

# The new models of decision in risk: A review of the critical literature

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#### The new models of decision in risk: A review of the critical literature

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**Abstract:** The aim of the risk decision theory is to describe the behavior of agents in the face of several random prospects. Since it is difficult to describe these preferences, we seek to represent them. The use of a representative function of preferences has been for a long time, the usual method of describing behavior in a random context. The obvious advantage of this method is that it allows including these data in a formalized model and, by extension, to understand the optimization process underlying any decision.

The determination of the representative function of preferences must be based on an axiomatic basis. From these axioms, an accurate specification of the value function will be derived. The purpose of this article is to examine the history of theories that have sought to determine a satisfactory criterion for responding to the risk decision problem and to analyze the contribution of these models.

**Key words:** Risk Aversion, Expected Utility (EU), Rank Dependent Expected Utility (RDEU), Gamble.

**JEL classification**: D81, C91.

#### Introduction

By exposing the famous problem known as the "Petersburg paradox", Daniel Bernoulli (1738) showed that the behavior of an economic agent averse to risk is characterized by a certain equivalent. This paradox examined by Nicolas Bernoulli (1713) challenged the validity of expected value.

The attitude towards the risk of the economic agent determines its utility function. For this reason, D. Bernoulli (1738) proposed replacing the criterion of expected value with

that of expected utility. The proposed function was 
$$u(x) = \beta \log \left(1 + \frac{x}{\alpha}\right)$$
 avec  $\alpha, \beta \rangle 0$ .

The mathematician Gabriel Cramer (1728), a contemporary of Bernoulli, arrived at a solution close to that worked out by the latter using another utility function of wealth:  $u(x) = \sqrt{x}$  which postulates, like the preceding one, a decrease in the marginal utility of wealth. Later, in 1947, with the work of Von Neumann and Morgenstern (VNM), that a theory of utility was defined. The progress of the article is as follows. Section 1 presents a synthesis of the theory of expected utility. In the second section, we present the risk decision models. The notion of risk aversion will be developed in Section 3. Finally, we conclude with the criticisms addressed to these models.

## 1. Theory of expected utility

The expected utility theorem states that, when confronted with a set of action lines with random results or, more generally, with a lottery set, an individual will choose the one whose expected utility is the highest, As far as its behavior respects five axioms: comparability, transitivity, strong independence, continuity and dominance.

This theory has two main qualities:

- First, it separates beliefs about sources of uncertainty, represented by probabilities on uncertain events and utility for payoff, represented by a utility function on the consequences.
- Second, the function representing the preferences is linear in probabilities. These qualities are at the origin of the success of the expected utility model as a means of representing the preferences of agents operating in an uncertain environment.

However, two questions arise:

- First, is it reasonable to assume that every agent is able to attribute a single probability distribution to any situation of uncertainty?
- Second, even when there is a probability distribution, does the agent behave according to the expected utility model?

The authors who have examined the model of expected utility have shown that the utility levels attached to the different earnings merely reflect an order of preference and should not be given any psychological or philosophical interpretation. Thus, the construction of the utility curve depends only on the initial payoff of the decision maker and his aversion towards risk. It should be emphasized that if the criterion of maximization of expected utility provides a theoretical answer to the problem of choice in an uncertainty situation, it will not make possible to choose the best investment, only if the characteristics of the utility function of the decision maker are perfectly identified. Indeed, if the attitude of the decision maker respects the axioms of VNM, the first derivative of the utility function will be positive and, according to Friedman and Savage (1948), the second derivative will be negative for a risk averse decision maker.

Other authors such as Vickrey (1945), Kaysen (1946), and Friedman & Savage (1948) wondered how the utility model could account for the behavior of all those who subscribed to insurance and purchased lottery tickets. Indeed, paying an insurance premium amounts to preferring a small loss with certainty rather than a large loss with low probability.

The first attitude seems, in their view, to attest the hypothesis of decline of marginal utility, while the latter seems to contradict it. Friedman & Savage (1948) provide the following answer: the utility function is first concave, then convex, and again concave, thus allowing attitudes previously considered contradictory.

This discussion will be eclipsed by the advent of the concept of independence. The idea is that "the order of preferences between two lotteries will not be changed if the two lotteries are combined with the same third lottery". This idea is the keystone of the expected utility model, and it is the axiom of independence that makes the utility function linear in probability. Although it was widely accepted later, it was nevertheless strongly questioned by Maurice Allais (1953), from its emergence. He demonstrates how difficult the axiom of independence to resist simple experiments. These experiments will for a long time be called "paradoxes", insofar as the observed violation of the axiom of independence was interpreted as an anomaly. Under the name of "Allais paradox", one implies an experiment leading to this violation. Hence, expected utility does not seem to represent the preferences of a majority of agents, but it must be consistent with the behavior of the agent and reflect his attitude towards risk.

#### 1.1. Risk aversion measures

Risk aversion is a central assumption in modern financial theory. Indeed, investors demand a higher remuneration as the risk of their financial locations is high.

Studies show that an investor has risk aversion if his utility function u is strictly concave. In other words, the marginal utility of wealth must be decreasing  $(u^{"}(.) \leq 0)$ . If

the marginal utility of wealth is increasing (resp. constant), the investor opts for the risk (resp. risk neutral).

Arrow (1965) and Friend & Blume (1975) have shown that absolute risk aversion decreases when an agent's payoff increases. As for Pratt (1964), he showed that relative aversion grows with payoff as a consequence of the fact that absolute aversion to risk is decreasing in payoff. This hypothesis has been questioned by Friend & Blume (1975) who have shown that the relative aversion is rather constant.

In the same context of uncertainty, Kimball (1990) introduces the notion of prudence as "the propensity of agents to arm themselves and prepare themselves for uncertainty". For him, prudence reflects how uncertainty affects decision variables. It analyzes problems in which the effect of risk is concerned with the marginal utility of agents and not with their total utility. Sandmo (1971) and Leland (1972) studied the saving decision in uncertainty. The results show that prudence indicates the intensity of the precautionary savings pattern. If future incomes are unpredictable, prudent agents save more to guard against changes in their future consumption. They conclude that this type of behavior is induced by the marginal convexity, i.e. it corresponds to a positive third derivative for the utility function VNM  $(u^{"}(.))$ 0). However, the convexity of the marginal utility is implied by the decline in the absolute risk aversion of Arrow (1965) and Pratt (1964). This property implies that absolute prudence is greater than absolute risk aversion. The analogy between Arrow's and Pratt's and Kimball's is fairly obvious. The first evaluates the concavity of the utility function, the second evaluates the convexity of the marginal utility. In order to clarify the difference in nature between the two concepts, Viala and Briys (1995) cite the case of the quadratic utility function which is representative of risk aversion behavior but not of prudent behavior. The marginal utility is linear in wealth  $(u^{"}(.)=0)$ . Consequently, if the agents associated with this utility function buy insurance, uncertainty never leads them to increase their savings in order to protect themselves against the hazards related to their future consumption. Indeed, if we consider the following quadratic utility function: U(r) = a +br - cr<sup>2</sup> where r is the rate of return perceived by the investor and a, b and c are constants with b and c strictly positive. If such a function can describe the attitude of an investor who is risk averse, since his second derivative is negative, it can only be used

if  $r \in \left[-\infty, \frac{b}{2c}\right]$  for the first derivative to be positive. This limitation constitutes a

disadvantage of the use of a quadratic utility function.

# 1.2. Criticals addressed to the expected utility model

In addition to the experimental violations of the axiom of independence, the expected utility model also raises a theoretical difficulty, namely the interpretation of the function u. Indeed, this function has two roles:

- The first is to express the decision maker's attitude towards risk (the concavity of u involving risk aversion).
- The second role consists in expressing the satisfaction of the results in the certain (the concavity of u implying a decreasing marginal utility of the payoff).

In particular, it is impossible in this model, as noted by Cohen and Tallon (2000), to represent an agent that would have both a decreasing marginal utility and a taste for risk. If the model of utility expectation has the merit of parsimony, it does not allow separating the representation of the attitude with regard to the risk of that with respect to the guaranteed payoff.

# 2. Risk decision models or "unexpected" utility theories

The theory of expected utility remains the reference of the representation of behavior in

the face of an exogenous uncertainty. It applies to the case of an uncertainty measured by a probability distribution. As soon as the distribution is not known and may depend on the investor's decision to better take account for the type of uncertainty prevailing on financial markets, the expected utility model becomes incapable of adequately representing the "The investor's attitude to risk. On the other hand, the results of several empirical and experimental studies challenge the predictions of the standard theory of VNM (1947). For example, one of the first experimental studies, that of Allais (1953), calls into question the hypothesis of linearity in probabilities. Ellsberg (1961) also shows that agents do not respect the axiom of independence, noting that utility weights are not probabilities. Hence, the emergence of new decision models that take account of this failure.

## 2.1. The Prospect Theory

The idea of prospect theory is to represent preferences by a function  $V: L \to \Re$  Such as for a lottery

$$L = (x_i, p_i)_{i=1,\dots,n} , V(L) = \sum_{i=1}^n \pi(p_i).u(x_i)$$
 [1]

where  $\pi$  is an increasing function of [0, 1] into [0, 1], with  $\pi(0) = 0$  and  $\pi(1) = 1$ .  $\pi(.)$  translates the transformation of probabilities for the agent whose preferences are represented. This type of transformation function makes it possible to take into account a possible "certainty effect". Thus, a discontinuity of  $\pi(.)$  on the left at point 1 would very conveniently translate the psychological change that results from the passage from an area of perfect certainty to a zone of risk.

This implies that near point 1,  $\pi(p)\langle p$ . The  $\pi(p_i)$  are called by Kahneman &

Tversky (1979), « decision weight ». They are no longer probabilities since  $\sum_{i=1}^{n} \pi(p_i)$  is

no longer necessarily equal to 1. Thus, the "paradoxes" of Allais are no longer necessarily paradoxes.

## 2.2. Theory of anticipated utility

Quiggin (1982) has taken up three axioms of VNM analysis: transitivity, first-order stochastic dominance, and continuity. To these three axioms, he added an axiom of independence weaker than that of the theory of expected utility.

This axiom is as follows:

Either a set of consequences C, and either a lottery  $L = (x_i, p_i)_{i=1,\dots,n}$  such as the consequence  $x_i$  is associated with the probability  $p_i$  with  $x_1 \le x_2 \le \dots \le x_n$ , then:

$$\forall L_1 = (x_i, p_i) \text{ et } L_2 = (x'_i, p_i) \text{ where } i = 1, 2, \dots, n$$

$$\exists c_i = CE\left\{(x_i, x_i'); \left(\frac{1}{2}, \frac{1}{2}\right)\right\}, \text{ and if } x_1^* = CE(L_1) \text{ and } x_2^* = CE(L_2) \text{ then}$$

$$(c_i, p_i) \sim \left\{ (x_1^*, x_2^*); \left(\frac{1}{2}, \frac{1}{2}\right) \right\}$$
 where CE(X) is the certain equivalent of X.

Thus Quiggin defines a functional utility V:  $L \to \Re$  such as:

$$1/V(L_1) \ge V(L_2)$$
 if and only if  $L_1 > L_2$ .

2/ For a lottery 
$$L = (x_i, p_i)_{i=1,...,n}$$
,  $V(L) = \sum_{i=1}^{n} \pi(p_i).u(x_i)$ 

where  $\pi$  is a non-decreasing function of [0,1] into [0,1] verifying the following assumptions:  $\pi(.)$  is concave on the interval  $[0, \frac{1}{2}] (\pi(p_i)) p_i$  and convex on the

interval 
$$[\frac{1}{2},1]$$
 ( $\pi(p_i)\langle p_i$ ) with  $\pi(\frac{1}{2}) = \frac{1}{2}$  and  $\pi(1) = 1$ .  
3/ V is unique to an affine transformation by. [2]

Quiggin shows that under these conditions, the first-order stochastic dominance is respected and that these hypotheses explain the paradoxes of Allais as well as the results of Friedman and Savage (1948) concerning the coexistence of the gamble and the insurance. They are also consistent with the experimental study by Kahneman and Tversky (1979), who observe that agents overweight low probabilities and underweight high probabilities.

## 2.3. Dual theory

Yaari (1987) proposed an alternative choice model to the expected utility model, which he described as dual theory. This theory evaluates the risky situation without transforming the final wealth into utility of wealth and modifying the probability distribution that defines the risk to which the individual is subjected. It is also noted that the attitude towards risk is no longer defined in the same way. Indeed, the criterion of expected utility expresses attitudes towards risk through the transformation of wealth. The dual theory and we will see later the RDEU theory define the attitudes towards the risk essentially through the transformation of the probabilities. The functional utility is

defined as follows: 
$$V(L) = x_1 + \sum_{i=2}^{n} [x_i - x_{i-1}] \pi \left( \sum_{j=i}^{n} p_j \right) = DT(L)$$
 [3]

However, in his paper, Yaari (1987) presented an implication of his model for simple portfolio choices and the result obtained is not very encouraging. Indeed, assuming that the investor has a choice between a safe asset and a risky asset whose return expectation exceeds the safe rate, Yaari obtains that the investor will never diversify, i.e. he will place all its fortune either in the safe asset or in the risky asset.

As a result, it can be argued that this model has a propensity to provide corner solutions that run counter to the idea of diversification. Later, Gayant (2004) shows the importance of the transformation of probabilities in the combination of risky assets.

## 2.4. Rank Dependent Expected Utility model (RDEU)

The RDEU model is a late response to the criticisms of the EU model formulated by Allais by presenting the probabilities in a nonlinear manner and taking into account the observations of Ellsberg (1961), by weakening the axiom of independence. The first one to have generalized the utility expectation by highlighting the merits of the transformation of the probabilities depending on the rank of the results is Quiggin (1982). The objective of Quiggin was to introduce a preference functional taking into account the transformation of probabilities and not suffering from any of the shortcomings of the descriptive model of Kahnman and Tversky (1979). This anomaly, namely the violation of stochastic dominance, was eliminated by replacing the transformation of the probability of each event by transforming the distribution of the decumulated probabilities of ordered events by increasing result. As a result, agent preferences cannot be represented by a classical expected utility function, but rather by a rank dependent expected utility function.

In accordance with the ranking dependent expected utility (RDEU) theory, the representative function of preferences is defined as follows:

For all X, Y random variables with values in a set of consequence or results

$$X \succ Y \Leftrightarrow V(X) \ge V(Y) \text{ with } V(X) = -\int_{M}^{M} u(x) d\pi(G_X(x))$$
 [4]

where the function u(.) is continue, differentiable and strictly increasing from [-M, M] to  $\Re$ , unique modulo a strictly positive affine transformation, and  $\pi(.)$  is a function continue, strictly increasing from [0, 1] into [0, 1]. Without loss of generality, we can assume that  $\pi(0) = 0$  and  $\pi(1) = 1$ ; furthermore  $\pi(.)$  is unique.

Note that, when  $X = L = (x_1, p_1; x_2, p_2; \dots; x_n, p_n)$  is a lottery with  $x_1 \le x_2 \le \dots \le x_n$ ,

$$V(L) = u(x_1) \cdot \pi \left(\sum_{i=1}^{n} p_i\right) + \left[u(x_2) - u(x_1)\right] \pi \left(\sum_{i=2}^{n} p_i\right) + \dots + \left[u(x_n) - u(x_{n-1})\right] \pi (p_n)$$

$$= u(x_1) + \sum_{i=2}^{n} \left[u(x_i) - u(x_{i-1})\right] \pi \left(\sum_{i=i}^{n} p_i\right)$$
[5]

The weighting coefficients  $\pi_i$  depend on the rank of the results  $x_i$ , hence the name given to this theory. These weights reflect the marginal contribution of  $\pi_i$  to the transformation function of the decumulated probabilities. In other words, the  $\pi_i$ , which are calculated by ordering the consequences from the most unfavorable to the most favorable, express a phenomenon of perception of probabilities or processing of risky information. As a result, the decision maker reasons his assessment of the theory by adding the following expectations:  $\mathbf{u}(x_1)$  (of which it is safe, so it weights by 1), the additional  $[\mathbf{u}(x_2) - \mathbf{u}(x_1)]$  which it weights by a transformation  $\pi(.)$  of the probability  $(p_2 + p_3 + \dots + p_n)$  to have at least this extra in addition to  $u(x_1)$ , ...., up to  $[[u(x_n) - u(x_{n-1})]$  which it weights by the same transformation  $\pi(.)$  of its probability  $p_n$ .

This representation of preferences is based on the assumption that there exists an objective probability distribution governing the appearance of states of nature in the second period. However, agents do not directly utilize this distribution to perform the expected utility calculations but first transform the objective probabilities. For stochastic dominance to be respected, it is necessary to consider the transform of the cumulative distribution. Consequently, the functional representing agent preferences is no longer linear with respect to the probabilities. In particular, the weights attached to each state of nature depend on the rank of these.

Note that, under this representation, in the case of a lottery with two results,  $V(L) = u(x_1) + \pi(p_2) [u(x_2) - u(x_1)]$ . This is the particular specification of the prospect theory.

The RDEU criterion is a generalization of expected utility. Indeed, it is seen that when the function  $\pi$  is the identity function ( $\pi(p) = p$ ,  $\forall p \in [0,1]$ ), the representative function of preferences becomes the classical expected utility. Thus, the relation [5] becomes:

$$V(L) = u(x_1) + \sum_{i=2}^{n} \left[ u(x_i) - u(x_{i-1}) \right] \left( \sum_{j=i}^{n} p_j \right) = \sum_{i=1}^{n} p_i u(x_i) = EU(L)$$
 [6]

Moreover, when u(x) = x, the RDEU theory is none other than the dual theory of Yaari. So, the relation [6] is only a special case of the relation [5]

As noted by J.P.Gayant (1995), the RDEU representation excludes any violation of first-order stochastic dominance. On the other hand, as its name indicates, this criterion

is characterized by the dependence on the rank of the results of the transformation of the probabilities. This means that the principle of invariance to the modification of a common consequence must not be postulated only when the replacement does not modify the order of the results.

Tallon (1997) points out that one of the advantages of the rank-dependent theory of expected utility, is that it makes it possible to distinguish several notions of aversion to risk. Indeed, he notes that it is possible to define, independently of any representation of preferences, a notion of weak risk aversion (according to which a decision maker prefers the expectation of the lottery to the lottery itself) and a notion of strong risk aversion (according to which an agent prefers a given lottery to a constant mean spread of the same lottery)

These two notions (Cohen, 1995) merge into the theory of expected utility and correspond to the concavity of the utility function in the certain. They are no longer equivalent in the context of rank-dependent utility.

Contrary to the criterion of expected utility, rank dependent expected utility theories distinguish attitudes towards wealth and attitude towards risk, to explain decision-making behavior. In this respect, they respond to experiments that stress that individuals underestimate or overestimate the probabilities of risk. i.e., they are optimistic or pessimistic (in relation to probabilities). To account for these behaviors, the RDEU theories have introduced in the calculation of the preference function a probability transformation function into the risk (Quiggin, 1982 and Yaari, 1987). These theories are able to predict cases often observed in reality or in experimental studies but unexplained by the EU theory.

## 3. Risk aversion and new decision models

The notion of risk aversion, which is a basic element in any economic application where the environment is not certain, has been questioned, especially with regard to the interpretation of certain results relating to the expected utility model. Thus, the emergence of new decision models necessitated a change in the characterization of risk aversion. Indeed, for a large number of economists, the notion of risk aversion is inseparable from the decline in marginal utility. If the two notions are confused in the expected utility model, they differ substantially in the general model of rank dependent expected utility. This observation is not without consequence. Indeed, the disappearance of the equivalence between the two notions in the RDEU model leads us to reconsider the notion of risk aversion in the EU model.

#### 3.1. Weak risk conversion

According to the Arrow and Pratt view, the notion of risk aversion is implicitly a "weak risk" notion.

**Definition: weak risk aversion :** An agent is weakly opposed to the risk if he prefers to any lottery the gain of his expected value with certainty.

Under the assumptions of the expected utility model, there is equivalence between this notion of risk aversion and the concavity of the utility function.

Chateuneuf & Cohen (1994) and Chateuneuf, Cohen & Meilijson (1997) show that a decision maker, satisfying the RDEU model and characterized by a utility function u(.) continuously differentiable and concave, is weakly opposed to the risk if and only if its function of transformation of the probabilities f(.) satisfied  $f(p) \le p$ ,  $\forall p \in [0,1]$ . They demonstrate that if  $u(x) = 1 - (1-x)^n$  with  $n \ge 1$ , then it is weakly adverse to the risk if and only if its transformation function f(.) satisfied  $f(p) \ge 1 - (1-p)^n$ ,  $\forall p \in [0,1]$ .

## 3.2. Strong risk aversion

Rothschild and Stiglitz (1970) define a stronger notion. It is based on the concept of mean preserving spread: Given two random variables X and Y, whose probability distributions are  $L_X$  and  $L_Y$ , Y is deduced from X by a mean preserving spread if:

$$1/ E(L_x) = E(L_y)$$

$$2/ \forall T \in [-M; M], \int_{-M}^{T} \operatorname{prob}\{X \setminus t\} dt \leq \int_{-M}^{T} \operatorname{prob}\{Y \setminus t\} dt$$
 [7]

(The expression 2 / is the definition of second-order stochastic dominance (X dominates Y)).

## **Definition: Strong risk aversion**

An agent is strongly opposed to the risk if between all pairs of random variables such that one is deduced from the other by a mean preserving spread, it always prefers the least "spread".

Under the hypotheses of the expected utility model, there is also an equivalence between this notion of risk aversion and the concavity of the utility function and thus the equivalence between the notion of weak aversion and that of strong aversion. It is the questioning of the expected utility model, which will confer on the distinction aversion weak-strong all pertinence. Under the assumptions of the RDEU model, the two notions differ. In fact, before the emergence of this generalization, the distinction between these notions seemed unfounded. Thus, the generalization of expected utility leads not only to a careful consideration of the notion of risk aversion, but also to a reexamination of the interpretations made over the past half century within the expected utility model.

In addition, Quiggin (1992) defines a new notion of "mean monotone preserving spread" to define a new notion of risk aversion, intermediate between weak aversion and strong aversion. A comparison of these three concepts can be found in the utility model and in the RDEU model (Cohen, 1995).

#### 4. Reviews addressed to these models

Several experimental studies (Bouyssou, 1984, Munier, 1989; Abdellaoui et al., 2007) have shown that individuals, confronted with simple risky choices, generally behave in contradiction with the hypothesis of linearity in probability and, consequently, in violation of the axiom of independence.

On the other hand, Schoemaker (1991) has shown that the RDEU model provides a predictive improvement on the utility model only in the case of losses and does nothing to improve earnings. Gayant (1995) found that the Schoemaker study (1991) is not convincing, since the latter uses, in constructing these tests, implicit assumptions that go beyond the risk framework.

Camerer (1992), Starmer (1992) and Abdellaoui & Munier (1994) lead series of experiments in the representation proposed by Machina (1982), known as the "Marshak-Machina Triangle". The triangle used by Marschak (1950) and Machina (1982) represents all possible lotteries with three fixed outcomes  $x_1, x_2$  et  $x_3$  ordered:  $x_1 \langle x_2 \langle x_3 \rangle$ .

The abscissa axis carries the probability  $p_1$  of the payment  $x_1$ , and the axis of the ordinates carries the probability  $p_3$  of the payment  $x_3$ . The sides of the triangle are chosen equal to the unit. Every point in the triangle gives  $p_2$  as the length of the horizontal segment that separates it from the hypotenuse. By describing the triangle, one describes visually all the possible distributions with fixed supports  $x_1$ ,  $x_2$  et  $x_3$ . Munier (1989) has detailed this type of experimentation and the use of its graphic representation to explain the various "paradoxes".

If Starmer (1992) concludes that an alternative model to the utility model is necessary, it can not arbitrate between non-additive utility and regret theory (Loomes and Sugden, 1982).

Camerer (1992) and Abdellaoui & Munier (1994) also attest to the inability of the classical model to represent the preferences of a majority of agents. They delimit in the triangle of Marshak-Machina, "zones" where the expected utility is valid and "zones" where it is not. Abdellaoui & Munier even determine an area where the RDEU model is invalidated.

While there has not been much work on estimating the probability transformation function, the estimates of Tversky and Kahneman (1992) have been consistent with the intuition of Quiggin (1982) (the small probabilities are "over-weight" and the high probabilities are "under-weight").

Another criticism, of an experimental nature, is addressed to these models. Indeed, experimental protocols are always defined from questions in order to know the degree of risk aversion of individuals. This information is necessary for at least two reasons: the first is to more precisely identify individual decisions and the second to verify theoretical predictions.

However, the degree of risk aversion may not have the same value from one theory to another. Indeed, according to the expected utility model, attitudes to risk are calculated from a transformation of wealth, whereas in dual theory, they are determined as a function of the transformation of probabilities.

In order for agents to reveal their preferences, researchers often propose: a fair or actuarial game or a risky situation. Subsequently, they check whether the agents pay a premium higher or lower than that which defines the risk neutrality according to the criterion of expected utility. If the price is higher, agents are averse towards risk, otherwise they like risk.

In this context, we should note that monetary incentives in economics, in order to carry out such experiments, are commonly used to encourage the subject to reveal his preferences. Some economists who think that they may be the source of the inconsistencies of certain answers criticize these methods. In fact, Battalio et al. (1990) and Etchart & Haridon (2011) conclude that individuals have more risk aversion in the presence of monetary incentives.

Moreover, the formulation of questions can influence individuals in their choice.

Finally, the complexity of these models and especially the results of the tests they have been subjected to allow us to say that these models have not yet provided definitive answers allowing to conclude their superiority with respect to the expected utility model (Chateauneuf et al., 2005 & Trabelsi, 2006).

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