Time-varying Fiscal Multipliers Identified by Systematic Component: A Bayesian Approach to TVP-SVAR model

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Identified by Systematic Component:
A Bayesian Approach to TVP-SVAR model *

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Abstract

This study estimates time varying fiscal multipliers from the aspect of fiscal policy rules derived from the systematic component along the line of "Agnostic Identification Procedure" proposed by Caldara and Kamps (2017) for the US economy between 1952:Q1-2018:Q1. To do so, we adopt time-varying parameter structural vector autoregressive (TVP-SVAR) with MCMC procedure by a Bayesian approach, and identify both of government spending and tax cut shocks using the zero and sign restrictions method proposed by Arias, Rubio-Ramirez and Waggoner (2018). And we compare those values with time varying version identified by standard sign restriction along the line of Mountford and Uhlig (2009). Our estimation reports that time-varying fiscal multipliers of output by government spending rule could be nearly double for one year but decline to unity after eight years, and seem to have been very stable for long terms such as sixty years. By contrast, those of tax cut rule are more fluctuate and negative for long run except the 1990's.

Keywords: Bayesian estimation, time-varying-parameter Structual VAR, Sign and Zero Restrictions,

JEL Classifications: C32, E32, E62

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1 Introduction

The effectiveness of fiscal policy has been the subject of a long-standing debate among economists and policymakers. Even though governments around the world often implement fiscal stimulus packages in hope that they will counter the economic downturn, in fact, there is still no clear-cut answer to how and why the fiscal policy can have an impact on the economy. Theoretically, neoclassical and New Keynesian models can lead to the exact opposite implications about the policy increasing public spending or cutting taxes. While empirical investigation could be expected to shed light on the debate, almost all existing identification schemes at least partly rely on the assumptions suggested by the theoretical models, and the different estimation methods or identification strategies lead to substantial disagreement on the sign and the size of fiscal multipliers. [1]

Another problem of assessing the effect of fiscal policy is that even if we can choose one specific empirical approach, results may vary as the sample period varies. The possibility of structural change or time variation of transmission mechanism of the fiscal policy can be important problems for the government, since the massive fiscal stimulus are always put in place when the severe economic downturn happens, which tends to come with significant structural changes. Indeed, several empirical studies suggests the shift of fiscal multipliers with business cycles (Tagkalakis (2008), Auerbach and Gorodnichenko (2012), Bachmann and Sims (2012), Candelon and Lieb (2013) and Caggiano et al. (2015)) and the volume of public debt level (Favero et al. (2011), Corsetti et al. (2012) and Ilzetzki et al. (2013)).

In this paper, we uncover changes in the effects of government spending shocks in the US over the period 1952-2018. To address the issues mentioned above, our methods have two distinct features. First, to identify both of government spending and tax cut shocks in a parsimonious and a data-consistent way, we use the systematic component identification scheme proposed by Caldara and Kamps (2017) and Arias et al. (2018). According to Caldara and Kamps (2017), frequently used identification method, which includes recursive ordering assumptions on the structural shocks, sign restriction on the impulse response and Proxy SVAR approach, can be characterized by the different type of the restrictions on the systematic component of fiscal policy represented as the rules relating policy instruments (e.g. government spending, tax rate) to macroeconomic conditions. In these existing methods, the sign and the size of coefficients on non-policy variables (e.g. output, inflation rate, interest rate) of such policy rule are implicitly assigned by the elements outside the data, for example economic theory, timing assumption, or estimation methods itself. In contrast, the systematic component identification approach explicitly uses information of directly estimated policy rule, so the imposed identification assumption is based on the evidence consistent with the data. In

[1] For a comprehensive survey, see Chinn (2013).

this paper, we impose the sign and zero restriction on the contemporaneous response of government spending to the changes of other non-policy variables based on the estimation results of fiscal spending and tax rules by Caldara and Kamps (2017). The method allows us to be free of any a priori assumption about the response of non-policy variables to the fiscal spending shocks, to adopt an agnostic position concerning the still controversial topics such as the existence and size of crowding effects of government expenditure and to provide useful information to test various theoretical implications the different models have suggested.

Second, we document the time-variation of fiscal spending multiplier using the tools of Bayesian time-varying parameters VAR (TVP-VAR) model. As argued in Cogley and Sargent (2005) and Primiceri (2005), the model has great flexibility in terms of capturing non-linearities and time heterogeneity and outperforms simpler methods including subsample or rolling-windows estimation by allowing us to estimate, not impose a priori, the number and the timing of the breaks. Although TVP-VAR models have been already used in a large number of papers focusing on monetary policy (Cogley and Sargent (2001), Cogley and Sargent (2005), Primiceri (2005), Baumeister, and Benati (2013), Belongia and Ireland (2016)), much less work has been done on the fiscal policy analysis. Kirchner, Cimadomo, and Hauptmeier (2010) investigates changes in the impact of EURO area's government spending shocks using a recursive identification scheme. Pereira and Lopes (2014) estimates the TVP-VAR model with Blanchard and Perotti (2002) type restriction using the US data before the global financial crisis. To best of our knowledge, this is the first paper to combine the systematic component approach with the TVP-VAR framework and to document the time-varying fiscal spending multipliers in the US.

Using five endogenous variables, i.e., (1) government spending, (2) real GDP (3) tax revenue (4) inflation (5) nominal interest rate, we estimate for the US economy between 1952:Q1-2018:Q1. The main findings are as follows. Time-varying fiscal multipliers of output by government spending rule could be nearly double for one year but decline to unity after eight years, and seem to have been very stable for sixty years. By contrast, those of tax cut rule are negative except period of the 1990's. We verify that this method is quite useful to do this end by compared with identifications by only sign restrictions, since the contemporaneous elasticity of output and inflation, whose signs and zero restriction this method can control, is thought to strongly affect both size and direction of fiscal multipliers.

The remaining of this paper is organized as follows. Section 2 describes about the TVP-VARs as well as identifications of fiscal policy shocks. Estimated results including time-varying impulse responses of the policy shocks and time-varying fiscal multipliers are reported in Section 3. Section 4 concludes. The methods of the identification with sign and zero restrictions and a Bayesian inference including algorithms for MCMC simulation for TVP-VARs are described in four appendix sections.

2 Empirical Methodology

In this section, we describe empirical methodology measuring time variations of fiscal multipliers. In the first subsection, a TVP VAR model incorporated with stochastic volatilities (SV) in its disturbance terms is introduced as our backbone model. The distinguished advantage of the model is to be designed to make coefficients and the covariance matrix of innovations time-vary in terms of all aspects from the viewpoint of 'agnostic'. The second subsection describes how to identify two fiscal policy shocks using the systematic component approach. The final two subsections deals with calculating fiscal multipliers and data used for estimation. Again, the methods of the identification with sign and zero restrictions and a Bayesian inference including algorithms for MCMC simulation for TVP-VARs are described in four appendix sections.

2.1 Set up TVP-VAR-SV

Consider the p-th lag length structual vector autoregression (SVAR(p)) model defined as

$$A_{0,t}\mathbf{y}_t = A_{1,t}\mathbf{y}_{t-1} + \cdots + A_{p,t}\mathbf{y}_{t-p} + \Sigma_t\varepsilon_t,\ \varepsilon_t \sim N(0, I), \tag{1}$$

where $\mathbf{y}_t$ is a $k \times 1$ vector of observed variables, structural parameters $A_{i,t}$, $i = 1, ..., p$, are $k \times k$ matrices of time varying coefficients, and a contemporaneous matrix $A_{0,t}$ is invertible and decomposed into a orthogonal matrix Q_t , i.e., $Q_t Q_t' = I$, and a lower triangular matrix $A_{tr,t}$ such that $A_{0,t} = Q_t A_{tr,t}$, where

$$A_{tr,t} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a_{21,t} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ a_{k1,t} & \cdots & a_{kk-1,t} & 1 \end{bmatrix}.$$

The disturbance ε_t is a $k \times 1$ vector of structural shocks and a time-varying covariance matrix Σ_t is a diagonal matrix that contains the stochastic volatilities which reflect the changes of the independent structural shocks $\sigma_{i,t}$ such as

$$\Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{k,t} \end{bmatrix}.$$

And the p-th lag length reduced VAR (p) model corresponding to above SVAR model is given by

$$\mathbf{y}_t = B_{1,t}\mathbf{y}_{t-1} + \cdots + B_{p,t}\mathbf{y}_{t-p} + u_t,\ u_t \sim N(0, \Omega_t), \tag{2}$$

where $B_{i,t}$ is a time varying reduced-form parameters given by $B_{i,t} = A_{0,t}^{-1}A_{i,t}$, and u_t is a one-period ahead forecasting error: $u_t = A_{0,t}^{-1}\Sigma_t\varepsilon_t$, because $A_{0,t}\Omega A_{0,t}' = \Sigma_t\Sigma_t'$. And also we can rewrite the one-period ahead forecasting errors as $u_t = A_{tr,t}^{-1} Q_t \Sigma_t\varepsilon_t$, using $A_{0,t}^{-1} = A_{tr,t}^{-1} Q_t'$. Notice that Q_t is a random matrix so that we can select its value to make structural shocks identified to satisfy zero and sign restrictions, as explained in the next subsection.

Letting β_t be a stacked $k^2p \times 1$ vector of the elements in the rows of the $k \times k$ matrices of the $B_{1,t}, \cdots, B_{p,t}$, and a_t be the vector of non-zero and non-one elements of the lower triangular matrix $A_{tr,t}$. h_t is the logarithm of the diagonal elements of time varying volatilities matrix, $\ln \sigma_{j,t}^2$. The dynamics of the time varying parameters of the reduced form are following random walk process as below.

$$\beta_{t+1} = \beta_t + u_{\beta,t}, \tag{3}$$

$$a_{t+1} = a_t + u_{a,t}, \tag{4}$$

$$h_{t+1} = h_t + u_{h,t}, \tag{5}$$

where $\beta_t = (\beta_{11,t}, ..., \beta_{kk,t})$, $a_t = (a_{21,t}, ..., a_{kk-1,t})$ and $h_t = (h_{1,t}, ..., h_{k,t})$ with $h_{j,t} = \ln \sigma_{j,t}^2$ for $j = 1, ..., k$. And $u_{b,t}, u_{a,t}$, and $u_{h,t}$, are assumed to be normally distributed with a zero mean and diagonal covariance matrices, Σ_β, Σ_a, and Σ_h . The structural shocks are also assumed to independent with the time-varying parameters such as

$$\begin{bmatrix} \varepsilon_t \\ u_{\beta,t} \\ u_{a,t} \\ u_{h,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \Sigma_\beta^2 & 0 & 0 \\ 0 & 0 & \Sigma_a^2 & 0 \\ 0 & 0 & 0 & \Sigma_h^2 \end{bmatrix} \right). \tag{6}$$

2.2 Identification by Systematic Components

Fiscal Policy Rules

Systematic component approach to identify the structural shocks is first proposed by Caldara and Kamps (2017) and Arias et al. (2018). They focus on the fact that the identification of policy shocks implies the specification of the systematic component of policy, which describes how policy usually reacts to economic conditions.[2] As pointed out in Caldara and Kamps (2017), labeling a structural shock in the SVAR as the goverment spending shock is equivalent to specifying the same equation as the government spend-

[2]This fact is pointed out by Leeper, Sims, and Zha (1996), Leeper and Zha (2003), and Sims and Zha (2006a).

ing rule. Without loss of generality, we let the first shock be the government spending shock. The first equation of (1)

$$\mathbf{a}_{0,t,1}\mathbf{y}_t = \sum_{\ell=1}^{p} \mathbf{a}_{\ell,t,1}\mathbf{y}_{t-l} + \sigma_{1,t}\varepsilon_{1t} \tag{7}$$

is the time-varying government spending rule, where ε_{1t} denotes the first entry of ε_t, $\mathbf{a}_{\ell,t,1}$ denotes the first row of $A_{\ell,t}$ for $0 \leq \ell \leq p$, and $a_{\ell,t,ij}$ denotes the (i,j) entry of $A_{\ell,t}$. From equation (7), it is clear that restricting the systematic component of government spending policy is equivalent to restricting $\mathbf{a}_{\ell,t,1}$ for $0 \leq \ell \leq p$.

Our TVP-VAR model consists of five endogenous variables: government spending, defined as the sum of government consumption and investment (g_t); gross domestic product (y_t); consumer price inflation (π_t); the 3-month T-bill rate (r_t); and federal tax revenue (tax_t). We take the natural logarithm and extract a quadratic trend for all the variables in per capita terms except π_t and r_t. We estimate the model on quarterly data for the U.S. from 1952 to 2018.

Our government spending rule can be written as

$$g_t = \underbrace{\psi_{y,t}\, y_t + \psi_{r,t}\, r_t + \psi_{\pi,t}\, \pi_t + \psi_{tax,t}\, tax_t}_{systematic\, component} + \underbrace{\varepsilon_{g,t}}_{shock.} \tag{8}$$

where $\psi_{y,t} = -a_{0,t,11}^{-1}a_{0,t,12}$, $\psi_{\pi,t} = -a_{0,t,11}^{-1}a_{0,t,13}$, $\psi_{r,t} = -a_{0,t,11}^{-1}a_{0,t,14}$, and $\psi_{tax,t} = -a_{0,t,11}^{-1}a_{0,t,15}$. Since our identification concentrates on the contemporaneous structural parameters, we here abstract from lag variables. Based on the estimation results of Caldara and Kamps (2017), we impose the zero and sign restrictions on these systematic components as summarized in Table 1.

Table 1: Systematic Components Restrictions for Government Spending Rule

parameters	$\psi_{y,t}$	$\psi_{\pi,t}$	$\psi_{r,t}$	$\psi_{tax,t}$
Zero and Sign Restrictions	<0	<0	=0	=0

Notes: "=0" denotes zero restriction, and "<0" and ">0" stand for negative and positive restrictions, respectively.

Since all the parameters in (8) have clear rolls for the movement of policy variable, we can add some interpretations about the sign of them. For example, a negative elasticity of inflation rate in the government spending rule may reflect the fact that nominal government spending is not fully indexed to inflation in the U.S., so real government spending falls in response to an increase in inflation. The important thing here, however, is that we do NOT impose the restriction $\psi_\pi < 0$ based on this theoretical interpretation. Rather, we simply accept the fact that estimated coefficent on π_t in the government spending rule is negative, and draw on this empirical evidence to identify the spending shock. The same thing can be said to the other restrictions.

In similar way, our tax cut rule can be written as

$$Tax_t = \underbrace{\psi^t_{y,t}\, y_t + \psi^t_{r,t}\, r_t + \psi^t_{\pi,t}\, \pi_t + \psi^t_{g,t}\, Gov_t}_{systematic\, component} + \underbrace{\varepsilon_{tax,t}}_{shock.}, \tag{9}$$

where $\psi_{y,t} = -a^{-1}_{0,t,51} a_{0,t,12}$, $\psi_{\pi,t} = -a^{-1}_{0,t,15} a_{0,t,13}$, $\psi_{r,t} = -a^{-1}_{0,t,15} a_{0,t,14}$, and $\psi_{g,t} = -a^{-1}_{0,t,15} a_{0,t,11}$. Based on the estimation results of Caldara and Kamps (2017), we impose the sign restrictions on these systematic components as summarized in Table 2.

Table 2: Systematic Components Restrictions for Tax Cut Rule

parameters	$\psi_{y}.$	ψ_r	ψ_π	ψ_g
Sign Restrictions	$>$**0**	$>$**0**	$>$**0**	$<$**0**

Notes: "<0" and ">0" stand for negative and positive restrictions, respectively.

It is worth noting that the main part of our restrictions are represented as sign and zero restrictions directly on the structural parameters, unlike a large number of studies using set identification impose sign restrictions on the impulse response functions. We do so using the Bayesian approach and the techniques developed in Arias, Rubio-Ramirez, and Waggoner (2018).

Impulse Response Functions (IRFs)

Next, we consider the derivation of IRFs in a standard VAR with constant structural parameters:A_0, A_+, following Arias, Rubio-Ramirez, and Waggoner (2018). Let $L_h(A_0, A_+)$ denote the IRF of the i-th variable to j-th structural shock at finite horizon h given by a $n \times n$ matrix as below.

$$\underbrace{IR_h(A_0, A_+)}_{n\times n} = (A_0^{-1} J' F^h J)'$$

where $A'_+ = [A'_1, \cdots, A'_p]$,

$$\underbrace{F}_{pn\times pn} = \begin{bmatrix} A_1 A_0^{-1} & I_n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{p-1} A_0^{-1} & 0 & \cdots & I_n \\ A_p A_0^{-1} & 0 & \cdots & 0 \end{bmatrix} \text{ and } \underbrace{J}_{pn\times n} = \begin{bmatrix} I_n \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

where I_n is a $n \times n$ identity matrix. Next, we apply them to the IRFs in the TVP-VARs. The IRFs: $L_h(A_0, A_+)$, can be rewritten as

$$\underbrace{IR_h(A_{t,0}, A_{t,+})}_{n\times n} = \left(A^{-1}_{t,0} J' \left(\prod_{i=t}^{t+h} F_i \right) J \right)^T,$$

where $A'_{t,+} = [A'_{t,1}, \cdots, A'_{t,p}]$,

$$\underbrace{F_t}_{pn \times pn} = \begin{bmatrix} A_{t,1}A_{t,0}^{-1} & I_n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{t,p-1}A_{t,0}^{-1} & 0 & \cdots & I_n \\ A_{t,p}A_{t,0}^{-1} & 0 & \cdots & 0 \end{bmatrix}.$$

Notice that the product of time-varying structural parameters: $A_{t,k}A_{t,0}^{-1}$ is equivalent to time-varying reduced-form parameters $B_{t,k}$ for $1 \leq k \leq p$.

Using the orthogonal matrix Q_t, the above IRF, $IR_h(A_0, A_+)= IR_h(A_{tr}Q, A_+)$, is transformed to $IR_h(A_{tr}, A_+Q')Q$, for horizons, $0 \leq h \leq \infty$. It indicates that the sets of structural parameters $(A_0,\, A_+)$ and $(A_{tr},\, A_+Q')$ are observationally equivalent so that we can replace A_0 with A_{tr} in the IRF. Accordingly, instead of A_0, the lower triangular matrix A_{tr} derived from Cholesky decomposition is used together with the matrix Q to be convenient to calculate. Let $f(A_0, A_+)$ be combination of contemporaneous matrix A_0 and the stacked IRF at horizon zero and long term: L, given by a $3n \times n$ matrix as below.

$$f(A_0, A_+) = \underbrace{\begin{bmatrix} A_0 \\ IR_0(A_0, A_+) \\ IR_L(A_0, A_+) \end{bmatrix}}_{3n \times n} = \underbrace{\begin{bmatrix} A_{tr}Q \\ IR_0(A_{tr}, A_+Q')Q \\ IR_L(A_{tr}, A_+Q')Q \end{bmatrix}}_{3n \times n}. \tag{10}$$

Using the function $f(A_0, A_+)$, we can identify the SVARs imposed from the zero and sign restrictions of the IRFs to the two fiscal policy shocks. From Tables 1 and 2, we impose the matrix A_0, while the restrictions of the IRFs are following Tables 3 and 4. Those tables show the zero and sign restrictions of government spending and tax cut rules for both of short and long terms, respectively. In the case of government spending rule, the positive shock immediately increases government spending and gradually converge to the zero for long run as Table 3. After disappearing of effect of this shock, variation of output also converges to the zero for long run. Meanwhile, the tax cut shock immediately decrease tax revenue and gradually converge to the zero for long run as Table 4. Similarly, the effect of output also disappears for long run.

Table 3: Sign and Zero Restrictions for IRF to Government Spending Shock

Structural Shocks		Endogenous Variables: Gov.	Output	π	R	Tax
Gov. Spend. Shock	Short Run	$>$**0**	**?**	**?**	**?**	?
	Long Run	**=0**	**=0**	**?**	**?**	?

Notes: "=0" denotes zero restriction, and "<0" and ">0" stand for negative and positive restrictions, respectively. "?" denotes no restriction.

Table 4: Sign and Zero Restrictions for IRF to Tax Cut Shock

Structural Shocks		Gov.	Tax	Output	π	R
Tax Cut	Short Run (0 Q)	**?**	<**0**	**?**	**?**	**?**
Shock	Long Run (20 Q)	**?**	**=0**	**=0**	**?**	**?**

Notes: "=0" denotes zero restriction, and "<0" and ">0" stand for negative and positive restrictions, respectively. "?" denotes no restriction.

2.3 Alternative Identification

We also introduce an alternative model in order to compare and evaluate the magnitude and direction of time varying fiscal multipliers. To do so, we adopt one of prevalent models which identify a fiscal policy shock by only sign restriction proposed by Mountford and Uhlig (2009). In this model, government spending shock is assumed to be orthogonal to tax, monetary policy and business cycle shocks. In addition, our compared model incorporate zeros restriction for long run to these four temporary shocks, since they are associated with demand shocks which indicates that they have effect on endogenous variables only in short run along the lines of Blanchard and Quah (1985). The zero and sign restriction are represented in Table 5.

Table 5: Zero and Sign Restrictions of Alternative Model

Shocks		Gov.	Output	π	r	tax
Gov Spend.	Short Run	>**0**	?	**?**	**?**	**?**
Shock	Long Run	**=0**	?	**?**	**?**	**?**
Tax	Short Run	**?**	?	**?**	**?**	>**0**
Shock	Long Run	**?**	?	**?**	**?**	**=0**
Business Cycles	Short Run	**?**	>**0**	**?**	**?**	**?**
Shock	Long Run	**?**	**=0**	**?**	**?**	**?**
Monetary Policy	Short Run	**?**	<**0**	<**0**	>**0**	**?**
Shock	Long Run	**?**	**=0**	**=0**	**=0**	**?**

Notes: "=0" denotes zero restriction, and "<0" and ">0" stand for negative and positive restrictions, respectively. "?" denotes no restriction.

2.4 Measuring Fiscal Multipliers

In this study, we calculate an impact fiscal multiplier and a present value fiscal multiplier following Mountford and Uhlig (2009). The impact fiscal multiplier at horizon i of structural shock s on endogenous variables y is defined as $IFM = \triangle y_{t+i}/\triangle g_t$, and is calculated from

$$\frac{e_y' L_i(A_0, A_+) q_s}{e_g' L_0(A_0, A_+) q_s} \frac{1}{(GOV/GDP)},$$

where g_t is the government spendings at period t, and (GOV/GDP) denotes the average share of the government expenditure in GDP over the sample period. In the similar way, the present value fiscal multiplier at horizon i of structural shock s on endogenous variables y is given by $CFM = \sum_{i=0}^{H} \beta^i \triangle y_{t+i} / \sum_{i=0}^{H} \beta^i \triangle g_{t+i}$, and is calculated from

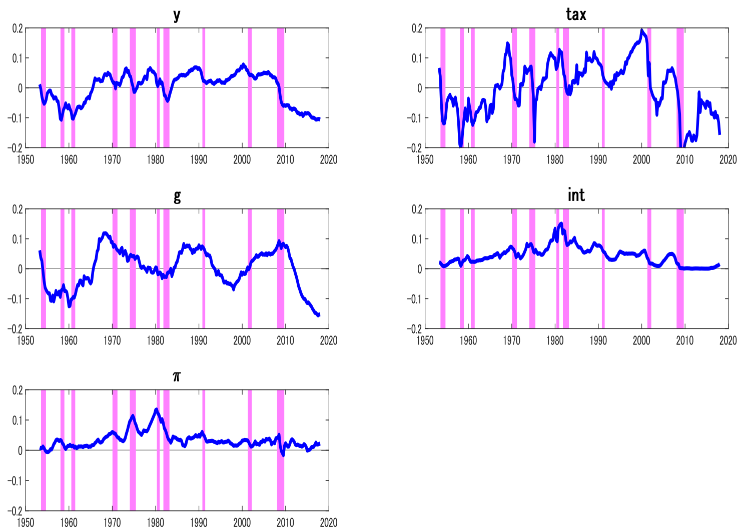
$$\frac{\sum_{i=0}^{H} \beta^i e_y' \, L_i(A_0, A_+) \, q_s}{\sum_{i=0}^{H} \beta^i e_g' \, L_i(A_0, A_+) \, q_s} \frac{1}{(GOV/GDP)},$$

where H is the number of horizon to measure the impact of the policy shock for a specified interval and β is a discount factor. In our simulation, we calculate four cases characterized from different horizons, i.e., $H = 4$, 8, 12, 20.

2.5 Data

We use the quarterly data from the U.S. for the period between 1952:Q1 and 2018:Q1. Following Caldra and Kamps (2017), we select the observed variables composed from five endogenous variables: government spending, defined as the sum of government consumption and investment (g_t); gross domestic product (y_t); consumer price inflation (π_t); the 3-month T-bill rate (r_t); and federal tax revenue (tax_t). We take the natural logarithm and extract a quadratic trend for all the variables in per capita terms except π_t and r_t. Data are shown as Figure 1.

Figure 1: Data



Notes: Our TVP-VAR model adopts seven endogenous variables: government spending, defined as the sum of government consumption and investment (g_t); gross domestic product (y_t); private consumption (c_t); private non-residential investment (inv_t); consumer price inflation (π_t); the 3-month T-bill rate (int_t); and federal tax revenue (tax_t). We take the natural logarithm and extract a quadratic trend for all the variables in per capita terms except π_t and r_t. We estimate the model on quarterly data for the U.S. from 1952:Q1 to 2018:Q1.

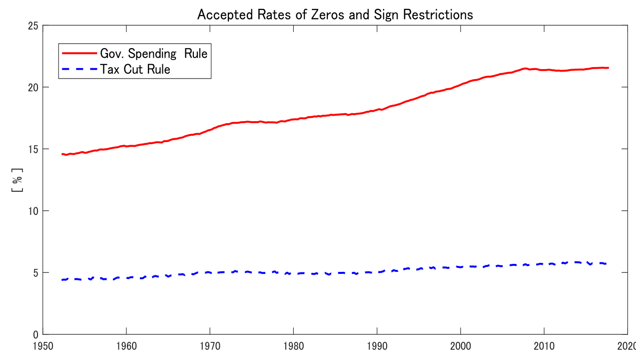
3 Evidence on Time-Varying Fiscal Multiplier

3.1 MCMC Simulations

As described in the previous section, we adopt the Bayesian estimation with MCMC simulation to obtain the posterior estimates satisfied both of zero and sign restrictions showed in Table 1, based on the algorithm 4 proposed by Arias et al. (2018). We run 240,000 MCMC simulations which consists of 15,000 iterations times 16 chains, discarding the first 5000 iterations of each chain to converge to the ergodic distribution, and sampling only draws satisfying the zero and sign restrictions out of the next 10,000 iterations of each chain. To calculate the effects of the IRF for the long run, we set $L = 80$ quarter (20 years) ahead in eq.(10).

Figure 2 shows the transition of the acceptance rates satisfying the zero and sign restrictions out of the 160,000 samples in each period. The acceptance rates of the identification by the government spending rule changes within the range of 15 % to over 20 % for the sample period, while those of identification by the tax cut rule show the range of around 5 %. These acceptance rates mean that approximately 30 thousands samples are used for posterior estimation of SVAR based on the government spending rule, and 8 thousands samples are used in the case of the tax cut rule, respectively. And we use these samples to calculate time-varying fiscal multipliers as shown in the following subsection.

Figure 2: Accepted Rates of Zero and Sign Restrictions



3.2 Time Variations of Systematic Components

Panel (a) of Figure 3 show the time series transition of poserior means of parameters of systematic components based on method by the government spending rule, while Panel (b) draws those of coefficients of a corresponding column out of contemporaneous matrix A_0 derived from identification by the tax cut rule.

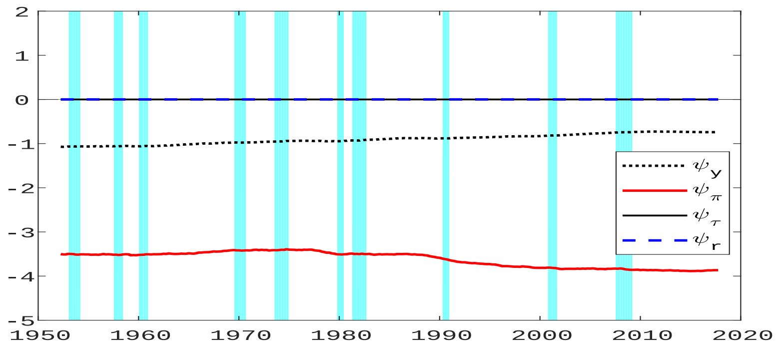
According to the paper by Caldara and Kamps (2017), the contemporaneous elasticity of government spending to output: ψ_y, and to inflation rate: ψ_π, are reported to be around -0.13 and -0.75, respectively. On the other hand, our estimations of time varying those elasticities show -1.0 and -3 through -4, respectively, which mean they are much higher absolute values than those fixed values reported by Caldara and Kamps (2017) over all of the sample period. Meanwhile, the posterior means of ψ_y are positive and around 1.5 over all sample period and those of ψ_π are around 2.5 for the tax cut rule, as Panel (b). Differences of sizes and signs of ψ_y andψ_π between two models are thought to be influence fiscal multipliers according to Caldara and Kamps (2017). In the following subsection, we examine this thing using time-varying parameter SVAR.

Figure 3: Contemporaneous Elasticities of Fiscal Policy Rules

(a) Government Spending Rule

$$g_t = \underbrace{\psi_{y,t}\, y_t + \psi_{r,t}\, r_t + \psi_{\pi,t}\, \pi_t + \psi_{tax,t}\, tax_t}_{systematic\, component} + \underbrace{\varepsilon_{gt}}_{shock.},$$

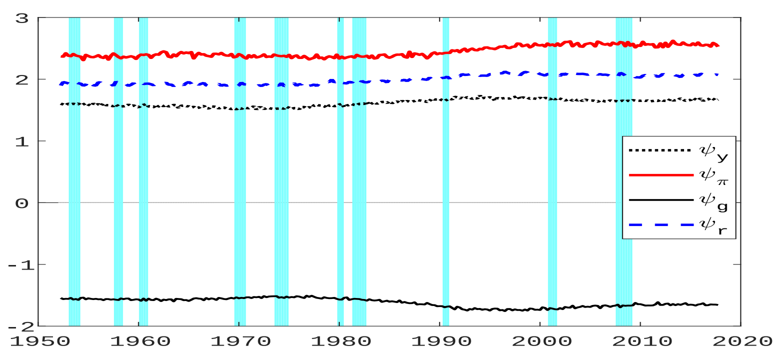
parameters	$\psi_{y}.$	ψ_r	ψ_π	ψ_{tax}
Sign and Zero Restrictions	$<$**0**	=**0**	$<$**0**	=**0**



(b) Tax Cut Rule

$$Tax_t = \underbrace{\psi^t_{y,t}\, y_t + \psi^t_{r,t}\, r_t + \psi^t_{\pi,t}\, \pi_t + \psi^t_{g,t}\, Gov_t}_{systematic\, component} + \underbrace{\varepsilon_{tax,t}}_{shock.},$$

parameters	$\psi_{y}.$	ψ_r	ψ_π	ψ_g
Sign Restrictions	$>$**0**	$>$**0**	$>$**0**	$<$**0**



Notes: The blue shade areas stand for the recessions reported by NBER.

3.3 Time Variations of Impulse Responses

Government Spending Shock

We draw two-dimension version of IRFs as Figure 4. The upper six graphs of Panel (a) in the figure depict the IRFs of real GDP to the government spending shock with respect to six different periods; i.e., 1960:Q1, 1970:Q1, 1990:Q1, 2000:Q1, 2010:Q1, while lower six graphs show time varying those of six different horizons; i.e., 1 Q ahead, 4Q ahead, 8Q ahead, 12Q ahead, 16Q ahead, 20 Q ahead for sample periods. The blue solid lines represent the posterior means of the IRF fixed by periods, while light blue shaded areas represent 68\% intervals. Similarly, the red solid lines and red shaded area are the posterior means and 68 % band of the IRF fixed by horizons, respectively. Panels (b), (c) and (d) depict the same kinds of graphs for tax revenue, interest rate, and inflation, respectively.

As shown in Panel (a), we can verify that the positive spending shock makes responses of real GDP for overall of horizon positive and the zero restriction of output for long run make itself converge to zero for all sample period. Meanwhile, we can see that familiar hump shape responses of the three variables, although there are neither sign nor zero restrictions for the three variables except government expenditure for contemporaneous response and for long run. These shapes are due to restriction by the systematic component of fiscal policy rules. As Panel (b), posterior means of contemporaneous response of tax revenue are also positive for all sample period although 68% bands hit the line of 0 % in around 15Q. And they seem to gradually converge to zero in spite of not imposing zero restrictions.

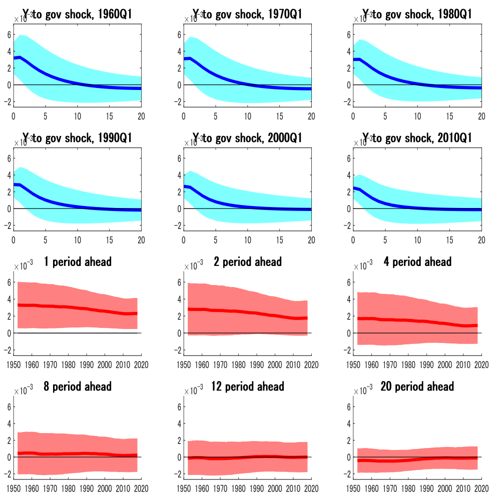
In the cases of interest rate and inflation, results of the IRFs are also positive to the government expenditure, and both seem to converge gradually to steady state. For over long sample period from 1952Q1 to 2018Q1, all of the IRFs have been very stable to the government spending shock.

Tax Cut Shock

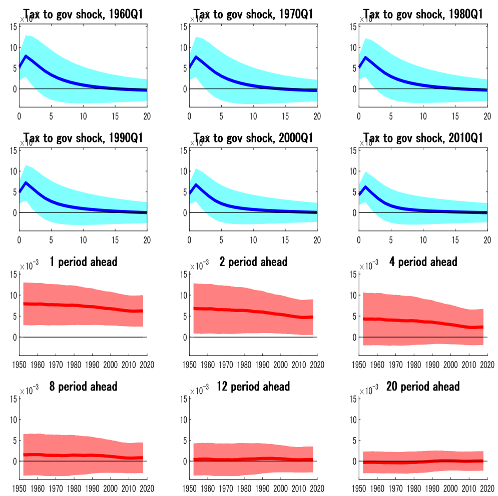
In the similar way, Figure 5 shows the IRFs of the four variables to the tax cut shock. The tax cut shock make responses of interest rate and inflation positive but those of government spending negative. On the other hand, the response of output are ambiguous, since 68 % band of the response cover both of positive and negative areas. However, we observe that posterior means of these responses go to negative for long run from 1960's to 1980's, while those change to be positive for middle terms in 1990's and 2000's.

Figure 4: Time Varying Impulse Response Functions to Government Spending Shock

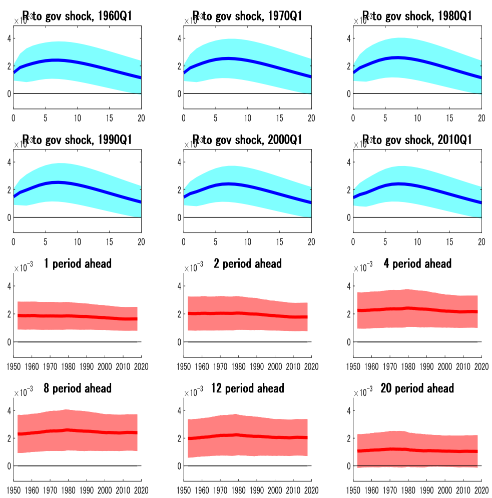
(a) Response of real GDP



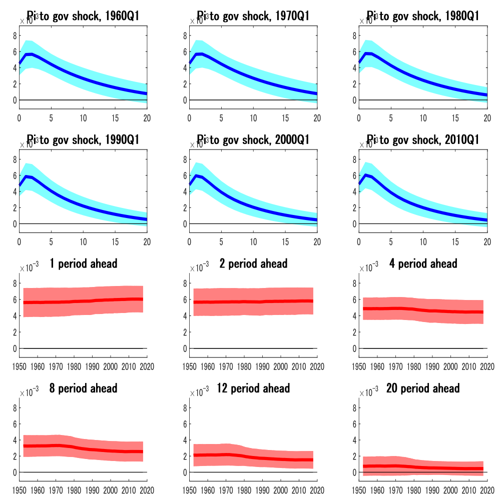
(b) Response of Tax Revenue



(c) Response of Interest Rate



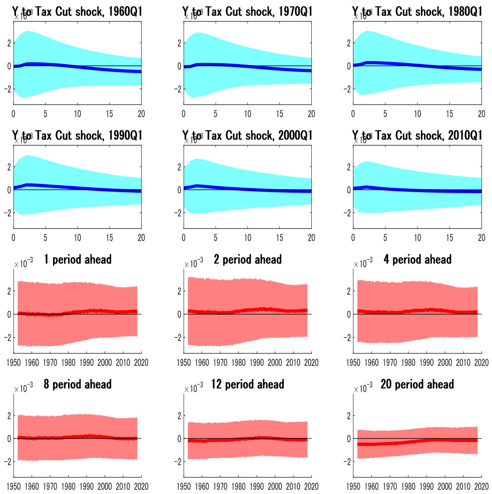
(d) Response of Inflation



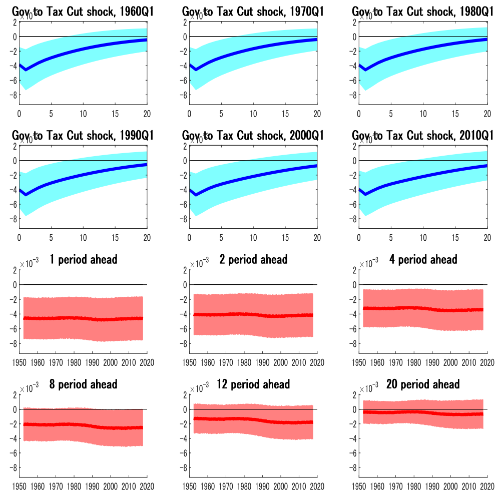
Notes: The blue solid lines represent the posterior means of the IRF fixed by periods, while light blue shaded areas represent 68% intervals. Similarly, the red solid lines and red shaded area are the posterior means and 68% band of the IRF fixed by horizons, respectively.

Figure 5: Time Varying Impulse Response Functions to Tax Cut Shock

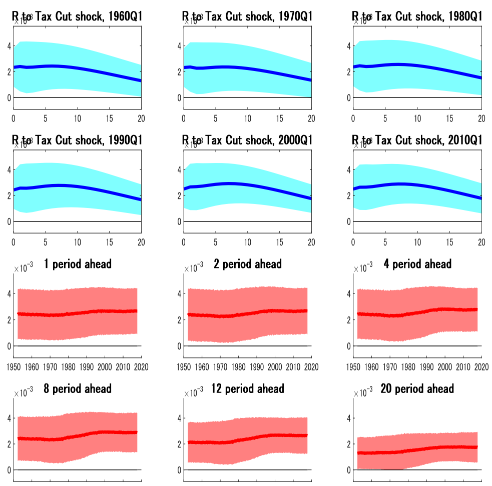
(a) Response of real GDP



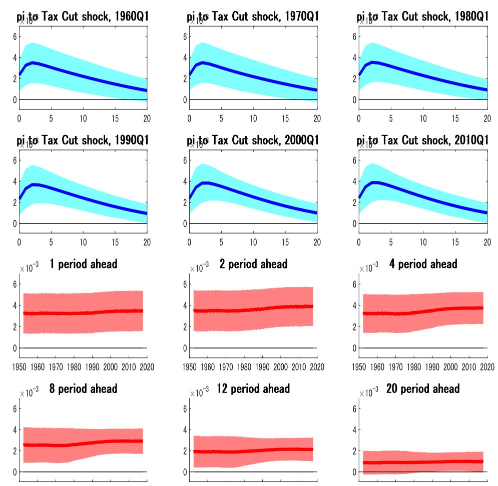
(b) Response of Gov. Spending



(c) Response of Interest Rate



(d) Response of Inflation



Notes: The blue solid lines represent the posterior means of the IRF fixed by periods, while light blue shaded areas represent 68% intervals. Similarly, the red solid lines and red shaded area are the posterior means and 68% band of the IRF fixed by horizons, respectively.

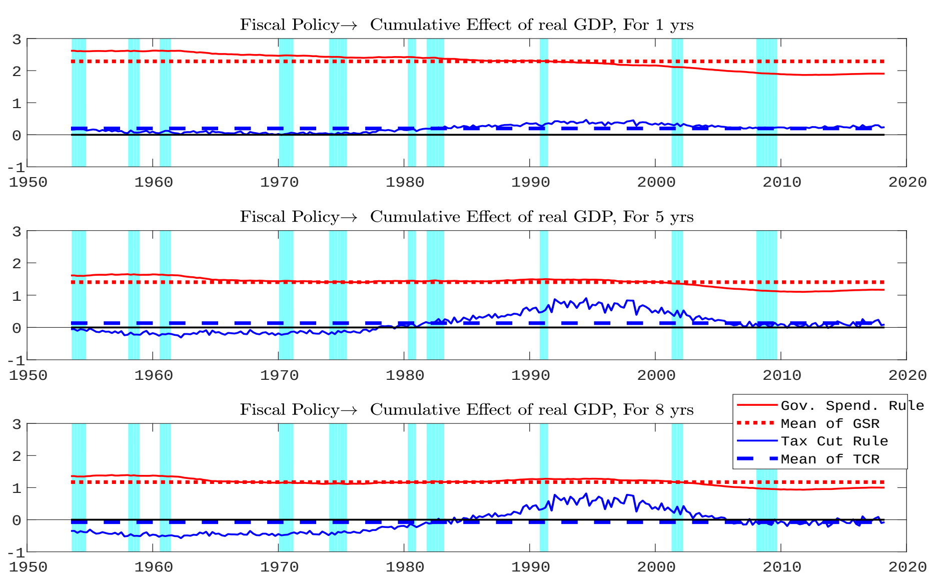
3.4 Time Variations of Fiscal Multipliers

Figures 6 through 8 depict the cumulative fiscal multipliers of real GDP, government spending and tax revenue, respectively. The upper, middle and lower graphs of each figure represent time-varying fiscal multipliers for period of one, five and eight years after both of the fiscal policy shocks hits the economy, respectively. The red line is the posterior means of time-varying fiscal multipliers identified by the government spending shock and the blue line is those by tax cut shock. The dotted lines of both colors are the average of time varying fiscal multiplier for all of the periods. Our paper pays attention for examining the extent to which real GDP is influenced by an increase of government expenditure by one percent. Therefore, we quantify the present discounted value of cumulative fiscal multipliers for both identifications of the policy shocks by Caldara and Kamps (2017).

As Figure 6, the time-varying fiscal multipliers of real GDP by government spending rule (the red solid line) could be nearly double for one year but decline to unity after eight years, and seem to have been very stable for sixty years. On the other hand, those of tax cut rule (the blue solid line) are positive but tiny for one year, and change to negative for more than five years except period of the 1990's.

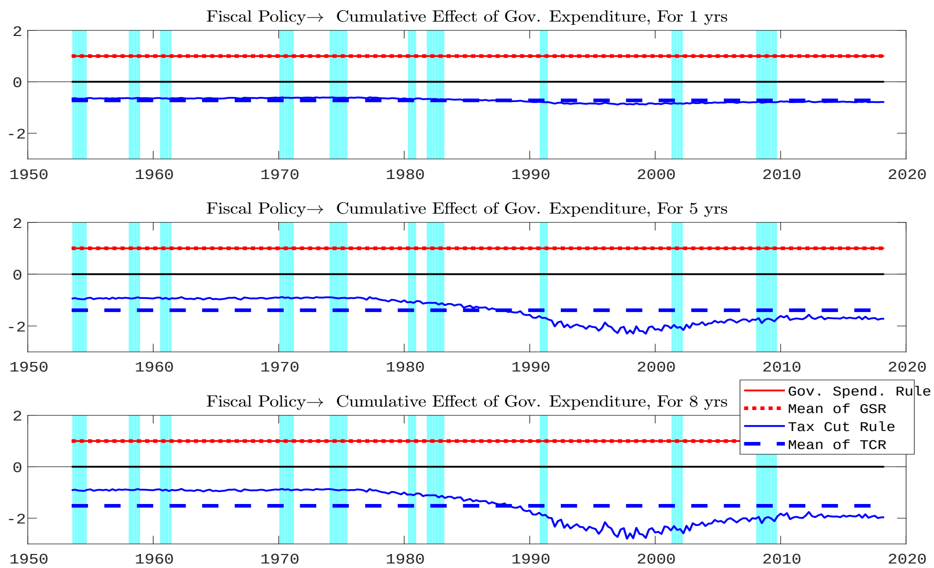
As Figure 7, the tax cut shock is likely to reduce the same size of government spending before 1990 for all three kinds of horizons, because the three cumulative multipliers (the blue solid line) show minus one. After the 1990's, the cumulative multipliers for five and eight years reach at two or over. It also indicates that a tax raise shock requires an increment of government spending more than the increment of tax. And as Figure 8, the government spending shock (the red solid line) increase the same size of increase of tax revenue for all three kinds of horizons for all over sample period. It means that response of tax revenue to the government spending shock have been stable.

Figure 6: Cumulative Fiscal Multipliers of Real GDP



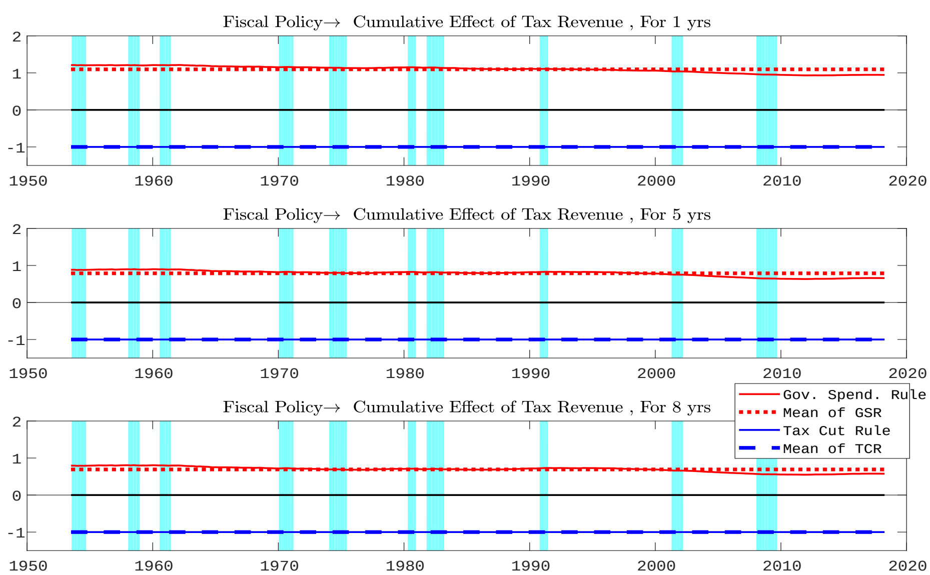
Notes: The blue shade areas stand for the recessions reported by NBER.

Figure 7: Cumulative Fiscal Multipliers of Government Spending



Notes: The blue shade areas stand for the recessions reported by NBER.

Figure 8: Cumulative Fiscal Multipliers of Tax Revenue



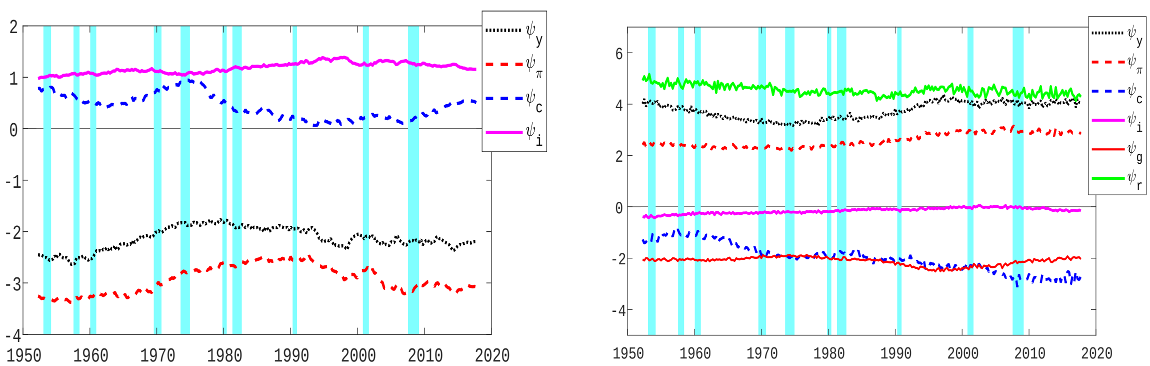
Notes: The blue shade areas stand for the recessions reported by NBER.

3.5 Robustness Check

Here, as robustness check, we expand our models from five endogenous variables to seven variables by adding private consumption and investment. Figure 9 show the posterior means of coefficents of systematic components based on fiscal policy rules, eq.(8) and eq.(9). Similar to the case of five variables model, the sign and zero restrictions of government spending rule and tax cut rule follow Table 1 through Table 4. By introducing private consumption and investment, coefficients of ψ_y of both rules become more than twice of five variables version, since the coefficients of private consumption and investment, ψ_c and ψ_i, have opposite signs against ψ_y . In other words, sizes of ψ_c and ψ_i work to balance out both of government spending and tax against impacts to increment of GDP. It suggests that we should also consider sign restrictions for ψ_c and ψ_i for both fiscal policy rules. However, increasing number of sign restrictions brings to decreasing in acceptance rate of Bayesian estimation as shown in Table 1, and it requires much more sampling number of MCMC procedure.

Figure 9: Contemporaneous Elasticities of Fiscal Policy Rules Using Seven Variables

(a) Government Spending Rule (b) Tax Cut Rule



Notes: Coefficients are based on fiscal policy rules, eq.(8) and eq.(9). The sign and zero restrictions of government spending rule and tax cut rule are following Table 1 through Table 4.

And Figure 10 shows cumulative multipliers of four variables, i.e., real GDP, tax revenue, private consumption and private investment, by government spending shocks identified by systematic component of government spending rule and by just sign restriction in Mountford and Uhlig (2009)'s style as described in Section 2.3. As Panel (a), the seven-variable-version of time-varying fiscal multipliers of real GDP by systematic component (the red solid and dashed lines) are very similar to the five-variable's one, since the averages for one year and for five years are around two and one, respectively. But, the size of multipliers of tax revenue are less than one, which means smaller than five-variable version as Panel (b). By contrast, multipliers by just sign restriction (the blue solid and dashed lines) are as small as 0.5 or less, even though they are positive. The size of multipliers of consumption and investment is new information, as shown in

Panels (c) and (d). We figure out that the response of private consumption to government spending is positive for short period but negative for long run. Meanwhile, the responses of private investment are negative for both of short and long run.

3.6 Discussion

Finally, we consider why these two fiscal policies have different effect on real GDP as Figure 6. Our previous estimations, which adopted a similar TVP-VAR with seven variables including private consumption and investment as Figure 9, found that there are a positive response of the consumption and a negative response of the investment to the positive government spending shock. In particular, these relationships become much stronger in the 1990's than other periods. Accordingly, we suggest that there are causal relations among those variables as below.

$$G \uparrow \Rightarrow C \uparrow + I \downarrow \Rightarrow Y \uparrow \Rightarrow T \uparrow = G \uparrow$$

A positive government spending shock generates positive feedback cycle of real GDP except private investment. These relations have recently been supported by a heterogeneous agent New Keynesian (HANK) approach (Kaplan et al. 2018). As Figure 6, these relations lies within all over sample period.

On the other hand, until the 1980's, there are causal relations between those variables and a tax cut shock as below.

$$T \downarrow \Rightarrow I \uparrow \Rightarrow \ G \downarrow = \ C \downarrow \Rightarrow \ \ Y \doteqdot 0$$

The impact of real GDP to tax cut had had limited magnitude for the period. It indicates that tax reduction policy did not show a significant effect on real economic activities under the Regan Administration, in spite of emerging budget deficit. However, after entering to the 1990's we can see that tax effect on private consumption and investment have got stronger written as

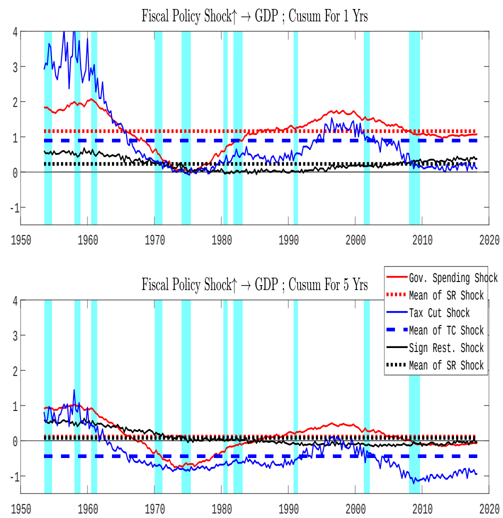
$$T \downarrow \Rightarrow I \uparrow\uparrow \Rightarrow \ G \downarrow = \ C \downarrow\downarrow \Rightarrow \ \ Y \uparrow$$

As we have known, in the actual US economy between 1992 and 2000 the Clinton Administration had taken the fiscal expansion policy including tax increase in order to resolve fiscal deficit produced in the previous presidential period. That is, opposite tax policy had been taken and, as result, it induces to both of massive decline of private investment and rise of private consumption because of increase of government spending generated from increment of tax revenue.

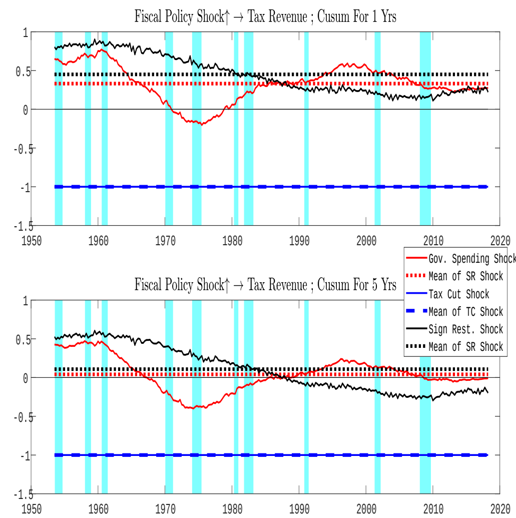
After the middle term of the 2000's, the administration has gone back to the Republican Party, and the effect of tax reduction also came back to the previous level. However,

Figure 10: Cumulative Fiscal Multipliers using Seven Variables

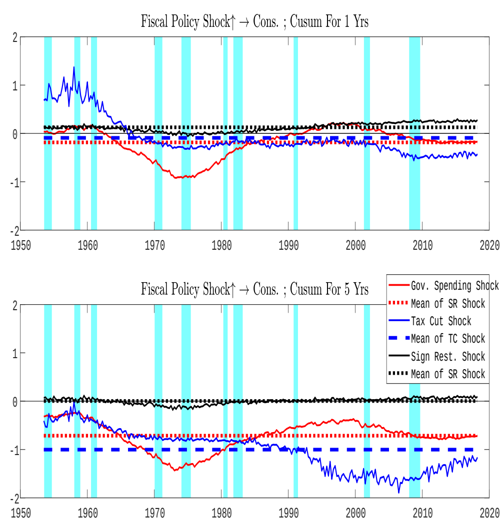
(a) real GDP



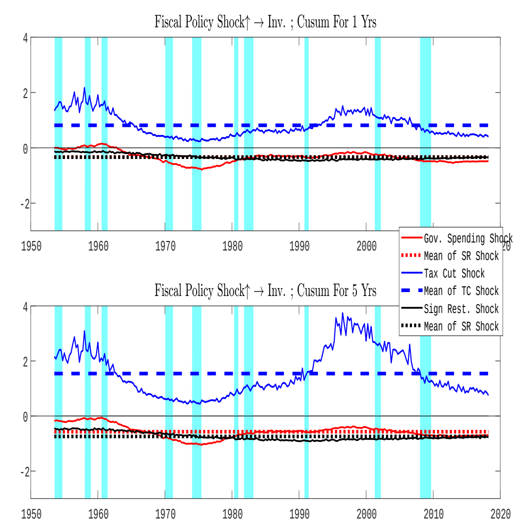
(b) Tax Revenue



(c) Private Consumption



(d) Private Investment



Notes: The blue shade areas stand for the recessions reported by NBER.

the fiscal multipliers of government spending shown stable until 2000 seem to have gradually declined since 2000.

From our estimations, we are wondering whether there is asymmetric effects between an increase and a decrease of tax. By contrast, there are symmetric effects between positive and negative government spending policies. In other words, a tax reduction policy brings no significant impact of real GDP with increase of investment and decrease of consumption, whereas a tax raise policy obviously declines real GDP by including massive decline of private investment despite expansion of government spending.

4 Conclusion

This study estimates time varying fiscal multipliers from the aspect of a fiscal policy rule derived from the systematic component along the line of "*Agnostic Identication Procedure*" proposed by Caldara and Kamps (2017) for the US economy between 1952:Q1-2018:Q1. To do so, we combine time-varying paramter vector autoregressive (TVP-VAR) with MCMC procedure by a Bayesian approach, and indentification of a fiscal policy shock using both of the zero and sign restrictions method proposed by Arias, Rubio-Ramirez and Waggoner (2018). (So we call our model a TVP-SVAR.)

And we compare those values with time varying version identified by standard sign restriction along the line of Mountford and Uhlig (2009). Our estimation reports that time-varying fiscal multipliers of output by government spending rule could be nearly double for one year but decline to unity after eight years, and seem to have been very stable for long terms such as sixty years. By contrast, those of tax cut rule are more fluctuate and negative for more than five years except period of the 1990's.

According to empirical results, we are wondering whether there is asymmetric effects between an increase and a decrease of tax, while we suppose that there are symmetric effects between positive and negative government spending policies. In other words, a tax reduction policy brings no significant impact of real GDP with increase of investment and decrease of consumption. By contrast, a tax raise policy obviously declines real GDP including massive decline of private investment despite of expansion of government spending.

We verify that this method is quite useful to do this end by compared with identifications by only sign restrictions, since the contemporaneous elasticity of output and inflation, whose signs and zero restriction this method can control, is thought to strongly affect both size and direction of fiscal multipliers.

A Appendix

A.1 Zero and Sign Restrictions

Zero restrictions

We consider how to impose the IRFs from the zero restrictions, using the manner by Arias, Rubio-Ramirez, and Waggoner (2018). Let Z_j denote a matrix in which the number of column is equal to the number of rows in $f(A_0, A_+)$ and j is the j-th structural shock imposing the zero restrictions. Using the orthogonal matrix Q_t, the product of the zero restrictions matrices and the IRF is transformed as below.

$$Z_j\, f(A_0Q,\, A_+Q)\, e_j = Z_j\, f(A_0,\, A_+)\, Q\, e_j = Z_j\, f(A_0,\, A_+)\, q_j,$$

where $q_j = Qe_j$. And then, the zero restrictions will hold if and only if

$$Z_j f(A_0, A_+)q_j = 0, \text{ for } 1 \leq j \leq n.$$

where n is number of endogenous variables. From Table 1 and Table 3, we set up the matrix of zero restrictions of government spending shock, Z_1, as

$$\underbrace{Z_1}_{R_z\times 3n} = \begin{array}{c} \begin{array}{ccccc:ccccc:ccccc} \mathbf{y} & \mathbf{g} & \pi & \mathbf{R} & \mathbf{t} & \mathbf{y} & \mathbf{g} & \pi & \mathbf{R} & \mathbf{t} & \mathbf{y} & \mathbf{g} & \pi & \mathbf{R} & \mathbf{t} \end{array} \\ \left[\begin{array}{ccccc:ccccc:ccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \end{array}$$

where elements corresponding to the endogenous variables imposed zero restrictions are set one, otherwise zero. The first n columns of the zero restriction matrix correspond to contemporaneous matrix A_0 , the second n columns correspond to the short run restriction; $LR_0(A_0, A_+)$, while the latter n columns of the matrix do to the long run restrictions: $LR_L(A_0, A_+)$. And the number of rows, R_Z, equals the number of the zero restrictions of the corresponding i-th shock shown in Tables 1 and 3. Notice that the the number of the zero restrictions is equal to the number of endogenous variables: n, less the ordinal number i of the i-th structual shock.

Sign restrictions

In the similar way to the above zero restrictions, sign restrictions can be implemented using a matrix expression. Let S_j be a matrix in which the number of column is equal to the number of rows in $f(A_0, A_+)$ and j is the j-th structural shock imposed the sign restrictions. Using the orthogonal matrix Q_t, the product of the sign restrictions matrices and the IRF is transformed as below.

$$S_j\, f(A_0Q, A_+Q)\, e_j = S_j f(A_0, A_+)Qe_j = S_j f(A_0, A_+)q_j,$$

And then, the sign restrictions will hold if and only if

$$S_j\, f(A_0, A_+)\, q_j > 0, \text{ for } 1 \leq j \leq n.$$

From Table 1 and Table 3, we set up the matrix of sign restrictions of government spending shock, S_1, as

$$\underbrace{S_1}_{R_S \times 3n} = \left[\begin{array}{ccccc:ccccc:ccccc} \mathbf{y} & \mathbf{g} & \pi & \mathbf{R} & \mathbf{t} & \mathbf{y} & \mathbf{g} & \pi & \mathbf{R} & \mathbf{t} & \mathbf{y} & \mathbf{g} & \pi & \mathbf{R} & \mathbf{t} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right],$$

where elements corresponding to the endogenous variables imposed the sign restrictions are set one, otherwise zeros. The first n columns of the sign restriction matrix correspond to contemporaneous matrix A_0 , the second n columns correspond to the short run restriction; $LR_0(A_0, A_+)$, while the latter n columns of the matrix do to the long run restrictions: $LR_L(A_0, A_+)$. And the number of rows, R_S, indicates the number of the sign restrictions of the corresponding i-th shock shown in Tables 1 and 3.

QR decomposition

Let $X = QR$ be the QR decomposition of a $n \times n$ matrix X. The $n \times n$ random matrix Q has the uniform distribution, i.e., $Q\,Q' = I$. and the $n \times n$ matrix R is a upper triangular matrix.

Let the matrix X be defined as

$$\underbrace{X_j(A_0, A_+)}_{n \times n} = \begin{bmatrix} Z_j f(A_0, A_+) \\ Q'_{j-1} \end{bmatrix}^T,$$

and the orthogonal matrix Q_j given from the QR decomposition of a $n{\times}n$ matrix $X_j(A_0, A_+)$ satisfies the zero restrictions, or $X_j(A_0, A_+)q_j = 0$ where $q_j = Q_j e_j$. By stacking them such as $Q = [q_1, \cdots, q_n]$, we obtain the rotation matrix Q to identify the SVAR model.

Algorithm for both restrictions

Finally, we show algorithm for both restrictions using the above QR decomposition. The sets of structural parameters are identified based on *Algorithm 4* by Arias et al. (2014) consisting of the following four steps.

1. Draw the sets of reduced-form parameters (B, Ω).

2. Using the QR decomposition mentioned above, draw an orthogonal matrix Q satisfies the zero restrictions, or $Z_j f(A_0, A_+)q_j = 0, \text{ for } 1 \leq j \leq n$.
3. Keep the draw if the sign restrictions are satisfied, or $S_j\, f(A_0, A_+)\, q_j > 0, \text{ for } 1 \leq j \leq n$, otherwise discard the draw.
4. Return to step 1 until the required number of draws from the posterior distribution conditional on the sign and zero restrictions has been obtained.

Here, we remark as follows. In Step 2 and Step 3, the structural parameters A_0 are observationally equivalent to the lower triangular matrix A_{tr}. So instead of A_0, we use A_{tr} derived from the inverse of Cholesky decomposition of Ω. And A_+ is derived from BA_{tr}.

A.2 Bayesian Estimation Methodology

State space model of TVP VARs

The TVP VARs are represented as state space models consisted of observation equations and state equations. In our model, the observation equation is Eq. (2) with observable variables y_t, and the state equations are Eq. (3), Eq.(4), and Eq.(5) with time-varying parameters, , regarded as state variables. And all parameters of the models are just three such as σ_β, σ_a and σ_h which determine covariances in Eq.(6).

Bayesian inference and MCMC Algorithm

Most of empirical studies dealing with TVP VARs have recently employed Bayesian inference via MCMC algorithm. Our study also follows them. There are four reasons to adopt the Bayesian estimation via the MCMC. First, its counterpart method: maximum likelihood estimation (MLE) method, is intractable to estimate because the state space model includes the nonlinear state equation (5) involved stochastic volatilities. Second, under the situation such as the uncertainty of parameters, the MCMC method is affordable to estimate simultaneously both of state variables and parameters. Third, the functions of both parameters and states variables such as the impulse response functions are also able to be sampled as the posterior distributions of the functions. Forth, all sampled parameters and state variables do not satisfied zero and sign restrictions. The impulse response functions just satisfied both restrictions are sampled as the products of the identified structural VAR.

In the state space model and the impulse response function involved the SVARs, draws generated iteratively from the following conditional posterior distributions of state variables and parameters must tend to convergence to the posterior joint distributions based on the property of Gibbs sampler. The MCMC algorithm estimating our model consists of the following nine steps.

1. Initialize parameters: Σ_β, Σ_a, Σ_h, and state variables: a_t, β_t, h_t.
2. Generate the state variables β_t given a_t, h_t, Σ_β, $\mathbf{y_t}$, from the conditional posterior distribution: $f(\beta_t|a_t, h_t, \Sigma_\beta, \mathbf{y_t})$.
3. Generate the parametersΣ_β given β_t, from the conditional posterior distribution: $f(\Sigma_\beta|\beta_t)$.
4. Generate the state variables a_t given β_t, h_t, Σ_a, $\mathbf{y_t}$, from the conditional posterior distribution: $f(a_t|\beta_t, h_t, \Sigma_a, \mathbf{y_t})$.
5. Generate the parametersΣ_a given a_t, from the conditional posterior distribution: $f(\Sigma_\alpha|\alpha_t)$.
6. Generate the state variables h_t given β_t, a_t, Σ_h, $\mathbf{y_t}$, from the conditional posterior distribution: $f(h_t|a_t, \beta_t, \Sigma_\beta, \mathbf{y_t})$.
7. Generate the parameters Σ_h, given h_t, from the conditional posterior distribution: $f(\Sigma_h|h_t)$.
8. Generate the IRFs: $f(A_0, A_+)$, based on the structural parameters: A_0, A_+, identified with zero and sign restrictions, given a_t, β_t, h_t, $\mathbf{y_t}$.
9. Return to step 2 until the required number of draws from the posterior distribution

Here, we remark some points of the above MCMC simulation. In Step 8, the identification of SVARs and generation of IRFs are implemented from the way described of Section 2.2. In Steps 2 and 4, the simulation smoother of de Jong and Shephard (1995) is used for drawing β_t and a_t . In Step 7, a nonlinear filtering method based on block-sampling method is used for sampling stochastic volatility h_t, following Shephard and Pitt (1997), Watanabe and Omori (2004) and Nakajima et a. (2011). These parts explaining the MCMC procedure generating parameters in reduced-form TVP-VARs are described in Appendix A1in more detail.

The priors of the parameters are specified as:$(\Sigma_\beta)_i^2 \sim IG(20, 10^{-4})$, $(\Sigma_a)_i^2 \sim IG(20, 10^{-4})$, and $(\Sigma_h)_i^2 \sim IG(20, 10^{-4})$, where subscript i denotes the i-th diagonal elements of the covariance matrices and IG an inverse-Gamma distribution. The initial state variables are set as $\beta_0 \sim N(0, 10I)$, $a_0 \sim N(0, 10I)$, and $h_0 \sim N(0, 10I)$.

A.3 MCMC procedure for TVP-VARs

In Section 2.3, we describe the nine steps of the MCMC algorithm estimating our model. Here, we focus on the steps generating parameters in reduced-form TVP-VARs. This section is described based on Appendix of Nakajima (2011) and Nakajima et al. (2011).

A.3.1 Generate the state variables β_t given a_t, h_t, Σ_β, Y_t, from the conditional posterior distribution: $f(\beta_t|a_t, h_t, \Sigma_\beta, Y_t)$.

To generate β_t from the conditional posterior distribution: $f(\beta_t|a_t, h_t, \Sigma_\beta, Y_t)$, we introduce the simulation smoother by de Jong and Shephard (1995) and Durbin and Koopman (2002) using the state space model with respect to β_t given by

$$y_t = X_t\beta_t + A_t^{-1}\Sigma_t\varepsilon_t,\ t = s+1, \cdots, n, \tag{11}$$

$$\beta_{t+1} = \beta_t + u_\beta,\ t = s+1, \cdots, n-1,$$

where β_s is set as μ_{β_0}, and $u_{\beta_s} \sim N(0, \Sigma_{\beta_0})$.

A.3.2 Generate the state variables a_t given β_t, h_t, Σ_a, Y_t , from the conditional posterior distribution: $f(a_t|\beta_t, h_t, \Sigma_a, Y_t)$.

To generate a_t from the conditional posterior distribution: $f(a_t|\beta_t, h_t, \Sigma_a, Y_t)$, the simulation smoother is also adopted from the following state space model,

$$\hat{y}_t = \hat{X}_t a_t + \Sigma_t\varepsilon_t,\ t = s+1, \cdots, n,$$

$$a_{t+1} = a_t + u_{at},\ t = s, \cdots, n-1,$$

where $a_s = \mu_{a0}$, $u_{as} \sim N(0, \Sigma_{a0})$, $\hat{y}_t = y_t - X_t\beta_t$, and

$$\hat{X}_t = \begin{bmatrix} 0 & \cdots & & & & & & 0 \\ -\hat{y}_{1t} & 0 & 0 & \cdots & & & & \vdots \\ 0 & -\hat{y}_{1t} & -\hat{y}_{2t} & 0 & \cdots & & & \\ 0 & 0 & 0 & -\hat{y}_{1t} & \cdots & & & \\ \vdots & & & & \ddots & 0 & \cdots & 0 \\ 0 & \cdots & & & 0 & -\hat{y}_{1t} & \cdots & -\hat{y}_{k-1t} \end{bmatrix},$$

for $t = s+1, \cdots, n$.

A.3.3 Generate the state variables h_t given β_t, a_t, Σ_h, Y_t, from the conditional posterior distribution: $f(h_t|a_t, \beta_t, \Sigma_\beta, Y_t)$.

To generate the stochastic volatility h_t from the conditional posterior distribution: $f(h_t|a_t, \beta_t, \Sigma_\beta, Y_t)$, we conduct the inference for $h_{jt}{}_{t=s+1}^{n}$ separately for j, because it is assumed that Σ_h and Σ_{h0} are diagonal matrices. Let y^*_{it} denote the i-th element of $A_t y_t$. Then, we can write:

$$y^*_{it} = exp(h_{it}/2)\varepsilon_{it},\ t = s+1, \cdots, n,$$

$$h_{i,t+1} = h_{it} + \eta_{it},\ t = s, \cdots, n-1,$$

$$\begin{pmatrix} \varepsilon_{it} \\ \eta_{it} \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0 \\ 0 & \nu_i^2 \end{pmatrix}\right),$$

where $\eta_{is} \sim N(0, \nu_{i0}^2)$, and ν_i^2 are the i-th diagonal elements of Σ_h and Σ_{h0}, respectively, and η_{it} is the i-th element of u_{ht}. We sample $h_t = (h_{i,s+1}, \cdots, h_{in})$ using the multi-move sampler developed by Shephard and Pitt (1997) and Watanabe and Omori (2004), the algorithm of which is described in the following subsection.

A.3.4 Generate the parameters Σ_α , Σ_β, andΣ_h.

To generate the parameter Σ_a given a_t, we draw the sample from the conditional posterior distribution: $\Sigma|a_t \sim IW(\hat{\nu}, \hat{\Omega}^{-1})$, where IW denotes the inverse-Wishart distribution, and $\hat{\nu} = \nu_0 + n - 1$, $\hat{\Omega} = \Omega_0 + \sum_{t=1}^{n-1}(a_{t+1} - a_t)(a_{t+1} - a_t)'$ in which the prior is set as $\Sigma \sim IW(\nu_0, \Omega_0^{-1})$. Sampling the diagonal elememts of Σ_β, Σ_h is also the same way to sample Σ_a.

A.4 Multi-Move Sampler of Stochastic Volatilities

This section is described based on Appendix of Nakajima (2011) and Nakajima et al. (2011). The algoritm of the multi-move sampler proposed by Shephard and Pitt (1997), Watanabe and Omori (2004) is adopted to generate draws of stochastic volatilities in the TVP-VARs from the conditional posterior distributions explainded in Appendix A2. We show the stochastic volatilities model again.

$$y_t^* = exp(h_t/2)\varepsilon_t,\ t = s+1, \cdots, n,$$

$$h_{t+1} = \phi h_t + \eta_t,\ t = s, \cdots, n-1,$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix}\right),$$

where y_t^* denote the i-th element of $A_t y_t$ shown in Eq.(11). For drawing a typical block such as$(h_r, \cdots, h_{r+d})$, we consider the draw of

$$\begin{aligned}(\eta_{r-1}, \cdots, \eta_{r+d-1}) &\sim \pi(\eta_{r-1}, \cdots, \eta_{r+d-1}|\omega) \\ &\propto \prod \frac{1}{e^{h_t/2}} \exp\left(\frac{y_t^{*2}}{2e^{h_t}}\right) \times \prod f(\eta_t) \times f(h_{r+d}) \end{aligned} \tag{12}$$

where

$$f(\eta_t) = \begin{cases} \exp\left\{-\frac{(1-\phi^2)\eta_0^2}{2\sigma_\eta^2}\right\} & (\text{if } t = 0), \\ \exp\left(-\frac{\eta_t^2}{2\sigma_\eta^2}\right) & (\text{if } t \geq 1), \end{cases}$$

$$f(h_{r+d}) = \begin{cases} \exp\left\{-\frac{(h_{r+d+1}-\phi h_{r+d})^2}{2\sigma_\eta^2}\right\} & (\text{if } r+d < n), \\ 1 & (\text{if } r+d = n), \end{cases}$$

and $\omega = (h_{r-1}, h_{r+d+1}, \beta, \gamma, \phi,)$. The posterior draw of $(h_r, \cdots, h_{r+d})$ can be obtained by running the state equation with the draw of $(\eta_{r-1}, \cdots, \eta_{r+d-1})$ given h_{r-1}.

We sample $(\eta_{r-1}, \cdots, \eta_{r+d-1})$ from the density (12) using the acceptance-rejection MH algorithm (Tierney, 1994; Chib and Greenberg, 1995) with the following proposal distribution constructed from the second-order Taylor expansion of

$$g(h_t) \equiv -\frac{h_t}{2} - \frac{y_t^{*2}}{2e^{h_t}},$$

around a certain point $\hat{h}_t$ which is given by

$$\begin{aligned} g(h_t) &\doteqdot g(h_t) + g'(\hat{h}_t)(h_t - \hat{h}_t) + \frac{1}{2}g''(\hat{h}_t)(h_t - \hat{h}_t)^2 \\ &\propto \frac{1}{2}g''(\hat{h}_t)\left\{h_t - \left(\hat{h}_t - \frac{g'(\hat{h}_t)}{g''(\hat{h}_t)}\right)\right\}^2, \end{aligned}$$

Here, the first and second derivatives are obtained such that

$$g'(\hat{h}_t) = -\frac{1}{2} + \frac{y_t^{*2}}{2e^{h_t}}, \; g''(\hat{h}_t) = -\frac{y_t^{*2}}{2e^{h_t}},$$

And the proposal density of $\pi(\eta_{r-1}, \cdots, \eta_{r+d-1}|\omega)$ is given by

$$q(\eta_{r-1}, \cdots, \eta_{r+d-1}|\omega) \propto \prod \exp\left\{-\frac{(h_t^* - h_t)^2}{2\sigma_t^{*2}}\right\} \times \prod f(\eta_t),$$

where

$$\sigma_t^{*2} = -\frac{1}{g''(\hat{h}_t)}, \; h_t^* = h_t + \sigma_t^{*2} g'(\hat{h}_t), \tag{13}$$

for $t = r, \cdots, r+d-1$, and $t = r+d$ in the case that $r+d = n$. Meanwhile, in the case that$r + d \leq n$,

$$\sigma_{r+d}^{*2} = \frac{1}{-g''(\hat{h}_{t+d}) + \phi^2/\sigma_\eta^2} \tag{14}$$

$$h_{r+d}^* = \sigma_{r+d}^{*2}\left\{g'(h_{r+d}) - g''(h_{r+d})h_{r+d} + h_{r+d}/\sigma_\eta^2\right\}, \tag{15}$$

for $t = r + d$. The proposal density of the AR-MH algorithm is derived from the following state space model,

$$h_t^* = h_t + \varsigma_t,\ t = s+1, \cdots, n,$$

$$h_{,t+1} = h_t + \eta_t,\ t = s, \cdots, n-1, \tag{16}$$

$$\begin{pmatrix} \varsigma_t \\ \eta_t \end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma_t^{*2} & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix}\right),$$

with $\eta_{r-1} \sim N(0, \sigma_\eta^2)$ when $r \geq 2$ and $\eta_s \sim N(0, \sigma_\eta^2/(1-\phi^2))$. Given ω, we draw candidate point of $(\eta_{r-1}, \cdots, \eta_{r+d-1})$ for AR-MH algorithm by running the simulation smoother over the state-space representation (16).

For realizing efficient drawings, we need to calculate the mode of the above posterior density for $(\hat{h}_r, \cdots, \hat{h}_{r+d})$. Numerically, we obtain the mode by iterating the following steps several times,

1. Initialize $(\hat{h}_r, \cdots, \hat{h}_{r+d})$.
2. Compute $(h_r^*, \cdots, h_{r+d}^*)$, and $(\sigma_r^*, \cdots, \sigma_{r+d}^*)$ by eq.(13) through eq.(15).
3. Run the simulation smoother for state space model eq.(16) with $(h_r^*, \cdots, h_{r+d}^*)$, and $(\sigma_r^*, \cdots, \sigma_{r+d}^*)$ as obervable variables. And Generate estimations $h_t^* = E(h_t|\omega)$ for $t = r, \cdots, r+d$.
4. Replace $(\hat{h}_r, \cdots, \hat{h}_{r+d})$ with $(h_r^*, \cdots, h_{r+d}^*)$.
5. Return to Step 2.

To implement a block sampling for h_t, they are devided into $K+1$ blocks, say, $(h_{k(i-1)}, \cdots, h_{k(i)})$ for $i = 1, \cdots, K+1$. Shephard and Pitt (1997) suggested to adopt stochastic knots for determining the positions of blocks: i, the rule of which is given by

$$k(i) = int\left[\frac{n(j+U_i)}{K+2}\right],$$

for $i = 1, \cdots, K$, where int is a function rounding to an integer value from the insight, and U_i is the random sample from the uniform distribution $U[0,1]$.

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