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Arefiev, Nikolay

State University - Higher School of Economics (Moscow),
EUREQua, Paris-1, Université - Sorbonne

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THE INCONSISTENCY PUZZLE RESOLVED: AN OMITTED VARIABLE

N. G. AREFIEV

MACROTEAM-HSE, MOSCOW

EUREQUA, PARIS-1, UNIVERSITÉ - SORBONNE

Abstract. We find that the contemporary version of the dynamic Ramsey problem omits one important variable that we take into consideration in this paper. The effect of introducing of this variable into the analysis of dynamic inconsistency is similar to that of introducing expected inflation into the Phillips curve: we show that only a policy surprise affects the attainable resource allocation set and the optimal policy. In contrast to Chamley (1986), we show that intensive capital income taxation at the beginning of optimal policy does not imply a lump-sum taxation of household wealth and cannot reduce the excess tax burden. We also demonstrate that the Ramsey policy is dynamically consistent even without commitment. We resolve the Ramsey problem and compare our results to those of Chamley on optimal capital income taxation.

: *Key words*: Inconsistency, Equilibrium policy, Optimal taxation

: *JEL classification*: E61, E62, H21

The optimal policy problem, as given by Fisher (1980) or Chamley (1986), omits one important variable that we take into consideration in this paper. In order to reveal this variable, we introduce another variable x, implicit initial household wealth lump-sum taxation due to policy revision, which we call expropriation. We demonstrate that the only reason for inconsistency is the government's desire to produce a positive expropriation, $x > 0$. Thereafter, we introduce a new variable, that was omitted by previous researchers, expected expropriation x^e. Introduction of this variable into the analysis allows us to rehabilitate the Fishers' (1980) benevolent government and to show that the Ramsey policy is, in fact, dynamically consistent.

The consequences of introduction of x^e into the analysis are similar to that of introducing expected inflation into the Phillips curve. Previous researchers implicitly assumed that the attainable resource allocation set and optimal policy depend only

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e-mail: nicolay_ arefiev@yahoo.com.

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Preliminary results of this research (the case of policy without expropriation) were presented in my paper, titled "Dynamic Consistency, Property Rights and the Benevolent Government", at 58th Econometric Society European Meeting, Stockholm, 2003.

on expropriation x. However, we show that the attainable resource allocation set and optimal policy depend on the difference between the actual value and the expected value of expropriation, which we call the expropriation surprise, $x^s = x - x^e$. This is the key result of this paper.

The central conclusion is that under rational expectations there is no reason for the dynamic inconsistency of the benevolent government. Indeed, if expected expropriation equals actual expropriation and $x^e = x$, then the expropriation surprise is zero: $x^s = 0$. We show that only a policy surprise x^s affects the attainable resource allocation and the optimal policy; consequently, the policy and the resource allocation under any value of x will be the same as under $x = 0$. This means that the government has no stimulus for inconsistency. In the Kydland and Prescott (1977) terminology, this means that the optimum coincides with the equilibrium.

In this paper we revise the Chamley (1986) - Judd (1985) result of short run intensive capital income taxation. This result holds if the government can commit to future policy, which means in our frameworks that $x^e = 0$. Under commitment, the Chamley-Judd policy may achieves a higher value of the objective function than a policy without commitment, as considered in this paper, because of a positive expropriation surprise $x^s > 0$.

However, a few arguments cast doubt on the Chamley-Judd result. The central argument is that the Chamley-Judd policy lacks a measure of realism. This policy implicitly assumes that the government can expropriate some part of household wealth without affecting expropriation expectations. In fact, if expected expropriation for some reasons differs from zero (either before or after the date the optimal policy is first implemented), then the attainable resource allocation set will differ from the one considered by Chamley; this renders impossible the implementation of the Chamley policy. In our framework, under rational expectations $x^s = 0$ and, consequently, the government cannot achieve a lower value of initial household wealth by means of intensive capital income taxation at the beginning of optimal policy. The only effect of intensive capital income taxation is an unnecessary consumption distortion.

Another argument against the Chamley policy is that for any given value of expropriation surprise x^s there exists a policy that is better than Chamley's: consumption taxation instead of capital income taxation can produce the desired value of x^s without producing the side effect of the unnecessary consumption distortion implied by Chamley's policy, and which we discuss in section 2.

Finally, the value of expropriation x in Chamley's framework is given quasi-exogenously. Chamley assumes that the consumption tax is zero and the capital income tax is bounded at 100%. If we relax either of these hypotheses, we get a higher value of expropriation than under the Chamley policy. Thus, the value of expropriation in Chamley's framework is determined by some *ad hock* hypothesis.

These arguments make us doubt that the policies of Chamley and Judd are in fact the policies that we are looking for. It is not surprising that these policies have not yet been implemented in any country.

We find the equilibrium policy ($x^s = 0$) and compare it with the one of Chamley. Under equilibrium policy, the Chamley-Judd result of zero capital income taxation may hold not only in the long run, but indeed from the very beginning of the optimal policy. However, our solution differs from the solution of Chamley and Judd in two respects. First, our solution is dynamically consistent. Second, our solution is not just an application of the Chamley-Judd's long run recommendation to the short run: at the date a policy revision is announced, the consumption and labor taxes should be adjusted in a special way in order to compensate the redistribution of wealth resulting from the abolition of capital income taxation.

We show our key result using the primal approach to optimal fiscal policy problem, developed by Ramsey (1927), Atkinson and Stiglitz (1980), Lucas and Stockey (1983), Chari and Kehoe (1998) and many others. Chari, Christiano and Kehoe (1996) apply the primal approach to optimal monetary policy problem; their method directly extends our results to the issue of inconsistency of optimal monetary policy.

Section 1 presents the model. We consider a continuous-time version of the neoclassical growth model with endogenous labor similar to the one used by Chamley. In sections 2 and 3 we show that both the attainable allocation set (section 2) and the optimal policy (section 3) depend only on the expropriation surprise x^s, but not on x and x^e separately. In section 4 we compare the equilibrium policy, which is defined as $x^s = 0$, with that of Chamley. Section 5 concludes.

1. Model

The representative household maximizes expected utility, which depends on consumption C and labor L.

$$\max_{[C,L]} E_0 \int_0^\infty e^{-\rho t} U(C, L)\, dt \tag{1}$$

We take the producer price of the final good to be equal to one. The consumer price of the final good is equal to $(1 + \tau_C)$, where τ_C is the consumption tax. The real wealth A consists of capital K and government debt B. The budget constraint is given by

$$\dot{A} = rA + WL - (1 + \tau_C) C \tag{2a}$$

$$\lim_{t\to\infty} A(t) e^{-\int_0^t r(z)dz} \geq 0 \tag{2b}$$

$$A_0 - given, \tag{2c}$$

where r and W are the after-tax equilibrium real rate of return and the real wage.

The co-state variable for equation (2a) is γ, and a is household wealth measured in units of the utility function, $a = \gamma A$.

We use the Dirac delta function and the Heaviside function to formalize the household wealth expropriation effect due to policy revision. Let $X(t)$ be the Heaviside function, which accounts for the accumulated wealth expropriation effect at date t. We assume that X is constant during the periods in which the policy is not revised, and if a revision takes place, it discontinuously jumps in order to account for the new wealth expropriation effect,

$$dX = \begin{cases} 0, & \text{if there is no policy revision at date } t \\ -\lim\limits_{dt\to 0+} \frac{a_{t+dt} - a_{t-dt}}{a_{t-dt}}, & \text{if there is a policy revision} \end{cases} \tag{3}$$

Let x be the derivative of X with respect to time,

$$x = \dot{X} \tag{4}$$

By definition, $x(t)$ is the Dirac delta function, with $x = 0$ on the intervals where the policy is not revised, and at the dates of policy revision the value of x tends to infinity. However, the integral of x is bounded on any time interval.

The household takes into account the fact that the policy may be revised. It expects that during dt there will be a revision of the policy with probability pdt. If a revision takes place, there is an implicit expropriation of $\phi \times a$ of the wealth, where ϕ is a random variable defined on $(-\infty, 1)$ with a distribution function $\xi(\phi)$. Let x^e be the expected expropriation rate per time,

$$x^e = p \int_{-\infty}^{1} \phi \xi(\phi) d\phi \tag{5}$$

Similarly to x, the variable x^e may tend to infinity at some particular points of time, but the integral of x^e on any time interval remains bounded. In contrast to x, x^e may be positive on some time intervals.

The accumulated expected wealth expropriation effect is $X^E(t)$,

$$X^E = \int_{-\infty}^{t} x^e_\tau d\tau \tag{6}$$

Expropriation surprise x^s and accumulated expropriation surprise X^s are introduced as follows:

$$\begin{aligned} x^s &= x - x^e && \text{(7a)} \\ X^s &= X - X^e && \text{(7b)} \end{aligned}$$

The first order conditions of the household problem are (see annex B for details):

$$\begin{aligned} u_C &= (1 + \tau_C)\,\gamma && \text{(8a)} \\ u_L &= -W\gamma && \text{(8b)} \\ \dot{\gamma} &= (\rho - r + x^s)\,\gamma && \text{(8c)} \end{aligned}$$

Neither the density function $\xi(\phi)$, nor the probability of expropriation p are present in the first order conditions (8); only x^s is important.

Production is not of any particular importance in problems of optimal taxation; see Judd (1999) for a discussion. We assume perfectly competitive markets and constant returns to scale, which implies that there is no profit. The production function depends on labor L and capital K, and is given by

$$Y = F(K, L) \tag{9}$$

The rate of depreciation is δ.

The capital income and labor taxes are τ_K and τ_L. The before-tax interest rate and wage are $\hat{r}$ and $\hat{W}$: $r = (1-\tau_K)\,\hat{r}$, and $W = (1-\tau_L)\,\hat{W}$. The firms' first-order conditions are given by

$$\hat{r} = F_K - \delta \tag{10a}$$

$$\hat{W} = F_L \tag{10b}$$

The government collects taxes to supply an exogenous amount of public goods G. Its budget constraint can be written as

$$\dot{B} = rB + G - \tau_C C - \tau_K \hat{r} K - \tau_L \hat{W} L \tag{11a}$$

$$\lim_{t\to\infty} B(t) e^{-\int_0^t r(z)dz} \leq 0 \tag{11b}$$

$$B_0 - given. \tag{11c}$$

Market clearing requires

$$\dot{K} = Y - C - G - \delta K \tag{12a}$$

$$K(t) \geq 0 \ \forall t \tag{12b}$$

$$K_0 - given. \tag{12c}$$

The representative household can not solve its optimization problem while fiscal policy is unknown. We suppose that there is a fiscal policy $\left[\widetilde{\tau}_C(t), \widetilde{\tau}_K(t), \widetilde{\tau}_L(t)\right]_{t\in[0,\infty)}$, which may be suboptimal, but which is given *ex ante*. The household solves its optimization problem assuming that this policy may be implemented. However, the household takes into accont that the government can revise the policy.

The government solves a modified Ramsey (1927) problem: it maximizes the household's utility (1) with respect to fiscal policy $\left[\tau_C(t), \tau_K(t), \tau_L(t)\right]_{t\in[0,\infty)}$ taking into consideration the wealth expropriation effects that occur if the optimal policy diverges from the ex ante policy.

2. Expropriation and attainable allocation set

The set of allocations that are attainable by the social planner (who finds the first-best allocation), is given by the resource constraint. This constraint may be found by substitution of the production function (9) into the market clearing condition (12):

$$\dot{K} = F(K, L) - C - G - \delta K \tag{13a}$$

$$K(t) \geq 0 \ \forall t \tag{13b}$$

$$K_0 - given. \tag{13c}$$

The implementability constraint ensures that the allocation that resolves the Ramsey problem can be decentralized without lamp-sum taxes. This constraint requires that, for a considered allocation $\left[C(t), L(t)\right]_{t\in[0,\infty)}$, there exists a vector of consumer prices that simultaneously satisfies the household's budget constraint and its first-order conditions[1]. We can derive the resource constraint from equations (2a) and (8). The expropriation-augmented implementability constraint is given by

$$\dot{a} = \rho a - U_C C - U_L L - x^s a, \tag{14}$$

There are two differences between the conventional implementability constraint (see, for example, Chari and Kehoe (1998)) and the inconsistency-augmented constraint (14). First, there is a new term $x^s a$ in (14). Secondly, for a given value of X_0, the value of a_0 is also given.

The government finds the equilibrium under ex-ante policy $[\widetilde{\tau}_C(t), \widetilde{\tau}_L(t), \widetilde{\tau}_K(t)]_{t\in[0,\infty)}$, and arrives at $\widetilde{a}_0$. The value of X_0 is historically given. Consequently, the initial conditions for (14) are given by

$$a_0 = \widetilde{a}_0 \tag{15a}$$

$$X_0 - \textit{given} \tag{15b}$$

Note, that a policy revision that produces a wealth expropriation effect at date 0 will change not only a_0, but X_0 as well.

The resource and implementability constraints with the initial and transversality conditions exactly describe the set of allocations that may be implemented in a decentralized economy without lump-sum taxes. The proof of this fact is well known in the litterature, see annex A for details.

Proposition 1. *The attainable resource allocation set depends on the expropriation surprise x^s, but not on x and x^e separately.*

Proof. The attainable resource allocation set is given by the resource constraint (13) and the expropriation-augmented implementability constraint (14). We see that only x^s enters into these constraints. □

Proposition 1 reveals why our conclusions differ from those of Chamley and Judd. Chamley and Judd implicitly assume that a positive value of x is possible only at $t = 0$, and $x^e = 0 \ \forall t \geq 0$. This is why they arrive at the result that the more the government expropriates at the beginning, the better the policy outcome.

However, expected expropriation affects the attainable resource allocation set, and there is no reasons to believe that x^e is always zero. If we assume rational expectations and, $x^e = x$, then $x^s = 0$ and the expropriation x does not affect the attainable resource allocation set.

3. Optimal policy for a given expropriation surprise

3.1. **The modified Ramsey problem.**

Assume that the vector $[x(t), x^e(t)]_{t\in[0,\infty)}$ is given exogenously. The optimal policy problem takes the form:

[1] In an economy with two goods, the implementability constraint coincides with the price-consumption curve.

$$\max_{[C(t),L(t)]} \int_0^\infty e^{-\rho t} U(C,L)\,dt \tag{16a}$$

$$\dot{K} = F(K,L) - C - G - \delta K \tag{16b}$$

$$\dot{a} = \rho a - U_C C - U_L L - x^s a \tag{16c}$$

$$\lim_{t\to\infty} a(t) e^{-\rho t} = 0 \tag{16d}$$

$$a_0 = \widetilde{a}_0 \tag{16e}$$

$$K_0 - \textit{given.} \tag{16f}$$

The co-state variable for the implementability constraint is λ (negative), and for the resource constraint μ (positive). The first-order conditions are

$$U_C(1-\lambda(1+H_C)) = \mu \tag{17a}$$

$$U_L(1-\lambda(1+H_L)) = -\mu F_L \tag{17b}$$

$$\dot{\lambda} = x^s \lambda \tag{17c}$$

$$\dot{\mu} = (\rho - (F_K - \delta))\,\mu, \tag{17d}$$

where the terms H_C and H_L are given by

$$H_C = \frac{U_{CC}}{U_C} C + \frac{U_{CL}}{U_C} L \tag{18a}$$

$$H_L = \frac{U_{CL}}{U_L} C + \frac{U_{LL}}{U_L} L \tag{18b}$$

The term H_i is a measure of the excess tax burden related to a particular form of taxation. It plays the same role as the inverse elasticity of demand in microeconomic analysis of the deadweight loss of taxation; see Atkinson and Stiglitz (1980). A possible interpretation of $(-\lambda)$ is the marginal excess burden of taxation measured in terms of utility.

3.2. **Optimal policy.** Equations (17) and the constraints to the Ramsey problem (16) give the resource allocation under the optimal policy. In order to determine the policy itself, it is necessary to combine the first order conditions of the household's problem (8) with the first order conditions of the optimal policy problem (17) taking into consideration the initial condition (16e). For convenience, we introduce here a determining cumulative tax set that uniquely determines all tax distortions. This set determines the optimal policy and consists of the following 3 cumulative taxes:

$$1 + T_{C/L} = \frac{\Phi_C}{\Phi_L} \tag{19a}$$

$$1 + T_{C(t+z)/C(t)} = \frac{\Phi_{C,t+z}}{\Phi_{C,t}} \exp\left[X^s_{t+z} - X^s_t\right] \tag{19b}$$

$$1 + T_{C(0)/A(0)} = (1 + \widetilde{\tau}_C(0)) \frac{U_C(C(0),L(0))}{U_C(\widetilde{C}(0),\widetilde{L}(0))}, \tag{19c}$$

where

$$\Phi_{C,t} = \left(1 - \lambda_t \left(1 + H_{C,t}\right)\right)^{-1} \tag{20a}$$

$$\Phi_L = \left(1 - \lambda \left(1 + H_L\right)\right)^{-1} \tag{20b}$$

and

$$1 + T_{C/L} = \frac{1 + \tau_C}{1 - \tau_L} \tag{21a}$$

$$1 + T_{C(t+z)/C(t)} = \frac{1 + \tau_C(t+z)}{1 + \tau_C(t)} \exp\left(\int_t^{t+z} \tau_K r(s) ds\right) \tag{21b}$$

$$1 + T_{C(0)/A(0)} = \left(1 + \tau_C(0)\right) \tag{21c}$$

Equation (19a) was found from (8a), (8b), (17a) and (17b), equation (19b) was found from (8a), (8c), (17a) and (17d), and equation (19c) was found from the definition $a = \gamma A$, the constraint (16e), and equation (8a).

There is an infinite number of policies that implement (19) and decentralize the optimal allocation. In order to get the only policy, we exogenously define the dynamics of one of the tax rates. Suppose that the consumption tax is constant, and its value is chosen to satisfy (19c):

$$1 + \tau_C = 1 + T_{C(0)/A(0)} \tag{22a}$$

In this case, the optimal capital and labor taxes are given by

$$1 - \tau_L = \frac{1 + \tau_C}{1 + T_{C/L}} \tag{22b}$$

$$\tau_K \hat{r} = \frac{\dot{T}_{C(t)/C(0)}}{1 + T_{C(t)/C(0)}} \tag{22c}$$

Proposition 2. *For any given dynamics of x^s, the solution to the optimal policy problem (16) is dynamically consistent.*

Proof. From (16) it can be immediately seen that, if x and x^e are given, the solution to the problem is dynamically consistent: all state variables are in fact *state* variables, which do not include forward-looking terms. If a formal argument is required, the consistency may be shown, for example, by comparing the solutions obtained by two alternative methods: the Pontriagin and Bellman principles. The Pontriagin principle maximizes the discounted value of the objective function and may be dynamically inconsistent. The Bellman principle recognizes that in the future there will be chosen a plan that is optimal for that period, and that resolves the consistency problem. From the fact that these two solutions are equivalent, it follows that the solution to the optimal policy problem is dynamically consistent. □

A special case is $x^s_t = 0\ \forall t$. An application of proposition 4 to this case is that under $x^s = 0$ the optimal policy is also dynamically consistent.

Proposition 3. *Optimal policy depends on expropriation surprise x^s, but not on x and x^e separately.*

Proof. See the equations that describe the optimal policy (19). □

Proposition 3 supplements proposition 3, and says that not only the attainable resource allocation set, but the optimal policy as well depends only on the expropriation surprise.

Propositions 3 and 5 encourage us to analyze the equilibrium policy, defined as $x_t^s = 0 \ \forall t \geq 0$, instead of the policy of Chamley (1986) and Judd (1985).

4. Equilibrium policy

Let us consider the case of *equilibrium policy*, $x_t^s = 0 \ \forall t \geq 0$. A special case of the equilibrium policy is *policy without expropriation*, where $x_t^e = x_t = 0 \ \forall t$.

For the case of equilibrium policy, our conclusions are similar to the long run conclusions of Chamley and Judd. If H_C is constant, the optimal cumulative tax $T_{C(t)/C(0)}$ is zero (see (19b), (20a) and (17c)). Equation (22c) shows that the optimal capital income tax in this case is also zero. This is possible in the two cases: either in the case of isoelastic preferences (for example, $U(C,L) = \frac{C^{1-\theta}-1}{1-\theta} + V(L)$), or if the economy is on the balanced growth path[2].

There are two differences between the equilibrium policy and the policy of Chamley and Judd. First, the equilibrium policy is dynamically consistent. Second, the optimal capital income tax under the equilibrium policy may be zero not only in the long run, but also in the short run.

However, our solution does not replicate the Chamley and Judd long run recommendations into the short run: in the short run the optimal consumption and labor taxes are adjusted in order to avoid any change in a_0. For example, a capital income tax reduction increases the shadow price of the household wealth. In order to compensate for this effect, the government needs to increase the consumption tax and to decrease the labor tax.

If we neglect certain second-order effects that we discuss in the next paragraph, then the required changes in consumption and labour taxes may be approximately calculated in the following manner. Suppose that a decrease in the capital income tax increases the after-tax interest rate by 10%. Then the capitalists become 10 % richer, and to compensate this effect, the consumer price of the final good $(1+\tau_C)$ should be increased by 10%. The new value of the labour tax should ensure that the infratemporal government budget constraint is satisfied.

This arithmetics works well in Y=AK - type models, and when the excess tax burden of distortionary taxation is not too high. However, in the Ramsey-Cass-Koopmans framework, the effect of a decrease of τ_K on a is lower, and requires a smaller increase of the consumption tax. In addition, the capital income tax reduction produces another effect: this tax should be substituted by others. This will increase the cumulative tax $T_{C/L}$, decrease the labor supply, decrease the before tax interest rate, and reduce a_0, and this requires a decrease in the consumption tax.

In the general case, it is not clear whether a particular tax should be increased or decreased.

Note that on the balanced growth path all taxes are constant. Consequently, the optimal debt to GDP ratio is also constant.

[2]These two cases are not too different: the balanced growth path is possible only if preferences are isoelastic in consumption for the *realized* allocation.

5. Conclusions

Previous papers implicitly assumed that the attainable allocation set and the optimal policy depend on the expropriation of household wealth due to a policy revision at the beginning of optimal policy. However, we show that only an expropriation surprise affects the attainable resource allocation set and the optimal policy.

If we knew exactly what affects expected expropriation, we could define the attainable set of expropriation surprises and maximize households' utility on this set. However, expectations of expropriation depend on a large number of factors, such as credibility, commitment, history, economic and cultural development, government debt, sunspots in terms of Azariadis, and so on. The exact relationships are unknown, so we cannot solve the problem of maximization.

However, a long discussion in 1970s and 1980s on the ways inflationary expectations are formed induced researchers to use rational expectations by default. The reason why the rational expectations hypothesis prevails in contemporary research is the weakness of the alternatives: any other particular hypothesis is worse than that of rational expectations. In our framework, under rational expectations $x = x^e$, wherefrom $x^s = 0$, and the government cannot affect the attainable allocation set by means of an implicit expropriation of household wealth at the beginning of the optimal policy.

Under $x^s = 0$ intensive capital income taxation at the beginning of the optimal policy does not imply a lump-sum taxation of households' initial wealth, and creates only an unnecessary consumption distortion. Thus, in contrast to the Chamley result, we show that intensive capital income taxation at the beginning of optimal policy is suboptimal.

The only reason for the inconsistency of the Chamley policy is the desire to produce a positive expropriation surprise. Under rational expectations $x^s = 0$ and therefor an expropriation surprise is impossible and the policy is dynamically consistent.

Appendix A. The attainable allocation set (comments to section 2)

The derivation of the attainable allocation set that we use in section 2 is well-known in the literature, see, for example, Lucas and Stockey (1983).

We get the resource (13) and implementability (14) constraints from conditions that are satisfied for any equilibrium allocation, consequently, they are also satisfied for any equilibrium allocation.

If an allocation $[C(t), L(t)]_{t\in[0,\infty)}$ satisfies equation (14), then, for any given strictly positive dynamics of one of the consumer prices $[r(t), \tau_c(t), W(t)]_{t\in[0,\infty)}$, there exists dynamics of the other prices such that the household will choose the given allocation. Indeed, the first-order conditions (8) and diffenitions $a = \gamma A$ and (3) give prices such that these conditions are satisfied, and substitution of these prices into the implementability constraint gives the households' budget constraint; consequently, the last is also satisfied.

If an allocation $[C(t), L(t)]_{t\in[0,\infty)}$ satisfies the resource constraint (13), then we can find the dynamics of the producer prices $\left(\hat{r}, \hat{W}\right)$ under which firms will choose an input-output vector such that the equilibrium market condition is satisfied. Indeed, from equation (13) and the initial conditions we can calculate the dynamics of K that gives the dynamics of output $Y = C + G + \dot{K} + \delta K$. Knowing the dynamics of Y, K, and L, the firms' first-order conditions can be used to find the prices $\left(\hat{r}, \hat{W}\right)$ under which the firms choose the considered allocation, $\hat{r} = F_K$ and $\hat{W} = F_L$.

If both constraints are satisfied, the government budget constraint is also satisfied by Walras' law. Thus, these constraints guarantee that there exist vectors of consumer and producer prices such that all equilibrium conditions are satisfied. The cumulative tax rates that decentralize the considered allocation may be found from the difference between the consumer and producer prices, for example,

$$1 + T_{C,L} = \frac{(1+\tau_C)}{W/\hat{W}}.$$

Appendix B. First-order conditions to the wealth expropriation-augmented household problem

Let $V(A(t), X(t), t)$ be the value function,

$$V(A(t), X(t), t) = \max_{[C,L]} E_t \int_t^{\infty} e^{-\rho\tau} U(C, L)\, d\tau \tag{23}$$

Taking into account that

$$E_t V(A(t+dt), X(t+dt), t+dt) = (1 - pdt)\, V(A(t+dt), X(t), t+dt) + pdt \int_{-\infty}^{1} V(A(t+dt), X(t)+\phi, t+dt)\xi(\phi)d\phi, \tag{24}$$

the Bellman equation can be written as:

$$\begin{aligned} 0 = \max_{[C,L]} \Big(& e^{-\rho t} U(C, L) + \\ & \frac{V(A(t+dt), X(t), t+dt) - V(A(t), X(t), t)}{dt} + \\ & p \int_{-\infty}^{1} \left(V(A(t+dt), X(t)+\phi, t+dt) - V(A(t+dt), X(t), t+dt)\right) \xi(\phi)d\phi \Big) \end{aligned} \tag{25}$$

We will use a Taylor decomposition for the second term, and substitute $\dot{A}$ from (2). Taking the limit as $dt \to 0+$, this gives:

$$\begin{aligned} 0 = \max_{[C,L]} \Big(& e^{-\rho t} U(C, L) + \\ & V_A(A(t), X(t), t)(rA + WL - (1+\tau_C)C) + V_t(A(t), X(t), t) + \\ & p \int_{-\infty}^{1} \left(V(A(t), X(t)+\phi, t) - V(A(t), X(t), t)\right) \xi(\phi)d\phi \Big) \end{aligned} \tag{26}$$

Equation (26) is the Bellman equation for the problem. The first-order conditions are:

$$e^{-\rho t}U_C = (1+\tau_C)V_A\left(A(t), s(t), t\right) \tag{27a}$$

$$e^{-\rho t}U_L = -WV_A\left(A(t), s(t), t\right) \tag{27b}$$

Let γ be the shadow price of the household's wealth:

$$\gamma = V_A e^{\rho t}, \tag{28}$$

then equations (27) give (8a) and (8b).

Application of the envelope theorem gives:

(29)

$$0 = V_{AA}(A(t), X(t), t)\dot{A} + rV_A(A(t), X(t), t) + V_{At}(A(t), X(t), t)$$
$$p\int\limits_{-\infty}^{1}\left(V_A(A(t), X(t)+\phi, t) - V_A(A(t), X(t), t)\right)\xi(\phi)d\phi\Bigg)$$

Differentiate (28) with respect to time:

$$\dot{\gamma} = \left(V_{AA}\dot{A} + V_{At}\right)e^{\rho t} + \rho\gamma \tag{30}$$

From equations (28), (29) and (30), taking into account (3), (4), and (5) we arrive at the last first-order condition to the expropriation-augmented household problem (8c).

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