Technological Progress and Optimum Labor Market Friction

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Abstract

This paper presents a stylized economy with a labor market characterized by search friction. Endogenous technological progress is the only source of growth. A single good is produced with the only factor of production, labor. It works as input in both production and R&D. The model derives the optimal job match that leads towards maximum long run gain, and this optimal can be achieved even without changing the economy's growth rate. Model proposes, higher payment in R&D is optimal for high growing countries and/or countries with high labor market efficiency. The results are robust with tax-financing government expenditure and under stochastic fluctuations.

Keywords: Search and Matching friction; Labor Market; Technological progress

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1. Introduction

"Signs America's Scientific Edge Is Slipping"[2]

---Farrel (Denver Post, 2006)

The above headline aptly captures the central concern of the present paper. A class of empirical literature suggests college enrollment decisions are a countercyclical phenomenon (Dellas and Sakellaris (2003)). It is estimated (Long (2004)) that college enrollment rates often increase as the unemployment rate grows. Such a trend is evident for both boys and girls (Clark (2011)). Even in the course of the recent recession (during 2010), college applications have increased significantly both in the USA and UK (Bell and Blanchflower (2011), Long (2013)). We need to posit a suitable theoretical framework, to account for these findings and the headline mentioned at the beginning. This paper builds a model that determines an optimal level of labor market friction such that the countercyclical nature of college enrollment, in turn, can create desired technological advancement. The major contribution of the paper is in its central argument: the labor market imperfection itself can be used to counter an already existent distortion to achieve optimality.

The existing discourse (started from Diamond (1965), Yaari (1965), Blanchard (1985)) shows that the short lived individuals bring an inefficiency in the system by not including 'the future' of the economy in their optimization exercise. Economists have discussed the relationship between R&D and the long run objective of the economy for quite a long time[3]. However, the discussion about the emergence of inefficiency in the allocation of resources in the R&D sector due to the finitely lived individuals is much thinner in the endogenous growth literature. In this context, this paper suggests that the optimal level of labor market friction helps the economy to achieve its long run goal. To restore efficient allocation in the model with finitely lived individuals, this paper neither depends on altruism (like Barro (1974) type growth models) nor it includes money (like, Samuelson (1958) type models).

This stylized model closely builds on a Pissarides type (Pissarides (2000)) search-matching framework of labor market imperfection. However, here, the implications of the labor market friction are somewhat different. The unmatched part of the workforce is considered as a *reserved*

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[2] Commentaries and accounts, both in the popular press and in serious academic discussions, have come up with the opinion that steadily USA is losing its comparative advantage in advanced research (Business Roundtable (2005), Galama and Hosek (2008) etc).

[3] See Romer (1986, 1994), Grossman and Helpman (1991), Howitt and Aghion (1998), etc.

labor endowment. Labor market frictions induce some of the workers to work in the R&D sector at a lower wage (Lassibille (2001), Marey (2002), Dupuy and Smits (2010)) and this, in turn, improves the future technological progress of the economy[4]. Hence, in this set up, labor market friction has a dual role. On the one hand, it restricts matching between workers and firms and promotes inefficiency. On the other hand, it encourages agents to join the R&D sector which in turn enhances the development of future technology. These two effects taken together give rise to the possibility of interior optimality. The role of the government is crucial in this model. Factor payment in R&D, i.e. R&D wage rate, is dependent on both the production sector wage and government's policy. The key idea of this set up is, once government alters the policy for R&D sector wage payment, effectively it manipulates the wage in the production sector because the labor market in production sector is constituted of search and matching friction where wage is determined through Nash Bargaining. Again, the R&D sector wage is also affected by private sector wage rate. This loop can bring efficiency in the model through counter distortion if government judiciously sets its policy towards R&D expenditure. The paper derives the 'optimum', find the suitable policy which leads the economy to that optimum and finally, discusses the robustness of the analysis under balance budget of the government and stochastic fluctuations in production.

In the long run the hypothetical economy achieves a constant growth rate. Moreover, growth rate of this hypothetical economy remains insulated from the labor market outcomes and depends on the marginal rate of technology upgrading due to investment in research activities. Hence, the optimal labor market outcome does not perturb the growth rate of the economy. Interestingly, optimal policy results show that the economies with higher growth rate need to pay higher wages to the R&D sector workers. Same is the case for the countries where the labor market is more efficient. These findings hint towards the possibilities of over investment in the wage payment of R&D sectors in the less growing or less developed (defined as higher level of labor market friction) countries, by imitating more developed economies (see, McGuinness (2006)).

[4] At this point, the author clarifies that the model is not aiming to introduce friction in the labor market from the lack of it, but the focus is to find the optimal level of friction while knowing the fact that the labor market is frictional in most of the countries. Policy directive is to keep the friction at the optimum level, and not to introduce friction afresh.

Interestingly, all the major results remain unaltered, if the model is extended by inclusion of a binding budget constraint for the government, or by allowing random fluctuation in the production process of the firm.

Few articles in the literature of trade union show that, in the presence of unionization, unemployment can cause a positive effect to generate R&D. Hence, government may strengthen the power of the labor union in the production sector, such that R&D sector grows, which in turn boosts the long run growth rate of the economy (Palokangas (2004), Lingens (2005)). However, there is a quite a large literature about growth and labor market friction. Most prominent contributions among many are Aghion and Howitt (1994), Mortensen and Pissarides (1998), King and Welling (1995), Meckl (2004) etc. All of the articles are thematically different from this present paper, and none of them tries to capture labor market as a tool of counter distortion to achieve optimality.

The plan of the paper is as follows. The next section explains the assumptions and the modeling details of this paper. Determination of the equilibrium values of all endogenous variables is obtained in Section 3. Section 4 finds the long run optimal path for the economy and suggests appropriate policy to achieve that path. Section 5 discusses the two extensions of the basic model. The first checks the robustness of the model under the balanced budget constraint for the government. The second shows the robustness of the results under stochastic fluctuations. The last section, namely section 6, summarizes the whole model and draws some concluding remarks.

2. The Basic Model

In this section, we build our benchmark model. The following sub-sections describe the details of this basic model, which is embedded in a discrete time framework.

2.1. Basic Structure and Time Sequence

The hypothetical economy comprises of individuals who live for two periods, infinitely lived firms and a benevolent government. Individuals are endowed with two unit of labor from which each unit can be supplied in each period inelastically. They receive utility from consumption at the end of their life span. Firms employ labor, as the only factor input, to produce a single good which is consumed by the agents of the economy. Product markets are perfectly competitive. However, the labor market consists of search and matching friction (see, Pissarides, 2000). Government provides an education system and carries R&D activity by educated work-force (which is produced by the education system) to improve the available technology of production in the economy[5].

Population mass of each generation is normalized to unity. Two generations are present in the economy at any particular time period: young and old. In the beginning, young individuals have two options: either they can search for jobs or they can take education, provided by the government. Individuals who decide to search for job, create a pool of job seeking workers. At the beginning of each period, vacant firms search for productive workers to undertake production. The pool of job seeking workers and the vacant firms interact through a Pissarides type random matching function. Vacant firms who get matched with the job seekers can commence production. Other vacant firms (who have failed to find workers) remain idle until the next matching process takes place. Firms employ worker for consecutive two time periods and one worker-firm match sustains until the worker is alive. This can also be interpreted as job firing rate is equal to one after two periods and within the two periods the firing rate is zero.

Government does R&D activity only with the help of educated workers. Therefore, individuals who have taken education during the first period can join R&D activity in the second period frictionlessly[6] and receives wage for one period. Technology available for production at any particular period t is developed as an end product of $(\mathrm{t}-1)^{\mathrm{th}}$ period's R&D activity. The workers who develop that new technology joined the economy and received their education at $(\mathrm{t}-2)^{\mathrm{th}}$ period.

2.2. Technological progress

Technology grows endogenously in this model. The progress of technology depends on the labor endowment allotted to the research activity. The gain from investing in research activity increases over time at an increasing rate. Available technology at period t is denoted as $\mathrm{A_t}$. Dynamics of technological progress is as follows: exposition

$$\mathrm{A_{t+n+1}} = \mathrm{A_{t+n}} + \psi^{\mathrm{t+n}}\Omega_{\mathrm{t+n}}. \tag{1}$$

[5] Literature burdens with the conflicting views about the effectiveness of public funding versus private funding towards enhancing overall productivity of the economy (See, David, Hall and Toole (2000), Hall and Reenen (2000), Feldman and Lichtenberg (1998) etc). We are taking the side of public expenditure of R&D since here we are interested in the overall technological improvement of the economy. There are countries where still public funding is higher than private funding in research (See Albuquerque (2011)).

[6] The extension of this baseline model is elaborated in Appendix 2 where the entry in R&D is also consisted of friction and taking education is costly. The main take away of the results are imitating the baseline model's finding. For more clean exposition of the argument, we abstract those two complications in the main text.

where, n is any natural number and $\psi > 1$. Ω indicates the amount of labor invested towards R&D at any period. ψ can be interpreted as the marginal contribution of labor investment towards technological change. Here it is assumed that the technology of a particular period depends on the previous period's technology level and the marginal progress of technology is increasing with time.

In this basic structure of the model, it is assumed that mapping between improvement in the research activity and increase in the productivity of the worker are non-stochastic. That is, improvement in research is certain, and if R&D improves then the increase in productivity of the economy is guaranteed in the next period.

2.3. Firms

Firms, at the beginning of any point in time, can be of two types: filled or vacant. Filled firms pay per period wage to the already hired laborers at the beginning of each period. This is the only cost for a filled firm. Per period productivity of each worker, employed at t, is A_t which remains unchanged for the two successive time periods. Vacant firms need to incur a positive cost of posting a vacancy for hiring a worker. We assume this vacancy posting cost is proportional[7] to the productivity of a worker at that period. That is, vacancy posting cost at period t is dA_t, say. After posting the vacancy a vacant firm faces a random matching function to get a worker and undertake production. The matching function which is considered here follows the properties of Pissarides type matching function which is increasing in its each argument, concave and homogenous of degree one. The specific form is assumed as the following (Stevens (2007))[8]:

$$M_t = \frac{\gamma u_t v_t}{u_t + v_t} \tag{2}$$

Where $0 < \gamma < 1$. u_t is the number of individuals searching for the job and v_t is the number of vacancies posted at t^{th} period. γ can be interpreted as the degree of overall matching efficiency of the labor market.

Probability of a successful matching at period t for a firm is,

$$M(\theta^{-1}, 1) \equiv \frac{M_t}{v_t} = \frac{\gamma}{1+\theta_t} \tag{3}$$

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[7]This assumption is commonly used in the literature. See Pissarides (chapter 1, 2000).

[8] The specific form of matching function is purely for algebraic clarity. Exact results can be obtained even with the general form.

Where $\theta(\equiv \frac{v}{u})$ is conventionally termed as market tightness.

Let V_t be the life time expected return from a vacant post from the t^{th} *period* onwards. There are two components in V_t: cost of posting a vacancy and expected return from that vacancy. Since firms are infinitely lived V_t can be represented in recursive form.

$$V_t = -dA_t + M(\theta_t^{-1}, 1)\left((A_t - w_t) + \frac{(A_t - w_t)}{1+r} + \frac{V_{t+2}}{(1+r)^2}\right) + \left(1 - M(\theta_t^{-1}, 1)\right)\left(\frac{V_{t+1}}{1+r}\right) \quad (4)$$

$r > 0$ is the interest rate, faced by the firm. w_t is the per period wage which is determined at the beginning of period t and prevails the same contract for the next two periods. The justification behind this assumption is, the technical knowhow for each matching taking place at the beginning of period t, is A_t and as we have explained earlier, firms can adopt technology only before commencing the production activity.

Perfectly competitive goods market allows firms to entry and to exist, freely. In equilibrium this implies, V_t to be zero, for all t. Thus using equation (4), we get:

$$M(\theta_t^{-1}, 1) = \frac{dA_t}{2(A_t - w_t)} \quad (5)$$

2.4. Workers

In this model a representative individual receives utility only from the consumption of the good. Further it is assumed that she consumes at the end of her life span. She does not have any bequest motive or no credit market exists to smooth the consumption over two periods. Utility depends linearly on consumption[9]. Therefore, only the income which she earns in her whole life time does matter in utilitarian terms, and now onwards we use wage earned as an alternative for her utility.

Individuals, who are born and look for the job at period t, face the same matching function as described in equation (2). The probability of getting an employment by one job seeker is, hence, the following:

$$M(1, \theta) \equiv \frac{M_t}{u_t} = \frac{\gamma}{\theta_t^{-1} + 1}. \quad (6)$$

[9] Introduction of effort cost can also generate similar results to which this model is particularly focusing.

A successful matching offers w_t as per period wage to the 'lucky' searcher. If one fails to get employed, as described earlier, she may join the government provided schooling or may remain unemployed. In either case, she does not receive any wage at period t. However, if she decides to take education (costless, for computational ease) then she has the option to join research activity and earn a positive wage, R_{t+1} (say), at period $(t+1)$. It is a simplifying assumption that there exists no friction when she joins R&D activity. On the other hand if she remains unemployed, she gets nothing. Clearly in this model remaining unemployed is a dominated strategy for any individual.[10]

2.5. Wages

Costly friction creates the possibility of rent seeking. Firm and worker settle a wage rate through Nash bargaining. Hence,

$$w_t = \underset{w_t}{\arg\max} \left(2(A_t - w_t)\right)^{\beta} (2w_t - R_{t+1})^{1-\beta}.$$

$0 < \beta < 1$ be the bargaining power of the firm. First parenthesis contains firm's share of output. Firm's opportunity cost of not entering into a productive matching is zero (as mentioned earlier, $V_t = 0$ for all t at equilibrium). Whereas worker's opportunity cost of accepting this match for the two consecutive periods is R_{t+1}, the wage rate for R&D activity at period $(t+1)$. Thus, worker's output share net from her opportunity cost is comprised in the second parenthesis of the above equation. After simplification,

$$w_t = (1-\beta)A_t + \frac{\beta}{2}R_{t+1}. \tag{7}$$

Equation 7 shows that the worker's return from a job match ($2w_t$) is a weighted average of the total return from a productive firm-worker matching ($2A_t$) and the worker's return from working in R&D sector (R_{t+1}). That is, $2w_t$ lies in between $2A_t$ and R_{t+1}. In this paper, the determination of R_{t+1}, the Government provided R&D wage rate is not modeled. For any arbitrary R_{t+1}, it can be higher, lower or equal to $2A_t$.

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[10] The costless education and frictionless R&D sector are not very restrictive assumptions for the present purpose. A simple extension of the basic model can imitate similar outcomes.

If $R_{t+1} > 2A_t$, then $2w_t > 2A_t$ as well (see equation 7). In one hand, all the individual will find it optimal to work in R&D and no one will search for job to produce good. Similarly the number of vacancy posting will be zero (since return from posting a vacancy will be negative).

If $R_{t+1} = 2A_t$, then one can solve the equilibrium number of workers needed for R&D activity to get A_t. However, that does not stop from equilibrium vacancy posting to become zero (see equation 5). Therefore, production will not take place.

$R_{t+1} < 2A_t$ (which implies $2w_t > R_{t+1}$ and $2A_t > 2w_t$ too) is the only interesting case. In this case equilibrium number of vacancy posting is positive (see equation 5). Here onwards the paper focuses only on this case.

Without loss of generality, following particular R_{t+1} (which is less than $2A_t$) is assumed in this paper for keeping the model simple and interesting: Government set the wage of the R&D sector, R_{t+1}, as proportional to the expected wage what the agent would get from the firm at period t if she gets matched. That is,

$$R_{t+1} = s2M_t w_t \tag{8}$$

where, $0 < s \leq 1$ is the proportionality parameter. As it is discussed in the introduction, often it has been seen that the return from working in the R&D sector is lower at a comparable skill level. In our model individual who works in the R&D sector, settles late (only for a single period) and receives lesser wage too. This makes the framework of the model more close to the real world (Lassibille (2001) etc.).

3. Equilibrium

This section proceeds to solve the model for a unique equilibrium and determines the value of all the endogenous variables.

First we determine the equilibrium wage rate substituting equation (8) into equation (7),

$$w_t = \frac{1-\beta}{1-\beta s M(1,\theta_t)} A_t. \tag{9}$$

Therefore, the wage rate depends positively on the policy parameter s, the technology level of the economy and the probability of getting a job, while the bargaining power of the firm affects wage rate negatively.

As we have argued in the previous section, for each individual the return from a job in the production sector is strictly higher than the return from R&D sector job. Note that, the person who fails to match with the firm, still gets the opportunity to enter into the R&D sector. Therefore, each individual chooses to search for the job at the beginning of her life span, as an equilibrium decision. Hence, in equilibrium

$$u_t = 1. \tag{10}$$

and,

$$\Omega_t = 1 - M_{t-1}. \text{ (Since, from equation (10), } M(1, \theta_t) = M_t) \tag{11}$$

From the matching function (equation (2)), assumed in this model, it can be shown that summation of the two probability values (i.e. $M(1, \theta_t)$ and $M(\theta_t^{-1}, 1)$, probability of getting a job and probability of getting a worker) is constant and same with the level of overall labor market efficiency parameter. However, this functional form is not binding to reach our results.

$$\text{Therefore, } M(\theta_t^{-1}, 1) + M(1, \theta_t) = \gamma. \tag{12}$$

Using equations (5), (10) and (12), therefore, a relation between wage rate and per-period matching of the economy can be established, in the following equation:

$$M_t = \gamma - \frac{dA_t}{2(A_t - w_t)}. \tag{13}$$

Substituting the value of w_t from equation (9) in equation (13) $M_t$can be expressed in terms of exogenous parameters.

$$M_t + \frac{d}{2\beta}\frac{1-\beta s M_t}{1-sM_t} = \gamma. \tag{14}$$

Left hand side (LHS) of the equation (14) is a positively sloped monotonic function with respect to M_t and right hand side (RHS) is independent of M_t.Therefore in M_t plane, LHS and RHS can intersect each other for suitable parametric restriction[11]. Hence, equation (14) solves for the time independent equilibrium value of overall employment level of the economy. Henceforth this market determined equilibrium value of M_t is denoted as M^* (where, $0 < M^* < 1$).This solves all the endogenous variables of the model.

Clearly, technological progress has no impact on M^*, but government spending on research activity has a negative influence on it. The intuition is as follows. A higher return from the R&D sector increases the outside option of the individual when he bargains with the firm. If the individual gets a higher return from research activity then it increases the outside option for her, when she bargains with the firm. This in turn reduces the return of the firm from a productive matching, and that implies, V_t becomes negative (recollect equation 4). V_t reaches the break-even point only if v falls, and hence, M^* goes down. Equilibrium level of matching (M^*) also depends on d, β and γ[12], but here we are focusing on the governments' policy parameter.

For the present purpose we rewrite M^* as:

$$M^* = M^*(s;\ d, \beta, \gamma). \tag{15}$$

where, $\frac{\partial M^*}{\partial s} < 0$ and $M^*(0; d, \beta, \gamma) = \gamma - \frac{d}{2\beta}$ (note, we already assumed $\left(\gamma - \frac{d}{2\beta}\right) > 0$ for the existence of the equilibrium).

4. Long Term Gain vis-à-vis Short Term Loss

In this section we introduce a social planner (government may play this role as well) who is concerned about the long run goal. GDP at a particular time point, t, is strictly increasing with M^*. That is, if the labor market friction reaches its minimum then per-period GDP reaches its maximum. In this model GDP at period t is the following:

$$GDP_t = (A_t + A_{t-1})M^*. \tag{16}$$

Given all available information, clearly GDP_t takes the maximum value for $M^* = 1$.

[11] Necessary parametric restriction is $d < 2\gamma\beta$.

[12] Effect of other three parameters on M^* are the following: $\frac{\partial M^*}{\partial d} < 0$, $\frac{\partial M^*}{\partial \beta} > 0$ and $\frac{\partial M^*}{\partial \gamma} > 0$.

However, the long run present discounted sum of GDP values from time period t is denoted as Γ_t and is equal to:

$$\Gamma_t = \sum_{n=0}^{\infty} \delta^n M(A_{t+n-1} + A_{t+n}). \tag{17}$$

where, the discount rate is $\delta \in (0,1)$. Incorporating equation (11) we can rewrite equation (1) for the time independent M as follows.

$$A_{t+n+1} = A_{t+n} + \psi^{t+n}(1-M). \tag{18}$$

We reduce the equation (17) further by using equation (18) and assuming $\delta\psi < 1$, as:

$$\Gamma_t = \frac{M}{1-\delta}\left(2A_{t-1} + (1-M)(1+\delta)\left(\frac{\psi^{t-1}}{1-\psi\delta}\right)\right). \tag{19}$$

As the social planner is concerned about the long run benefit of the economy, the objective of her is to maximize equation (19). From t^{th} period onwards, an optimal path for the economy is to be chosen by obtaining an appropriate overall matching level of the economy, M^{**}. Therefore to maximize Γ_t, we set $\frac{\partial \Gamma_t}{\partial M} = 0$ (from equation (19)) and solve for M^{**}[13].

$$M^{**} = \frac{1}{2} + \frac{A_{t-1}}{\psi^{t-1}}\left(\frac{1-\psi\delta}{1+\delta}\right). \tag{20}$$

It can also be shown that[14] (using equation (18)),

$$\frac{A_t}{\psi^t} = \frac{1-M}{\psi-1}. \tag{21}$$

Suitable substitution of equation (21) into equation (20) solves for M^{**} which maximizes Γ_t, and represented as the following. The typical expression of this optimal matching level is,

$$M^{**} = \frac{1}{2}\left(1 + \frac{1-\psi\delta}{\psi-\delta}\right) < 1. \tag{22}$$

Therefore, $M^{**} \in (0,1)$ is the socially optimal level of matching. That is, to achieve the long run objective the benevolent social planner needs to choose an optimal policy such that $M^{**} \in (0,1)$ can be achieved. Note that, M^{**} does not depend on s.Since, it has been shown (in section 3) that M^{*} is monotonically falling with respect to s and M^{**} is independent of s, for specific parametric

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[13] At M^{**} second order condition is also satisfied

[14] Assumption: if $n \to \infty$, then $A_{t-n} \to 0$.

restrictions (i.e. if, $\gamma - \frac{d}{2\beta} > M^{**}$), M^* and M^{**} cut each other in (M, s) plane. Hence, social planner may achieve M^{**} by suitably choosing s^* such that,

$$M^{**} = M^*(s^*; d, \beta, \gamma) \tag{23}$$

Figure 1

4.1. Growth and Comparative Statics.

This hypothetical economy grows at a constant rate and equal to,

$$g \equiv \frac{\Delta GDP_{t+1}}{GDP_t} = \frac{\psi+1}{1+\frac{2 \quad A_t}{1-M\psi^t}} = \psi - 1. \text{ (Using equation (21))} \tag{24}$$

g does not depend on M either. That is, if the economy moves along the optimal path then also growth rate of the economy remains unperturbed. Hence, s^*can be set with a fixed per period growth rate. Note that, in this model, like in most of the endogenous growth literature, wage rate is growing at the same rate as the economy (see equation 9).

Turning to the comparative statics exercise, clearly as ψ increases growth rate of the economy increases. Interestingly, $\frac{\partial M^{**}}{\partial \psi} < 0$ from equation (22). That is, for high growing economy it is optimal to allocate more individuals to R&D. Thus, the model says, from figure 1, government of high growing economy in the steady state should pay high to the R&D sector workers.

Again, partial differentiation of equation (14) with respect to γ around M^*, shows that: $\frac{\partial M^*}{\partial \gamma} > 0$, for any value of s. Given all other parameters fixed, if the overall efficiency of the labor market increases then market determined equilibrium level of job match also rises. This result combined with the figure 1 gives us more insight. Since, for each s, M^* increases for higher γ, therefore in the figure 1, M^* curve shifts up. That leads to higher s^*. That is, economies with higher level of labor market efficiency should pay more to the R&D sector workers than the one with lesser labor market efficiency. On the contrary, in the less developed countries where labor market is less efficient, return from R&D needs to be lesser as compared to developed countries (consists of higher labor market efficiency).

These two observations taken tighter brings out an important discussion. By imitating high developed countries less developed or less growing economies may end up paying excess money to the R&D sector workers. Empirical literature draws attention to the prevalence of investment in over-education in several less growing or less developed countries (see, McGuinness 2006). Therefore, when the present paper focuses on the issue of the optimal level of labor market friction (which also implies the optimal level of payment to the R&D sector workers), then it indicates the possibility of over-payment in the R&D sector too.

5. Discussions

In the previous sections we develop a model with a very simple set up. This section demonstrates with two examples that, the simple set up is not very restrictive to our findings. The discussion is kept short, because it does not bring any qualitative changes in the results of the model, but, nonetheless, demonstrates robustness.

5.1. Government with Budget Constraint

This section incorporates a budget constraint (henceforth BB) for the government, which has to be balanced. Here, government has the power to freely impose discriminatory tax on the individuals. Using that authority, government taxes once on a lump-sum basis, the individuals who get the job as a worker of a firm, and thus, finances the cost incurred for R&D activity. This tax rate, τ_t, is known to all at the beginning of any period, t (say). So, workers bargain on their effective wage after getting matched with a firm. Therefore, the new wage rate, w_t^τ, is:

$$w_t^\tau = \underset{w_t^\tau}{\arg\max} \left(2(A_t - w_t^\tau)\right)^\beta (2w_t^\tau - \tau_t - R_{t+1}^\tau)^{1-\beta}.$$

Equation (8) still holds in this set-up. That is, government pays researcher as a proportion of the expected wage what they may get if they would work in a firm. Using that we solve the new,

$$w_t^\tau = \frac{(1-\beta)A_t + \frac{\beta}{2}\tau_t}{1-\beta s M_t^\tau}. \tag{25}$$

Government in each period spends the entire revenue, earned from taxation, in research activity. Hence, the budget balance constraint is the following:

$$\tau M_t^\tau = (1 - M_t^\tau)R_{t+1}^\tau \tag{26}$$

Substituting equation (8) in equation (26) and then replacing w_t^τ from equation (25), equilibrium tax rate can be solved:

$$\tau_t^* = \frac{2s(1-\beta)}{1-s\beta} A_t(1 - M_t^\tau). \tag{27}$$

For solving the equilibrium value of market determined overall successful matching we would recall equation (13). For the present purpose the equation is rewritten with new notation:

$$M_t^\tau = \gamma - \frac{dA_t}{2(A_t - w_t^\tau)}. \tag{28}$$

It is straight forward to understand, in this version also all individuals first search for a job in a firm, and then who ever fails to get employment receives education and joins R&D activity at the second period. Therefore, equations (10) and (11) remain unchanged. Now, first substituting equation (27) into equation (25) and then, replacing the value of w_t^τ back into equation (28) we get the following equation which can solve for the equilibrium $0 < M^{\tau*} < 1$:

$$M_t^\tau + \frac{d}{2\beta}\left[\frac{1-sM_t^\tau}{1-\beta s M_t^\tau} + \frac{(1-M_t^\tau)s(1-\beta)}{(1-s\beta)(1-\beta s M)}\right]^{-1} = \gamma. \tag{29}$$

LHS of the above equation has a positive relation with M_t^τ. Hence, for $\frac{d}{2\beta}\left(\frac{1-s\beta}{1-s}\right) < \gamma$, there exists a unique solution for M^{τ^*}. Note that, $M^* < M^{\tau*}$. Moreover, $\frac{\partial M^{\tau^*}}{\partial s} < 0$ and $M^{\tau*}(0; d, \beta, \gamma) > 0$ still holds.

Since, BB policy is a redistributive instrument, GDP value of any period does not change compared to the benchmark case (without BB constraint). Hence the optimal $\Gamma_t^*(M^{**})$ and growth rate of the economy remain unchanged. However, to reach that $s^{\tau*}$ is to be set such that $M^{\tau^*} = M^{**}$, and it is not difficult to show that $s^* > s^{\tau*}$.

Figure 2

5.2. Stochastic Fluctuation in Output

This section discusses the model further by allowing stochastic fluctuation. This is to check how the results vary from the basic model when economy varies from one state to another. Intuitive interpretation of this extension says, the effect of research output on the productivity of the firm is uncertain. Say, the productivity depends on a random parameter, ϕ. Whatever be the technological outcome, the usefulness on the productivity also depends on ϕ too. To make the analysis simple it is assumed that ϕ can take only two positive values: $\phi_t = \{\phi_1, \ \phi_2\}$, and follows a stationary Markov process, with the transition matrix $\Pi_{2\times 2}$[15]. To avoid the cumbersome analysis, we once again ignore the budget constraint of the government.

Instead of A_t, let the productivity from a successful matching at period t is now:

$$\Psi_t = A_t \phi_t. \tag{30}$$

Value of ϕ_t is known to all only at the beginning of the period t. That is, at the beginning of a time period firms and workers both know what is going to be the productivity level of a new matching at that period.

It is straight forward to prove that although wage fluctuates with stochastic shocks equation (14) still holds. Thus one can solve for M^*. Therefore, overall per period matching level of the economy remains the same and time invariant. This is because, in this hypothetical economy, matched firms and workers are absorbing the entire effect of the productivity shock.

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[15] $\Pi = \begin{bmatrix} \bar{\pi} & 1-\bar{\pi} \\ \underline{\pi} & 1-\underline{\pi} \end{bmatrix}$. Where, $0 < \Pi_{ij} < 1$. Therefore, if at time period t, ϕ takes the value ϕ_1 then the probability of repeating ϕ_1 at period (t+1) is $\bar{\pi}$ and the probability of a change to ϕ_2 is $1 - \bar{\pi}$. On the other hand, if ϕ is ϕ_2 at period t, then at period (t+1), ϕ takes the value ϕ_1 with the probability $\underline{\pi}$ and it remains ϕ_2 with the probability $1 - \underline{\pi}$.

Determination of long run optimal is now based on the maximization of the expected time discounted long run sum of GDP values. Therefore the objective function for this analysis is,

$$E\Gamma_t = \sum_l p_l^* \left(E_{t-1}^l \Gamma_t\right). \tag{32}$$

Where, $l = \{1, 2\}$ and $E_{t-1}^l \Gamma_t$ is the long run time discounted sum of expected GDP values given the value of ϕ at the period (t-1). Stationary state value of $P(\phi = \phi_l)$ at any period is denoted as p_l^*. A typical solution of $p_1^* = \frac{\bar{\pi}}{\underline{\pi} + (1-\bar{\pi})}$ and $p_2 = \frac{1-\bar{\pi}}{\underline{\pi} + (1-\bar{\pi})}$.

Given $(\phi = \phi_l)$, following equation specifies $E_{t-1}^l \Gamma_t$ as, ,

$$E_{t-1}^l \Gamma_t = E_{t-1}^l \left[\sum_{n=0}^{\infty} \delta^n M(\Psi_{t+n-1} + \Psi_{t+n})\right].$$

Kolmogorov-Chapman[16] equation helps to reduce the above mentioned infinite sum[17] and then by maximizing (32) with respect to M, we obtain:

$$M_\phi^{**} = 1 - \frac{1}{2 + \frac{1}{(1+\delta)(\psi-1)}\left(\frac{pA_1 + qA_2}{pB_1 + qB_2}\right)}. \tag{33}$$

Where, $A_l = \phi_l + (1+\delta)\Pi(l;:)(\mathbf{I} - \delta\Pi)^{-1}[\phi]$ and $B_l = \Pi(l;:)(\mathbf{I} - \delta\psi\Pi)^{-1}(\mathbf{I} - \delta\Pi)^{-1}[\phi]$. Correspondingly, government can solve s_ϕ^{**} such that $M^* = M_\phi^{**}$, (recall, market determined equilibrium matching, M^*, is negatively related with s).

In this version also, expected growth rate does not depend on the overall matching level of the economy and ψ has an unambiguously positive impact on growth rate. Hence government can choose the optimal path without hampering the growth rate. Expected growth rate is determined as,

$$Eg_t = \sum_l p_l^* E_{t-1}^l g_t \tag{34}$$

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[16]Kolmogorov-Chapman equation: $\Pi^{(n+m)} = \Pi^{(n)}\Pi^{(m)}$. $(n+m)$, (n) and (m) are denoted as steps of transition.

[17]$E_{t-1}^l \Gamma_t = M\left[A_{t-1}\left[\phi_l + (1+\delta)\Pi(l;:)_{1\times 2}[\mathbf{I} - \delta\Pi]^{-1}{}_{2\times 2}[\phi]_{2\times 1}\right]\right] + M(1-M)\psi^{t-1}(1+\delta)\left[\Pi(l;:)_{1\times 2}[\mathbf{I} - \delta\psi\Pi]^{-1}{}_{2\times 2}[\mathbf{I} - \delta\Pi]^{-1}{}_{2\times 2}[\phi]_{2\times 1}\right]$.

where, $E^{l}_{t-1}g_t = \frac{\Pi(l;:)\Pi\phi\psi^2 - \phi_l}{\Pi(l;:)\phi\psi + \phi_l}$.

Note that, expected growth rate increases if ψ increases. That leads to fall in M^{**}_{ϕ} also. Hence, s^{*}_{ϕ} increases.

6. Conclusion

This model views labor market friction from a very different perspective compared to existing literature. After the introduction of search-matching framework, neo-classical general equilibrium models got a rigorous micro foundation for an imperfect labor market which enabled them to comment on different macroeconomic problems. Most of the contributions in this literature showed how positive level of friction in the labor market can drag the economy away from perfect competitive solutions. Our model, in contrast, looks for a positive role of labor market friction as a counter distortion in a model with endogenous technological progress and two period lived individuals. It suggests, instead of reducing friction arbitrarily, economy can gain in long run if there exits an appropriate level of positive labor market friction. Economy can achieve that even without changing its constant steady state growth rate. The paper proposes, higher payment in R&D sector is optimal for high growing countries and/or countries with high labor market efficiency.

It has been shown that the results are consistent for self-financing government. That is, the main results remain unchanged if the government has to maintain a balanced budget. In one other extension, we allow random fluctuations in the research output and productivity mapping which shifts the productivity of a successful matching from one state to another. It is shown that the model is robust to introduction of stochastic fluctuations also.

Appendix 1

In the sub-section 2.5, we have claimed that the assumption made in equation 8 does not reduce the generality of the model. Following argument is made in support of that claim.

Using equation 7, for any R_{t+1}, equation 5 can be written as following

$$M(\theta_t^{-1}, 1) = \frac{dA_t}{2\left(\beta A_t - \frac{\beta}{2} R_{t+1}\right)}$$

or, $M(\theta_t^{-1}, 1) = \frac{d}{2\beta\left(1-\frac{R_{t+1}}{2A_t}\right)}$

Using equation (12) and the above equation,

$$M_t = \gamma - \frac{d}{2\beta\left(1-\frac{R_{t+1}}{2A_t}\right)}$$

As it is argued in sub-section 2.5, $0 < \frac{R_{t+1}}{2A_t} < 1$ for all t. Note that, $\frac{R_{t+1}}{2A_t}$ is negatively related to M_t and for certain restriction on γ, β and d, $0 < M_t < 1$ for all $0 < \frac{R_{t+1}}{2A_t} < 1$. These two properties of the above equation generates most of the important results of the paper. In this paper, $\frac{R_{t+1}}{2A_t}$ has been replaced by a specific $0 < s_t < 1$. To make the model more interesting and to ensure the convergence, the assumption in equation 8 is made.

Appendix 2

Consider the baseline model, but relaxing the assumptions of costless education and frictionless R&D sector. To make the model more comprehensive let us introduce a labor-input for providing the education. That is, not only to improve the technology level but also to develop educated labor force economy needs educated laborers (namely, 'teachers'). Therefore, the changed set up is the following. Agents who take education at period t, have to pay η_t. R&D sector hires λ proportion of the educated labor force. Rests are hired to provide education to the next generation (here, one can introduce some friction and generate an educated unemployed labor pull. That will introduce only another parameter but otherwise analysis will be the same). Both types of educated workers are receiving wage for only one period (i.e., for t+1th period) and the wages are denoted as R_{t+1} and Z_{t+1}, respectively. Budget balancedness for R&D sector has already been discussed above. Here, the discussion focuses on the additional cost that has to be incurred by the Government by paying the wage to the 'teachers'. To finance that budget, it is assumed that the revenue generated from the agents who pay the cost for the education, is distributed to the "teachers". Either the cost (η_t) or the wage payment of the 'teachers' (Z_t) can be considered as given. Here, the Z_t is considered as exogenously given, and is set similarly as R_{t+1}. Since all the other mechanisms of the model are exactly the same, here we avoid the repetition of argument that has been made about the assumption of R_{t+1}, and hence of Z_{t+1}.

In this set up the key equations which come up along with all major equations of Section 2, are the following.

Bargaining wage equation: $w_t = (1-\beta)A_t + \frac{\beta}{2}(\lambda R_{t+1} + (1-\lambda)Z_{t+1} - \eta_t)$. (43)

Second parenthesis involves the net expected opportunity cost for the laborer to join the production sector.

Budget equation for ‘teachers’: $N_t\eta_t = (1-\lambda)N_{t-1}Z_t$. (44)

Where, N_t denotes the number of agents willing to take education at period t. Note that ‘teachers’ who receive wage at period t have taken education at period $(t-1)$ and become ‘teacher’ with probability $(1-\lambda)$ at period t. Therefore, the right hand side of equation (44) represents the cost for paying teachers and the left hand side is the revenue from the agents who are willing to take education at period t.

Using similar argument as stated in baseline model, in equilibrium,[18]

$N_t = 1 - M_t$. (45)

Let us assume, similar to R_{t+1} in equation (8), $Z_{t+1} = s'M_t w_t$. (46)

Where, $0 < s' < 1$.

Rearranging equation (5) and using equation (10) and (12): $M_t = \gamma - \frac{d}{2(1-w_t/A_t)}$. (47)

From equation (43), $\frac{w_t}{A_t}$ can be expressed in terms of $\left\{\frac{w_{t-1}}{A_{t-1}}, w_{t-1}, M_t, M_{t-1}, t\right\}$ using equations (1), (8), (10), (45) and (46). Specifically the equation is the following,

$$\frac{w_t}{A_t}\left(1 - \frac{\beta}{2}(\lambda s + (1-\lambda)s')M_t\right) = (1-\beta) - \frac{\beta}{2}(1-\lambda)\left(\frac{1-M_{t-1}}{1-M_t}\right)s'M_{t-1}\left(\frac{1}{\frac{A_{t-1}}{w_{t-1}} + \psi^{t-1} * \frac{1-M_{t-1}}{w_{t-1}}}\right). \quad (48)$$

[18] Note that, since the cost associated with getting education makes R&D ever less lucrative.

Note that, equation (47) says, M_t is a function of $\frac{w_t}{A_t}$ only. Therefore, equation (48) returns $\frac{w_t}{A_t}$ as a function of $\frac{w_{t-1}}{A_{t-1}}$, w_{t-1} and t.

To solve the steady state, first we define, there exists a steady state and then we check the consistency.

Say, at the steady state, $\frac{w_t}{A_t} = \omega$, (49)

and hence, $M_t = M(\omega) = \gamma - \frac{d}{2(1-\omega)}$. (50)

Therefore, equation (48) becomes,

$$\omega\left(1-\frac{\beta}{2}(\lambda s+(1-\lambda)s')M\right) = (1-\beta)-\frac{\beta}{2}(1-\lambda)s'M\left(\frac{1}{\frac{1}{\omega}+\psi^{t-1}\frac{1-M}{w_{t-1}}}\right). \quad (51)$$

Now, one can solve for w_{t-1} and divide that by A_{t-1}. Using similar condition as of equation (21)[19] following can be obtained,

$$\omega^2\left(1-\frac{\beta}{2}\lambda sM\right)-\omega\left(1-\beta-\frac{(\psi-1)}{\lambda}\left(1-\frac{\beta}{2}(\lambda s+(1-\lambda)s')M\right)\right)-\frac{(\psi-1)(1-\beta)}{\lambda}=0. \quad (52)$$

It is clear that RHS of equation (52) is a third order polynomial (using the value of $M(\omega)$ from equation (50)) of ω. Therefore, there exists at least one real root of ω and parametric specification guarantees its value to be positive. Using the value of ω, steady state M can be solved too.

Thus, $M^* = M(s, s')$. (53)

Rest of the analysis is similar to the basic model described in section 4 and the main results hold.

[19] Now $\frac{\psi^t}{A_t} = \frac{\psi-1}{\lambda(1-M)}$.

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Figures

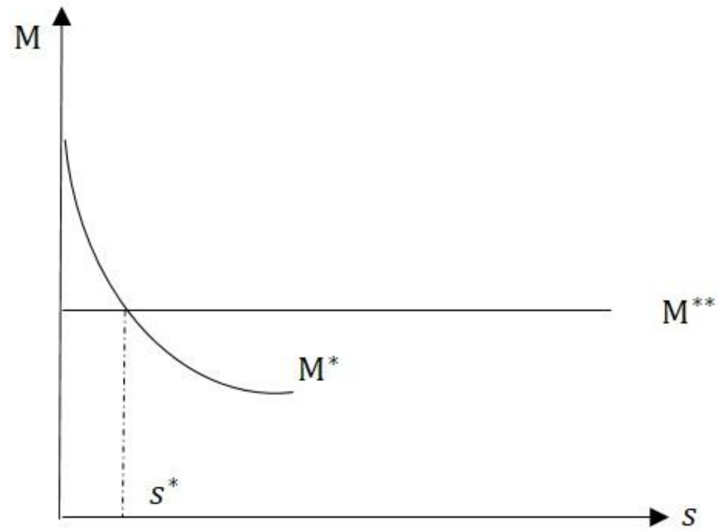

Figure 1

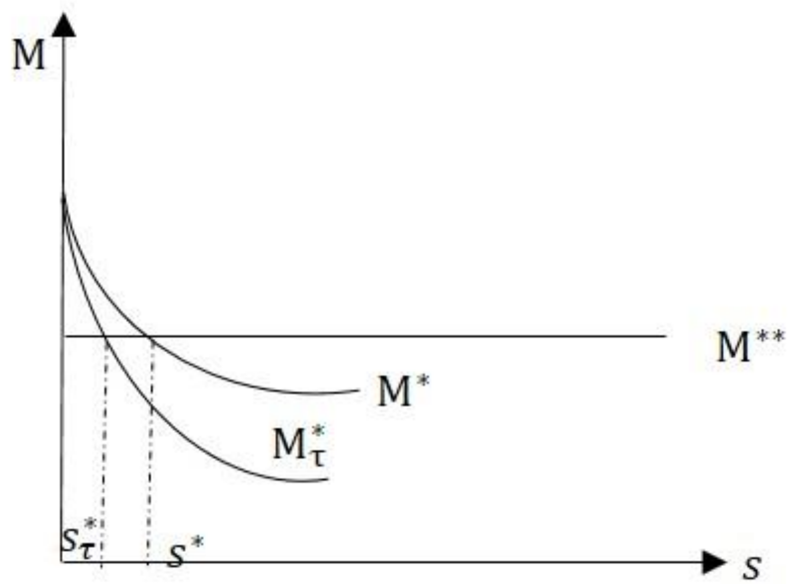

Figure 2