathbb{E}_t \left[\Lambda_{t,t+1} \underbrace{x_{t+1}}_{\leq 1} \right]}_{\geq 1}} R^D_{t+1}. \tag{C.2}$$

In principle, this model is qualitatively consistent with the flow of funds as shown in Figure 2. However, this model cannot deliver quantitatively important bank run risk, since unconstrained retail banks can easily and cheaply absorb the liquidated assets of shadow banks in a systemic bank default.

Case III: Financially Constrained Retail and Shadow Banks If both retail and shadow banks are constrained, there will be a spread between the return on retail loans issued by retail banks and the return on deposits, since retail banks cannot lever up to the point where returns equalize:

$$\mathbb{E}_t\left[\Lambda_{t,t+1}\frac{R^A_{t+1}}{Q_t+f^R_t}\right] > \mathbb{E}_t\left[\Lambda_{t,t+1}\right] R^D_{t+1},$$

and

$$\mathbb{E}_t\left[\Lambda_{t,t+1}x_{t+1}R^B_{t+1}\right] > \mathbb{E}_t\left[\Lambda_{t,t+1}\right] R^D_{t+1}.$$

Moreover, retail banks are willing to accept a lower return on wholesale loans compared to retail loans, since they need less net worth to finance a given amount of wholesale loans compared to retail loans:

$$\gamma\mathbb{E}_t\left[\Lambda_{t,t+1}\Omega^R_{t+1}\left(\frac{R^A_{t+1}}{Q_t+f^R_t}-R^D_{t+1}\right)\right] = \mathbb{E}_t\left[\Lambda_{t,t+1}\Omega^R_{t+1}\left(x_{t+1}R^B_{t+1}-R^D_{t+1}\right)\right].$$

The marginal value of an additional unit of net worth Ω^R_{t+1} shows up, since for financially constrained banks, Ω^R_{t+1} is stochastic and in general $\Omega^R_{t+1} > 1$. Hence, the return on wholesale loans is given by

$$R^B_{t+1} = \frac{\gamma\mathbb{E}_t\left[\Lambda_{t,t+1}\Omega^R_{t+1}\frac{R^A_{t+1}}{Q_t+f^R_t}\right] + (1-\gamma)\mathbb{E}_t\left[\Lambda_{t,t+1}\Omega^R_{t+1}\right] R^D_{t+1}}{\mathbb{E}_t\left[\Lambda_{t,t+1}\Omega^R_{t+1}x_{t+1}\right]}. \tag{C.3}$$

The numerator represents the cost of an additional unit of wholesale loans, which consists of the opportunity cost of reducing retail lending by γ units and the cost of raising an additional $1-\gamma$ units of deposits. Moreover, there is an adjustment for the risk-adjusted loss in default in the denominator. In contrast to equation C.2 in the previous case, this term now contains the value of an additional unit of net worth to the shadow bank. Hence, in this model, it is not only borrower balance sheet characteristics, but also lender balance sheet characteristics which determine the equilibrium on the wholesale funding market. In particular, in states in which net worth is particularly scarce for retail banks, e.g. due to a large loss, the spread of the retail and wholesale lending rates over the deposit rate will rise.

D Computation

D.1 Solution

We solve the model nonlinearly using a time iteration algorithm. Solving the model nonlinearly is important, because bank runs can lead to large deviations from steady state, where perturbation algorithms are inaccurate.

The state space of the model is $\mathcal{S} = (N^R, N^S, K, Z)$ in the *"no bank run"* equilibrium and $\mathcal{S}^* = (N^{R,*}, K, Z)$ in the *"bank run"* equilibrium. We approximate the consumption policy functions $C^H(\mathcal{S})$, $V^R(\mathcal{S})$, $V^S(\mathcal{S})$, $C^{H,*}(\mathcal{S}^*)$ and $V^{R,*}(\mathcal{S}^*)$ and the capital prices $Q(\mathcal{S})$ and $Q^*(\mathcal{S}^*)$ using fourth order polynomials. We compute the polynomial coefficients by imposing that the polynomial approximations must be equal to the original functions on the grid. Specifically, denoting the polynomial coefficients by α and the polynomials by $\Pi(\mathcal{S})$, we impose for example for the consumption of households

$$\Pi(\mathcal{S}_i)\alpha_{C^H} = C^H(\mathcal{S}_i) \quad i = 1, \ldots, N. \tag{D.1}$$

for all N grid points. We use a Smolyak grid with order $\mu = 5$ for the endogenous and exogenous states. We compute the Smolyak grid and polynomials using the toolbox by Judd, Maliar, Maliar, and Valero (2014).

One slight complication of the model is that the future net worth values N_R and N_S, depends on $Q(\mathcal{S})$. This implies that, for example, the household net worth for a given function $Q(.)$ must be computed as a solution to the nonlinear function[24]

$$N^{R,\prime} = \left[(r^K + (1-\delta)Q(N^{R,\prime}, N^{S,\prime}, K', Z'))K^{R,\prime} + R^{B,\prime}B' - R^{D,\prime}D'\right](1-\sigma^R)+\upsilon^R K. \tag{D.2}$$

With this in mind, we will now outline our solution algorithm for the *"no*

[24] In principle, Π^Q is also a function of the states. We ignore this here for the sake of exposition. We do however account for this correctly in the code.

bank run" equilibrium. Suppose we are in iteration k and have initial guesses for the no-run consumption policy functions $C^H_{(k)}(\mathcal{S}), V^R_{(k)}(\mathcal{S})$, and $V^S_{(k)}(\mathcal{S})$ and the capital price function $Q_{(k)}(\mathcal{S})$ as well as values for the future net worth $N^{R,\prime}_{(k)}$ and $N^{S,\prime}_{(k)}$.

1. Given the value functions and the future net worth, compute the future value functions and capital prices as

$$
\begin{aligned}
C^{H\prime}_{(k)} &= C^H_{(k)}(\mathcal{S}'_{(k)}),\\
Q'_{(k)} &= Q_{(k)}(\mathcal{S}'_{(k)}),
\end{aligned}
$$

   and so on.

2. Compute the new values for $(K^{H,\prime}, K^{R,\prime}, K^{S,\prime}, D', B', R^{D,\prime}, R^{B,\prime}, Q)$ for all grid points $i = 1, \dots, N$ using the first order conditions B.1, B.2, B.9, B.10, B.3, 2.25 and the leverage constraints 2.5 and 2.6. Compute the future net worth where necessary according to

$$
\tilde{N}^{R,\prime}_{(k+1)} = \left[(r^{K,\prime} + (1-\delta)Q_{(k)}(N^{R,\prime}_{(k)}, N^{S,\prime}_{(k)}, K', Z'))K^{H,\prime} + R^{B,\prime}B' - R^{D,\prime}D'\right](1-\sigma^R)+\upsilon^R K'. \tag{D.3}
$$

   We compute expectations using Gauss-Hermite quadrature. Note that for each quadrature node Z', a different value of $\tilde{N}^{R,\prime}_{(k+1)}$ must be computed.

3. Using the new policies and prices, update the consumption function of the household using equation 2.34, and the value functions for the retail and shadow banks using equation 2.1.

4. Update the next period net worth values using D.3, with some attenuation: $N^{R,\prime}_{(k+1)} = (1-\iota)N^{R,\prime}_{(k)} + \iota\tilde{N}^{R,\prime}_{(k+1)}$, with $\iota = 0.5$.

5. Repeat until the errors in the consumption, capital price and net worth values on the grid are small. We iterate until the maximum error in consumption is smaller than 1e-5 and the maximum error in the net worth is smaller than 1e-5.

If bank runs are unanticipated, we can first solve for the *"no bank run"* equilibrium and then afterwards for the *"bank run"* equilibrium. Importantly, expectations during a bank run are taken over the future *"bank run"* and *"no bank run"* states. It is therefore necessary to keep track of two sets of net worth values, $N^{R,'}_{(k)}$ and $N^{R,',*}_{(k)}$. Otherwise, the algorithm works in the same way as for the *"no bank run"* equilibrium. For the anticipated run case, we use the unanticipated run case as initial guess and solve jointly for the *"no bank run"* and *"bank run"* policy functions.