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Nakagawa, Shintaro

Konan University

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On the Maximum Number of Players Voluntarily Contributing to Two or More Public Goods

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Shintaro Nakagawa*

Abstract

Cornes and Itaya (2010) showed that in a two-player game of voluntary provision of two public goods if the players have different preferences, and if both players simultaneously make positive contributions to both public goods, the system of equations representing the Nash equilibrium is overdetermined. We extend this proposition to a model of voluntary provision of two or more public goods and show that if the players have different preferences, and if the number of players who contribute simultaneously to two or more public goods is more than the number of public goods, the system representing the Nash equilibrium is overdetermined. This result implies that in a large group, the share of players contributing to multiple public goods may well be quite small and the majority of the players may contribute to at the most one public good.

Keywords: Voluntary provision, multiple public goods

JEL Classification: H41

* Corresponding author. Faculty of Economics, Konan University, 8-9-1 Okamoto, Higashinada-ku, Kobe 658-8501, Japan. Email address: shintaro@konan-u.ac.jp

1. Introduction

Voluntary provision of public goods has been an important issue in the literature of public economics. Compared to voluntary provision of one public good, voluntary provision of two or more public goods has been paid limited attention, except for several papers such as Kemp (1984); Bergstrom et al. (1986); Cornes and Schweinberger (1996); Dasgupta and Kanbur (2005); Cornes and Itaya (2010); and Ihori et al. (2014). However, it has not been sufficiently investigated how many players contribute to two or more public goods in a Nash equilibrium.[1]

One exception is Cornes and Itaya (2010). Cornes and Itaya considered an economy consisting of two players, one private good, and two voluntarily provided public goods. They then showed that if the players have different preferences, and if both players simultaneously make positive contributions to both public goods, the system of equations representing the Nash equilibrium consists of equations strictly more than unknown variables, i.e., the system of equations is overdetermined. Thus, they claimed "there "almost surely" does not exist a Nash equilibrium in which both players simultaneously make positive contributions to both public goods" (Cornes and Itaya (2010), Proposition 2(i), p. 369).

However, they did not extend their proposition to more general settings. There remain unsolved questions such as "How is their proposition modified if we introduce a new player to the model?" or "How about if we introduce another public good?"

In this paper, we extend their model to a model in which many players voluntarily contribute to many public goods. We assume that H players voluntarily contribute to J public goods and that every player has different preferences. Then, we show that the system of equations representing a Nash equilibrium wherein J players contribute to two or more public goods is overdetermined.

For example, consider an economy wherein players voluntarily provide two public goods. In this economy, a system of equations representing a Nash equilibrium wherein two players voluntarily contribute to both public goods is overdetermined irrespective of the number of players. Even if there exist one hundred players, a Nash

[1] The identification of contributors in one public good model has been investigated in the literature (McGuire and Groth 1985; McGuire 1991; Andreoni and McGuire 1993).

equilibrium wherein two players contribute to both public goods is unlikely to exist.

Our result implies that if many players voluntarily contribute to a limited number of public goods, a limited number of players contribute to two or more public goods and the majority of players contribute to at the most only one public good.

2. Model

Let us consider an economy consists of *H* players. They consume *I* private goods and voluntarily provide *J* public goods. Let c_i^h be the amount of private good *i* consumed by player *h*, G_j the amount of public good *j*, g_j^h the contribution to public good *j* by player *h*, Y^h the income of player *h*, q_i the price of private good *i*, and p_j the unit cost of contribution to public good *j*. We assume that Y^h, q_i, and p_j are positive constants, and that each player considers the contributions made by other players as given. Following Bergstrom et al. (1986), a Nash equilibrium of this model is defined as follows.

Definition: A *Nash equilibrium* of this model is a vector of private goods consumption and contributions to public goods, (c_i^{h*}, g_j^{h*}), for $h = 1,\ldots,H$, $i = 1,\ldots,I$, $j = 1,\ldots,J$, such that for each player $h \in \{1,\ldots,H\}$, it solves the following problem:

$$\max_{c_1^h,\ldots,c_I^h,g_1^h,\ldots,g_J^h} U^h\left(c_1^h,\ldots,c_I^h,G_1,\ldots,G_J\right)$$

subject to

$$G_j = \sum_{h=1}^{H} g_j^h, \qquad \text{for } j = 1,\ldots,J, \tag{1}$$

$$Y^h = \sum_{i=1}^{I} q_i c_i^h + \sum_{j=1}^{J} p_j g_j^h, \tag{2}$$

where $U^h(.)$ is strictly increasing, strictly quasi-concave, and twice continuously differentiable in all arguments.

Following Cornes and Itaya (2010), we assume that all goods are essential, or $\lim_{c_i^h \to 0} \partial U^h / \partial c_i^h = \infty$ for $i = 1,\ldots,I$ and $\lim_{G_j \to 0} \partial U^h / \partial G_j = \infty$ for $j = 1,\ldots,J$.

We now focus on how many players simultaneously contribute to two or more public goods. Let us consider a list of public goods to which player h contributes positive amounts. Let S^{h*} be a set of indexes of public goods that player h contributes. Index j is in S^{h*} if and only if player h contributes a positive amount to public good j:

$$S^{h*} = \{ j \mid g_j^{h*} > 0 \}. \tag{3}$$

For example, if player 1 contributes positive amounts to public goods 1 and 2 and does not contribute to the other public goods, we obtain $S^{1*} = \{1, 2\}$. Then, we classify the players based on the size of S^{h*} as follows:

Definition: Player h is a *noncontributor* if S^{h*} is empty. He/she is a *unilateral contributor* if the size of S^{h*} is one. He/she is a *multilateral contributor* if the size of S^{h*} is more than one.

Next, we consider how many players are multilateral contributors. Let us form groups of players according to how many public goods they contribute. We define the group of noncontributors as N_0^* and that of unilateral contributors as N_1^*. In general, we define the group of players contributing to k types of public goods as N_k^*. Note that the members of group N_k^* may differ in the combination of public goods they contribute: one member might contribute to public goods $1, ..., k$, while another member might contribute to public goods $2, ..., k+1$. We denote the size of group N_k^* by n_k^*. By definition, we obtain the following:

$$H = \sum_{k=0}^{J} n_k^*. \tag{4}$$

The left-hand side of (4) is the number of players in this model, while the right-hand side is the sum of the number of players contributing to k public goods from $k = 0$ to $k = J$.

Then, we have our main result as the following proposition:

Proposition 1: Let us suppose that there is a Nash equilibrium of this model, (c_i^{h*}, g_j^{h*}), for $h = 1, \ldots, H$, $i = 1, \ldots, I, j = 1, \ldots, J$. We assume that the number of players contributing to k types of public goods in the equilibrium, n_k^*, satisfies the following inequality:

$$\sum_{k=2}^{J} (k-1) n_k^* \geq J. \tag{5}$$

Then, the system of equations representing the Nash equilibrium is overdetermined.

Proof: The Lagrangian function of player h's utility maximization problem is defined as follows:

$$L^h = U^h\left(c_1^h, \ldots, c_I^h, G_1, \ldots, G_J\right) + \lambda^h \left\{ Y^h - \sum_{i=1}^{I} q_i c_i^h - \sum_{j=1}^{J} p_j g_j^h \right\} + \sum_{j=1}^{J} \mu_j^h g_j^h. \tag{6}$$

We denote the Lagrange multipliers in the Nash equilibrium by λ^{h*} and μ_j^{h*} ($j = 1, \ldots, J$). Then, $c_i^*, g_j^*, \lambda^{h*}$, and μ_j^{h*} satisfy the first order conditions for $h's$ utility maximization, which are given as follows:

$$\frac{\partial U^h}{\partial c_i^h} - \lambda^{h*} q_i = 0, \quad \text{for } i = 1, \ldots, I. \tag{7}$$

$$\frac{\partial U^h}{\partial G_j} - \lambda^{h*} p_j + \mu_j^{h*} = 0, \quad \text{for } j = 1, \ldots, J. \tag{8}$$

$$\mu_j^{h*} g_j^{h*} = 0, \quad \text{for } j = 1, \ldots, J. \tag{9}$$

Solving (7) for private good 1, we obtain the following:

$$\lambda^{h*} = \frac{1}{q_1} \frac{\partial U^h}{\partial c_1^h}. \tag{10}$$

Substituting (10) in (7), we obtain the following:

$$\frac{\partial U^h}{\partial c_i^h} = \frac{q_i}{q_1} \frac{\partial U^h}{\partial c_1^h}, \text{ for } i = 2, \ldots, I. \tag{11}$$

If public good j is in set S^{h*}, which is the set of indexes of public goods to which player h contributes, this follows:

$$\frac{\partial U^h}{\partial G_j} = \frac{p_j}{q_1}\frac{\partial U^h}{\partial c_1^h}, \text{ for } j \in S^{h*}. \tag{12}$$

If player h is a noncontributor, h's budget constraint becomes the following:

$$Y^h = \sum_{i=1}^{I} q_i c_i^{h*}, \text{ for } h \in N_0^*. \tag{13}$$

By aggregating the budget constraints of unilateral and multilateral contributors, we obtain the following:

$$\sum_{h \notin N_0^*} Y^h = \sum_{h \notin N_0^*} \sum_{i=1}^{I} q_i c_i^{h*} + \sum_{j=1}^{J} p_j G_j^*, \tag{14}$$

where G_j^* is the amount of public good j provided in the Nash equilibrium.

The Nash equilibrium levels of private goods consumption and public goods provision, (c_i^{h*}, G_j^*), must solve the system of (11), (12), (13), and (14).

Table 1. The number of equations to be solved in the Nash equilibrium

Expression	**Total number of equations**
(11)	$(I-1)H$
(12)	$\sum_{k=1}^{J} k n_k^*$
(13)	n_0^*
(14)	1
Total	$HI + 1 + \sum_{k=2}^{J} (k-1) n_k^*$

In Table 1, we count the numbers of equations in this system. Expression (11) indicates the first-order conditions on private goods consumption. There are $I-1$ equations for each player, and there are H players in this economy. Thus, we have a total of $(I-1)H$ equations. Expression (12) represents the first-order condition for player h to make a positive contribution to public good j. If player h is a unilateral contributor, he/she has one equation on the efficiency of his/her contribution. Counting these equations for every unilateral contributor, we have n_1^* equations. If player h contributes to k types of public goods, there exist k equations corresponding to the public goods the player contributes. Counting the equations for every player contributing to k types of public goods, we obtain kn_k^* equations. Thus, we obtain a total of $\sum_{k=1}^{J} kn_k^*$ equations. Expression (13) signifies the budget constraints for noncontributors. Because there are n_0^* noncontributors, we obtain n_0^* equations. Expression (14) describes the budget constraint for unilateral and multilateral contributors. Summing up the budget constraints for all players contributing to at least one public good, we obtain only one equation. Computing the total number of equations, we obtain the following:

$$\begin{aligned}(I-1)H+\sum_{k=1}^{J} kn_k^* + n_0^* + 1 &= HI-(H-n_0^*)+\sum_{k=1}^{J} kn_k^* + 1\\ &= HI-\sum_{k=1}^{J} n_k^* + \sum_{k=1}^{J} kn_k^* + 1\\ &= HI+\sum_{k=1}^{J}(k-1)n_k^* + 1\\ &= HI+1+\sum_{k=2}^{J}(k-1)n_k^*. \qquad (15)\end{aligned}$$

The unknown variables of the system of equations are c_i^{h*} and G_j^*. We have HI variables for c_i^{h*} and J variables for G_j^*. As a total, we have $HI+J$ unknown variables.

If expression (5) holds, we obtain the following:

$$HI+1+\sum_{k=2}^{J}(k-1)n_k^* \geq HI+1+J > HI+J, \qquad (16)$$

which means that the number of equations is more than the number of unknown variables. Thus, the system of equations representing the Nash equilibrium is overdetermined.

Proposition 1 is a generalized version of Cornes and Itaya's (2010) proposition

2(i). Suppose that there are two public goods, $J = 2$. Then, expression (5) becomes

$$n_2^* \geq 2. \tag{17}$$

The left-hand side of (17) is the number of players contributing to both public goods. Then, Proposition 1 means that if two or more players contributing to both public goods, the system of equations representing the Nash equilibrium is overdetermined.

Proposition 1 also shows how many multilateral contributors can exist in a model with more than two public goods. Interestingly, the number of multilateral contributors is bounded not by the number of players but by the number of public goods. If players consume J public goods in an economy, any Nash equilibrium wherein J players contribute to two or more public goods is unlikely to exist because the system of equations representing the equilibrium is overdetermined.

Unlike the number of multilateral contributors, the number of unilateral contributors and that of noncontributors are not bounded. What is the difference between the multilateral and unilateral contributors? Let us suppose that player h contributes to public goods 1 and 2. In this case, the first-order conditions for the player include one more equation than that for a unilateral contributor because player h allocates his/her contributions to public goods 1 and 2. Thus, if there are more multilateral contributors, there are more equations to be solved. However, the number of unknown variables is fixed. Thus, if the system of equations is not overdetermined, the number of multilateral contributors is bounded.

From Proposition 1, we immediately obtain the following corollary:

Corollary 1: In the following Nash equilibria, the system of equations representing the equilibrium is overdetermined:

(i) Equilibrium with J or more multilateral contributors.

(ii) Equilibrium wherein two or more players contribute to all the public goods.

(iii) Equilibrium wherein one player contributes to all the public goods, and at least one player contributes to two or more public goods.

3. Conclusion

In this paper, we have constructed a model in which H players voluntarily contribute to J public goods. Our proposition implies that unless the system is overdetermined, the number of multilateral contributors, who contribute to two or more public goods, is at the most $J-1$. Thus, in a large group, the share of players contributing to multiple public goods may be quite small, and the majority of the group members may contribute to at the most one public good.

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