# A Supermultiplier Stock-Flow Consistent model: the return of the paradoxes of thrift and costs in the long run?

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**A Supermultiplier Stock-Flow Consistent model: the "return" of the paradoxes of thrift and costs in the long run?**

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## Abstract

Supermultiplier models have been recently brought to the post-Keynesian debate. Yet these models still rely on quite simple economic assumptions, being mostly flow models which omit the financial determinants of autonomous expenditures. Since the output growth rate converges in the long run to the exogenously given growth rate of "non-capacity creating" autonomous expenditure and the utilization rate moves towards the normal utilization rate, the paradoxes of thrift and costs remain valid only as level effects (average growth rates). This paper investigates whether the core conclusions of supermultiplier models hold in a more complex economic framework, described by means of a supermultiplier SFC model, in which private business investment is assumed to be completely induced by income while the autonomous expenditure component - in this case consumption out of wealth - becomes endogenous. The results of the numerical simulation experiments suggest that the paradox of thrift can remain valid in terms of growth effects and that a lower profit share can also be associated to a higher accumulation rate, though with a lower profit rate.

**Keywords**: Supermultiplier; SFC model; autonomous expenditures; paradoxes of thrift and costs; growth theories.
**JEL classification codes**: B59, E11, E12, E25, O41.

## 1 Introduction

Supermultiplier models, as conceived by Sraffian authors (Serrano, 1995a; Bortis, 1997), keep the "Keynesian hypothesis" (Lavoie, 2014, 359), emphasizing the idea that growth can be demand-led even in the long run. This is made possible through the introduction of a "non capacity creating" autonomous expenditure which grows at an

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exogenously given rate and towards which capital accumulation rate will converge in the long run, as business investment is completely induced by income. One of the consequences of these assumptions is that, since both capital accumulation and utilization rates converge to given exogenous values, the paradox of thrift and the paradox of costs are no longer valid in terms of growth effects in the long run, but only in short and medium runs. That is to say growth rates can be higher during the traverse period, but not in the fully adjusted position of the system, when utilization rates reach the normal level.

Supermultiplier models have been recently brought to the post-Keynesian debate by Allain (2015b) and Lavoie (2016). Besides, some empirical evidence for its main results is provided by Girardi and Pariboni (2015). However, these models still do not properly account for the interactions between financial stocks and flows,which - as we sustain here - could lead to different results regarding the paradoxes in the long run.

The aim of the paper is to verify whether these key results of the supermultiplier model – that is, that the paradoxes only hold for level effects in the long run – remain valid in a more financially complex economic framework with interactions between financial stocks and flows. To accomplish this we propose to build an Stock Flow Consistent model keeping the supermultiplier approach essentials – namely, the autonomous expenditure component, induced business investment and the Harrodian investment behavior through which firms react to the discrepancies between actual and desired utilization rates. We adopt a consumption function found in most post-Keynesian models, in which households consume a proportion of their wages and of their wealth. The consumption out of wealth is the autonomous expenditure component of this economy and since the dynamics of household wealth is endogenous to the system, we can say that at least part of the autonomous expenditure component is also endogenous to the model.[1]

Besides this introduction, the paper is organized as follows. The second section of the paper briefly reviews the heterodox growth models literature debate over the utilization rate and the consequences of each model, culminating in the proposal of these alternative (mainly supermultiplier) growth models. In subsection 2.1, we further discuss the features and results of the recent supermultiplier models and highlight the lack of financial complexity, which motivates the building of the model in the third section. In section 3, we present the framework of the model as well as short and long run equilibrium conditions. Following this, in the fourth section three numerical simulation experiments are carried out. The experiments are a reduction in firms' mark-up (subsection 4.1); an increase in the propensity to consume out of wages (subsection 4.2); and, finally, an

[1] Fiebiger and Lavoie (2016) call these expenditures 'semi-autonomous' since it would be unrealistic to consider that any of the effective demand components could be fully autonomous in the real world.

increase in the autonomous expenditure component (an increase in the propensity to consume out of wealth) (subsection 4.3). Still in this section, we make a general assessment of the shared results of the shocks (subsection 4.4). Section 5 concludes the paper.

## 2 Heterodox Demand-led growth models

Heterodox growth theories, as well as the neoclassical model of growth[2], have emerged as an attempt to get around the instability presented by Harrod's model (Kregel, 1980; White, 2008; Fazzari et al., 2013; Cesaratto, 2015). One of the issues raised by Harrod (1939) is that the *steady growth state* of the model is unstable because deviations of the growth rate of the economy from the "warranted growth rate" will make the path explode or collapse (Fazzari et al., 2013).

Accordingly, the models based on the Cambridge equation (Kaldor, 1961; Robinson, 1962) avoided instability assuming endogenous income distribution, what makes it possible for the system to reach the exogenously given utilization rate in the long run. However, in these models, a higher profit share was associated to higher accumulation and profit rates[3], which means that getting around instability had come at the cost of not reproducing the stylized fact that a higher capital accumulation rate can be accommodated through an increase in the utilization rate without changing income distribution between wages and profits (Cesaratto, 2015).

During the 1980s, some central features of Cambridge models began to be more fiercely questioned, such as full employment, the endogenous income distribution and the contradiction between short and long run dynamics [4] (Lavoie, 2014). These controversies originated the first neo-Kaleckian models, as put forward by Rowthorn (1981), Dutt (1984) and Amadeo (1986), which extended the effective demand principle to the long run without assuming full capacity utilization and price mechanisms to bring about the adjustment between investment and savings (Amadeo, 1987; Skott, 2010; Hein et al., 2012).

These neo-Kaleckian models considered income distribution to be exogenous, so changes in the capital accumulation rate would take place through the endogenous capac-

[2]As we focus on heterodox growth theories, namely theories in which growth is led by demand and in which autonomous components of demand can also play a role in the long run, we do not deal with neoclassical growth theories.

[3]While a higher propensity to save would reduce them both, in consonance with the paradox of thrift.

[4]Contradiction between short run and long run behavior of the economy refers to the fact that in the short run quantities change to adjust output to demand, through the endogenous utilization rate, while in the long run, capacity is at its full level, so prices must change to equal output to demand (Lavoie, 2014, p.347-359).

ity utilization rate, even in the long run. They came up with two particularly interesting features at the same time: the paradox of thrift and the paradox of costs. The paradox of thrift says that an increase in the saving rate [5] would lead to lower capital accumulation, profit and utilization rates in the new equilibrium. The paradox of costs - in the version initially presented by Rowthorn (1981) - means that an increase in real wages after a fall in firms' mark-up would boost consumption and lead to a higher utilization rate and, consequently, to higher capacity accumulation and profit rates (Dutt, 2011; Lavoie, 2014). One could say that the paradoxes that emerge from the canonical neo-Kaleckian model are dynamic paradoxes or paradoxes in terms of "growth effects". The initial paradox of thrift as presented by Keynes (1936) referred to the negative effect of a higher propensity to save on the level of output. Likewise, the paradox of costs as put forward by Kalecki (1969) considered only the static effect of a decrease in wages on firms' level of profit (Lavoie, 2014).

The neo-Kaleckian approach, despite its predominance among post-Keynesian authors, has been repeatedly criticized for not dealing with the Harrodian instability issue. The point is: since the utilization rate is endogenous in the long run it could be permanently higher or lower than the normal or planned utilization rate. In neo-Kaleckian models, long run accumulation is ultimately exogenous, so a higher utilization rate does not affect investment plans and, consequently, firms do not revise the trend growth of sales even with a persistently higher or lower demand. For authors from other heterodox strands, as some Sraffians, deviations between actual and normal utilization should foster changes in growth expectations and in investment decisions, giving rise to Harrodian instability. [6](Hein et al., 2011, 2012; Lavoie, 2014).

As an alternative to neo-Kaleckian models, Serrano (1995a) and Bortis (1997) proposed the so-called "Sraffian" supermultiplier model, in which long run growth is demand-led and capacity utilization converges towards the normal or planned levels, by means of the adjustment of the marginal propensity to invest of private firms. This is made possible by the introduction of an autonomous demand component growing at an exogenously given rate while private business investment is assumed to be induced by income without losing the Keynesian causality of investment to savings. The full inducement of private business investment addresses the criticism that firms must reevaluate their expected long run growth rate, when the utilization rate diverges from the normal one. The approach solves a previously impossible trinity, harmonizing the Keynesian

[5]Since in most neo-Kaleckian models, as a simplification, workers spend all their income and only capitalists save, they usually refer to the saving rate of capitalists.

[6]"(...) It seems unrealistic to assume that the growth rate of sales expected by firms, which is captured by the parameter $\gamma$ in the investment function, stays at the same value forever. Overtime, it should slowly adjust to past changes in the growth rate of sales (...)" (Nah and Lavoie, 2017, p.14).

hypothesis, exogenous income distribution and the long run balance between productive capacity and the aggregate demand Cesaratto (2015).

The first versions of the supermultiplier model (Serrano, 1995a; Bortis, 1997) lacked a clear depiction of the endogenous mechanism by means of which the utilization rate tends towards its normal rate. However, a Harrodian mechanism through which the propensity to invest becomes endogenous and changes according to the discrepancy between the actual utilization and the normal one was included in a recent version of the supermultiplier by Freitas and Serrano (2015), which means at least a conditional solution to the Harrod's knife-edge instability problem (Allain, 2015b; Lavoie, 2016). In a similar fashion to this latest version, the supermultiplier model was brought to the post-Keynesian debate, within the neo-Kaleckian framework, by Allain (2015b,a) and by Lavoie (2016). They both combine a "non-capacity creating" autonomous expenditure component which grows at an exogenously given rate towards which the rate of capital accumulation will converge in the long run and the Harrodian adjustment mechanism.

These supermultiplier models bring into the scene a whole new spectrum of demand-led models, which could enrich the post-Keynesian literature, since in most neo-Kaleckian models, private business investment is the demand component which leads growth, while there is no reason why this should always be the case [7]. From the canonical version of the neo-Kaleckian model (Dutt, 2011; Hein et al., 2011) to its most popular variant, the "post-Kaleckian" (Lavoie, 2014) model of Bhaduri and Marglin (1990), this remains as a predominant feature. This also applies to the more financially complex stock-flow consistent (SFC from now on) models. To be fair, we can find some models in which government expenditures assume the leading role of growth, as in chapter 11 of Godley and Lavoie (2007)'s book. However, to our best knowledge, only recently the implications of different growth-regimes - as for, instance, consumption-led ones - started to be explored [8].

In the next subsection, we deal with these recent supermultiplier growth models and how they still need to add some financial complexity to the economic framework to do justice to the post-Keynesian debate about the roles of money and finance on the

[7]The recent U.S. experience suggests that consumption, for instance, can autonomously grow in relation to current income to a large extent and for a considerable period of time (Guttmann and Plihon, 2008; Cynamon and Fazzari, 2008; Barba and Pivetti, 2008; Bibow, 2010; Lavoie, 2014; Allain, 2014). The "*funding effect*" (see Brown, 2007) of some institutional arrangements put forward by financial innovation, as well as consumer credit with real estate collateral, are good examples of how consumption can grow independently of current income growth. In Fazzari et al. (2013)'s words: "(...) the rising importance of finance for consumer spending strongly suggests that consumption dynamics could play a much more important role in demand growth than is the case with the passive income based consumption (...) (Fazzari et al., 2013, p. 19).

[8]Apart from the balance of payment constrained growth models, in which net exports lead growth. However, these models are too partial: most of them do not even include investment decisions, and were thus excluded from this paper (see Blecker, 2009).

**Table 1:** Balance sheet of Supermultiplier models

| Assets | Households | Firms | Goverment$^{\diamond}$ | $\sum$ |
|---|---|---|---|---|
| 1. Fixed Capital | | $+K_f$ | | $+K_f$ |
| 2. Equities$^{\dagger}$ | $+E$ | $-E$ | | 0 |
| 3. Govt. Bills$^{\diamond}$ | $+B$ | | $-B$ | 0 |
| 4. Net worth | $V_h$ | $V_f$ | $-B$ | $+K_f$ |

Note 1: The papers considered in this table are the following: Allain (2015b,a); Freitas and Serrano (2015); Dutt (2016); Lavoie (2016); Hein (2016); Nah and Lavoie (2017).
Note 2: The white cells are part of the models in all papers considered in this table.
$^{\dagger}$ Equities are included only in Hein (2016).
$^{\diamond}$ Government bills are included only in Dutt (2016) and Hein (2016).

dynamics of capitalism.

## 2.1 The lack of financial determinants in supermultiplier growth models

According to Lavoie (2010), one of the main reservations of post-Keynesians about the supermultiplier approach is that it does not include the financial features of the economy, differently from several neo-Kaleckian models, which in the 1990s begun to link the financial and real sides of the economy[9] (Lavoie, 2006; Dutt, 2011). b. This has not been significantly altered by recently published papers such as Freitas and Serrano (2015); Lavoie (2016); Allain (2015a); Dutt (2016); Hein (2016).

So far these supermultiplier growth models rely on quite simple economies in order to obtain analytical solutions. Most of them have only two or three sectors, firms and households (investment and consumption) and, more recently, government or the foreign sector; they typically feature only one kind of real asset (the capital stock) and one kind of financial asset (government debt), if at all. In tables 1 and 2, in which we present respectively the balance sheet and the transactions flow matrix of the recent models in this literature, we can notice they are mainly flow models, not paying enough attention to the interaction between stocks of financial assets and income flows.

We are aware that increasing complexity and dealing with both the real and the financial sides of the economy might not have been the goals of these models so far. Yet the inclusion of financial determinants and the analysis of debt and deficit dynamics is starting to gain momentum (see Dutt, 2015; 2016 and Hein, 2016). In Allain (2015b) government expenditures lead growth in the long run, but the government budget deficit is balanced, so there is neither government debt nor interest payments accruing from

[9]For more on how neo-Kaleckians include financial issues in their models see Dutt (2011) and on how neo-Kaleckians deal with the impacts of financialization on these models see Hein (2011).

**Table 2:** Transactions flow matrix of Supermultiplier models

| | Households | Firms | | Government[†] | Foreign sector[⋆] | $\sum$ |
|---|---|---|---|---|---|---|
| | | Current | Capital | | | |
| 1. Consumption | $-C$ | $+C$ | | | | 0 |
| 2. Investment | | $+I$ | $-I$ | | | 0 |
| 3. Govt. expenditures[†] | | $+G$ | | $-G$ | | 0 |
| 4. Exports[⋆] | | $+X$ | | | $-X$ | 0 |
| 5. Imports[⋆] | | $-M$ | | | $+M$ | 0 |
| 6. Wages | $+W$ | $-W$ | | | | 0 |
| 5. Taxes[†] | $-T$ | | | $+T$ | | 0 |
| 7. Profit | $+FD$ | $-F$ | $+FU$ | | | 0 |
| 8. Interest[⋄] | $+iB$ | | | $-iB$ | | 0 |
| 9. Subtotal | $S_h$ | 0 | $S_f$ | $S_g$ | $S_{ex}$ | 0 |

Note 1: The models considered are the same ones of table 1.
Note 2: The white cells are part of the models in all papers considered in this table.
[†] The Government sector and government expenditures are included in Allain (2015b,a), Dutt (2016) and Hein (2016). Taxes are included in Allain (2015b); Dutt (2016), but not in Hein (2016).
[⋆] The foreign sector and exports and imports are included only in Nah and Lavoie (2017).
[⋄] Interest payments on bills are included only in Dutt (2016) and Hein (2016).

government bills. On the other hand, Dutt (2015, 2016) and Hein (2016) address the effects of debt dynamics on income inequalities in a system where government plays the leading role of growth.

Dutt (2016) highlights how the supermultiplier mechanism impacts public debt[10]: an increase in the growth rate of autonomous government expenditures leads to a higher accumulation rate during the transition, which means a reduction in the government deficit to capital ratio and consequently leads to a reduction in the debt to capital ratio, due to the increase in income and taxation, reducing the financial needs of the government. The lower debt to capital ratio also means a reduction in the financial income received by capitalists as a share of capital, thus reducing income inequality; in turn, Hein (2016) does not deal with taxation issues,focusing on the ambiguous effect of an increase in the debt to capital ratio on the pre-tax functional distribution of income: a higher deficit pushes activity, thus increasing production and income from real activity (reducing the financial income share). On the other hand, the consequent increase in government debt to capital ratio increases the financial income share received from interest payments.

In table 3, we exhibit the main features and results of these models. The ultimate source of growth varies: it is consumption out of credit [11] in Freitas and Serrano (2015),

[10] Dutt (2016) also shows how debt dynamics changes long run stability conditions - the growth rate of government expenditure should be lower than the normalized saving rate and higher than the after tax interest rate for stability to hold.

[11] Girardi and Pariboni (2015) find some evidence that consumption out of credit bears a close corre-

it is the capitalists' consumption in Lavoie (2016) and government expenditures in Allain (2015b).In (Allain, 2015a), the author proposes an interesting model in which subsistence consumption, through a redistributive mechanism between employed and unemployed workers, works as the autonomous variable growing at the exogenous population growth rate [12]. Most of these models explicitly deal with the Harrodian instability problem, by means of an adjustment mechanism of the expected trend growth rate of sales or (in the case of Freitas and Serrano (2015)), of the propensity to invest, which makes the utilization rate converge to the normal rate. In both adjustment mechanisms presented, Harrodian instability is needed for the utilization to converge to the normal one, as long as it is not too strong. Therefore, the adjustment of the expected trend growth rate (or propensity to invest) by firms must be slow.

Despite conciliating the autonomous expenditure component with some financial complexity - through government debt dynamics - it is important to stress that neither Hein (2016) nor Dutt (2016) deeply discuss the Harrodian instability issue. Hein (2016) assumes that the normal utilization is not precisely defined in a world of uncertainty or that it adjusts endogenously to the actual utilization rate. Indeed, Hein (2016) keeps the usual neo-Kaleckian investment function, in which animal spirits is exogenous and capacity utilization adjusts endogenously to the changes in aggregate demand even in the long run. Differently, Dutt (2016) considers that firms have rational expectations and assume that the trend growth rate of sales equals the growth rate of the autonomous demand component chosen.

As far as we know, a more "complete" stock-flow consistent supermultiplier model, which deals with Harrodian instability issues and which is concerned with growth dynamics, is still rare. In Dos Santos and Zezza (2008), the authors already suggest that it could be interesting to study an investment function with a Harrodian mechanism, according to which firms would adjust their investment demand to stabilize the capacity utilization, within an SFC framework. More recently, we can find three papers which include an investment function of the accelerator type in an SFC framework. Both Bortz (2014); Leite (2015) provide an investment function which makes investment endogenous and dependent on income but they rely on the assumption that government expenditures are completely exogenous, so their dynamics will be closely related to the supermultiplier models described in this section. Pedrosa and Macedo e Silva (2014) also provides a model in which investment is endogenous and in which government expenditures are a fraction

lation to the GDP and that GDP growth precedes the increase in household consumption credit. Based on this, they question whether this variable should be considered autonomous in the long run.

[12]In this paper, Allain claims to have a solution also to the second of Harrod's problem since the growth rate in the long run also matches the natural rate of growth.

of the capital stock, this means the dynamics of the model is closely related to the one presented by the model proposed here. However, the purpose of the authors is to analyse the government debt dynamics, which is not our focus here.

As Freitas and Serrano (2015) acknowledge it, it is essential to focus on the financial determinants and on the dynamics of the different "non-capacity creating" components of autonomous demand which could take on the leading role on growth. Allain (2015b) also suggests that the results of supermultiplier models may vary according to the autonomous expenditure chosen as the growth engine. Hein (2016) stresses that the insights provided by his model should be examined and assessed in "(...) more complex models, which might include taxes and thus the post-tax distribution of income, more complicated investment functions, explicitly considering the issue of investment finance for example, wealth-based and debt-based consumption, or a foreign sector" (Hein, 2016, p.20).

However, in the (still too simple) supermultiplier models summarized in table 3, the choice of the engine of growth seems to be inconsequential. The accumulation rate will converge to the exogenously given growth rate of the leading variable, whatever it is. A decrease in the propensity to save, for instance, will increase the *level* of output but will not permanently effect the growth rate of the economy, since the capital accumulation rate will converge towards the exogenously given growth rate of autonomous consumption or government expenditures. The same applies to the paradox of costs. In the case of (e.g.) a reduction in the profit share, the level of output and the level of profits will be higher as a consequence of the increase in household expenditures, but the rate of profits will be lower since the utilization rate converges to the normal utilization rate. [13] [14]

As mentioned by Lavoie (2016), although the paradoxes of thrift and costs are lost as growth effects in supermultiplier models, they still hold if redefined as level effects. This also means that during the traverse from one steady state to the other, growth rates change, being higher or lower on *average*. However, the disappearance of the growth effects reflects the assumption that "non-capacity creating autonomous expenditures" are completely exogenous. A different picture may emerge if, by means of a more complete description of the feedbacks between financial stocks and flows, one allows for a specific engine of growth to become partially endogenous to the model. This is what we propose in sections 3 and 4.

[13] To be fair, in Nah and Lavoie (2017) there are some different short and medium run effects, as the sensitivity of the real exchange rate due to changes in income distribution may give rise to wage or profit-led regimes (table 3).

[14] Dejuán (2014) also proposes a supermultiplier model in which net exports lead growth but, differently from Nah and Lavoie (2017), does not analyze the impacts of the sensitivity of real exchange rates to income distribution, which could change the short and medium run results of the model.

**Table 3:** Supermultiplier models features and results

| Model | $Y$ | $Z$ | Investment behavior | Results of a decrease in $s$ | | Results of a decrease in $\pi$ | |
|---|---|---|---|---|---|---|---|
| | | | | $Y$ | $g_k$, $g_y$,$u$ | $Y$ | $g_k$, $g_y$,$u$ |
| Allain (2015b) | $C+I+G$ | Government expenditures | Harrodian instability mechanism | Permanent + | Transient + | Permanent + | Transient + |
| Allain (2015a) | $C+I$ | Subsistence consumption | Harrodian instability mechanism | Permanent + | Transient + | Permanent + | Transient + |
| Dutt (2016) | $C+I+G$ | Government expenditures | Rational expectations | Permanent + | Transient + | Permanent + | Transient + |
| Hein (2016) | $C+I+G$ | Government expenditures | Animal spirits, endogenous $u$ | Permanent + | Transient + | Permanent + | Transient + |
| Freitas and Serrano (2015) | $C+I$ | Consumption financed by credit | Harrodian instability mechanism | Permanent + | Transient + | Permanent + | Transient + |
| Lavoie (2016) | $C+I$ | Capitalist's consumption | Harrodian instability mechanism | Permanent + | Transient + | Permanent + | Transient + |
| Nah and Lavoie (2016) | $C+I+XL$ | Net exports | Harrodian instability mechanism | Permanent + | Transient + | Permanent +/- | Transient +/- |

Legend: $Y$ is for output, $Z$ for the autonomous expenditure component, $g_k$ for capital accumulation rate, $g_y$ for outuput growth, $u$ for utilization rate, $s$ for propensity to save and $\pi$ for the profit share.

# 3 A Supermultiplier Stock-Flow Consistent model

Based on the brief review of the previous section, we propose to build an SFC model in which the "non-capacity creating" autonomous expenditure component is the consumption out of household wealth and in which private business investment is totally induced. Since household wealth is endogenous to the model, it follows that the autonomous expenditure component is also endogenous.[15]

In the next subsections we present the framework of the model, the short run equilibrium condition, the dynamics equations and the long run equilibrium conditions.

## 3.1 Framework of the model

In the present subsection, we describe our SFC model that attempts to incorporate some of the Supermultiplier approach features. Table 4 presents the balance sheet of the four institutional sectors featured: households, firms, government and banks. The model deals with a pure credit closed economy without inflation (price level is stable and equals the unity). This is so because introducing a Central Bank and/or inflation would make the model unnecessarily complex for the initial purpose we have in mind. Of course, we allow for the price of equity to change in order to account for household capital gains or losses.

*Banks* lend to firms and receive deposits from households. As banks do not make profits, deposits earn the same interest rate of loans granted to firms. Firms sell equities to households and are not credit constrained, for banks grant all demand for loans. As prices are held constant, one can assume that a monetary authority determines the real interest rate, as in Ryoo and Skott (2013). *Households* in this economy hold three kinds

[15] While the notion of exogeneity vs. endogeneity to the model can be clearly defined, the notion of autonomy vs. inducement seems to me somewhat arbitrary. In supermultiplier models, investment is not necessarily induced by current income. In Cesaratto et al. (2003, p. 42), induced investment is a function of the "expected average rate of growth of normal effective demand over the life of the investment that is currently being installed".

**Table 4:** Balance sheet matrix

| Assets | Household | Firms | Banks | Government | $\sum$ |
|---|---|---|---|---|---|
| 1. Deposits | $+M$ | | $-M$ | | 0 |
| 2. Loans | | $-L$ | $+L$ | | 0 |
| 3. Fixed capital | | $+K_f$ | | | $+K_f$ |
| 4. Equities | $+pe.E$ | $-pe.E$ | | | 0 |
| 5. Government Bills | $+B$ | | | $-B$ | 0 |
| 6. Net worth | $V_h$ | $V_f$ | 0 | $-B$ | $+K_f$ |

**Table 5:** Transactions and Flow of Funds matrix

| | Household | Firms Current | Firms Capital | Banks | Government | $\sum$ |
|---|---|---|---|---|---|---|
| 1. Consumption | $-C$ | $+C$ | | | | 0 |
| 2. Investment | | $+I$ | $-I$ | | | 0 |
| 3. Government expenditures | | $+G$ | | | $-G$ | 0 |
| 4. Wages | $+W$ | $-W$ | | | | 0 |
| 5. Taxes | $-T$ | | | | $+T$ | 0 |
| 6. Profit | $+FD$ | $-F$ | $+FU$ | | | 0 |
| 7. Deposits interest | $+i_r.M_{-1}$ | | | $-i_r.M_{-1}$ | | 0 |
| 8. Loans interest | | $-i_r.L_{-1}$ | | $+i_r.L_{-1}$ | | 0 |
| 9. Bills interest | $+i_r.B_{-1}$ | | | | $-i_r.B_{-1}$ | 0 |
| 10. Subtotal | $S_h$ | 0 | $S_f$ | 0 | $S_g$ | 0 |
| 11. $\Delta$ Deposits | $-\Delta M$ | | | $+\Delta M$ | | 0 |
| 12. $\Delta$ Loans | | | $+\Delta L$ | $-\Delta L$ | | 0 |
| 13. $\Delta$ Equity | $-pe.\Delta E$ | | $+pe.\Delta E$ | | | 0 |
| 14. $\Delta$ Bills | $-\Delta B$ | | | | $+\Delta B$ | 0 |
| 15. $\sum$ | 0 | 0 | 0 | 0 | 0 | 0 |

of assets. They buy equity from productive firms and bills issued by the government and hold the rest of their wealth in the form of deposits at banks.

Table 5 shows the transactions between institutional sectors in its first part and the flow of funds in the second part. At this point we can describe the transactions of each sector and the behavioral assumptions.

**Government** issues bills to finance its expenditures that are not covered by taxation of household income [16]. Besides, government bills, firms' loans and household deposits yield the same interest rate. Government expenditures are a fraction of aggregate income at the beginning of the period (equation 2) [17]. In equation 1, which shows how government

[16] As in Le Heron and Mouakil (2008) the government only taxes household (not firms) income.

[17] Since many countries pursue austerity measures and we are not focusing on fiscal policy, considering government expenditures as procyclical should not be a problem, as in Le Heron and Mouakil (2008).

debt evolves over time, $i_r$ is the real rate of interest, $G$ is the government expenditure, $T$ is the taxation of household income and $B_{-1}$ is the stock of bills issued by the government and held by households at the beginning of the period. In equations 2 and 3 respectively $\sigma$ represents the ratio of government expenditures to past income [18] and $\tau$ represents the ratio of taxes on household income.

$$B = B_{-1} + G - T + i_r.B_{-1} \tag{1}$$

$$G = \sigma.Y_{-1} \tag{2}$$

$$T = \tau.Y_h \tag{3}$$

**Household** income comprehends wages and financial income (interest on deposits and bills and dividends) (equation 4). The wage share of income is defined by equation 5, in which $\pi$ is firms' profit share. Household disposable income is defined as the after-tax household income (equation 6). Households consume a fraction ($\alpha_1$) of their after-tax wages and a fraction ($\alpha_2$) of their stock of wealth at the beginning of the period (equation 7), as in Dos Santos and Zezza (2008). Consumption out of wealth (i.e. "capitalist consumption") represents the autonomous expenditure component. Despite being autonomous (in relation to current income), it is endogenous to the model, since it depends on household wealth, so we can analyse its dynamics through household wealth dynamics. Household savings are defined by equation 8. In the model, financial income does not affect consumption directly, but through its effect on wealth.

$$Y_h = W + FD + i_r.(B_{-1} + M_{-1}) \tag{4}$$

$$W = (1 - \pi).Y \tag{5}$$

$$Y_d = (1 - \tau).Y_h \tag{6}$$

$$C = \alpha_1(1 - \tau).W + \alpha_2 V_{h_{-1}} \tag{7}$$

$$S_h = Y_d - C \tag{8}$$

Following Dos Santos and Zezza (2008), we suppose that the proportion of household

[18] Since we are building the model in a discrete time framework, one may wonder whether the stability of the model would depend on the lagged effect of income on government expenditures. We have tested the model for government expenditures based on current income (results can be provided upon request) and the changes we observe, would also be observed in theoretical analytical models. If government expenditures depended on current income, the short run effects of the supermultiplier on the model would be amplified. This means the model would present a higher growth rate, requiring a slower adjustment of the trend growth of sales or the propensity to invest to keep instability away. This is the same effect observed in the models of Allain (2015a); Lavoie (2016); Freitas and Serrano (2015).

wealth allocated in equities ($\lambda$) depends positively on the expectation of return ($\lambda_0$) and negatively on the real interest rate (equation 9). The stock of equities issued is decided by firms. As households buy all equities issued by firms, the price of equities ($pe$) come into play to clear the market (equation 10). To avoid indetermination, since bills and deposits have the same remuneration rate, we suppose that households buy all government bills (Ryoo and Skott, 2013; Pedrosa and Macedo e Silva, 2014).

$$\lambda = \lambda_0 - i_r \tag{9}$$

$$pe = \frac{\lambda . V_h}{E} \tag{10}$$

The stock of wealth changes due to household savings and due to capital gains (equation 11). As households are assumed to buy all bills issued by the government, deposits share in wealth must be treated as a residual (equation 12)

$$V_h = V_{h_{-1}} + S_h + \Delta pe . E_{-1} \tag{11}$$

$$M = M_{-1} + S_h - pe . \Delta E - \Delta B \tag{12}$$

**Firms** decide the mark-up ($\mu$) on wage costs. The mark-up on costs defines functional income distribution (Lavoie and Godley, 2001), as in traditional neo-Kaleckian models (equation 13). Firms must also make their investment decisions and this is where the supermultiplier approach comes properly into the scene. Aggregate investment of firms is induced by output (equation 14) (Serrano, 1995a; Freitas and Serrano, 2015). Firms as a whole have a marginal propensity to invest out of income ($h$), which is endogenous to the model and reacts to discrepancies between the utilization rate ($u$) and the normal utilization rate ($u_n$) (equation 15), following a Harrodian adjustment mechanism (see Lavoie, 2016; Freitas and Serrano, 2015), in which $\gamma$ represents the speed of adjustment of the propensity to invest to the discrepancies between the actual utilization rate and the desired utilization rate.

$$\pi = \frac{\mu}{(1+\mu)} \tag{13}$$

$$I = h.Y \tag{14}$$

$$\Delta h = \begin{cases} h_{-1}.\gamma.(u - u_n), & \text{if } |u - u_n| > x \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

Since we agree with Sraffian and Classical authors when they say that the utilization rate cannot be "anywhere" in the long run, but also agree with the neo-Kaleckians when they point that there is no reason for firms to choose a specific number for the utilization rate, we believe that adopting a range, out of which the propensity to invest reacts, is a satisfying option, as suggested by Hein et al. (2012). As highlighted by Dutt (2011), in a world of uncertainty, firms may want to keep their investment strategy unchanged if the capacity utilization is within a reasonable band. This corridor is represented by the parameter $x$ (equation 15).[19]

The change in the stock of capital is given by equation 16 and differs from the flow of investment because we include capital depreciation in the model ($\delta$). The actual utilization rate is given by the ratio of output to full-capacity output (equation 18) and full-capacity output (equation 17) is determined by the ratio of the initial capital stock to the given capital-output ratio ($v$). From these equations, we can draw the actual rate of growth of the capital stock (equation 19).

$$K = K_{-1} - \delta K_{-1} + I \tag{16}$$

$$Y_{fc} = \frac{K_{-1}}{v} \tag{17}$$

$$u = \frac{Y}{Y_{fc}} \tag{18}$$

$$g_k = \frac{hu}{v} - \delta \tag{19}$$

Firms must still decide how they will finance their investment. We suppose firms finance their investment through retained earnings, equity issuance and bank loans, which are assumed to clear firms' demand for funds (equation 20).[20] Equities are a fixed proportion ($a$) of the capital stock at the beginning of the period (equation 21). Firms retain a fraction of their profit ($s_f$) discounting the payment of interest on loans (equation 22) and distribute the rest of net profit to households in the form of dividends (equation 23). Total net profit is the sum of retained and distributed profits (equation 24). Gross profit is given by equation 25.

$$L = L_{-1} + I - FU - pe.\Delta E \tag{20}$$

$$E = a.K_{-1} \tag{21}$$

[19]From equation 15, we observe that, for a given adjustment parameter $\gamma$, any element that influences the utilization rate (see equation 29), when the latter is out of the inertia zone, will influence the short-run propensity to invest in a pretty standard way. It will increase, for instance, in household propensities to (respectively) consume and spend, as it will decrease (given the Kaleckian features of our model) in the profit share.

[20]As in many SFC models (see Lavoie and Godley (2001); Godley and Lavoie (2007); Zezza (2008)), we present firms' loans as the buffer of the sector, considering, as a matter of simplification, that firms exhaust their internal funds before recurring to external funding for investment. However, this simplification is not suitable for analysing firms' process of accumulating debt and their likelihood of becoming more fragile.

$$FU = s_f(\pi.Y - i_r L_{-1}) \tag{22}$$

$$FD = (1 - s_f)(\pi.Y - i_r.L_{-1}) \tag{23}$$

$$F = \pi.Y - i_r.L_{-1} \tag{24}$$

$$F_g = \pi Y \tag{25}$$

Normalizing equation 24 by the stock of capital at the beginning of the period, we get what we can call a net profit rate (equation 26). Gross profit rate (equation 27) is attained through the same procedure for equation 25.

$$r_n = \pi \frac{u}{v} - i_r \frac{l_{-1}}{(1 + g_{k-1})} \tag{26}$$

$$r_g = \pi \frac{u}{v} \tag{27}$$

After presenting the framework of the model, we can move on to the short run goods market equilibrium and to the dynamic equations of the system.

## 3.2 Short-run goods market equilibrium

In our closed economic system, real output is the sum of household consumption, firms investment and government expenditures (equation 28). If we substitute equations 7, 14 and 2 in equation 28, normalize it by the opening stock of capital and make some algebraical rearrangements, we get the short run equilibrium utilization rate (equation 29). The term $\alpha_2 v h_{-1}$, which represents the normalized consumption out of wealth or capitalist consumption, is the truly autonomous expenditure component of this system (the $z$ component). The supermultiplier appears on the RHS of equation 29 within the parenthesis and shows the effect of induced consumption, induced investment and government expenditures on the level of output. The essence of the supermultiplier approach is maintained as the level of output and the utilization rate in the short run are determined by an autonomous component of demand, which is not private business investment, times the supermultiplier (see Freitas and Serrano, 2015).

$$Y = C + I + G \tag{28}$$

$$u^* = \left( \frac{1}{(1 + g_{k-1})[1 - h - \alpha_1(1 - \tau)(1 - \pi)] - \sigma} \right) \alpha_2 v_{h-1} v \tag{29}$$

Assuming, as in neo-Kaleckian models, that the model presents Keynesian stability, savings should react more than investment to changes in output and capacity, which means that for the denominator of equation 29 to be positive the following condition should be satisfied:

$$1 - \alpha_1(1-\tau)(1-\pi) - \frac{\sigma}{(1+g_{k_{-1}})} > h \tag{30}$$

## 3.3 Dynamic equations and Steady State ratios

We can now obtain the dynamic equations of government debt, household wealth and firms' loans normalized by the capital stock [21] at the beginning of the period. This step is important in order to give us the long run equilibrium ratios, or the steady growth ratios of the stocks. Dividing equation 1 by the lagged capital stock and making some algebraic manipulation, we get the normalized stock of *government debt* (equation 31). We can notice that the stock of government debt, in the short run, depends positively on the stock of debt at the beginning of the period and on the after-tax interest rate (which remunerates bills held by households). It also depends positively on the government propensity to spend and on the profit share, since retained profits are not taxed. On the other hand, the taxation of distributed profits, firms' normalized stock of loans at the beginning of the period and the capital accumulation rate have a negative effect on government debt to capital ratio. The current capital utilization rate also has a negative on the normalized stock of government debt, considering that the taxation of distributed profits assumes positive values ($\tau(1-s_f\pi)$). The intuition is that since in the short run government expenditures depend on past income, past capacity utilization should have a positive effect on government debt to capital ratio but not the current capacity utilization, which increases government revenues.[22]

$$b_t = \frac{b_{-1}[1+i_r(1-\tau)] + \sigma u_{-1} - \tau i_r s_f l_{-1}}{(1+g_{k_{-1}})} - \tau(1-s_f\pi)\frac{u}{v} \tag{31}$$

The same procedure is applied to firms' loans. We divide equation 20 by the lagged capital stock and get equation 32. Firms' loans to capital ratio depends positively on the loans at the beginning of the period, on the interest rates they pay on this initial stock and on the propensity to invest. As long as the propensity to invest is larger than the retained earnings share, the effect of capacity utilization will also be positive. Firms' loans relate negatively to retained profits and to the capital accumulation rate. If the growth rate

[21] The normalized stocks of government and firms' debt will also be referred to respectively as government bills to capital ratio and firms' loans to capital ratio, since bills and loans are the sole components of these institutional sectors' debts.

[22] The effects presented here are drawn based on reasonable and positive values for the parameters, as well as on the assumption that the model presents Keynesian stability and that the steady growth ratios converge to a stable equilibrium, which requires as a necessary condition for the denominator of the equilibrium ratios to be positive.

assumes positive values, which is the case in normal times, the portion of wealth in the form of equities exerts a negative impact over firms' loans.

$$l_t = \frac{(1 + s_f i_r)l_{-1} + \lambda v_h}{1 + g_{k_{-1}}} + (h - s_f \pi)\frac{u}{v} - \lambda v_h \tag{32}$$

When it comes to the normalized stock of household wealth, the algebra gets slightly more complicated. The normalized stock of wealth (equation 33) is obtained by the division of equation 11 by the lagged capital stock. The short run stock of wealth is positively affected by the stock of wealth at the beginning of the period, by the after-tax dividend income, by the after-tax savings out of wages, by the interest households receive over the stock of government bills and by the amount of interest firms pay on loans (which is the same they receive on deposits). The normalized stock of wealth negatively relates to the consumed portion of wealth at the beginning of the period and to the consumed portion of after-tax wages. The equities share on wealth has an ambiguous and transient effect, since its effects vanish in the long run. The effect of the capital accumulation rate on the normalized stock of wealth depends on the value of the parameters of the model.

$$v_h = \frac{(1 - \alpha_2 - \lambda)v_{h_{-1}} + (1 - \tau)[(1 - \alpha_1 + \pi(\alpha_1 - s_f))\frac{u}{v}(1 + g_{k_{-1}}) + s_f i_r l_{-1} + i_r b_{-1}]}{1 + g_{k_{-1}} - \lambda} \tag{33}$$

As we are testing whether the supermultiplier results hold in a more complex economic system, we have to deal with long run equilibrium normalized stocks, in which all growth rates follow the growth rate of the autonomous expenditure component (34) and in which the utilization rate converges to the normal utilization rate, or gets into the inertia zone (35).

$$g^* = g_k = g_{v_h} = g_b = g_l \tag{34}$$

$$u^* \simeq u_n \tag{35}$$

Given conditions 34 and 35, and thus considering that all stocks grow at the same rate, normalized stocks at the beginning of the period equal normalized stocks at the end of the period (thus $\Delta b_t = 0$) in the long run. The normalized government debt (31) can be rewritten as:

$$b^* = \frac{[\sigma - \tau(1 - s_f \pi)(1 + g^*)]\frac{u_n}{v} - \tau i_r s_f l^*}{g^* - i_r(1 - \tau)} \tag{31A}$$

We notice that, *cet.par.*, an increase in the propensity to spend of the government ($\sigma$) increases the steady state value of the debt to capital ratio. The same is true for the profit share ($\pi$). While the firms' loans steady state ratio affects negatively the government debt

- which means that when the government diminishes its debt, firms increase their leverage -, the normal utilization level has a positive effect on the government debt ratio, as long as the term between brackets – $[\sigma - \tau(1 - s_f\pi)(1 + g^*)]$ – is greater than zero. We consider this is the case, assuming as an stylized fact that both government and firms' debt are positive.

Following the same steps for firms, we arrive at the long run normalized stock of loans:

$$l^* = \frac{(1 + g^*)[(h - s_f\pi)\frac{u_n}{v}] - g_k\lambda v_h}{g^* - s_f i_r} \tag{32A}$$

In the steady state, as in the short run, firms need less borrowed funds to finance the same amount of investment, the larger the proportion of household wealth ($\lambda$) allocated in equities. The opposite happens with the utilization rate, *cet.par.*, a higher normal utilization rate implies a larger ratio of loans. The greater the propensity to invest and the smaller the profit share, the greater will be the firms loans to capital ratio in the long run.

The equilibrium stock of household wealth-to-capital ratio (from equation 33) is negatively influenced by the propensity to consume out of wealth ($\alpha_2$) and by the propensity to consume out of after-tax wages ($\alpha_1$). The higher is firms' debt ratio, the higher will be the wealth ratio, because of the higher financial income received by households, which will also increase with the interest rate on bills and deposits. The same applies to the government debt ratio. Other things equal, higher values for the normal utilization rate and for the profit share also translate into a higher steady state ratio of wealth. As in the short run, the effect of the growth rate on the steady state ratio of wealth depends on the combination of the parameters of the model.

$$v_h^* = \frac{(1 - \tau)[(1 - \alpha_1 + \pi(\alpha_1 - s_f))\frac{u_n}{v}(1 + g_k) + i_r b + s_f i_r l]}{g_k + \alpha_2} \tag{33A}$$

Solving equation 29 for equilibrium values, considering that conditions 34 and 35 are satisfied and, consequently, $\Delta h = 0$, we come to the equation for the long run growth rate of this economy:

$$g^* = \frac{\alpha_2 v_h v + u_n\sigma}{u_n[1 - h - \alpha_1(1 - \tau)(1 - \pi)]} - 1 \tag{36}$$

In equation 36, we can observe that income distribution ($\pi$) as well as the other components of the supermultiplier – propensity to invest ($h$), propensities to consume ($\alpha_1$, $\alpha_2$) and propensity to spend of the government ($\sigma$) – can have permanent effects on growth. Accordingly, the normalized consumption out of wealth also influences the rate of growth

**Table 6:** Effects of the shocks

| | Reduction in $\mu(\pi)$ | | Increase in $\alpha_1$ | | Increase in $\alpha_2$ | |
|---|---|---|---|---|---|---|
| | Short run | Long run | Short run | Long run | Short run | Long run |
| $g$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $u$ | $+$ | $=$ | $+$ | $=$ | $+$ | $=$ |
| $r_g$ | $-$ | $-$ | $+$ | $=$ | $+$ | $=$ |
| $r_n$ | $-$ | $-$ | $+$ | $-$ | $+$ | $-$ |

$(\alpha_2 v_h)$.

After presenting the level and the dynamic equations and the short and long run equilibrium conditions, we can move on to the simulation experiments to test for the long run effects of the model.

# 4 Experiments

From the *steady growth state*, we run some simulation experiments to evaluate the long run features of the model. The first shock is a decrease in the mark-up, which means an increase in the wage share, in order to assess whether the paradox of costs holds in terms of level and growth effects, considering the initial values and parameters of the model.[23] The second shock is an increase in the propensity to consume out of after-tax wages ($\alpha_1$) (a reduction in the propensity to save) in order to assess whether the paradox of thrift holds in terms of level and growth effects. At last, we shock the autonomous consumption component, through an increase in the propensity to consume out of wealth ($\alpha_2$), to analyze how it changes the dynamics of the economy in the long run. The results of the shocks are summarized in table 6.[24]

## 4.1 The paradox of costs

A decrease in the mark-up raises the wage share and leads to a higher consumption out of wages, which translates into a higher income and activity level. The increase in capacity utilization following the increase in consumption and income makes firms change their expectation of growth, which raises their propensity to invest, increasing the rate of capital accumulation as we can see in figure 1(a). We also observe that as the rate of growth of household wealth during the transition is lower than the capital accumulation

[23] The parameters and long run values of the variables are presented in table 7 in Appendix A.

[24] All numerical simulations were computed using R and Eviews 9 software. The programming codes of the simulations are available upon request.

rate (figure 1a), the ratio of household wealth to capital will be lower in comparison to the baseline (figure 2a). As in the original supermultiplier model (Serrano, 1995a), as investment increases in relation to output, through a higher propensity to invest $h$ (figure 1c), the autonomous expenditure component (consumption out of wealth) $z$ loses relative weight on output (figure 1d). From figure 1(b), we note that the utilization rate converges towards the desired rate in the long run, through the adjustment of the propensity to invest.

From equations 31 and 31A, we know that the reduction in the profit share contributes directly to reduce government debt ratio. Besides that, a reduction in the normalized stock of bills contributes to reduce itself further since the amount of interest the government pays on bills (to households) also decreases. The increase in the utilization rate following the boost in activity at the same time raises the ratio of bills to capital (since government spends a constant fraction of output) and has a negative effect on that ratio through the increase on taxes (figure 2a). In the short run, the government budget deficit falls sharply but as output increases it stabilizes at a higher level compared to the baseline (figure 2c). The increase in firms loans and the higher accumulation rate also contribute to reduce the government bills to capital ratio.

In the case of firms, the higher loans to capital ratio is due to the increase in the propensity to invest along with the lower amount of wealth to capital which reduces total equities as a source of finance in comparison to the baseline (figure 2d). These effects compensate the impact of a higher accumulation rate in reducing firms loans - through the increase in profit income (figure 2a).

Household wealth to capital ratio is negatively influenced by the initial reduction in the profit share, which diminishes the financial income accruing from firms' dividends and diminishes the immediate need for the government to issue bills since taxation from household wage income increases. This more than compensates the effect of the increase in the interest payments households receive from deposits (figure 1a). The growth rate of wealth is higher in comparison to the baseline due to the overall increase in income and activity following the higher wage share (figure 2a).

Regarding the gross and net profit rates of firms (figure 2b), it is clear that since the utilization rate converges to a desired rate or range, both rates decrease in relation to the baseline. In the short run, the positive effect of an increase in income and utilization is not enough to compensate the reduction of firms profit share. However, gross and net profit levels increase in relation to the baseline [25].

[25] One could say that, as firms have more than one goal in the long run, they may be willing to cut profit rates in order to grow and to increase their market shares (Lavoie, 2014).As pointed by one of anonymous

Based on these results, we realize that income distribution can influence growth in the long run, even if the utilization rate converges to the desired rate or range. This is made possible by the inclusion of the endogenous autonomous expenditure component in the model, which means that there are factors other than the utilization rate through which income distribution can affect output growth. Yet the profit rate cannot increase in the long run, since the profit share decreases and the utilization rate goes back to its normal range. In this case, even if the model only presents the paradox of costs in terms of level effects, it is still possible to say that income distribution has a permanent impact on growth in the long run.

## 4.2 The paradox of thrift

Following an increase in the propensity to consume out of wages ($\alpha_1$), consumption increases and leads to an increase on output and capacity utilization. This leads to an increase in the propensity to invest of firms and in the capital accumulation rate (figures 3a,4c). Consumption out of wealth loses participation in income (figure 4d), with capital accumulation growing faster than wealth, as in the first simulation experiment. The difference here is that in the short run, the reduction in workers' propensity to save impacts negatively households savings and, consequently, their wealth (figure 3a). However, as soon as consumption affects activity, the higher income will raise the financial income received by households, which contributes positively to wealth growth.

The government debt to capital ratio also decreases as household income - due to an increase in dividend payment and wages -, taxation and capital accumulation increase. In the short run, as firms' loans ratio decreases, the debt ratio falls at a slower pace. In the long run, the reduction in the payment of interest to households, due to the lower debt ratio, and the higher accumulation rate together with a higher loans to capital ratio make the debt to capital ratio decrease even further (figures 3a, 3c).

Differently from the previous experiment, in this case, firms' loans to capital ratio falls in the short run and stabilizes at a higher ratio in the long run in comparison to the baseline. As firms slowly increase their propensity to invest as a reaction to the higher level of activity, the sharp increase in the accumulation rate negatively impacts the loans to capital ratio, compensating the increase in the propensity to invest and the decrease in the amount of equities due to the lower ratio of household wealth to capital (figures 3a, 4a, 4c). Still, in the long run, since the accumulation rate converges towards the growth

referees, the results regarding the rate of profit could also be related to a fallacy of composition between the decisions of firms at the micro level and aggregate macro results. For more on this, see Hein and Van Treeck (2008).

**Figure 1:** Effects of an increase in real wages (reduction in $\mu$)

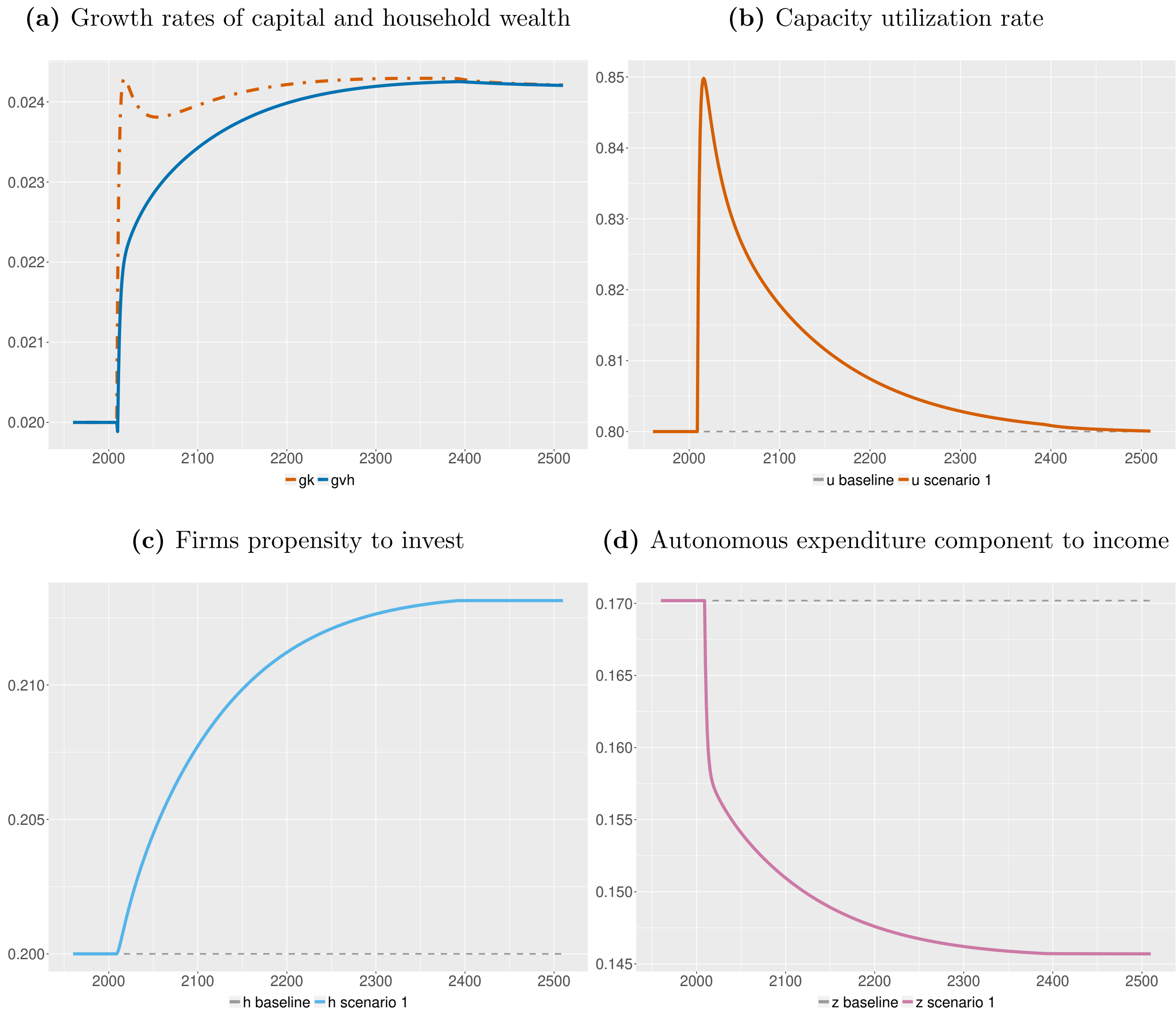
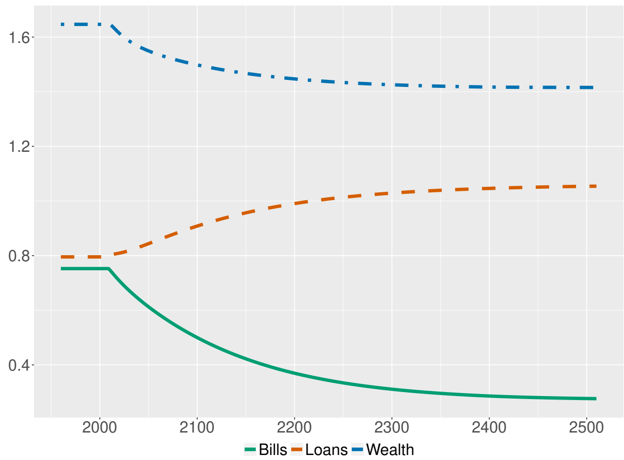
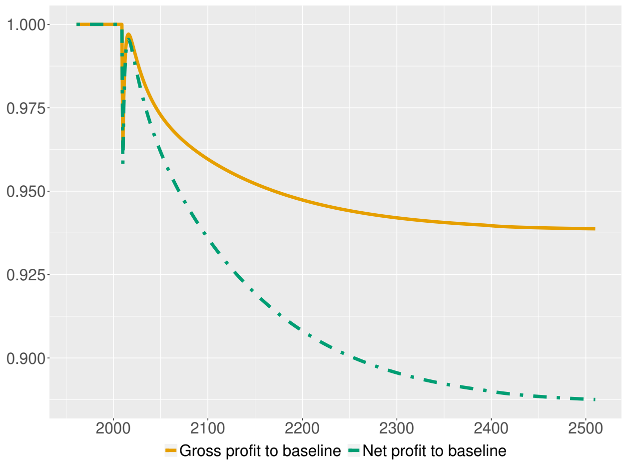


**Figure 2:** Effects of an increase in real wages (reduction in $\mu$)

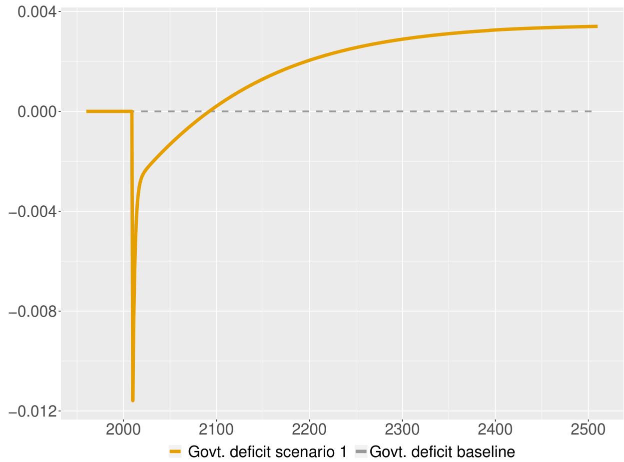
**(a)** Steady growth state ratios



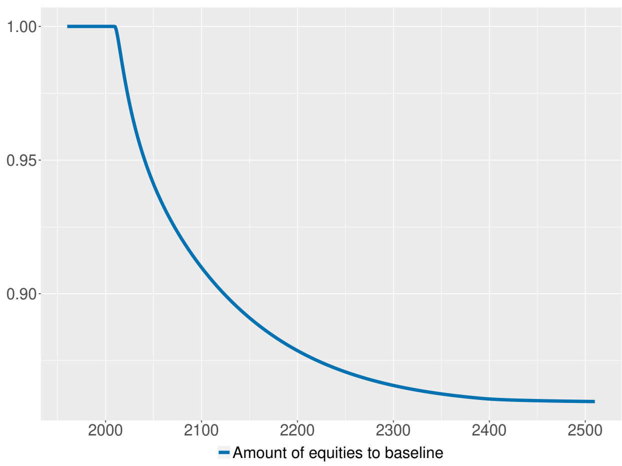

**(b)** Profit rates




**(c)** Government budget deficit




**(d)** Amount of equities



rate of wealth, stabilizing at lower position in comparison to the initial shock (higher than the baseline), the value it assumes is not enough to cover the effects of the propensity to invest and of the amount of equities on the loans to capital ratio.

Household wealth to capital suffers the negative impact of the lower normalized stock of government bills, since interest payments decrease, and also the negative impact of the lower interests on deposits, as a result of lower firms' loans ratio. However, as income and capacity utilization increase, they have a positive effect on wealth, even if wealth grows at a lower rate than capital accumulation. In addition to this, in the long run, as firms' loans attain a higher position in comparison to the baseline, they positively influence wealth (figures 3a, 4b).

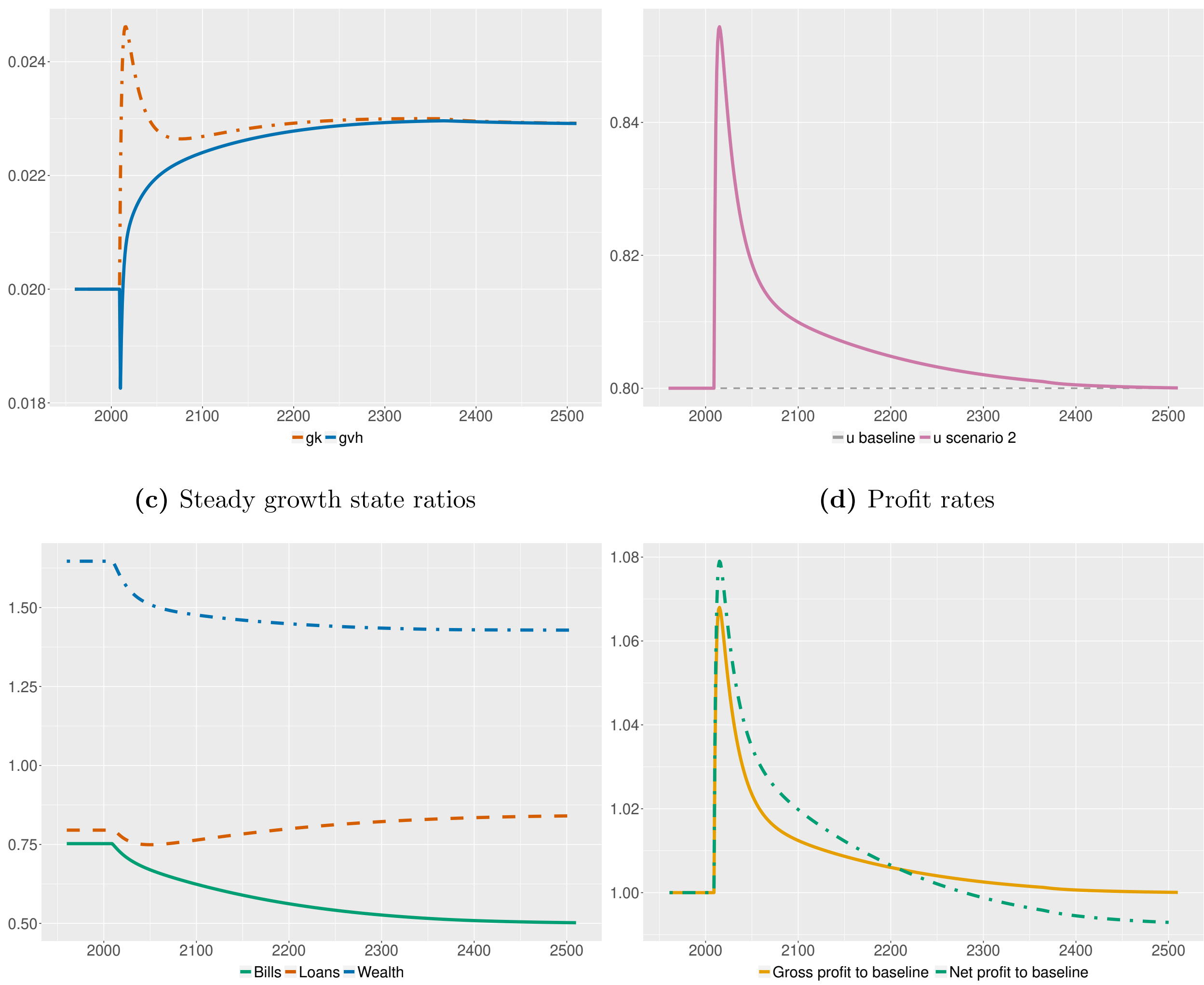
Gross and net profit rates increase in relation to their baseline values due to the temporary increase in the utilization rate. As the utilization rate converges to its desired level, and there are no changes in the profit share, the gross profit rate goes back to its baseline value. The net profit rate decreases as the ratio of loans to capital rests at a higher level, which means that a larger part of profits is destined to the payment of interest on loans (figure 3d).

In sum, we observe that the paradox of thrift in terms of growth effects is still valid in the long run in this framework in which there is an autonomous expenditure component working as an "attractor" to growth and with the utilization rate converging to a desired range. This happens because the reduction in the propensity to save stimulates the economy, boosting consumption from wages, which entails both a higher output level in relation to the baseline, and a higher growth rate in the long run, through the supermultiplier. Differently from neo-Kaleckian models, in which the effect happens through the utilization rate, raising the level of activity and the accumulation rate, in this model the effect happens through the utilization rate in the short run; however, the accumulation rate depends on the output, which ultimately depends on the autonomous expenditure component (consumption out of wealth) and on its multiplier which is permanently increased, raising the overall rate of growth of the economy.

## 4.3 A shock to the propensity to consume out of wealth

An increase in the propensity to consume out of wealth increases consumption, which reduces household savings and, consequently, household wealth in the short run. Differently from the previous experiment, the autonomous component of demand increases relatively to income, but as soon as the effect on capacity kicks in, consumption out of wealth decreases in relation to output (figure 5d). As in earlier experiments, the higher

**Figure 3:** Effects of an increase in the propensity to consume out of income ($\alpha_1$)




**(a)** Growth rates of capital and household wealth

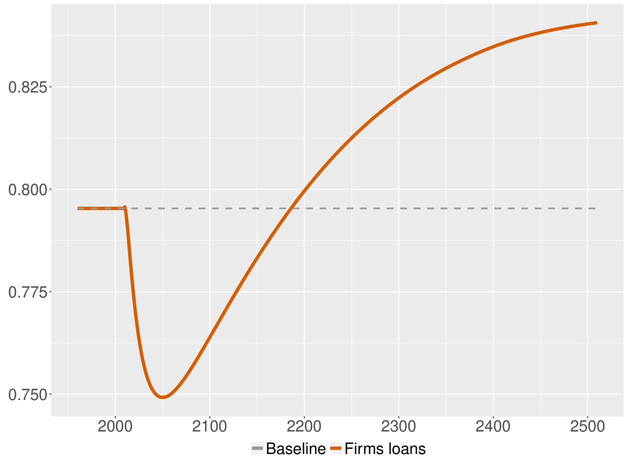
**(b)** Capacity utilization rate

**(c)** Steady growth state ratios

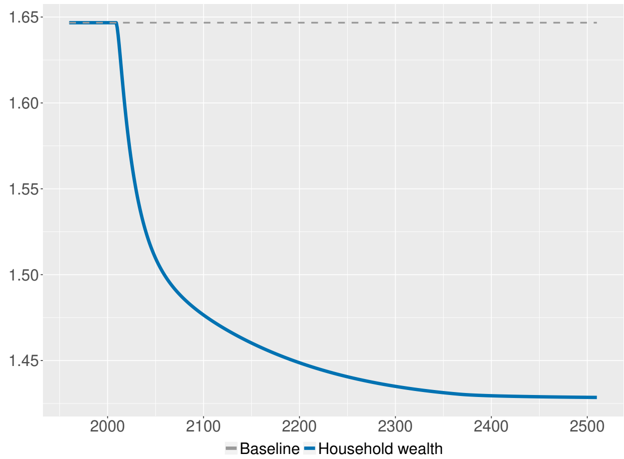
**(d)** Profit rates

**Figure 4:** Effects of an increase in the propensity to consume out of income ($\alpha_1$)

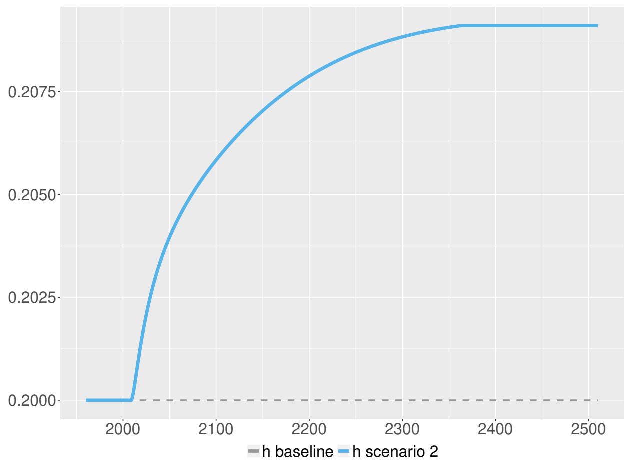
**(a)** Firms loans ratio




**(b)** Househould wealth to capital ratio




**(c)** Propensity to invest




**(d)** Autonomous expenditure component to income

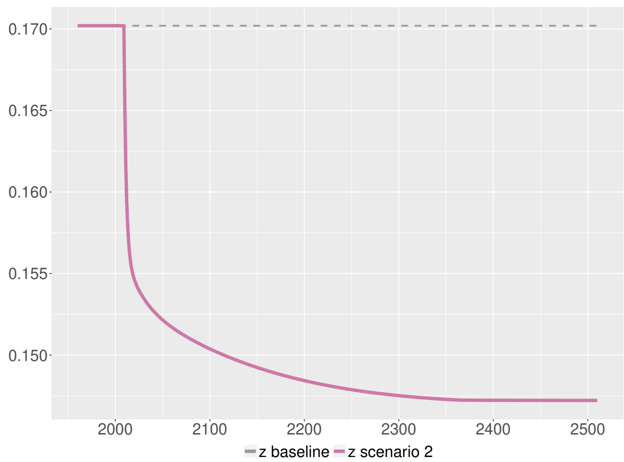
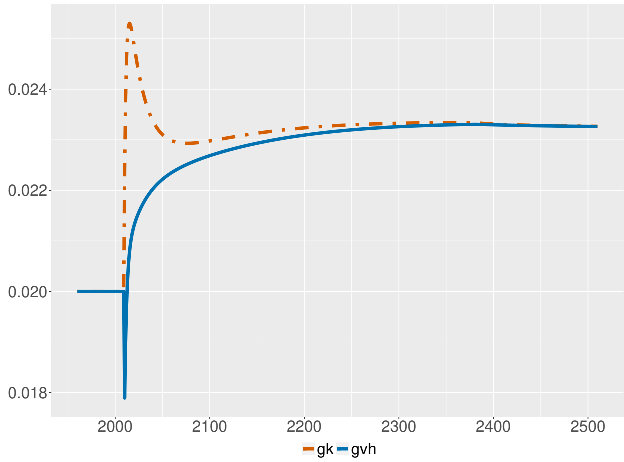
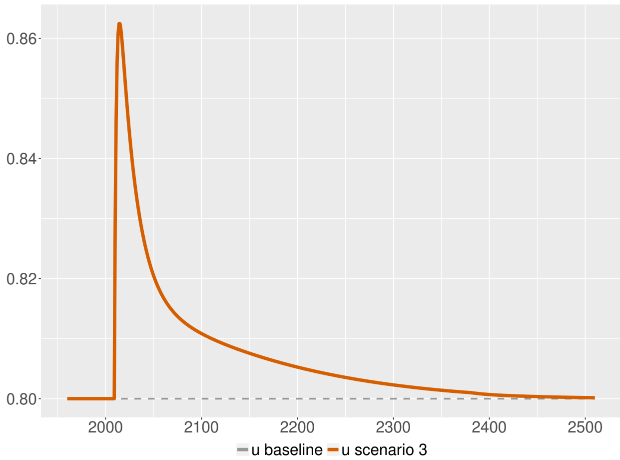


**Figure 5:** Effects of an increase in the autonomous expenditure component (increase in $\alpha_2$)

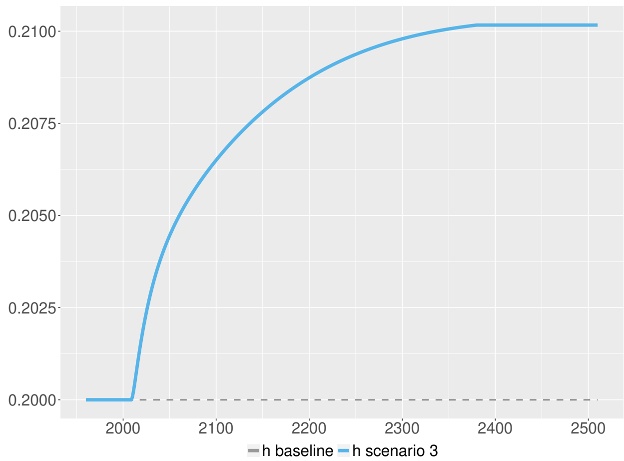
**(a)** Growth rates of capital and household wealth



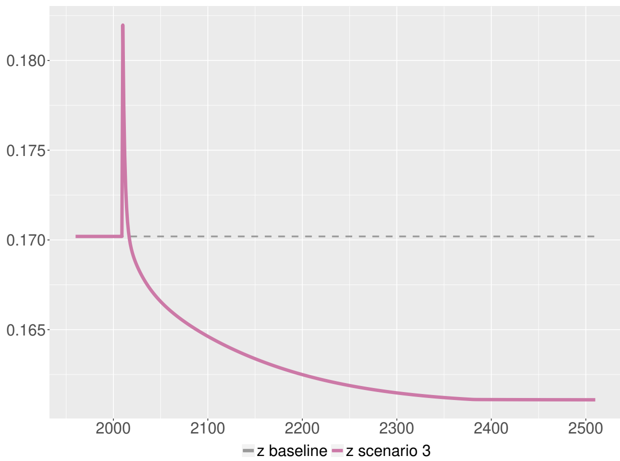

**(b)** Capacity utilization rate




**(c)** Firms propensity to invest




**(d)** Autonomous expenditure component to income




utilization rate (figure 5b) leads firms to increase their propensity to invest, which increases the accumulation rate at a faster pace than that of household wealth (figures 5a, 5c). The effects on the ratios of government bills to capital, firms loans to capital and household wealth to capital are very similar to the effects of a shock in the propensity to consume out of wages. Government bills and household wealth to capital ratios stabilize at lower positions in comparison to the baseline, while the firms loans ratio decreases in the short run but increases in the long run. The gross profit rate increases in the short run but goes back to its baseline rate while the net profit rate decreases as the amount of interest payment on loans increases in the long run.

### 4.4 An assessment of the shocks

All the scenarios have in common the fact that changes in exogenous parameters alter (directly or indirectly) the supermultiplier, and thus affect the long run growth rate as well. Therefore, as long as the autonomous expenditure component is endogenous to the model, the effects of the shocks are not restricted to the transition period. The experiments then show that while the utilization converges back to its desired level or range, the adjustments of a shock to income distribution or to the propensity to save can be absorbed through an endogenous change in the growth rate. It is worth mentioning that this happens without the loss of the Keynesian causality, since the adjustment of capacity to demand occurs through changes in the autonomous component of demand, whose share in income falls when investment rises.

It goes without saying that most of the results of our experiments were only made possible by the adoption of an SFC framework. In the original supermultiplier approach, autonomous demand growth is given once and for all – or until it is exogenously changed. This exogeneity makes it impossible to establish the connections between a change in the propensity to invest and the determinants of the autonomous expenditure (household wealth, in the case of this paper). Moreover, the omission of financial variables prevents the evaluation of the effects of an increase in capital accumulation and in the autonomous expenditure growth rate on the financial stocks of the economy (loans, bills, household wealth). It also prevents understanding that a permanent (say) increase in the supermultiplier allows for a permanent increase in the growth rate of wealth in the same direction despite the reduction of the household wealth to capital ratio.

## 5 Final remarks

As we have seen, so far supermultiplier models do not deal properly with financial issues. They do not take into consideration the interactions between financial stocks and flows and how such interactions could impact growth in the long run. Since the growth rate of the autonomous expenditure component is exogenously given, these models do not allow for the emergence of the paradoxes of thrift and costs in terms of growth effects. It does not matter which "non capacity creating" autonomous expenditure is leading growth in the long run, whether consumption, government expenditures, or net exports, only the level effects of changes in income distribution and in the propensity to save will last. However, when we allow for the autonomous expenditure component to be endogenous, as in the model we built here, which depends on household wealth, we also allow for feedbacks between financial income and financial stocks, as for feedbacks between the latter and the capital stock. Changes in income distribution and in the propensity to save will permanently affect the growth rate of the economy, through the supermultiplier, and through the dynamics of household wealth to capital ratio.

The main results obtained through the experiments of section 4 can be summarized as follows:

(i) An essential claim of the Supermultiplier approach is that a higher growth rate of the autonomous expenditure component is associated with a higher investment to income ratio. This assumption still holds for a more complex model even if the autonomous expenditure component is endogenous to the economic system. As the autonomous expenditure component grows at a faster pace it increases the income which will stimulate more expenditures, say by increasing consumption out of wages, and these higher expenditures will boost investment, as the utilization of capacity rises. As investment accelerates induced by income, the investment share increases relatively to income while the reverse happens to the autonomous expenditure component share (Serrano, 1995b);

(ii) The paradox of costs is still valid in terms of level effects. A reduction in the mark-up of firms (lower profit share) leads to lower profit rate, but to a higher level of profits in the long run, as a consequence of the higher capital accumulation. However, differently from other supermultiplier models, a higher wage share has a permanent growth effect, through the supermultiplier mechanism. The discrepancy between actual and desired utilization rates promotes a permanent increase in the propensity to invest. This, along with the higher wage share, compensates the effect of a lower wealth-to-capital ratio on the growth rate of this economy;

(iii) The paradox of thrift is valid both in level and in growth terms in the long run. An increase in the propensity to consume out of after-tax wages permanently affects the growth rate of the economy in the long run through the supermultiplier;

(iv) The relation between stocks and flows matter, since an increase in the propensity to invest contributes to increase the stock of firms' debt to capital. This implies that the propensity to invest can find a constraint in the values it can assume, coming from the amount of loans firms borrow in order to finance this same investment and which also depends on the how the propensity to invest will impact the accumulation rate, in order to compensate the higher loans-to-capital ratio;

(v) The behaviour of the autonomous component reveals once more the centrality of stock and flow interactions, for consumption out of wealth is influenced by the government debt ratio, by firms' propensity to invest and by the capital accumulation rate;

Needless to say, the discussion presented here could be enriched in several ways. The first one concerns the generality of our conclusions, which should be evaluated by means of a stability analysis of the model and a sensitivity analysis of the parameters to verify for which range of (economically meaningful) parameters the paradoxes remain valid in the long run. Second, it would be important to move to an open economy setting; it is well known that the paradox of costs may not hold when international transactions are taken into account. Third, it would also be important to test the same hypothesis for an economy with a more complex financial system, for instance incorporating consumer credit and assuming a more "active" and profit-earning banking sector, including the possibility of credit rationing.

A final and possibly important front which would require further research refers to the implication of specific growth engines. There is no reason to assume that a consumption-led growth regime will be as durable as (say) a government- or an export-led one. Each growth engine will feature specific interactions between stocks and flows, will face specific financial constraints and will present different stability conditions.

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# Appendix A

**Table 7:** Parameters and long run value of variables

| Parameters/variables | Baseline | Scenario 1 | Scenario 2 | Scenario 3 |
|---|---|---|---|---|
| $\tau$ | 0.37 | 0.37 | 0.37 | 0.37 |
| $\sigma$ | 0.34 | 0.34 | 0.34 | 0.34 |
| $\alpha_1$ | 0.8 | 0.8 | 0.84 | 0.8 |
| $\alpha_2$ | 0.03373529 | 0.03373529 | 0.03373529 | 0.0374 |
| $a$ | 0.1 | 0.1 | 0.1 | 0.1 |
| $x$ | 0.001 | 0.001 | 0.001 | 0.001 |
| $\lambda_0$ | 0.08 | 0.08 | 0.08 | 0.08 |
| $pe$ | 0.9880216 | 0.8493563 | 0.8571246 | 0.846402 |
| $\delta$ | 0.044 | 0.044 | 0.044 | 0.044 |
| $i_r$ | 0.02 | 0.02 | 0.02 | 0.02 |
| $v$ | 2.5 | 2.5 | 2.5 | 2.5 |
| $\mu$ | 0.7 | 0.63 | 0.7 | 0.7 |
| $\pi$ | 0.4117647 | 0.3865031 | 0.4117647 | 0.4117647 |
| $s_f$ | 0.4 | 0.4 | 0.4 | 0.4 |
| $\gamma$ | 0.014 | 0.014 | 0.014 | 0.014 |
| $h$ | 0.2 | 0.2131375 | 0.2091036 | 0.2101661 |
| $u \simeq u_n$ | 0.8 | 0.8 | 0.8 | 0.8 |
| $m^* = l^*$ | 0.7953297 | 1.054224 | 0.840638 | 0.8442299 |
| $b^*$ | 0.7525708 | 0.276434 | 0.5021906 | 0.4817999 |
| $v_h^*$ | 1.646703 | 1.415594 | 1.428541 | 1.41067 |
| $g^*$ | 0.02 | 0.02421 | 0.02291702 | 0.023161 |