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Welfare Effects of Certification under Latent Adverse Selection*

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Abstract

Asymmetric information is a classic example of market failure that undermines the efficiency associated with perfectly competitive market outcomes: the “lemons” market. Credible certification, that substantiates unobservable characteristics of products that consumers value, is often considered a potential solution to such market failure. This paper examines welfare effects of certification in markets in which there is asymmetric information, but without an adverse selection problem. We analyze the market equilibrium when the certification technology becomes available and contrast this with the equilibrium without certification. We find that despite an improvement in allocative efficiency, overall welfare may decrease due to the possibility of certification when such certification is either costly or inaccurate. In fact, most of these results are not derived from the direct welfare cost of certification, but rather from certification’s effect on the market(s).

Keywords: credible certification, welfare-reducing certification, asymmetric information, adverse selection.

JEL Classifications: D8, D4, L1

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1 Introduction

Asymmetric information is a classic example of market failure that undermines the efficiency associated with perfectly competitive market outcomes—the “lemons” market ([Akerlof, 1970](#)). This issue has recently regained more detailed scrutiny with the rise of “organic,” “fair trade,” “local,” “sustainable” etc. products. In markets for such products, the concept of product quality in consumer preferences has extended to include the process of production and distribution. When such properties are claimed by firms, along with specific corporate philosophies or policies, it is very hard or impossible for consumers to ascertain the quality or the process of production of those products even after their consumption experience in many cases. Due to the aforementioned credence good nature,¹ quality assurance mechanisms, such as offering warranties (e.g., see [Spence, 1977](#)) and building up a reputation (e.g., see [Klein and Leffler, 1981](#); and [Shapiro, 1983](#)), are not expected to function well to address the information problem in those markets.² Accordingly, credible (third-party) certification is often considered the only potential solution for overcoming informational asymmetries. In addition, while “quality” in the traditional, lemon’s sense (vertical differentiation) may be hard to measure, consumers’ valuation of the production process, as in the examples above, makes certification more easily quantifiable. Although the degree to which such claims are backed up by formal certification or regulated by law varies, various certification schemes are in use in many marketplaces.

In spite of the seeming attractiveness of credible certification, it is still important to substantiate whether introducing certification increases or decreases social welfare through a formal analysis, especially considering its popularity. To this end, in this paper we consider markets in which goods have unobservable characteristics that consumers value. We analyze the market equilibrium when a technology is available that credibly verifies the relevant attributes of products and contrast this with the equilibrium without certification. We assume that the certification technology could be costly and imperfect. One effect the certification technology has is to potentially create two markets (market segmentation)—one with certified products and the other without—while without certification there is only one pooled market. As a result, one has to consider both the firm’s decision as to whether to seek certification as well as the consumer’s choice as to which market to patronize.

We find that certification may be welfare-reducing when the certification technology is costly or imperfect. This implies that even when the certification technology is perfectly accurate—and

¹For more detailed definition and discussion of credence goods see, for example, [Darby and Karni \(1973\)](#) and [Dulleck and Kerschbamer \(2006\)](#).

²See [Bonroy and Constantatos \(2015\)](#) and [Dranove and Jin \(2010\)](#) for detailed discussion on this point.

therefore is able to resolve informational asymmetries in the market place—overall welfare may decrease due to the possibility of certification when such certification is costly. Similarly, even when the certification technology is costless, welfare may decrease when such certification is quite imperfect.

The underlying reason for this is that while the certification process admits better information and therefore increases efficiency by reallocating high quality goods to consumers with relatively higher valuations, it also brings a negative impact on the average quality of non-certified goods. The latter effect can push some high quality producers out of the non-certified market or the entire group (a market collapse). In contrast, absent any certification, the increased presence of products with desirable characteristics provide a sufficiently strong positive externality to sustain an equilibrium that entails a larger number of high-quality products. When the certification technology is relatively costly or imperfect, only a small portion of high type producers are present in the certified market in equilibrium, and this may still cause the non-certified market to collapse, which results in a quite low welfare level. Most of these results are not a result from the direct welfare cost of certification, but rather from certification’s effect on the market(s).

2 Related Literature

This paper contributes to the literature on certification (or labeling), or more broadly, quality disclosure. There are two streams of the certification literature. The first strand focuses on the role of strategic certification intermediaries. Some studies aim to investigate how competition between certifiers affects market outcomes including optimal pricing schemes, the amount of information transmitted to consumers or quality provision (e.g., [Albano and Lizzeri, 2001](#); [Fischer and Lyon, 2014](#); [Hvide, 2009](#); and [Lizzeri, 1999](#)). Some other papers (e.g., [Benabou and Laroque, 1992](#)) explore whether a reputation concern can mitigate the incentive problems of certification intermediaries. Some of these papers have something in common with ours in that they show a potential source of inefficiency associated with certification. However, while they emphasize the role of strategic certifiers as a source of inefficiency, we are more interested in sellers’ incentives for getting certified and their incidence on markets taking certification environments as given.

The other strand of this literature focuses on a seller’s incentive for quality disclosure. Since the “unraveling result”³ was presented by [Grossman \(1981\)](#) and [Milgrom \(1981\)](#), many subsequent studies have explored the subject of when and why unraveling fails to hold (e.g., see [De and Narbar, 1991](#); [Guo and Zhao, 2009](#); [Grubb, 2011](#); [Hotz and Xiao, 2013](#); [Shavell, 1994](#); and [Viscusi,](#)

³The unraveling result refers to the situation in which each seller voluntarily discloses her quality to consumers for differentiating herself from other sellers with a lower quality if quality disclosure is credible and costless.

1978). Closely related with such work, several authors also studied whether mandatory disclosure laws enhance or hinder efficiency when voluntary disclosure fails to occur (e.g., see [Bar-Isaac et al., 2012](#); [Board, 2009](#); [Gavazza and Lizzeri, 2007](#); [Harbaugh et al., 2011](#); [Jovanovic, 1982](#); and [Matthews and Postlewaite, 1985](#)). Though these papers share several features with us, the most important difference in ours is that we compare the situations with and without certification, rather than comparing a situation in which sellers voluntarily choose to get certified with another situation in which sellers must get certified to sell.⁴

There are only a relatively small number of studies that focus on welfare effects of certification. The authors of these studies point that welfare may decrease with the availability of certification mostly in the context of Eco-labeling, as we do in this paper. Among these studies, [Baltzer \(2012\)](#), [Bonroy and Constantatos \(2008\)](#) and [Zago and Pick \(2004\)](#) differ from ours in that the underlying mechanisms that derive welfare decreasing results in their work are other than exacerbated adverse selection problem by certification. The most closely related papers to this one are [Mason and Sterbenz, 1994](#) and [Mason, 2011](#). Taking into account sellers' incentive for opting for certification, the above two papers show that certification can aggravate adverse selection, and thus certification maybe welfare reducing. However, the main drivers of the exacerbated adverse selection problem in their approaches and our approach are different: unlike they are more focused on the mimicking incentives of low type sellers, we are more interested when certification causes the collapse of the non-certified market. More importantly, we assume a downward sloping demand curve in order to seriously consider allocative efficiency, associated with certification, which is an important factor when examining welfare effects of certification.⁵

The remainder of this paper proceeds as follows. Section 2 describes the base model. To explain the value of information, in section 3, we compare two benchmarks: full information and no information case. We derive a certification equilibrium and conduct a welfare analysis in Section 4. Finally, Section 5 concludes. All omitted proofs are in the appendix.

3 The Base Model

We consider markets in which there is asymmetric information between consumers and firms about the product quality.

⁴For more examples, see cited papers in [Dranove and Jin \(2010\)](#).

⁵[Creane \(1998\)](#) examines quality certification in an international trade setting, finding that importing (consuming) welfare decreases with certification. However, global (total) welfare is increasing.

3.1 Consumers

In order to introduce certification (which implies a segmentation of the market and hence sorting of demand according to distinct quality/grades) later in the model, a richer modeling structure for demand is needed. Following [Mussa and Rosen \(1978\)](#), we adopt the vertical differentiation model of gradations g with a continuum of consumer types each of whom has a unit demand. Consumers are of total mass 1 and distributed on $[\underline{\theta}, \bar{\theta}]$ according to the strictly increasing cdf, $F(\theta)$. Then, a type- θ consumer's net utility, when paying price p , is given by

$$U(g, p|\theta) = \theta g - p,$$

while a consumer gets 0 when buying nothing. All consumers are price takers.

3.2 Firms

On the supply side suppose there is a mass (measure) n of firms which are also all price takers. A firm produces (or serves) either high (\bar{g}) or low (\underline{g}) quality product (or service). Since we do not focus on a moral hazard problem but on an adverse selection problem, we assume these quality levels are exogenously given. A proportion of high grade producers is indexed by $\gamma_0 \in (0, 1)$, and $\gamma_0 n$ firms can produce a unit of high quality at cost of \bar{c} . The rest of producers, $(1 - \gamma_0)n$, can produce a unit of low quality at cost of \underline{c} . Each seller knows the true quality of her own product, but consumers do not. In order to make the analysis of the lower grade market simpler when a segmentation of the market arises (under full information or with certification), let us normalize $\underline{g} = \underline{c} = 0$.

3.3 Two Benchmarks

In the rest of this section, we study the equilibrium configurations when there is no certification technology (No Information/No Certification) and when the quality of product is observable to consumers (Full Information) in turn as two benchmark cases before delving into equilibrium configurations with certification. By comparing welfare under the two benchmarks, we will explain why certification can potentially be valuable to the society.

3.3.1 No Information (No Certification) Equilibrium

Without any quality certification the market is subject to the law of one price, therefore only overall market demand for different compositions of quality on offer is required. Consider the supply

side first. No high quality goods are offered if the price is below their production costs. More specifically,

$$q^S(p) = \begin{cases} \kappa(1 - \gamma_0)n, \kappa \in [0, 1] & \text{if } p = 0, \\ (1 - \gamma_0)n & \text{if } p \in (0, \bar{c}), \\ (1 - \gamma_0 + \kappa\gamma_0)n, \kappa \in [0, 1] & \text{if } p = \bar{c}, \\ n & \text{if } p > \bar{c}. \end{cases} \quad (1)$$

Since consumers rationally anticipate the supply schedules, the demand structure is given by

$$q^D(p) = \begin{cases} 1 - F(p/\gamma\bar{g}) & \text{if } p \geq \bar{c}, \\ 0 & \text{if } p < \bar{c}, \end{cases}$$

in which γ is beliefs of consumers, and so $\gamma\bar{g}$ represents ex-ante average quality in the market. To make the analysis non-trivial, we take the following assumption.

Assumption 1: $0 < \bar{c} < \gamma_0\bar{g}\bar{\theta}$

Assumption 1 specifies cost structure under which adverse selection need not necessarily happen. The last inequality implies that at least some trade of high quality given the prior belief is efficient. The potential for adverse selection comes from the first inequality, which implies that all high producers are driven out if consumers believe that there are no high type in the market. This is because in that case even for the highest type the consumer valuation would be lower than \bar{c} .

Since we are interested in inquiring about if certification can decrease social welfare, we further assume that the number of firms is relatively small so that there is asymmetric information in the market but not necessarily an adverse selection problem.

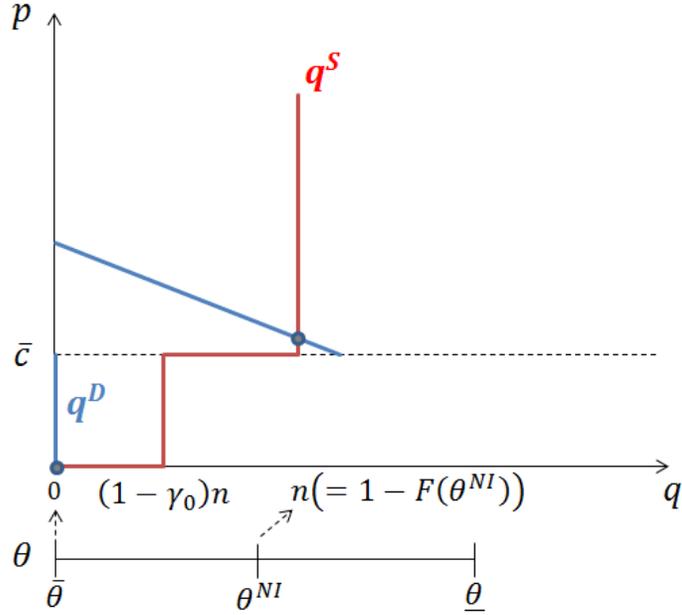
Assumption 2: $n \leq 1 - F(\bar{c}/\gamma_0\bar{g})$

Under Assumption 2, in any equilibrium in which a positive quantity is traded (at an equilibrium price greater than \bar{c}), rational expectation implies that consumer beliefs must be the same as prior beliefs, i.e., $\gamma = \gamma_0$, and all n firms serve consumers.⁶ As seen in Figure 1, although there exists a trivial one in which no transaction occurs ($p^{NI} = 0$ and $q^{NI} = 0$) as well in general, we select the equilibrium with the higher quantity and the higher level of welfare.

Thus, in equilibrium

⁶In principle, γ could differ from γ_0 if n is sufficiently large because there may be some high quality producers staying out of the market in equilibrium when the market price is equal to \bar{c} . See [Creane and Jeitschko \(2016\)](#) for more on this point.

Figure 1: No Information Equilibrium



Note: The demand curves need not necessarily be linear.

$$\begin{aligned} q^{NI} &= n; & p^{NI} &= \gamma_0 \bar{g} H(1 - n) \\ \pi_H^{NI} &= p^{NI} - \bar{c}; & \pi_L^{NI} &= p^{NI} \end{aligned}$$

where $H(\tau) \equiv F^{-1}(\tau)$ a quantile function defined on $[0, 1)$.⁷

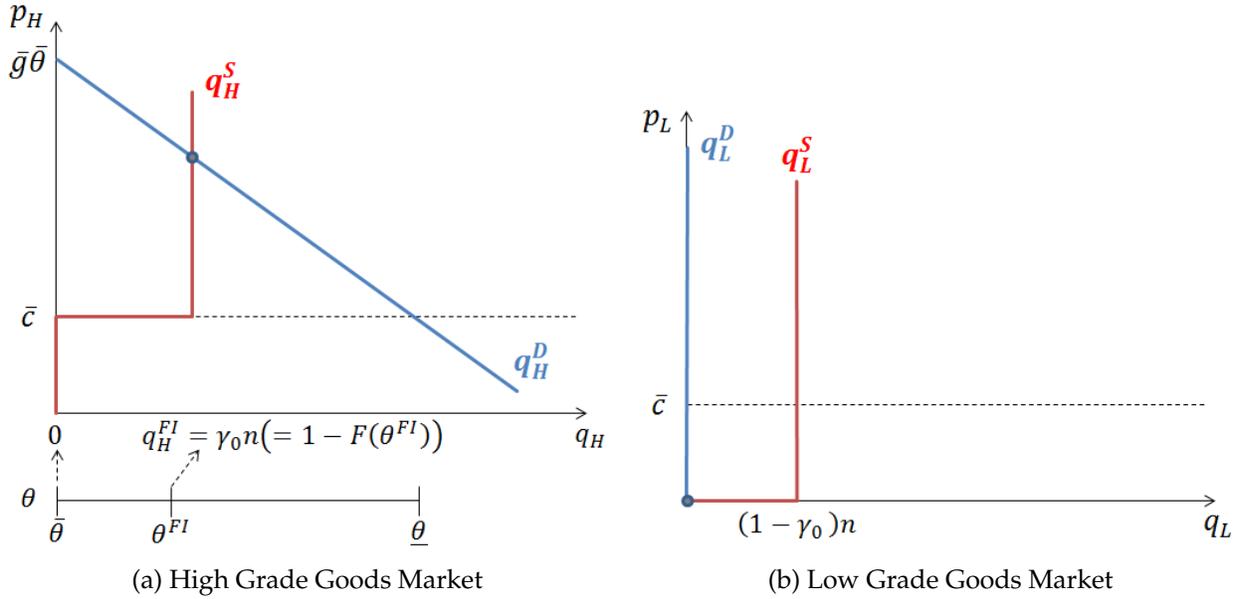
3.3.2 Full Information Equilibrium

Under full information assumption, firms will be sorted into two distinct groups. And thus, we have to examine the two separate markets. In the low grade market (market L), it is optimal for low grade producers to produce as long as the market price (p_L) is greater than their production cost $\underline{c} = 0$. All low quality firms are indifferent between producing or not when $p_L = 0$. Therefore, the supply correspondence in market L is given by,

$$q_L^S(p_L | n) = \begin{cases} \kappa(1 - \gamma_0)n, \kappa \in [0, 1] & \text{if } p_L = 0, \\ (1 - \gamma_0)n & \text{if } p_L > 0. \end{cases}$$

⁷ $q^{NI} = n$, or $\theta^{NI} = H(1 - n)$.

Figure 2: Full Information Equilibrium



Note: The demand functions need not necessarily be linear.

However, it is optimal for any type of consumer, θ , not to purchase in this market if p_L is positive because $U(\underline{g}, p_L | \theta) = -p_L < 0$ for all consumer types. This implies the demand correspondence in market L is given by,

$$q_L^D(p_L | n) = \begin{cases} \in [0, 1] & \text{if } p_L = 0, \\ 0 & \text{if } p_L > 0. \end{cases}$$

In principle, the equilibrium quantity can vary though the unique equilibrium price is 0 ($p_L = 0$). In all such situations, we assume there is no transaction by setting $q_L^{FI} = 0$ and $p_L^{FI} = 0$. Note that the normalization assumption in market L ($\underline{g} = \underline{c} = 0$) results in the normalization of surplus in that market as well. ⁸

Now consider the high grade market (market H). The supply schedule can be derived in a

⁸This does not harm generality much for total welfare analysis. However, when it comes to consumer surplus, this simplifying assumption makes a difference. With the normalization assumption, the surplus of the marginal consumer in market H is 0. In contrast, if we assume $\bar{g} > \bar{c}$, so that if some lower quality products can be served in an equilibrium, the marginal consumer of the higher grade market should get some surplus since now she has an outside option of buying from the lower grade market. Therefore, consumer surplus from the high grade market may decrease with the normalization in market L .

similar way as we did for market L :

$$q_H^S(p_H|n) = \begin{cases} 0 & \text{if } p_H < \bar{c}, \\ \kappa\gamma_0 n, \kappa \in [0, 1] & \text{if } p_H = \bar{c}, \\ \gamma_0 n & \text{if } p_H > \bar{c}. \end{cases}$$

In order to derive demand, note that a type- θ consumer would purchase given p_H if and only if

$$U(\bar{g}, p_H|\theta) = \bar{g}\theta - p_H \geq 0.$$

If we define the marginal consumer who is indifferent between buying or not as $\theta_H^{FI} \equiv p_H/\bar{g}$, consumers with $\theta \geq \theta_H^{FI}$ buy high grade goods, while the remainder ($\theta < \theta_H^{FI}$) do not. The demand schedule is thus given by

$$q_H^D(p_H) = \begin{cases} 1 - F(p_H/\bar{g}) & \text{if } p_H \in [0, \bar{g}\bar{\theta}], \\ 0 & \text{if } p_H > \bar{g}\bar{\theta}. \end{cases}$$

Assumption 2 implies that under full information all high types serve in the market,⁹ and thus the equilibrium outcome can be summarized as follows,

$$\begin{aligned} q_H^{FI} &= \gamma_0 n; & q_L^{FI} &= 0; \\ p_H^{FI} &= \bar{g}H(1 - \gamma_0 n) > \bar{c}; & p_L^{FI} &= 0; \\ \pi_H^{FI} &= p_H^{FI} - \bar{c}; & \pi_L^{FI} &= 0, \end{aligned}$$

where π_H^{FI} and π_L^{FI} stand for equilibrium profits of high and low quality firms under full information respectively.¹⁰

3.3.3 Welfare Comparison

Having the equilibrium configurations in the two benchmarks, we show that information increases social welfare. We take social welfare as the sum of firm profits and consumer surplus. Since price paid by consumers is just transferred to firms, total welfare under no information and full

⁹Mathematically, this is because Assumption 2 implies $n < 1 - F(\bar{c}/\gamma_0\bar{g}) < (1/\gamma_0)[1 - F(\bar{c}/\bar{g})]$ and thus, $n \leq (1/\gamma_0)[1 - F(\bar{c}/\bar{g})] \iff p_H^{FI} = \bar{g}H(1 - \gamma_0 n) \geq \bar{c}$.

¹⁰ $q_H^{FI} = \gamma_0 n$, or $\theta^{FI} = H(1 - \gamma_0 n)$.

information can be written as gross consumer benefits net of production costs:

$$\begin{aligned} W^{NI} &= \int_{\theta^{NI}}^{\bar{\theta}} \gamma_0 \bar{g} \theta dF(\theta) - \gamma_0 n \bar{c}, \\ W^{FI} &= \int_{\theta_H^{FI}}^{\bar{\theta}} \bar{g} \theta dF(\theta) - \gamma_0 n \bar{c}. \end{aligned}$$

Proposition 1 (*Information Increases Welfare*) *Welfare is strictly greater under full information than under no information regardless of a proportion of high grade producers, i.e.,*

$$W^{FI} > W^{NI}, \quad \forall \gamma_0 \in (0, 1).$$

Proof: Subtracting W^{NI} from W^{FI} and rewriting it shows a trade-off between the two situations as follows:

$$\begin{aligned} W^{FI} - W^{NI} &= \int_{\theta_H^{FI}}^{\bar{\theta}} \bar{g} \theta dF(\theta) - \int_{\theta^{NI}}^{\bar{\theta}} \gamma_0 \bar{g} \theta dF(\theta) \\ &= \underbrace{\left\{ \int_{\theta_H^{FI}}^{\bar{\theta}} \bar{g} \theta dF(\theta) - \int_{\theta_H^{FI}}^{\bar{\theta}} \gamma_0 \bar{g} \theta dF(\theta) \right\}}_{\substack{\text{Gains from reallocating} \\ \text{high quality to } \theta \geq \theta_H^{FI}}} - \underbrace{\int_{\theta^{NI}}^{\theta_H^{FI}} \gamma_0 \bar{g} \theta dF(\theta)}_{\substack{\text{Losses from not served} \\ \text{consumers } (\theta^{NI} < \theta < \theta_H^{FI})}}. \end{aligned}$$

Now we show that the above expression is always greater than 0. Note that $\int_{\theta_1}^{\theta_2} \theta dF(\theta) = \theta_2 F(\theta_2) - \theta_1 F(\theta_1) - \int_{\theta_1}^{\theta_2} F(\theta) d\theta = \int_{\tau_1}^{\tau_2} H(\tau) d\tau$ for any arbitrary $\theta_1 < \theta_2$ and corresponding $\tau_1 < \tau_2$. The first equality follows from integration by parts and the second from the definition of $H(\cdot)$. This implies,

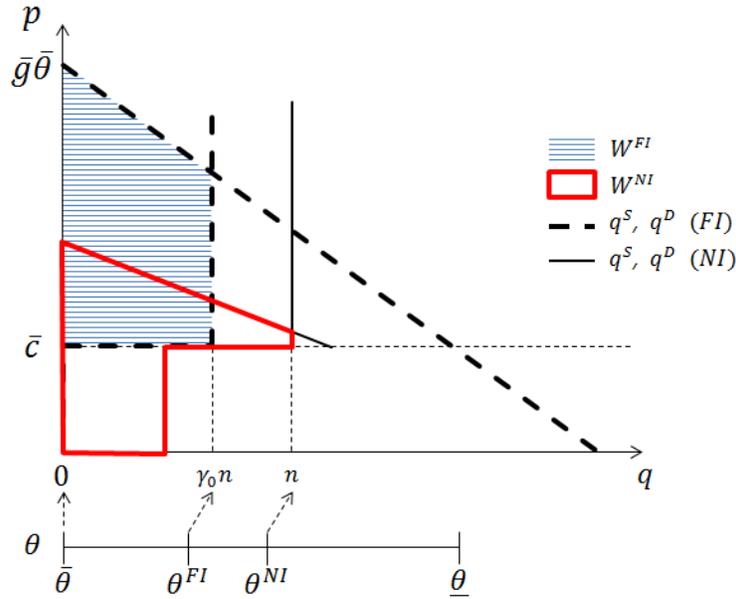
$$W^{FI} - W^{NI} = \bar{g} \left[(1 - \gamma_0) \int_{\tau_H^{FI}}^1 H(\theta) d\theta - \gamma_0 \int_{\tau^{NI}}^{\tau_H^{FI}} H(\theta) d\theta \right],$$

where $\theta_H^{FI} = H(\tau_H^{FI})$ and $\theta^{NI} = H(\tau^{NI})$. Moreover, since $(\tau_2 - \tau_1)H(\tau_1) < \int_{\tau_1}^{\tau_2} H(\tau) d\tau < (\tau_2 - \tau_1)H(\tau_2)$ for any increasing $H(\cdot)$ ¹¹, we have

$$\begin{aligned} W^{FI} - W^{NI} &> \bar{g} \left[(1 - \gamma_0)(1 - \tau_H^{FI})\theta_H^{FI} - \gamma_0(\tau_H^{FI} - \tau^{NI})\theta_H^{FI} \right] \\ &= \bar{g} \left[(1 - \gamma_0)\gamma_0 n \theta_H^{FI} - \gamma_0(1 - \gamma_0)n \theta_H^{FI} \right] \\ &= 0. \end{aligned}$$

¹¹Since $F(\cdot)$ is strictly increasing, so is $H(\cdot)$.

Figure 3: Welfare Comparison between *NI* and *FI* Benchmarks



Note: The demand function needs not necessarily be linear. ■

Proposition 1 states that restoring full information improves welfare. The intuition behind this result is straightforward. Even though the market is not subject to adverse selection under no information, the market still suffers from an allocation inefficiency associated with asymmetric information. This is because those with relatively higher valuations may obtain a low quality product and vice versa. When we move from the no information case to the full information case, while the aggregate production costs remain the same, allocative efficiency is restored because now high quality goods are allocated only to the group of consumers with relatively higher valuations. To understand this point further, refer to Figure 3. The measure of $\gamma_0 n$ consumers above θ_H^{FI} gain at least $\bar{g}(1 - \gamma_0)\theta_H^{FI}$ each from the reallocation of goods, while the measure of $(1 - \gamma_0)n$ consumers between θ^{NI} and θ_H^{FI} lose at most $\gamma_0 \bar{g}\theta_H^{FI}$. Hence, welfare under full information is always greater.

4 Certification Equilibrium

The previous section shows that information increases social welfare. One immediate implication of the result is that perfect and cost-less certification, which coincides with the full information

benchmark, is welfare enhancing. However, what if the certification technology is costly or imperfect? Are there any cases in which welfare with certification is even lower than that under no information? In order to answer these questions, we derive a price equilibrium given certification decisions of firms and then characterize a certification equilibrium. The welfare analysis follows next and shows that certification may actually decrease social welfare.

For simplicity and tractability of the model, we employ further assumptions. Throughout this section, let us assume that consumers are uniformly distributed on the line segment $[0, 1]$, i.e.,

$$F(\theta) = \theta$$

on $[0, 1]$, and $H(\tau) = \tau$. Also, for simplicity we normalize $\bar{g} = 1$. To reduce the equilibrium constellations to consider, we also restrict our interests to the case in which Assumption 2 binds and denote such measure of producers by \bar{n} . With the uniform distribution assumption, note that $\bar{n} = 1 - \bar{c}/\gamma_0$.

4.1 Certification Environment

We consider a certification market in which all certifiers are homogeneous in their certification costs and accuracy of certification test. More specifically, each certifier needs to incur the same fixed cost of certification, $z \geq 0$. We further assume that the certification market is perfectly competitive so that the equilibrium price for certification is always equal to z .

Certification technology may be imperfect in the following sense. Let \tilde{g} denote the grade discovered by the certification test. Then,

$$\begin{aligned} Pr \{ \tilde{g} = \bar{g} | g = \bar{g} \} &= x; & Pr \{ \tilde{g} = \bar{g} | g = \underline{g} \} &= 0; \\ Pr \{ \tilde{g} = \underline{g} | g = \bar{g} \} &= 1 - x; & Pr \{ \tilde{g} = \underline{g} | g = \underline{g} \} &= 1, \end{aligned}$$

where $0 < x \leq 1$. So, the test is informative in the sense that the test perfectly weeds low grades out and identifies a high type with some precision. Put differently, the certification test commits a type-I error with probability $1 - x$ but not a type-II error. However, x can be lower than $1/2$. Certification is imperfect unless $x = 1$.

If the test reveals that a seller is of high type ($\tilde{g} = \bar{g}$), she gets certified (*Crt*). Otherwise ($\tilde{g} = \underline{g}$), the seller can only sell in the pool of sellers who did not apply for certification and applied but failed to get certification (*NC*). Put differently, consumers can only condition on whether the good is certified or not certified. Finally, the sellers cannot shop around for certification, and so the

first certification result sticks. This assumption precludes the possibility that the sellers use an imperfect and cost-less certification repeatedly in such a way that full information outcome may be replicated.

We consider the following two-period model when certification is an available option for sellers:

Period 1 (Certification Decision): In the first period each seller makes a decision whether to opt for certification test or not.

Period 2 (Walrasian Price Formation): In the second period, with there being asymmetric information partially resolved through the certification (given sellers' certification decisions and outcomes) process, trade takes place in which Walrasian mechanism forms prices in both markets: certified (p^{Crt}) and non-certified (p^{NC}) market. Depending on offered prices in each market, high and low grade producers decide whether to produce at production costs of \bar{c} and 0 respectively (produce to demand) or to shut down, and consumers choose to buy in the Crt market or in the NC market or not to buy.

In fact, firms need to go through a certification process before producing in many cases. For example, to get the USDA Organic label, a farm must develop and implement an organic system plan first and then hire a USDA-accredited certifying agent to get inspected before producing. Note that in this case the certification cost, z , can include all expenses for adopting the organic system as well as an agency fee.¹²

4.2 Price Equilibrium

Given the assumptions on certification environment, there is no chance that the low type is identified as the high type by the test. Thus, without loss of generality, we can assume that the low type never seeks certification, and the price equilibrium constellations depend only on the high producers' certification decisions. Let α denote the proportion of high type sellers who choose to opt for certification. Then, there are three cases to consider: a pooling equilibrium ($\alpha = 0$), a semi-separating equilibrium ($\alpha \in (0, 1)$) and a fully separating equilibrium ($\alpha = 1$). In a pooling equilibrium in which no high type seeks certification ($\alpha = 0$), the resulting outcomes coincide with the no information benchmarks, and there is nothing to be analyzed further. In a semi-separating equilibrium or a fully separating one, certification environment gives rise to the following two interesting features. First, third party certification results in market segmentation. Second, if a large portion of high grade firms congregate in one of the two markets, this may give rise to the

¹²See <https://www.ams.usda.gov/services/organic-certification/becoming-certified> (last retrieved on May 1, 2017).

collapse of the other market due to the intensified adverse selection even if there is no adverse selection problem in the beginning when all firms are pooled together in one market.

In order to derive a price equilibrium for each $\alpha \in (0, 1]$, write down the supply schedules in each market which are similar to (1) with the modification of population of high and low grade producers. In the Crt market there are $\alpha x \gamma_0 \bar{n}$ measure of high producers deciding whether to produce or not. Similarly, in the NC market, the rest of producers, the pool of $(1 - \alpha x) \gamma_0 \bar{n}$ measure of high types and $(1 - \gamma_0) \bar{n}$ of low types, make the same decisions. And thus, supply schedules in the two markets are given by

$$q^{Crt/S}(p^{Crt}) = \begin{cases} 0 & \text{if } p^{Crt} \in [0, \bar{c}), \\ \rho \alpha x \gamma_0 \bar{n}, \rho \in [0, 1] & \text{if } p^{Crt} = \bar{c}, \\ \alpha x \gamma_0 \bar{n} & \text{if } p^{Crt} > \bar{c}; \end{cases} \quad (2)$$

$$q^{NC/S}(p^{NC}) = \begin{cases} \kappa(1 - \gamma_0) \bar{n}, \kappa \in [0, 1] & \text{if } p^{NC} = 0, \\ (1 - \gamma_0) \bar{n} & \text{if } p^{NC} \in (0, \bar{c}), \\ [(1 - \gamma_0) + \kappa(1 - \alpha x) \gamma_0] \bar{n}, \kappa \in [0, 1] & \text{if } p^{NC} = \bar{c}, \\ [(1 - \gamma_0) + (1 - \alpha x) \gamma_0] \bar{n} & \text{if } p^{NC} > \bar{c}. \end{cases} \quad (3)$$

Since consumers rationally expect the supply schedules, the posterior belief for the Crt market is given by

$$\gamma^{Crt} \equiv \gamma(\bar{g}|Crt) = \frac{\alpha x \rho \gamma_0}{\alpha x \rho \gamma_0} = 1$$

as long as $p^{Crt} \geq \bar{c}$ and $\gamma(1|Crt) = 0$ otherwise. Note that in the NC market low grade producers are willing to sell at all positive prices while high types would only do so when $p^{NC} \geq \bar{c}$, which implies in any equilibrium associated with some transaction and with shutting down by some firms, the exiting firms must be of high type. From this argument we know that,

$$\gamma^{NC} \equiv \gamma(\bar{g}|NC) = \frac{(1 - \alpha x) \kappa \gamma_0}{(1 - \alpha x) \kappa \gamma_0 + (1 - \gamma_0)} \leq \frac{(1 - \alpha x) \gamma_0}{(1 - \alpha x) \gamma_0 + (1 - \gamma_0)}$$

in which κ is 1 for $p^{NC} > \bar{c}$, any number in $[0, 1]$ for $p^{NC} = \bar{c}$ and 0 for $p^{NC} < \bar{c}$. Thus, $\gamma^{Crt} = 1 > \gamma_0 \geq \gamma^{NC}$ for all $\alpha \in (0, 1]$, which basically means that certification allows high quality producers to differentiate themselves from low quality producers with some probability while lowering the average quality in the NC market.

Given these beliefs and two price levels - p^{Crt} and p^{NC} , consumers are divided into three sepa-

rate groups depending on the following decision rule: a type- θ consumer

$$\begin{cases} \text{buys in market } i & \text{if } \gamma^i \theta - p^i \geq \max\{\gamma^j \theta - p^j, 0\}, \\ \text{does not buy} & \text{if } 0 \geq \max\{\gamma^{Crt} \theta - p^{Crt}, \gamma^{NC} \theta - p^{NC}\}, \end{cases} \quad (4)$$

where $i, j \in \{Crt, NC\}$ and $i \neq j$.

After defining a price equilibrium formally, we will derive some preliminary results.

Definition 1 *A price equilibrium in the second period, given high types' certification decision (α) and certification precision (x), is a quadruple of $(p^{Crt}, \gamma^{Crt}, p^{NC}, \gamma^{NC})$ such that*

1. *Walrasian Market Clearing in Both Markets: given p^{Crt} and p^{NC} , the quantities supplied, determined by (2) and (3), equal to the quantities demanded, derived by (4) and beliefs: γ^{Crt} and γ^{NC} , in each market.*
2. *Consistent Beliefs: there exists ρ (and κ) $\in [0, 1]$ describing high grade firms' production decisions in (2) (and (3)) which implies equilibrium quantity in the certified market (and non-certified market) and γ^{Crt} (and γ^{NC}).*

Note that there could be two potential types of price equilibria: with only one active market and the collapse of the other, and with two active markets. Let's denote the former one as "1-market" equilibrium and the latter as "2-market." To verify the equilibrium characteristics and its uniqueness, first we derive some conditions that equilibrium prices should meet.

Lemma 1 *In any price equilibrium in which some high quality goods are traded,*

1. *the equilibrium price in the Crt market is always greater than that in the NC market (i.e., $p^{Crt} > p^{NC}$);*
2. *if some high quality producers shut down, the equilibrium price for that market must be equal to \bar{c} .*

There are several important implications of Lemma 1. First, since p^{Crt} is higher than p^{NC} in equilibrium (and $\gamma^{Crt} > \gamma^{NC}$), there is no 1-market equilibrium in which transaction occurs only in the NC market. This implies that 1-market and 2-market equilibria have to result in the price structures $p^{Crt} \geq \bar{c} > p^{NC} = 0$ and $p^{Crt} > p^{NC} \geq \bar{c}$ respectively.

Second, there exists a marginal consumer who is indifferent between buying in Crt market and NC market. The cutoff consumer's type θ^{Crt} is derived from the following condition:

$$\gamma^{Crt} \theta^{Crt} - p^{Crt} = \gamma^{NC} \theta^{Crt} - p^{NC}. \quad (5)$$

Without considering the possibility of not buying, all consumers with $\theta \geq \theta^{Crt}$ are affiliated with the Crt market while the remainder belongs to the NC market.

Third, all high types with a certification should serve ($\rho = 1$) in both 1-market and 2-market equilibria. To see this, recall that the equilibrium price under no information with \bar{n} is equal to \bar{c} implying the margin is 0 for the high grade producers. Now, there are only $\alpha x \gamma_0 \bar{n}$ number of producers in the Crt market, and they serve consumers with relatively high valuations from the top (i.e., $\theta \in [\theta^{Crt}, 1]$). Also, note that each consumer's willingness to pay is also higher than that under no information because $\gamma^{Crt} > \gamma_0$. Therefore, it must be the case, $p^{Crt} > \bar{c}$ and $\rho = 1$.

Fourth, in any 2-market equilibria, $p^{NC} = \bar{c}$ and $\kappa < 1$. In no information benchmark, θ^{NI} -type consumer's willingness to pay, $\gamma_0 \theta^{NI}$, was equal to \bar{c} . Now since $\gamma^{NC} < \gamma_0$, his willingness to pay is lower than \bar{c} , and so some high types have to shut down. It is optimal that some high types choose to sell and the others do not only when those two actions yield the same payoffs of 0 to them, which implies $p^{NC} = \bar{c}$.

If both type of equilibria coexist, even though the previous claim implies that given α , x and \bar{n} , social welfare from the Crt market is invariant with the equilibrium quantity equal to $\alpha x \gamma_0 \bar{n}$, we may have multiple price equilibria due to the nature of the NC market. Hence, equilibrium selection is necessary to make a prediction about optimal certification decisions. To be consistent with the no information benchmark and as we want to show welfare under certification might be lower than under no information, we select the most favorable equilibrium in terms of welfare. Note that if there exist both 1-market and 2-market equilibrium, the welfare maximizing one is the 2-market equilibrium. Otherwise, the welfare maximizing equilibrium is the 1-market one. The lemma below characterizes such price equilibria.

Lemma 2 *The price equilibrium constellations are as follows.*

1. *If the production cost of high quality good is relatively high (i.e., $\bar{c} \in [\gamma_0^2, \gamma_0)$), the NC market always collapses ($\kappa = 0$), so a 1-market equilibrium is the unique welfare maximizing price equilibrium for all $\alpha \in (0, 1]$ and $x \in (0, 1]$; and*
2. *if the production cost of high quality good is relatively low (i.e., $\bar{c} \in (0, \gamma_0^2)$), there exists a threshold ξ such that*
 - (a) *for $\alpha x \in (\xi, 1]$, the NC market always collapses, so a 1-market equilibrium is the unique welfare maximizing price equilibrium; and*
 - (b) *for $\alpha x \in (0, \xi]$, a 2-market equilibrium with $\kappa(\alpha, x) \in (0, 1)$ is the unique welfare maximizing price equilibrium where $\kappa(\alpha, x)$ is the larger root solving $p^{NC} = \bar{c}$.*

The lemma states conditions under which we have 1-market and 2-market equilibria. In order to grasp a deeper understanding, first we explain two different negative effects of high type's departure, from the pool of entire producers to the Crt market, on equilibrium price in the NC market. Not only it causes the average quality in the NC market (γ^{NC}) to fall (rotation in the NC market demand curve), but also it skims consumers with highest valuations from the pool (left shift of the NC market demand curve). However, that does not necessarily result in the collapse of the NC market if some portion of high type shut down. With a downward sloping demand curve, this exit by high grade producers has two countervailing effects on equilibrium price down in the NC market: p^{NC} might increase due to the decrease in quantity supplied (move along the NC market demand curve) but decrease due to the further drop in the average quality in the NC market (further rotation in the NC market demand curve). If the former effect dominates the latter, 2-market equilibria may emerge. Therefore, as stated in Lemma 2 price equilibrium configurations crucially depend on the magnitude of \bar{c} . 2-market equilibria do not emerge at all when \bar{c} is relatively large (or \bar{n} relatively small) because, roughly speaking, the smaller \bar{n} , the less potential for such positive (quantity) effects on p^{NC} .

Given any certification decisions (characterized by α), the corresponding welfare-maximizing price equilibrium is unique and well defined. Corollary 1 formally summarizes price equilibrium outcomes.

Corollary 1 *The welfare maximizing price equilibrium outcomes are as follows (given α and x).*

1. *When only 1-market equilibria exist,*

$$\begin{aligned} \gamma^{Crt} &= 1; & \gamma^{NC} &= 0; \\ \theta^{Crt} &= 1 - \alpha x \gamma_0 \bar{n}; & \theta^{NC} &= \theta^{Crt}; \\ p^{Crt} &= \theta^{Crt}; & p^{NC} &= 0; \\ \pi_H^{Crt} &= p^{Crt} - \bar{c}; & \pi_H^{NC} &= \pi_L^{NC} = 0. \end{aligned}$$

2. *When 1-market and 2-market equilibria coexist,*

$$\begin{aligned} \gamma^{Crt} &= 1; & \gamma^{NC} &= \gamma^{NC}(\kappa(\alpha, x)); \\ \theta^{Crt} &= 1 - \alpha x \gamma_0 \bar{n}; & \theta^{NC} &= 1 - [1 - (1 - \alpha x) \{1 - \kappa(\alpha, x)\}] \bar{n}; \\ p^{Crt} &= (1 - \gamma^{NC})\theta^{Crt} - p^{NC}; & p^{NC} &= \gamma^{NC}\theta^{NC} = \bar{c}; \\ \pi_H^{Crt} &= p^{Crt} - \bar{c}; & \pi_H^{NC} &= 0; \\ & & \pi_L^{NC} &= p^{NC}; \end{aligned}$$

where $\kappa(\alpha, x)$ is defined as in Lemma 2, and θ^{Crt} and θ^{NC} represent the marginal consumer in the Crt and NC market respectively.

4.3 Certification Equilibrium and Welfare Analysis

Assuming all aspects and results of the model up to now are common knowledge to the agents, now we explore optimal decisions of high types on certification and conduct welfare analysis. As stated earlier, we are interested in checking whether availability of certification is socially beneficial or not. In order to highlight the welfare decreasing result without involving 2-market equilibria, first we consider the case 1 in Lemma 2 where the collapse of the NC market arises whenever some high types get certification. Next, we confirm that similar results also hold for the other cases in which 2-market equilibria may emerge.

4.3.1 Analysis when only 1-market Equilibria Exist

Throughout this subsection, assume that $\bar{c} \in [\gamma_0^2, \gamma_0)$ is relatively large so that even an arbitrarily small exit of high quality triggers the collapse of the NC market. In period 1, each high type will decide whether to get certified or not given a pair of a certification cost and a test precision, $(z, x) \in \Omega \equiv \mathbb{R}_+ \times (0, 1]$. From the two benchmark cases and Corollary 1, given all other agents' strategies (given α), an individual high type's payoff of opting for certification is given by

$$E\Pi_H(\text{Cert}|\alpha) = x \{(1 - \alpha x \gamma_0 \bar{n}) - \bar{c}\} - z$$

because she gets π_H^{Crt} only with probability x and 0 with the complementary probability, but pays z regardless of the certification outcome. In contrast, the payoff of not getting certified is always given by $E\Pi_H(\text{No Cert}|\alpha) = 0$ regardless of α because in either case she ends up getting $\pi_H^{NC} = 0$.

The optimal decision rule for a high type is derived by comparing the payoffs with and without certification. However, since in our model each agent is atomic, technically it is hard for us to imagine how each individual's deviation affects the equilibrium prices. To this end, we define the certification equilibrium as follows which is particularly useful for the analysis associated with 2-market equilibria:

Definition 2 Given (z, x) , α^* characterizes a certification equilibrium if there is no profitable ϵ -deviation among the high grade producers. A profitable ϵ -deviation means that there exists an arbitrarily small positive number $\epsilon \approx 0$ such that $E\Pi_H(\text{Cert}|\alpha^*) > E\Pi_H(\text{No Cert}|\alpha^* - \epsilon)$ and $E\Pi_H(\text{No Cert}|\alpha^*) > E\Pi_H(\text{Cert}|\alpha^* + \epsilon)$.

Since here $E\Pi_H(\text{Cert}|\alpha)$ and $E\Pi_H(\text{No Cert}|\alpha)$ are monotone and constant with respect to α respectively, we have three different certification equilibrium configurations depending on the costs of certification fixing the certification precision. Given x , when the certification cost is relatively small, a fully separating equilibrium arises ($\alpha^* = 1$). When the certification cost is relatively high, a pooling equilibrium arises ($\alpha^* = 0$). When the certification cost is intermediate, a semi-separating equilibrium arises with $\alpha^* \in (0, 1)$ in which all high quality producers are indifferent between getting certified or not, i.e., $E\Pi_H(\text{Cert}|\alpha^*) = E\Pi_H(\text{No Cert}|\alpha^*)$. The preceding arguments are summarized in the following proposition.

Proposition 2 *Suppose only 1-market equilibria exist (i.e., $\bar{c} \in [\gamma_0^2, \gamma_0)$). Then, given $x \in (0, 1]$, in equilibrium all high types apply for certification ($\alpha^* = 1$) when z is relatively small, only some of them ($\alpha^* \in (0, 1)$) when z is intermediate and no high types ($\alpha^* = 0$) when z is relatively large. Specifically,*

$$\alpha^* = \begin{cases} 1 & \text{if } z \in [0, \tilde{z}(x)], \\ \frac{1}{x\gamma_0\bar{n}} \left[1 - \left(\frac{z}{x} + \bar{c}\right)\right] & \text{if } z \in (\tilde{z}(x), \hat{z}(x)), \\ 0 & \text{if } z \in [\hat{z}(x), \infty), \end{cases}$$

where $\tilde{z}(x) \equiv x \{(1 - x\gamma_0\bar{n}) - \bar{c}\}$ and $\hat{z}(x) \equiv x(1 - \bar{c})$.

As each $(z, x) \in \Omega$ has a unique certification equilibrium, we can define three mutually exclusive subsets of Ω in which fully separating, semi-separating, and pooling equilibria arise respectively:

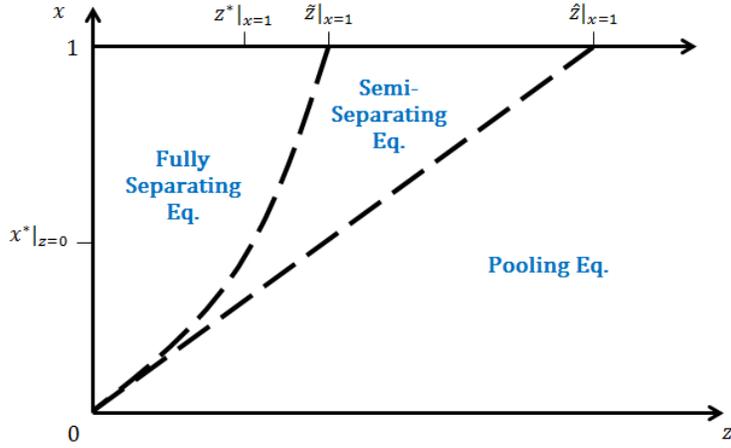
$$\begin{aligned} \Omega^{FS} &\equiv \{(z, x) \in \Omega | z \leq \tilde{z}(x) \text{ and } x \in (0, 1]\}, \\ \Omega^{SS} &\equiv \{(z, x) \in \Omega | \tilde{z}(x) < z < \hat{z}(x) \text{ and } x \in (0, 1]\}, \\ \Omega^P &\equiv \{(z, x) \in \Omega | \hat{z}(x) \leq z \text{ and } x \in (0, 1]\}. \end{aligned}$$

Figure 4 illustrates the three subsets of Ω on the $z - x$ plane. A simple comparative statics analysis suggests that more high quality producers seek certification, ceteris paribus, as z decreases on parameter spaces in which a semi-separating equilibrium emerges (i.e., $\forall (z, x) \in \Omega^{SS}$).

For (z, x) in Ω^{FS} or Ω^{SS} , the welfare function defined as the sum of consumer surplus and firms' expected profits net of certification costs can be written as

$$W^{Crt}(z, x) = \int_{\theta^{Crt}}^1 \theta d\theta - \alpha^* x\gamma_0\bar{n}\bar{c} - \alpha^* \gamma_0\bar{n}z$$

Figure 4: Certification Equilibrium Constellations

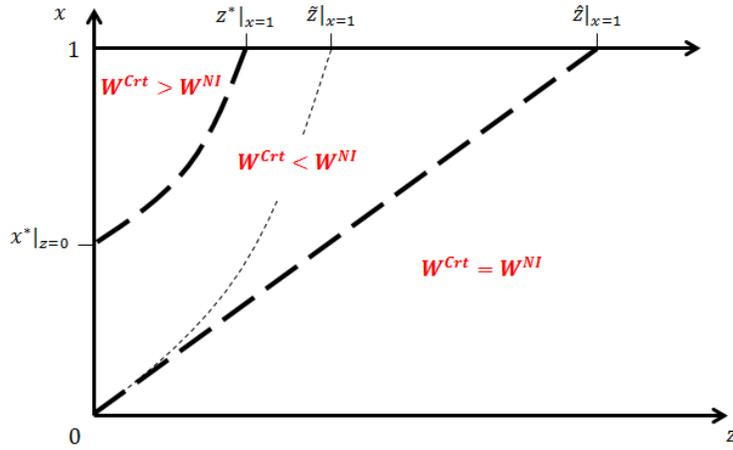


Note: Vertical and horizontal axes represent accuracy of the certification technology and the certification cost respectively.

where $\theta^{Crt} = 1 - \alpha^* x \gamma_0 \bar{n}$ and α^* as defined in Proposition 2. Otherwise, i.e., $(z, x) \in \Omega^P$, $W^{Crt}(z, x) = W^{NI}$. Depending on the parameter values, we have three qualitatively different results. If certification environments are similar enough to the full information benchmark (put differently, if z is low, and x is high), then welfare with certification is greater than welfare under no information ($W^{Crt}(z, x) > W^{NI}$). In this situation, certification is profitable for all high type producers. Even though certification causes the NC market collapse and accrues certification costs, allocative efficiency enhanced by certification outweighs such loss. In the opposite extreme cases, if the cost of certification is too large or the certification technology is too inaccurate, nobody has an incentive to opt for certification. Thus, welfare with certification coincides with welfare under no information ($W^{Crt}(z, x) = W^{NI}$).

The most interesting case is the intermediate area in between the above two extremes in which welfare with certification is lower than welfare under no information ($W^{Crt}(z, x) < W^{NI}$). In order to formally state these results, let us define $\Omega^{NI} \equiv \{(z, x) \in \Omega^{FS} | W^{Crt}(z, x) = W^{NI}\}$ and upper contour set of welfare level W^{NI} as $\Omega^{NI+} \equiv \{(z, x) \in \Omega^{FS} | W^{Crt}(z, x) > W^{NI}\}$.

Figure 5: Welfare Comparisons



Note: Vertical and horizontal axes represent accuracy of the certification technology and the certification cost respectively. $z^*|_{x=1}$ represents a value such that $W^{Crt}(z^*, 1) = W^{NI}$. In a similar fashion, $x^*|_{z=0}$ represent a value such that $W^{Crt}(0, x^*) = W^{NI}$.

Proposition 3 Suppose only 1-market equilibria exist (i.e., $\bar{c} \in [\gamma_0^2, \gamma_0)$). Then,

$$W^{Crt}(z, x) \begin{cases} > W^{NI} & \text{if } (z, x) \in \Omega^{NI+}, \\ = W^{NI} & \text{if } (z, x) \in \Omega^{NI} \cup \Omega^P, \\ < W^{NI} & \text{otherwise,} \end{cases}$$

where Ω^{NI+} is a convex proper subset of Ω^{FS} which is located around the point $(0, 1)$ as shown in Figure 5.

Figure 5 shows a graphical illustration of Proposition 3. It is worth noting that it is not per se certification cost which makes W^{Crt} lower than W^{NI} . As we discussed in the full information benchmark, certification brings welfare gain from reallocating high quality to consumers whose value for the good is relatively high. However, if only a small measure of high grade producers are selected into the *Crt* market, such welfare gain with certification will be negligible. Moreover, given that we consider a parameter space on which the *NC* market always collapses with certification, welfare loss will be huge because most of high type producers now shut down. In other words, the main driving force for a welfare-decreasing result is adverse selection caused by a negative impact of certification on the *NC* market. To see this point clearly, consider the vertical axis in Figure 5. If the certification technology is relatively inaccurate ($x < x^*|_{z=0}$), we see $W^{Crt} < W^{NI}$ even though the certification cost is 0.

4.3.2 Analysis when 1-market and 2-market Equilibria Coexist

We now turn our attention to the other case $\bar{c} \in (0, \gamma_0^2)$ so that 2-market equilibria may exist. For simplicity, we investigate only the following two special cases in turn: “a cost-less ($z = 0$), but imperfect ($x \in (0, 1]$) test” and “a costly ($z \in [0, \infty)$), but perfect ($x = 1$) one”.

Cost-less ($z = 0$), but Imperfect ($x \in (0, 1]$) Test

In this case, from Corollary 1 one can easily see that $\pi_H^{Crt} > 0$ and $\pi_H^{NC} = 0$ for all $\alpha > 0$ (i.e., regardless of whether there exists a 2-market equilibrium or not). Then, since certification is free here, expected payoffs of high types are always higher with certification, i.e., $E\Pi_H(\text{Cert}|\alpha) = x\pi_H^{Crt} > E\Pi_H(\text{No Cert}|\alpha) = 0$ for all $\alpha > 0$, which implies the unique certification equilibrium is a fully separating one ($\alpha^* = 1$).

What remains unknown for welfare calculation is to check when we would have a 2-market equilibria. From Lemma 2, 2-market equilibria are expected for only small x values. Let κ^- and κ^+ the smaller and larger solutions solving for

$$p^{NC}(\kappa|\alpha^* = 1, x \dots) = \bar{c}$$

respectively. Then, if real roots of the above equation exist, then according to our equilibrium selection criteria, we choose $\kappa(\alpha^* = 1, x) = \kappa^+$. Hence, welfare in this case is given by

$$W^{Crt}(x) = \underbrace{\int_{\theta^{Crt}}^1 \theta d\theta - x\gamma_0 \bar{n}\bar{c}}_{\text{Welfare from } Crt \text{ market}} + \underbrace{\int_{\theta^{NC}}^{\theta^{Crt}} \gamma^{NC} \theta d\theta - (1-x)(1-\kappa^+) \gamma_0 \bar{n}\bar{c}}_{\text{Welfare from } NC \text{ market}}$$

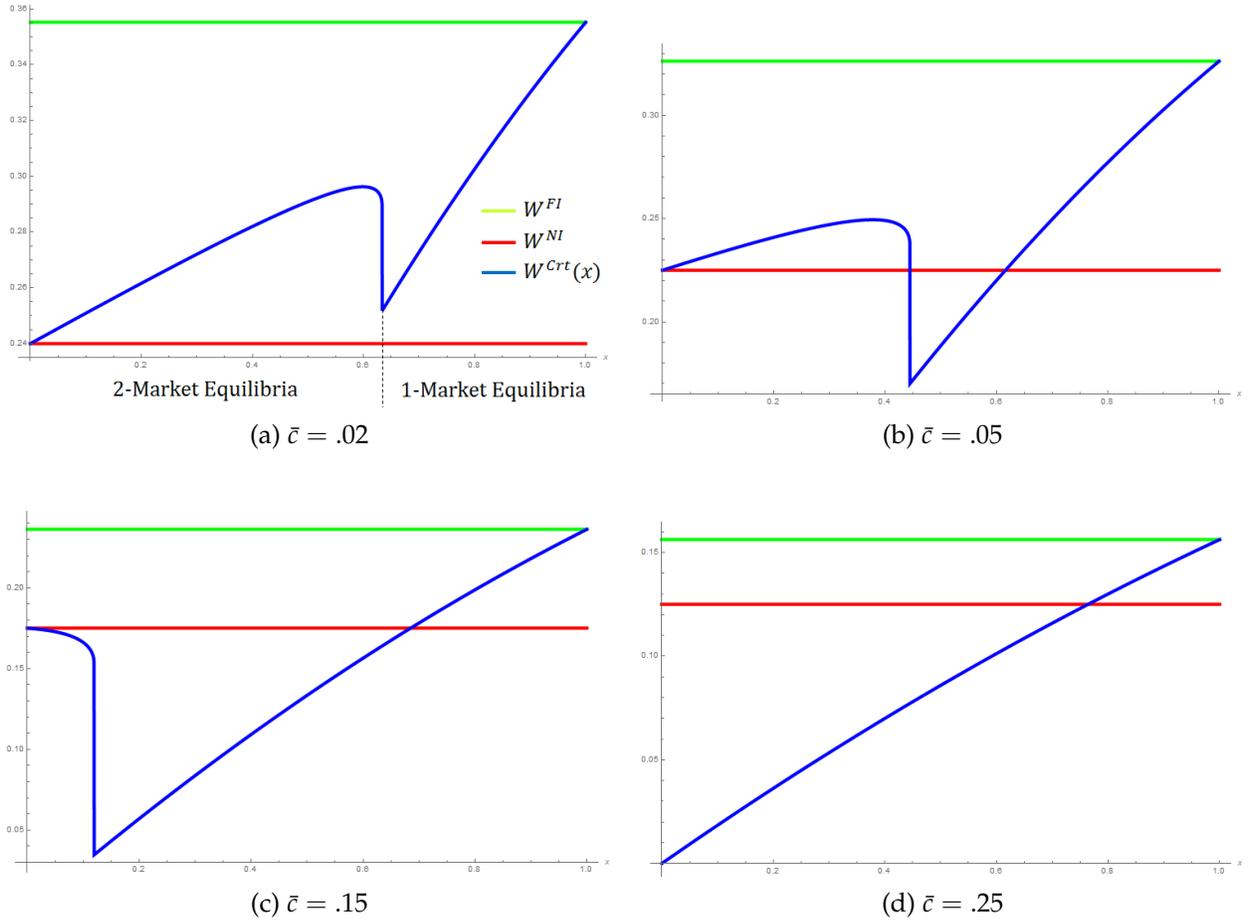
where all corresponding variables are defined as in Lemma 2-2 with $\kappa(\alpha^* = 1, x) = \kappa^+$. If we have just 1-market equilibria, welfare is given by

$$W^{Crt}(x) = \int_{\theta^{Crt}}^1 \theta d\theta - x\gamma_0 \bar{n}\bar{c},$$

where all corresponding variables are defined as in Lemma 2-1.

We demonstrate welfare comparisons of a 2-market equilibrium through simulations when $\gamma_0 = .5$ for various values of production cost \bar{c} in Figure 6. In all 3 panels except $\bar{c} = .25 (= \gamma_0^2)$, we have cutoffs of test precision determining before which 2-market equilibria emerge and otherwise 1-market equilibria. Put differently, if test is more precise than the cutoffs, certification results in the collapse of the NC market in equilibrium. The simulation shows that W^{Crt} could be lower than W^{NI} here too as long as production costs are not too small.

Figure 6: Welfare Comparisons under Cost-less, but Imperfect Certification when $\gamma_0 = .5$



Note: Vertical and horizontal axes represent welfare levels and the certification accuracy respectively.

Costly ($z \in [0, \infty)$), but Perfect ($x = 1$) Test

Now we study the case of costly but perfect certification. More specifically, consider the case where $\gamma_0 = .5$ and $\bar{c} = .1$. Then, depending on certification decisions, whole trajectories of equilibrium prices and profit levels of high grade producers are as shown in Figure 7. As seen in the case of cost-less and imperfect certification, 2-market equilibria prevail if only small number of high types choose to get certified (relatively small α). This causes discontinuities in the price and profit functions.

Given the price equilibrium outcomes, optimal certification decisions depend on the sizes of $E\Pi_H(\text{Cert}|\alpha) = \pi_H^{Crt} - z$ and $E\Pi_H(\text{No Cert}|\alpha) = 0$. Note that if the certification cost is relatively small ($z \leq \tilde{z}$), a fully separating equilibrium is obtained (refer to the right panel of Figure 7). Similarly, if the certification cost is relatively large ($z \geq \hat{z}$), a pooling equilibrium results. Otherwise ($\tilde{z} < z < \hat{z}$), a certification equilibrium candidate must satisfy the following condition: $\pi_H^{Crt}(\alpha^*) = z$. Note that for values of z slightly above \tilde{z} , there might be two equilibrium candidates (α) making high types indifferent between opting for certification and not doing so. However, the certification equilibrium associated with the smaller candidate of the two is not ϵ -stable because high types currently not opting for certification expect the equilibrium price and their profits to increase if a small measure of them deviate. Therefore, the certification equilibrium is characterized by α^* such that $\pi_H^{Crt}(\alpha^*) = z$ and $\partial \pi_H^{Crt} / \partial \alpha|_{\alpha=\alpha^*} < 0$.

One thing to note is that here the highest certification cost supporting a semi-separating equilibrium, \hat{z} , is lower than the one we would have obtained if we did not consider 2-market equilibria (which would have been given as $\hat{z} = 1 - \bar{c} = .9$ as in Proposition 2). Whenever 2-market equilibrium exist, the price in *Crt* market should be set in such a way leaving the exact same amount of surplus to the marginal type consumer (θ^{Crt}) which he would get when purchasing in the *NC* market instead. For this reason, when 2-market equilibria exist, the upper bound supporting a semi-separating certification equilibrium gets smaller.

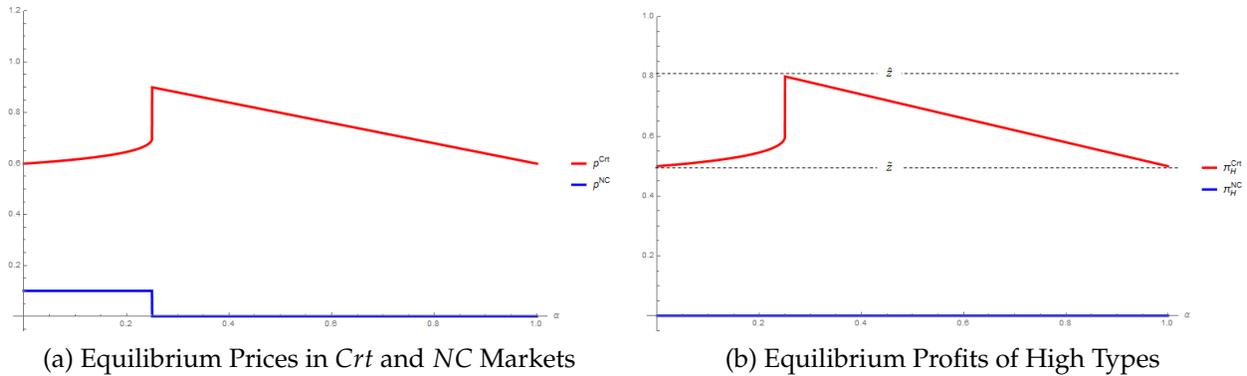
Welfare calculations again depend on the type of equilibrium. If we have 2-market equilibria,

$$W^{Crt}(z) = \underbrace{\int_{\theta^{Crt}}^1 \theta d\theta - \alpha^* \gamma_0 \bar{n} \bar{c}}_{\text{Welfare from } Crt \text{ market}} + \underbrace{\int_{\theta^{NC}}^{\theta^{Crt}} \gamma^{NC} \theta d\theta - (1 - \alpha^*)(1 - \kappa^+) \gamma_0 \bar{n} \bar{c}}_{\text{Welfare from } NC \text{ market}} - \underbrace{\alpha^* \gamma_0 \bar{n} z}_{\text{Certification Costs}},$$

where all corresponding variables are defined as in Lemma 2-2 with $\kappa(\alpha^*, x = 1) = \kappa^+$ and Proposition 2. If we have just 1-market equilibria,

$$W^{Crt}(z) = \int_{\theta^{Crt}}^1 \theta d\theta - \alpha^* \gamma_0 \bar{n} (\bar{c} + z),$$

Figure 7: Price Equilibrium Outcomes under Costly, but Perfect Certification when $\gamma_0 = .5$ and $\bar{c} = .1$



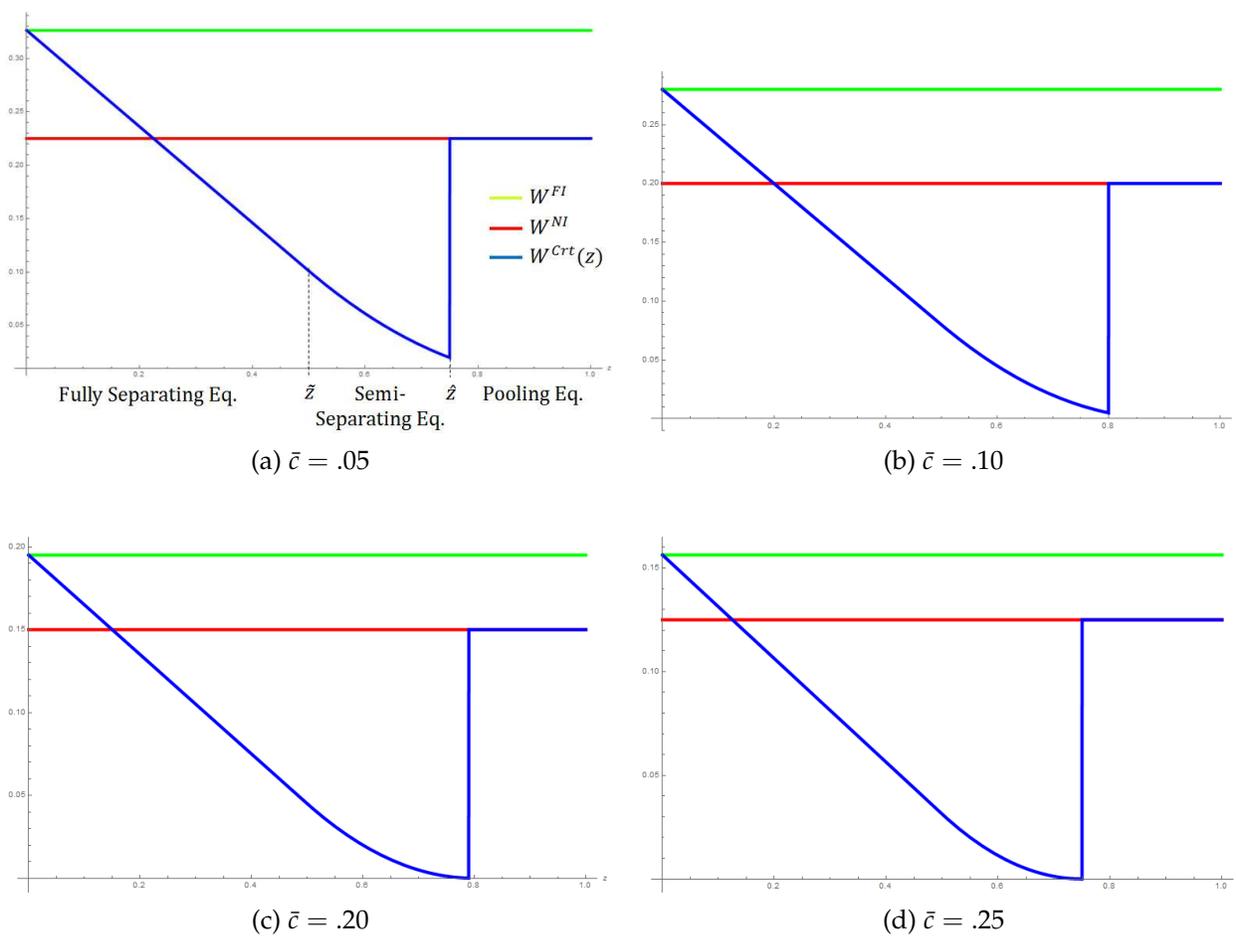
Note: Vertical and horizontal axes represent price and profit levels and measure of high grade producers opting for certification respectively.

where all corresponding variables are defined as in Lemma 2-1 and Proposition 2. Figure 8 summarizes simulation results regarding welfare levels for various values of production cost. It confirms once again that welfare could be lower with certification. In all cases, as the certification cost, z , increases from 0 to a certain threshold, welfare with certification monotonically decreases from the full information level to the one lower than the no information level. Beyond the threshold, it becomes the same as the no information level.

5 Conclusion

It is well-known that markets with asymmetric information about quality or other vertically differentiating goods characteristics suffer from allocative inefficiencies. Overcoming these asymmetries can lead to welfare increases. However, this understanding overlooks the close relationships between markets. Allowing for certification may create the co-existence of both a certified and a non-certified market. Both consumers and firms subsequently self-select into one market, the other market, or opting out. Total welfare is a function of these self-selections, and thus welfare implications cannot be considered in isolation. If certification is imperfect or costly, then positive externalities of high quality in non-certified markets can be destroyed without a complete offsetting through added welfare in the certified markets. This implies subtle policy-implications when evaluating the introduction of government certification programs and regulations of industry self-governance with respect to certification programs.

Figure 8: Welfare Comparisons under Costly, but Perfect Certification when $\gamma_0 = .5$



Note: Vertical and horizontal axes represent welfare levels and certification costs respectively.

We close this paper by discussing one feasible extension that may offer additional interesting insights. It would be interesting to add a first stage to the game in which firms make their entry decisions. This will allow us to see long-run consequences of certification depending on different certification environments, which have received little interest in most of existing studies. We leave this idea for future research to consider.

Appendix

Proof of Proposition 1

Proof of proposition 1 is in the text of the paper. ■

Proof of Lemma 1

Let us show $p^{Crt} > p^{NC}$ first. Suppose there exist a price equilibrium where at least some high quality goods are traded and $p^{Crt} \leq p^{NC}$. Then, since $\gamma^{Crt} \geq \gamma^{NC}$,

$$\gamma^{Crt}\theta - p^{Crt} > \gamma^{NC}\theta - p^{NC} \forall \theta \in [0, 1].$$

Then, all consumers $\theta \geq p^{Crt}/\gamma^{Crt}$ wants to buy in *Crt*. Having some equilibrium output of high quality goods, it must be the case that the cutoff type should be in the interior of support of consumer types i.e., $p^{Crt}/\gamma^{Crt} \in (0, \bar{\theta})$. Otherwise, there is no demand for both markets, and such an equilibrium does not exist. Thus, it is sufficient to only consider the following range of $p^{Crt} \in (0, \gamma^{Crt})$: If $0 < p^{Crt} < \bar{c}$, there is no supply in *Crt* market even though there is demand, so such prices are not market clearing. If $\bar{c} \leq p^{Crt} < \gamma^{Crt}$, there are always some low quality producers willing to sell in *NC* since $p^{NC} \geq p^{Crt} \geq \bar{c} > 0$ while there is no demand at all for that market. Hence, these prices are not market clearing either, which implies $p^{Crt} > p^{NC}$ must be true.

The second argument is obvious from the binary feature of firm's optimal decision rule. ■

Proof of Lemma 2

Given \bar{n} (or \bar{c}) and $\alpha \in (0, 1)$, for a 2-market equilibrium to exist, the following conditions must hold:

1. Existence of marginal types (θ^{Crt}) between buying in *Crt* market and buying in *NC* market:

$$\gamma^{Crt}\theta^{Crt} - p^{Crt} = \gamma^{NC}\theta^{Crt} - p^{NC};$$

2. Existence of marginal types (θ^{NC}) between buying in *NC* market and not buying:

$$\gamma^{NC}\theta^{NC} - p^{NC} = 0;$$

3. Market clearing condition in *Crt* market

$$q^{Crt/D} \equiv 1 - \theta^{Crt} = \alpha\gamma_0\bar{n} \equiv q^{Crt/S};$$

4. Market clearing condition in *NC* market

$$q^{NC/D} \equiv \theta^{Crt} - \theta^{NC} = [(1 - \alpha x)\kappa\gamma_0 + (1 - \gamma_0)]\bar{n} \equiv q^{NC/S};$$

5. Consistent beliefs: $\gamma^{Crt} = 1, \gamma^{NC} = \frac{(1-\alpha x)\kappa\gamma_0}{(1-\alpha x)\kappa\gamma_0+(1-\gamma_0)} \leq \frac{(1-\alpha x)\gamma_0}{(1-\alpha x)\gamma_0+(1-\gamma_0)}$ where $\kappa \in [0, 1]$.

6. From high grade firm's indifferent condition between selling in *NC* and being inactive and the definition of \bar{n} , for all $\kappa \in [0, 1]$

$$p^{NC} = \bar{c}.$$

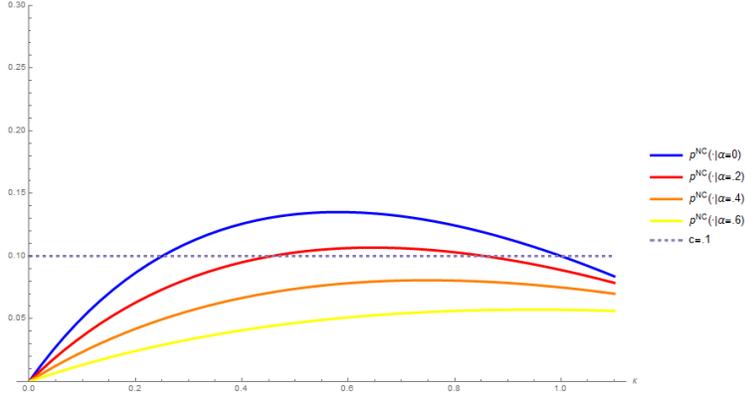
From the condition 3, $\theta^{Crt} = 1 - \alpha\gamma_0\bar{n}$. Plugging this in the condition 4, we end up having $\theta^{NC} = \{1 - [1 - (1 - \alpha x)(1 - \kappa)\gamma_0]\bar{n}\}$. Since p^{Crt} can be adjusted according to the condition 1 once p^{NC} is determined, the existence of a 2-market equilibrium pins down to find a proper κ satisfying $p^{NC} = \bar{c}$. More specifically, from the condition 2, 5 and 6, if there exists $\kappa \in [0, 1]$ which solves the following equation,

$$p^{NC}(\kappa|\alpha, x \dots) \equiv \frac{(1 - \alpha x)\gamma_0\kappa}{(1 - \alpha x)\gamma_0\kappa + (1 - \gamma_0)} \{1 - [1 - (1 - \alpha x)(1 - \kappa)\gamma_0]\bar{n}\} = \bar{c},$$

a 2-market equilibrium exists. Before solving the equation, let us show that welfare is maximized with the largest value $\kappa(\alpha, x \dots)$ satisfying the above equation. First, note that welfare generated from the *Crt* market will be the same no matter what kind of equilibrium we have because q^{Crt} and γ^{Crt} are invariant. What only changes in the *Crt* market is p^{Crt} which is a net transfer from consumers to producers over different kinds of equilibria. Therefore, whenever a 2-market equilibrium exists, a 1-market equilibrium is not a welfare maximizing one. Then, it remains to pick up a 2-market equilibrium resulting in the highest welfare level, which is obviously the largest value $\kappa(\alpha, x \dots)$ such that $p^{NC} = \bar{c}$.

The first and second derivative of $p^{NC}(\kappa|\alpha, x \dots)$ with respect to κ are respectively,

Figure 9: Certification Equilibrium Constellations



Note: We assume that $\gamma_0 = .5$, $\bar{c} = .1$ and $x = 1$. Vertical and horizontal axes represent p^{NC} and κ respectively.

$$\frac{\partial p^{NC}}{\partial \kappa} = \frac{\left\{ \begin{array}{l} \gamma_0(1-x\alpha) \times [1-\gamma_0 - \{\gamma_0^2(1-x\alpha)^2\kappa^2 \\ + 2\gamma_0(1-\gamma_0)(1-x\alpha)\kappa + (1-\gamma_0)(1-\gamma_0(1-x\alpha))\} \bar{n}] \end{array} \right\}}{[1-\gamma_0 + \gamma_0(1-x\alpha)\kappa]^2}$$

$$\frac{\partial^2 p^{NC}}{\partial \kappa^2} = -\frac{2\gamma_0^2(1-\gamma_0)(1-\gamma_0 x \alpha \bar{n})(1-x\alpha)^2}{[1-\gamma_0 + \gamma_0(1-x\alpha)\kappa]^3} < 0.$$

Note that $p^{NC}(\kappa|\alpha, \dots)$ has a similar shape with a concave quadratic curve passing through the origin around which the slope of the curve is positive. Furthermore, as α increases from 0 to 1, this curve gets closer to the horizontal axis fixing κ and has a (weakly) larger argmax of the function (i.e., $\partial p^{NC}/\partial \alpha < 0$ given κ for all $\kappa \in [0, 1]$ and $\partial(\arg \max_{\kappa} p^{NC})/\partial \alpha \geq 0$). Even though, Figure 9 is drawn for the case where $\gamma_0 = .5$, $\bar{c} = .1$ (so $\bar{n} = .8$) and $x = 1$, the shape of p^{NC} is similar for all other parameter values. Also, we will have the same properties when it comes to x rather than α because $p^{NC}(\kappa|\alpha, x, \dots)$ is symmetric with respect to α and x .

Note that $p^{NC}(\kappa = 1, \alpha = 0) = \bar{c}$ always by construction. So, if

$$\left. \frac{\partial p^{NC}}{\partial \kappa} \right|_{\alpha=0, \kappa=1} \geq 0,$$

2-market equilibria do not exist. Now observe that

$$\begin{aligned} \text{sign} \left(\frac{\partial p^{NC}}{\partial \kappa} \Big|_{\alpha=0, \kappa=1} \right) &= \text{sign} \left([1 - \gamma_0 - \{\gamma_0^2 + 2\gamma_0(1 - \gamma_0) + (1 - \gamma_0)^2\} \bar{n}] \right) \\ &= \text{sign} (1 - \gamma_0 - \bar{n}). \end{aligned}$$

Therefore, $\frac{\partial p^{NC}}{\partial \kappa} \Big|_{\alpha=0, \kappa=1} \geq 0 \iff 1 - \gamma_0 - \bar{n} \geq 0$ which is equivalent to $\bar{c} \geq \gamma_0^2$.

Even though we may have some 2-market equilibria if $\bar{c} < \gamma_0^2$, that is the case only for small α . Finally, whenever a 2-market equilibria exists, the welfare maximizing $\kappa(\alpha)$ is the larger root of the equation $p^{NC}(\kappa|\alpha, \dots) = \bar{c}$. Now it completes the proof to define ζ as a positive number making $p^{NC}(\kappa|\alpha x = \zeta, \dots) = \bar{c}$ have only one solution. ■

Proof of Proposition 2

Obvious from the discussion in the text and thus omitted. ■

Proof of Proposition 3

We will walk through a sequence of steps leading to the desired results.

Step 1: $W^{Crt}(z, x)$ is strictly decreasing in z and strictly increasing in x on Ω^{FS} since $\partial W^{Crt}(z, x) / \partial z = -x\gamma_0\bar{n} < 0$ and

$$\begin{aligned} \partial W^{Crt}(z, x) / \partial x &= (1 - x\gamma_0\bar{n})\gamma_0\bar{n} - \gamma_0\bar{n}\bar{c} \\ &= \{(1 - x\gamma_0) - \bar{c}(1 - x)\} \gamma_0\bar{n} \\ &> (1 - \bar{c})(1 - x) \gamma_0\bar{n} \\ &> 0 \end{aligned}$$

where the first inequality follows from $\gamma_0 < 1$, and the second inequality follows from Assumption 1.

Step 2: If x is too small, $W^{Crt}(z, x) < W^{NI}$ regardless of z on Ω^{FS} . To see this, fix $z = 0$. Since $W^{Crt}(0, 1) = W^{FI} > \lim_{x \rightarrow 0} W^{Crt}(0, x) = 0$ and $\frac{\partial W^{Crt}(z, x)}{\partial x} > 0$, there exists a unique x^* such that $W^{Crt}(0, x) \leq W^{NI}$ if and only if $x \leq x^*$ by the Intermediate Value Theorem. Then, for all $(z, x) \in \Omega^{FS}$ such that $x < x^*$, $W^{Crt}(z, x) < W^{NI}$ since $W^{Crt}(z, x)$ strictly decreases in z .

Step 3: For each $x > x^*$ on Ω^{FS} , there exists $z^*(x) < \tilde{z}(x)$ such that $W^{Crt}(z, x) \leq W^{NI}$ if and

only if $z \geq z^*(x)$. First, let us show that $\lim_{z \rightarrow \tilde{z}(x)} W^{Crt}(z, x) < W^{NI}$.

$$\begin{aligned}
\lim_{z \rightarrow \tilde{z}(x)} W^{Crt}(z, x) - W^{NI} &= (1/2) \left[1 - (1 - x\gamma_0\bar{n})^2 \right] - x\gamma_0\bar{n}\bar{c} - \gamma_0\bar{n}\tilde{z}(x) \\
&\quad - \left\{ (1/2)\gamma_0 \left[1 - (1 - \bar{n})^2 \right] - \gamma_0\bar{n}\bar{c} \right\} \\
&= \gamma_0\bar{n} \left\{ (1/2) \left[x^2\gamma_0\bar{n} - (2 - \bar{n}) \right] + \bar{c} \right\} \\
&< \gamma_0\bar{n} \left\{ (1/2) \left[\bar{n} - (2 - \bar{n}) \right] + \bar{c} \right\} \\
&= -\gamma_0\bar{n} (1/\gamma_0 - 1) \bar{c} \\
&< 0
\end{aligned}$$

where the second equality follows from arranging terms; third equality follows from arranging terms after plugging in $1 - (\bar{c}/\gamma_0)$ for \bar{n} ; the first inequality follows from $x^2\gamma_0 < 1$. Along with this results and the fact that $W^{Crt}(0, x) > W^{NI}$ for all $x > x^*$, we proved the point again by the Intermediate Value Theorem.

Step 4: From Step 2 and 3, $\Omega^{NI+} = \{(z, x) \in \Omega^{FS} | z < z^*(x) \text{ and } x > x^*\}$ and $\Omega^{NI+} \subsetneq \Omega^{FS}$.

Step 5: For all $(z, x) \in \Omega^{SS}$, $W^{Crt}(z, x) < W^{NI}$. The welfare was defined as the sum of consumer surplus. Compared to the fully separating case, now less amount of consumers are served in equilibrium given x while consumer valuation for the good remains the same as before so that consumer surplus decreases. In addition to this, the aggregate expected profits are lower in semi-separating equilibrium because by definition producers earn 0 now while their profits are positive in fully separating equilibria. In sum, $W^{Crt}(z, x)$ is lower than $W^{Crt}(\tilde{z}(x), x)$, which implies $W^{Crt}(z, x) < W^{NI}$ for all $z \in (\tilde{z}(x), \hat{z}(x))$ given x . ■

References

- Akerlof, G. A.** 1970. "The Market for "Lemons": Quality Uncertainty and the Market Mechanism." *The Quarterly Journal of Economics*, 84(3): 488–500.
- Albano, G. L., and A. Lizzeri.** 2001. "Strategic Certification and Provision of Quality." *International Economic Review*, 42(1): 267–283.
- Baltzer, K.** 2012. "Standards vs. Labels with Imperfect Competition and Asymmetric Information." *Economics Letters*, 114(1): 61–63.
- Bar-Isaac, H., G. Caruana, and V. Cuñat.** 2012. "Information Gathering Externalities for a Multi-Attribute Good." *The Journal of Industrial Economics*, 60(1): 162–185.
- Benabou, R., and G. Laroque.** 1992. "Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility." *Quarterly Journal of Economics*, 107(3): 921–958.
- Board, O.** 2009. "Competition and Disclosure." *The Journal of Industrial Economics*, 57(1): 197–213.

- Bonroy, O., and C. Constantatos.** 2008. "On the Use of Labels in Credence Goods Markets." *Journal of Regulatory Economics*, 33(3): 237–252.
- , and ———. 2015. "On the Economics of Labels: How Their Introduction Affects the Functioning of Markets and the Welfare of All Participants." *American Journal of Agricultural Economics*, 97(1): 239–259.
- Creane, A.** 1998. "Ignorance Is Bliss as Trade Policy." *Review of International Economics*, 6(4): 616–624.
- , and **T. D. Jeitschko.** 2016. "Endogenous Entry in Markets with Unobserved Quality." *The Journal of Industrial Economics*, 64(3): 494–519.
- Darby, M. R., and E. Karni.** 1973. "Free Competition and the Optimal Amount of Fraud." *The Journal of Law & Economics*, 16(1): 67–88.
- De, S., and P. Nabar.** 1991. "Economic Implications of Imperfect Quality Certification." *Economics Letters*, 37(4): 333–337.
- Dranove, D., and G. Z. Jin.** 2010. "Quality Disclosure and Certification: Theory and Practice." *Journal of Economic Literature*, 48(4): 935–963.
- Dulleck, U., and R. Kerschbamer.** 2006. "On Doctors, Mechanics, and Computer Specialists: The Economics of Credence Goods." *Journal of Economic Literature*, 44(March): 5–42.
- Fischer, C., and T. P. Lyon.** 2014. "Competing Environmental Labels." *Journal of Economics & Management Strategy*, 23(3): 692–716.
- Gavazza, A., and A. Lizzeri.** 2007. "The Perils of Transparency in Bureaucracies." *American Economic Review*, 97(2): 300–305.
- Grossman, S. J.** 1981. "The Informational Role of Warranties and Private Disclosure about Product Quality." *The Journal of Law and Economics*, 24(3): 461–483.
- Grubb, M. D.** 2011. "Developing a Reputation for Reticence." *Journal of Economics & Management Strategy*, 20(1): 225–268.
- Guo, L., and Y. Zhao.** 2009. "Voluntary Quality Disclosure and Market Interaction." *Marketing Science*, 28(3): 488–501.
- Harbaugh, R., J. W. Maxwell, and B. Roussillon.** 2011. "Label Confusion: The Groucho Effect of Uncertain Standards." *Management Science*, 57(9): 1512–1527.
- Hotz, V. J., and M. Xiao.** 2013. "Strategic Information Disclosure: The Case of Multiattribute Products with Heterogeneous Consumers." *Economic Inquiry*, 51(1): 865–881.
- Hvide, H. K.** 2009. "Oligopolistic Certification." *The B.E. Journal of Theoretical Economics*, 9(1): 1–21.
- Jovanovic, B.** 1982. "Truthful Disclosure of Information." *Bell Journal of Economics*, 13(1): 36–44.
- Klein, B., and K. B. Leffler.** 1981. "The Role of Market Forces in Assuring Contractual Performance." *The Journal of Political Economy*, 89(4): 615–641.
- Lizzeri, A.** 1999. "Information Revelation and Certification Intermediaries." *The RAND Journal of Economics*, 30(2): 214–231.

- Mason, C. F.** 2011. "Eco-Labeling and Market Equilibria with Noisy Certification Tests." *Environmental and Resource Economics*, 48(4): 537–560.
- , **and F. P. Sterbenz.** 1994. "Imperfect Product Testing and Market Size." *International Economic Review*, 35(1): 61–86.
- Matthews, S., and A. Postlewaite.** 1985. "Quality Testing and Disclosure." *The RAND Journal of Economics*, 16(3): 328–340.
- Milgrom, P. R.** 1981. "Good News and Bad News: Representation Theorems and Applications." *The Bell Journal of Economics*, 12(2): 380–391.
- Mussa, M., and S. Rosen.** 1978. "Monopoly and Product Quality." *Journal of Economic Theory*, 18(2): 301–317.
- Shapiro, C.** 1983. "Premiums for High Quality Products as Returns to Reputations." *The Quarterly Journal of Economics*, 98(4): 659–680.
- Shavell, S.** 1994. "Acquisition and Disclosure of Information Prior to Sale." *The RAND Journal of Economics*, 25(1): 20–36.
- Spence, M.** 1977. "Consumer Misperceptions, Product Failure and Producer Liability." *The Review of Economic Studies*, 44(3): 561–572.
- Viscusi, W. K.** 1978. "A Note on "Lemons" Markets with Quality Certification." *Bell Journal of Economics*, 9(1): 277–279.
- Zago, A. M., and D. Pick.** 2004. "Labeling Policies in Food Markets: Private Incentives, Public Intervention, and Welfare Effects." *Journal of Agricultural and Resource Economics*, 29(1): 150–165.