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ABSTRACT

We analyze Nash equilibrium in fiscal competition with tax and public investment between symmetric regions. We show that given the opposite strategic nature of tax (strategic complement) and public investment (strategic substitute), there is possibility of multiple equilibria. We find that if strategic substitute effect dominates strategic complement effect, then both regions have first mover advantage in a timing game and simultaneous move Nash equilibrium (early, early) emerges; otherwise sequential move equilibria-(early, late) and (late, early) emerges. Also, sequential move Nash equilibria are Pareto improving than simultaneous move outcome. Lastly, race-to-the-bottom in taxes is restricted in sequential move equilibria.

Keywords: *Capital taxation; Public investment; Tax competition; Joint strategic substitutes; Joint strategic complements*

JEL Code: H25, F21, R50, H41, H73

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1. Introduction

In the fiscal competition for mobile capital, governments compete to attract capital flows using tax (Wilson, 1986; Zodrow and Mieszkowski, 1986; Wildasin, 1988) as well as non-tax instruments[2] (Hindriks, Peralta and Weber, 2008; Dembour and Wauthy, 2009; Pieretti and Zanaj, 2011). There is plenty of empirical evidence for use of tax and non-tax instruments to attract mobile capital across countries and regions within a country. Hauptmeier et al. (2012), analyze the strategic nature of tax and public input competition, using the data from Germany municipal regions. They find that tax rate in a region is inversely affected by public input provision in the other region. To counter the tax rate decrease in a region, other region can decrease the tax rate or can increase its provision for public input good. Benassy-Quere et al. (2007), in the context of European countries, provide empirical support for the joint role of taxes and public input goods in attracting foreign direct investment. They document that the role of productivity enhancing public goods is as large as the role of taxes in determining capital flows. Further, countries, which set high tax rates, can attract capital by providing higher level of public good. Bellak et al. (2009) corroborates the findings of Benassy-Quere et al. (2007), that taxes and infrastructure (telecommunication, electricity, transport and production facilities) are determinants of FDI in central and Eastern European Countries (CEEC). They also show that tax elasticities are decreasing function of infrastructure facilities. This means that for higher infrastructure levels, regions can charge higher taxes and vice-versa. Venkatesan and Verma (2000), in the context of Indian federal system, demonstrates that there is evidence of industrial policy based competition among Indian states, using industrial policy and FDI data from 1991-

[2] One of the widely used instruments apart from taxation is public investment in terms of infrastructure, electricity, law and order conditions, ease of doing business and so forth. These goods are considered to be productivity enhancing and therefore making the region attractive for capital inflows.

97. They document that there is wide variation in tax rates as well as in non-tax incentives[3] provided by state governments, which determine the capital flows to these states and there is positive correlation between the incentives provided by the governments and inflow of capital. It is evident from these empirical studies that both tax rates and public investments provided by the governments are important factors in determining allocation of mobile capital across regions.

On the other hand, the theoretical literature on tax and non-tax instruments in regional competition for mobile capital is relatively new and sparse. One of the key issues in this literature remains to understand the interaction and interdependence of tax and non-tax instruments and their implication on capital allocation, race-to-the-bottom in tax rates, provision of productivity enhancing public goods and social welfare across regions. A handful of studies have attempted to answer these questions in different settings. Hindriks, Peralta and Weber (2008) analyze the interaction between capital taxation and public investment provision in a sequential choice setting where first public investment choice is made followed by capital taxation. They show that tax competition distorts public investment level and reduces the level of public good provision. In a Hotelling type model of sequential choice of tax and public investment competition, Dembour and Wauthy (2009), find that nature of regional spillover affects the optimal public investment level as well as tax intensity. Pieretti and Zanaj (2011) in an asymmetric regions setting show that, small jurisdictions can compete with larger regions by providing higher level of public input without engaging in tax undercutting. One of the key aspects of these studies is that tax and non-tax (instruments) choices are made in different stages of the competition between the regions. This has led to understanding of only the indirect effect

[3] They characterizing these incentives in three groups: a) financial incentives (investment subsidy; financing of technology, land etc; special packages for large projects), b) fiscal incentives (export based incentives, sales tax incentives, stamp and registration duty subsidy, free electricity etc.) and c) other incentives (single window clearance, road and transport facilities etc.).

of tax and non-tax instruments on each other without direct interaction in level of tax rates and public investment choices. One argument put forward for the same remains that public investment decision being irreversible in nature is taken at first stage followed by tax choice by the regions. But this may not be the case always.

Regions can choose their tax and public investment policies in yearly budgets and can be simultaneously determined. This plausible case in a real world situation remains interesting from multiple perspectives. First, this suggests that tax and non-tax instruments can strategically interact when chosen simultaneously and can affect the optimal levels of each other. To elaborate further, tax competition is strategic complement in nature (à la Bertrand) and public investment competition is considered strategic substitute in nature (à la Cournot) (Kempf and Rota-Graziosi, 2010a; Kempf and Rota-Graziosi, 2010b). A simultaneous choice of them would lead to opposite effect on social welfares of the regions. In other words, we can see a spectrum of outcomes ranging from one effect dominating the other and vice-versa. Second, this setting would do away with the balanced budget conditions for regions. Thus suggesting that there can be possibility of both surplus as well as deficit for the regions while setting both tax and non-tax instruments simultaneously. This is also closer to real world setting.

Given the above discussion, this paper analyzes the nature and consequences of interregional fiscal competition for mobile capital in terms of simultaneous choice of tax rate and level of public investment by the regions. Further, to contribute to the growing literature on endogenous choice of fiscal instruments and strategic timings of regions (Kempf and Rota-Graziosi, 2010a; Ogawa, 2013; Hindriks and Nishimura, 2015), we also endogenize the timings moves by regions in tax and public investment competition.

One study that closely relates to ours is Kawachi et al. (2015), where sequential versus simultaneous choice of tax and non-tax instruments is considered, one at a time and then sequentially. A key difference in their framework is that they consider timing of regions while considering either just one instrument (tax or public investment) or when both instruments are considered public investment choice is still before tax and not jointly. In our setting, we are interested in the joint simultaneous choice of tax and public investment level by a region; and then the effect of sequential timing of regions in deciding the strategies jointly.

The main contributions of our study are as follows. First, we consider the joint and simultaneous decision of tax and public investment as opposed to sequential decision for public investment and taxation as in Hindriks, Peralta and Weber (2008), Dembour and Wauthy (2009) and Pieretti and Zanaj (2011). This helps us in understanding the interaction of tax and public investment strategies and their joint impact on attracting mobile capital. Second, we endogenize the leadership in joint tax and public investment competition by extending the formulation of Kempf and Rota-Graziosi (2010)'s timing game for multiple strategy space. Lastly, but not least, we also contribute to the literature on the nature and classification of strategic effects in competitive environment, based on the seminal work of Bulow, Geanakoplos, and Klemperer (1985). We analyze the joint effect of tax and public investment which individually are of different strategic nature i.e. strategic complements and strategic substitutes respectively. To the best of our knowledge, this is the first paper that analyzes the joint effect of opposite type of strategic choices in a model.

The main findings from our model are as follows. We show that, under plausible parametric conditions, sequential move sub game perfect Nash equilibrium is welfare improving for both the regions. Second, we demonstrate that in tax and public investment competition there is a trade off in tax (strategic complements) and public investment (strategic substitutes) effect and there is possibility of both first mover and second mover advantage. To formally analyze this, we introduce the concepts of joint strategic substitutes (i.e. strategic substitute effect dominates strategic complement effect) and joint strategic complements (i.e. strategic complement effect dominates strategic substitute effect). We find that if joint strategic substitute effect holds, then simultaneous move equilibrium prevails and if joint strategic complements effect holds, then two sequential move equilibria hold and are Pareto superior for both the regions compared to the simultaneous move equilibrium. Third, given that there is possibility of multiple equilibria, we discuss the issue of equilibrium selection using Pareto dominance and risk dominance criteria (Harsanyi and Selten, 1986). We are not able to select any of these sequential equilibria based on risk or Pareto dominance criteria due to symmetric nature of outcomes. Further, we show that in sequential game equilibrium, welfare as well as tax rates in both the regions are higher compared to simultaneous move game. Thus, it restricts the race-to-the-bottom in tax rates.

The rest of the paper is organized as follows. Section 2 explains the basic framework of the model and properties of tax and public investment strategies. Section 3 examines the implications of tax and public investment competition in both simultaneous and sequential moves by the regions. Section 4 deals with the issue of endogenous leadership in case of tax and public investment competition and analyzes the joint strategic effects of multiple strategies. Section 5 concludes with policy implications and discussion.

2. The Basic Model

We consider two regions: region-1 and region-2, competing for foreign owned mobile capital. Total available mobile capital is assumed to be 1, which is exogenously determined. Both the regions strategically decide the level of public investment (g) and the tax rate (t) on investment capital (x) in order to maximize their respective welfare. Public investment (g) facilitates production and thus enhances the productivity of investment capital (x); whereas higher tax (t) discourages capital inflows. Let (g_i, t_i) denote the pair of public investment and tax rate to be chosen by region i, and x_i denote the investment capital attracted in region i, where $i = 1,2$. Following Hindriks, Peralta and Weber (2008), we consider that the production functions of region-1 and region-2 are given by (1) and (2), respectively.

$$F_1(x_1, g_1) = (\gamma + g_1 + \theta g_2)x_1 - \frac{\delta}{2}x_1^2 \quad (1)$$

$$F_2(x_2, g_2) = (\gamma + g_2 + \theta g_1)x_2 - \frac{\delta}{2}x_2^2 \quad (2)$$

In equations (1) and (2), γ (> 0) is the technology parameter, δ (> 0) is the rate of decline in the marginal productivity of capital and θ $(0 \leq \theta \leq 1)$ is the spillover effect of public investment in one region on the other region. Here, $\theta = 1$ $(\theta = 0)$ corresponds to the case of perfect spillover (no spillover). Note that the capital and public investment are complementary in the production function. Clearly, the production functions are increasing, twice continuously differentiable and concave in the level of capital. The provision of productivity enhancing public investment (g) by both regions, involves cost which is assumed to be a convex quadratic function, $c_i(g_i) = \frac{{g_i}^2}{2}$, indicating an increasing marginal cost of provision for public investment.

Following Laussel and Le Breton (1998), Hindriks, Peralta and Weber (2008) and Kempf and Rota-Graziosi (2010a), the objective functions of region-1 and region-2, in terms of social welfare, are as follows:

$$W_1 = \left(F_1 - x_1 F'_{1,x_1}(x_1, g_1)\right) + (t_1 x_1) \quad - \left(\frac{g_1^2}{2}\right) = \frac{\delta}{2} x_1^2 + \quad (t_1 x_1) \quad - \left(\frac{g_1^2}{2}\right) \qquad (3)$$

$$W_2 = \left(F_2 - x_2 F'_{2,x_2}(x_2, g_2)\right) + (t_2 x_2) \quad - \left(\frac{g_2^2}{2}\right) = \frac{\delta}{2} x_2^2 + (t_2 x_2) \quad - \left(\frac{g_2^2}{2}\right) \qquad (4)$$

Each region is maximizing the sum of return to immobile factors and tax revenue, net of the cost of public investment provision. The regions are maximizing only returns on immobile factors and tax revenue, since capital is assumed to be foreign owned.[4] In our model, we assume that $0 \leq \theta \leq 1$ and $\delta > \frac{(1-\theta)^2}{2}$. They lead to satisfaction of the non negativity constraints on outcomes of public investment, tax rates, capital, and welfare level. Also, second order and stability conditions of the model are satisfied with these conditions.

We assume that capital market is perfectly competitive and there is no arbitrage possibility. Therefore, capital market clearance condition can be written as follows.

$$F'_{1,x_1}(x_1, g_1) - t_1 = F'_{2,x_2}(x_2, g_2) - t_2 > 0 \qquad (5)$$

$$x_1 + x_2 = 1 \qquad (6)$$

[4] Laussel and Le Breton (1998) argues that such objective functions can also be justified by considering that majority of the citizens are consumers (labour) and not investment capital owners. Therefore the regions are concerned about the welfare of the representative median voter. As highlighted by one of the reviewers, in case of our model, explicit modeling of multidimensional policy space and political set up would be tedious. De Donder et al. (2012) discuss the majority voting rules in context of multidimensional policy spaces.

Condition (5) states that net marginal returns to capital for both the regions should be equal for capital market clearing. Further, marginal return to capital is considered to be positive, leading to condition (6) i.e. all the capital is allocated between the regions and no idle capital is available.

In the next sub-section, we provide the properties of tax rate and public investment in the context of fiscal competition between regions.

2.1 Characterizing fiscal competition between regions

Before moving on to solve the model, we provide some key characteristics of the strategic variables i.e. tax rate and public investment.

In our fiscal competition model, at the first stage, regions decide their strategic variables in a non-cooperative manner, either simultaneously or sequentially. In the second stage, the foreign capital owners based on these decisions allocate their capital in either of the regions.

Note that, irrespective of whether the two regions are engaged in simultaneous or sequential competition, stage 2 equilibrium would remain the same. To solve this stage we optimize (5) with constraints (1), (2) and (6). The stage 2 equilibrium capital allocation is as follows:

$$x_1 = \frac{\delta + (1-\theta)(g_1 - g_2) - t_1 + t_2}{2\delta} \tag{7a}$$

$$x_2 = \frac{\delta + (1-\theta)(g_2 - g_1) - t_2 + t_1}{2\delta} \tag{7b}$$

For any given tax rate and level of public investment, amount of mobile capital allocated in region-1 and region-2 are given by (7a) and (7b), respectively. Clearly, we can see that, $\frac{\partial x_i}{\partial t_i} < 0, \frac{\partial x_i}{\partial t_j} > 0 \ \forall \ i \neq j$, i.e. higher tax rate in a region leads to lower capital allocation in that region and higher capital allocation in the other region. Also note that if $\theta \neq 1$, i.e., if spillover of public investment is not perfect, $\frac{\partial x_i}{\partial g_i} > 0, \frac{\partial x_i}{\partial g_j} < 0 \;\; \forall \; i \neq j$. It implies that public investment in a region positively (negatively) affects level of capital allocated to that region (other region), unless the spillover effect of public investment is perfect. Therefore, it is evident that, tax rate and level of public investment of a region have opposing effects on the inflow of investment capital in that region, as argued before. We can also see that, if tax rate in region-2 (say) decreases, then region-1 has an option not to decrease its tax rate and instead spend more on public investment. This indicates that the regions have alternate choices in the presence of public investment, other than entering into a tax undercutting war leading to a race-to-the-bottom. We discuss this issue in subsequent section.

Now, we consider (a) the properties of tax rates and (b) properties of public investment, in stage 1. Substituting the expressions for x_1 and x_2 from (7a) and (7b) in (3) and (4), we obtain $W_1(.)$ and $W_2(.)$. From the first order condition with respect to tax rates, we get the tax reaction functions of region-1 and region-2, as follows:

$$TR_1: \;\; t_1(t_2) = \frac{1}{3}(\delta + (1-\theta)(g_1 - g_2) + t_2) \tag{8a}$$

$$TR_2: \;\; t_2(t_1) = \frac{1}{3}(\delta - (1-\theta)(g_1 - g_2) + t_1) \tag{8b}$$

Based on tax reaction functions, we characterize some properties of tax rates.

Lemma 1 (Complements) -Tax rates are complements in nature: *Welfare of region-1 is increasing in t_2 and welfare of region-2 is increasing in t_1.*

Proof: See appendix A1.

This means that if region-1 (say) increases tax rate, region-2's welfare increases due to higher capital flow to region-2.

Lemma 2 (Strategic complements): *Tax rates are strategic complements and reaction functions of the regions are positively sloped.*

Proof: See appendix A2.

Therefore, interregional fiscal competition in terms of tax rates is à la Bertrand in nature. A decrease in tax rate in region-1 induces the region-2 to set lower tax rate and vice versa. Also, note that lower tax rate in one region, compared to that of its rival region, leads to higher capital flow in that region and that, in turn, leads to higher welfare of that region, ceteris paribus. It implies that, in case of tax competition, there is a possibility of race-to-the-bottom in tax rates.

Given the strategic complements nature of tax rates, we can show that in a sequential move tax competition game, regions will have second mover advantage (Kempf and Rota-Graziosi, 2010a).

Proposition 1 (Second mover advantage): *Under lemma 1 and Lemma 2, region-1 as well as region-2 always prefers to be the follower, rather than a leader, in case of sequential move tax competition.*

Proof: See appendix A3.

Proposition 1 implies that both the regions have unilateral incentive to become follower in a tax competition game.

Next, we consider the properties of public investment in a fiscal competition game. Let us first consider that, given the tax rates, the two regions decide their levels of public investments simultaneously and independently. As before, substituting the expressions for x_1 and x_2 from (7a) and (7b) in (3) and (4), we obtain $W_1(.)$ and $W_2(.)$. Differentiating the welfare functions $W_1(.)$ and $W_2(.)$ with respect to public investment level g_1 and g_2 respectively, we get the resultant public investment reaction functions of region-1 and region-2, respectively, as follows.

$$G1:\ \ g_1(g_2) = \frac{(1-\theta)(\delta-(1-\theta)g_2+t_1+t_2)}{4\delta-(1-\theta)^2} \tag{9a}$$

$$G2:\ \ g_2(g_1) = \frac{(1-\theta)(\delta-(1-\theta)g_1+t_1+t_2)}{4\delta-(1-\theta)^2} \tag{9b}$$

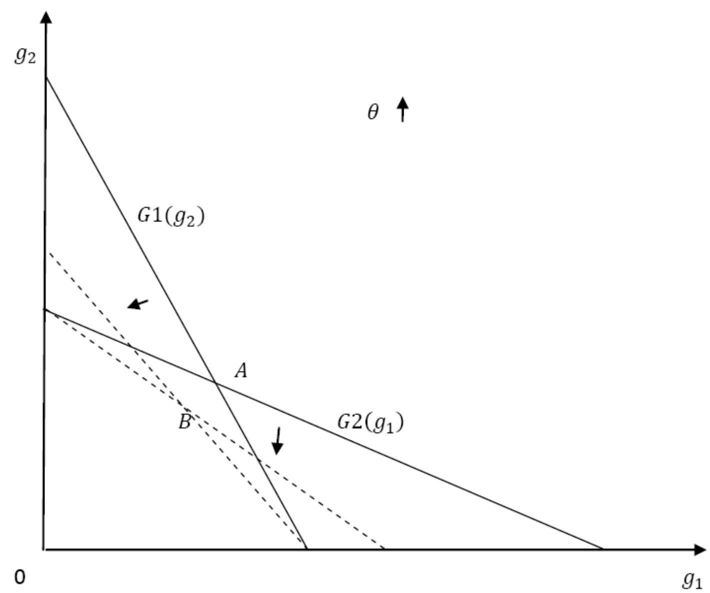

Figure 1: Public investment reaction functions

It is easy to check that the reactions functions are negatively sloped and nature of competition is à la Cournot. We depict the public investment reaction functions of the two regions in Figure 1. Note that the δ parameter affects the intercept as well as slope term in the reaction function and causes outward shifts of the reaction functions. Higher δ leads to outward shift of the reaction functions and higher spending on public investment and lower welfare. If θ increases, the reaction functions rotates inwards, as in figure 1.

Now, we show some important characteristics of public investment in the given framework, which are useful for further analysis.

Lemma 3 (Substitutes): Levels of public investment are substitutes. W*elfare of region-1,* W_1*, is decreasing in* g_2 *and welfare of region-2,* W_2*, is decreasing in* g_1*.*

Proof: See appendix A4.

Lemma 4 (Strategic Substitutes): *Levels of public investments are strategic substitutes and the corresponding reaction functions of the two regions are downward sloping.*

Proof: See appendix A5.

Now, we show that if regions engage in public investment competition in a sequential move game, then regions prefer to be the leader compared to being the follower.

Proposition 2 (First mover advantage): *Under Lemma 3 and Lemma 4, both the regions always prefer to be the leader rather than a follower in interregional fiscal competition in terms of public investments, given the exogenously determined tax rates.*

Proof: See appendix A6.

Clearly, we can say that both the regions prefer to be the leader in a public investment competition game, because of the strategic substitute nature of the public investment.

In the next section, we analyze the combined effect of joint choice of tax and public investment in a simultaneous as well as sequential move fiscal competition game.

3. Fiscal Competition: tax and public Investment

In this section, we analyze multidimensional nature of fiscal competition for mobile capital between the regions. Having understood the strategic nature of both tax and public investment, the key question we intend to answer is that, when each region has two strategies (tax rate and level of public investment) of opposite nature, what would be the outcome of fiscal competition, in terms of tax rates, provision of public good as well as regional social welfare. Further, if timing is involved in fiscal competition, will a region prefer to be the leader or the follower? What would be the Nash equilibrium in the timing game? To answer these questions, we solve the simultaneous as well as sequential game.

The stages of the multidimensional competition game involved are as follows.

Stage 1: Region-1 and region-2 decide the respective tax rates (t_1, t_2), and amount of public investment (g_1, g_2). Each region decides its tax rate as well as the level of public investment at the same time.

a) If the regions move simultaneously, then tax rates and levels of public investments are decided by both the regions simultaneously and independently.

b) Alternatively, if regions move sequentially, leader region (say region-1) decides its tax rate and level of public investment first, and the follower region (say region-2) decides its tax rate and level of public investment next.

Stage 2: Mobile capital is allocated between the regions through a perfectly competitive capital market, based on their respective tax rates and public investment levels.

We solve the game using the standard backward induction method, considering (a) simultaneous move game and (b) sequential move game, in stage 1, separately.

3.1 Simultaneous move game

Note that, in this case also allocation of mobile capital in stage 2 is given by (7a) and (7b) from the previous section.

$$x_1(t_1, t_2, g_1, g_2) = \frac{\delta + (1-\theta)(g_1 - g_2) - t_1 + t_2}{2\delta} \tag{7a}$$

$$x_2(t_1, t_2, g_1, g_2) = \frac{\delta + (1-\theta)(g_2 - g_1) - t_2 + t_1}{2\delta} \tag{7b}$$

Now, from (7a), (7b), (3) and (4) we get $W_1(t_1, g_1)$ and $W_2(t_2, g_2)$, respectively. Therefore, the problem of the region-1 and the region-2 can be written as follows:

$$\underset{t_1, g_1}{\text{Max}}\ W_1(.) = \frac{1}{8\delta}\big((-4\delta + (1-\theta)^2)g_1^2 - (1-\theta)\big)g_1(2\delta - 2(1-\theta)g_2 + 2t_1 + 2t_2)$$
$$+ (\delta + (-1+\theta)g_2 - t_1 + t_2)(\delta + (-1+\theta)g_2 + 3t_1 + t_2)$$

$$\underset{t_2, g_2}{\text{Max}}\ W_2(.) = \frac{1}{8\delta}((1-\theta)^2 g_1^2 + (-4\delta + (1-\theta)^2)g_2^2 + (-1+\theta)g_2(-2\delta - 2t_1 - 2t_2) + (-\delta - t_1 - 3t_2)(-\delta$$
$$- t_1 + t_2) + (-1+\theta)g_1(2\delta - 2(-1+\theta)g_2 + 2t_1 + 2t_2))$$

The first order conditions for maximization of the above two objectives are as follows:

$$\frac{\partial W_1}{\partial t_1} = 0 \Longrightarrow t_1 = \frac{(\delta + (1-\theta)(g_1 - g_2) + t_2)}{3} \tag{10a}$$

$$\frac{\partial W_2}{\partial t_2} = 0 \Longrightarrow t_2 = \frac{(\delta - (1-\theta)(g_1 - g_2) + t_1)}{3} \tag{10b}$$

$$\frac{\partial W_1}{\partial g_1} = 0 \Longrightarrow g_1 = \frac{(1-\theta)(\delta - (1-\theta)g_2 + t_1 + t_2)}{4\delta - (1-\theta)^2} \tag{11a}$$

$$\frac{\partial W_2}{\partial g_2} = 0 \Longrightarrow g_2 = \frac{(1-\theta)(\delta - (1-\theta)g_1 + t_1 + t_2)}{4\delta - (1-\theta)^2} \tag{11b}$$

The values of the capital, tax rates, public investment and welfare for both the regions in symmetric sub game perfect Nash equilibrium, are as follows:

$$t_{1N} = t_{2N} = \frac{\delta}{2};\ g_{1N} = g_{2N} = \frac{1-\theta}{2};\ \ x_{1N} = x_{2N} = \frac{1}{2} \text{ and } W_{1N} = W_{2N} = \frac{1}{8}(3\delta - (1-\theta)^2) \tag{12}$$

Since the two regions are symmetric, each region gets equal share of mobile capital in equilibrium. Tax rates and levels of public investments are also same in the two regions.

3.2 Sequential move game

Coming to the case of sequential move game, we intend to complement and extend the analysis of Kempf and Rota-Graziosi (2010a) who demonstrates that in a sequential move pure tax competition for mobile capital, both the regions levy higher tax rates compared to that of simultaneous move pure tax competition. They argue that due to change in the timing of the moves, regions improve their welfare level and there is restriction in race-to-the-bottom in tax rates. Does this result hold in case of multidimensional fiscal competition? We attempt to answer this question.

The present analysis is particularly interesting because tax rates and levels of public investments are of opposite strategic nature. If both the strategies were strategic complements, it would have been easy to see that the results of Kempf and Rota-Graziosi (2010a) were likely to hold even in case of tax and public investment competition. However, it is not straightforward to understand the implications, when strategies are of opposite nature.

Without any loss of generality, we assume that region-1 is leader and region-2 is follower. Needless to say that equilibrium outcome of stage 2 remains the same as before. Now, in stage 1, we solve the problem of the follower region first, using backward induction method.

We get the reaction functions of follower (region-2) from first order conditions- (10b) and (11b). Now, we solve the problem of leader (region-1). The leader anticipates the strategies of the follower region and includes the reaction function of the follower in its problem, then chooses for the optimal tax rate and public investment to maximize its regional welfare.

$$\underset{t_1,g_1}{\mathrm{Max}}\, W_1(t_1,t_2,g_1,g_2)$$

$$= \frac{1}{32\delta}\Big(4(-4\delta+(1-\theta)^2)g_1^2 - 4(-1+\theta)g_1\big(2\delta+2(-1+\theta)g_2+2t_1+2t_2\big)$$

$$+ \ \big(2\delta+\ 2(-1+\theta)g_2-2t_1+2t_2\big)\big(2\delta+2(-1+\theta)g_2+6t_1+2t_2\big)\Big)$$

subject to the constraints (10b) and (11b).

Solving the above problem, we get the equilibrium level of public investment and tax rate of leader as follows:

$$g_{1L}{}^{*} = \frac{(2\delta-(1-\theta)^2)(1-\theta)}{(5\delta-3(1-\theta)^2)} \quad \text{and} \quad t_{1L}{}^{*} = \frac{(2\delta-(1-\theta)^2)^2}{(5\delta-3(1-\theta)^2)} \qquad (13a)$$

Substituting the equilibrium values of leader region's tax rate and public investment in (10b) and (11b), we get the optimal level of public investment and the tax rate of the follower region as follows:

$$g_{2F}{}^{*} = \frac{(3\delta-2(1-\theta)^2)(1-\theta)}{(5\delta-3(1-\theta)^2)} \quad and \; t_{2F}{}^{*} = \frac{\delta(3\delta-2(1-\theta)^2)}{(5\delta-3(1-\theta)^2)} \tag{13b}$$

The corresponding regional welfare and capital allocation, in equilibrium, are

$$W_{1L}{}^{*} = \frac{(2\delta-(1-\theta)^2)^2}{2(5\delta-3(1-\theta)^2)}\;,\; W_{2F}{}^{*} = \frac{(3\delta-(1-\theta)^2)(-6\delta+4(1-\theta)^2)^2}{8(5\delta-3(1-\theta)^2)^2}; \tag{13c}$$

$$x_{1L}{}^{*} = \frac{(2\delta-(1-\theta)^2)}{(5\delta-3(1-\theta)^2)}, \qquad x_{2F}{}^{*} = \frac{(3\delta-2(1-\theta)^2)}{(5\delta-3(1-\theta)^2)} \tag{13d}$$

To satisfy the stability condition, we need $\delta > \frac{(1-\theta)^2}{2}$, and for the non negativity constraints we must have $\delta > \frac{2(1-\theta)^2}{3}$ and $0 \leq \theta < 1$. Therefore, we assume that $\delta > \frac{2(1-\theta)^2}{3}$ and $0 \leq \theta < 1$.

In the next section, we use the results of this section from the simultaneous and sequential move games, to solve a timing game between the regions to endogenize the choice of leadership.

4. Endogenizing leadership in fiscal competition: a timing game

Now, we turn to endogenize the regions' decision to be a leader or a follower in case of sequential move multidimensional fiscal competition. First, we analyze whether regions are better off in case of sequential move competition than in case of simultaneous move competition and address the issue of endogenous leadership, by considering a timing game. Next, we further analyze the interplay of the two strategic variables – tax rate and public investment and offer explanation for the equilibrium outcomes of the timing game. In the timing game, in the initial stage (say, stage 0), both the regions simultaneously and independently decide whether to be the leader (early) or to be the follower (late). That is, in stage 0, each region decides whether to move *early* or *late*.

If both the regions decide to move *early* or they both decide to move *late* then, the game becomes a simultaneous move game in stage 1. Alternatively, if one region opts to move *early* and the other region opts to move *late*, we have a sequential move game (i.e. Stackelberg game) in stage 1. The player who moves *early* becomes leader and the player moving *late* is follower. Finally, in stage 2, allocation of mobile capital between the two regions is determined through competitive capital market as in section 3. Since the decision in stage 0 is about the time of move, we refer this extended game as a timing game.

We solve the above mentioned timing game by standard backward induction method. Note that the stage 2 equilibrium outcomes would be same as that in Section 3. In stage 1, (a) the equilibrium outcomes are same as given by (12), if both the regions decide to move *early* or both decide to move *late* in stage 0; (b) the equilibrium outcomes are as given by (13c), if one region decides to move *early* and the other region decides to move *late* in stage 0. Note that, since regions are symmetric, the subscripts 1 and 2 (in 1L and 2F) are interchangeable in (13c).

Now, we can represent stage 0 of the timing game in normal form as follows.

		Region-2	
		Early	Late
Region-1	Early	W_{1N}, W_{2N}	W_{1L}, W_{2F}
	Late	W_{1F}, W_{2L}	W_{1N}, W_{2N}

Table 1: A timing game

Comparing the payoffs of the above game corresponding to alternative pairs of strategies, we observe that

(a) if $\delta_1 < \delta \leq \delta_2$ and $0 \leq \theta < 1$ then, $W_{iL} > W_{iN} = W_{jN} > W_{jF}$; and

(b) if $\delta > \delta_2$ and $0 \leq \theta < 1$ then, $W_{jF} > W_{iL} > W_{iN} = W_{jN}$,

where $\delta_1 = \frac{2}{3}(1-\theta)^2$ and $\delta_2 = (1-\theta)^2$ and $i \neq j = 1, 2$. Note that, by assumption, $\delta > \delta_1$ and $0 \leq \theta < 1$. Therefore, δ_2 is the critical value of interest. Note that, welfare of any region i ($i = 1$, 2) can be expressed as $W_i = \frac{\delta x_i^2}{2} + t_i x_i - \frac{g_i^2}{2}$. Therefore, $\delta = \frac{\partial^2 W_i}{\partial x_i^2}$. That is, we can interpret δ as the rate of increase in marginal welfare of a region due to change in allocated capital in that region. So, higher is the value of δ, higher is the increase in welfare (both at level and in marginal terms) due to increase in capital flow to a region. In other words, regions value the mobile capital more, if δ is higher.

So, if the regions value mobile capital less than a critical level and the spillover effect of public investment is less than perfect, i.e., when $0 \leq \theta < 1$ and $\delta_1 < \delta \leq \delta_2$, the leader gets higher payoff (welfare) compared to the simultaneous move game outcome as well as the follower region's payoffs, but the follower gets lesser payoff compared to the simultaneous case. Clearly, in this case there is first mover advantage. So both the regions would want to become the leader in sequential game. In such a situation, the game in stage 1 turns out to be simultaneous move game, since in stage 0 the equilibrium strategy pair of the two region is (*early, early*).

Alternatively, if the regions value mobile capital more than a critical level and the spillover effect of public investment is less than perfect, i.e., when $0 \leq \theta < 1$ *and* $\delta > \delta_2$, both the leader and follower get higher payoffs compared to the simultaneous move situation. So both the regions prefer to have a sequential move game. In this case, the follower's welfare is higher compared to the leader's welfare. Clearly, there is a second move advantage in this case. So, each region wants to become a follower. But, even if one region becomes leader, it attains a higher welfare level compared to the simultaneous move game. If one region opts to move *early*, then other region prefers to move *late* and vice versa. In this situation there are two subgame perfect Nash equilibria- (*late, early*) and (*early, late*). We summarize these results in the following proposition.

Proposition 3: *In the timing game with two choice variables of each region, there are three equilibria as follows.*

a) If $0 \leq \theta < 1$ *and* $\delta_1 < \delta \leq \delta_2$ *, there is only one subgame perfect Nash equilibrium,(early , early).*

b) If $0 \leq \theta < 1$ *and* $\delta > \delta_2$*, there are two subgame perfect Nash equilibria, (late, early) and (early, late). These are also Stackelberg equilibria of the game and are Pareto improving equilibria for both the regions.*

We have shown that, since tax rates are strategic complements, there is second mover advantage in pure tax competition (Proposition 1). On the other hand, since levels of public investments are strategic substitutes, there is first mover advantage in case of pure public investment competition (Proposition 2). Now, Proposition 3 implies that, in case of competition in tax rates and levels of public investments, if the regions value mobile capital less than a critical level ($\delta \leq \delta_2$), regions have a first mover advantage and the resultant game is simultaneous move game. In contrast, if the regions value mobile capital more than a critical level ($\delta > \delta_2$), regions have the second mover advantage and the resultant game is a sequential move game. Therefore, it is not necessary that there will be second mover advantage for at least one region and sequential move game need not necessarily be Pareto superior to simultaneous move game, if we allow for larger strategy space that includes both tax rate and level of public investment. Clearly, the results of Kempf and Rota-Graziosi (2010a), where pure tax competition has been considered, emerge as a special case in the present analysis. In case of multidimensional fiscal competition for mobile capital, whether Stackelberg equilibrium will emerge as the sub game perfect Nash equilibrium or not, that depends on the rate of change in marginal welfare of the regions due to change in capital allocation.

Note that, we can say that whether a region is a leader (loses in tax rates decision and gains in public investment decision) or a follower (loses in public investment decision and gains in tax rate decision), it loses in one strategic variable's decision and gains in the other. It indicates that the relative strength of the two strategic effects, strategic substitute effect via public investment and strategic complement effect via tax rate, determines the equilibrium of the timing game.

Therefore, in order to understand the mechanism behind the changes in the equilibrium as the magnitude of the parameter δ changes, we need to examine the interplay between the strategic complement effect of tax rates and strategic substitute effect of levels of public investments. We suspect that when strategic substitutes effect dominates strategic complement effect, we get the first equilibrium *(early, early)* because of first mover advantage; on the other hand, when strategic complements effect is stronger than the strategic substitute effect, there is second mover advantage and we get two Stackelberg equilibria, (*late, early*) and (*early, late*). These results can be compared with industrial organization literature (Hamilton and Slutsky, 1990; Robson, 1990). In case of endogenous timing price (quantity) competition game between two firms, Stackelberg (Cournot) outcome is Pareto superior for both the both firms. But our results differs from them in the context of strategy space (we consider two strategic variables, instead of only one strategic variable).

In the next sub-section, we untangle the joint effect of two strategies. We note here that, to the best of our knowledge, in the relevant literature, the nature and impact of joint effect of two or more number of opposite strategies has not been discussed so far. Therefore, from a theoretical point view as well, it is important to formalize the concept of joint effect of multiple strategies, which are of opposite nature, and to examine the implication of joint effect of multiple strategies on equilibrium outcomes.

4.1 Joint strategic substitute and joint strategic complement

Let us first define the 'Joint strategic substitute' and 'Joint strategic complement' as follows.

Definition: In case of multidimensional fiscal competition between players, i.e., when players can choose more than one variable (of opposite nature) strategically, if strategic substitute effect dominates the strategic complements effect, the joint nature of strategic variables is '*Joint Strategic Substitute* (JSS)' and effect being JSS effect (JSSE). On the other hand, if strategic complement effect dominates strategic substitute effect, the joint nature of strategic variables is '*Joint Strategic Complement* (JSC)' and effect being JSC effect (JSCE).

Now, in the multidimensional competition game, there are two choices where regions either prefer to become leader or follower. Clearly, the joint effect of the two strategic variables, tax rate and level of public investment, is the difference between the leader and the follower region's payoffs in the sequential move game. We can express the joint effect (JE) of the two strategic variables on region i's (i = *1, 2*) welfare as follows.

$$JE_i = W_i\left(g_{iL}, t_{iL}, g_{jF}, t_{jF}\right) - W_i\left(g_{iF}, t_{iF}, g_{jL}, t_{jL}\right);\ i, j = 1, 2,\ \ i \neq j \tag{14}$$

Without any loss of generality, let us consider that i =*1* and j =*2*. Now using the welfare function of region-1, we can decompose the JE_1 as follows.

$$JE_1 = W_1(g_{1L}, t_{1L}, g_{2F}, t_{2F}) - W_1(g_{1F}, t_{1F}, g_{2L}, t_{2L})$$

$$= [W_1(g_{1L}, t_{1L}, g_{2F}, t_{2F}) - W_1(g_{1F}, t_{1L}, g_{2L}, t_{2F})] - [W_1(g_{1F}, t_{1F}, g_{2L}, t_{2L}) - W_1(g_{1F}, t_{1L}, g_{2L}, t_{2F})]$$

$$= [SSE_1] - [SCE_1]$$

Here, in the first part strategy differs in terms of public investment (strategic substitute) and in the second part, it is the tax rates that differ (strategic complement).

Clearly, $JE_1 > 0$, if $SSE_1 > SCE_1$ and a region prefers to be the leader due to first mover advantage of public investment being larger that second mover advantage of tax rate.

Now, plugging the values of welfare we get the following.

$$JE_1 = [\left\{\frac{(-\delta + (1-\theta)^2)(7\delta^2 - 6\delta(1-\theta)^2 + (1-\theta)^4)(1-\theta)^2}{2\delta(5\delta - 3(1-\theta)^2)^2}\right\}$$

$$-\left\{\frac{(-\delta + (1-\theta)^2)^2(7\delta^2 - 6\delta(1-\theta)^2 + (1-\theta)^4)}{2\delta(5\delta - 3(1-\theta)^2)^2}\right\}]$$

$$\Rightarrow JE_1 = \frac{(-7\delta^3 + 13\delta^2(1-\theta)^2 - 7\delta(1-\theta)^4 + (1-\theta)^6)}{2\delta(5\delta - 3(1-\theta)^2)^2}$$

It is easy to check that,

(a) $JE_1 > 0$, if $\delta < \delta_2 = (1-\theta)^2$.

(b) $JE_1 = 0$, if $\delta = \delta_2 = (1-\theta)^2$.

(c) $JE_1 < 0, if\ \delta > \delta_2 = (1-\theta)^2$.

Therefore, if $\delta < \delta_2$, SSE dominates SCE, i.e., there is JSSE, and the regions prefer to be the leader. In contrast, if $\delta > \delta_2$, SCE dominates SSE, i.e., there is JSCE, and regions prefer to be follower. Note that, this is true for region-2 as well.

Proposition 4: *In the multidimensional fiscal competition game:*

a) Both the regions would have first mover advantage if there is joint strategic substitute effect i.e. SSE >SCE and Nash equilibrium in the timing game would be {early, early};

b) On the other hand regions would have second mover advantage and there would be joint strategic complement effect i.e. SCE > SSE and Nash equilibrium would be either {early, late} or {late, early}, a la Kempf and Rota-Graziosi (2010a).

In appendix A7 of this paper, we also provide an alternative explanation of joint strategic substitute and complement effect using the reaction functions of region's strategies (tax and public investment).

As is clear, the critical value of δ plays a crucial role in determining whether simultaneous or sequential move equilibria would prevail. One plausible explanation for this result is as follows.

The value of δ affects the strategic nature of tax rates and public investment through their reaction functions. In case of tax rates, a higher value of δ, leads to higher intercept of tax reaction function which in turn leads to higher equilibrium tax rates, keeping other things constant. For the public investment, on the other side, there is effect both on intercept and slope. Higher δ leads to regions being more aggressive with over investment as the outcome, keeping other things constant. Keeping these two effects in mind, in the joint choice of tax and public investment, at the lower values of δ $(< \delta_2)$, the strategic substitute effect of public investment remain larger, as compared to strategic complements effect of tax rates and therefore, first mover advantage holds. This leads to {early, early} as the Nash equilibrium of the timing game. On the other hand, for larger values of $\delta (> \delta_2)$, the effect on tax reaction function is larger as compared to public investment reaction functions, thus leading to second mover advantage due to joint strategic effect tilting towards strategic complements .

In the next sub-section, we attempt to resolve the choice of one of the equilibrium in case of multiple equilibria of the timing game.

4.2 Equilibrium Selection: Pareto vs. Risk Dominance

As noted earlier, there are multiple equilibria of the timing game, when JSCE holds, (Proposition 3, Part (b)). Now the question arises, how do we select one of the two equilibria? In the literature on selection of equilibrium, there are two criteria which are widely used to rank them. One is Pareto dominance criterion and another one is risk dominance criterion. Pareto dominance criterion is applicable when at least one player is better off without worsening the others. On the other hand, an equilibrium risk-dominates the other when former is less risky than the latter, that is risk-dominant equilibrium is the one for which product of deviation losses is the largest (Harsanyi and Selten, 1988).

In our case, from Proposition 3 part (b), equilibrium (*early, late*) (region-1 leads and region-2 follows) risk dominates equilibrium (*late, early*) (region-2 leads and region-1 follows), if the former is associated with larger product of deviation losses: $\varphi = (W_{1L} - W_{1N})(W_{2F} - W_{2N}) - (W_{1F} - W_{1N})(W_{2L} - W_{2N}) > 0$. That is, the equilibrium *(early, late)* risk dominates *(late, early)*, if the product of welfare losses of the two regions by deviating from (early, late) is greater than the product of welfare losses of the two regions by deviating from *(late, early),* to the *(early, early)* or *(late, late)*.

Now, since the regions are assumed to be symmetric, $W_{1L} = W_{2L}, W_{1F} = W_{2F}$ and $W_{1N} = W_{2N}$. Therefore, in the present context, we have φ =0. It implies that none of the two equilibrium risk dominates the other. Therefore, we cannot select any one of the two equilibria by the risk dominance criterion. Also, it is straightforward to observe that we cannot select any equilibrium on the basis of the Pareto dominance criterion either.

4.3 Equilibrium outcomes: tax and public investment level

In this subsection, we discuss the level of tax rates and public investment in both regions under alternative scenarios i.e. simultaneous move equilibrium {early, early} and {early, late} or {late, early}.

Table 2: Ranking of equilibrium outcomes

δ	$\delta \leq \delta_2 = (1-\theta)^2$	$\delta > \delta_2 = (1-\theta)^2$
Joint Effect	$JSSE_i$ holds , if $SSE_i > SCE_i$	$JSCE_i$ holds, if $SSE_i < SCE_i$
Nash equilibrium	{early, early}	{early, late} or {late, early}
Welfare	$W_{1L} > W_{1N} = W_{2N} > W_{2F}$	$W_{2F} > W_{1L} > W_{1N} = W_{2N}$
Capital	$x_{1L} \geq x_{1N} = x_{2N} \geq x_{2F}$	$x_{2F} > x_{1N} = x_{2N} > x_{1L}$
Public Investment	$g_{1L} \geq g_{1N} = g_{2N} \geq g_{2F}$	$g_{2F} > g_{1N} = g_{2N} > g_{1L}$
Taxes	$t_{1N} = t_{2N} \geq t_{1L} \geq t_{2F}$	$t_{1L} > t_{2F} > t_{1N} = t_{2N}$

As can be seen from the table 2, in case of {early, early} equilibrium, there is symmetry in the outcomes. Interestingly, tax rates are higher than the case of Stackelberg outcome (leader and follower regions) and thus indicate restrictions in race-to-the-bottom. On the other hand, in case of sequential move equilibrium, tax rates of both the regions are higher than simultaneous game. This indicates that irrespective of the strategy space, sequential move of regions leads to restriction in the race-to-the-bottom (similar to Kempf and Rota-Graziosi, 2010a). Further, leader region's public investment level is lower than the follower region but the reverse order holds for tax rates. In terms of capital mobility, follower region wins over the leader region, with impact of higher public investment overcoming the negative impact of higher tax rates in that region. For further details on the results, please refer to appendix A8 of the paper.

5. Conclusion and discussion

We demonstrate the nature of Nash equilibrium in a multidimensional fiscal competition for foreign owned mobile capital if the strategic choices (tax and public investment) are of opposite nature. We first show that in a pure tax competition game, tax rates are strategic complements and each of the two regions prefers to be follower, because of second mover advantage. In contrast, public investments are strategic substitutes and the regions prefer to be leader in sequential move game.

In a fiscal competition with both tax rate and public investment, we examine the consequences of interplay between these two choice variables. We demonstrate that, since tax rate and level of public investment are of opposite strategic nature, there is a trade off involved. As a result, the possibility of both first mover advantage and second mover advantage exists, unlike in case of uni-dimensional competition. This is a new result.

Next, we formalize the concepts of joint strategic substitute effect and joint strategic complements effect. We show that if there is a joint strategic substitute effect, both the regions prefers to be the leader and, thus, the resultant outcome is simultaneous move Nash equilibrium. On the other hand, if there is a joint strategic complement effect, sequential move equilibrium is Pareto superior than the simultaneous move equilibrium. However, in the latter case there are two sequential move equilibria, which are identical from the point of view of both Pareto dominance and risk dominance criterion.

We also provide ranking of the equilibrium outcomes in alternative scenarios. We show that, if there is joint strategic complements effect, capital allocated and level of public investment in the follower (leader) region is higher than that in case of simultaneous move equilibrium. But, the equilibrium tax rate (welfare) of leader region is higher (lower) than that of the follower region, and equilibrium tax rate and welfare of both leader region and follower region are higher (lower) than that in case of simultaneous move game. These results are similar to that in Kempf and Rota-Graziosi (2010a). In sequential move game, race-to-the-bottom is controlled and further, there is also limited restriction on the overspending on public investment.

For further research some of the extension of this study can be on the following lines. First, whether results of the model is likely to remain robust to increasing the numbers regions competing with each other. For the sake of brevity, we just provide our intuition for the same. In our model, increasing the number of regions, would have two effects from tax and public investment strategies side. On the one hand, large number of regions would intensify the competition in tax rates leading to race-to-the-bottom. On the other hand, in public investment competition, due to spillover effects and strategic substitute nature of public investment, there would be reduction in public investment level of the regions. These two effects when combined would reduce the welfare level of each region in the simultaneous move equilibrium. Now the intuition in a dynamic setting would be slightly tedious to provide. The reason being as follows. In N regions sequential game, one region would be the leader where as N-1 regions would be the follower. Now there would be two sets of effects generated. One, we have already discussed in the earlier case i.e. follower regions would compete with each other in a simultaneous choice game, but leader would have to pre-empt and act according to the competition among the followers as well as based on the strategic effect of followers on his/her strategic choice of tax rates and public investment. A detailed analysis would help, in answering whether sequential effect (leading to higher tax rate) would dominate the simultaneous effect (leading to lower tax rates) or not. Further, a similar argument holds for public investment level also.

Second extension could be in terms of departure from absentee capital owners to resident owners of capital and their impact on outcomes of the model. Ogawa (2013) analyzed the extension of Kempf and Rota-Graziosi (2010a) to the case of heterogeneous capital ownership (from absentee ownership to resident ownership), the author finds that in the case of resident ownership, instead of equilibrium corresponding to Stackelberg outcome, simultaneous move outcome emerges as sub game perfect Nash equilibrium. On the other hand, if there is absentee ownership then Stackelberg outcome would emerge. Later on Kempf and Rota-Graziosi (2015) argued that based on the strategic nature of tax rates, there can be emergence of both simultaneous and sequential outcomes with various capital ownership structure. Keeping this in mind, in our context, we can argue that resident ownership of capital is more likely to tilt results towards simultaneous outcomes as compared to sequential outcome. A detailed analysis would surely help in understanding this scenario better and is left for future research.

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APPENDIX A

A1: Tax rates are complements in nature.

Proof: We have, $W_1 = \left(\frac{\delta x_1{}^2}{2}\right) - \left(\frac{g_1^2}{2}\right) + (t_1 x_1)$ and $W_2 = \left(\frac{\delta x_2{}^2}{2}\right) - \left(\frac{g_2^2}{2}\right) + (t_2 x_2)$. Differentiating W_1 and W_2 with respect to t_2 *and* t_1 respectively, we get

$$\frac{\partial W_1}{\partial t_2} = (t_1 + \delta x_1)\frac{\partial x_1}{\partial t_2} \tag{A1.a}$$

$$\frac{\partial W_2}{\partial t_1} = (t_2 + \delta x_2)\frac{\partial x_2}{\partial t_1} \tag{A1.b}$$

Now, from equation (7a) and (7b), we get $\frac{\partial x_1}{\partial t_2} = \frac{\partial x_2}{\partial t_1} = \frac{1}{2\delta} > 0$. Substituting theses values in (A1.a) and (A1.b), we have, $\frac{\partial W_1}{\partial t_2} = \frac{1}{2\delta}(t_1 + \delta x_1) > 0$ and $\frac{\partial W_2}{\partial t_1} = \frac{1}{2\delta}(t_2 + \delta x_2) > 0$. QED.

A2: (Strategic complements)

Proof: It is easy to check that $\frac{\partial^2 W_1}{\partial t_2\, \partial t_1} = \frac{1}{2}\frac{\partial x_1}{\partial t_2} > 0$ and $\frac{\partial^2 W_2}{\partial t_1\, \partial t_2} = \frac{1}{2}\frac{\partial x_2}{\partial t_1} > 0$. That is, if there is an increase in region-2's (region-1's) tax rate, the negative marginal effect of region-1's (region-2's) tax rate on its own welfare decreases. Therefore, tax rates are strategic complements. From (10a) and (10b), we get the slopes of the tax reaction functions of region-1 and region-2, respectively, as follows, $\frac{\partial t_2}{\partial t_1}|_{\mathrm{T1}} = 3 > 0$ and $\frac{\partial t_2}{\partial t_1}|_{\mathrm{T2}} = \frac{1}{3} > 0$. QED.

A3: (Second mover advantage)

Proof: Since the two regions are symmetric, it is sufficient to show that region-1 always prefers to be the follower in sequential move tax competition. That is, we have to prove that $W_1(t_{1F}, t_{2L}) > W_1(t_{1L}, t_{2F})$, where (a) t_{1F} and t_{2L} are the equilibrium tax rates of region-1 and region-2, respectively, when region-1 is the follower and region-2 is the leader and (b) t_{1L} and t_{2F} are the equilibrium tax rates of region-1 and region-2, respectively, when region-1 is the leader and region-2 is the follower. The proof is as follows (follows from Kempf and Rota-Graziosi, 2010a):

In case of Stackelberg (sequential) competition in tax rates, we have $t_{1N} < t_{1F} < t_{1L}$. (see Kempf and Rota-Graziosi, 2010a for a detailed discussion) and same holds from region-2's perspective.

Now we can say that

$W_1(t_{1F}, t_{2L}) > W_1(t_{1L}, t_{2L}) > W_1(t_{1L}, t_{2F})$

In this, first inequality comes from the definition of Stackelberg equilibrium when region-1 is the follower. The second part of the inequality holds given the fact that $t_{2F} < t_{2L}$ and $\frac{\partial W_1(.)}{\partial t_2} > 0$ (due to complement nature of tax rates from Lemma 1). Thus the region-1 has a second mover advantage in tax rates. Due to the symmetry in the payoff of region-2, the second mover advantage also holds for region-2 also. QED.

A4: (Public investments are Substitutes)

Proof: We have $W_1 = \left(F_1 - x_1 F'_{1,x_1}(x_1, g_1)\right) - \left(\frac{g_1^2}{2}\right) + (t_1 x_1)$ and $W_2 = \left(F_2 - x_2 F'_{2,x_2}(x_2, g_2)\right) - \left(\frac{g_2^2}{2}\right) + (t_2 x_2)$. Using the production function (1) and (2), we can write the following.

$$W_1 = \left(\frac{\delta x_1{}^2}{2}\right) - \left(\frac{g_1^2}{2}\right) + (t_1 x_1)$$

$$W_2 = \left(\frac{\delta x_2{}^2}{2}\right) - \left(\frac{g_2^2}{2}\right) + (t_2 x_2)$$

Now differentiating W_1, W_2 with respect to g_2 *and* g_1 respectively, we get,

$$\frac{\partial W_1}{\partial g_2} = (t_1 + \delta x_1)\frac{\partial x_1}{\partial g_2}$$

$$\frac{\partial W_2}{\partial g_1} = (t_2 + \delta x_2)\frac{\partial x_2}{\partial g_1}$$

Differentiating (7a) and (7b) w.r.t. g_2 *and* g_1, respectively, we get $\frac{\partial x_1}{\partial g_2} = \frac{\partial x_2}{\partial g_1} = \frac{-(1-\theta)}{2\delta} < 0$

Substituting theses values in above equations, we get, $\frac{\partial W_1}{\partial g_2} = \frac{-(1-\theta)}{2\delta}(t_1 + \delta x_1) < 0$ and $\frac{\partial W_2}{\partial g_1} = \frac{-(1-\theta)}{2\delta}(t_2 + \delta x_2) < 0$. QED

A5: (Strategic Substitutes)

Proof: It is easy to check that $\frac{\partial^2 W_1}{\partial g_2 \partial g_1} = \frac{\partial x_1}{\partial g_1}\frac{\partial x_1}{\partial g_2} < 0$ and $\frac{\partial^2 W_2}{\partial g_1 \partial g_2} = \frac{\partial x_2}{\partial g_2}\frac{\partial x_2}{\partial g_1} < 0$. Therefore, levels of public investments are strategic substitutes. Now, using the public investment reaction functions of region-1 and region-2 from (9a) and (9b), we obtain,

$$\frac{\partial g_2}{\partial g_1} = -\frac{(1-\theta)^2}{4\delta - (1-\theta)^2} < 0$$

$$\frac{\partial g_1}{\partial g_2} = -\frac{(1-\theta)^2}{4\delta - (1-\theta)^2} < 0$$

We must have, $2\delta > (1-\theta)^2$ to satisfy the second order, stability and non negativity conditions. So, we can say that reaction curves in public investment choice are downward sloping. QED.

A6: Proposition 2 (First mover advantage)

Proof: Under Lemma 3 and Lemma 4, we can show that both the regions prefer to be the leader.

In case of Stackelberg game for public investment, $g_{1F} < g_{1N} < g_{1L}$ and the same holds for the region-2's public investment level under different conditions. With that in mind, we can say that:

$W_1(g_{1L}, g_{2F}) > W_1(g_{1F}, g_{2F}) > W_1(g_{1F}, g_{2L})$

In this, first inequality comes from the definition of Stackelberg equilibrium when region-1 is the leader. The second part of the inequality holds given the fact that $g_{2F} < g_{2L}$ and $\frac{\partial W_1(.)}{\partial g_2} < 0$ (due to substitute nature of public investment from Lemma 3). Thus the region-1 has a first mover advantage in public investment. Due to the symmetry in the payoff of region-2, the first mover advantage also holds for region-2 also. QED

A7: Alternate explanation of joint effects and its decomposition

To illustrate it further, note that tax reaction functions are strictly upward sloping and public investment reaction functions are strictly downward sloping (see Lemma 2 and Lemma 4). The slopes of tax reaction function in t_1-t_2 plane and the slope of the public investment reaction function in g_1-g_2 plane, of region-2 are, respectively, $\frac{\partial t_2}{\partial t_1} = 3$ and $\frac{\partial g_2}{\partial g_1} = -\frac{4\delta-(1-\theta)^2}{(1-\theta)^2}$. And, the slopes of tax reaction function in t_1-t_2 plane and the slope of the public investment reaction function in g_1-g_2 plane, of region-2 are, respectively, $\frac{\partial t_2}{\partial t_1} = \frac{1}{3}$ and $\frac{\partial g_2}{\partial g_1} = -\frac{(1-\theta)^2}{4\delta-(1-\theta)^2}$. Note that, in t_1-t_2 plane, flatter (steeper) the tax reaction function of region-1 (region-2), greater is the response of region-1 (region-2) for a unit change in its rival's tax rate. Similarly, in g_1-g_2 plane, flatter (steeper) the public investment reaction function of region-1 (region-2), greater is the response of region-1 (region-2) for a unit change in its rival's public investment.

In general, we can write the slope of the tax reaction functions of region-2, in t_1-t_2 plane, as

$$\frac{\partial t_2}{\partial t_1} = -\left(\frac{\frac{\partial^2 W_2}{\partial t_1 \partial t_2}}{\frac{\partial^2 W_2}{\partial {t_2}^2}}\right)$$

On the RHS, the denominator must be negative (due to the second order condition) and the numerator is the strategic complement effect of tax rates, which is positive.

Similarly, we can write the slope of the public investment reaction function of region-2, in g_1-g_2 plane, as $\frac{\partial g_2}{\partial g_1} = -\left(\frac{\frac{\partial^2 W_2}{\partial g_1 \partial g_2}}{\frac{\partial^2 W_2}{\partial {g_2}^2}}\right)$. Here, In the RHS, the denominator must be negative (by the second order condition for maximization) and the numerator is the strategic substitute effect of tax rates, which is negative.

Now, we argue that the relative magnitude of strategic effect of a choice variable, relative to that of the other choice variable, can be shown to be related to the relative magnitudes of the reaction functions.

Proposition 4: *In the sequential move multidimensional competition, the relation between the joint strategic effects and slopes of the reactions functions are as follows.*

a) *Joint Strategic Substitutes Effect (JSSE) holds, if the absolute slope of the public investment reaction function of region-2 (region-1) in g_1-g_2 plane is greater (smaller) than the absolute slope of the tax reaction function of region-2 (region1) in t_1-t_2 plane.*

b) *Joint Strategic Complements Effect (JSCE) holds, if the absolute slope of the public investment reaction function of region-2 (region-1) in g_1-g_2 plane is smaller (greater) than the absolute slope of the tax reaction function of region-2 (region-1) in t_1-t_2 plane.*

Proof: It is sufficient to show that the Proposition 4 is true for any one of the two regions due to symmetric nature. Let us consider region-2's reaction functions. Now, the absolute value of slope of the public investment reaction function of region-2, in g_1-g_2 plane, is greater than the absolute value of the slope of the tax reaction function of region-2, in t_1-t_2 plane, if

$$\left|\frac{\partial g_2}{\partial g_1}\right| > \left|\frac{\partial t_2}{\partial t_1}\right|$$

$$\Rightarrow \left|-\frac{\frac{\partial^2 W_2}{\partial g_1 \partial g_2}}{\frac{\partial^2 W_2}{\partial {g_2}^2}}\right| > \left|-(\frac{\frac{\partial^2 W_2}{\partial t_1 \partial t_2}}{\frac{\partial^2 W_2}{\partial {t_2}^2}})\right|$$

$$\Rightarrow \left(\frac{\frac{\partial^2 W_2}{\partial g_1 \partial g_2}}{\frac{\partial^2 W_2}{\partial {g_2}^2}}\right) > -\left(\frac{\frac{\partial^2 W_2}{\partial t_1 \partial t_2}}{\frac{\partial^2 W_2}{\partial {t_2}^2}}\right)$$

$$\Rightarrow \left(\frac{\frac{\partial^2 W_2}{\partial g_1 \partial g_2}}{\frac{\partial^2 W_2}{\partial {g_2}^2}}\right) + \left(\frac{\frac{\partial^2 W_2}{\partial t_1 \partial t_2}}{\frac{\partial^2 W_2}{\partial {t_2}^2}}\right) > 0$$

$$\Rightarrow \left[\frac{(1-\theta)^2}{4\delta-(1-\theta)^2} - \frac{1}{3}\right] > 0$$

$\Rightarrow \delta < (1-\theta)^2$

$$\Rightarrow \text{SSE} > SCE$$

That is, if for region-2 $\left|\frac{\partial g_2}{\partial g_1}\right| > \left|\frac{\partial t_2}{\partial t_1}\right|$, there is JSSE. Otherwise, if $\left|\frac{\partial g_2}{\partial g_1}\right| < \left|\frac{\partial t_2}{\partial t_1}\right|$, there is JSCE.

QED

Note that, if all the choice variables are strategic complements in nature, JSCE will always hold true. In Kempf and Rota-Graziosi (2010a), each region has one choice variable (tax rate) and tax rates are strategic complements, so there is JSCE and, thus, each region has second mover advantage. However, in general, JSCE effect need not necessarily hold true. In the present model, JSCE holds true, only if the absolute slope of the public investment reaction function of region-2 (region-1) in g_1-g_2 plane is smaller (greater) than the absolute slope of the tax reaction function of region-2 (region-1) in t_1-t_2 plane. Otherwise, JSSE holds true.

A8: Comparison of equilibrium outcomes between simultaneous and sequential game: tax rate, public investment, capital allocation and social welfare

In Table 2, we report the rankings welfare, capital allocation, level of public investment and tax rate of region-1 and region-2, in (*early, late*), (*late, early*) and simultaneous move equilibrium, assuming $0 \leq \theta < 1$, $\frac{2}{3}(1-\theta)^2 = \delta_1 \leq \delta$.

δ	$\delta \leq \delta_2 = (1-\theta)^2$	$\delta > \delta_2 = (1-\theta)^2$
Welfare	$W_{1L} > W_{1N} = W_{2N} > W_{2F}$	$W_{2F} > W_{1L} > W_{1N} = W_{2N}$
Capital	$x_{1L} \geq x_{1N} = x_{2N} \geq x_{2F}$	$x_{2F} > x_{1N} = x_{2N} > x_{1L}$
Pub. Investment	$g_{1L} \geq g_{1N} = g_{2N} \geq g_{2F}$	$g_{2F} > g_{1N} = g_{2N} > g_{1L}$
Taxes	$t_{1N} = t_{2N} \geq t_{1L} \geq t_{2F}$	$t_{1L} > t_{2F} > t_{1N} = t_{2N}$

Table 2: Ranking of equilibrium outcomes

Case 1: $\boldsymbol{\delta \leq \delta_2}$

In this case, tax rates (strategic complement) effect is less than public investments (strategic substitute) effect and, thus, joint strategic substitute effect holds true. So, there is first mover advantage. If region-1 moves *early* and region-2 moves *late*, public investment in region-1 is higher than that in region-2. The reason being that region-1 is more aggressive in setting public investment due to JSSE. But, both leader and follower region's tax rates are lower as compared to the simultaneous move game. The reason is that, in region-1, the capital elasticity of tax rates in the sequential game is higher than simultaneous move game ($e\_seq(x_{t1}) = \frac{(1-\theta)^2}{2\delta} - 1 > e\_sim(x_{t1}) = -\frac{1}{2}$). For region-2 which has the same level of capital elasticity of tax in both the games ($e\_seq(x_{t2}) = e\_sim(x_{t2}) = -\frac{1}{2}$), the level of public investment is lower than the earlier game, so region-2 being a follower charges tax rate even lower than region-1. Moreover region-2 is the follower in tax rates which causes lower tax rate as compared to region-1. In total, the strategic effect of tax rates (SCE) is dominated by strategic effect of public investment (SSE). So, the higher public investment in region-1 over compensates the negative effect of tax rate. This causes higher allocation of capital in region-1 and lower in region-2, compared to the simultaneous move game. These are new results.

Case 2: $\delta > \delta_2$

In this case, we have a Pareto superior situation in the sequential move game, because both the regions are getting higher payoffs compared to their simultaneous game payoffs, irrespective of the role of the region as a leader or a follower. For this case, we have joint strategic complements effect and regions prefer to become the follower. As we can see that region-2 is providing higher level of public investment than region-1. There are higher tax rates because, regions' tax reaction functions shifts outwards due higher δ. So region-1 as well as region-2 charges higher tax rates than the simultaneous move game. Moreover, region-2 is levying lower tax rate than region-1, due to higher sensitivity of capital to tax rates in region-2. This leads to a higher capital allocation to region-2 ($\frac{\partial x_{2F}}{\partial g_{2F}} > 0, \frac{\partial x_{2F}}{\partial t_{2F}} < 0$) as well as higher welfare levels in region-2. These results are similar to Kempf and Rota-Graziosi (2010a), in the sense that sequential game equilibrium is providing higher welfare level for both the regions, irrespective of the role as a leader or a follower.

A9. Discussion on empirical estimation of fiscal competition

To further extend the scope of this study, an empirical investigation of strategic choice of regions in fiscal competition can be undertaken. To empirically estimate the strategic tax competition models, one need to essentially model and estimate the reaction functions of the regions corresponding to tax, public investment or joint choice games either in static (simultaneous) or dynamic (sequential) setting[5]. To test the hypothesis of our model and check whether key parameter threshold of δ in our model, we can focus on equations, 7 (a & b), 10 (a & b) and 11 (a & b). Say from equation 7 (a &b), we can say that the relation between the own tax rates and capital level in a region is negative, keeping other things constant and further the corresponding coefficient estimated would be $\frac{\partial x}{\partial t} = \frac{-1}{2\delta}$. On the other hand for public investment and capital level, the coefficient would depend on regional spillover θ and change in marginal productivity of capital δ. Further, in estimating the strategic interaction of tax rates and public investment, value of δ is estimated through intercept terms and both intercept and coefficient terms respectively. While estimating the empirical model, a few caveats should be kept in mind, in our model we assume symmetry of regions to focus on the strategic interaction more, which may not be true in empirical settings. Further, it may so happen in that in empirical setting, there are both simultaneous and sequential competition outcomes are visible between various pairs of regions/countries and therefore, a more general model need to be structured taking motivation from the existing model. This remains beyond the scope of this paper, but can be analyzed in future research.

[5] For a detailed discussion on estimation and methodological issues, one can refer to Altshuler and Goodspeed (2015).