CAPM: A Tale of Two Versions

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Abstract

Categorization is the mental operation by which brain classifies objects and events. We do not experience the world as a series of unique events. Rather, we make sense of our experiences within a framework of categories that represent prior knowledge. Given that categorization is the core of cognition, we argue that the traditional view that each firm is viewed in isolation needs to be altered. Instead, like every other object they ever come across, investors view each firm within a framework of categories that represent prior knowledge. This involves sorting a firm into a category based on a subset of firm-attributes. Such categorization-relevant attributes are refined whereas other firm-attributes are confounded with the category-exemplar. Two versions of CAPM arise as a result. In the first version, the relationship between average excess return and stock beta is flat (possibly negative). Value effect and size premium (controlling for quality) arise in this version. In the second version, the relationship is strongly positive. The two-version CAPM accounts for several recent empirical findings including fundamentally different intraday vs overnight behavior, as well as behavior on macroeconomic announcement days. The tug-of-war dynamics of the two versions also suggest that momentum is expected to be an overnight phenomenon, which is consistent with empirical findings. We argue that, perhaps, our best shot at observing classical CAPM in its full glory is a laboratory experiment with subjects who have difficulty categorizing (such as in autism spectrum disorders).

Keywords: CAPM, Categorization, Value Effect, Betting-Against-Beta, Size Effect.

JEL Classification: G12, G41

CAPM: A tale of two versions

Consider the following two empirical observations: Firstly, stock prices behave very differently with respect to their sensitivity to market risk (beta) at specific times. Typically, average excess return and beta relationship is flatter than expected (Frazzini and Pedersen 2014, Fama and French 2004, Black, Jensen, and Scholes 1972). It could even be negative[1]. However, during specific times, this relationship is strongly positive, such as on days when macroeconomic announcements are made (Savor and Wilson 2014) or during the night (Hendershott, Livdan, and Rosch 2018). Secondly, a hue, which is halfway between yellow and orange, is seen as yellow on a banana and orange on a carrot (Mitterer and de Ruiter 2008). In this article, we argue that the two observations are driven by the same underlying mechanism.

The second observation is an example of the implications of categorization for color calibration. In this article, we argue that the first observation is also due to categorization, which gives rise to two versions of CAPM. In one version, the relationship between expected return and stock beta is flatter than expected or could even be negative, whereas in the second version, this relationship is strongly positive.

Categorization is the mental operation by which brain classifies objects and events. We do not experience the world as a series of unique events. Rather, we make sense of our experiences within a framework of categories that represent prior knowledge. That is, new information is only understood in the context of prior knowledge. Describing categorization, Cohen and Lefebvre (2005) write, "*This operation is the basis of construction of our knowledge of the world. It is the most basic phenomenon of cognition, and consequently the most fundamental problem of cognitive science."* To cognize is to categorize (Harnad 2017). Our daily lives are

[1] Cohen, Polk, and Vuolteenaho (2005), and Jylha (2018)

dependent on our ability to form categories, and inefficiencies in category-formation have been associated with autism spectrum disorders (ASD) (see Church et al (2010)).

It is well-recognized in cognitive science literature that categorization is driven by selective attention where some aspects in the information-environment are sharply attended-to while others are attenuated.[2] The upside of categorization is that the attributes that rely on the information aspects in sharp focus (the basis for categorization) get refined. The downside of categorization is that the attributes dependent on the attenuated aspects get confounded with the corresponding attributes of the category-exemplar.

Both sides are readily seen in various examples of categorization. Mitterer and de Ruiter (2008) present participants with drawings of banana and carrots filled with a hue halfway between yellow and orange. Drawings are immediately recognized as banana or carrot based on shape (shape-attribute is refined as either a banana-shape or a carrot-shape). That's the upside of categorization. However, the other attribute, color, gets confounded with the color of the category exemplar. This confounding creates a perception of yellow when the hue is viewed on a banana. Similarly, it creates a perception of orange when the same hue is seen on a carrot. This cofounding is the downside of categorization.

Making categorization-induced inferences is a general perceptual strategy used by the brain. When a racially ambiguous face has been categorized as either Hispanic or Black (based on hair, so hair attribute is refined), then the complexion attribute gets confounded with the complexion of the category-exemplar leading to the same complexion being perceived as lighter on a Hispanic face than on a Black face (Maclin and Malpass 2001, 2003). Similarly, a sound half way between "s" and

[2] Medin & Schaffer (1978), Nosofsky (1986), Pazzani (1991), Hampton (1995), Nosofsky (1992), Smith and Minda (1998)

“f” is heard as “s” if the environmental cue is refined as a horse and as “f” if the environmental cue is refined as a giraffe (Norris et al 2001, 2006).

To take an example from our daily lives, imagine you go to a park and you spot a dog. You may instinctively attend to the “ownership” aspect, and if you see a person accompanying the dog, you may categorize the dog as a pet. Paying attention to the “ownership” aspect refines the ownership attribute. The refined “ownership” attribute enables useful inferences. That’s the upside of categorization. The downside is that, if the category-exemplar of pet dog in your memory has a passive demeanour, you may underestimate the aggressiveness of the dog in question. No wonder we continue to see occasional dog bite cases.

Despite recognition across the allied disciplines of cognitive science, neuroscience, and psychology that categorization defines how we think (Cohen and Lefebvre 2017), economics and finance literature is largely silent on how it matters for decision-making in their domain. An exception is found in behavioral literature, where the downside of categorization is formalized as categorization-bias, capturing the notion that objects in the same category are deemed more similar (objects in different categories are deemed more different) than they actually are (Mullainathan 2000, Hong, Stein and Yu 2007, Mullainathan et al 2008). However, a more nuanced view, which admits both the upside as well as the downside, is lacking.

Here, in accord with cognitive science literature, we present a view of categorization that has both an upside as well as a downside, and apply this nuanced perspective to the capital asset pricing model (CAPM). If categorization is fundamental to how our brains make sense of information, then investor behaviour, like any other domain of human behaviour, should also be viewed through this lens. This means that the traditional view that each firm is viewed in isolation needs to be altered. When an investor encounters a new firm, she views it within a framework of categories that represent prior knowledge. This involves sorting the firm into a category based on some firm-attributes. Categorization refines such attributes. Other attributes are confounded with the corresponding attributes of the category-exemplar. This attention-attenuation mechanism associated with categorization gives rise to two versions of CAPM.

In finance literature, news is often classified as either earnings (cash flows) news or discount rate (cost of capital) news. [3] Both declining earnings and higher discount rates destroy investor wealth; however, as argued in Campbell and Vuolteenaho (2004), a higher discount rate also means higher returns on investment opportunities, so part of the loss is mitigated, making “bad earnings news” a stronger destroyer of wealth. Empirically, Chen et al (2013) find that most of the stock price movements are driven by earnings news. Consistent with this, when analysts revise their stock recommendations, market prices respond twice as strongly when the revisions are due to revisions in earnings estimates (Kecskes et al 2016). It is hardly any surprise that market participants generally consider earnings news to be most important (Basu et al 2013, Graham et al 2005).

Despite the importance given to earnings news, there are specific times when the discount rate news clearly dominates, such as on days when macroeconomic announcements about interest rates, inflation, or unemployment are made, or during the night, when the local market is closed whereas markets abroad (benchmarks for calibrating discount rates) are open. We show that, depending on which type of news is dominant, different versions of CAPM are obtained with categorization.

While categorizing firms, if investors pay more attention to the earnings aspect, then the earnings estimates are sharpened whereas the discount-rate gets confounded with the category-exemplar. This leads to a version of CAPM, in which a flatter or even negative relationship between stock beta and expected excess returns arise. Betting-against-beta anomaly (Black 1972, Frazzini and Pedersen 2014) is

[3] Campbell and Vuolteenaho (2004), Campbell and Mei (1993), Campbell (1991), Campbell and Shiller (1988)

observed along with the value effect, as well as the size premium after controlling for quality (consistent with the findings in Asness et al 2018). We argue that this is the default version which typically prevails.

Looking ahead at the results, the first version of CAPM takes the following form:

$$E(r_i) - r = \alpha_i + \beta_i E(r_M - r)$$

where r_M, r, and r_i are market portfolio, risk-free, and stock returns respectively. The only difference between this version and classical CAPM is the appearance of alpha or α_i. It is this additional term which drives all the results:

$$\alpha_i = \frac{E(r_M - r)(d_i - c)}{(1 + c)} \qquad (typical\ firm) \qquad (1.1)$$

$$\alpha_i = -\frac{E(r_M - r)c}{(1 + c)} \qquad (category\ exemplar) \qquad (1.1a)$$

where $0 < c < 1$ is a constant in a cross-section, and $d_i > 0$ has the following properties:

$$(1)\ \frac{\partial d_i}{\partial \beta_i} < 0\ (high\ alpha\ of\ low\ beta)$$

$$(2)\ \frac{\partial d_i}{\partial P_i} < 0\ (similar\ to\ value\ effect); P_i\ is\ stock\ price$$

$$(3)\ \frac{\partial d_i}{\partial \sigma^2(P_i + D_i)} < 0\ ;\ D_i is\ dividend\ (size\ premium\ controlling\ for\ quality)$$

(1) is clearly high-alpha-of-low-beta, and (2) is high-alpha-of-low-price similar to value effect. We interpret (3) as size-premium controlling for quality (Asness et al 2018). This is because small-cap stocks with safe, steady earnings and low leverage generally have the smallest payoff volatility in the market. Interesting, in this version of CAPM, the relationship between excess return and stock beta can also be negative as $\frac{\partial d_i}{\partial \beta_i} < 0$.

$$\frac{\partial E(r_i)}{\partial \beta_i} = \frac{E(r_M - r)}{(1+c)}\frac{\partial d_i}{\partial \beta_i} + E(r_M - r) < 0 \; if \; \left|\frac{\partial d_i}{\partial \beta_i}\right| > 1 + c$$

As c is generally quite small, the relationship between excess return and stock beta is quite possibly negative. Recent studies generally find such a negative relationship (Savoir and Wilson 2014, Hendershott et al 2018 and references there in). Furthermore, category-exemplars have the lowest alphas in this version (as can be seen from 1.1a).

While categorizing firms, if investors pay more attention to the discount rate aspect, then the discount-rates are refined whereas earnings estimates are confounded with the category-exemplar. A second version of CAPM arises. In this version, there is a strong positive relationship between beta and expected excess return.

The second version of CAPM has the following form:

$$E(r_i) - r = \alpha_i + \beta_i E(r_M - r)$$

$\alpha_i = h - e_i$ (typical firm) where $e_i > 0$ and h is a constant in a cross-section.

$\alpha_i = h$ (category-exemplar)

$$(1)\frac{\partial e_i}{\partial \beta_i} < 0 \quad (High\ alpha\ of\ high\ beta)$$

$$(2)\frac{\partial e_i}{\partial P_i} < 0 \quad (Growth\ stocks\ do\ better)$$

So, in this second version, alpha rises with beta. This makes the relationship between excess return and stock beta strongly positive. Also, $\frac{\partial e_i}{\partial P_i} < 0$, so growth stocks do better in this version than value stocks. Furthermore, category-exemplars have the highest alphas in this version. It is interesting to note that the stocks that do better in the first version (value, low beta) generally do worse in the second version consistent with the tug-of-war dynamics documented in Lou, Polk, and Skouras (2018).

One way to make sense of the co-existence of two versions is to classify investors as either earnings-focused or discount-rate focused, similar to the bull and bear classifications. Both investor types co-exist; however, which version is reflected in price dynamics depends on which investor type is dominant. When earnings news dominates (intraday), earnings-focused investors trade more actively and influence prices in the process, whereas when discount rate news is dominant (overnight), then the discount-rate focused investors mostly affect price behavior through trading.

On days of macroeconomic announcements about interest rates, inflation, and unemployment, the discount rate news is naturally in focus. Hence, this second version of CAPM is likely to be observed on macroeconomic announcement days. Indeed, in accord with the second version, Savor and Wilson (2014) find that on such days the relationship between average excess returns and stock beta is strongly positive, along with growth stocks doing better.

During the night, the local market is closed, whereas major markets abroad are open. In general, if a major market abroad closes sharply lower (higher) then the local market responds by opening lower (higher) as well. This suggests that the world markets help re-calibrate the discount rates for the local market at open (Ammer and Mei (1996) find that risk premiums rather than fundamental variables account for most of the co-movements across national indices). At open, increased participation of discount-rate focused investors implies that the second CAPM version is likely to be observed overnight (from close-to-open). Consistent with this, Hendershott et al (2018) find that overnight, there is a strong positive relationship between stock betas and average excess returns.

The first version generally dominates intraday. As this version comes with size and value effects, the prediction is that size and value are primarily intraday phenomena. Indeed, this is exactly what Lou, Polk, and Skouras (2018) find.

We show that, all else equal, discount-rate focused investors have higher willingness-to-pay than earnings focused investors. If discount-rate investors are primarily overnight traders whereas earnings focused investors are active intraday, then one expects prices to typically rise overnight from close-to-open and fall intraday between open-to-close. Consistent with this prediction, recent work by Kelly and Clark (2011) suggests that returns are higher overnight than intraday.

Momentum trading takes a long position in recent high return earners while shorting recent worst return performers. As the focus of momentum strategies is on returns or discount rates, momentum traders are discount-rate focused investors who are mostly active overnight. Consequently, one expects momentum to be primarily an overnight phenomenon. Indeed, this is what Lou et al (2018) find.

Can we ever observe the original CAPM instead of a version of it? Because categorization and associated attention-attenuation mechanism (selective attention) is such a fundamental aspect of cognition, it never turns-off in a healthy brain. Hence, the classical CAPM is unlikely to be ever observed. We catch glimpses of it in various versions depending on which type of news/ investor type dominates. However, among ASD sufferers, there is a breakdown in categorization ability (Gastgeb and Strauss 2012, Church et al 2010). So, perhaps a laboratory experiment with high functioning ASD sufferers (and limited informational complexity) is our best shot at observing CAPM in its full glory.

2. Adjusting CAPM for categorization

As discussed in the introduction, when information about an object or an event reaches the human brain, it makes sense of it within a framework of categories that represent prior knowledge. This involves sorting that object or event in a category based on a subset of attributes. Such categorization-relevant attributes gets refined, whereas other (categorization-irrelevant) attributes get confounded with the corresponding attributes of the category-exemplar.

Treating financial information the same, we argue that firms are not viewed in isolation. Rather, investors view them within a framework of categories that represent prior knowledge. This involves sorting a firm into a category based on a subset of attributes. While categorizing firms, if investors focus more on the earnings-aspect then earnings-estimates are sharpened whereas the discount-rates are confounded with the category-exemplar. The reverse happens if the discount-rate aspect is categorization-relevant.

As discussed in the introduction, financial information is generally classified as either earnings information or discount-rate information. There are several reasons to expect that investors typically pay more attention to earnings news:

1) Both declining earnings and higher discount rates destroy investor wealth; however, as argued in Campbell and Vuolteenaho (2004), a higher discount rate also means higher returns on investment opportunities, so part of the loss is mitigated, making “bad earnings news” a stronger destroyer of wealth.

2) Empirically, earnings news drives most stock price movements (Chen et al 2013).

3) Market participants consider earnings news to be most important (Basu et al 2013, Graham et al 2005).

4) When analysts revise their stock recommendations, market prices respond twice as strongly when the revisions are due to revisions in earnings estimates (Kecskes et al 2016).

Given these reasons, we consider our baseline case to be the one in which earnings aspect is categorization-relevant. This case is examined next. The alternate case in which the discount-rate aspect is categorization-relevant is examined in section 2.3.

2.1 Baseline case: Earnings news is categorization-relevant

To adjust CAPM for categorization, we use the same starting point as in Frazzini and Pedersen (2014). Consider an overlapping-generations (OLG) economy in which agents with wealth W_t are born in each period t and live for two periods.

Each period t, young agents invest in stocks and the risk-free asset to maximize utility:

$$\max n'(E_t(P_{t+1} + D_{t+1}) - (1+r)P_t) - \frac{\gamma}{2}n'\theta_t n \tag{2.1}$$

where n is the vector representing the number of shares of each type in the portfolio, P_t is the vector of prices, D_t is the vector of dividends, r is the risk-free rate, γ captures risk-aversion, and θ_t is the variance-covariance matrix of $P_{t+1} + D_{t+1}$.

From the first-order-condition of utility maximization of agent i:

$$n_i = \frac{1}{\gamma_i}\theta^{-1}(E_t(P_{t+1} + D_{t+1}) - (1+r)P_t)$$

In equilibrium, demand equals supply:

$$\sum_i n_i = n^*$$

It follows that:

$$n^* = \frac{1}{\gamma}\theta^{-1}(E_t(P_{t+1} + D_{t+1}) - (1+r)P_t) \tag{2.2}$$

where the aggregate risk aversion, γ is defined as $\frac{1}{\gamma} = \sum_i \frac{1}{\gamma_i}$

Solving (2.2) for equilibrium price:

$$P_t = \frac{E_t(P_{t+1} + D_{t+1}) - \gamma\theta_t n^*}{1+r} \tag{2.3}$$

By choosing an appropriate risk-premium, δ_t, one may write:

$$P_t = \frac{E_t(P_{t+1} + D_{t+1}) - \gamma\theta_t n^*}{1+r} = \frac{E_t(P_{t+1} + D_{t+1})}{1+r+\delta_t} \tag{2.4}$$

where $\delta_t = f(\gamma, \theta, n^*)$.

The R.H.S of (2.4) shows that stock price movements can be attributed either to earnings news which affects the numerator, $E_t(P_{t+1} + D_{t+1})$, or it can be attributed to the discount rate news which affects the denominator, $1 + r + \delta_t$.

We start by considering the simplest case in which investors divide assets into only two categories: risky and risk-free. That is, all risky stocks are placed in one category. To illustrate the implications for CAPM, initially assume that there are only two risky stocks belonging to firms L and S.

From (2.3):

$$P_{Lt} = \frac{E_t\big(P_{L(t+1)} + D_{L(t+1)}\big) - \gamma n_L^* \sigma_{Lt}^2 - \gamma n_S^* \sigma_{LS}}{1+r} \tag{2.5}$$

$$P_{St} = \frac{E_t\big(P_{S(t+1)} + D_{S(t+1)}\big) - \gamma n_S^* \sigma_{St}^2 - \gamma n_L^* \sigma_{LS}}{1+r} \tag{2.6}$$

where σ_{Lt}^2 and σ_{St}^2 are payoff variances of L and S respectively, and σ_{LS} is their covariance. Assuming that γ, r, and n^* are constant, investors form expectations regarding the following attributes of $L's\ stock$: $\big(P_{L(t+1)} + D_{L(t+1)}, \sigma_{Lt}^2, \sigma_{LS}\big)$. Similarly, they form expectations about the following attributes of $S'stock$: $\big(P_{S(t+1)} + D_{S(t+1)}, \sigma_{St}^2, \sigma_{LS}\big)$

Firm L is analyzed first. We assume rational expectations about future earnings as well as volatility of earnings of firm L. And, these rational expectations translate into rational expectations about all three attributes of $L's$ stock: $\big(P_{L(t+1)} + D_{L(t+1)}, \sigma_{Lt}^2, \sigma_{LS}\big)$.

Firm S is analyzed next, and is co-categorized with firm L, which is the category-exemplar. Assuming that earnings aspect is focused on while categorizing implies that earnings estimate is refined. There are several ways in which categorization-induced inferences improve the earnings estimate. Most obvious is size comparison which would be refined further based on how similar the two firms are. We assume that such categorization-induced inferences lead to rational expectations regarding the estimated earnings of firm S. The downside is the confounding of earnings volatility of firm S with firm L.

Defining π_S and π_L as the earnings of S and L respectively:

Upside of categorization: $E_t(\pi_s)$ is rational.

Downside of categorization: $\sigma_{st}^{C2}(\pi_S) = m\sigma_{St}^2(\pi_S) + (1-m)\sigma_{Lt}^2(\pi_L)$

where $0 \leq m \leq 1$ captures the degree of confounding. There is no confounding when $m = 1$. The confounding is maximum when $m = 0$.

This confounding of earnings volatility confounds stock payoff volatility, as investors consider stock price (inclusive of dividends) to be a function of earnings per share or EPS:

$$\frac{\sigma_{st}^{C2}(\pi_S)}{n_S^{*2}} = \frac{m\sigma_{St}^2(\pi_S)}{n_S^{*2}} + (1-m)\frac{\sigma_{Lt}^2(\pi_L)}{n_S^{*2}}\frac{n_L^{*2}}{n_L^{*2}}$$

$$\Rightarrow \sigma_{st}^{C2}(EPS_S) = m\sigma_{St}^2(EPS_S) + (1-m)\sigma_{Lt}^2(EPS_L)\frac{n_L^{*2}}{n_S^{*2}}$$

$$\Rightarrow \sigma_{st}^{C2}\left(P_{S(t+1)} + D_{S(t+1)}\right) \approx$$
$$m\sigma_{St}^2\left(P_{S(t+1)} + D_{S(t+1)}\right) + (1-m)\sigma_{Lt}^2\left(P_{L(t+1)} + D_{L(t+1)}\right)\frac{n_L^{*2}}{n_S^{*2}} \quad (2.7)$$

Substituting (2.7) in (2.6):

$$P_{St} = \frac{E_t\left(P_{S(t+1)} + D_{S(t+1)}\right) - \gamma n_S^* m\sigma_{St}^2 - \gamma n_S^*(1-m)\sigma_{Lt}^2\frac{n_L^{*2}}{n_S^{*2}} - \gamma n_L^*\sigma_{LSt}}{1+r} \quad (2.8)$$

Adding and subtracting $\gamma n_S^*\sigma_{St}^2$ to the numerator and using $cov\left(P_{S(t+1)} + D_{S(t+1)}, n_S^*\left(P_{S(t+1)} + D_{S(t+1)}\right) + n_L^*\left(P_{L(t+1)} + D_{L(t+1)}\right)\right) = n_S^*\sigma_{St}^2 + n_L^*\sigma_{LSt}$ with a further substitution of $X_{S(t+1)} = P_{S(t+1)} + D_{S(t+1)}$ and $X_{L(t+1)} = P_{L(t+1)} + D_{L(t+1)}$ leads to:

$$P_{St} = \frac{E_t\left(X_{S(t+1)}\right) - \gamma\left[cov\left(X_{S(t+1)}, n_S^*X_{S(t+1)} + n_L^*X_{L(t+1)}\right) + n_S^*(1-m)\left(\sigma_{Lt}^2\frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)\right]}{1+r}$$

$$(2.9)$$

In terms of expected returns:

$$E_t(r_s) = r + \frac{\gamma}{P_{St}}\left[Cov\left(X_{S(t+1)}, n_s^* X_{S(t+1)} + n_L^* X_{L(t+1)}\right) + n_S^*(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)\right] \quad (2.10)$$

The additional term on the R.H.S of (2.10), $n_S^*(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)$, is due to the confounding of the earnings-variance of S with the earnings-variance of L. This term disappears if rational expectations are formed regarding variance: $m = 1$

The expected return of L is the usual expression with rational expectations:

$$E_t(r_L) = r + \frac{\gamma}{P_{Lt}}\left[Cov\left(X_{L(t+1)}, n_s^* X_{S(t+1)} + n_L^* X_{L(t+1)}\right)\right] \quad (2.11)$$

To obtain the expected return on the market portfolio, multiply (2.10) by $\frac{n_S^* P_{St}}{n_S^* P_{St} + n_L^* P_{Lt}}$ and (2.11) by $\frac{n_L^* P_{Lt}}{n_S^* P_{St} + n_L^* P_{Lt}}$ and add the two equations:

$$E_t(r_M) = r + \frac{\gamma}{n_S^* P_{St} + n_L^* P_{Lt}}\left[Var\left(n_s^* X_{S(t+1)} + n_L^* X_{L(t+1)}\right) + n_S^{*2}(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)\right] \quad (2.12)$$

Denoting the price of market portfolio as $P_{Mt} = n_S^* P_{St} + n_L^* P_{Lt}$, the associated next period payoff as $X_{M(t+1)} = n_s^* X_{S(t+1)} + n_L^* X_{L(t+1)}$, and solving (2.12) for γ leads to:

$$\gamma = \frac{(E_t(r_M) - r)P_{Mt}}{Var\left(X_{M(t+1)}\right) + n_S^{*2}(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)} \quad (2.13)$$

Substituting (2.13) in (2.10) leads to:

$$E_t(r_S) = r + [E_t(r_M) - r] \cdot \frac{Cov(r_S, r_M) + \dfrac{n_S^*(1-m)\left(\sigma_{Lt}^2 \dfrac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)}{P_{St}P_{Mt}}}{Var(r_M) + \dfrac{n_S^{*2}(1-m)\left(\sigma_{Lt}^2 \dfrac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)}{P_{Mt}^2}} \qquad (2.14)$$

Substituting (2.13) in (2.11) leads to:

$$E_t(r_L) = r + [E_t(r_M) - r] \cdot \frac{Cov(r_L, r_M)}{Var(r_M) + \dfrac{n_S^{*2}(1-m)\left(\sigma_{Lt}^2 \dfrac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)}{P_{Mt}^2}} \qquad (2.15)$$

(2.14) and (2.15) are the categorization-adjusted CAPM expressions for S and L respectively when variance is the confounded attribute. If there is no confounding of variance, that is, when $m = 1$, the traditional CAPM expression is obtained.

It is straightforward to generalize to the case of Q categories of risky stocks with K stocks (qk with $k = 1,2,3, \dots, K$) plus one exemplar qL in each category q: [4]

$$E_t\left(r_{qk}\right) = r + [E_t(r_M) - r] \cdot \frac{Cov\left(r_{qk}, r_M\right) + \dfrac{n_{qk}^*(1-m)\left(\sigma_{qLt}^2 \dfrac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{P_{qkt}P_{Mt}}}{Var(r_M) + \sum_{q=1}^{Q}\sum_{k=1}^{K} \dfrac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^2 \dfrac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{P_{Mt}^2}} \qquad (2.16)$$

[4] Siddiqi (2018) derives equivalent adjusted-CAPM expressions by assuming that exemplar firms are starting points for analysing other firms with anchoring-and-adjustment heuristic preventing full adjustments. He simply assumes, somewhat unsatisfyingly, that anchoring bias in variance is larger than the anchoring bias in earnings level. In contrast, in this article, we directly utilize the general categorization theory and consider both the upside and the downside of categorization in full generality. The general treatment here allows the two version of CAPM to readily emerge.

$$E_t(r_{qL}) = r + [E_t(r_M) - r] \cdot \frac{Cov(r_{qL}, r_M)}{Var(r_M) + \sum_{q=1}^{Q}\sum_{k=1}^{K} \frac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^2 \frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{P_{Mt}^2}} \quad (2.17)$$

If there is no confounding, then (2.16) and (2.17) converge to the classical CAPM.

It is clear from the above that adjusting CAPM for categorization of firms in investors' brains somewhat changes the CAPM; however, the general form remains the same. To see the impact of the changes more clearly, it is useful to split the adjusted-CAPM into alpha and beta components. This is done next.

2.2 Splitting into Alpha and Beta

Splitting (2.16) into beta (exposure to market) and alpha (excess return not explained by beta) leads to the following expressions for stock k in category q (see appendix A):

$$E_t(r_{qk}) - r = \alpha_{qk} + \beta_{qk}[E_t(r_M) - r] \quad (2.18)$$

where $\alpha_{qk} = \frac{[E_t(r_M)-r]}{(1+c)}(d_{qk} - c)$, $\beta_{qk} = \frac{Cov(r_{qk}, r_M)}{Var(r_M)}$

$$c = \sum_{q=1}^{Q}\sum_{k=1}^{K} \frac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^2 \frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{Var(X_{Mt})}$$

$$d_{qk} = \frac{n_{qk}^{*}(1-m)\left(\sigma_{qLt}^2 \frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{P_{qkt}P_{Mt}Cov(r_{qkt}, r_{Mt})}$$

Similarly, for the category-exemplar (from 2.17), alpha is:

$$\alpha_{qL} = -\frac{[E_t(r_M) - r]}{(1+c)}c \quad (2.18a)$$

By definition, exemplar-firms are the basis around which categories are built. In general, the largest firms in the market get most of investor, analyst, and media attention; hence, are natural category-exemplars for the marginal investor. As earnings-variance scales with size, one expects the exemplar firm to have the largest earnings-variance in its category, which makes d_{qk} (and c) positive:

$$\sigma_{qL}^2(earnings) \geq \sigma_{qk}^2(earnings) \;\; \forall k \; k = 1,2,\ldots\ldots,K$$

$$\Rightarrow \sigma_{qL}^2(EPS)\frac{n_{qL}^{*2}}{n_{qk}^{*2}} \geq \sigma_{qk}^2(EPS)$$

$$\Rightarrow \sigma_{qLt}^2\big(P_{qL(t+1)} + d_{qL(t+1)}\big)\frac{n_{qL}^{*2}}{n_{qk}^{*2}} \geq \sigma_{qk}^2\big(P_{qk(t+1)} + d_{qk(t+1)}\big)$$

$$\Rightarrow d_{qk} > 0$$

The general form of CAPM with categorization is the same as with classical CAPM with appearance of alpha in (2.18) being the only difference. There are several interesting implications of the properties of alpha, and these implications align very well with several well-known anomalies with classical CAPM. One can see betting-against-beta, value effect, as well as an analogue of the size premium in this version of CAPM.

Proposition 1 shows that alpha is higher for a low-beta stock when compared with a high-beta stock. That is, high-alpha is associated with low-beta, and low-alpha is associated with high-beta.

Proposition 1 (high beta is low alpha):

In CAPM adjusted for categorization (when earnings aspect is categorization-relevant), α falls as β rises.

Proof:

$$\alpha = \frac{[E_t(r_M) - r]}{(1+c)}\big(d_{qk} - c\big)$$

where $d_{qk} = \frac{n_{qk}^*(1-m)\left(\sigma_{qLt}^2\frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{P_{qkt}P_{Mt}Cov(r_{qkt}, r_{Mt})}$

$$\Rightarrow d_{qk} = \frac{n_{qk}^{*}(1-m)\left(\sigma_{qLt}^{2}\frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^{2}\right)}{Var(r_{Mt})P_{qkt}P_{Mt}\beta_{qk}}$$

$$\Rightarrow \frac{\partial d_{qk}}{\partial \beta_{qk}} = -\frac{n_{qk}^{*}(1-m)\left(\sigma_{qLt}^{2}\frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^{2}\right)}{Var(r_{Mt})P_{qkt}P_{Mt}{\beta_{qk}}^{2}} < 0 \qquad (2.19)$$

Hence, alpha falls as beta rises and alpha rises as beta falls.

■

Corollary 1.1: Category-exemplars (largest firms) have the lowest alphas in their respective categories

Empirically, intraday, not only alpha falls as beta rises, but the effect is strong enough to make the relationship between intraday average excess return and stock beta negative (Savor and Wilson 2014 ,Hendershott, Livdan, and Rosch 2018).

In version one of CAPM presented here, not only alpha falls as beta rises, but it could quite plausibly fall rapidly enough to make the relationship negative:

$$\frac{\partial\left(E(r_{qk}) - r\right)}{\partial \beta_{qk}} = \frac{[E(r_M) - r]}{(1+c)} \cdot \frac{\partial d_{qk}}{\partial \beta_{qk}} + [E(r_M) - r]$$

$$\Rightarrow \frac{\partial\left(E(r_{qk}) - r\right)}{\partial \beta_{qk}} < 0 \; if \; \left|\frac{\partial d_{qk}}{\partial \beta_{qk}}\right| > 1 + c$$

That is, if $\frac{n_{qk}^{*}(1-m)\left(\sigma_{qLt}^{2}\frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^{2}\right)}{Var(r_{Mt})P_{qkt}P_{Mt}\beta_{qk}^{2}} > 1 + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^{2}\frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^{2}\right)}{Var(X_{Mt})}$

With a little re-arrangement in the L.H.S, the above condition can be expressed as:

$$\frac{1}{\beta_{qk}^{2}}\left\{\sum_{j=1}^{Q}\sum_{i=1}^{K}\frac{n_{ji}^{*2}(1-m)\left(\sigma_{jLt}^{2}\frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{jit}^{2}\right)\frac{P_{ji}}{P_{qk}}}{Var(X_{Mt})}\right\} > 1 + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^{2}\frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^{2}\right)}{Var(X_{Mt})} \qquad (2.20)$$

It is easy to see that (2.20) can hold in the data for a plausible range of parameter values.

It also follows (by straightforward inspection) that in this version of CAPM, category-exemplars (largest firms) have the lowest alphas. That is, they are expected to perform the worst intraday, when the first version is likely to dominate.

Next, we consider the characteristics of a factor that is long in low-beta stocks funded by short-selling high-beta stocks. Suppose the portfolio of low-beta stocks has an alpha of α^L, whereas the portfolio of high-beta stocks has an alpha of α^H.

We construct a betting-against-beta (BAB) factor as:

$$r_t^{BAB} = \alpha^L - \alpha^H \tag{2.21}$$

Proposition 2 describes the predictions of categorization-adjusted CAPM regarding the BAB factor.

Proposition 2 (positive expected return of BAB):

The expected excess return from a self-financing BAB factor is positive

$$\mathbf{E_t(r^{BAB}) = \frac{[E_t(r_M) - r]}{(1 + c)} \cdot (d^L - d^H) \geq 0}$$

and tends to increase in the market risk-premium and the gap between the beta values of low-beta and high-beta portfolios.

Proof:

The alpha of the low-beta portfolio is: $\frac{[E_t(r_M)-r]}{(1+c)}(d^L - c)$. Similarly, the alpha of the high-beta portfolio is: $\frac{[E_t(r_M)-r]}{(1+c)} \cdot (d^H - c)$. Taking expectations in (2.21) and substituting from the above yields: $E_t(r^{BAB}) = \frac{[E_t(r_M)-r]}{(1+c)} \cdot (d^L - d^H)$.

As d falls when β rises, the above expression is positive. In general, larger the gap between β^L and β_H, greater is the distance between d^L and d^H.

■

$\frac{\partial d_{qk}}{\partial \sigma_{qk}^2}$

The results in proposition 2 are similar to the results derived in Frazzini and Pedersen (2014). However, the two approaches are very different. Frazzini and Pedersen (2014) derive these results based on a CAPM framework with borrowing, cash, and margin constraints and here the results follow from categorization of firms when the earnings aspect is categorization-relevant. The empirical support in Frazzini and Pedersen (2014) could be interpreted as support for the version one of CAPM developed here.

Proposition 3 shows that the well-known value effect could potentially be due to categorization as well. The value effect is the finding that value stocks (stocks with low market price relative to fundamentals) tend to outperform growth stocks (stocks with high market price relative to fundamentals).

Proposition 3 (value effect):

Alpha from value stocks is higher than the alpha from growth stocks*.*

Proof:

Follows directly from (2.18) by noting that $\frac{\partial \boldsymbol{d_{qk}}}{\partial \boldsymbol{P_{qk}}} < \mathbf{0}$

■

Proposition 4 shows how alpha varies with payoff volatility.

Proposition 4 (size-effect when quality is controlled):

Alpha is higher for low payoff-volatility stocks

Proof.

$\frac{\partial \boldsymbol{d_{qk}}}{\partial \boldsymbol{\sigma_{qk}^2}} < \mathbf{0}$. That is, alpha falls as payoff-volatility rises. Controlling for quality, small-cap stocks have low payoff-volatility; hence, higher alpha ■

Asness et al (2018) show that size-effect emerges after controlling for quality. Stocks that are safe and profitable are considered quality stocks. Small-cap stocks have smaller prices but that does not automatically translate into smaller payoff-volatility as some small-cap stocks are low quality or junk stocks with uncertain earnings. Smaller prices of small-caps only translate into smaller payoff-volatility if they are of high quality. That is, if they deliver stable earnings. Hence, proposition 4 establishes a size-effect after controlling for quality in a manner consistent with the findings in Asness et al (2018).

2.3 CAPM when discount rate news is categorization-relevant

If relatively more attention is paid to the discount rate aspect while categorizing firms together, then expectations about volatility of earnings are refined due to comparison with the earnings-volatility of the category-exemplar. This is the upside of categorization. However, expectations about earnings level are confounded. This is the downside.

For a firm k in category q, which is categorized with the exemplar-firm L, the upside of categorization is improved expectations (rational expectations) about earnings-volatility. That is, $\sigma_t^2\left(\pi_{qk(t+1)}\right)$ is rational, where $\pi_{qk(t+1)}$ is next period earnings.

Downside of categorization is that earnings-expectations are confounded with the earnings-expectations of the category-exemplar:

$$E_t^C\left(\pi_{qk(t+1)}\right) = mE_t\left(\pi_{qk(t+1)}\right) + (1-m)E_t\left(\pi_{qL(t+1)}\right)$$

where $0 \leq m \leq 1$ captures the degree of confounding. There is no confounding when $m = 1$. The confounding is maximum when $m = 0$.

Essentially following the same steps as in the last section:

$$E_t^C\left(\frac{\pi_{qk(t+1)}}{n_{qk}^*}\right) = mE_t\left(\frac{\pi_{qk(t+1)}}{n_{qk}^*}\right) + (1-m)E_t\left(\frac{\pi_{qL(t+1)}}{n_{qL}^*}\right)\frac{n_{qL}^*}{n_{qk}^*}$$

$$\Rightarrow E_t^C\left(EPS_{qk(t+1)}\right) = mE_t\left(EPS_{qk(t+1)}\right) + (1-m)E_t\left(EPS_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*}$$

Assuming that investors consider next period price (inclusive of dividends) to be some function of next period EPS:

$$E_t\left(\left(P_{qk(t+1)} + D_{qk(t+1)}\right)^C\right) \approx mE_t\left(P_{qk(t+1)} + D_{qk(t+1)}\right) + (1-m)E_t\left(P_{qL(t+1)} + D_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*}$$

$$\Rightarrow E_t\left(\left(X_{qk(t+1)}\right)^C\right) \approx mE_t\left(X_{qk(t+1)}\right) + (1-m)E_t\left(X_{qL(t+1)}\right)\frac{n_L^*}{n_S^*} \qquad (2.22)$$

where $X = P + D$ has been used above.

By following a similar set of steps as in section 2.1, the CAPM expressions for a firm k in category q and the exemplar-firm L in category$q$ are obtained:

$$E_t(r_{qk}) = r + \frac{Cov(r_{qk}, r_M)}{Var(r_M)}\left[(E_t(r_M) - r) + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^*(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{Mt}}\right] - \frac{(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{qkt}} \qquad (2.23)$$

$$E_t(r_{qL}) = r + \frac{Cov(r_{qL}, r_M)}{Var(r_M)}\left[(E_t(r_M) - r) + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^*(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{Mt}}\right] \qquad (2.24)$$

As expected, the classical CAPM expression is obtained from (2.23) and (2.24) if there is no confounding: $m = 1$.

Splitting (2.23) into alpha and beta (see appendix B):

$$E_t(r_{qk}) - r = \alpha_{qk} + \frac{Cov(r_{qk}, r_M)}{Var(r_M)} \left[(E_t(r_M) - r)\right] \tag{2.25}$$

$$\alpha_{qk} = h_t - e_{qkt}$$

$$h_t = \sum_{q=1}^{Q} \sum_{k=1}^{K} \left\{ \frac{n_{qk}^*(1-m)\left(E_t\left(X_{qL(t+1)}\right) \frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right) \right)}{P_{Mt}} \right\}$$

$$e_{qkt} = \frac{(1-m)\left(E_t\left(X_{qL(t+1)}\right) \frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right) \right)}{P_{qkt}\beta_{qkt}} > 0$$

For category exemplars: $\alpha = h_t$

Proposition 5 shows that, when discount rate aspect is paid relatively more attention than the earnings aspect, then high beta stocks have high alpha, and low beta stocks have low alpha. That is, alpha and beta move together, creating a steeper relationship between average excess return and beta. Furthermore, category-exemplars (largest firms) have the highest alphas in their respective categories. This is the complete opposite of what happens when earnings aspects is categorization-relevant.

Proposition 5 (High alpha of high beta)

When discount rate aspect is paid relatively more attention than the earnings aspect, alpha increases with beta in a given cross-section.

Proof:

Follows directly by realizing that $\frac{\partial e_{qkt}}{\partial \beta_{qk}} < \mathbf{0}$ ■

Corollary 5.1: Category-exemplars (largest firms) have the highest alphas in their respective categories.

Proposition 5 shows if discount rate aspect is paid more attention, then alpha rises with beta. This is in sharp contrast with the baseline case (when earnings aspect is paid more attention) as in that case alpha falls with beta. On days when macroeconomic announcements about interest rate, inflation, and unemployment are made, discount rate news is naturally in focus, making investors who consider discount-rate aspect to be categorization-relevant marginal. Hence, one expects alpha to rise with beta on such days creating a steeper relationship between average returns and stock beta. This is consistent with empirical evidence (Savor and Wilson 2014). Similarly, discount rate news (coming from other markets that are open) is expected to be more important during the night when the local market is closed. Consistent with the prediction here, Hendershott et al (2018) find that the relationship between average return and beta is strongly positive during the night.

Proposition 6 shows that, if discount rate aspect is paid more attention, then growth stocks are expected to do better than value stocks. Again, this is consistent with empirical evidence in Savor and Wilson (2014).

Proposition 6 (growth effect)

When discount rate aspect is paid more attention than the earnings aspect, stocks with high market prices relative to fundamentals do better than stocks with low market prices relative to fundamentals.

Proof:

Follows directly by realizing that $\frac{\partial e_{qkt}}{\partial P_{qk}} < \mathbf{0}$

■

The two versions of CAPM have quite opposite predictions. In the first version (propositions 1-4), alpha falls with beta, and we observe the value effect and the size premium (controlling for quality). In the second version, alpha rises with beta, and growth stocks do better (propositions 5-6). The two versions represent different clienteles or investor types. The first version corresponds to earnings-focused investors, whereas the second one corresponds to discount-rate focused investors. As discussed earlier, the first version is expected to dominate intraday whereas the second version is expected to dominate overnight. This creates interesting tug-of-war dynamics between the two investor types, which are discussed next.

3. Tug-of-War Dynamics

Lou et al (2018) report a series of intriguing empirical findings:

1) Overnight clienteles are fundamentally different than intraday clienteles, which is based on the robust finding that a hedge portfolio (best overnight performers minus the worst overnight performers) continues to perform well overnight in the future while performing poorly intraday.

2) Size and value are only observed intraday.

3) In general, strategies that do well intraday show opposite results overnight.

4) Momentum returns are earned overnight.

These findings are consistent with CAPM having two versions as developed here, with one version being dominant intraday whereas the other version holding sway overnight. When the local market is closed but other major markets abroad are open, then the discount-rate news is naturally in focus, which makes discount-rate focused investors dominant. Hence, the second version prevails overnight. However, when the market is open, the baseline case in which earnings-focused investors dominate is restored (unless it's a macroeconomic announcement day when interest rate, inflation, or unemployment information is released). This is the source of the tug-of-war dynamics.

As the first version prevails intraday only, value and size effects are intraday phenomena with the effects reversing overnight (partially). This should be true across all strategies and not just for value and size portfolios due to the opposing behavior of the two versions. To illustrate, lets' examine a strategy in which one goes long low-equity-issuance stocks and shorts high-equity-issuance stocks. Intraday (version one of CAPM) this strategy has a positive alpha. This is because in version one: $\frac{\boldsymbol{\partial \alpha_{qk}}}{\boldsymbol{\partial n_{qk}}} < \mathbf{0}$. However, overnight (version two of CAPM) this strategy has a negative alpha because, in version two: $\frac{\boldsymbol{\partial \alpha_{qk}}}{\boldsymbol{\partial n_{qk}}} > \mathbf{0}$. This is exactly what Lou et al (2018) find.

Momentum trading is about buying past winners and shorting past losers. Winners are stocks with the highest past returns and losers are stocks with the lowest past returns. Hence, by definition, momentum traders are discount-rate or return focused. As discount-rate focused investors generally dominate from close-to-open, one expects momentum effect to be an overnight phenomenon. Consistent with this prediction, Lou et al (2018) report that momentum returns are mostly earned overnight.

Proposition 7 shows that discount-rate focused traders have a higher willingness-to-pay than earnings focused traders all else equal.

Proposition 7: Discount-rate focused investors have higher willingness-to-pay than earnings focused investors all else equal.

Proof:

Confounding of earnings-variance of a firm with the category-exemplar lowers an investor's willingness-to-pay:

$$\frac{E_t\left(P_{S(t+1)} + D_{S(t+1)}\right) - \gamma n_s m \sigma_{St}^2 - \gamma n_s (1-m) \sigma_{Lt}^2 \frac{n_L^2}{n_S^2} - \gamma n_L \sigma_{LSt}}{1+r} < \frac{E_t\left(P_{S(t+1)} + D_{S(t+1)}\right) - \gamma n_s \sigma_{St}^2 - \gamma n_L \sigma_{LS}}{1+r}$$

Confounding of expected earnings-level of a firm with the category exemplar increases an investor's willingness-to-pay:

$$\frac{mE_t\left(P_{S(t+1)} + D_{S(t+1)}\right) + (1-m)E_t\left(P_{\boldsymbol{L}(t+1)} + D_{\boldsymbol{L}(t+1)}\right)\frac{n_L}{n_S} - \gamma n_s \sigma_{St}^2 - \gamma n_L \sigma_{LS}}{1+r} > \frac{E_t\left(P_{S(t+1)} + D_{S(t+1)}\right) - \gamma n_s \sigma_{St}^2 - \gamma n_L \sigma_{LS}}{1+r}$$

This is because an exemplar firm is expected to be the largest firm in its category with the highest expected earnings and volatility of earnings as these values generally scale with size.

■

If discount-rate investors are primarily overnight traders whereas earnings focused investors are active intraday, then one expects prices to typically rise overnight from close-to-open and fall intraday between open-to-close. Consistent with this prediction, Kelly and Clark (2011) find this pattern in returns.

4. Discussion and Conclusions

Categorization is the core of cognition and the fuel and fire of thinking. It is the basis of construction of our knowledge of the world, and is critically important in inference and decision-making. In this article, we explore the implications of categorization for CAPM. The defining feature of categorization in the human brain is selective attention in which some aspects in the information environment are paid more attention than others. Such aspects are the basis for categorization.

We argue that, just like other objects or events, firms are also not viewed in isolation. Rather, investors make sense of them within a framework of categories that represent prior knowledge. This involves sorting a firm into a category based on a subset of firm-attributes. Attributes attended-to are refined, whereas the other attributes get confounded with the corresponding attributes of the category-exemplar.

We show that this process gives rise to two versions of CAPM. In one version, the earnings aspect is paid more attention than discount rate aspect, and in the second version, the discount rate aspect is paid relatively more attention than earnings aspect. In the first version, the relationship between excess return and stock beta is flat and it could even turn negative. Profitability of betting-against-beta, value effect, and size-premium controlling for quality arise in this version. In the second version, the relationship between excess return and stock beta is strongly positive and growth stocks do better. We argue that the first version is typically seen intraday, whereas the second version is seen during days of macroeconomic announcements and during the night.

Apart from explaining the changing relationship between excess return and beta, several other predictions of the two-version approach also hold in the data:

1) In general, strategies that do better overnight perform poorly intraday and vice versa.

2) Size and value are primarily intraday phenomena.

3) Momentum returns are earned overnight.

Categorization never turns-off in a heathy brain. So, the classical CAPM is unlikely to be ever observed. However, as discussed earlier, inefficiencies in categorization has been associated with ASD. Perhaps, our best shot at observing CAPM in its full glory is a laboratory experiment with high functioning ASD sufferers.

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Appendix A

Beta-adjusted return from categorization-adjusted CAPM is:

$$\frac{E_t(r_{qk})-r}{\frac{Cov(r_{qk},r_M)}{Var(r_M)}}=[E_t(r_M)-r]\cdot\left\{\frac{1+\frac{n_{qk}^{*}(1-m)\left(\sigma_{qLt}^2\frac{n_{qL}^{*2}}{n_{qk}^{*2}}-\sigma_{qkt}^2\right)}{P_{qkt}P_{Mt}Cov(r_{qk},r_M)}}{1+\sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^2\frac{n_{qL}^{*2}}{n_{qk}^{*2}}-\sigma_{qkt}^2\right)}{Var(X_M)}}\right\} \qquad (A1)$$

It follows that alpha is:

$$\alpha_{qk}=[E_t(r_M)-r]\cdot\left\{\frac{1+\frac{n_{qk}^{*}(1-m)\left(\sigma_{qLt}^2\frac{n_{qL}^{*2}}{n_{qk}^{*2}}-\sigma_{qkt}^2\right)}{P_{qkt}P_{Mt}Cov(r_{qk},r_M)}}{1+\sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^2\frac{n_{qL}^{*2}}{n_{qk}^{*2}}-\sigma_{qkt}^2\right)}{Var(X_M)}}\right\}-[E_t(r_M)-r]$$

$$\Rightarrow \alpha_{qk} = [E_t(r_M) - r] \cdot \left\{\frac{1 + d_{qk}}{1 + c}\right\} - [E_t(r_M) - r]$$

$$\Rightarrow \alpha_{qk} = \frac{[E_t(r_M) - r]}{(1 + c)}\left(d_{qk} - c\right)$$

Appendix B

$$\frac{E_t\left(r_{qk}\right) - r}{\frac{Cov\left(r_{qk}, r_M\right)}{Var(r_M)}} = \left[(E_t(r_M) - r) + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^*(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{Mt}}\right] - \frac{(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{qkt}\beta_{qk}} \qquad (B1)$$

It follows that alpha is:

$$\alpha_{qk} = \left[(E_t(r_M) - r) + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^*(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{Mt}}\right] - \frac{(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{qkt}\beta_{qk}} - (E_t(r_M) - r)$$

$$\Rightarrow \alpha_{qk} = h_t - e_{qkt}$$