Student-Project-Resource Matching-Allocation Problems: Game Theoretic Analysis

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Student-Project-Resource Matching-Allocation Problems: Game Theoretic Analysis

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In this work, we consider a three sided student-project-resource matching-allocation problem, in which students have preferences on projects, and projects on students. While students are many-to-one matched to projects, indivisible resources are many-to-one allocated to projects whose capacities are thus endogenously determined by the sum of resources allocated to them. Traditionally, this problem is divided into two separate problems: (1) resources are allocated to projects based on some expectations (resource allocation problem), and (2) students are matched to projects based on the capacities determined in the previous problem (matching problem). Although both problems are well-understood, unless the expectations used in the first problem are correct, we obtain a suboptimal outcome. Thus, it is desirable to solve this problem as a whole without dividing it in two.

In this paper, we first show that a stable (i.e., fair and nonwasteful) matching does not exist in general (nonwastefulness is a criterion related to efficiency). Then, we show that no strategyproof mechanism satisfies fairness and very weak efficiency requirements. Given this impossibility result, we develop a new strategyproof mechanism that strikes a good balance between fairness and efficiency, which is assessed by experiments.

1 INTRODUCTION

In this paper, we introduce a simple, but fundamental problem, which we call a Student-Project-Resource matching-allocation problem (SPR).[1] On one hand, this problem can be considered a two-sided, many-to-one matching problem [30], since students are matched to projects based on their preferences. On the other hand, it is also a discrete resource allocation problem [24], since resources are allocated to each project. However, unlike the standard setting of two-sided many-to-one matching, where the capacity of each project is exogenously determined, we assume capacities are endogenously determined by the resource allocation.

If the mechanism designer knows the preferences of students, she can allocate resources to projects using combinatorial optimization techniques. If the capacity of each project is determined, even if the mechanism designer does not know the preferences of students beforehand, she can find a matching that satisfies desirable properties (e.g., stability) with a strategyproof mechanism [9], i.e., students voluntarily disclose their true preferences. However, the mechanism designer usually does not know their preferences. Thus, a common practice is to determine the resource allocation part, based on some expectations or past data and fix the capacities of projects. Then, the actual matching of students to projects is determined by a matching mechanism. In this approach, if the expectations used in the first problem are incorrect, the outcome is suboptimal: excess demand of seats and excess supply may coexist in the same matching-allocation.

One real-life instance where this practice is applied is the nursery-school waiting list problem [28]. In Japanese municipalities, the procedure for matching children (i.e., students) and nurses (resources) to publicly certified nursery schools is as follows. First, the matching authority announces the quotas of all publicly certified nursery schools for each age group (projects). Allocation of the resources (nurses) within a school to each age group[2] is determined based on estimates. Next, based

[1]The student-project-room allocation problem is an instance that belongs to a previously proposed problem class [12]. Our model is a strict generalization; multiple resources (instead of one room) can be allocated to each project. It does not belong to the well-behaved class called $M^{\natural}$-convex set.

[2]Each age group in a nursery school corresponds to a project in our model.

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on the quotas for each age group, the actual assignment is determined by a matching mechanism. The major shortcoming of this procedure is that in the resulting matching, excess demand and excess supply may coexist in one school. This is because the local authorities must determine the quotas for each age group of all the schools before they know the actual demand for them. To avoid such inefficiency, this problem should be solved as a whole without dividing it in two.

Related work. This paper follows a stream of works dealing with constrained matching. Two-sided matching has been attracting considerable attention from AI and TCS researchers [3, 16, 17, 21]. A standard market deals with maximum quotas, i.e., capacity limits that cannot be exceeded. However, many real-world matching markets are subject to a variety of distributional constraints [23], including regional maximum quotas, which restrict the total number of students assigned to a set of schools [20], minimum quotas, which guarantee that a certain number of students are assigned to each school [8, 11, 15, 31, 32], and diversity constraints [7, 14, 22, 25]. A similar model was recently considered [19], but with a compact representation scheme which handles exponentially many students and induces intrinsically different computational problems.

There exist several works on three-sided matching problems [2, 18, 27] where three types of players/agents, e.g., males, females, and pets, are matched. Although their model might look superficially similar to our model, they are fundamentally different. In the student-project allocation problem [1], students are matched to projects, while each project is offered by a lecturer. A student has a preference over projects, and a lecturer has a preference over students. Each lecturer has her capacity limit. This problem can be considered as a standard two-sided matching problem with distributional constraints. More specifically, this problem is equivalent to a two-sided matching problem with regional maximum quotas [23]. A 3/2-approximation algorithm exists for the student-project allocation problem [5], and one can also obtain super-stability, despite ties [29]. In our model, a resource is not an agent/player; it has no preference over projects/students. Also, a project/student has no preference over resources; a project just needs to be allocated enough resources to accommodate applying students.

Our contribution. We introduce a student-to-project matching problem that endogenously handles the resource allocation problem defining the capacity of projects. We first show that a stable (i.e., fair and nonwasteful) matching does not exist in general (nonwastefulness is a criterion related to efficiency). Then, we show that no strategyproof mechanism can satisfy fairness and very weak requirements on efficiency. Hence, we carefully design a strategyproof mechanism called Sample, Vote, and Deferred Acceptance (SVDA), and show numerically that SVDA strikes a good balance between fairness and efficiency.

2 MODEL

In this section, we introduce necessary definitions and notations.

Definition 2.1 (Student-Project-Resource (SPR) Instance). It is a tuple $(S, P, R, \succ_S, \succ_P, T_R, q_R)$.

- $S = \{s_1, \dots, s_{|S|}\}$ is a set of students.
- $P = \{p_1, \dots, p_{|P|}\}$ is a set of projects.
- $R = \{r_1, \dots, r_{|R|}\}$ is a set of resources.
- $\succ_S = (\succ_s)_{s \in S}$ are the students' preferences over set $P \cup \{\emptyset\}$.
- $\succ_P = (\succ_p)_{p \in P}$ are the projects' preferences over set $S \cup \{\emptyset\}$.
- Resource r has capacity $q_r \in \mathbb{N}_{>0}$, and $q_R = (q_r)_{r \in R}$.
- Resource r is compatible with $T_r \subseteq P$, and $T_R = (T_r)_{r \in R}$.

For soundness,[3] every preference $\succ_p$ may extend to 2^S in a non-specified manner such that:

- $\forall s, s' \in S, \forall S' \subseteq S \setminus \{s, s'\}, s \succ_p s' \Leftrightarrow S' \cup \{s\} \succ_p S' \cup \{s'\}$ (responsiveness) and
- $\forall s \in S, \forall S' \subseteq S \setminus \{s\}, s \succ_p \emptyset \Leftrightarrow S' \cup \{s\} \succ_p S'$ (separability).

Contract $(s, p) \in S \times P$ means that student s is matched to project p. Contract (s, p) is acceptable for student s (resp. project p) if $p \succ_s \emptyset$ holds (resp. $s \succ_p \emptyset$). The contract is acceptable when both hold. W.l.o.g., we define set of contracts $X \subseteq S \times P$ by $(s, p) \in X$ if and only if it is acceptable for p.[4]

Definition 2.2 (Matching). A matching is a subset $Y \subseteq X$, where for every student $s \in S$, subset $Y_s = \{(s, p) \in Y \mid p \in P\}$ satisfies $|Y_s| \leq 1$, and either

- $Y_s = \emptyset$, or
- $Y_s = \{(s, p)\}$ and $p \succ_s \emptyset$, holds.

For a matching Y, let $Y(s) \in P \cup \{\emptyset\}$ denote the project s is matched, and $Y(p) \subseteq S$ denote the set of students assigned to project p.

Definition 2.3 (Allocation). An allocation $\mu : R \longrightarrow P$ maps each resource r to a project $\mu(r) \in T_r$. (A resource is indivisible.) Let $q_\mu(p) = \sum_{r \in \mu^{-1}(p)} q_r$.[5]

Definition 2.4 (Feasibility). A feasible matching (Y, μ) is a couple of a matching and an allocation where for every project $p \in P$, it holds that $|Y(p)| \leq q_\mu(p)$.

In other words, matching Y is feasible with allocation μ if each project p is allocated enough resources by μ to accommodate $Y(p)$. We say Y is feasible if there exists μ such that (Y, μ) is feasible.

Traditionally (e.g. with fixed quotas), for feasible matching (Y, μ) and $(s, p) \in X \setminus Y$, we say student s *claims an empty seat* of p if $p \succ_s Y(s)$ and matching $Y \setminus \{(s, Y(s))\} \cup \{(s, p)\}$ is feasible with *same* allocation μ. However, in our setting, since the distributional constraint is endogenous and as flexible as allocations are, the definition of nonwastefulness uses this flexibility, as follows.

Definition 2.5 (Nonwastefulness). Given feasible matching (Y, μ), a contract $(s, p) \in X \setminus Y$ is a claiming pair if and only if:

- student s has preference $p \succ_s Y(s)$, and
- matching $Y \setminus \{(s, Y(s))\} \cup \{(s, p)\}$ is feasible with some *possibly new* allocation μ'.

A feasible matching (Y, μ) is nonwasteful if it has no claiming pair.

In other words, (s, p) is a claiming pair if it is possible to move s to a more preferred project p while keeping the assignment of other students unchanged with allocation μ'. Note that μ' can be different from μ. Thus, (s, p) can be a claiming pair even if moving her to p is impossible with the current allocation μ, but it becomes possible with a different/better allocation μ'.

Definition 2.6 (Fairness). Given feasible matching (Y, μ), contract $(s, p) \in X \setminus Y$ is an envious pair if and only if:

- student s has preference $p \succ_s Y(s)$, and
- there exists student $s' \in Y(p)$ such that p prefers $s \succ_p s'$.[6]

We also say s has justified envy toward s' when the above conditions hold. A feasible matching (Y, μ) is fair if it has no envious pair (equivalently, no student has justified envy).

[3]Without these properties, this work is still valid, though a claiming or envious pair (s, p) may not necessarily make sense.
[4]For designing a strategyproof mechanism, we assume each $\succ_s$ is private information of s, while the rest of parameters are public. Thus, X does not need to be part of the input, since it is characterized by projects' preferences.
[5]For $\mu^{-1}(p) = \emptyset$, we assume that an empty sum equals zero.
[6]Note that matching $(Y \setminus \{(s, Y(s)), (s', Y(s'))\}) \cup \{(s, p)\}$ is still feasible with same allocation μ.

In other words, student s has justified envy toward s', if s' is assigned to project p, although s prefers p over her current project $Y(s)$ and project p also prefers s over s'.

Definition 2.7 (Stability). A feasible matching (Y, μ) is stable if it is nonwasteful and fair (no claiming/envious pair).

Definition 2.8 (Pareto Efficiency). Matching Y is Pareto dominated by Y' if all students weakly prefer Y' over Y and at least one student strictly prefers Y'. A feasible matching is Pareto efficient if no feasible matching Pareto dominates it.

Pareto efficiency implies nonwastefulness (not vice versa).

Definition 2.9 (Mechanism). Given any SPR instance, a mechanism outputs a feasible matching (Y, μ). If a mechanism always obtains a feasible matching that satisfies property A (e.g., fairness), we say this mechanism is A (e.g., fair). A mechanism is strategyproof if no student gains by reporting a preference different from her true one.

An SPR belongs to a general class of problems, where distributional constraints satisfy a condition called *heredity*[7] [12]. Three general strategyproof mechanisms exist in this context [12]. *First*, Serial Dictatorship (SD) obtains a Pareto efficient (thus also nonwasteful) matching. SD matches students one by one, based on a fixed ordering. Let Y denote the current (partial) matching. For next student s from the fixed order, SD chooses $(s, p) \in X$ and add it to Y, where p is her most preferred project s.t. $Y \cup \{(s, p)\}$ is feasible with some allocation μ'. Unfortunately, SD is computationally expensive[8] and unfair. *Second*, Artificial Caps Deferred Acceptance (ACDA) obtains a fair matching in polynomial-time. The idea is to fix a resource allocation μ and run the well-known Deferred Acceptance (DA) [9]. In DA, each student first applies to her most preferred project. Then each project deferred accepts applicants up to its capacity limit based on its preference and the rest of the students are rejected. Then a rejected student applies to her second choice, and so on.[9] However, ACDA is inefficient since μ is chosen independently from students' preferences. *Third*, Adaptive DA (ADA) is a mixture of SD and DA, which satisfies nonwastefulness. In ADA, several students are assigned simultaneously and compete with each other. However, it is computationally as expensive as SD. Also, it assumes that the capacity limit of each project is given; otherwise, it becomes identical to SD.

Example 2.10. Nonwastefulness and fairness are incompatible since there exists an instance with no stable matching. Let us show a simple example with two students s_a, s_b, two projects p_a, p_b, and a unitary resource compatible with both. Students' preferences are $p_a \succ_{s_a} p_b$ and $p_b \succ_{s_b} p_a$. Projects' are $s_b \succ_{p_a} s_a$ and $s_a \succ_{p_b} s_b$. By symmetry, assume the resource is allocated to p_a. From fairness, s_b must be allocated to p_a. Then (s_b, p_b) becomes a claiming pair.[10]

3 STRATEGYPROOF MECHANISM

In this section, we discuss how to develop a strategyproof mechanism that can strike a good balance between fairness and efficiency.

[7]Heredity means that if matching Y is feasible, then any of its subsets are also feasible. An SPR satisfies this property.
[8]It requires to solve SPR/FA (see below) $O(|X|)$ times.
[9]Each project deferred accepts applying students, without distinguishing newly applied and already deferred accepted students.
[10]We use this example as a building block in the next section.

3.1 Impossibility Theorem

Let us prove an impossibility theorem that shows (full) fairness is not compatible with very mild conditions on efficiency in a strategyproof mechanism.

We introduce two conditions that related to efficiency. The first one is called *weak nonwastefulness.*

Definition 3.1 (Weak Nonwastefulness). For feasible matching (Y, μ), student s is a *strongly claiming student* if $Y(s) = \emptyset$, and for any feasible matching (Y, μ'), s claims an empty seat of some project p (p can be different for each μ'). A feasible matching is weakly nonwasteful if it has no strongly claiming student.

In other words, student s is a strongly claiming student if she is currently unassigned, and under any feasible resource allocation, there exists some project p such that s claims an empty seat of p.

To define another concept called *resource efficiency*, we first define *unanimous preferences.*

Definition 3.2 (Unanimous Preference). Students unanimously prefer p over p' if for every $s \in S$, $(s, p) \in X$ and $p \succ_s p'$ hold.

This condition means that project p accepts all students and all students prefer p over p'. If students unanimously prefer p over p', allocating any resource (which is compatible with both p and p') to p' is *inefficient* in terms of students' welfare. The following formalizes this intuition.

Definition 3.3 (Resource Efficiency). Resource allocation μ is resource efficient if any resource r, such that $p', p \in T_r$ and students unanimously prefer p over p', is not allocated to p'. A mechanism is resource efficient if it always returns a resource efficient allocation.

Nonwastefulness clearly implies weak nonwastefulness. Also, Pareto efficiency implies nonwastefulness. The following theorem shows that Pareto efficiency also implies resource efficiency.

Theorem 3.4. *If feasible matching (Y, μ) is Pareto efficient, then there exists allocation μ' s.t. (Y, μ') is feasible and μ' is resource efficient.*

Proof. For the sake of contradiction, assume Y is Pareto efficient, all students unanimously prefer p over p', but for any μ such that (Y, μ) is feasible, there exists resource r such that $p, p' \in T_r$, is allocated to p'. Consider μ' obtained from μ, such that r is re-assigned to p for If (Y, μ') is feasible, we repeated the same procedure. Then, eventually, (Y, μ') becomes infeasible (otherwise, we obtain resource efficient μ', which contradicts our assumption). Since students unanimously prefer p over p', any student assigned to p' is acceptable for p and prefers p over p'. Let us consider another matching Y', in which some students are moved from p' to p such that (Y', μ') becomes feasible. Then, the moved students prefer Y' over Y (and other students are indifferent). This contradicts our assumption that Y is Pareto efficient. □

Now we are ready to introduce our impossibility theorem.

Theorem 3.5. *No mechanism exists that is fair, weakly nonwasteful, resource efficient, and strategyproof.*

Example 3.6. Assume three students s_1, s_2, s_3 and three projects p_1, p_2, p_3. There is one resource r with $q_r = 2$ and $T_r = \{p_1, p_2, p_3\}$. Preferences are:

$$\begin{aligned}
s_1 &: p_2 \succ p_3 \succ p_1 \succ \emptyset, \quad & p_1 &: s_1 \succ s_2 \succ s_3 \succ \emptyset,\\
s_2 &: p_3 \succ p_1 \succ p_2 \succ \emptyset, \quad & p_2 &: s_2 \succ s_3 \succ s_1 \succ \emptyset,\\
s_3 &: p_1 \succ p_2 \succ p_3 \succ \emptyset, \quad & p_3 &: s_3 \succ s_1 \succ s_2 \succ \emptyset.
\end{aligned}$$

Proof. If r is not allocated to any project and all students are unmatched, weak nonwastefulness is violated. Thus, r must be allocated to some project (this is also true for any student preferences as long as all projects are acceptable). From weak nonwastefulness and fairness, possible matchings are: allocating s_1 and s_2 to p_1, allocating s_2 and s_3 to p_2, or allocating s_3 and s_1 to p_3. From the symmetry, we can assume s_1 and s_2 are allocated to p_1 without loss of generality. Then, let us examine the case where s_3's preference is changed to: $p_3 \succ p_1 \succ p_2 \succ \emptyset$. From resource efficiency, r cannot be allocated to p_1, since all students prefer p_3 over p_1. If r is allocated to p_2 (or p_3), then from fairness and weak nonwastefulness, s_3 must be allocated to p_2 (or p_3). This violates strategyproofness since s_3 is not allocated to any project in the original situation. □

Resource efficiency and weak nonwastefulness are independent properties. For example, ACDA satisfies weak nonwastefulness; under the resource allocation used in ACDA, no student constitutes a claiming pair, while it does not satisfy resource efficiency, i.e., it may allocate a resource to an unanimously less preferred project. Also, assume a mechanism does not assign any student to any project, while no resource is allocated to any project p' if students unanimously prefer another project p. Then, it is trivially strategyproof and satisfies resource efficiency. However, this mechanism does not satisfy weak nonwastefulness in general.

3.2 Sample and Vote Deferred Acceptance (SVDA)

Given that ACDA is too inefficient, and SD/ADA are too unfair and computationally expensive (need to verify feasibility $O(|S \times P|)$ times), we develop a new strategyproof mechanism called Sample and Vote Deferred Acceptance (SVDA). Its basic idea is to determine resource allocation μ based on the preferences of sampled students. Then we run DA based on μ. The whole mechanism is designed carefully to guarantee strategyproofness. The idea of dividing students/participants into two groups and utilizing the information obtained by one group to appropriately set parameters for the mechanism applied to another group, is a popular technique to guarantee strategyproofness in auction domains [4, 10].

Mechanism 1 (Sample and Vote Deferred Acceptance (SVDA))**.**

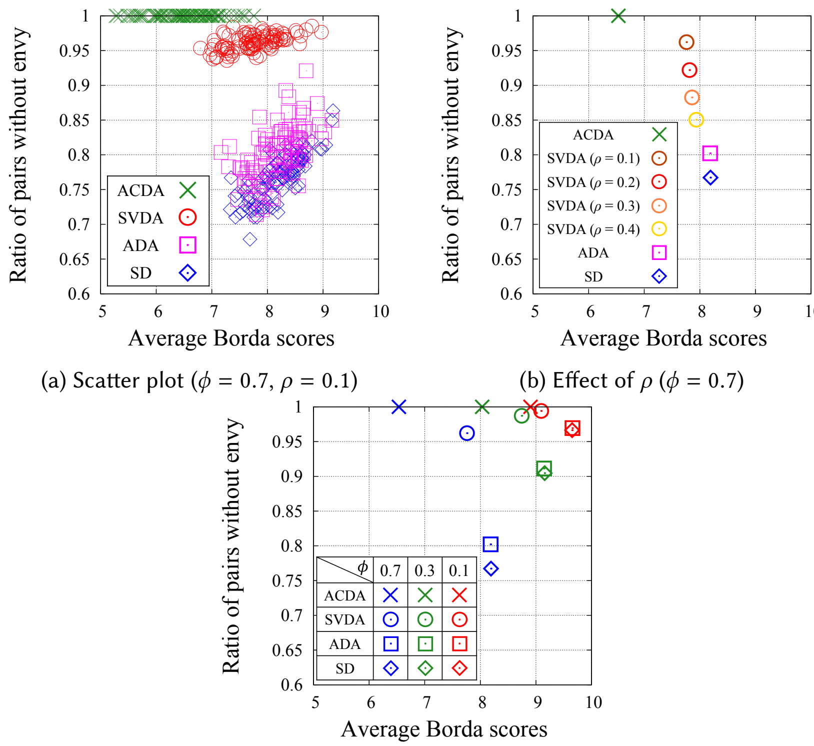
- **Step 1:** Select $S' \subseteq S$, which we call the sampled students. We call $S \setminus S'$ the regular students. Then run SD and find (partial) matching $Y_{S'}$ for S'.
- **Step 2:** Allocate $R' \subseteq R$ to projects such that $Y_{S'}$ is feasible and R' is minimal: no $R'' \subsetneq R'$ makes $Y_{S'}$ feasible.
- **Step 3:** Allocate $R \setminus R'$ based on the preferences of S'. Then run DA for $S \setminus S'$. The capacity of p is $q_\mu(p) - |Y_{S'}(p)|$, where μ is the current resource allocation.

To decide allocation $R \setminus R'$ based on the preferences of S', we use the following simple method. For each r, each $s \in S'$ (hypothetically) votes for candidates T_r based on $\succ_s$, where each project obtains the Borda score based on $\succ_s$. Then r is allocated to the winner.[11]

Let us show an example. Four students s_1, s_2, s_3, s_4, four projects p_1, p_2, p_3, p_4, and two resources r_1, r_2, where $T_{r_1} = \{p_1, p_2\}$, $T_{r_2} = \{p_3, p_4\}$, and $q_{r_1} = 2$, $q_{r_2} = 1$. For s_1, s_2, and s_3, preferences are: $p_1 \succ p_2 \succ p_3 \succ p_4$, and for s_4, preference is: $p_4 \succ p_3 \succ p_2 \succ p_1$. The preferences of all the projects are identical: $s_4 \succ s_3 \succ s_2 \succ s_1$.

In Step 1, assume $S' = \{s_1\}$, i.e., s_1 is the only sampled student. In SD, s_1 is matched to her first-choice project p_1. In Step 2, the minimal allocation to make $\{(s_1, p_1)\}$ feasible is to allocate r_1 to p_1. Thus, $R' = \{r_1\}$. In Step 3, the allocation of $R \setminus R' = \{r_2\}$ is determined by the preference

[11] Details of this voting procedure, e.g., whether a student can vote for a project to which she is unacceptable or not, or how a tie is broken, do not affect the theoretical properties of SVDA. Thus, they can be determined in an arbitrary way.



(a) Scatter plot ($\phi = 0.7$, $\rho = 0.1$)

(b) Effect of ρ ($\phi = 0.7$)

(c) Effect of ϕ ($\rho = 0.1$)

Fig. 1. Tradeoff between efficiency/fairness

of s_1. Then, r_2 is allocated to p_3. The capacities of p_1, p_2, p_3, and p_4 are respectively $1, 0, 1$, and 0. Then, $S \setminus S' = \{s_2, s_3, s_4\}$ are matched by DA. Thus, s_2 is unmatched, s_3 is matched to p_1, and s_4 is matched to p_3. It is not fair, since (s_2, p_1) is an envious pair. Nonwastefulness is not satisfied either, since (s_4, p_4) is a claiming pair, while it satisfies weak nonwastefulness and fairness within regular students.

Theorem 3.7. *SVDA is strategyproof, resource efficient, weakly nonwasteful and fair among $S \setminus S'$, i.e., no regular student has justified envy toward another regular student.*

Proof. SVDA is clearly strategyproof for S' since SD is strategyproof. SVDA is strategyproof for $S \setminus S'$ since DA is strategyproof and the capacity of each project is determined exogenously for $S \setminus S'$. It satisfies resource efficiency. Assume students unanimously prefer p over p'. When assigning S', any resource r s.t. $p, p' \in T_r$ never be allocated to p', since students applies to p before applying to p'. Also, it never wins in the voting procedure. It satisfies weak nonwastefulness since if student s is not matched to any project, she cannot claim an empty seat of any project in current allocation μ. Also, since DA is fair, no regular student has justified envy toward another regular student. □

SVDA needs to verify feasibility $O(|P||S'|)$ times in Step 1. However, when $|S'|$ is small, such a feasibility problem is trivially "yes" in most cases. Also, state-of-the-art IP solvers, e.g., Gurobi optimizer [13] can handle fairly large-scale feasibility problems.

4 EXPERIMENTAL EVALUATION

We considered a market with $|S| = 200$ students, $|P| = 10$ projects and $|R| = 20$ resources.[12] For each resource r, we randomly generated T_r such that each project p is included in T_r with probability 0.2. The capacity of each resource is 1, 5, 10, 15 or 20 (the number of resources for each capacity is 4). Student preferences are generated with the Mallows model [6, 26, 33]. In this model, a student preference $\succ_s$ is drawn with probability $\Pr(\succ_s)$:

$$\Pr(\succ_s) = \frac{\exp(-\phi \cdot d(\succ_s, \succ_{\widehat{s}}))}{\sum_{\succ'_s} \exp(-\phi \cdot d(\succ'_s, \succ_{\widehat{s}}))}.$$

Here $\phi \in \mathbb{R}$ denotes a spread parameter, $\succ_{\widehat{s}}$ is a central preference (uniformly randomly chosen from all possible preferences in our experiment), and $d(\succ_s, \succ_{\widehat{s}})$ represents the Kendall tau distance between $\succ_s$ and $\succ_{\widehat{s}}$. The Kendall tau distance is equal to the number of ordered pairs in $\succ_s$ that are inconsistent with those in $\succ_{\widehat{s}}$. When $\phi = 0$, Mallows model becomes identical to the uniform distribution and, as ϕ increases, quickly converges to the constant distribution returning $\succ_{\widehat{s}}$. The preference of each project is drawn uniformly at random. We created 100 instances for each parameter setting and compared SVDA with ACDA, SD and ADA. As described earlier, ADA needs a capacity limit for each project p. We set this value to $\sum_{r|p \in T_r} q_r$, which is the largest capacity when all of the shared resources are allocated to it.[13]

To illustrate the tradeoff between efficiency and fairness, we plotted the results of the obtained matching in a two-dimensional space in Figure 1, where the x-axis shows the average Borda scores of the students, i.e., if a student is assigned to her i-th choice project, her score is $|P| - i + 1$, and the y-axis shows the ratio of student pairs that did not form envious pairs. Thus, points located north-east are preferable. Each point represents the result of one instance for one mechanism. For SVDA, we set the ratio of sampled students $\rho = |S'|/|S|$ to 0.1. Figure 1 (a) illustrates that SVDA strikes a good balance between efficiency and fairness. Also, in Figure 1 (b), we show the average for 100 problem instances for each mechanism. For SVDA, we vary ρ from 0.1 to 0.4. When ρ is small, SVDA is similar to ACDA. By increasing ρ, it gradually becomes similar to SD. Thus, by controlling parameter ρ, we can further fine-tune the balance. In Figure 1 (c), we vary spread parameter ϕ from 0.1 to 0.7. When ϕ is large, the competition among students becomes more severe and resource allocation significantly affects the students welfare. When ϕ is small, the difference among mechanisms becomes smaller. One might argue that SVDA works only when sampled students resemble regular students. This is true to some extent, but this result shows that when students are diverse, all mechanisms work reasonably well. We also run experiments with different voting procedures (single majority and Copeland) and found quite similar results.

5 CONCLUSIONS AND PROSPECTS

We introduced a student-to-project matching problem that endogenously handles the resource allocation problem defining the capacity of projects. We showed that a fair and nonwasteful matching does not exist in general. Moreover, we showed that it is impossible to design a mechanism that is fair, strategyproof, and satisfies very mild efficiency properties. Then we developed mechanism SVDA that is strategyproof and strikes a good balance between fairness and efficiency. Future works include theoretically identifying an optimal sample size for achieving a given criterion to balance between fairness and efficiency, and dealing with various constraints on the allocation of resources, e.g., the total number of resources allocated to each project is bounded.

[12] Since SD and ADA are computationally expensive, running experiments for larger markets is time-consuming, while SVDA (and ACDA) can handle much larger markets.

[13] Since this capacity is large and not binding in many cases, ADA resembles SD.

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