CAPM: A Tale of Two Versions

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Abstract

Given that categorization is the core of cognition, we argue that investors do not view firms in isolation. Rather, they view them within a framework of categories that represent prior knowledge. This involves sorting a given firm into a category and using categorization-induced inferences to form earnings and discount-rate expectations. If earnings-aspect is categorization-relevant, then earnings estimates are refined, whereas discount-rates are confounded with the category-exemplar. The opposite happens when discount-rates are categorization relevant. Earnings-focused approach, predominantly used by institutional investors, leads to a version of CAPM in which the relationship between average excess return and stock beta is flat (possibly negative). Value effect and size premium (controlling for quality) arise in this version. Discount-rate focused approach, typically used by individual investors, leads to a second version in which the relationship is strongly positive with growth stocks doing better. The two-version CAPM accounts for several recent empirical findings including fundamentally different intraday vs overnight behavior, as well as behavior on macroeconomic announcement days. Momentum is expected to be an overnight phenomenon, which is consistent with empirical findings. We argue that, perhaps, our best shot at observing classical CAPM in its full glory is a laboratory experiment with subjects who have difficulty categorizing (such as in autism spectrum disorders).

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JEL Classification: G12, G41

CAPM: A tale of two versions

Consider the following two empirical observations: Firstly, stock prices behave very differently with respect to their sensitivity to market risk (beta) at specific times. Typically, average excess return and beta relationship is flatter than expected (Frazzini and Pedersen 2014, Fama and French 2004, Black, Jensen, and Scholes 1972). It could even be negative[1]. However, during specific times, this relationship is strongly positive, such as on days when macroeconomic announcements are made (Savor and Wilson 2014) or during the night (Hendershott, Livdan, and Rosch 2018). Secondly, a hue, which is halfway between yellow and orange, is seen as yellow on a banana and orange on a carrot (Mitterer and de Ruiter 2008). In this article, we argue that the two observations are driven by the same underlying mechanism.

The second observation is an example of the implications of categorization for color calibration. In this article, we argue that the first observation is also due to categorization, which gives rise to two versions of CAPM. In one version, the relationship between expected return and stock beta is flatter than expected or could even be negative, whereas in the second version, this relationship is strongly positive.

Categorization is the mental operation by which brain classifies objects and events. We do not experience the world as a series of unique events. Rather, we make sense of our experiences within a framework of categories that represent prior knowledge. That is, new information is only understood in the context of prior knowledge. Describing categorization, Cohen and Lefebvre (2005) write, "*This operation is the basis of construction of our knowledge of the world. It is the most basic phenomenon of cognition, and consequently the most fundamental problem of cognitive science."* To cognize is to categorize (Harnad 2017). Our daily lives are dependent on our ability to form categories, and inefficiencies in category-formation have been associated with autism spectrum disorders (ASD) (see Church et al (2010)).

[1] Cohen, Polk, and Vuolteenaho (2005), and Jylha (2018)

It is well-recognized in cognitive science literature that categorization is driven by selective attention where some aspects in the information-environment are sharply attended-to.[2] Categorization-induced inferences refine such aspects while confounding others. Hence, categorization has both an upside and a downside.

Both sides are readily seen in various examples of categorization. Mitterer and de Ruiter (2008) present participants with drawings of banana and carrots filled with a hue halfway between yellow and orange. The subjects are asked to first identify a drawing and then state its color. Drawings are categorized as banana or carrot based on shape enabling useful inferences. That's the upside of categorization. However, the other attribute, color, gets confounded with the color of the category exemplar, so the same hue is seen as yellow on a banana and orange on a carrot.[3] That's the downside of categorization.

To take an example from our daily lives, imagine you go to a park and you spot a dog. You may attend to the "ownership" aspect, and if you see a person accompanying the dog, you may categorize the dog as a pet. This categorization leads to useful inferences such as who is responsible for dog's behavior. That's the upside of categorization. The downside is that, if the category-exemplar of pet dog in your memory has a passive demeanour, you may underestimate the aggressiveness of the dog in question. No wonder we continue to see occasional dog bite cases.

Despite recognition across the allied disciplines of cognitive science, neuroscience, and psychology that categorization defines how we think (Cohen and Lefebvre 2017), economics and finance literature is largely silent on how it matters for decision-making in their domain. An exception is found in behavioral literature, where the downside of categorization is formalized as categorization-bias, capturing the notion that objects in the same category are deemed more similar (objects in different categories are deemed more

[2] Medin & Schaffer (1978), Nosofsky (1986), Pazzani (1991), Hampton (1995), Nosofsky (1992), Smith and Minda (1998)

[3] Making categorization-induced inferences is a general strategy used by the brain. When a racially ambiguous face has been categorized as either Hispanic or Black (based on hair, so hair attribute is refined), then the complexion attribute gets confounded with the complexion of the category-exemplar leading to the same complexion being perceived as lighter on a Hispanic face than on a Black face (Maclin and Malpass 2001, 2003). Similarly, a sound half way between "s" and "f" is heard as "s" if the environmental cue is refined as a horse and as "f" if the environmental cue is refined as a giraffe (Norris et al 2001, 2006).

different) than they actually are (Mullainathan 2000, Hong, Stein and Yu 2007, Mullainathan et al 2008). However, a more nuanced view, which admits both the upside as well as the downside, is lacking.

Here, in accord with cognitive science literature, we present a view of categorization that has both an upside as well as a downside, and apply this nuanced perspective to the capital asset pricing model (CAPM). If categorization is fundamental to how our brains make sense of information, then investor behavior, like any other domain of human behaviour, should also be viewed through this lens. This means that the traditional view that each firm is viewed in isolation needs to be altered. When an investor considers a firm, she views it within a framework of categories that represent prior knowledge. This involves sorting a given firm into a category based on attributes that are deemed categorization-relevant. Categorization-induced inferences help refine such attributes while confounding categorization-irrelevant attributes with the category-exemplar.

Valuation requires estimating earnings (cash-flows) potential and estimating discount-rates. Even among firms that sell similar products (same sector) some may have more similar earnings potential, whereas other may have more similar discount-rates. The former type may include firms with similar earnings-related fundamentals but very different levels of debt ratio and equity betas. Also, their multiples (generally related to inverse of the discount-rate) such as P/E, EV/Sales or EV/EBITDA could be very different.[4] The latter type may include firms with similar debt ratios and equity betas or similar P/E and EV/EBITDA but quite different earnings or cash-flows fundamentals. We argue that, an earnings-focused approach, such as discounted cash-flows (DCF), tends to categorize the former type of firms together, whereas, the relative valuation approach (RV) based on multiples such as P/E or EV/EBITDA tends to categorize the latter types of firms together. In other words, the choice of a valuation approach introduces a bias in how firms are categorized.

This bias in the way firms are categorized together affects the quality of categorization-induced inferences along the two dimensions of earnings and discount-rates. When earnings-aspect is categorization-relevant, then categorization-induced inferences

[4] P/E=Price-per-share/Earnings-per-share,
EV/EBITDA=Enterprise value/Earnings before interest, taxes, debt, and amortization.
EV/Sales=Enterprise value/Sales

related to earnings are of higher quality than such inferences related to discount-rates, and vice versa.

In this paper, we take discounted cash-flows (DCF) as the prototype of an earnings-potential focused approach, and valuation by multiples or relative valuation (RV) as the prototype discount-rate focused approach. However, the analysis is general and applies to any versions of them.

In an earnings-focused approach such as DCF, the first step is estimating earning (cash-flows) potential. Categorization happens at this step. If a given firm is categorized with another firm with similar earnings-potential, then categorization-induced inferences help refine the earnings estimate. Some common examples of categorization-induced inferences related to earnings in the DCF approach are: 1) A firm selling a similar product is expected to face a declining price of its product. The same thing is likely to happen to the price of the given firm’s brand. 2) A prominent similar firm grew rapidly for only 4 years. So the given firm, even though small and rapidly growing, is expected to have its growth rate plateau within 4 years. Such inferences add value. This is the upside of categorization.

However, if two firms that have been categorized together based on earnings potential have very different discount-rates, then using the discount-rate of one firm as a starting point for the other firm may not be helpful. If fact, it may only muddy the waters.

In sharp contrast, the RV approach is focused on estimating multiples such as P/E or EV/EBITDA. Such multiples are proxies for inverse of discount rates (further discussion in Section 2.1). This suggests that discount-rate aspect is categorization-relevant in the RV approach. However, if firms have been categorized together based on P/E, then their earnings potential may be very different. Implication being that discount-rates are refined, whereas the earnings estimates are confounded in the RV approach.

We show that when earnings aspect is categorization-relevant (as in DCF analysis), a version of CAPM is obtained, which displays a flatter or even negative relationship between stock beta and expected excess returns. Betting-against-beta anomaly (Black 1972, Frazzini and Pedersen 2014) is observed along with the value effect, as well as the size premium after controlling for quality (consistent with the findings in Asness et al 2018). We argue that this is the default version which typically prevails.

Looking ahead at the results, the first version of CAPM takes the following form:

$$E(r_i) - r = \alpha_i + \beta_i E(r_M - r)$$

where r_M, r, and r_i are market portfolio, risk-free, and stock returns respectively. The only difference between this version and classical CAPM is the appearance of alpha or α_i.

It is this additional term which drives all the results:

$$\alpha_i = \frac{E(r_M - r)(d_i - c)}{(1 + c)} \qquad (typical\ firm) \qquad (1.1)$$

$$\alpha_i = -\frac{E(r_M - r)c}{(1 + c)} \qquad (category\ exemplar) \qquad (1.1a)$$

where $0 < c < 1$ is a constant in a cross-section, and $d_i > 0$ has the following properties:

$$(1)\ \frac{\partial d_i}{\partial \beta_i} < 0\ (high\ alpha\ of\ low\ beta)$$

$$(2)\ \frac{\partial d_i}{\partial P_i} < 0\ (similar\ to\ value\ effect); P_i\ is\ stock\ price$$

$$(3)\ \frac{\partial d_i}{\partial \sigma^2(P_i + D_i)} < 0\ ;\ D_i is\ dividend\ (size\ premium\ controlling\ for\ quality)$$

(1) is clearly high-alpha-of-low-beta, and (2) is high-alpha-of-low-price similar to value effect. We interpret (3) as size-premium controlling for quality (Asness et al 2018). This is because small-cap stocks with safe, steady earnings and low leverage generally have the smallest payoff volatility in the market. Interesting, in this version of CAPM, the relationship between excess return and stock beta can also be negative as $\frac{\partial d_i}{\partial \beta_i} < 0$.

$$\frac{\partial E(r_i)}{\partial \beta_i} = \frac{E(r_M - r)}{(1 + c)} \frac{\partial d_i}{\partial \beta_i} + E(r_M - r) < 0\ if\ \left|\frac{\partial d_i}{\partial \beta_i}\right| > 1 + c$$

As c is generally quite small, the relationship between excess return and stock beta is quite possibly negative. Recent studies generally find such a negative relationship (Savoir and

Wilson 2014, Hendershott et al 2018 and references there in). Furthermore, category-exemplars have the lowest alphas in this version (as can be seen from 1.1a).

While categorizing firms, if investors are focused on the discount rate aspect (as in RV analysis), then the discount-rates are refined whereas earnings estimates are confounded with the category-exemplar. A second version of CAPM arises. In this version, there is a strong positive relationship between beta and expected excess return.

The second version of CAPM has the following form:

$E(r_i) - r = \alpha_i + \beta_i E(r_M - r)$

$\alpha_i = h - e_i$ (typical firm) where $e_i > 0$ and h is a constant in a cross-section.

$\alpha_i = h$ (category-exemplar)

$$(1)\frac{\partial e_i}{\partial \beta_i} < 0 \quad (High\ alpha\ of\ high\ beta)$$

$$(2)\frac{\partial e_i}{\partial P_i} < 0 \quad (Growth\ stocks\ do\ better)$$

So, in this second version, alpha rises with beta. This makes the relationship between excess return and stock beta strongly positive. Also, $\frac{\partial e_i}{\partial P_i} < 0$, so growth stocks do better in this version than value stocks. Furthermore, category-exemplars have the highest alphas in this version. It is interesting to note that the stocks that do better in the first version (value, low beta) generally do worse in the second version consistent with the tug-of-war dynamics documented in Lou, Polk, and Skouras (2018).

One way to make sense of the co-existence of two versions is to classify investors as either earnings-focused or discount rate-focused. If earnings-focused investors dominate, then the first version is observed. If the discount-rate-focused investors dominate, then the second version is observed. Note, that earnings-focused approach (such as DCF) is typically employed by large institutional investors, whereas RV approach is associated with individual investors (and with sell-side equity analysts who publish research reports for individual investors). [5]

If institutional investors are earnings-focused and individual investors are discount rate-focused, then the trading behavior of each type can be observed to make specific predictions:

1) Institutional investors typically avoid trading at the open and prefer to trade in the afternoon near the market close (Lou et al 2018 and references therein). The objective is to time the trade when the market is most liquid to avoid any adverse price impact. This means that trade at open is dominated by individual investors. So, one expects to see the relationship between stock beta and average return to be strongly positive (second version) overnight and flat or even negative (first version) intraday. Indeed, this is what Hendershott et al (2018) find.

2) Institutional traders typically trade in the right direction prior to macroeconomic announcement days (suggesting superior information) with institutional trading volume falling sharply on macro-announcement days (Hendershott, Livdan, and Schurhoff 2015). As trade on such days is dominated by individual investors, one expects to see a strongly positive relationship (second version) on macro-announcement days. Indeed, this is what Savor and Wilson (2014) find.

3) The first version generally dominates intraday due to institutional investors being dominant. As the corresponding CAPM version comes with size and value effects, the prediction is that size and value are primarily intraday phenomena. Indeed, this is exactly what Lou et al (2018) find.

4) We show that, all else equal, discount rate-focused investors have higher willingness-to-pay than earnings-focused investors. If discount rate-focused investors dominate trade at open, whereas earnings-focused investors are active intraday, then one expects prices to typically rise overnight from close-to-open and fall intraday between open-to-close. Consistent with this prediction, Kelly and Clark (2011) suggest that returns are indeed higher overnight than intraday.

[5] Asquith, P., M. Mikhail, and A. Au (2005), "Information Content of Equity Analyst Reports." *Journal of Financial Economics*, 75, (2005), pp. 245–282.

5) If momentum traders, who buy past winners and short past losers, are primarily individual investors, then one expects momentum to be an overnight phenomenon observed between close-to-open. This is because individual traders dominate trade at or near open. Lou et al (2018) find that momentum is indeed an overnight phenomenon.

The alignment of such a diverse range of predictions with empirical evidence strongly suggests that categorization matters for financial markets.

If we only observe a specific version at a given time, does it follow that classical CAPM can never be observed? Because categorization is such a fundamental aspect of cognition, it never turns-off in a healthy brain. Hence, the classical CAPM is unlikely to be ever observed. We catch glimpses of it in various versions depending on which type of news/ investor type dominates. However, among ASD sufferers, there is a breakdown in categorization ability (Gastgeb and Strauss 2012, Church et al 2010). So, perhaps a laboratory experiment with high functioning ASD sufferers (and limited informational complexity) is our best shot at observing CAPM in its full glory.

2. Adjusting CAPM for categorization

As discussed in the introduction, when information about an object or an event reaches the human brain, it makes sense of it within a framework of categories that represent prior knowledge. This involves sorting that object or event into a category based on selective attention to some aspects. Categorization-induced inferences refine categorization-relevant attributes, while confounding categorization-irrelevant attributes with the category-exemplar.

Treating financial information the same, we argue that firms are not viewed in isolation. Rather, investors view them within a framework of categories that represent prior knowledge. This involves sorting a firm into a category based on a subset of attributes. While categorizing firms, if investors focus on the earnings-aspect then earnings-estimates are sharpened whereas the discount-rates are confounded with the category-exemplar. The reverse happens if the discount-rate aspect is categorization-relevant. As discussed in the introduction, an earnings-focused approach (such as DCF), typically associated with institutional investors, suggests categorization with earnings aspect being categorization-relevant. This gives rise to version one of CAPM. This version is discussed in sections 2.1 and 2.2. RV approach (typically associated with individual investors) gives rise to version two of CAPM, which is discussed in section 2.3.

To adjust CAPM for categorization, we use the same starting point as in Frazzini and Pedersen (2014). Consider an overlapping-generations (OLG) economy in which agents with wealth W_t are born in each period t and live for two periods.

Each period t, young agents invest in stocks and the risk-free asset to maximize utility:

$$\max n'(E_t(P_{t+1}+D_{t+1})-(1+r)P_t)-\frac{\gamma}{2}n'\theta_t n \tag{2.1}$$

where n is the vector representing the number of shares of each type in the portfolio, P_t is the vector of prices, D_t is the vector of dividends, r is the risk-free rate, γ captures risk-aversion, and θ_t is the variance-covariance matrix of $P_{t+1}+D_{t+1}$.

From the first-order-condition of utility maximization of agent i:

$$n_i=\frac{1}{\gamma_i}\theta^{-1}(E_t(P_{t+1}+D_{t+1})-(1+r)P_t)$$

In equilibrium, demand equals supply:

$$\sum_i n_i=n^*$$

It follows that:

$$n^*=\frac{1}{\gamma}\theta^{-1}(E_t(P_{t+1}+D_{t+1})-(1+r)P_t) \tag{2.2}$$

where the aggregate risk aversion, γ is defined as $\frac{1}{\gamma}=\sum_i\frac{1}{\gamma_i}$

Solving (2.2) for equilibrium price:

$$P_t=\frac{E_t(P_{t+1}+D_{t+1})-\gamma\theta_t n^*}{1+r} \tag{2.3}$$

By choosing an appropriate risk-premium, δ_t, one may write:

$$P_t = \frac{E_t(P_{t+1} + D_{t+1}) - \gamma\theta_t n^*}{1+r} = \frac{E_t(P_{t+1} + D_{t+1})}{1 + r + \delta_t} \tag{2.4}$$

where $\delta_t = f(\gamma, \theta, n^*)$.

The R.H.S of (2.4) shows that stock price movements can be attributed either to earnings news which affects the numerator, $E_t(P_{t+1} + D_{t+1})$, or it can be attributed to the discount-rate news which affects the denominator, $1 + r + \delta_t$.

Discount rate-focused approach: Focus is on forecasting multiples such as P/E ratio. As this multiple is related to the inverse of the discount-rate, discount-rate is categorization-relevant in this approach. It follows that categorization-induced inferences refine $\frac{1}{1+r+\delta}$ while confounding $E_t(P_{t+1} + D_{t+1})$ with the category-exemplar.

Earnings-focused approach: Focus is on earnings potential. As earnings aspect is categorization-relevant, $E_t(P_{t+1} + D_{t+1})$ is refined via categorization-induced inferences while $\frac{1}{1+r+\delta}$ is confounded with the category-exemplar.

2.1 Earnings aspect is categorization-relevant

We start by considering the simplest case first in which investors divide assets into only two categories: risky and risk-free. That is, all risky stocks are placed in one category. To illustrate the implications for CAPM, initially assume that there are only two risky stocks belonging to firms L and S.

From (2.3):

$$P_{Lt} = \frac{E_t\left(P_{L(t+1)} + D_{L(t+1)}\right) - \gamma n_L^* \sigma_{Lt}^2 - \gamma n_S^* \sigma_{LS}}{1+r} \tag{2.5}$$

$$P_{St} = \frac{E_t\left(P_{S(t+1)} + D_{S(t+1)}\right) - \gamma n_S^* \sigma_{St}^2 - \gamma n_L^* \sigma_{LS}}{1+r} \tag{2.6}$$

where σ^2_{Lt} and σ^2_{St} are payoff variances of L and S respectively, and σ_{LS} is their covariance. Assuming that γ, r, and n^* are constant, investors form expectations regarding the following attributes of $L's\ stock$: $\left(P_{L(t+1)} + D_{L(t+1)}, \sigma^2_{Lt}, \sigma_{LS}\right)$. Similarly, they form expectations about the following attributes of $S's\ stock$: $\left(P_{S(t+1)} + D_{S(t+1)}, \sigma^2_{St}, \sigma_{LS}\right)$

At the firm level, L is analyzed first. We assume rational expectations about future earnings as well as volatility of earnings of firm L. And, these rational expectations translate into rational expectations about all three attributes of $L's$ stock: $\left(P_{L(t+1)} + D_{L(t+1)}, \sigma^2_{Lt}, \sigma_{LS}\right)$.

Firm S is analyzed next, and is co-categorized with firm L, which is the category-exemplar. Assume that their earnings-related fundamentals are similar while their risks are quite different. So, categorization induced inferences refine the earnings estimate while confounding the volatility of earnings.

Defining π_S and π_L as the total earnings of firm S and L respectively:

Upside of categorization: $E_t(\pi_s)$ is rational.

Downside of categorization: $\sigma^{C2}_{st}(\pi_S) = m\sigma^2_{St}(\pi_S) + (1-m)\sigma^2_{Lt}(\pi_L)$

where $0 \leq m \leq 1$ captures the degree of confounding. There is no confounding when $m = 1$. The confounding is maximum when $m = 0$.

This confounding of earnings volatility confounds stock payoff volatility, as investors consider stock price (inclusive of dividends) to be a function of earnings per share or EPS:

$$\frac{\sigma^{C2}_{st}(\pi_S)}{n^{*2}_S} = \frac{m\sigma^2_{St}(\pi_S)}{n^{*2}_S} + (1-m)\frac{\sigma^2_{Lt}(\pi_L)}{n^{*2}_S}\frac{n^{*2}_L}{n^{*2}_L}$$

$$\Rightarrow \sigma^{C2}_{st}(EPS_S) = m\sigma^2_{St}(EPS_S) + (1-m)\sigma^2_{Lt}(EPS_L)\frac{n^{*2}_L}{n^{*2}_S}$$

$$\Rightarrow \sigma^{C2}_{st}\left(P_{S(t+1)} + D_{S(t+1)}\right) \approx$$

$$m\sigma^2_{St}\left(P_{S(t+1)} + D_{S(t+1)}\right) + (1-m)\sigma^2_{Lt}\left(P_{L(t+1)} + D_{L(t+1)}\right)\frac{n^{*2}_L}{n^{*2}_S} \qquad (2.7)$$

Substituting (2.7) in (2.6):

$$P_{St} = \frac{E_t\left(P_{S(t+1)} + D_{S(t+1)}\right) - \gamma n_S^* m \sigma_{St}^2 - \gamma n_S^*(1-m)\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \gamma n_L^* \sigma_{LSt}}{1+r} \tag{2.8}$$

Adding and subtracting $\gamma n_S^* \sigma_{St}^2$ to the numerator and using

$cov\left(P_{S(t+1)} + D_{S(t+1)}, n_S^*\left(P_{S(t+1)} + D_{S(t+1)}\right) + n_L^*\left(P_{L(t+1)} + D_{L(t+1)}\right)\right) = n_S^* \sigma_{St}^2 + n_L^* \sigma_{LSt}$

with a further substitution of $X_{S(t+1)} = P_{S(t+1)} + D_{S(t+1)}$ and $X_{L(t+1)} = P_{L(t+1)} + D_{L(t+1)}$ leads to:

$$P_{St} = \frac{E_t\left(X_{S(t+1)}\right) - \gamma\left[cov\left(X_{S(t+1)}, n_S^* X_{S(t+1)} + n_L^* X_{L(t+1)}\right) + n_S^*(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)\right]}{1+r}$$

(2.9)

In terms of expected returns:

$$E_t(r_s) = r + \frac{\gamma}{P_{St}}\left[Cov\left(X_{S(t+1)}, n_s^* X_{S(t+1)} + n_L^* X_{L(t+1)}\right) + n_S^*(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)\right] \tag{2.10}$$

The additional term on the R.H.S of (2.10), $n_S^*(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)$, is due to the confounding of the earnings-variance of S with the earnings-variance of L. This term disappears if rational expectations are formed regarding variance: $m = 1$

The expected return of L is the usual expression with rational expectations:

$$E_t(r_L) = r + \frac{\gamma}{P_{Lt}}\left[Cov\left(X_{L(t+1)}, n_s^* X_{S(t+1)} + n_L^* X_{L(t+1)}\right)\right] \tag{2.11}$$

To obtain the expected return on the market portfolio, multiply (2.10) by $\frac{n_S^* P_{St}}{n_S^* P_{St} + n_L^* P_{Lt}}$ and (2.11) by $\frac{n_L^* P_{Lt}}{n_S^* P_{St} + n_L^* P_{Lt}}$ and add the two equations:

$$E_t(r_M) = r + \frac{\gamma}{n_S^* P_{St} + n_L^* P_{Lt}} \left[Var\left(n_S^* X_{S(t+1)} + n_L^* X_{L(t+1)}\right) \right.$$
$$\left. + n_S^{*2}(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)\right] \qquad (2.12)$$

Denoting the price of market portfolio as $P_{Mt} = n_S^* P_{St} + n_L^* P_{Lt}$, the associated next period payoff as $X_{M(t+1)} = n_S^* X_{S(t+1)} + n_L^* X_{L(t+1)}$, and solving (2.12) for γ leads to:

$$\gamma = \frac{(E_t(r_M) - r)P_{Mt}}{Var\left(X_{M(t+1)}\right) + n_S^{*2}(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)} \qquad (2.13)$$

Substituting (2.13) in (2.10) leads to:

$$E_t(r_S) = r + [E_t(r_M) - r]$$
$$\cdot \frac{Cov(r_S, r_M) + \frac{n_S^*(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)}{P_{St} P_{Mt}}}{Var(r_M) + \frac{n_S^{*2}(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)}{P_{Mt}^2}} \qquad (2.14)$$

Substituting (2.13) in (2.11) leads to:

$$E_t(r_L) = r + [E_t(r_M) - r]$$
$$\cdot \frac{Cov(r_L, r_M)}{Var(r_M) + \frac{n_S^{*2}(1-m)\left(\sigma_{Lt}^2 \frac{n_L^{*2}}{n_S^{*2}} - \sigma_{St}^2\right)}{P_{Mt}^2}} \qquad (2.15)$$

(2.14) and (2.15) are the categorization-adjusted CAPM expressions for S and L respectively when variance is the confounded attribute. If there is no confounding of variance, that is, when $m = 1$, the traditional CAPM expression is obtained.

It is straightforward to generalize to the case of Q categories of risky stocks with K stocks (qk with $k = 1,2,3, \dots, K$) plus one exemplar qL in each category q: [6]

$$E_t(r_{qk}) = r + [E_t(r_M) - r] \cdot \frac{Cov(r_{qk}, r_M) + \dfrac{n_{qk}^{*}(1-m)\left(\sigma_{qLt}^2 \dfrac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{P_{qkt} P_{Mt}}}{Var(r_M) + \sum_{q=1}^{Q} \sum_{k=1}^{K} \dfrac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^2 \dfrac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{P_{Mt}^2}} \tag{2.16}$$

$$E_t(r_{qL}) = r + [E_t(r_M) - r] \cdot \frac{Cov(r_{qL}, r_M)}{Var(r_M) + \sum_{q=1}^{Q} \sum_{k=1}^{K} \dfrac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^2 \dfrac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{P_{Mt}^2}} \tag{2.17}$$

If there is no confounding, then (2.16) and (2.17) converge to the classical CAPM.

It is clear from the above that adjusting CAPM for categorization of firms in investors' brains somewhat changes the CAPM; however, the general form remains the same. To see the impact of the changes more clearly, it is useful to split the adjusted-CAPM into alpha and beta components. This is done next.

2.2 Splitting into Alpha and Beta

Splitting (2.16) into beta (exposure to market) and alpha (excess return not explained by beta) leads to the following expressions for stock k in category q (see appendix A):

$$E_t(r_{qk}) - r = \alpha_{qk} + \beta_{qk}[E_t(r_M) - r] \tag{2.18}$$

[6] Siddiqi (2018) derives equivalent adjusted-CAPM expressions by assuming that exemplar firms are starting points for analysing other firms with anchoring-and-adjustment heuristic preventing full adjustments. He simply assumes, somewhat unsatisfyingly, that anchoring bias in variance is larger than the anchoring bias in earnings level. In contrast, in this article, we directly utilize the general categorization theory and consider both the upside and the downside of categorization in full generality. The general treatment here allows the two version of CAPM to readily emerge.

where $\alpha_{qk} = \frac{[E_t(r_M)-r]}{(1+c)}\left(d_{qk} - c\right)$, $\beta_{qk} = \frac{Cov(r_{qk}, r_M)}{Var(r_M)}$

$$c = \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^2\frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{Var(X_{Mt})}$$

$$d_{qk} = \frac{n_{qk}^{*}(1-m)\left(\sigma_{qLt}^2\frac{n_{qL}^{*2}}{n_{qk}^{*2}} - \sigma_{qkt}^2\right)}{P_{qkt}P_{Mt}Cov(r_{qkt}, r_{Mt})}$$

Similarly, for the category-exemplar (from 2.17), alpha is:

$$\alpha_{qL} = -\frac{[E_t(r_M) - r]}{(1+c)}c \tag{2.18a}$$

By definition, exemplar-firms are the basis around which categories are built. In general, the largest firms in the market get most of investor, analyst, and media attention; hence, are natural category-exemplars for the marginal investor. As earnings-variance scales with size, one expects the exemplar firm to have the largest earnings-variance in its category, which makes d_{qk} (and c) positive:

$$\sigma_{qL}^2(earnings) \geq \sigma_{qk}^2(earnings) \;\; \forall k \; k = 1,2, \ldots\ldots, K$$

$$\Rightarrow \sigma_{qL}^2(EPS)\frac{n_{qL}^{*2}}{n_{qk}^{*2}} \geq \sigma_{qk}^2(EPS)$$

$$\Rightarrow \sigma_{qLt}^2\left(P_{qL(t+1)} + d_{qL(t+1)}\right)\frac{n_{qL}^{*2}}{n_{qk}^{*2}} \geq \sigma_{qk}^2\left(P_{qk(t+1)} + d_{qk(t+1)}\right)$$

$$\Rightarrow d_{qk} > 0$$

The general form of CAPM with categorization is the same as with classical CAPM with appearance of alpha in (2.18) being the only difference. There are several interesting implications of the properties of alpha, and these implications align very well with several well-known anomalies with classical CAPM. One can see betting-against-beta, value effect, as well as an analogue of the size premium in this version of CAPM.

Proposition 1 shows that alpha is higher for a low-beta stock when compared with a high-beta stock. That is, high-alpha is associated with low-beta, and low-alpha is associated with high-beta.

Proposition 1 (high beta is low alpha):

In CAPM adjusted for categorization (when earnings aspect is categorization-relevant), α falls as β rises.

Proof:

$$\alpha = \frac{[E_t(r_M) - r]}{(1+c)}\left(d_{qk} - c\right)$$

where $d_{qk} = \frac{n^*_{qk}(1-m)\left(\sigma^2_{qLt}\frac{n^{*2}_{qL}}{n^{*2}_{qk}} - \sigma^2_{qkt}\right)}{P_{qkt}P_{Mt}Cov(r_{qkt}, r_{Mt})}$

$$\Rightarrow d_{qk} = \frac{n^*_{qk}(1-m)\left(\sigma^2_{qLt}\frac{n^{*2}_{qL}}{n^{*2}_{qk}} - \sigma^2_{qkt}\right)}{Var(r_{Mt})P_{qkt}P_{Mt}\beta_{qk}}$$

$$\Rightarrow \frac{\partial d_{qk}}{\partial \beta_{qk}} = -\frac{n^*_{qk}(1-m)\left(\sigma^2_{qLt}\frac{n^{*2}_{qL}}{n^{*2}_{qk}} - \sigma^2_{qkt}\right)}{Var(r_{Mt})P_{qkt}P_{Mt}{\beta_{qk}}^2} < 0 \tag{2.19}$$

Hence, alpha falls as beta rises and alpha rises as beta falls.

■

Corollary 1.1: Category-exemplars (largest firms) have the lowest alphas in their respective categories

Empirically, intraday, not only alpha falls as beta rises, but the effect is strong enough to make the relationship between intraday average excess return and stock beta negative (Savor and Wilson 2014 ,Hendershott, Livdan, and Rosch 2018).

In version one of CAPM presented here, not only alpha falls as beta rises, but it could quite plausibly fall rapidly enough to make the relationship negative:

$$\frac{\partial\left(E\left(r_{qk}\right)-r\right)}{\partial\beta_{qk}}=\frac{[E(r_M)-r]}{(1+c)}\cdot\frac{\partial d_{qk}}{\partial\beta_{qk}}+[E(r_M)-r]$$

$$\Rightarrow\frac{\partial\left(E\left(r_{qk}\right)-r\right)}{\partial\beta_{qk}}<0\ if\ \left|\frac{\partial d_{qk}}{\partial\beta_{qk}}\right|>1+c$$

That is, if $\frac{n^*_{qk}(1-m)\left(\sigma^2_{qLt}\frac{n^{*2}_{qL}}{n^{*2}_{qk}}-\sigma^2_{qkt}\right)}{Var(r_{Mt})P_{qkt}P_{Mt}\beta^2_{qk}}>1+\sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n^{*2}_{qk}(1-m)\left(\sigma^2_{qLt}\frac{n^{*2}_{qL}}{n^{*2}_{qk}}-\sigma^2_{qkt}\right)}{Var(X_{Mt})}$

With a little re-arrangement in the L.H.S, the above condition can be expressed as:

$$\frac{1}{\beta^2_{qk}}\left\{\sum_{j=1}^{Q}\sum_{i=1}^{K}\frac{n^{*2}_{ji}(1-m)\left(\sigma^2_{jLt}\frac{n^{*2}_{qL}}{n^{*2}_{qk}}-\sigma^2_{jit}\right)\frac{P_{ji}}{P_{qk}}}{Var(X_{Mt})}\right\}>1+\sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n^{*2}_{qk}(1-m)\left(\sigma^2_{qLt}\frac{n^{*2}_{qL}}{n^{*2}_{qk}}-\sigma^2_{qkt}\right)}{Var(X_{Mt})}\qquad(2.20)$$

It is easy to see that (2.20) can hold in the data for a plausible range of parameter values.

It also follows (by straightforward inspection) that in this version of CAPM, category-exemplars (largest firms) have the lowest alphas. That is, they are expected to perform the worst, when the first version is likely to dominate. Next, we consider the characteristics of a factor that is long in low-beta stocks funded by short-selling high-beta stocks. Suppose the portfolio of low-beta stocks has an alpha of α^L, whereas the portfolio of high-beta stocks has an alpha of α^H.

We construct a betting-against-beta (BAB) factor as:

$$r_t^{BAB}=\alpha^L-\alpha^H \qquad (2.21)$$

Proposition 2 describes the predictions of categorization-adjusted CAPM regarding the BAB factor.

Proposition 2 (positive expected return of BAB):

The expected excess return from a self-financing BAB factor is positive

$$\mathbf{E_t(r^{BAB}) = \frac{[E_t(r_M) - r]}{(1+c)} \cdot (d^L - d^H) \geq 0}$$

and tends to increase in the market risk-premium and the gap between the beta values of low-beta and high-beta portfolios.

Proof:

The alpha of the low-beta portfolio is: $\frac{[E_t(r_M)-r]}{(1+c)}(d^L - c)$. Similarly, the alpha of the high-beta portfolio is: $\frac{[E_t(r_M)-r]}{(1+c)} \cdot (d^H - c)$. Taking expectations in (2.21) and substituting from the above yields: $E_t(r^{BAB}) = \frac{[E_t(r_M)-r]}{(1+c)} \cdot (d^L - d^H)$.

As d falls when β rises, the above expression is positive. In general, larger the gap between β^L and β_H, greater is the distance between d^L and d^H.

■

The results in proposition 2 are similar to the results derived in Frazzini and Pedersen (2014). However, the two approaches are very different. Frazzini and Pedersen (2014) derive these results based on a CAPM framework with borrowing, cash, and margin constraints and here the results follow from categorization of firms when the earnings aspect is categorization-relevant. The empirical support in Frazzini and Pedersen (2014) could be interpreted as support for the version one of CAPM developed here.

Proposition 3 shows that the well-known value effect could potentially be due to categorization as well. The value effect is the finding that value stocks (stocks with low market price relative to fundamentals) tend to outperform growth stocks (stocks with high market price relative to fundamentals).

Proposition 3 (value effect):

Alpha from value stocks is higher than the alpha from growth stocks.

Proof:

Follows directly from (2.18) by noting that $\frac{\partial \boldsymbol{d}_{qk}}{\partial \boldsymbol{P}_{qk}} < \mathbf{0}$

■

Proposition 4 shows how alpha varies with payoff volatility.

Proposition 4 (size-effect when quality is controlled):

Alpha is higher for low payoff-volatility stocks

Proof.

$\frac{\partial \boldsymbol{d}_{qk}}{\partial \boldsymbol{\sigma}_{qk}^{2}} < \mathbf{0}$. That is, alpha falls as payoff-volatility rises.

■

Asness et al (2018) show that size-effect emerges after controlling for quality. Stocks that are safe and profitable are considered quality stocks. Small-cap stocks have smaller prices but that does not automatically translate into smaller payoff-volatility as some small-cap stocks are low quality or junk stocks with uncertain earnings. Smaller prices of small-caps only translate into smaller payoff-volatility if they are of high quality. That is, if they deliver stable earnings. Hence, proposition 4 establishes a size-effect after controlling for quality in a manner consistent with the findings in Asness et al (2018).

2.3 Discount-rate aspect is categorization-relevant

If risk or discount-rate is categorization-relevant, then, at firm-level, expectations about risk or volatility of earnings are refined due to categorization-induced inferences.[7] This is the upside of categorization. However, expectations about earnings level are confounded. This is the downside.

For a firm k in category q, which is categorized with the exemplar-firm L, the upside of categorization is improved expectations (rational expectations) about earnings-volatility. That is, $\sigma_t^2\left(\pi_{qk(t+1)}\right)$ is rational, where $\pi_{qk(t+1)}$ is next period earnings.

Downside of categorization is that earnings-expectations are confounded with the earnings-expectations of the category-exemplar:

$$E_t^C\left(\pi_{qk(t+1)}\right) = mE_t\left(\pi_{qk(t+1)}\right) + (1-m)E_t\left(\pi_{qL(t+1)}\right)$$

where $0 \leq m \leq 1$ captures the degree of confounding. There is no confounding when $m = 1$. The confounding is maximum when $m = 0$.

Essentially following the same steps as in the last section:

$$E_t^C\left(\frac{\pi_{qk(t+1)}}{n_{qk}^*}\right) = mE_t\left(\frac{\pi_{qk(t+1)}}{n_{qk}^*}\right) + (1-m)E_t\left(\frac{\pi_{qL(t+1)}}{n_{qL}^*}\right)\frac{n_{qL}^*}{n_{qk}^*}$$

$$\Rightarrow E_t^C\left(EPS_{qk(t+1)}\right) = mE_t\left(EPS_{qk(t+1)}\right) + (1-m)E_t\left(EPS_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*}$$

Assuming that investors consider next period price (inclusive of dividends) to be some function of next period EPS:

$$E_t\left(\left(P_{qk(t+1)} + D_{qk(t+1)}\right)^C\right)$$
$$\approx mE_t\left(P_{qk(t+1)} + D_{qk(t+1)}\right) + (1-m)E_t\left(P_{qL(t+1)} + D_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*}$$

[7] Instead of working with earnings-volatility at the firm-level, we can equivalently work with share-level equity discount-rates by realizing that a refinement of discount-rate is a refinement of payoff covariance with the aggregate market. This requires refinement in share-payoff volatility, which in turn follows from refinement in earnings-volatility at the firm-level. However, to maintain consistency with the previous sections, we choose to work with firm-level earnings-volatility.

$$\Rightarrow E_t\left(\left(X_{qk(t+1)}\right)^C\right) \approx mE_t\left(X_{qk(t+1)}\right) + (1-m)E_t\left(X_{qL(t+1)}\right)\frac{n_L^*}{n_S^*} \tag{2.22}$$

where $X = P + D$ has been used above.

By following a similar set of steps as in section 2.1, the CAPM expressions for a firm k in category q and the exemplar-firm L in category$q$ are obtained:

$$E_t(r_{qk}) = r$$

$$+\frac{Cov(r_{qk}, r_M)}{Var(r_M)}\left[(E_t(r_M) - r) + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^*(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{Mt}}\right]$$

$$-\frac{(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{qkt}} \tag{2.23}$$

$$E_t(r_{qL}) = r$$

$$+\frac{Cov(r_{qL}, r_M)}{Var(r_M)}\left[(E_t(r_M) - r) + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^*(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{Mt}}\right] \tag{2.24}$$

As expected, the classical CAPM expression is obtained from (2.23) and (2.24) if there is no confounding: $m = 1$.

Splitting (2.23) into alpha and beta (see appendix B):

$$E_t(r_{qk}) - r = \alpha_{qk} + \frac{Cov(r_{qk}, r_M)}{Var(r_M)}\,[(E_t(r_M) - r)] \tag{2.25}$$

$$\alpha_{qk} = h_t - e_{qkt}$$

$$h_t = \sum_{q=1}^{Q} \sum_{k=1}^{K} \left\{ \frac{n_{qk}^*(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{Mt}} \right\}$$

$$e_{qkt} = \frac{(1-m)\left(E_t\left(X_{qL(t+1)}\right)\frac{n_{qL}^*}{n_{qk}^*} - E_t\left(X_{qk(t+1)}\right)\right)}{P_{qkt}\beta_{qkt}} > 0$$

For category exemplars: $\alpha = h_t$

Proposition 5 shows that, when discount rate aspect is categorization-relevant, then high beta stocks have high alpha, and low beta stocks have low alpha. That is, alpha and beta move together, creating a steeper relationship between average excess return and beta. Furthermore, category-exemplars (largest firms) have the highest alphas in their respective categories. This is the complete opposite of what happens when earnings aspects is categorization-relevant.

Proposition 5 (High alpha of high beta)

When discount rate aspect is categorization-relevant, alpha increases with beta in a given cross-section.

Proof:

Follows directly by realizing that $\boldsymbol{\frac{\partial e_{qkt}}{\partial \beta_{qk}} < 0}$

■

Corollary 5.1: Category-exemplars (largest firms) have the highest alphas in their respective categories.

Proposition 5 shows if discount rate aspect is categorization-relevant, then alpha rises with beta. This is in sharp contrast with the baseline case (when earnings aspect is categorization-relevant) as in that case alpha falls with beta. Proposition 6 shows that, if discount rate aspect is categorization-relevant, then growth stocks are expected to do better than value stocks.

Proposition 6 (growth effect)

When discount rate aspect is categorization-relevant, stocks with high market prices relative to fundamentals do better than stocks with low market prices relative to fundamentals.

Proof:

Follows directly by realizing that $\frac{\partial \boldsymbol{e_{qkt}}}{\partial \boldsymbol{P_{qk}}} < \mathbf{0}$

■

The two versions of CAPM have quite opposite predictions. In the first version (propositions 1-4), alpha falls with beta, and we observe the value effect and the size premium (controlling for quality). In the second version, alpha rises with beta, and growth stocks do better (propositions 5-6). The two versions represent different clienteles or investor types. The first version corresponds to institutional investors who are earnings-focused, whereas the second one corresponds to individual investors who are discount rate-focused. As discussed earlier, the first version is expected to dominate intraday whereas the second version is expected to dominate overnight. This creates interesting tug-of-war dynamics between the two investor types, which are discussed next.

$\frac{\partial \alpha_{qk}}{\partial n_{qk}}$

3. Tug-of-War Dynamics

Lou et al (2018) report a series of intriguing empirical findings:

1) Overnight clienteles are fundamentally different than intraday clienteles, which is based on the robust finding that a hedge portfolio (best overnight performers minus the worst overnight performers) continues to perform well overnight in the future while performing poorly intraday.

2) Size and value are only observed intraday.

3) In general, strategies that do well intraday show opposite results overnight.

4) Momentum returns are earned overnight.

These findings are consistent with CAPM having two versions as developed here, with one version being dominant intraday whereas the other version holding sway overnight. Typically, institutional investors are earnings-focused, whereas individual investors are discount-rate or RV-focused. The absence of institutional investors from trading at open makes individual investors dominant. Consequently, the second version (associated with the RV approach) dominates overnight. As institutional investors dominate trading intraday, the first version (associated with the earnings-focused approach) dominates intraday. As size and value effects are only associated with the first version which prevails intraday, size and value are intraday phenomenon only.

In the two-version CAPM, in general, the strategies that do well in the first version do poorly in the second one. To illustrate, lets' examine a strategy in which one goes long low-equity-issuance stocks and shorts high-equity-issuance stocks. Intraday (version one of CAPM) this strategy has a positive alpha. This is because in version one: $\frac{\partial \alpha_{qk}}{\partial n_{qk}} < \mathbf{0}$. However, overnight (version two of CAPM) this strategy has a negative alpha because, in version two: $\frac{\partial \alpha_{qk}}{\partial n_{qk}} > \mathbf{0}$. This is exactly what Lou et al (2018) find.

Momentum trading is about buying past winners and shorting past losers. If momentum traders are individual traders who dominate at open, one expects momentum effect to be an overnight phenomenon. Consistent with this prediction, Lou et al (2018) report that momentum returns are mostly earned overnight.

Proposition 7 shows that discount-rate focused traders have a higher willingness-to-pay than earnings focused traders all else equal.

Proposition 7: Discount-rate focused investors have higher willingness-to-pay than earnings focused investors all else equal.

Proof:

Confounding of earnings-variance of a firm with the category-exemplar lowers an investor's willingness-to-pay:

$$\frac{E_t\left(P_{S(t+1)} + D_{S(t+1)}\right) - \gamma n_s m\sigma_{St}^2 - \gamma n_s (1-m)\sigma_{Lt}^2 \frac{n_L^2}{n_S^2} - \gamma n_L \sigma_{LSt}}{1+r} < \frac{E_t\left(P_{S(t+1)} + D_{S(t+1)}\right) - \gamma n_s \sigma_{St}^2 - \gamma n_L \sigma_{LS}}{1+r}$$

Confounding of expected earnings-level of a firm with the category exemplar increases an investor's willingness-to-pay:

$$\frac{mE_t\left(P_{S(t+1)} + D_{S(t+1)}\right) + (1-m)E_t\left(P_{L(t+1)} + D_{L(t+1)}\right)\frac{n_L}{n_S} - \gamma n_s \sigma_{St}^2 - \gamma n_L \sigma_{LS}}{1+r} > \frac{E_t\left(P_{S(t+1)} + D_{S(t+1)}\right) - \gamma n_s \sigma_{St}^2 - \gamma n_L \sigma_{LS}}{1+r}$$

This is because an exemplar firm is expected to be the largest firm in its category with the highest expected earnings and volatility of earnings as these values generally scale with size.

■

If discount-rate investors are primarily overnight traders whereas earnings focused investors are active intraday, then one expects prices to typically rise overnight from close-to-open and fall intraday between open-to-close. Consistent with this prediction, Kelly and Clark (2011) find this pattern in returns.

4. Discussion and Conclusions

Categorization is the core of cognition and the fuel and fire of thinking. It is the basis of construction of our knowledge of the world, and is critically important in inference and decision-making. In this article, we explore the implications of categorization for CAPM. The defining feature of categorization in the human brain is selective attention in which some aspects in the information environment are paid more attention than others. Such aspects are the basis for categorization.

We argue that, just like other objects or events, firms are also not viewed in isolation. Rather, investors make sense of them within a framework of categories that represent prior knowledge. This involves sorting a firm into a category based on the categorization-relevant aspect. Categorization-induced inferences refine such aspect, while confounding categorization-irrelevant aspect with the corresponding attribute of the category-exemplar.

We show that this process gives rise to two versions of CAPM. In one version, the earnings-aspect is categorization-relevant, and in the second version, the discount rate aspect is categorization-relevant. In the first version, the relationship between excess return and stock beta is flat and it could even turn negative. Profitability of betting-against-beta, value effect, and size-premium controlling for quality arise in this version. In the second version, the relationship between excess return and stock beta is strongly positive and growth stocks do better. We argue that the first version is typically seen intraday, whereas the second version is seen during days of macroeconomic announcements and during the night.

Apart from explaining the changing relationship between excess return and beta, several other predictions of the two-version approach also hold in the data:

1) In general, strategies that do better overnight perform poorly intraday and vice versa.

2) Size and value are primarily intraday phenomena.

3) Momentum returns are earned overnight.

Categorization never turns-off in a heathy brain. So, the classical CAPM is unlikely to be ever observed. However, as discussed earlier, inefficiencies in categorization has been associated with ASD. Perhaps, our best shot at observing CAPM in its full glory is a laboratory experiment with high functioning ASD sufferers.

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Appendix A

Beta-adjusted return from categorization-adjusted CAPM is:

$$\frac{E_t\left(r_{qk}\right)-r}{\frac{Cov\left(r_{qk},r_M\right)}{Var(r_M)}}=[E_t(r_M)-r] \cdot\left\{\frac{1+\frac{n_{qk}^{*}(1-m)\left(\sigma_{qLt}^2\frac{n_{qL}^{*2}}{n_{qk}^{*2}}-\sigma_{qkt}^2\right)}{P_{qkt}P_{Mt}Cov\left(r_{qk},r_M\right)}}{1+\sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n_{qk}^{*2}(1-m)\left(\sigma_{qLt}^2\frac{n_{qL}^{*2}}{n_{qk}^{*2}}-\sigma_{qkt}^2\right)}{Var(X_M)}}\right\} \qquad (A1)$$

It follows that alpha is:

$$\alpha_{qk} = [E_t(r_M) - r] \cdot \left\{ \frac{1 + \frac{n^*_{qk}(1-m)\left(\sigma^2_{qLt}\frac{n^{*2}_{qL}}{n^{*2}_{qk}} - \sigma^2_{qkt}\right)}{P_{qkt}P_{Mt}Cov(r_{qk}, r_M)}}{1 + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n^{*2}_{qk}(1-m)\left(\sigma^2_{qLt}\frac{n^{*2}_{qL}}{n^{*2}_{qk}} - \sigma^2_{qkt}\right)}{Var(X_M)}} \right\} - [E_t(r_M) - r]$$

$$\Rightarrow \alpha_{qk} = [E_t(r_M) - r] \cdot \left\{\frac{1 + d_{qk}}{1 + c}\right\} - [E_t(r_M) - r]$$

$$\Rightarrow \alpha_{qk} = \frac{[E_t(r_M) - r]}{(1 + c)}(d_{qk} - c)$$

Appendix B

$$\frac{E_t(r_{qk}) - r}{\frac{Cov(r_{qk}, r_M)}{Var(r_M)}} = \left[(E_t(r_M) - r) + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n^*_{qk}(1-m)\left(E_t(X_{qL(t+1)})\frac{n^*_{qL}}{n^*_{qk}} - E_t(X_{qk(t+1)})\right)}{P_{Mt}}\right] - \frac{(1-m)\left(E_t(X_{qL(t+1)})\frac{n^*_{qL}}{n^*_{qk}} - E_t(X_{qk(t+1)})\right)}{P_{qkt}\beta_{qk}} \qquad (B1)$$

It follows that alpha is:

$$\alpha_{qk} = \left[(E_t(r_M) - r) + \sum_{q=1}^{Q}\sum_{k=1}^{K}\frac{n^*_{qk}(1-m)\left(E_t(X_{qL(t+1)})\frac{n^*_{qL}}{n^*_{qk}} - E_t(X_{qk(t+1)})\right)}{P_{Mt}}\right] - \frac{(1-m)\left(E_t(X_{qL(t+1)})\frac{n^*_{qL}}{n^*_{qk}} - E_t(X_{qk(t+1)})\right)}{P_{qkt}\beta_{qk}} - (E_t(r_M) - r)$$

$$\Rightarrow \alpha_{qk} = h_t - e_{qkt}$$