FIR-GEM: A SOE-DSGE Model for fiscal policy analysis in Ireland

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FIR-GEM: A SOE-DSGE Model for fiscal policy analysis in Ireland

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Abstract

This paper presents FIR-GEM: Fiscal IRish General Equilibrium Model. FIR-GEM is a small open economy DSGE model designed as fiscal toolkit for fiscal policy analysis in Ireland. To illustrate the model’s potential for fiscal policy analysis, we conduct three types of experiments. First, we analyse the fiscal transmission mechanism through which Irish fiscal policy affects the Irish economy. Second, we compute fiscal multipliers for the main tax-spending instruments, namely government consumption, public investment, public wage bill, public transfers, consumption, labour and capital tax. We focus on a fiscal policy stimulus that is either implemented through spending increases or tax cuts. Third, we perform robustness analysis on key structural characteristics that can affect quantitatively the size of fiscal multipliers. We find that the size of fiscal multipliers in the Irish economy heavily depends on its degree of openness, the method of fiscal financing employed, the elasticity of the sovereign risk premia to Irish debt dynamics and the flexibility of Irish labour and product markets.

Keywords: Fiscal policy, DSGE, Ireland, Openness.

JEL classification: E62, F41, F42

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1 Introduction

**FIR-GEM: Fiscal IRish General Equilibrium Model** is a small open economy dynamic stochastic general equilibrium model (SOE-DSGE) that attempts to capture the main features of the Irish economy. The primary aim of FIR-GEM is to serve as a fiscal policy toolkit for fiscal policy analysis in Ireland. The present model belongs to the class of medium-scale DSGE models that are widely used in policy institutions. These models are based on microeconomic foundations and economic agent’s intertemporal choice. The general equilibrium framework captures the interaction between policy actions and private agent’s economic behaviour. These features are vital for fiscal policy analysis. Fiscal policymaking can utilize a rich menu of tax and spending instruments that could result in a wide range of macroeconomic outcomes. Fiscal actions do not only affect private agents’ current economic decisions but their economic behaviour over time (intertemporal choices) by influencing their expectations about future fiscal policy. This makes fiscal policy analysis a complex task (see Leeper (2010)). To evaluate and rank alternative fiscal policies, research economists should take into account an explicit analysis of the structure of the economy, private agent’s expectations and the dynamic adjustment of their economic behaviour to those fiscal policies.

In addition, any macroeconomic and fiscal policy analysis should take into account the specific structure of the Irish economy. In the next paragraphs, we summarize some structural characteristics of the Irish economy that the present model is designed to capture.

First, a key structural characteristic of Ireland is its exceptional degree of openness. Ireland’s openness is reflected in a number of key macroeconomic aggregates. In particular, the larger size of the Irish tradable sector vis-à-vis the non-tradable sector. For example, the ratio of the value added in the tradable sector to the value added in both sectors averages 59% over the period 2001 to 2014. Moreover, the Irish tradable sector is dominated by foreign affiliated firms (Multinational Enterprises), this is reflected in the sector specificity and export-orientation of the tradable sector in our model.

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1 For example European Commission DG ECFIN uses the Quest III model, see Ratto, Roeger, and in ’t Veld (2009) and the Global Multi-Country Model (GM), see Albionico et al. (2017). The ECB uses the New Area Wide Model (NAWM), see Warne, Coenen, and Christoffel (2008) and Coenen et al. (2018). While several European countries have developed DSGE models, e.g. REMS, see Bosa et al. (2010), and FiMOD for Spain, see Stahler and Thomas (2012). BoGGEM for Greece, see Papageorgiou (2014), GEAR for Germany see Gadatsch et al. (2015), AINO 2.0 see Kilponen et al. (2016) and many others.

2 For a thorough discussion on the current state and role of DSGE models in policymaking see Gurkaynak and Tille (2017), Reis (2017) and Christiano, Eichenbaum, and Trabandt (2018).


5 The Irish tradable sector is dominated by foreign affiliated firms (Multinational Enterprises), this is reflected in the sector specificity and export-orientation of the tradable sector in our model.
sector is highly export-oriented\textsuperscript{6}, for example the exports to GDP ratio averages 96% between 2001 and 2014\textsuperscript{7}. Domestic consumption and production heavily rely on imports while the Irish trade surplus averages 14% as a share of GDP over the period 2001 to 2014. In order to capture these characteristics of the Irish economy, we incorporate two sectors of domestic private production, i.e. we distinguish between the tradable and the non-tradable sectors. The factors of production are sector specific while sectoral reallocation entails production costs (as in Uribe and Schmitt-Grohe (2017)).

Second, Ireland is modelled as a small open economy participating in a currency union (Eurozone). This implies that households and government can participate in global financial and capital markets but their behaviour cannot influence the world interest rate\textsuperscript{8}. As a result, we follow Schmitt-Grohe and Uribe (2003) and assume that the nominal interest rate faced by domestic residents in the world financial markets is an increasing function of the deviation of the Irish public debt to GDP ratio from a threshold level (for similar modelling see Garcia-Cicco, Pancrazi, and Uribe (2010) and Philippopoulos, Varthalitis, and Vassilatos (2017)). This assumption is empirically relevant and has non trivial implications for the efficacy of fiscal policy. Being a member of the Eurozone implies the loss of monetary independence and a fixed nominal exchange rate regime for the Irish economy. Thus the only macroeconomic policy tool available is fiscal policy.

Third, Irish fiscal policy over the period 2001 to 2014 is characterized by relatively low automatic stabilizers.\textsuperscript{9} Indicatively, government expenditures and tax revenues as shares of GDP are among the lowest within Eurozone countries, they amount to 38% and 34\%\textsuperscript{10} of GDP between 2001 and 2014.\textsuperscript{11} The present model incorporates a rich menu of fiscal policy instruments. In particular, the Government has four spending instruments at its disposal, namely government consumption, public investment, public wages and agent-specific public transfers and three tax instruments, namely consumption, labour and capital taxes. In addition, the Government can issue domestic and foreign public debt (along with taxes levied on households) which are

\textsuperscript{6}For more details on the composition of the Irish tradable sector see Barry and Bergin (2012) and Barry and Bergin (2018).
\textsuperscript{7}Although this figure reduces to 52\% in value added terms for 2001-2011, it still highlights the importance of exports in the Irish economy.
\textsuperscript{8}As is well known, a small open economy with an exogenous world interest rate induces non-stationary dynamics.
\textsuperscript{9}For e.g. Kostarakos and Varthalitis (2019) compare effective tax rates in Ireland with Eurozone average and find that Irish ETRs rank amongst the lowest.
\textsuperscript{10}We also express Irish fiscal aggregates as GNI shares, since GDP and GNI differ by 15\% on average over 2001-2014. Although the gap between Eurozone averages and Ireland closes, Irish fiscal aggregates remain amongst the lowest within Eurozone countries. For example government expenditures and tax revenues amount to 45\% and 39\% in Ireland while Eurozone averages are 48\% and 45\% respectively.
\textsuperscript{11}Ireland recently implemented a front-loaded fiscal consolidation package via mostly expenditure cuts (for more details on the Irish fiscal consolidation see McCarthy (2015) and Larch et al. (2016)). Irish public debt to GDP ratio peaked to 120\% in 2012, but post 2014 Ireland succeeded in stabilizing domestic public finances and restoring access to international financial markets.
used to finance public expenditures. We adopt a rule-like approach to policy in that fiscal policy is conducted via simple and implementable fiscal policy rules.\textsuperscript{12} Here, all the main tax-spending instruments are allowed to react to the public debt to GDP ratio and the level of the deficit so as to ensure fiscal sustainability.\textsuperscript{13} In addition, a firm in the public sector utilizes goods purchased from the private sector, public employment and public capital to produce a good that provides both welfare-enhancing and productivity-enhancing services.\textsuperscript{14}

Fourth, we incorporate in the model several features that quantitatively matter for the fiscal transmission mechanism and are empirically relevant (see e.g. in Zubairy (2014) and in Leeper, Traum, and Walker (2017)). Namely households with non-Ricardian behaviour (as in e.g. Gali, Lopez-Salido, and Valles (2007)), real frictions and nominal rigidities, while, we allow for complementarity/substitutability between private/public consumption and productivity-enhancing public goods.

We calibrate the model using Irish annual data over the period 2001-2014\textsuperscript{15}. To illustrate the model's ability to assess fiscal policy, we conduct three types of simulations. First, we use the model to examine the fiscal transmission mechanism through which Irish fiscal policy affects the Irish economy. Second, we compute fiscal multipliers for the main tax-spending instruments, namely government consumption, public investment, public wage bill, public transfers, consumption, labour and capital tax. We focus on a fiscal stimulus policy that is either implemented through spending increases or tax cuts. Third, we perform robustness analysis on structural characteristics that can affect qualitatively and quantitatively the size of fiscal multipliers in the Irish case. These include the degree of openness, alternative fiscal financing methods, the sensitivity of the international nominal rate at which Ireland borrows from the rest of the world to Irish public debt dynamics, complementarity/substitutability of public and private consumption, flexibility of the Irish labour and product markets.

The main results are as follows: first Irish fiscal multipliers are expected to be smaller in magnitude than most EU countries due to the degree of openness of the domestic economy and the large influence of the tradable sector. Second fiscal policy affects the composition of aggregate output. The fiscal stimulus works

\textsuperscript{12}In Schmitt-Grohe and Uribe (2005) and (2007) “simple” and “implementable” means that policy can easily and effectively be communicated to the public; that is policy instruments react to a small number of easily observed macroeconomic indicators.\textsuperscript{13}Most European countries set their policy by following some type of fiscal rules so this is an empirically relevant assumption (see European Commission 2012).\textsuperscript{14}For DSGE models that incorporate a public production function see e.g. Forni et al. (2010), Papageorgiou (2014), Economides et al. (2013) and (2017).\textsuperscript{15}We focus over the period 2001-2014 since there are well documented problems with Irish national accounts after 2014 for more details see Fitzgerald (2018).

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solely through the non-tradable sector while the tradable sector remains unaffected or contracts in size. The latter is crucial in the Irish case where the tradable sector is significantly larger than the non-tradable sector. Third a fiscal stimulus via spending produces more output than a stimulus via tax cuts in the short run; that is spending multipliers are consistently larger than tax multipliers. Fourth, in terms of the effect on GDP in the first year, the most effective Irish fiscal instruments are as follows: public investment, government consumption, consumption taxes, capital taxes, public transfers, public wages and, finally, labour taxes. Fifth, a fiscal expansion via spending is expected to have a negative effect on the competitiveness of the domestic economy. Our results show a deterioration of the Irish external balance in the early years of the stimulus era; that is the fiscal stimulus is likely to crowd out exports and at the same time crowd in imports. Sixth, income tax cuts induce a smaller effect on the Irish external balance. Capital and labour tax cuts are expected to reduce production costs and prices in Ireland vis-à-vis the rest of the world leading to an improvement in the competitiveness of the Irish economy. As such, a fiscal expansion via tax cuts induces supply-side effects that take time to materialize (i.e. multipliers are smaller in the short run) but their effects are long lasting. Seventh, the method of fiscal financing is crucial for the efficacy of a fiscal stimulus. A spending stimulus financed via tax increases mitigates the positive effect on GDP.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 solves the model. Section 4 develops the calibration strategy and presents the steady state solution. Section 5 analyses the fiscal transmission mechanism of the model. Section 6 quantifies fiscal multipliers in the Irish case. Section 7 conducts a robustness analysis, while Section 8 concludes and discusses possible avenues for future research. An appendix presents details of the model.

2 Related Literature

This paper contributes to the literature on medium scale SOE-DSGE models for fiscal policy analysis in policy institutions. Our work emphasizes the role played by the degree of openness in the fiscal transmission mechanism in Ireland and quantifies fiscal multipliers for the main tax-spending Irish fiscal instruments. There are three papers that quantify fiscal multipliers for Ireland, in particular, Clancy, Jacquinot, and Lozej (2016) compute the government consumption and investment multiplier for Ireland and Slovenia using a global DSGE
model (EAGLE), Bergin and García-Rodríguez (2019) use the ESRI COSMO large-scale macroeconometric model\textsuperscript{16} to compute fiscal multipliers for tax-spending Irish fiscal instruments and Ivory, Casey, and Conroy (2019) estimate spending multipliers using a suite of VAR-type models. To the best of our knowledge this is the first paper that quantifies all the main tax-spending multipliers for Ireland using a medium scale SOE-DSGE model with a rich fiscal sector, analyses the associated fiscal transmission mechanism, provides an Irish fiscal instrument ranking with respect to their effect in the Irish GDP and computes the effects of fiscal policy on the composition of aggregate output and the competitiveness of the Irish economy.

DSGE models for Ireland\textsuperscript{17} include EIRE Mod, see Clancy and Merola (2016b), which however does not incorporate an explicit fiscal sector. Klein and Ventura (2018) develop a growth model for Ireland to study the Ireland’s remarkable historical economic performance over 1980-2005 focusing on the role of fiscal policy while Ahearne, Kydland, and Wynne (2006) study Ireland’s depression episode over the period from 1973 to 1985.

This paper also contributes to the vast literature on fiscal multipliers using DSGE models by quantifying fiscal multipliers in Ireland for the main tax-spending instruments\textsuperscript{18}. For example, a similar study, Kilponen et al. (2015), compare tax-spending multipliers across fourteen countries in Europe. Our contribution also lies in the field of fiscal policy effects on the trade balance and the composition of output, e.g. Monacelli and Perotti (2008) and Monacelli and Perotti (2010) study the effect of government spending on trade balance and international relative prices\textsuperscript{19}.

\textsuperscript{16}Bergin et al. (2017) develop a large-scale macroeconometric model estimated for Ireland in the spirit of NiGEM model developed by National Institute of Economic and Social Research.

\textsuperscript{17}Clancy and Merola (2016a) and Lozej, Onorante, and Rannenberg (2018) develop SOE-DSGE models with financial frictions for Ireland focusing on macroprudential policies.

\textsuperscript{18}The literature on fiscal multipliers is voluminous for a detailed review see Battini et al. (2014) and the references therein. Some selective references are: Coenen, Straub, and Trabandt (2013) use a suite of DSGE models to compute multipliers, Leeper, Traum, and Walker (2017) use bayesian techniques to quantify the size of multipliers across different model specifications, Zubairy (2014) and Drautzburg and Uhlig (2015) use estimated models to compute fiscal multipliers for U.S economy. Christiano, Eichenbaum, and Rebelo (2011) compute spending multipliers when the zero lower bound on the nominal interest rate binds, while Canzoneri, Collard, Dellas, and Diba (2016) find asymmetric multipliers over the business cycle.

\textsuperscript{19}Our results for Ireland with the remarkable degree of openness confirm empirical studies, like Benetrix and Lane (2010) and Ilzetzki, Mendoza, and Vegh (2013), that fiscal multipliers in open economies are smaller than in closed economies.
3 A Small Open Economy Model

3.1 Informal description of the model

This section develops a small open economy dynamic general equilibrium model (SOE-DSGE) with a rich fiscal sector calibrated for Ireland. This model is designed as a fiscal policy toolkit for Ireland, and thus contains several key features seeking to resemble the structure of the Irish economy and, hence, be suitable for fiscal policy analysis. The model: (a) distinguishes between tradable and non-tradable production sectors; (b) allows for sector-specificity of factor inputs; (c) incorporates heterogeneous agents; (d) empirically relevant nominal and real frictions; (e) debt-elastic interest rate; (f) delegated monetary policy (Ireland is a member of a currency union) and independent national fiscal authority and (g) it allows for an explicit fiscal sector with a rich menu of spending-tax fiscal instruments, explicit fiscal rules and a public production function.

This model belongs to the class of small open economies and thus incorporates several open economy features. In particular, households and government can participate in international financial markets. To ensure stationarity we assume that the international interest faced by domestic borrowers is debt elastic, as in e.g. Schmitt-Grohe and Uribe (2003). Moreover, domestic economic agents can engage in international trade, thus they can consume and invest in imports; while a share of domestic production is exported to the rest of the world.

The model consists of three types of economic agents: households, firms and a government. We incorporate two type of households: first, forward-looking optimizing agents which have access to domestic and international financial and capital markets while receiving dividends from domestic firms. These households are referred to as Ricardians or Savers. Second, financially constrained agents which do not have access to financial and capital markets, that is they live hand to mouth and each period consume all of their after tax disposable income. These households are referred to as non-Ricardians or non-Savers. The introduction of the latter type of households in the model has non-trivial effects in the transmission mechanism of fiscal policy actions (see e.g. Gali, Lopez-Salido, and Valles (2007), Cespedes, Fornero, and Gali (2011) and Leeper, Traum, and Walker (2017)). Non-Savers are relatively more prone to changes in government expenditures or/and taxes since they cannot smooth out changes in their disposable income over time. Both type of households provide labour services to the three sectors of the economy, namely tradable, non-tradable and public sectors while
they optimally allocate hours worked among these sectors. Both types of households pay consumption taxes and receive household-specific public transfers, Ricardians pay labour and income taxes while non-Ricardians pay only labour taxes.

The model incorporates private and public production. There are two stages of private production. In the final stage, the final good, that is used for private and public consumption and investment, is produced. There are two firms namely a final good and a composite tradable good producer at this level. The final good producer utilizes the composite tradable and the intermediate non-tradable good to produce the final good. Similarly, the composite tradable good producer utilizes the home produced tradable good and the imported good to produce the composite tradable good.

In the intermediate stage, the intermediate tradable and non-tradable bundles are produced. There are $N^i$ intermediate non-tradable firms. Each non-tradable firm indexed by $i$ hires labour and rents physical capital from households to produce a differentiated variety $i$. A non-tradable distributor combines all varieties, $i = 1..N^i$, into an intermediate non-tradable bundle. Similarly, there are $N^j$ intermediate home tradable firms. Each home tradable firm indexed by $j$ hires labour and rents physical capital from households to produce a differentiated variety $j$. A tradable distributor combines all varieties, $j = 1..N^j$, into an intermediate home tradable bundle.

Firms in the public sector use goods purchased from the private sector, public employment and public capital to produce a good that provides both utility-enhancing and productivity-enhancing services. The associated public spending inputs are set exogenously by the government.

In terms of economic policy, Ireland is a member of the Eurozone thus we focus on a monetary policy regime in which the nominal exchange rate is fixed and there is no monetary policy independence (this mimics membership in a currency union). Fiscal policy is conducted via simple fiscal policy rules.

### 3.2 Households

The economy is populated by $N$ number of households. The population is comprised of two types of households, Ricardian households (or Savers) indexed by the upperscript $r = 1..N^r$ and non-Ricardians (or Non Savers) indexed by the upperscript $nr = 1..N^{nr}$ where $N^r + N^{nr} = N$. 

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3.2.1 Ricardian Households (Savers)

Preferences and Constraints There are \(N^r\) Ricardians/Savers indexed by the superscript \(r = 1...N^r\).

Each household \(r\) maximizes its expected discounted lifetime utility, \(V^r_0\), in any given period \(t\):

\[
V^r_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U^r \left( \bar{c}^r_t, l^H_t, l^{NT}_t, l^P_t \right)
\]

where \(\bar{c}^r_t \equiv c^r_t + \vartheta^g y^r_t\) denotes composite consumption comprising of \(c^r_t\) consumption of the final good (defined in section 3.3.1 below) and \(y^r_t\) consumption of public good per capita produced by a state firm (defined in section 3.4.2), \(\vartheta^g > (\prec) 0\), measures the degree of substitutability (complementarity) between public and private consumption, \(l^H_t, l^{NT}_t\) and \(l^P_t\) denote hours of work in the tradable, non-tradable and public sectors\(^{20}\), respectively, \(0 < \beta < 1\) is a subjective discount factor and \(E_0\) is the rational expectations operator conditional on information at time 0. Each household’s sequential budget constraint in period \(t\) is given by (in nominal terms):

\[
P_t \left(1 + \tau^r_t\right) c^r_t + P_t x^H_t + P_t x^{NT}_t + P_t b^r_t + S_t P_t f^r_t + \Phi^r (f^r_t, f^{\tau^r}_t)
\]

\[
= (1 - \tau^r_t) P_t \left( w^H_t l^H_t + w^{NT}_t l^{NT}_t + w^P_t l^P_t \right) + (1 - \tau^k_t) P_t \left( \omega^r_t + k^{NT}_t \right) + R_{t-1} P_{t-1} b^r_{t-1} + Q_{t-1} S_{t-1} P_{t-1} f^{\tau^r}_{t-1} - P_t \tau^r_t
\]

where \(P_t\) is the nominal price of the final good, \(l^H_t, l^{NT}_t, l^P_t, b^r_t\) and \(f^r_t, f^{\tau^r}_t\) are the real value of the end-of-period domestic government bonds and internationally traded assets (the latter is expressed in foreign currency) respectively\(^{21}\), \(S_t\) is the nominal exchange rate defined as the domestic currency price of one unit of foreign currency, \(R_{t-1}, Q_{t-1} \geq 1\) denote the gross nominal return of domestic government bonds and international assets between \(t - 1\) and \(t\) respectively, \(\tau^r_t, \tau^a_t, \tau^k_t\) are consumption, labour and capital tax rates respectively, \(\tau^l_t\) is public transfers.

\(^{20}\)Our modelling implies that each household is comprised of many members which can be employed in all three sectors. Then, each household allocates its members to each sector by maximizing its lifetime utility, for similar modelling see Uribe and Schmitt-Grohe (2017). See Ardagna (2001), Forni, Gerali, and Psani (2010), Economides, Papageorgiou, Philippopoulos, and Vassilatos (2013) and Papageorgiou (2014), Economides, Papageorgiou, and Philippopoulos (2017) for models which include public employment.

\(^{21}\)For simplicity and notational convenience and without loss of generality, all quantities and relative prices will be expressed in terms of the final good as in Uribe and Schmitt-Grohe (2017).
targeted to Ricardian household $r$. Finally, borrowing on the international market entails an adjustment cost $\Phi^*()$. The laws of motion for physical capital in tradable and non-tradable sectors are given by:

$$k_{t}^{H,r} = \left(1 - \delta^H\right) k_{t-1}^{H,r} + x_{t}^{H,r} - \Phi^H\left(k_{t}^{H,r}, k_{t-1}^{H,r}\right) \tag{3}$$

$$k_{t}^{NT,r} = \left(1 - \delta^{NT}\right) k_{t-1}^{NT,r} + x_{t}^{NT,r} - \Phi^{NT}\left(k_{t}^{NT,r}, k_{t-1}^{NT,r}\right) \tag{4}$$

where $\delta^H$ and $\delta^{NT}$, $\Phi^H()$ and $\Phi^{NT}()$ are sector specific depreciation rates and adjustment costs respectively. Functional forms of the period utility and the adjustment costs are specified in Appendix G.

A key element of the Irish macroeconomic structure is the presence of two distinct sectors of production, i.e. the tradable and non-tradable sector. Both have different structural characteristics. We allow for sector specificity by including features that aim to slow down the sectoral re-allocation of factors of production, i.e. labour and physical capital (as in e.g. Uribe and Schmitt-Grohe (2017)). To do this, we, first, allow for imperfect substitutability of labour across different sectors by introducing sector-specific hours worked as separate arguments in the utility function. Second, we allow for sector-specific depreciation rates and capital adjustment costs in the associated laws of motion (3) and (4). Both elements imply that factor movements among sectors entails costs; the magnitude of these costs are calibrated to reflect the relevant Irish data.

**Choice of allocations** Each household $r$ maximizes its lifetime utility (1) in any given period $t$ by choosing purchases of the final consumption good, $c_{t}^{r}$, hours of work in the tradable, $l_{t}^{H,r}$, non-tradable sector, $l_{t}^{NT,r}$, and public sector, $l_{t}^{P,r}$, the end-of-period physical capital stocks, $k_{t}^{H,r}$, and $k_{t}^{NT,r}$, the end-of-period holdings of domestic government bond, $b_{t}^{r}$, and international traded assets expressed in foreign currency, $f_{t}^{r}$, subject to the constraint (2) (in which we incorporate constraints (3) and (4)). The Lagrange multiplier associated with constraint (2) is $\Lambda_{t}^{r}$. The first-order conditions with respect to $c_{t}^{r}$, $l_{t}^{H,r}$, $l_{t}^{NT,r}$, $l_{t}^{P,r}$, $k_{t}^{H,r}$, $k_{t}^{NT,r}$, $b_{t}^{r}$ and $f_{t}^{r}$ are given by:

$$\frac{\partial U_{t}^{r}}{\partial c_{t}^{r}} = \Lambda_{t}^{r} \left(1 + \tau_{t}^{r}\right) \tag{5}$$
3.2.2 Non-Ricardian Households (Non-savers)

In line with the empirical evidence see e.g. in Gali, Lopez-Salido, and Valles (2007), Cespedes, Fornero, and Gali (2011) and Leeper, Traum, and Walker (2017), we incorporate a fraction of financially constrained households which we refer to as non-Ricardian households or non-Savers.

Preferences and Constraints Each non-Ricardian household \( nr \) has the same preferences as Ricardian households and chooses \( c_{t}^{nr}, l_{t}^{H,nr}, l_{t}^{NT,nr} \) and \( l_{t}^{P,nr} \) to maximize its expected discounted lifetime utility, \( V_{0}^{nr} \):

\[
V_{0}^{nr} \equiv \mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} U \left( c_{t}^{nr}, l_{t}^{H,nr}, l_{t}^{NT,nr}, l_{t}^{P,nr} \right)
\]

subject to the sequential budget constraint in period \( t \) (in nominal terms):

\[
(1 + \tau_{t}^{n}) P_{t} c_{t}^{nr} = (1 - \tau_{t}^{n}) P_{t} \left( w_{t}^{H} l_{t}^{H,nr} + w_{t}^{NT} l_{t}^{NT,nr} + w_{t}^{P} l_{t}^{P,nr} \right) - P_{t} \tau_{t}^{nr}
\]
public sectors; but they have no access to capital or financial markets. In other words, they live hand-to-mouth and consume their after tax labour income plus targeted government lump-sum transfers, $P_{t}^{1,nr} < 0$.

**Choice of allocations** Each household $nr$ maximizes its lifetime utility (13) in any given period $t$ by choosing purchases of the final good, $c_{t}^{nr}$, hours of work in the tradable, $l_{t}^{H,nr}$, non-tradable sector, $l_{t}^{NT,nr}$, and public sector, $l_{t}^{P,nr}$ subject to the constraint (14). The Lagrange multiplier associated with constraint (14) is $\Lambda_{t}^{nr}$. The first-order conditions with respect to $c_{t}^{nr}$, $l_{t}^{H,nr}$, $l_{t}^{NT,nr}$, $l_{t}^{P,nr}$ are:

$$\frac{\partial U_{t}^{nr}}{\partial c_{t}^{nr}} = \Lambda_{t}^{nr} (1 + \tau_{t}^{n})$$

(15)

$$- \frac{\partial U_{t}^{nr}}{\partial l_{t}^{H,nr}} = \Lambda_{t}^{nr} (1 - \tau_{t}^{n}) w_{t}^{H}$$

(16)

$$- \frac{\partial U_{t}^{nr}}{\partial l_{t}^{NT,nr}} = \Lambda_{t}^{nr} (1 - \tau_{t}^{n}) w_{t}^{NT}$$

(17)

$$- \frac{\partial U_{t}^{nr}}{\partial l_{t}^{P,nr}} = \Lambda_{t}^{nr} (1 - \tau_{t}^{n}) w_{t}^{P}$$

(18)

### 3.3 Firms

There are two stages of private production. In the final stage, the final good that is used for private and public consumption and investment is produced. There are two firms namely a final good and a composite tradable good producer (the associated problems are solved in sections 3.3.1 and 3.3.2). The final good producer utilizes the composite tradable and the single intermediate non-tradable good to produce the final good. Similarly, the composite tradable good producer utilizes the home produced tradable good and the imported good to produce the composite tradable good.

In the intermediate stage, the intermediate non-tradable and tradable bundles are produced (the associated problems are solved in sections 3.3.3 and 3.3.4). There are $N^{i}$ intermediate non-tradable firms, each non-tradable firm indexed by $i$ hires labour and rents physical capital from households to produce a differeriated variety $i$. A non-tradable distributor combines all varieties, $i = 1..N^{i}$, into an intermediate non-tradable bundle. Similarly, there are $N^{j}$ intermediate home tradable firms, each home tradable firm indexed by $j$ hires labour and rents physical capital from households to produce a differeriated variety $j$. A tradable distributor
combines all varieties, \( j = 1..N^j \), into an intermediate home tradable bundle.

### 3.3.1 Final good producer

In this section, we solve the problem of the final good producer in per capita terms\(^{22}\). The final good is produced using a non-tradable good, \( y_{t}^{NT} \), and a composite tradable good, \( y_{t}^{T} \), via a CES technology:

\[
y_{t} = \left[ (v)^{\frac{1}{\zeta}} \left( y_{t}^{T} \right)^{\frac{\zeta - 1}{\zeta}} + (1 - v)^{\frac{1}{\lambda}} \left( y_{t}^{NT} \right)^{\frac{\zeta - 1}{\lambda}} \right]^{\frac{\lambda}{\zeta - 1}} \tag{19}\]

where \( \zeta \) is the intratemporal elasticity of substitution between the composite tradable good and the non-tradable good, and \( v \in (0, 1) \) is a share parameter governing the share of the composite tradable input and the non-tradable input in the production of the final good. The producer of the final good behaves competitively and maximizes its profits given by:

\[
P_{t} y_{t} - P_{t}^{T} y_{t}^{T} - P_{t}^{NT} y_{t}^{NT}\]

demand functions for the composite tradable good and the non-tradable good are given by:

\[
\frac{\partial y_{t}}{\partial y_{t}^{T}} = \frac{P_{t}^{T}}{P_{t}} \tag{20}
\]

\[
\frac{\partial y_{t}}{\partial y_{t}^{NT}} = \frac{P_{t}^{NT}}{P_{t}} \tag{21}
\]

Combining (20) and (21) yields:

\[
y_{t}^{T} = \frac{v}{1 - v} \left( \frac{P_{t}^{T}}{P_{t}^{NT}} \right)^{\frac{\zeta}{\zeta - 1}} y_{t}^{NT} \tag{22}
\]

while the associated price index is:

\[
P_{t} = \left[ v \left( P_{t}^{T} \right)^{1 - \zeta} + (1 - v) \left( P_{t}^{NT} \right)^{1 - \zeta} \right]^{\frac{1}{1 - \zeta}} \tag{23}
\]

\(^{22}\)Notice that throughout the paper small case letters denote per capita (firm) quantities, \( z_{t} \equiv \frac{z_{t}}{N_{t}} \), while capital case letters denote, \( Z_{t} \), aggregate quantities unless otherwise stated.
3.3.2 Composite tradable good producer

The composite tradable good is produced using the domestic absorption of the home tradable good, \( y_{t}^{H,d} \), and an imported good, \( y_{t}^{F} \), via a CES technology:

\[
y_{t}^{T} = \left( \nu^{H} \right)^{\frac{1}{\zeta^{H}}} \left( y_{t}^{H,d} \right)^{\frac{\zeta^{H}}{\zeta^{H}-1}} + \left( 1 - \nu^{H} \right)^{\frac{1}{\zeta^{H}}} \left( y_{t}^{F} \right)^{\frac{\zeta^{H}}{\zeta^{H}-1}} \right)^{\frac{\zeta^{H}}{\zeta^{H}-1}} \tag{24}
\]

where \( \zeta^{H} \) is the intratemporal elasticity of subsitution between the domestic absorption of home tradable good and the imported good and \( \nu^{H} \in (0, 1] \) denotes a share parameter that determines the share of the domestic absorption of the home tradable good vis-à-vis the imported good. Also it determines implicitly, the share of the home tradable good which is exported to the rest of the world. This parameter can capture key features of the Irish economy like the export orientation of home tradable production and the share of imported inputs in the production of the composite and the final good. The producer of the composite tradable good behaves competitively and maximizes its profits given by:

\[ P_{t}^{T} y_{t}^{T} - P_{t}^{H} y_{t}^{H} - P_{t}^{F} y_{t}^{F} \]

demand functions for the domestic tradable good and imported good are given by:

\[
\frac{\partial y_{t}^{T}}{\partial y_{t}^{H}} = \frac{P_{t}^{H}}{P_{t}^{T}} \quad \tag{25}
\]

\[
\frac{\partial y_{t}^{T}}{\partial y_{t}^{NT}} = \frac{P_{t}^{F}}{P_{t}^{T}} \quad \tag{26}
\]

Combining (25) and (26) yields:

\[
y_{t}^{H,d} = \frac{\nu^{H}}{1 - \nu^{H}} \left( \frac{P_{t}^{H}}{P_{t}^{F}} \right)^{-\zeta^{H}} y_{t}^{F} \tag{27}
\]

while the associated price index is:

\[
P_{t}^{T} = \left[ \nu^{H} \left( \frac{P_{t}^{H}}{P_{t}^{F}} \right)^{1-\zeta^{H}} + (1 - \nu^{H}) \left( \frac{P_{t}^{F}}{P_{t}^{H}} \right)^{1-\zeta^{H}} \right]^{\frac{1}{1-\zeta^{H}}} \tag{28}
\]
where

\[ P_t^F = S_t P_t^* \]  \hfill (29)

In the next two subsections we explain how the tradable and non-tradable goods are produced.

### 3.3.3 Non-tradable sector

**Non-tradable good distributor** A non-tradable good distributor combines varieties, \( i = 1..N^i \), of the intermediate non-tradable goods, \( y_{t,NT,i} \), into a composite non-tradable good, \( Y_{t,NT} \), using a Dixit-Stiglitz aggregator:

\[
Y_{t,NT} = \left( \sum_{i=1}^{N^i} \left( y_{t,NT,i} \right)^{\varepsilon_{NT}} \right)^{-\frac{1}{\varepsilon_{NT}}}
\]

where \( Y_{t,NT} \) and \( y_{t,NT} \equiv \frac{Y_{t,NT}}{N^i} \) denote aggregate and per capita quantity respectively, \( \varepsilon_{NT} > 0 \) is the elasticity of substitution across goods \( i \). The non-tradable good distributor maximizes its profits by choosing, \( Y_{t,NT} \), while taking prices, \( P_{t,NT} \) and \( P_{t,NT,i} \), as given:

\[
P_{t,NT} Y_{t,NT} - \sum_{i=1}^{N^i} P_{t,NT,i} y_{t,NT,i}
\]

The optimality condition yields a downward sloping demand function for each intermediate good of variety \( i \):

\[
y_{t,NT,i} = \left( P_{t,NT} \right)^{-\varepsilon_{NT}} y_{t,NT} \left( \frac{P_{t,NT,i}}{P_{t,NT}} \right)^{-\varepsilon_{NT}}
\]

where the associated price index is \( P_{t,NT} \equiv \left( \sum_{i=1}^{N^i} \left( P_{t,NT,i} \right)^{1-\varepsilon_{NT}} \left( \frac{1}{1-\varepsilon_{NT}} \right) \right) \).

**Intermediate non-tradable goods firms** There are \( N^i \) intermediate non-tradable good firms indexed by the superscript \( i \). Each intermediate non-tradable good firm \( i \) supplies variety \( i \) by solving a two-step problem.

First, intermediate firm \( i \) minimizes its cost by choosing its factor inputs \( k_{t-1,NT,i} \) and \( l_{t,NT,i} \):

\[
\Psi \left( y_{t,NT,i} \right) = \min_{\{k_{t-1,NT,i}, l_{t,NT,i}\}} \left\{ P_{t,NT} k_{t-1,NT,i} + P_{t,NT} l_{t,NT,i} \right\}
\]

taking prices as given and subject to the production function:
\[ y_{NT,i} = A_{NT}^{NT} \left( \frac{y_t}{y_t^g} \right)^{x_{NT}} \left( \left( \frac{k_{t-1}^{NT,i}}{l_{t-1}^{NT,i}} \right)^{a_{NT}} \left( \frac{l_{t}^{NT,i}}{l_{t-1}^{NT,i}} \right)^{1-a_{NT}} \right)^{x_2^{NT}} \]  

(30)

Each intermediate non-tradable good firm \( i \) produces a differentiated product \( i \) utilising as inputs, \( y_t^g \), per firm public good (see section 3.4.2) which is used as an intermediate input in the private production, \( k_{t-1}^{NT,i} \), physical capital rented from households in fully competitive capital markets and labour services rented from households, \( l_{t}^{NT,i} \), in fully competitive labour markets.\(^{23}\) \( A_{NT}^{NT} \) is a scale parameter that measures productivity in the non-tradable sector. \( x_1^{NT} > 0 \) is a parameter that determines the share of the public good as an intermediate productive input, \( x_2^{NT} \in [0,1] \) determines the share of private productive inputs.\(^{24}\) While \( a^{NT} \) and \( 1-a^{NT} \) are structural parameters related to capital and labour income share in the non-tradable sector.

The first order conditions are given by (where \( \Lambda_{NT,i}^{NT} \) is the Lagrange multiplier associated with (30)):

\[ P_{t}^{k} = \Lambda_{i}^{NT,i} a^{NT} x_2^{NT} y_t^{NT,i} \left( \frac{k_{t-1}^{NT,i}}{l_{t-1}^{NT,i}} \right) \]

(31)

\[ P_{t}^{w} = \Lambda_{i}^{NT,i} \left( 1 - a^{NT} \right) x_2^{NT} y_t^{NT,i} \left( \frac{l_{t}^{NT,i}}{l_{t-1}^{NT,i}} \right) \]

(32)

Plugging the conditional factor demands into the nominal cost function we get the minimum nominal cost function for any given level of production, \( \Psi \left( y_t^{NT,i} \right) \). It can be shown that the associated Lagrange multiplier is equal to the nominal marginal cost \( \Lambda_{i}^{NT,i} = \frac{\partial \Psi \left( y_t^{NT,i} \right)}{\partial y_t} \). Nominal profits of firm \( i \) can be written as:

\[ P_{i}^{\text{NT},i} = P_{t}^{\text{NT},i} y_t^{NT,i} - P_{t}^{k} k_{t-1}^{NT,i} - P_{t}^{w} l_{t}^{NT,i} - \frac{\phi^{NT}}{2} \left( \frac{P_{t}^{\text{NT},i}}{P_{t-1}^{\text{NT},i}} - 1 \right)^2 P_{t}^{\text{NT}} y_t^{NT} \]

(33)

In the second step, each firm \( i \) chooses its price, \( P_{t}^{\text{NT},i} \), to maximize its nominal profits facing Rotemberg-type

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\(^{23}\)Labour markets in Ireland are generally acknowledged as being among the most flexible in OECD countries (see McQuinn and Varthalitis (2018) for a comparison of Ireland labour market flexibility indicators with OECD and EU averages) and Babecky et al. (2010) for a comparison of wage rigidities across European countries. In addition, Ireland has a Social partnership model that promotes coordination in wage setting. This coordination approach enables wages to adjust to economy wide shocks. Thus, in the present model we assume perfectly competitive labour markets while we abstract from any form of nominal or real wage rigidity.

\(^{24}\)In the benchmark calibration, we set \( x_2^{NT} = 1 \), which yields a production function à la Baxter and King (1993), \( y_t^{NT,i} = A_{NT}^{NT} \left( \frac{y_t^g}{y_t^g} \right)^{x_1^{NT}} \left( \frac{k_{t-1}^{NT,i}}{l_{t-1}^{NT,i}} \right)^{a_{NT}} \left( \frac{l_{t}^{NT,i}}{l_{t-1}^{NT,i}} \right)^{1-a_{NT}} \). When we set \( x_2^{NT} = 1 - x_1^{NT} \) we allow for complementarity between public and private factor inputs.
nominal rigidities (as in e.g. Bi et al. (2013)): 

\[
\max_{P_{t}^{NT,i}} \sum_{t=0}^{\infty} E_{0,t} \left\{ P_{t}^{NT,i} y_{t}^{NT,i} - \Psi \left( y_{t}^{NT,i} \right) - \frac{\phi_{NT}^{2}}{2} \left( \frac{P_{t}^{NT,i}}{P_{t-1}^{NT,i}} - 1 \right) \right\}^{2} P_{t}^{NT} y_{t}^{NT}
\]

subject to demand for each variety \(i\):

\[
y_{t}^{NT,i} = \left[ \frac{P_{t}^{NT,i}}{P_{t}^{NT}} \right]^{-\varepsilon_{NT}} y_{t}^{NT}
\]

After imposing symmetry, i.e. \(y_{t}^{NT} = y_{t}^{NT,i}\) and \(P_{t}^{NT,i} = P_{t}^{NT}\) the profit maximizing condition yields:

\[
-\phi_{NT}^{NT} \left( \frac{P_{t}^{NT}}{P_{t-1}^{NT}} \frac{P_{t}}{P_{t-1}} - 1 \right) P_{t}^{NT} y_{t}^{NT} \frac{P_{t}^{NT}}{P_{t-1}^{NT}} P_{t} - \frac{\beta}{\lambda_{t}^{H}} \left( \frac{P_{t+1}^{NT}}{P_{t}} - 1 \right) P_{t+1}^{NT} y_{t+1}^{NT} \frac{P_{t+1}^{NT}}{P_{t}} P_{t+1} = 0
\]

where \(\psi^{NT}\) denotes real marginal cost.

### 3.3.4 Tradable good sector

**Home tradable good distributor** A home tradable good distributor combines varieties \(j = 1..N^{j}\) of the intermediate tradable goods, \(y_{t}^{H,j}\), into a composite tradable good \(Y_{t}^{H}\) using a Dixit-Stiglitz aggregator:

\[
Y_{t}^{H} = \left( \sum_{j=1}^{N^{j}} \left( y_{t}^{H,j} \right) \right)^{\frac{1}{\varepsilon^{H}+\lambda_{t}^{H}}}^{\frac{\varepsilon^{H}}{\varepsilon^{H}+\lambda_{t}^{H}}}
\]

where \(Y_{t}^{H}\) and \(y_{t}^{H}\) denote aggregate and per capita quantity respectively; \(\varepsilon^{H} > 0\) is the elasticity of substitution across goods \(j\). The tradable good distributor maximizes profits by choosing, \(Y_{t}^{H}\), while taking prices, \(P_{t}^{H}\) and \(P_{t}^{H,j}\), as given:

\[
P_{t}^{H} Y_{t}^{H} - \sum_{j=1}^{N^{j}} P_{t}^{H,j} y_{t}^{H,j} = 0
\]

The optimality condition yields a downward slopping demand function for each intermediate good of variety \(j\):

\[
y_{t}^{H,j} = \left[ \frac{P_{t}^{H,j}}{P_{t}^{H}} \right]^{-\varepsilon^{H}} y_{t}^{H}
\]
where the associated price index is $P_t^H \equiv \left( \sum_{j=1}^{N^j} \left( P_t^{H,j} \right)^{1-\varepsilon^H} \right)^{\frac{1}{1-\varepsilon^H}}$.

### 3.3.5 Intermediate tradable good firms

There are $N^j$ intermediate non-tradable good firms indexed by the superscript $j$. Each intermediate tradable good firm $j$ supplies variety $j$ by solving a two-step problem. First, intermediate firm $j$ minimizes its cost by choosing its factor inputs $k_{t-1}^{H,j}$ and $l_{t}^{H,j}$:

$$
\Psi \left( y_{t}^{H,j} \right) = \min_{\left\{ k_{t-1}^{H,j}, l_{t}^{H,j} \right\}} \left\{ P_t^{r_k} k_{t-1}^{H,j} + P_t^{w_l} l_{t}^{H,j} \right\}
$$

taking prices as given and subject to the production function:

$$
y_{t}^{H,j} = A_t^{H} \left\{ y_{t}^g \right\}^\varepsilon^H \left\{ \left( \left( l_{t}^{H,j} \right)^a H_t \right)^{1-a H} \right\}^{\frac{\varepsilon^H}{1-a H}} \tag{35}
$$

Each intermediate tradable good firm $j$ produces a differentiated product $j$ utilising as inputs, $y_{t}^g$, per firm public good (see section 3.4.2) which as before is used as an intermediate productive input, $k_{t-1}^{H,j}$, physical capital rented from households in fully competitive capital markets and labour services rented from households, $l_{t}^{H,j}$. $A_t^{H}$, measure productivity in the tradable sector. $\varepsilon_1^H > 0$ is a parameter that determines the share of public productive input, $\varepsilon_2^H \in [0,1]$, determines the share of private productive inputs. While $a^H$ and $1-a^H$ are structural parameters related to capital and labour income share in the non-tradable sector. The first order conditions are given by (where $\Lambda_{t}^{H,j,i}$ is the Lagrange multiplier associated with (35)):

$$
P_t^{r_k} = \Lambda_{t}^{H,j,i} \left( \frac{y_{t}^g}{k_{t-1}^{H,j}} \right) \tag{36}
$$

$$
P_t^{w_l} = \Lambda_{t}^{H,j,i} \left( 1 - a^H \right) \left( \frac{y_{t}^g}{l_{t}^{H,j}} \right) \tag{37}
$$

Plugging the conditional factor demands into the nominal cost function we get the minimum nominal cost function for any given level of production, $\Psi \left( y_{t}^{H,j} \right)$. It can be shown that the associated Lagrange multiplier
is equal to the nominal marginal cost $\lambda_t^{H,j} = \frac{\partial \psi(y_t^{H,j})}{\partial y_t^{H,j}}$. Nominal profits of firm $j$ can be written as:

$$P_t \omega_t^{H,j} = P_t^{H,j} y_t^{H,j} - P_t^{H,k} k_{t-1}^{H,j} - P_t w_t^{H,j} y_t^{H,j} - \frac{\phi_H}{2} \left( \frac{P_t^{H,j}}{P_t^{H,j-1}} - 1 \right)^2 P_t^{H} y_t^{H}$$

In the second step, each firm $i$ chooses its price, $P_t^{H,j}$, to maximize its nominal profits facing Rotemberg-type nominal rigidities:

$$\max_{P_t^{NT,j}} \sum_{t=0}^{\infty} E_0 \Xi_{0,t} \left\{ P_t^{H,j} y_t^{H,j} - \psi \left( y_t^{H,j} \right) - \frac{\phi_H}{2} \left( \frac{P_t^{H,j}}{P_t^{H,j-1}} - 1 \right)^2 P_t^{H} y_t^{H} \right\}$$

subject to demand for each variety $j$:

$$y_t^{H,j} = \left[ \frac{P_t^{H,j}}{P_t^{H}} \right]^{-\epsilon_H} y_t^{H}$$

After imposing symmetry, i.e. $y_t^{H} = y_t^{H,j}$ and $P_t^{H,j} = P_t^{H}$ the profit maximizing condition yields:

$$-\phi_H \left( \frac{P_t^{H}}{P_t^{H,j}} \frac{P_t}{P_{t-1}} - 1 \right) P_t^{H} y_t^{H} \frac{P_t^{H}}{P_t^{H,j}} \frac{P_t}{P_{t-1}} + \beta \frac{\lambda_{t+1}}{\lambda_{t+1}} \frac{P_t}{P_{t+1}} \frac{P_t^{H}}{P_t^{H,j}} \left( \frac{P_t^{H,j}}{P_t^{H,j-1}} - 1 \right) P_t^{H} y_{t+1}^{H} \frac{P_t^{H,j}}{P_t^{H,j-1}} \frac{P_t}{P_{t+1}} \frac{P_t^{H}}{P_t^{H,j}} y_{t+1}^{H} = 0$$

where $\psi_{t}^{H'}$ denotes real marginal cost.

### 3.4 Government

#### 3.4.1 Government Budget Constraint

The sequential government budget constraint in real per capita terms is written as:

$$d_t = R_{t-1} \lambda_t^d d_{t-1} + Q_t-1 \frac{S_t}{S_{t-1}} (1 - \lambda_t^q) d_{t-1} + g_t^i + g_{t}^w + \frac{w_t}{1 - \tau_t^r} \tau_t^r \leq 0$$

where $d_t \equiv \frac{D_t}{N}$ is real per capita total public debt and $\lambda_t^q = \frac{P_t^q D_t}{P_t^H D_t}$ and $(1 - \lambda_t^q) \equiv \frac{S_t^q F_t^q}{P_t^H D_t}$ are shares of total public debt held by domestic and foreign households respectively, $g_t^i$, $g_t^w$, $w_t$ and $\tau_t^r$, $\tau_t^l$, $\tau_t^{l,lr} < 0$ are government consumption, investment, public wage bill and public transfers in real and per capita terms,
\[ \tau_t^l = \nu^r \tau_t^l + \nu^{nr} \tau_t^{nr}, \] and \( \tau_t \) are total tax revenues in real and per capita terms defined as:

\[
\tau_t \equiv \tau_t^l \left( \nu c_t^r + \nu^{nr} c_t^{nr} \right) + \nu^p \tau_t^p \left( w_{tt}^{NT, H, r} + w_{tt}^{NT, NT, r} + w_{tt}^{P, P, r} \right) + \tau_t^p \left( w_{tt}^{H, H, NR} + w_{tt}^{NT, NT, NR} + w_{tt}^{P, P, NR} \right) + \tau_t^p \left( \omega_{tt}^{H, r} + \omega_{tt}^{NT, NT, r} + \omega_{tt}^{NT, NR, r} \right) + \nu^p \nu^p \left( \lambda_t^r + \lambda_t^{nr} \right) \tag{41} \]

Notice that the public wage bill is given by (in real and per capital terms):

\[
g_t^w \equiv w_{tt}^P l_t^q \tag{42} \]

Thus, the Government has nine fiscal policy instruments, \( g_t^c, g_t^l, g_t^w, \tau_t^l, \tau_t^p, \tau_t^{nr}, \tau_t^h, \delta_t, \lambda_t^p \) at its disposal. In each period fiscal policy can set eight policy instruments exogenously while one needs to adjust residually to satisfy the government budget constraint. In what follows, unless otherwise stated the residual fiscal policy instrument is public debt, \( d_t \). For more details see Appendix C.

### 3.4.2 Production of public goods-services

A single public firm produces a public good utilizing purchases of private goods, \( g_t^c \), public capital, \( k_{t-1}^g \), and labour services rented from households, \( l_t^p \), via the following technology (as in Economides, Papageorgiou, and Philippopoulos (2017)):

\[
y_t^g = A_t \left( k_{t-1}^g \right)^{a_1} \left( l_t^p \right)^{a_2} \left( g_t^c \right)^{1-a_1-a_2} \tag{43} \]

where \( g_t^g \equiv \frac{Y_t^g}{N_t}, k_t^g \equiv \frac{k_{t-1}^g}{N_t}, l_t^p \equiv \frac{l_t^p}{N_t} \) and \( g_t^c \) denote per capita quantities and \( a_1, a_2 \in (0, 1) \) are parameters that measure the associated shares of public productive inputs. Public capital law of motion is given by:

\[
k_t^g = (1 - \delta^g) k_{t-1}^g + g_t^l \tag{44} \]

where \( 0 < \delta^g < 1 \) is the depreciation rate of public capital stock.

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\(^{25}\)Where \( \nu^r \equiv \frac{N^r}{N} \) and \( \nu^{nr} \equiv \frac{N^{nr}}{N} \) are population shares.
3.5 Market clearing conditions

In this section we solve for a symmetric equilibrium in per capita terms. Without loss of generality we set \(N^i = N^j = N\) and \(\nu^r \equiv \frac{N^r}{N}\), \(\nu^{nr} \equiv \frac{N^{nr}}{N}\) are Savers and Non-Savers population shares. Below, we present the market clearing conditions by market, i.e. the final good, tradable and non-tradable goods markets, labour markets, capital and bonds markets. In the final good market the market clearing condition yields:

\[
y_t = \nu^r c_t^r + \nu^r x_t^{H,r} + \nu^r x_t^{NT,r} + \nu^{nr} c_t^{nr} + g_c^t + g_i^t \tag{45}
\]

The market clearing condition in the tradable good market yields:

\[
y_t^H = y_t^{H,d} + x_t \tag{46}
\]

where \(y_t^{H,d} \equiv \frac{\gamma^{H,d}}{N}\) and \(x_t \equiv \frac{\gamma}{N}\) denote domestic absorption the home tradable produced good and exports per capita. For the non-tradable good the market clearing condition is \(y_t^{NT} = \frac{1}{N} \sum_{i=1}^{N^i} y_t^{NT;i}\). In capital markets:

\[
\frac{1}{N} \sum_{j=1}^{N^j} k_t^{H,j} = k_t^{H,j} = \frac{1}{N} \sum_{r=1}^{N^r} k_t^{H,r} = \nu^r k_t^{H,r}
\]

\[
\frac{1}{N} \sum_{i=1}^{N^i} k_t^{NT,i} = k_t^{NT,i} = \frac{1}{N} \sum_{r=1}^{N^r} k_t^{NT,r} = \nu^{nr} k_t^{NT,r}
\]

In the labour market of the home tradable good the market clearing condition yields:

\[
l_t^H = \nu^r l_t^{H,r} + \nu^{nr} l_t^{H,cr} \tag{47}
\]

In the labour market of the non-tradable good the market clearing condition yields:

\[
l_t^{NT} = \nu^r l_t^{H,r} + \nu^{nr} l_t^{H,cr} \tag{48}
\]
The market clearing condition in the labour market of the public good is:

$$l_t^g = v^r l_t^{P,r} + v^{nr} l_t^{P,nr}$$  \hspace{1cm} (49)$$

The market clearing condition in the domestic government bonds market is:

$$\sum_{r=0}^{N^r} b_t^r = N_t^r b_t$$  \hspace{1cm} (50)$$

Notice that aggregating total profits in the two sectors across firms and households yields $$\sum_{r=1}^{N^i} \omega_t^{H,i} = N^i \omega_t^{H,i}$$ and $$\sum_{r=1}^{N^r} \omega_t^{NT,r} = N^r \omega_t^{NT,r} = \sum_{j=1}^{N^j} \omega_t^{NT,j} = N^j \omega_t^{NT,j}$$. For more details on the aggregation and the market clearing conditions see Appendices A and B respectively.

### 3.6 The evolution of net foreign debt

Combining the aggregate Ricardian household budget constraint with the government budget constraint and substituting the definitions for profits in the tradable and non-tradable sector, the market clearing conditions for final good, tradable and non-tradable goods, labour and capital markets and the aggregate budget constraint of non-Ricardian households yields a dynamic equation that governs the evolution of net foreign debt (assets) (for more details see Appendix D). The evolution of net foreign debt in per capita terms is given by:

$$S_t P_t^* f_t^{*g} - S_t P_t^* v^r f_t^{*r} = Q_{t-1} S_t P_{t-1}^* f_{t-1}^{*g} - Q_{t-1} S_t P_{t-1}^* v^r f_{t-1}^{*r} + P_t^F y_t^F - P_t^H y_t^H + \Phi^* (f_t^*, f_t^{*r}) + \Phi^{NT} \left( \frac{p_t^{NT}}{p_t} \frac{P_t}{P_{t-1}} - 1 \right)^2 p_t^{NT} y_t^{NT}$$ \hspace{1cm} (51)$$

where $$f_t^{*g} = \frac{F_t^g}{N}$$ and $$y_t^F = \frac{Y_t^F}{N}$$ denote per capita quantities. $$S_t P_t^* (f_t^{*g} - v^r f_t^{*r})$$ is net external debt. A positive (negative) value implies that the small open economy is a net debtor (creditor). The trade balance is defined as $$P_t^H x_t - P_t^F y_t^F$$. 

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3.7 Definition of GDP

For our quantitative analysis we need to define a measure of aggregate domestic output, $y_t^{gdp}$. In the present model we incorporate public employment which yields income from public wages, thus, in order to be consistent with national accounts definitions we include the public wage bill in the definition of aggregate domestic output following Forni, Gerali, and Pisani (2010) and Papageorgiou (2014). Nominal GDP, $P_t y_t^{gdp}$, at current prices and per capita terms is given by:

$$P_t y_t^{gdp} \equiv P_t (\nu^r c_t^r + \nu^{nr} c_t^{nr}) + P_t \nu^r (x_t^H + x_t^{NT}) + P_t (g_t^c + g_t^i + g_t^w) + P_t^H x_t - P_t^F y_t^F$$

where using the definition of zero profit conditions, clearing market conditions for the final good and the tradable good yields:

$$y_t^{gdp} = p_t^H y_t^H + p_t^{NT} y_t^{NT} + g_t^w$$

(52)

In what follows, we use, $P_t y_t^{gdp}$, to express several theoretical variables as GDP shares.

3.8 Monetary and Fiscal policy regimes

To solve the model we need to specify the monetary and fiscal policy regimes.

3.8.1 Monetary policy and exchange rate regime

Ireland is a member of a currency union; thus we solve for a monetary regime without monetary independence and a fixed exchange rate regime. In particular, we assume that the nominal depreciation rate, $\epsilon_t \equiv \frac{S_t}{S_{t-1}}$, is exogenously set while at the same time the nominal interest rate on domestic government bonds, $R_t$, becomes an endogenous variable (for similar modelling see Philippopoulos, Varthalitis, and Vassilatos (2017)).

3.8.2 Fiscal policy rules

The Irish Government can follow an independent fiscal policy. In this paper we follow common practice in the related literature and we adopt a rule-like approach to policy. That is the main spending-tax policy instruments react to the debt-to-GDP ratio while fiscal persistence is captured by including an autoregressive term. For our quantitative analysis we express all spending instruments as shares of steady state GDP, $y^{gdp}$.
as it is defined in section 3.7, namely, the ratio of government consumption to GDP, \( s^g_{c} = \frac{g^c}{y_{gdp}} \), the ratio of public investment to GDP, \( s^g_{i} = \frac{g^i}{y_{gdp}} \), the ratio of public wages to GDP, \( s^w_{t} = \frac{w^t}{y_{gdp}} \), the ratio of total public transfers to GDP, \( s^l_{t} = \frac{l^t}{y_{gdp}} \). Then the associated fiscal rules are given by:

\[
\begin{align*}
\frac{s^g_{c} - s^g_{c}}{s^g_{c}} &= \rho^g_{c} \left( s^g_{c,t-1} - s^g_{c} \right) - \gamma^g_{c} \left( \frac{d_t}{y_{gdp}} - \frac{d_t}{y_{gdp}} \right) + \varepsilon^g_{t} \quad (53) \\
\frac{s^g_{i} - s^g_{i}}{s^g_{i}} &= \rho^g_{i} \left( s^g_{i,t-1} - s^g_{i} \right) - \gamma^g_{i} \left( \frac{d_t}{y_{gdp}} - \frac{d_t}{y_{gdp}} \right) + \varepsilon^g_{i} \quad (54) \\
\frac{s^w_{t} - s^w_{t}}{s^w_{t}} &= \rho^w \left( s^w_{t-1} - s^w_{t} \right) - \gamma^w \left( \frac{d_t}{y_{gdp}} - \frac{d_t}{y_{gdp}} \right) + \varepsilon^w \quad (55) \\
\frac{s^l_{t} - s^l_{t}}{s^l_{t}} &= \rho^l \left( s^l_{t-1} - s^l_{t} \right) - \gamma^l \left( \frac{d_t}{y_{gdp}} - \frac{d_t}{y_{gdp}} \right) + \varepsilon^l \quad (56) \\
\frac{\tau^c_{t} - \tau^c_{t}}{\tau^c_{t}} &= \rho^c \left( \tau^c_{t-1} - \tau^c_{t} \right) + \gamma^c \left( \frac{d_t}{y_{gdp}} - \frac{d_t}{y_{gdp}} \right) + \varepsilon^c \quad (57) \\
\frac{\tau^k_{t} - \tau^k_{t}}{\tau^k_{t}} &= \rho^k \left( \tau^k_{t-1} - \tau^k_{t} \right) + \gamma^k \left( \frac{d_t}{y_{gdp}} - \frac{d_t}{y_{gdp}} \right) + \varepsilon^k \quad (58) \\
\frac{\tau^n_{t} - \tau^n_{t}}{\tau^n_{t}} &= \rho^n \left( \tau^n_{t-1} - \tau^n_{t} \right) + \gamma^n \left( \frac{d_t}{y_{gdp}} - \frac{d_t}{y_{gdp}} \right) + \varepsilon^n \quad (59)
\end{align*}
\]

where \( \rho^g_{c}, \rho^g_{i}, \rho^w, \rho^l, \rho^c, \rho^k, \rho^n \in [0, 1] \) are autoregressive coefficients, \( \gamma^g_{c}, \gamma^g_{i}, \gamma^w, \gamma^l, \gamma^c, \gamma^k, \gamma^n \geq 0 \) are feedback policy coefficients on public debt to GDP ratio while variables without time subscript denote policy target values. Finally, \( \varepsilon^g_{c,t}, \varepsilon^g_{i,t}, \varepsilon^w_{t}, \varepsilon^l_{t}, \varepsilon^c_{t}, \varepsilon^k_{t}, \varepsilon^n_{t} \) are iid fiscal shocks that capture discretionary changes in fiscal policy instruments. In section 7.3 we augment these rules to study alternative fiscal financing schemes.

### 3.9 Closing the Small Open Economy

As is well known, to avoid non-stationarity and convergence to a well defined steady state we need to depart from the benchmark small open economy model (see Schmitt-Grohe and Uribe (2003)). In this paper, we endogenize the world interest rate, i.e. the nominal interest rate at which the domestic country borrows from the international capital markets, \( Q_t \). Following Schmitt-Grohe and Uribe (2003), Garcia-Cicco, Pancrazi, and
Uribe (2010) and Philippopoulos, Varthalitis, and Vassilatos (2017) we assume that the small open economy risk premium is an increasing function of the end-of-period total public debt as a share of nominal GDP, \( \frac{P_t d_t}{P_t y_t} \), when this share exceeds an exogenous certain threshold \( D \). The equation governing the sovereign risk premia is:

\[
Q_t = Q_t^* + \psi^d \left( \frac{P_t d_t}{P_t y_t} - D \right) + \epsilon^q_{t-1} - 1
\]  

(60)

where \( Q_t^* \) denotes the world interest rate plus the time-invariant component of the Irish sovereign premia and is exogenously determined, \( \psi^d \) is a parameter which measures the elasticity of the interest rate with respect to deviations of the total public debt to GDP ratio from its threshold value, \( \epsilon^q_t \), as follows:

\[
\log(\epsilon^q_t) = \rho^q \log(\epsilon^q_{t-1}) + \epsilon^q_t
\]

where \( \rho^q \in (0, 1) \) is a parameter measuring the persistence of world interest rate shocks and \( \epsilon^q_t \) is an iid shock.

The terms of trade are defined as the relative price of exports in terms of imports:

\[
tot_t = \frac{P_{tH}}{P_{tI}}
\]  

(61)

Following Philippopoulos, Varthalitis, and Vassilatos (2017) we assume that world demand for exports is exogenous and thus the terms of trade become an endogenous variable. That is, domestic exports are given by:

\[
x_t = \rho^x x_{t-1} + (1 - \rho^x) \left( \frac{tot_t}{tot} \right)^{-\gamma^x}
\]  

(62)

where \( 0 < \rho^x < 1 \) is a parameter that governs the persistence of exports while exports are also function of deviations in the terms of trade from its steady state value. The latter term ensures dynamic stability and allows exports to have an endogenous feedback from changes in the relative price of Irish exports. Where, \( \gamma^x > 0 \), implies that an increase in the relative price of exports to imports results in a decrease in the world demand for the home produced tradable good.
3.10 Decentralized Equilibrium

The Decentralized Equilibrium is a set of 47 processes \( c_t, \Lambda_t^r, \lambda_t^H, p_{nt}^r, p_t^r, h_t^H, k_{nt}^r, f_{nt}^r, c_t^r, \Lambda_t^{nr}, h_t^{nr}, \)

\( l_t^{nt, nr}, l_t^p, n_{nt}^r, n_t^H, \psi_t^H, \omega_t^H, \lambda_t, w_t^H, y_t^H, y_t^H, \omega_t^{nt}, \lambda_t^{nt, k}, w_t^{nt}, k_t^g, y_t^g, w_t^P, d_t, \tau_t, y_t, \)

\( y_t^F, y_t^d, Q_t, R_t, p_t^H, p_t^{nt}, p_t^T, p_t^F, x_t^H, x_t^{nt}, x_t, s_{nt}^r, \) satisfying the definitions of the output shares of government spending in section 3.8.2 and the fiscal rules (53)-(59) given the exogenous variables \( P_t^r, A_t^H, A_t^{nt}, A_t^g \) and \( \epsilon_t^{26} \) and initial conditions for the state variables. The full DE system is presented in Appendices F and H.

4 Calibration and steady-state solution

This section calibrates the model for Irish economy using annual data over 2001-2014, unless otherwise stated.

We employ data from various sources, namely ESRI database, CSO, Eurostat and OECD-TiVA (details are in Appendix I). In the present model, there are 38 parameters that need to be calibrated: \( \beta, \sigma, \eta^H, \eta^{NT}, \eta^g, \lambda^H, \lambda^{NT}, \lambda^g, \beta^g, \beta^H, \epsilon^H, \psi^H, \nu, \lambda^c, \zeta^H, a^H, a^{nt}, \kappa^H, \kappa^{NT}, \delta^H, \delta^{NT}, \delta^g, \lambda^r, Q^*, \bar{a}, \phi^H, \phi^{NT}, \phi^*, \nu^r, \nu^{nr}, \rho^x, \gamma^x, \theta^H, \theta^{NT}, A^H, A^{NT}, A^g. \) In addition, there are 8 feedback policy coefficients in the associated fiscal rules \( \gamma^{g,e}, \gamma^{g,i}, \gamma^{s,e}, \gamma^{s,i}, \gamma^{s,nr}, \gamma^r, \gamma^x, \gamma^{nt} \) as well as 7 steady-state values for the fiscal policy variables, \( s^{g,e}, s^{g,i}, s^{s,e}, s^{s,i}, s^{s,nr}, \tau^r, \tau^x, \tau^{nt} \). We assign values to the parameters of the model in three different ways: (a) based on parameters widely used in related DSGE models, (b) parameters set to match first moments of the Irish data and (c) parameters set to match second moments of the Irish data. The time unit is a year.

4.1 Parameters widely used in related DSGE models

We employ conventional parameter values used in the DSGE literature for the fifteen structural parameters that belong to this category. In particular the inverse of the elasticity of intertemporal substitution, \( \sigma \), is set equal to 2, the preference parameter which measures the degree of substitutability/complementarity between private and public goods, \( \theta^g \), is set equal to 0 in the benchmark calibration (in section 7.4 we relax this assumption), the inverse of the Frisch elasticity of labour supply for tradable, \( \eta^H \), non-tradable, \( \eta^{NT} \), and

\[ \text{The exchange depreciation rate is exogenous since Ireland participates in a currency union, for simplicity we assume } \epsilon_t \equiv 1. \]
public, $\eta^g$, hours worked is set equal to 2, the intratemporal elasticity of substitution between the composite tradable good and the non-tradable good, $\zeta$, is set equal to 0.5, the intratemporal elasticity of substitution between the domestic absorption of the home produced tradable good and the imported good, $\zeta^H$, is set equal to 1. $\chi^H = \chi^{NT} = \chi^g$ are set equal to 4 so as the weighted average of hours worked to be equal to 0.4, 0.38 and 0.31 in the tradable, non-tradable and public sector respectively. The elasticities of substitution among the different intermediate good varieties in the tradable, $\varepsilon^H$, and non-tradable sector, $\varepsilon^{NT}$, yield price markups equal to 1.1 and 1.4 respectively which are consistent with the fact that the tradable sector is more competitive than the non-tradable sector in Eurozone countries (see also Papageorgiou and Vourvachaki (2017) and Sajedi (2018)). Finally the scale parameters, $A^H, A^{NT}, A^g$ are normalized to 1.

4.2 Parameters set to match first moments of the Irish data

The fifteen structural parameters in this category are $\beta, \nu, \nu^H, a^H, a^{NT}, \kappa^H, \kappa^{NT}, \delta^H, \delta^{NT}, \delta^g, \lambda^g, Q^*, \delta, \nu^r, \nu^{nr}$. The value of the time preference rate is implied by equation (12), $\beta = 1/Q$, where in steady state $Q = R = Q^* = 1.043$ (inflation is normalized to 1). $Q^*$ is the sum of the world interest rate and the invariant component of Ireland’s interest rate premium. In turn, $R = Q = 1.043$ follows from setting the gross interest rate equal to the average value of the real interest rate plus the invariant component of the Irish sovereign premium. Structural parameters $\nu$ and $\nu^H$ capture the degree of openness of the Irish economy. In particular, $1 - \nu$, governs the share of the non-tradable input in the production of the final good, thus this implicitly determines the size of the non-tradable sector vis-a-vis the tradable sector in gross value added terms. To calibrate, $\nu$, we target the ratio of the gross value added produced in the non-tradable sector to the sum of value added produced in the tradable and non-tradable sectors. This share is equal to 41% in the Irish economy. To do this, we add the following restriction when we solve for the steady state solution of the model:

$$\frac{P^{NT} y^{NT}}{P^H y^H + P^{NT} y^{NT}} = 0.41$$

27 We define Ireland’s sovereign risk premium as the difference between Ireland’s and Germany’s nominal interest rates on 10 year maturity government bond. Real interest rate for Ireland and Germany are computed employing data on nominal interest rate on government bonds from Eurostat deflated with HICP. In particular, we employ the "EMU convergence criterion series - annual data [irt_lt_mcby_a]" and "HICP (2015 = 100) - annual data (average index and rate of change) [prc_hicp_aind]" for the nominal interest rate and HICP respectively over the period 2001-2008. We focus on that period since we do not want our results to be distorted by the extreme values of the 2008-2010 Irish debt crisis.
This implies that the associated tradable share, \( \frac{PHyH}{PHyH + PNTyNT} \), is equal to 59%. The parameter, \( \nu^H \), governs the share of domestic absorption of home produced tradable good vis-a-vis the imported good and implicitly determines the share of the home produced tradable good that it is exported abroad. To calibrate this parameter we target the value added export share in GDP which is equal to 52\%. Thus we impose:

\[
\frac{PHx}{PYgdp} = 0.52
\]  \( (64) \)

The parameter, \( 1 - a^H \), is calibrated to match the average value of the labour share in the tradable sector over 2001-2014 which is equal to 0.39:

\[
\frac{PW^Hl^H}{PHy^N^T} = 0.39
\]  \( (65) \)

which implies that \( a^H \) equal to 0.571 is consistent with the capital intensity of the tradable sector. Similarly, the labour share in the non-tradable sector, \( 1 - a^{NT} \), is calibrated to match the average value of the labour share in the non-tradable sector we observe in the Irish data, so we impose:

\[
\frac{PW^{NT}l^{NT}}{PNTy^{NT}} = 0.54
\]

which in turn yields \( a^{NT} \) equal to 0.244 indicating the labour intensity of the non-tradable sector\(^{29} \). The shares of public capital in the production functions of both sectors, \( k^H \) and \( k^{NT} \), are set equal to the average public investment to GDP ratio found in the data as in Baxter and King (1993), i.e. are set equal to 0.035. The depreciation rates, \( \delta^H \), \( \delta^{NT} \), \( \delta^g \), are calibrated by constructing time series for the private and public capital stock employing the methodology in Coenen, Karadi, Schmidt, and Warne (2018) and Gogos, Mylonidis, Papageorgiou, and Vassilatos (2014). The associated values are \( \delta^H = 0.071 \), \( \delta^{NT} = 0.051 \) and \( \delta^g = 0.0741 \) (see Appendix I for details). The threshold value, \( D \), above which the sovereign risk premia emerge, is set equal to 60\%. That is the average value of the Irish public debt to GDP ratio between 2001 and 2014 and also coincides with the limit imposed by the Maastricht Criteria for all EU countries. Finally, we set the fraction

\[28\] In the model exports and imports are value added while national accounts provide data on gross exports and imports which include intermediate goods. For that reason, we employ data from OECD-TiVA database which provide data on exports in value added (time series Domestic value added embodied in foreign final demand "FFD\_DVA"). Irish data are only available for 2001-2011. We calibrate the model to match exports expressed in value added terms and the trade balance as share of GDP and thus we obtain residually a value for imports.

\[29\] Our calibration is consistent with the declining labour share observed in Irish data see OECD (2018).
of "Savers" to total population, $\nu^r$, equal to 0.7, which is consistent with data reported in the Irish module of the Household Finance and Consumption Survey\(^{30}\) (2013) and in line with values reported in previous studies, e.g. Forni et al. (2009), Coenen, Straub, and Trabandt (2013), Papageorgiou (2014).

Table 1: Parameter values in (a) and (b)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Implied Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9588</td>
<td>time discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>inverse of elasticity of substitution in consumption</td>
</tr>
<tr>
<td>$\delta^g$</td>
<td>0</td>
<td>substitutability/complementarity between public and private consumption</td>
</tr>
<tr>
<td>$\delta^H$</td>
<td>2</td>
<td>inverse of Frisch labour elasticity in the tradable sector</td>
</tr>
<tr>
<td>$\psi^{NT}$</td>
<td>2</td>
<td>inverse of Frisch labour elasticity in the non-tradable sector</td>
</tr>
<tr>
<td>$\eta^p$</td>
<td>2</td>
<td>inverse of Frisch labour elasticity in public sector</td>
</tr>
<tr>
<td>$\chi^H, \chi^{NT}, \chi^g$</td>
<td>4</td>
<td>preference parameter related to work effort (all sectors)</td>
</tr>
<tr>
<td>$\delta^H$</td>
<td>0.071</td>
<td>capital depreciation rate in the tradable sector</td>
</tr>
<tr>
<td>$\delta^{NT}$</td>
<td>0.051</td>
<td>capital depreciation rate in the non-tradable sector</td>
</tr>
<tr>
<td>$\delta^g$</td>
<td>0.0741</td>
<td>capital depreciation rate in the public sector</td>
</tr>
<tr>
<td>$\sigma^H$</td>
<td>0.571</td>
<td>share of physical capital in the tradable sector</td>
</tr>
<tr>
<td>$\sigma^{NT}$</td>
<td>0.244</td>
<td>share of physical capital in the non-tradable sector</td>
</tr>
<tr>
<td>$\sigma^g$</td>
<td>0.183</td>
<td>share of public capital in the public sector</td>
</tr>
<tr>
<td>$\sigma^g$</td>
<td>0.542</td>
<td>share of public labour in the public sector</td>
</tr>
<tr>
<td>$\kappa^H$</td>
<td>0.035</td>
<td>public capital elasticity in the production function (tradable)</td>
</tr>
<tr>
<td>$\kappa^{NT}$</td>
<td>0.035</td>
<td>public capital elasticity in the production function (non-tradable)</td>
</tr>
<tr>
<td>$\varepsilon^H$</td>
<td>11</td>
<td>price elasticity of demand in the tradable sector</td>
</tr>
<tr>
<td>$\varepsilon^{NT}$</td>
<td>3.5</td>
<td>price elasticity of demand in the non-tradable sector</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.5817</td>
<td>share of tradable in the production of the final good</td>
</tr>
<tr>
<td>$\nu^H$</td>
<td>0.03</td>
<td>share of domestic tradable in the production of the composite tradable good</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.5</td>
<td>elasticity of substitution between the composite tradable and the non-tradable good</td>
</tr>
<tr>
<td>$\zeta^H$</td>
<td>1</td>
<td>elasticity of substitution between domestic tradable and imported good</td>
</tr>
<tr>
<td>$\nu^r$</td>
<td>0.7</td>
<td>total population share of &quot;Savers&quot;</td>
</tr>
<tr>
<td>$\nu^{nr}$</td>
<td>0.3</td>
<td>total population share of &quot;non-Savers&quot;</td>
</tr>
<tr>
<td>$\lambda^g$</td>
<td>0.5</td>
<td>share of public debt held by foreign investors</td>
</tr>
<tr>
<td>$A^H, A^{NT}, A^g$</td>
<td>1</td>
<td>productivity/scale parameter(s) (all sectors)</td>
</tr>
<tr>
<td>$\mathcal{D}$</td>
<td>0.6</td>
<td>debt to GDP threshold value</td>
</tr>
</tbody>
</table>

---

\(^{30}\)In Household Finance and Consumption Survey (2013) is reported that 88.6% of Irish households own a savings account while 56.8% have access to some form of loans. These definitions are closely related to the definition of "Ricardians/Savers" in our model, thus we take a value close to their average, i.e. 0.7.
4.3 Parameters set to match second moments of the Irish data

The parameters $\phi^H, \phi^{NT}, \phi^x, \psi^d, \gamma^x$ and $\rho^x$ are calibrated to capture the second moments properties of key endogenous variables of the model. The theoretical second moments are computed conditional on tradable and non-tradable TFP shocks\(^{31}\). The parameters $\phi^H, \phi^{NT}$ are calibrated to mimic the volatilities of physical capital time series in the tradable and non-tradable sectors. The parameter, $\phi^x$ and $\psi^d$, are calibrated to match the volatility of the trade balance observed in Irish data as well as to ensure that the solution of the model is dynamically stable\(^{32}\). The parameters, $\gamma^x$ and $\rho^x$, are calibrated to force the volatility and persistence of exports implied by the model to be as close as possible to the actual volatility and persistence of Irish exports. Finally $\phi^H$ and $\phi^{NT}$ are calibrated based on the study of Druant et. al (2009) which along with the associated elasticities of substitution among differentiated varieties in the tradable and non-tradable sectors implies that firms adjust their prices every 1.1 and 1.2 years in the tradable and non-tradable sectors respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Implied Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^H$</td>
<td>71</td>
<td>Rotemberg parameter in the tradable sector</td>
</tr>
<tr>
<td>$\phi^{NT}$</td>
<td>165</td>
<td>Rotemberg parameter in the non-tradable sector</td>
</tr>
<tr>
<td>$\phi^x$</td>
<td>1.56</td>
<td>capital adjustment cost in the tradable sector</td>
</tr>
<tr>
<td>$\rho^x$</td>
<td>0.39</td>
<td>capital adjustment cost in the non-tradable sector</td>
</tr>
<tr>
<td>$\phi^*$</td>
<td>0.01</td>
<td>adjustment cost in international borrowing</td>
</tr>
<tr>
<td>$\psi^d$</td>
<td>0.002</td>
<td>risk premium coefficient on total public debt to GDP ratio</td>
</tr>
</tbody>
</table>

4.4 Fiscal data

To set the long run values of fiscal variables we employ data from Eurostat. Regarding spending instruments, the long-run value of public spending on goods and services, $s^{G,c}$, public investment, $s^{G,i}$, public wage bill, $s^w$, and public transfers, $s^t$, as shares of output are set equal to their data averages over the period 2001 to 2014. We set the long run values of tax instruments equal to the associated effective tax rates, i.e. consumption, $\tau^c$, capital, $\tau^k$, and labour, $\tau^n$, tax rates are set equal to the associated 2001-2014 average effective tax rates.

\(^{31}\)In particular, we calibrate the parameters that govern the productivity process in the tradable and non-tradable sectors, i.e. $\rho^H, \rho^{NT}, \sigma^H, \sigma^{NT}$ to match the volatility and persistence of the actual Irish real GDP as well as mimic as close as possible the volatilities and persistence observed in the GVA in the tradable and non-tradable sectors respectively. TFP follow an AR(1) process, i.e. $\log A_t = \rho \log A_{t-1} + \epsilon^A$.

\(^{32}\)The calibrated value for the parameter, $\psi^d$, that governs the debt elasticity of nominal interest implies that a 1% increase in debt to GDP ratio above its threshold value results in a 0.2% increase in the risk premium. This calibration implies similar dynamics for the risk premium as in HERMES-13 see Bergin et al. (2013).
The effective tax rates are constructed following the methodology in Mendoza, Razin, and Tesar (1994) (more details on the methodology are reported in Kostarakos and Varthalitis (2019)). Finally, we set $\lambda^g = 0.56$ which implies that 56% of total public debt is held by foreign investors.

<table>
<thead>
<tr>
<th>Fiscal variable</th>
<th>Implied Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g$</td>
<td>0.051</td>
<td>government purchases of goods and services to GDP</td>
</tr>
<tr>
<td>$s^g$</td>
<td>0.035</td>
<td>public investment to GDP</td>
</tr>
<tr>
<td>$s^w$</td>
<td>0.1</td>
<td>public wage bill to GDP</td>
</tr>
<tr>
<td>$s^t$</td>
<td>-0.18</td>
<td>public transfers to GDP</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.243</td>
<td>effective tax rate on consumption</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.2</td>
<td>effective tax rate on capital</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>0.354</td>
<td>effective tax rate on labour</td>
</tr>
<tr>
<td>$\lambda^g$</td>
<td>0.56</td>
<td>share of public debt held by Irish residents</td>
</tr>
</tbody>
</table>

### 4.5 Steady-state solution

Table 4 presents the numerical solution of this system when we use the parameter values and policy instruments in Tables 1-3. We compare our solution with some key macroeconomic ratios observed in the Irish data.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^g + c^g + c^g$</td>
<td>Private consumption to GDP</td>
<td>0.48</td>
<td>0.48</td>
</tr>
<tr>
<td>$y^{NT}$</td>
<td>Share of tradables to total GVA in the private sector</td>
<td>0.59</td>
<td>0.59</td>
</tr>
<tr>
<td>$y^{NT}$</td>
<td>Share of non-tradables to total GVA in the private sector</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>$y^{NT}$</td>
<td>Trade balance to GDP</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>$y^{NT}$</td>
<td>Investment to GDP</td>
<td>0.16</td>
<td>0.21</td>
</tr>
<tr>
<td>$y^{NT}$</td>
<td>Labour share in the non-tradable sector</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>$y^{NT}$</td>
<td>Labour share in the tradable sector</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>$y^{NT}$</td>
<td>Exports to GDP</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>$y^{NT}$</td>
<td>Physical capital to GDP</td>
<td>2.5</td>
<td>2.07</td>
</tr>
</tbody>
</table>

The Annual Report on Public Debt (2018) indicates that 56% of total Irish public debt is held by foreign investors.
5 Fiscal policy transmission mechanism

In this section we present the impulse response functions of the key endogenous variables of the model to temporary discretionary fiscal changes in the main fiscal policy instruments. This provides insight into the transmission channel of fiscal policy changes in the Irish economy. To do this, we implement exogenous fiscal shocks to the main tax-spending instruments in equations (53-59). For comparison purposes we set the persistence parameter in each fiscal rule equal to 0.8 while the magnitude of the fiscal shocks is 1% of the pre stimulus GDP on impact. In what follows, we assume perfect foresight which means that the entire path of fiscal actions is fully anticipated by households and firms\textsuperscript{34}.

Due to the structure of the present model which is designed to resemble some key structural characteristics of the Irish economy, the sign and the magnitude of the effect as well as the transmission mechanism of the fiscal stimulus differs between the tradable and the non-tradable sector. Our results indicate that a fiscal stimulus increases Irish GDP; however this works solely through the non-tradable sector while the tradable sector shrinks. This is highly dependent on the degree of openness of the Irish economy.

5.1 Spending shocks

We start by studying the effects of a temporary discretionary fiscal change in government consumption, $s^{g,c}$. Figure 1 illustrates the dynamic responses of the key Irish macroeconomic variables when we implement an exogenous shock to government consumption. A fiscal stimulus via government consumption causes an increase in Irish GDP. This aggregate increase can be solely attributed to the stimulative effects that $s^{g,c}$ induces in the non-tradable sector (see the impulse response of $p^{NT}y^{NT}$) while the tradable sector contracts initially and eventually increases (see the impulse response of $p^{H}y^{H}$). However, the effects on the tradable sector are quantitatively small. Since the government fiscal shock results in different sectoral dynamic responses, we organize our discussion of the fiscal transmission mechanism around the impacts for the two sectors, i.e. non-tradable and tradable.

Regarding the non-tradable sector, firms increase production of the non-tradable good to meet the increased domestic demand stemming from a government consumption stimulus. To produce this additional output, \textsuperscript{34}To solve the model numerically we use the Non-Linear solver of Dynare; in particular the algorithm that uses a Newton-type method to solve the simultaneous equation system.
they rent physical capital and hire labour, i.e. private investment, $x^{NT}$, and hours worked, $l^{NT}$, in the non-tradable sector increase. The increased demand for productive factor inputs in the non-tradable sector causes an increase in the associated factor prices, i.e. private wages, $w^{NT}$, and return on physical capital, $r^{NT}$, which subsequently lead to upward pressures in the sectoral price, $p^{NT}$. The increase in the relative price of the non-tradable sector implies a deterioration in the competitiveness of the Irish economy vis-à-vis the rest of the world. This also can be seen by the impulse response of $p^F$, which in our model is the real exchange rate. A decrease in $p^F$ (i.e. real appreciation) means that foreign prices decrease vis-à-vis the domestic price of the final good and as a result imports increase, $p^F y^F$.

The tradable sector contracts vis-à-vis the non-tradable sector. By construction the Government allocates its expenditures both in the home produced and imported tradable goods. The impulse response functions show that government consumption crowds out exports, $p^H x^H$, while crowds in imports, $p^F y^F$. As a result, the trade balance deteriorates in response to a positive government consumption shock (this is consistent with empirical evidence see e.g. in Benetrix and Lane (2009) and Lane (2010)). This negative effect on the Ireland’s trade balance reverses any positive effect from the fiscal stimulus on tradable production. As a result, factor inputs shrink, namely private investment, $x^H$, and hours worked, $l^H$, and this exerts downwards pressures on sectoral factor prices, $p^H$. This reduction in factor prices gradually improves the terms of trade and shifts back resources to the tradable sector once the fiscal stimulus comes to an end; thus tradable output moves slightly upwards however this increase is quantitatively small.

The effect of a fiscal stimulus on aggregate private consumption depends on the weighted response of "Ricardians/Savers" and "Non-Ricardians/Non-Savers" consumption. A fiscal stimulus causes a negative wealth effect for "Ricardians/Savers" households. This works as follows, higher government consumption increases the debt-to-output ratio (see the dynamic response of $d/y_{gdp}$); in response to the deviation of debt from its target level fiscal policy reduces public transfers see the dynamic response of $s_l$ (for alternative fiscal financing schemes see section 7.3). Since "Ricardians/Savers" can smooth their lifetime consumption path through borrowing/lending, they reduce current consumption, $c^r$, to compensate for the future income loss caused by reduction in public transfers. On the other hand, "Non-Ricardians/Non-Savers" live hand to mouth which means that they consume any additional temporary income produced by the fiscal stimulus. As a result they increase current consumption, $c^{nr}$, over the fiscal stimulus period while they decrease future consumption,
i.e. once the fiscal stimulus comes to an end.

Figure 1: Dynamic responses to a government consumption shock

Similarly, Figure 2 illustrates the dynamic effects on Irish macroeconomic variables of a discretionary temporary increase in public investment. As it is expected, the transmission channel is similar to the case of a policy intervention via government consumption\textsuperscript{35}. In the short run the qualitative and quantitative effects are almost identical with those of an increase in government consumption. However, we observe some quantitative differences in the medium run, mainly because public investment increases the public capital stock; thus, the productive effects of a fiscal stimulus through public investment are usually more persistent and long lasting.

\textsuperscript{35}In the present model both government consumption and investment are used as productive inputs in public production see equation (43).
Figure 3 illustrates the dynamic effects on Irish macroeconomic variables from an increase in public wage bill which in the present model specification implies an increase in public wages\textsuperscript{36}. As with previous spending instruments, an increase in public wages boosts aggregate domestic output\textsuperscript{37}, $y_{gdp}$; however the magnitude and transmission mechanism through which an increase in public wages affects the Irish economy differs from the previous spending categories. An increase in public wages implies a positive change in the disposable income of both types of households\textsuperscript{38}. As a result current aggregate consumption increases. As expected, the effect on private consumption is higher in magnitude for "non-Savers" due to their financial constraints. The increase in public wages and in private consumption fuel upward pressures in private sector wages and prices, as can be observed from the impulse responses of $w^H$, $w^{NT}$, $p^H$ and $p^{NT}$. This adversely impacts the competitiveness of the Irish economy vis-à-vis the rest of the world relatively more than the previous spending instruments, as can be inferred from the prolonged increase in the terms of trade, real exchange rate appreciation and the associated decline in exports, $p^Hx$. The negative effect on the Irish trade balance is more prolonged.

\textsuperscript{36} Government sets the public wage bill, $g^w = \bar w^g \bar l^g$, then both type of households optimally allocate labour hours worked to the three sectors.

\textsuperscript{37} The quantitatively significant effect on aggregate output, $y^{gdp}$, arises from its definition (see section 3.7). That is, the aggregate GDP is defined as the sum of private production and public wages (for similar modelling and findings see Forni et al. (2010), Stahler and Thomas (2012) and Papageorgiou (2014)).

\textsuperscript{38} Recall that both type of households have members that work in the public sector.
5.2 Tax shocks

In this section we turn to the dynamic effects on the Irish economy of temporary discretionary fiscal changes in tax instruments, namely consumption, capital and labour tax. Figure 4 depicts the effects on the Irish macroeconomic variables of a fiscal shock on consumption tax. A temporary decrease in the consumption tax rate makes consumption purchases relatively cheaper, thus both types of households increase current consumption, i.e. \( c_r \) and \( c_{nr} \) increase. As noted previously, the increase in consumption of "Non-Ricardians/Non-Savers" is larger in magnitude. However, increasing private consumption puts upward pressures on domestic factor and product prices leading to a deterioration in the Irish terms of trade and a reduction in exports. In other words, a cut in consumption tax is similar to an increase in government consumption, i.e. crowds out exports and crowds in imports. The Irish trade balance deteriorates and the tradable sector temporarily shrinks. As above, any increase in GDP stems solely from the increase in the non-tradable production; and at the same time crowds out private investment, hours worked and production in the tradable sector.
Figure 5 summarizes the dynamic response of the Irish economy to a temporary discretionary decrease in the capital tax rate. Cuts in capital taxes fuel private investment in both sectors, however this increase takes more time to materialize and is more prolonged in the tradable sector due to the capital intensity of this sector (e.g. compare the impulse responses of, \( x^H \) with \( x^{NT} \)). On the other hand, in the labour intensive non-tradable sector the increase in investment (and physical capital) contemporaneously increases hours worked and as a result non-tradable production rises on impact.

Capital tax cuts can have significant effects on the international competitiveness of the Irish economy. In particular, Figure 5 shows that a decrease in the capital tax rate causes a long-lasting improvement in the Irish terms of trade. Although the terms of trade increases on impact due to the sluggish price adjustment of (see \( p^H \)) then experiences a prolonged reduction. Irish exports, \( p^H x \), follow a similar path, i.e. they decrease on impact but afterwards persistently increase. Thus, although the trade balance decreases\(^{39}\) (as in all other fiscal shocks due to the reliance of the Irish economy on imports); the reduction is smaller than when the fiscal stimulus is implemented via spending instruments. Finally, cuts in capital taxes induce a direct and prolonged increase in the disposable income of "Ricardians/Savers" (recall that they earn capital income); whereas they increase only temporarily the labour income of "non-Ricardians/non-Savers" because private wages in both

\(^{39}\) In Figures 1-6 we present percentage deviations of the trade balance to GDP ratio, \( tb_t/y_t^{gdp} \), from its steady state value. This ratio is reduced first because trade balance reduces, i.e. exports decrease while imports increase, second due to the increase in GDP.
sectors increase. As is apparent, the effect on "Savers" consumption, $c^r$, is smaller on impact but lasts longer as "Ricardians/Savers" can save/invest part of the current increase in their disposable income and use it to retain a higher level of consumption over longer horizon. While the effect on "Non-Savers", $c^{nr}$, is larger on impact but short-lived.

Figure 5: Dynamic responses to a capital tax shock

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Figure 6 presents the impulse responses of the Irish key macroeconomic variables to a temporary discretionary decrease in labour taxes. Reduction in labour taxes induce a positive wealth effect on both type of households which increase their current consumption, $c^r$ and $c^{nr}$. In addition, the lower labour tax leads to an increase in the after tax wage that households receive in all sectors, and incentivize them to substitute leisure with hours worked, i.e. hours worked increase in both sectors, $l^H$, $l^{NT}$ and $l^g$ (this is the intratemporal substitution effect). The increased labour supply in both sectors leads to lower equilibrium wages, $w^H$ and $w^{NT}$, which also exerts downward pressure on the returns to capital and marginal costs. Equilibrium factor prices fall and as a result the international competitiveness of the Irish economy improves. This is reflected in the dynamic path of the terms of trade and the associated increase in exports.

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40 Although pre-tax wages, $w^H$ and $w^{NT}$, fall in Figure 6, after tax wages, $(1 - \tau^p) w^H$ and $(1 - \tau^p) w^{NT}$, increase due to the decrease in labour tax. The same holds for after tax public wage, i.e. $(1 - \tau^p) w^p$.  

38
Figure 6: Dynamic responses to a labour tax shock

6 Fiscal Multipliers in Ireland

6.1 Definition

Fiscal multipliers can be measured in several ways. Following e.g. Zubairy (2014) and Leeper, Traum, and Walker (2017), we focus on present-value multipliers which embody the full dynamics of exogenous fiscal actions and discount future changes in macroeconomic variables. The present value multiplier of any government spending instrument over a $k$-period horizon is defined as:

$$PV_{\mathcal{F}^g}(k) = \sum_{j=0}^{k} \frac{\prod_{i=0}^{k} \left( \frac{R_i}{I_{t+j+1}} \right)^{-1}}{E_t} \Delta g_{t+j}$$

where $R_i$ is the model-based simulated real interest rate in period $t$. $\Delta g^{dp}_{t+j}$ is the change in output over a $k$-period horizon produced by an exogenous change in spending instruments, $\mathcal{F}^g_k \equiv \left\{ g_t, g_t^l, g_t^w, \tau_t \right\}$, over the same $k$-period horizon. $\Delta g_{t+j}$ is defined as the change in the level of government spending over the
same period. We define $g_t$ the sum of government consumption, public investment and the public wage bill\footnote{Notice that we exclude public transfers from $g_t$ since the latter are used to react to debt deviations from its target; thus any change in public transfers is not related to the discretionary fiscal stimulus. When we compute public transfers multiplier $g_t$ includes public transfers.}. Similarly, we define the present value multipliers of tax instruments:

$$PV_{\mathcal{F}}^T(k) \equiv \frac{\sum_{j=0}^{k} \left( \frac{R_{t+j}}{\Pi_{t+j+1}} \right)^{-1} \Delta y_{gdp}^{t+j}}{\sum_{j=0}^{k} \left( \frac{R_{t+j}}{\Pi_{t+j+1}} \right)^{-1} \Delta \tau_{t+j}}$$  \hspace{1cm} (67)

where $\mathcal{F}^T \equiv \{ \tau^c_t, \tau^k_t, \tau^n_t \}$ represents a tax instrument while $\Delta \tau_{t+j}$ is the change in total tax revenues where tax revenues are defined as the sum of consumption, capital and labour income taxes (as in equation (41)). By setting $k = 0$, the present value multiplier becomes equal to the impact multiplier.

### 6.2 Fiscal stimulus scenario

In this section we develop our main fiscal scenario. In what follows, we focus on the fiscal stimulus policy which is defined as a discretionary fiscal change in a particular spending or tax instrument. For reasons of comparison, the size of the fiscal shock is normalized to represent an increase in government expenditures or a decrease in total tax revenues equal to 1% of steady-state (pre-stimulus) GDP for three years. To ensure fiscal sustainability, Irish fiscal policy is set such that total public transfers react to deviations in the debt-to-output ratio from its target level (for alternative fiscal financing schemes see section 7.3). To do this, we set the fiscal policy coefficient on public transfers in the associated fiscal rule (in equation 56) equal to $\gamma^f = 0.1$ while to keep the remaining fiscal policy instruments constant we set the remaining feedback coefficients and persistence parameters equal to 0. We examine one fiscal instrument at a time while the residual policy instrument is total public debt. As above, we assume perfect foresight.

### 6.3 Output multipliers

Table 5 computes the present value output multipliers, $y_{gdp}$, by fiscal instrument when we set $k = 1, 2, 3$. For example, in the second row we compute the implied increase in GDP produced by a three year fiscal stimulus through an associated increase in government consumption. Our model suggests that a 1% increase
in government consumption for the next three years produces a 0.59% increase in GDP in the first year while the cumulative discounted change is equal to a 0.42% and 0.34% increase in the second and third year respectively.

<table>
<thead>
<tr>
<th>Description</th>
<th>Instrument</th>
<th>1st year</th>
<th>2nd year</th>
<th>3rd year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov. consumption</td>
<td>$s^{g,c}$</td>
<td>0.59</td>
<td>0.42</td>
<td>0.34</td>
</tr>
<tr>
<td>Gov. investment</td>
<td>$s^{g,i}$</td>
<td>0.62</td>
<td>0.37</td>
<td>0.25</td>
</tr>
<tr>
<td>Public wages</td>
<td>$s^{w}$</td>
<td>1.16</td>
<td>1.06</td>
<td>1.01</td>
</tr>
<tr>
<td>Public transfers</td>
<td>$s^{f}$</td>
<td>0.24</td>
<td>0.07</td>
<td>-0.02</td>
</tr>
<tr>
<td>Consumption tax</td>
<td>$\tau^{c}$</td>
<td>0.51</td>
<td>0.2</td>
<td>0.08</td>
</tr>
<tr>
<td>Capital tax</td>
<td>$\tau^{k}$</td>
<td>0.44</td>
<td>0.07</td>
<td>-0.05</td>
</tr>
<tr>
<td>Labour tax</td>
<td>$\tau^{n}$</td>
<td>0.09</td>
<td>0.2</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Some key results arising from Table 5 are as follows: First, spending multipliers are in general higher in the short-run than tax multipliers which is consistent with findings in other empirical and theoretical studies (for a collection of findings across models and methodologies see in Batini et al. (2014)). Impact multipliers indicate that the stimulative effects of a fiscal expansion are larger in the case of government consumption and public investment. In particular, government consumption and public investment impact multipliers are equal to 0.59 and 0.62. The latter values tend to be smaller than spending multipliers estimates that have been reported for the Euro Area. The relatively smaller output effect of a government spending stimulus can be attributed to several key structural characteristics of the Irish economy namely its openness, the relatively large influence of the tradable sector in the aggregate economy, the reliance of the Irish economy on imports and the sensitivity of sovereign risk premia to public debt dynamics (see analysis in sections 7.1 and 7.2 below).

Our model implies that households and the government can allocate their purchases among domestic tradable, non-tradable and imported goods. Thus, a component of government spending may lead to an increase in imports through a direct and an indirect channel. First, the government can directly purchase goods from domestic sources which are produced using imports or indirectly increase economic agents’ incomes who in turn spend part of this additional income on imported goods. The magnitude of this depends on the
effect of fiscal changes on the trade balance. Our results suggest that a fiscal stimulus via government spending crowds out exports which combined with the large size and the export-orientation of the Irish tradable sector can explain the smaller magnitude of Irish spending multipliers. This is reflected in the dynamic responses of the real exchange rate, the terms of trade and the trade balance which imply that the competitiveness of the Irish economy deteriorates (see section 5 above for an analysis on the fiscal transmission mechanism).

Regarding tax multipliers, a cut in consumption tax causes a positive domestic demand effect in the Irish economy. Aggregate private consumption increases; however as explained above part of it results in larger imports. The increased domestic demand boosts the non-tradable sector which leads to upward pressures in factor inputs in both sectors and finally in domestic prices. The competitiveness of the Irish economy deteriorates, exports are crowding out and the tradable sector contracts vis-à-vis the non-tradable sector.

Temporary income tax cuts (capital and labour) take relatively more time to accumulate and from a magnitude perspective produce consistently lower impact multipliers. Income tax cuts have both demand- and supply-side effects and stimulate private consumption and investment and in almost all cases employment. In addition, income taxes affect equilibrium factor prices directly meaning that they reduce marginal costs especially in the tradable sector. The reduction in prices in the tradable sector improves the competitiveness of the Irish economy. As a result, Irish exports increase under both income tax cuts while the trade balance decreases less than in all the other cases. To measure the compositional effects of a fiscal stimulus in the Irish economy we compute multipliers for other key Irish macroeconomic variables in the next section.

6.4 Other key multipliers

So far we have used a single measure to gauge the effects of discretionary fiscal actions, namely the aggregate output multiplier. Although the output multiplier enables fiscal policymakers to see the overall effects on the Irish economy, usually it is useful to quantify multipliers of other key endogenous macroeconomic variables. This is of particular importance when fiscal policy aims to target specific sectors or/and cohorts of the population when forming its policy. It is well known that different fiscal policy instruments can have different implications in different sectors and/or for agents of the economy (see e.g. Leeper (2010)). The openness and the export/import-oriented nature of the Irish economy make this analysis essential for designing well executed fiscal policies.
To assess these effects in Tables 6 and 7 we quantify fiscal multipliers for the sectoral outputs, consumption, total and sectoral investment, agent-specific consumption and net exports by fiscal instrument (spending and tax respectively). We compute these multipliers implementing the same fiscal scenario analysed in section 6.2. To save space we only present the impact multiplier for each endogenous variable. Tables 6 and 7 illustrate that the stimulative effects on aggregate output solely comes from the non-tradable sector; whereas a fiscal stimulus leaves the tradable sector unaffected and this holds across all available fiscal instruments. This can be explained by the net exports multiplier computed in the last rows of Table 6 and 7. A fiscal stimulus causes a decrease in the trade balance. This can be explained by several reasons including the reliance of the Irish economy on imports, the deterioration in the terms of trade and the resulting deterioration in domestic competitiveness. That is, a relatively large share of the additional disposable income that may arise through spending hikes or tax cuts will result in higher levels of imports, while Irish exports become relatively more expensive.

Table 6: Spending Impact Multipliers

<table>
<thead>
<tr>
<th>Variable</th>
<th>$g^{s,c}$</th>
<th>$g^{s,i}$</th>
<th>$s^w$</th>
<th>$s^l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.59</td>
<td>0.62</td>
<td>1.16</td>
<td>0.24</td>
</tr>
<tr>
<td>Tradable</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Non-tradable</td>
<td>0.6</td>
<td>0.61</td>
<td>0.15</td>
<td>0.22</td>
</tr>
<tr>
<td>Aggregate consumption</td>
<td>0.03</td>
<td>0.07</td>
<td>0.187</td>
<td>0.27</td>
</tr>
<tr>
<td>&quot;Savers&quot; consumption</td>
<td>-0.1</td>
<td>-0.1</td>
<td>0.004</td>
<td>0</td>
</tr>
<tr>
<td>&quot;Non-Savers&quot; consumption</td>
<td>0.12</td>
<td>0.17</td>
<td>0.183</td>
<td>0.27</td>
</tr>
<tr>
<td>Total Investment</td>
<td>0.22</td>
<td>0.2</td>
<td>0.15</td>
<td>0.23</td>
</tr>
<tr>
<td>Investment in the tradable sector</td>
<td>-0.1</td>
<td>-0.12</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Investment in the non-tradable sector</td>
<td>0.32</td>
<td>0.32</td>
<td>0.14</td>
<td>0.21</td>
</tr>
<tr>
<td>Net Exports</td>
<td>-0.6</td>
<td>-0.59</td>
<td>-0.14</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

$42$ Net exports are defined as $p^H x - p^F y^F$. 

43
Table 7: Tax Impact Multipliers

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \tau^c )</th>
<th>( \tau^k )</th>
<th>( \tau^n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.51</td>
<td>0.44</td>
<td>0.09</td>
</tr>
<tr>
<td>Tradable</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.03</td>
</tr>
<tr>
<td>Non-tradable</td>
<td>0.49</td>
<td>0.44</td>
<td>0.12</td>
</tr>
<tr>
<td>Aggregate consumption</td>
<td>0.86</td>
<td>0.19</td>
<td>0.17</td>
</tr>
<tr>
<td>&quot;Savers&quot; consumption</td>
<td>0.4</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>&quot;Non-Savers&quot; consumption</td>
<td>0.46</td>
<td>0.16</td>
<td>0.13</td>
</tr>
<tr>
<td>Investment</td>
<td>0.16</td>
<td>0.94</td>
<td>0.09</td>
</tr>
<tr>
<td>Investment in the tradable sector</td>
<td>-0.07</td>
<td>0.56</td>
<td>0.06</td>
</tr>
<tr>
<td>Investment in the non-tradable sector</td>
<td>0.23</td>
<td>0.38</td>
<td>0.04</td>
</tr>
<tr>
<td>Net Exports</td>
<td>-0.46</td>
<td>-0.41</td>
<td>-0.15</td>
</tr>
</tbody>
</table>

7 Robustness analysis

In this section, we perform a robustness analysis along two dimensions of the model that determine the size of fiscal multipliers. First, we conduct a sensitivity analysis with respect to key structural parameters of the model. In doing so, we focus on parameters that are required to replicate some key structural characteristics of the Irish economy and have quantitatively significant effects on fiscal multipliers, e.g. the degree of openness and the sensitivity of sovereign premia to public debt dynamics. Second, we consider alternative fiscal financing scenarios, namely a spending- and a tax-financed budget neutral fiscal stimulus and compare them with our main fiscal scenario.

7.1 Degree of openness

One of the most important structural characteristics of the Irish economy is its exceptional open nature. In particular, Ireland is among the most open economies globally. This results in the following characteristics: (i) a larger share of tradable vis-à-vis the non-tradable sector, (ii) reliance on imports both for production and consumption, and (iii) an export-oriented tradable sector, meaning that the larger share of gross value
added produced in this sector is exported to the rest of the world while the domestic absorption is much smaller. These Irish specific characteristics affect the size of fiscal multipliers qualitatively and quantitatively.

To examine the significance of the degree of openness on our results we perform a robustness analysis with respect to two key structural parameters of the present model. First, the parameter that governs the long run share of tradable vis-à-vis the non-tradable production (denoted as $v$). Second, the parameter that governs the long run share of domestic absorption of the home produced tradable good vis-à-vis the imported good (denoted as $v^H$). This parameter implicitly determines the share of exports on tradable production in gross value added terms.

Figure 7 presents the impulse response functions of aggregate output, net exports, tradable and non-tradable output to a temporary discretionary fiscal shock in government consumption (for comparison we adopt the main fiscal scenario in section 5) when we vary the structural parameter, $v$. In particular, we simulate the main fiscal scenario in three economies. First, when the associated parameter is calibrated to reflect the shares of tradable and non-tradable sectors in the Irish economy ($v = 0.5817$). Then, we examine when there is a balanced economy between the tradable and non-non-tradable sectors ($v = 0.5$) and finally an economy with a larger share of the non-tradable sector ($v = 0.3$). As the parameter $v$ decreases the economy depends more on the non-tradable sector which implies a smaller degree of openness.

The dynamic paths for the three economies are illustrated in Figure 7. Our results suggest that the output multiplier is smaller when the economy relies relatively more on the tradable sector. The calibration based on the Irish data delivers the smallest output multiplier which is consistent with the very open nature of the Irish economy. That is, the fiscal stimulus is crowding out exports and crowding in imports (see the dynamic response of net exports). This causes a contraction in the tradable sector (especially when this sector is export-oriented as in the Irish case - see next paragraph). The larger the share of the tradable sector, the smaller the size of the spending multiplier.
To further explore the degree of openness channel on spending multipliers, Figure 7 depicts the associated impulse response functions to a government consumption shock (again as it is defined in section 5) when we vary the structural parameter governing the share of imports in the production of the composite tradable good (and implicitly the share of exports in domestic tradable production). As above, we solve for three economies, i.e. for an economy in which the largest share of domestic tradable production is exported to the rest of the world while at the same time domestic investment and consumption depends heavily on imports ($v_H = 0.03$). This is the value calibrated using Irish data. We also solve the model, $v_H = 0.3$ for and $v_H = 0.5$, the latter parameterizations result in a larger share of domestic absorption of the tradable output and at the same time consumption and investment relies less on imports. In other words, Irish residents consume more of home produced tradable goods and less imported goods; this implies a smaller degree of openness.

As above, the dynamic path of aggregate output indicates that the fiscal multiplier is getting smaller as we decrease $v_H$. The economic intuition is the following, government consumption stimulates domestic demand while as indicated above crowds out net exports. Thus, the multiplier decreases (increases) as a smaller (larger) share of home tradable production is consumed by domestic agents (see dynamic response of tradable output). In addition, as $v_H$ increases, Irish residents and firms utilize less imports and more from the domestically produced intermediate goods. In general, the smaller the $v_H$, the more export oriented is the home tradable sector and import driven domestic demand and as a result the smaller the size of the output multiplier.
7.2 Sensitivity of sovereign risk premia to debt dynamics

Ireland recently exited a programme of financial support in which it originally entered due to unsustainable public debt dynamics and high sovereign premia. Over the period 2010 to 2013 the Irish sovereign risk premia rose sharply while the Irish government was unable to borrow from international financial markets. In light of this economic environment, the effect of the fiscal stimulus on public debt dynamics and the sovereign premia is very important in determining the size of output multiplier. To illustrate this, Figure 9 presents the results of a sensitivity analysis with respect to the parameter in equation (60), $\psi^d$, that governs the sensitivity of the nominal interest rate, at which Ireland borrows from the international capital markets to deviations of the public debt to output ratio from a threshold value. Our results indicate that a larger parameter, that results in an increased sovereign premium, entails higher cost of international borrowing (higher nominal rates) for an identical government spending increase. This increases the crowding out channel of the fiscal stimulus and, as a result, reduces the size of the output multiplier.
7.3 Method of fiscal financing

In the benchmark fiscal scenario fiscal stimulus is financed by a gradual decrease in total public transfers and a mild increase in government debt which means that all the remaining fiscal instruments are kept constant to their historical data averages. That is, we set the reaction of the public transfers to debt in the associated fiscal rule (in equation 56) equal to 0.1 while the remaining feedback policy coefficients are set equal to 0. As pointed out in e.g. Leeper (2010), the size of multipliers depends on the method of fiscal financing schemes. To this end, in this section we compare three different fiscal financing scenarios of an increase in government consumption, namely our main scenario which means public transfers and debt-financed fiscal stimulus along with two budget neutral fiscal scenarios, i.e. a spending- and a tax-financed scenario. To simulate these scenarios, we augment the fiscal rules in equations (53-59) to react to the level of public deficit/surplus. In particular, fiscal rules are written as:

\[ f_t^g - f^g = \rho^g (f_{t-1}^g - f^g) - \gamma^g_d \left( \frac{d_t}{y_{t}^{gap}} - \frac{d}{y_{t}^{gap}} \right) - \gamma^g_d (df_t - df) + \varepsilon^g_t \]  

\[ \tau_t - \tau = \rho^\tau (\tau_{t-1} - \tau) + \gamma^\tau_d \left( \frac{d_t}{y_{t}^{gap}} - \frac{d}{y_{t}^{gap}} \right) + \gamma^\tau_d (df_t - df) + \varepsilon^\tau_t \]

where \( f_t^g \) are spending instruments and \( df_t \equiv g_t^c + g_t^i + g_t^w - \tau_t^c \) and \( df_t > (\leq) 0 \) implies that a country runs a fiscal deficit (surplus). In the spending-financed scenario, the increase in government consumption is budget
neutral and is financed through a decrease in government investment. To simulate this fiscal scenario we set \( \gamma^I_d = 1, \gamma^{df,i} = -9 \) and \( \gamma^{df,l} = 9 \). Thus, total government spending remains constant (see the red line with the circle marker in Figure 10) so the deficit/surplus and public debt remains unchanged. Similarly, in the tax-financed scenario, the increase in government consumption is again budget neutral but now is financed through an increase in tax revenues. To simulate this fiscal scenario we set \( \gamma^{c,e}_d = \gamma^{c,k}_d = \gamma^{c,n}_d = 0.7 \), and \( \gamma^{c,e}_{df} = \gamma^{c,k}_{df} = \gamma^{c,n}_{df} = 20 \). In this case, total government spending increases (see the yellow dashed line) but at the same time tax revenues increase so as the deficit/surplus and public debt remains constant. We observe in the first panel of Figure 10 that the main fiscal scenario yields the larger multiplier on impact. A government consumption expansion financed through cuts of other spending instruments or tax hikes mitigates the stimulative effects of an expansion in government consumption. In addition, although tax hikes are temporary and tax rates return to its pre-stimulus value after three years, the effect on output over longer horizons is negative. Using taxes (especially distortionary income taxes) to finance fiscal expansions induces negative supply-side effects which dampen the effect of spending stimulus over the longer run. These findings are consistent with similar studies that estimate negative fiscal multipliers over the long run when fiscal stimulus is financed through taxes (see e.g. Zubairy (2014)).

Figure 10: Alternative fiscal financing schemes
### 7.4 Other structural parameters

We conduct sensitivity analysis with respect to several key structural parameters that determine the size of fiscal multiplier, namely the degree of complementarity/substitutability of public and private consumption, $\psi^g \in [-0.24, 0.24]$, parameters that govern capital adjustment costs in both sectors, $\phi^H \in [0.1, 1]$ and $\phi^{NT} \in [0.1, 4]$, Rotemberg-type price rigidity parameters in the tradable and non-tradable sector, $\theta^H \in [20, 91]$, and, $\theta^{NT} \in [20, 165]$, the elasticity of substitution between the composite tradable and non-tradable good $\zeta \in [0.1, 2.5]$, the share of "non-Ricardians/Non-Savers" in Irish population, $\nu^{nr} \in [0.12, 0.5]$. For comparison, we also include results associated with structural parameters analysed in section 7.1.

In Table 8, we compute ranges of the first year fiscal multipliers by varying one structural parameter at a time. In particular, we report the first year multiplier for the values of the associated parameter in the first column, for example, the government consumption, $s^g$, first year multiplier varies from 0.2 to 1.12 as we decrease $\nu$ from 0.8 to 0.3 ceteris paribus. Our robustness analysis confirms that the relatively smaller size of the Ireland’s fiscal multipliers can be mostly attributed to structural parameters associated with its degree of openness, e.g. $\nu$, $\nu^H$, $\zeta$, then to parameters related to the flexibility of the Irish labour and product markets.

<table>
<thead>
<tr>
<th>Structural parameter</th>
<th>$s^g,c$</th>
<th>$s^g,i$</th>
<th>$\tau^c$</th>
<th>$\tau^h$</th>
<th>$\tau^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu \in [0.8, 0.3]$</td>
<td>[0.2,1.12]</td>
<td>[0.23,1.14]</td>
<td>[0.18,0.99]</td>
<td>[0.16,0.83]</td>
<td>[0.0,0.19]</td>
</tr>
<tr>
<td>$\nu^H \in [0.01, 0.8]$</td>
<td>[0.57,0.98]</td>
<td>[0.6,0.92]</td>
<td>[0.5,0.73]</td>
<td>[0.43,0.66]</td>
<td>[0.08,0.4]</td>
</tr>
<tr>
<td>$\zeta \in [2.5, 0.1]$</td>
<td>[0.24,0.71]</td>
<td>[0.28,0.74]</td>
<td>[0.21,0.64]</td>
<td>[0.195,0.55]</td>
<td>[0.03,0.11]</td>
</tr>
<tr>
<td>$\theta^g \in [0.24, -0.24]$</td>
<td>[0.53, 0.64]</td>
<td>[0.63,0.6]</td>
<td>[0.54,0.48]</td>
<td>[0.44,0.44]</td>
<td>[0.08,0.1]</td>
</tr>
<tr>
<td>$\phi^H \left(\phi^{NT}\right) \in [1 (4), 0.1 (0.4)]$</td>
<td>[0.54, 0.63]</td>
<td>[0.57,0.65]</td>
<td>[0.47,0.54]</td>
<td>[0.24,0.89]</td>
<td>[0.07,0.1]</td>
</tr>
<tr>
<td>$\nu^{nr} \in [0.12, 0.5]$</td>
<td>[0.54,0.68]</td>
<td>[0.56,0.74]</td>
<td>[0.36,1.18]</td>
<td>[0.36,0.69]</td>
<td>[0.01,0.3]</td>
</tr>
<tr>
<td>$\theta^H \left(\theta^{NT}\right) \in [20 (20), 91 (185)]$</td>
<td>[0.41,0.59]</td>
<td>[0.41,0.62]</td>
<td>[0.28,0.52]</td>
<td>[0.29,0.44]</td>
<td>[0.15,0.08]</td>
</tr>
</tbody>
</table>

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43 To save space in Table 8 we report results for government consumption and investment and the main tax rates. Results for the remaining fiscal instruments are available upon request.

44 We also solve for a version of FIR-GEM that incorporates real wage rigidity in all sectors, results are available upon request.

45 Sensitivity analysis with respect to other structural parameters are available upon request.
8 Conclusions

This paper develops FIR-GEM, a medium-scale small open economy DSGE model calibrated for the Irish economy. FIR-GEM is designed as a fiscal toolkit for fiscal policy simulations in the domestic economy. We analyse the transmission mechanism through which Irish fiscal policy affects the economy focusing on key structural characteristics of the Irish economy like its degree of openness. We also quantify Irish fiscal multipliers for a rich menu of fiscal policy instruments. We find that in the Irish economy fiscal multipliers are smaller in magnitude than most EU countries due to the degree of openness of the domestic economy and the large influence of the tradable sector. Fiscal policy changes result in changes in the composition of GDP and the external balance of the Irish economy. A fiscal stimulus is likely to expand the non-tradable sector vis-à-vis the tradable sector. A fiscal expansion via expenditures may negatively impact the Irish external balance by causing a deterioration in Irish competitiveness; while a fiscal expansion via income tax cuts may have a smaller effect on the Irish external balance as production costs and prices are likely to be reduced. This improves the competitiveness of the Irish economy and results in long lasting effects on GDP. In terms of the effect on GDP in the first year, the most effective Irish fiscal instruments are as follows: public investment, government consumption, consumption taxes, capital taxes, public transfers, public wages and, finally, labour taxes. We also find that the method of fiscal financing is crucial for the efficacy of a fiscal stimulus. A spending stimulus financed via tax increases mitigates the positive effect on GDP.

Finally, we discuss directions for future research. Ireland’s tradable sector consists of domestic firms and foreign affiliated firms (Multinational enterprises (MNEs)). MNEs production heavily depends on inputs such as R&D, IT software, brands which are complementary with high skilled labour. To capture MNEs’ distinct production characteristics, a natural extention is to incorporate technology and intangible capital as in e.g. McGrattan and Prescott (2009) and Klein and Ventura (2018) along with higher degree of capital-skill complementarity in the spirit of Krusell et al. (2000). Finally, Irish labour market is heavily affected by international migration inflows and outflows. Both features entail non trivial effects for Irish fiscal policy. We leave both these extensions for future research.
References


A  Aggregation

Aggregating quantities across "Ricardians/Savers" households $r$:

$$
\sum_{i=1}^{N^r} c_t^r = N^r c_t^r, \quad \sum_{i=1}^{N^r} x_{t}^{H,r} = N^r x_{t}^{H,r}, \quad \sum_{i=1}^{N^r} x_{t}^{NT,r} = N^r x_{t}^{NT,r}, \quad \sum_{i=1}^{N^r} k_t^{H,r} = N^r k_t^{H,r}, \quad \sum_{i=1}^{N^r} k_t^{NT,r} = N^r k_t^{NT,r},
$$

$$
\sum_{i=1}^{N^r} i_t^{H,r} = N^r i_t^{H,r}, \quad \sum_{i=1}^{N^r} i_t^{NT,r} = N^r i_t^{NT,r}, \quad \sum_{i=1}^{N^r} \omega_{t}^{H,r} = N^r \omega_{t}^{H,r}, \quad \sum_{i=1}^{N^r} \omega_{t}^{NT,r} = N^r \omega_{t}^{NT,r}, \quad \sum_{i=1}^{N^r} b_t^r = N^r b_t^r, \quad \sum_{i=1}^{N^r} f_t^r = N^r f_t^r.
$$

Similarly across "non-Ricardians/non-Savers" households $nr$:

$$
\sum_{i=1}^{N^{nr}} c_t^{nr} = N^{nr} c_t^{nr}, \quad \sum_{i=1}^{N^{nr}} i_t^{H,nr} = N^{nr} i_t^{H,nr}, \quad \sum_{i=1}^{N^{nr}} i_t^{NT,nr} = N^{nr} i_t^{NT,nr}, \quad \sum_{i=1}^{N^{nr}} \omega_{t}^{H,nr} = N^{nr} \omega_{t}^{H,nr}, \quad \sum_{i=1}^{N^{nr}} \omega_{t}^{NT,nr} = N^{nr} \omega_{t}^{NT,nr}, \quad \sum_{i=1}^{N^{nr}} b_t^{nr} = N^{nr} b_t^{nr}, \quad \sum_{i=1}^{N^{nr}} f_t^{nr} = N^{nr} f_t^{nr}.
$$

Aggregation across intermediate non-tradable firms $i$:

$$
\sum_{i=1}^{N^i} y_{t}^{NT,i} = N^i y_{t}^{NT,i}, \quad \sum_{i=1}^{N^i} k_{t}^{NT,i} = N^i k_{t}^{NT,i}, \quad \sum_{i=1}^{N^i} i_{t}^{NT,i} = N^i i_{t}^{NT,i}.
$$

Aggregation across intermediate tradable firms $j$:

$$
\sum_{j=1}^{N^j} y_{t}^{NT,j} = N^j y_{t}^{NT,j}, \quad \sum_{j=1}^{N^j} k_{t}^{NT,j} = N^j k_{t}^{NT,j}, \quad \sum_{j=1}^{N^j} i_{t}^{NT,j} = N^j i_{t}^{NT,j}.
$$

Small case letters denote per capita or per firm quantities. For example, the final good in aggregate terms is denoted as $Y_t$, and in per capita terms is $y_t \equiv \frac{Y_t}{N}$.

B  Market clearing conditions

We solve for a symmetric equilibrium in per capita terms. Without loss of generality we set $N^i = N^j = N$ and $\nu^r = \frac{N^r}{N}$ and $\nu^{nr} = \frac{N^{nr}}{N} = 1 - \nu^r$. Below, we present the clearing market conditions by market, i.e. the final good, tradable and non-tradable goods markets, labour market, capital and domestic bonds market. Market clearing condition in the final good market yields:

$$
y_t = \nu^r c_t^r + \nu^r x_{t}^{H,r} + \nu^r x_{t}^{NT,r} + \nu^{nr} c_t^{nr} + g_t^r + g_t^{nr} \tag{70}
$$

In the tradable good market (in aggregate terms):

$$
\sum_{j=1}^{N^j} y_{t}^{NT,j} = N^j y_{t}^{NT,j} = Y_t^{H,d} + X_t \tag{71}
$$
where $Y_{t}^{H,d}$ is domestic absorption of home produced tradable good and $X_{t}$ demand from the rest-of-the world (i.e. exports). Equation (71) in per capita terms is written:

$$y_{t}^{H} = y_{t}^{H,d} + x_{t} \quad \text{(72)}$$

where $y_{t}^{H,d} \equiv \frac{Y_{t}^{H,d}}{N}$ and $x_{t} \equiv \frac{X_{t}}{N}$. In the non-tradable good market (in aggregate terms):

$$Y_{t}^{NT} = \sum_{i=1}^{N} y_{i}^{NT,i} = N^{i} y_{i}^{NT,i}$$

where $y_{t}^{NT} \equiv \frac{Y_{t}^{NT}}{N}$ denotes per capita quantity. In capital markets:

$$\sum_{j=1}^{N^{j}} k_{t}^{H,j} = N^{j} k_{t}^{H,i} = \sum_{r=1}^{N^{r}} k_{t}^{H,r} = N^{r} k_{t}^{H,r}$$

$$\sum_{i=1}^{N^{i}} k_{t}^{NT,i} = N^{i} k_{t}^{NT,i} = \sum_{r=1}^{N^{r}} k_{t}^{NT,r} = N^{r} k_{t}^{NT,r}$$

In labour market of the home tradable good:

$$\sum_{i=1}^{N^{i}} \ell_{i}^{H,i} = \sum_{r=1}^{N^{r}} \ell_{i}^{H,r} + \sum_{nr=1}^{N^{nr}} \ell_{i}^{H,nr}$$

which yields:

$$N^{i} \ell_{i}^{H,i} = N^{r} \ell_{i}^{H,r} + N^{nr} \ell_{i}^{H,nr} \quad \text{(73)}$$

In labour market of the non-tradable good:

$$\sum_{j=1}^{N^{j}} \ell_{j}^{NT,j} = \sum_{r=1}^{N^{r}} \ell_{j}^{NT,r} + \sum_{nr=1}^{N^{nr}} \ell_{j}^{NT,nr}$$

which yields:

$$N^{j} \ell_{j}^{NT,j} = N^{r} \ell_{i}^{H,r} + N^{nr} \ell_{i}^{H,nr} \quad \text{(74)}$$
In the labour market of the public good:

\[ L^g_t = N^r l^P_t + N^{nr} l^{P, nr}_t \]  

(75)

and \( b^g_t \equiv \frac{L^g_t}{N^r} \). In domestic government bonds market:

\[ \sum_{r=0}^{N^r} b^r_t = N^r b_t \]  

(76)

Notice that aggregating total profits in the two sectors across firms and households yield \( \sum_{r=1}^{N^r} \omega_{H,r}^t = N^r \omega_{H}^t = \sum_{i=1}^{N^i} \omega_{H,i}^t = N^i \omega_{H}^t \) and \( \sum_{r=1}^{N^r} \omega_{NT,r}^t = N^r \omega_{NT}^t = \sum_{j=1}^{N^j} \omega_{NT,j}^t = N^j \omega_{NT}^t \).

C Government Budget Constraint

The sequential government budget constraint in nominal and aggregate terms is written:

\[ P_t B_t + S_t P^{*}_t F^g_t = R_{t-1} P_{t-1} B_{t-1} + Q_{t-1} S_t P^{*}_{t-1} F^{g}_{t-1} + P_t G_t - P_t T_t \]  

(77)

where \( P_t G_t \) is total government spending expressed in units of the final good:

\[ P_t G_t \equiv P_t G^c_t + P_t G^i_t + P_t G^w_t - P_t T^d_r - P_t T^d_{nr} \]  

(78)

Total expenditures decomposes into government consumption, \( G^c_t \), government investment, \( G^i_t \), public wage bill, \( G^w_t \), total public transfers, \( P_t T^d_t \equiv -P_t T^d_r - P_t T^d_{nr} \), where \( T^d_r, T^d_{nr} < 0 \) are agent-specific public transfers.

For convenience we define total public debt expressed in domestic currency as \( P_t D_t \equiv P_t B_t + S_t P^{*}_t F^g_t \). Where the share held by domestic households (Irish residents) is defined as \( \lambda^g_t \equiv \frac{P_t B_t}{P_t D_t} \) and the share held by foreigners (non-Irish residents) is defined as \( (1 - \lambda^g) \equiv \frac{S_t P^{*}_t F^g_t}{P_t D_t} \).
In per capita and real terms is written:

\[ P_t T_t \equiv P_t \tau_t^c \left( N_t^c e_t^c + N_t^{cr} c_t^{cr} \right) + P_t \tau_t^N N_t^c \left( w_t^H H_t + w_t^{NT} i_t^{NT,r} + w_t^P P_t^r \right) + P_t \tau_t^{NR} \left( w_t^{H,nr} H_t + w_t^{NT,nr} i_t^{NT,nr} + w_t^{P,nr} P_t^{n,ar} \right) + \tau_t^k N_t \left( r_t^{k1} k_{t-1} + \omega_t^k + r_t^{NT,k} k_{t-1}^{NT} + \omega_t^{NT,r} \right) \]  

(79)

Using the definition of total public debt, we can re-write (77) as follows:

\[ P_t D_t = R_{t-1} \lambda_t^0 P_{t-1} D_{t-1} + Q_{t-1} \frac{S_t}{S_{t-1}} (1 - \lambda_t^0) P_{t-1} D_{t-1} + P_t G_t - P_t T_t \]

In per capita and real terms is written:

\[ d_t = R_{t-1} \lambda_t^0 d_{t-1} + Q_{t-1} \frac{S_t}{S_{t-1}} (1 - \lambda_t^0) d_{t-1} + g_t^c + g_t^i + g_t^w - \tau_t^c - \tau_t \]  

(80)

where \( d_t \equiv \frac{D_t}{N_t^c} \), \( g_t^c \equiv \frac{G_t^c}{N_t^c} \), \( g_t^i \equiv \frac{G_t^i}{N_t^i} \), \( g_t^w \equiv \frac{G_t^w}{N_t^w} \), \( \tau_t^c \equiv \frac{T_t^c}{N_t^c} \), \( \tau_t \equiv \frac{T_t}{N_t} \) where tax revenues in real and per capita terms are:

\[ \tau_t \equiv \tau_t^c \left( N_t^c e_t^c + N_t^{cr} c_t^{cr} \right) + \tau_t^N N_t^c \left( w_t^H H_t + w_t^{NT} i_t^{NT,r} + w_t^P P_t^r \right) + \tau_t^{NR} \left( w_t^{H,nr} H_t + w_t^{NT,nr} i_t^{NT,nr} + w_t^{P,nr} P_t^{n,ar} \right) + \tau_t^k N_t \left( r_t^{k1} k_{t-1} + \omega_t^k + r_t^{NT,k} k_{t-1}^{NT} + \omega_t^{NT,r} \right) \]  

(81)

**D The evolution of net foreign debt (assets)**

In this appendix we derive the equation (51) that governs the evolution of net foreign debt (assets). First, we aggregate the Ricardian household budget constraint over \( r \):

\[ P_t \left( 1 + \tau_t \right) N_t^c e_t^c + P_t N_t^c x_t^H + P_t N_t^c x_t^{NT,r} + P_t N_t^c x_t^{NT} + P_t N_t^c b_t^r + S_t P_t N_t^c f_t^{cr} + N_t^c \Phi \left( f_t^r, f_t^* \right) \]

\[ = (1 - \tau_t^c) P_t N_t^c \left( w_t^H H_t + w_t^{NT} i_t^{NT,r} + w_t^P P_t^r \right) + (1 - \tau_t^{NR}) P_t N_t^c \left( r_t^{k1} k_{t-1} + \omega_t^k + \omega_t^{NT,r} \right) \]

\[ + (1 - \tau_t^k) P_t N_t^c \left( r_t^{k1} k_{t-1} + \omega_t^k \right) + R_{t-1} P_{t-1} N_t^c b_{t-1} + Q_{t-1} S_t P_t^* N_t^c f_t^{cr} - P_t N_t^c x_t^{cr} \]  

(82)
Recall that the government budget constraint in aggregate terms is written as:

\[ P_t B_t + S_t P_t^* F_t^{*g} = R_{t-1} P_{t-1} B_{t-1} + Q_{t-1} S_t P_{t-1}^* F_{t-1}^{*g} \]
\[ + P_t G_t^r + P_t G_t^i + P_t G_t^{nr} - P_t N^r \tau_{t}^{r} - P_t N^r \tau_{t}^{nr} \]
\[ - P_t \tau_{t}^{i} \left( N^r c_t^{i} + N^{nr} c_t^{nr} \right) - P_t \tau_{t}^{nr} N^r \left( w_t H_t^{r} + w_t^{NT} t^{NT,r} + w_t^{P} t^{P,r} \right) \]
\[ - P_t \tau_{t}^{i} N^{nr} \left( w_t H_t^{r} + w_t^{NT} t^{NT, nr} + w_t^{P} t^{P, nr} \right) \]
\[- \tau_{t}^{i} N^r P_t \left( r_t^{H,k} k_{t-1}^{H,i} + \omega_t^{H,r} + r_t^{NT,k} k_{t-1}^{NT,i} + \omega_t^{NT,r} \right) \] (83)

Subsequently, using the fact that in equilibrium \( P_t B_t = P_t N^r b_t \) we solve (83) for \( P_t B_t \) to substitute out this term from (82). We aggregate profits across firms and Ricardians households, which implies that \( \sum_{i=1}^{N_t} \omega_{t}^{NT,i} = N^r \omega_{t}^{NT,r} = N^r \omega_{t}^{NT,r} \) and \( \sum_{j=1}^{N_t} \omega_{t}^{H,j} = N^j \omega_{t}^{H,j} = \sum_{r=1}^{N_t} \omega_{t}^{NT,r} = N^r \omega_{t}^{NT,r} \). Thus, the definitions of aggregate profits are written as:

\[ P_t N^r \omega_{t}^{NT,r} = P_t N^r \omega_{t}^{NT,i} = P_t^{NT} N_t^{y_t^{NT,i}} - P_t^{NT,k} N_t^{i} k_{t-1}^{NT,i} - P_t w_t^{NT} N_t^{i} t^{NT,i} - N^r \phi_{t}^{NT,i} \left( \frac{P_t^{NT}}{P_{t-1}^{NT}} - 1 \right)^2 P_t^{NT} y_t^{NT} \] (84)

\[ P_t N^r \omega_{t}^{H,r} = P_t N^r \omega_{t}^{H,i} = P_t^{H} N_t^{y_t^{H,i}} - P_t^{H,k} N_t^{i} k_{t-1}^{H,i} - P_t w_t^{H} N_t^{i} t^{H,i} - N^j \phi_{t}^{H,j} \left( \frac{P_t^{H}}{P_{t-1}^{H}} - 1 \right)^2 P_t^{H} y_t^{H} \] (85)

Subsequently, we substitute the equations (84) and (85) into (83), then, the marketing clearing conditions, \( N_t^{i} k_{t-1}^{H,i} = N^r k_{t-1}^{H,i}, N_t^{i} k_{t-1}^{NT,i} = N^r k_{t-1}^{NT,i}, N_t^{i} t^{NT,i} = N^r t^{NT,i} + N^{nr} t^{NT, nr}, N_t^{i} t^{H,i} = N^r t^{H,i} + N^{nr} t^{H, nr} \) and the aggregate non-Ricardian budget constraint and we substitute the definition of the public wage bill,

\[ P_t G_t^i = N^r P_t w_t^{P} t^{P, r} + N^{nr} P_t w_t^{P} t^{P, nr}. \] The equation (82) is written:

\[-S_t P_t^* F_t^{*g} + Q_{t-1} S_t P_t^* F_{t-1}^{*g} + P_t G_t^r + P_t G_t^i + P_t G_t^{nr} \]
\[ P_t N^r \omega_{t}^{r} + P_t N^r c_t^{r} + P_t N^r c_t^{nr} + S_t P_t^* N^r f_t^{r} \]
\[ = Q_{t-1} S_t P_t^* N^r f_{t-1}^{r} + P_t^{NT} N_t^{i} y_t^{NT,i} + P_t^{H} N_t^{i} y_t^{H,j} - N^r \phi_{t}^{NT,i} \left( \frac{P_t^{NT}}{P_{t-1}^{NT}} - 1 \right)^2 P_t^{NT} y_t^{NT} \]
\[- N^r \phi_{t}^{NT,i} \left( \frac{P_t^{H}}{P_{t-1}^{H}} - 1 \right)^2 P_t^{H} y_t^{H} - N^r \Phi^* \left( f_t^{r} + f_t^{*} \right) \] (86)

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In turn, we use the zero profit conditions for the final good and the composite tradable good firms:

\[ P_t Y_t = P_t^H Y_t^H + P_t^{NT} Y_t^{NT} = P_t^H Y_t^{H,d} + P_t^F Y_t^F + P_t^{NT} Y_t^{NT} \]

(87)

and the clearing market condition for the domestic tradable good:

\[ P_t^H Y_t^H = P_t^H \left( Y_t^{H,d} + X_t \right) = P_t^H N_j y_t^{H,d} \]

(88)

where \( X_t \) are aggregate exports and \( Y_t^{H,d} \) aggregate domestic absorption of the home tradable good. Non-tradable output is written:

\[ P_t^{NT} N_j y_t^{NT,j} = P_t^{NT} Y_t^{NT} = P_t Y_t - P_t^H Y_t^{H,d} - P_t^F Y_t^F \]

(89)

Plugging (87) and (88) into (86) we have that:

\[ -S_t P_t^* F_t^* + Q_{t-1} S_t P_{t-1}^* F_{t-1}^* + P_t G_t^e + P_t G_t^i + P_t N^r c_t^{nr} \]

\[ P_t N^r c_t^e + P_t N^r x_t^{H,r} + P_t N^r x_t^{NT,r} + S_t P_t^* N^r f_t^r \]

\[ = Q_{t-1} S_t P_{t-1}^* N^r f_{t-1}^r + P_t Y_t - P_t^H Y_t^{H,d} - P_t^F Y_t^F + P_t \left( Y_t^{H,d} + X_t \right) \]

\[ -N_j \frac{\phi_{NT,j}}{2} \left( \frac{P_t^{NT}}{P_t^{NT,j}} - 1 \right)^2 P_t^{NT,NT} - N_j \frac{\phi_H}{2} \left( \frac{P_t^H}{P_t^{NT,j}} - 1 \right)^2 P_t^H y_t^H - N^r \Phi^* \left( f_t^*, f_t^* \right) \]

(90)

Using the clearing market condition for the final good:

\[ P_t Y_t = P_t N^r c_t^e + P_t N^r c_t^{nr} + P_t N^r x_t^{H,r} + P_t N^r x_t^{NT,r} + P_t G_t^e + P_t G_t^i \]

(91)

(which is written in real per capita terms \( y_t = \nu^r c_t^e + \nu^r x_t^{H,r} + \nu^r x_t^{NT,r} + \nu^{nr} c_t^{nr} + g_t^e + g_t^i \)).

The evolution of net foreign debt in aggregate terms is given by:

\[ S_t P_t^* F_t^* - S_t P_{t-1}^* F_{t-1}^* = Q_{t-1} S_t P_{t-1}^* F_{t-1}^* - Q_{t-1} S_t P_{t-1}^* N^r f_{t-1}^r \]

\[ +P_t^F Y_t^F - P_t^H X_t \]

\[ +N_j \frac{\phi_{NT,j}}{2} \left( \frac{P_t^{NT}}{P_t^{NT,j}} - 1 \right)^2 P_t^{NT,NT} + N_j \frac{\phi_H}{2} \left( \frac{P_t^H}{P_t^{NT,j}} - 1 \right)^2 P_t^H y_t^H + N^r \Phi^* \left( f_t^*, f_t^* \right) \]

(92)
In real and capita terms:

\[ \frac{S_i P_i^t}{P_i^t} (f_i^g - \frac{N^r}{N^r} f_i^{r,t}) = Q_{i-1} \frac{S_i P_i^{t-1}}{P_i^{t-1}} (f_i^g - \frac{N^r}{N^r} f_i^{r,t-1}) + P_i^F y_i^F - P_i^H x_t + \frac{\phi_{N^T,i}}{2} \left( \frac{P_i^{N^T,i}}{P_i^{t-1}} - 1 \right)^2 P_i^{N^T,i} y_i^{NT} + \frac{\phi_{H,i}}{2} \left( \frac{P_i^H}{P_i^{t-1}} - 1 \right)^2 P_i^H y_i^H + \frac{N^r}{N^r} \Phi^* (f_i^r, f^*) \]  

where \( f_i^{g,r} = \frac{F_i^{g,r}}{N^r}, y_i^F = \frac{Y_i^F}{N^r} \) and \( x_t = \frac{X_t}{N^r} \) are per capita imports and exports respectively.

### E Profit maximization

In this appendix we solve the profit maximization problem of intermediate firms in the tradable sector. The profit maximization problem in the non-tradable sector is similar. Each firm \( i \) chooses its price, \( P_{i,N^T,i} \), to maximize its the expected sum of nominal profits facing Rotemberg-type nominal rigidities:

\[ \max_{P_{i,N^T,i}} E_0 \sum_{t=0}^{\infty} \Xi_{0,t} \left\{ P_{i,N^T,i} y_i^{N^T,i} - \Psi \left( y_i^{N^T,i} \right) - \frac{\phi_{NT}}{2} \left( \frac{P_{i,N^T,i}}{P_{i,N^T,i-1}} - 1 \right)^2 P_{i,N^T,i} y_i^{NT} \right\} \]  

where \( \Xi_{0,t} \) is a stochastic discount factor, while \( \Psi \left( y_i^{N^T,i} \right) \) denotes the total nominal cost, subject to demand for each variety \( i \):

\[ y_i^{N^T,i} = \left[ \frac{P_{i,N^T,i}}{P_i^t} \right]^{\varepsilon_{NT}} y_i^{NT} \]  

The first order condition yields:

\[ E_0 \Xi_{0,t} \left\{ \left( 1 - \varepsilon_{NT} \right) \left[ \frac{P_{i,N^T,i}}{P_i^t} \right]^{-\varepsilon_{NT}} y_i^{NT} - \left( -\varepsilon_{NT} \right) \Psi' \left( y_i^{N^T,i} \right) \left[ \frac{P_{i,N^T,i}}{P_i^t} \right]^{-\varepsilon_{NT}} \frac{1}{P_i^{t-1}} y_i^{NT} \right\} - E_0 \Xi_{0,t+1} \left\{ \phi_{NT} \left( \frac{P_{i,N^T,i}}{P_i^t} - 1 \right) P_{i,N^T,i} y_i^{NT} y_i^{NT+1} \left( -\frac{1}{P_i^{t+1}} \right)^2 P_{i,N^T,i+1} \right\} = 0 \]

Assuming a symmetric equilibrium, i.e. \( P_{i,N^T,i} \equiv P_{i,N^T} \) and \( y_i^{N^T} \equiv y_i^{N^T,i} \) and multiply with \( P_{i,N^T} \):

\[ E_0 \Xi_{0,t} \left\{ \left( 1 - \varepsilon_{NT} \right) P_{i,N^T} y_i^{NT} + \varepsilon_{NT} \Psi' \left( y_i^{NT} \right) \right\} - E_0 \Xi_{0,t} \left\{ \phi_{NT} \left( \frac{P_{i,N^T}}{P_i^{t-1}} - 1 \right) P_{i,N^T} y_i^{NT} P_{i,N^T,i} y_i^{NT} y_i^{NT+1} \left( -\frac{1}{P_i^{t+1}} \right)^2 P_{i,N^T,i+1} \right\} = 0 \]  

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Divide with $P_t$ and substituting the stochastic discount factor $\Xi_{0,t+1} \equiv \frac{1}{P_t} = \frac{\nu_{t+1}}{\nu_t} P_t$: 

$$E_0 \left\{ \left(1 - \nu^{NT} \right) p_{t}^{NT} y_{t} + \nu^{NT} \psi \left( \cdot \right) y_{t}^{NT} \right\} - \phi^{NT} \left( \frac{p_{t}^{NT}}{p_{t-1}^{NT}} \right) p_{t}^{NT} y_{t}^{NT} p_{t}^{NT} + \beta \frac{\nu_{t+1}}{\nu_t} P_t \left\{ \phi^{NT} \left( \frac{p_{t+1}^{NT}}{p_t} p_{t+1} \right) - 1 \right\} p_{t+1}^{NT} y_{t+1}^{NT} p_{t+1}^{NT} P_{t+1}^{NT} = 0 \quad (97)$$

where $p_{t}^{NT} \equiv \frac{p_{t}^{NT}}{P_t}$ and $\psi \left( \cdot \right) \equiv \frac{\psi \left( \cdot \right)}{P_t}$. 

F Decentralized Equilibrium (in nominal terms)

The Decentralized Equilibrium is a set of 58 processes $c_t, \lambda_t^H, l_t^H, l_t^{NT, H}, p_t^{H, r}, k_t^{NT, r}, f_t^{r, H}, c_t^{NT}, \lambda_t^{NT}, l_t^{NT, r}, l_t^{NT, r}, l_t^{NT, r}, l_t^g y_t^H, \psi^H, \omega_t^H, t_t^{H, k}, w_t^H, y_t^{NT}, \psi_t^{NT}, \omega_t^{NT}, t_t^{NT, k}, w_t^{NT}, k_t^g, y_t^P, w_t, d_t, \tau_t, y_t, y_t^H, y_t^F, y_t^{g, P}, Q_t, R_t, P_t^H, P_t^{NT}, P_t^T, P_t, P_t^F, x_t^H, x_t^{NT}, x_t, tot_t g_t, g_t, g_t^w, t_t^1, s_t^w, s_t^g, s_t^c, s_t^k, \tau_t^1, \tau_t^c, \tau_t^k, \tau_t^n$ satisfying the following 58 equations, given the exogenous variables $P_t^*, A_t^H, A_t^{NT}, A_t^g$ and $\frac{S_t}{S_{t-1}}$ and initial conditions for the state variables:

$$\frac{\partial U^r_t}{\partial c_t} = \lambda_t^r \left(1 + \tau_t^r \right) \quad (F.1.)$$

$$\frac{\partial U^r_t}{\partial l_t^{H, r}} = \lambda_t^r \left(1 - \tau_t^n \right) w_t^H \quad (F.2.)$$

$$\frac{\partial U^r_t}{\partial l_t^{NT, r}} = \lambda_t^r \left(1 - \tau_t^n \right) w_t^{NT} \quad (F.3.)$$

$$\frac{\partial U^r_t}{\partial p_t^{r, r}} = \lambda_t^r \left(1 - \tau_t^n \right) w_t^P \quad (F.4.)$$

$$\lambda_t^r \left(1 + \frac{\phi \left( k_t^H, k_{t-1}^H \right)}{\partial k_t^H} \right) = E_0 \beta \lambda_{t+1}^r \left(1 - \delta^H + (1 - \tau_{t+1}^H) r_{t+1}^{H, k} - \frac{\phi \left( k_{t+1}^{NT}, k_t^H \right)}{\partial k_t^H} \right) \quad (F.5.)$$

$$\lambda_t^r \left(1 + \frac{\phi^{NT} \left( k_t^{NT}, k_{t-1}^{NT} \right)}{\partial k_t^{NT}} \right) = E_0 \beta \lambda_{t+1}^r \left(1 - \delta^{NT} + (1 - \tau_{t+1}^k) r_{t+1}^{NT, k} - \frac{\phi^{NT} \left( k_{t+1}^{NT}, k_t^{NT} \right)}{\partial k_t^{NT}} \right) \quad (F.6.)$$

$$\lambda_t^r = E_0 \beta \lambda_{t+1}^r R_t \frac{P_t}{P_t+1} \quad (F.7.)$$

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\[ \Lambda \left( \frac{S_P^r}{P_t} + \frac{\partial \phi^r(f_{tr}^r, f_{tr}^{\prime r})}{\partial f_{tr}^{\prime r}} \right) = E_0 \beta Q_1 \Lambda_{t+1} \frac{S_{t+1} P_{t+1}^r}{P_{t+1}^r} \]  

(F.8.)

\[ \frac{\partial U_{tr}^{nr}}{\partial c_{tr}^{nr}} = \Lambda_t^{nr} (1 + \tau_t) \]  

(F.9.)

\[ - \frac{\partial U_{tr}^{nr}}{\partial H_t^{nr}} = \Lambda_t^{nr} (1 - \tau_t^w) w_t^H \]  

(F.10.)

\[ - \frac{\partial U_{tr}^{nr}}{\partial NT_t^{nr}} = \Lambda_t^{nr} (1 - \tau_t^w) w_t^{NT} \]  

(F.11.)

\[ - \frac{\partial U_{tr}^{nr}}{\partial H_t} = \Lambda_t^{nr} (1 - \tau_t^w) w_t^P \]  

(F.12.)

\[(1 + \tau_t) P_t c_{tr}^{nr} = (1 - \tau_t^w) P_t \left( w_t^H H_t^{nr} + w_t^{NT} H_t^{NT,.nr} + w_t^P P_t^{nr} \right) - P_{t,r}^{1,nr} \]  

(F.13.)

\[ y_t = \left[ (v)^{\zeta} (y_t^{1,n})^{\zeta_{\zeta^{1,n}}} + (1 - v)^{\zeta} (y_t^{NT})^{\zeta_{\zeta^{1,n}}} \right]^{\zeta_{\zeta^{1,n}}} \]  

(F.14.)

\[ y_t^{T} = \frac{v}{1 - v} \left( \frac{P_t^T}{P_t^{NT}} \right)^{-\zeta} y_t^{NT} \]  

(F.15.)

\[ P_t = \left[ v \left( P_t^T \right)^{1-\zeta} + (1 - v) \left( P_t^{NT} \right)^{1-\zeta} \right]^{\zeta_{\zeta^{1,n}}} \]  

(F.16.)

\[ y_t^{T} = \left[ (v^{H})^{\zeta_{\zeta^{1,n}}} (y_t^{H,d})^{\zeta_{\zeta^{1,n}}^{1,m}} + (1 - v^{H})^{\zeta_{\zeta^{1,n}}} (y_t^{F})^{\zeta_{\zeta^{1,n}}^{1,m}} \right]^{\zeta_{\zeta^{1,n}}^{1,m}} \]  

(F.17.)

\[ y_t^{H,d} = \left. \frac{v^{H}}{1 - v^{H}} \left( \frac{P_t^{H}}{P_t^{F}} \right)^{-\zeta_{\zeta^{1,n}}} \right. y_t^{F} \]  

(F.18.)

\[ P_t^{T} = \left[ v^{H} \left( P_t^{H} \right)^{1-\zeta_{\zeta^{1,n}}} + (1 - v^{H}) \left( P_t^{F} \right)^{1-\zeta_{\zeta^{1,n}}} \right]^{\zeta_{\zeta^{1,n}}^{1,m}} \]  

(F.19.)

\[ P_t^{F} = S_t P_t^{r} \]  

(F.20.)

\[ y_t^{NT,i} = A_t^{NT} \left( y_t^{g} \right)^{x_{1}^{NT}} \left( k_{i-1}^{NT,i} \right)^{g^{NT}} \left( l_t^{NT,i} \right)^{1-a^{NT,i} x_{2}^{NT}} \]  

(F.21.)
\[ P_{t<i}^k = \Psi_t^{NT'} a^{NT} x_t^{NT,i} \frac{y_t^{NT}}{k_{t-1}^i} \]  
(F.22.)

\[ P_{t,i} w_{t,i}^{NT} = \Psi_t^{NT'} (1 - a^{NT}) x_t^{NT} \frac{y_t^{NT}}{l_{t,i}^i} \]  
(F.23.)

\[ P_{1,i}^{Nj} \omega_{t,i}^{NT,i} = P_{1,i}^{NT,i} \frac{N^j}{N} y_{t,i}^{NT,i} - P_{r,t}^i \frac{N^i}{N} k_{t-1}^i - P_{1,i} w_{t,i}^{NT} \frac{N^i}{N} l_{t,i}^i \]  
(F.24.)

\[ \left\{ (1 - \varepsilon^{NT}) P_{t,i}^{NT} y_{t,i}^{NT} + \varepsilon^{NT} \psi_t^{NT'} y_{t,i}^{NT} \right\} \]

\[ -\phi^{NT} \left( \frac{p_{t,i}^{NT}}{p_{1,i}^{NT+1}} \frac{P_{i}}{P_{i+1}} - 1 \right) P_{1,i}^{NT} y_{t,i}^{NT} \left( \frac{p_{t,i}^{NT}}{p_{1,i}^{NT+1}} \frac{P_{i}}{P_{i+1}} + \frac{N^j}{N} \frac{p_{t,i}^{NT+1}}{P_{i+1}} \right) \left\{ \phi^{NT} \left( \frac{p_{t,i}^{NT+1}}{p_{1,i}^{NT+1}} \frac{P_{i+1}}{P_{i}} - 1 \right) P_{t+1,i}^{NT} y_{t+1,i}^{NT} \frac{p_{t,i}^{NT+1}}{P_{i+1}} \right\} = 0 \]
(F.25.)

\[ y_{t,i}^{H,j} = A_i^H \left\{ y_{t,i}^{H} \right\}^{1-H} \left( \left( \frac{H}{l_{t,i}^i} \right)^{H-1} \right) \frac{1}{N^{l_{t,i}^i}} \]
(F.26.)

\[ P_{t,i} w_{t,i}^{H} = \Psi_t^{H'} (1 - a^{H}) x_t^{H} \frac{y_t^{H}}{N^{l_{t,i}^i}} \]
(F.27.)

\[ P_{1,i}^{Nj} \omega_{t,i}^{H,j} = P_{t,i}^{H} \frac{N^j}{N} y_{t,i}^{H,j} - P_{r,t}^i \frac{N^j}{N} k_{t-1}^i - P_{1,i} w_{t,i}^{H} \frac{N^j}{N} l_{t,i}^j \]
(F.28.)

\[ \left\{ (1 - \varepsilon^{H}) P_{t,i}^{H} y_{t,i}^{H} + \varepsilon^{H} \psi_t^{H'} y_{t,i}^{H} \right\} \]

\[ -\phi^{H} \left( \frac{p_{t,i}^{H}}{p_{1,i}^{H+1}} \frac{P_{i}}{P_{i+1}} - 1 \right) P_{1,i}^{H} y_{t,i}^{H} \left( \frac{p_{t,i}^{H}}{p_{1,i}^{H+1}} \frac{P_{i}}{P_{i+1}} + \frac{N^j}{N} \frac{p_{t,i}^{H+1}}{P_{i+1}} \right) \left\{ \phi^{H} \left( \frac{p_{t,i}^{H+1}}{p_{1,i}^{H+1}} \frac{P_{i+1}}{P_{i}} - 1 \right) P_{t+1,i}^{H} y_{t+1,i}^{H} \frac{p_{t,i}^{H+1}}{P_{i+1}} \right\} = 0 \]
(F.29.)

\[ P_{t,d_1} = R_{t-1} \lambda^d d_{t-1} + Q_{t-1} \frac{S_i}{S_{t-1}} (1 - \lambda^d) P_{t-1} d_{t-1} + P_t g_{i,t} + P_t g_{i,t}^o + P_t g_{i,t}^u - P_t r_{i,t} - P_t r_{i,t}^l - P_t r_{i,t}^l + r_{i,t}^l \]  
(F.30.)

\[ P_{t,r_1} \equiv \tau_t^i P_{t} \left( N^i c_t^i + N^{nr} c_t^{nr} \right) + \tau_t^i N^i \left( w_{t}^{H} t_{t}^{H,r} + w_{t}^{NT} t_{t}^{NT,r} + w_{t}^{P} P_{t}^{r} \right) \]

\[ + \tau_t^i N^{nr} \left( w_{t}^{H} t_{t}^{H,nr} + w_{t}^{NT} t_{t}^{NT,nr} + w_{t}^{P} P_{t}^{nr} \right) + \tau_t^i N^j \left( k_{t}^{H,k} h_{t-1}^{H,r} + \omega_{t}^{H,r} + r_{t}^{H,k} k_{t-1}^{NT,r} + \omega_{t}^{NT,r} \right) \]
(F.31.)

\[ k_t^i = (1 - \delta^g) k_{t-1}^i + g_t^i \]  
(F.32.)

\[ y_t^g = A_i^g \left( k_{t-1}^i \right)^{\eta^g} \left( \frac{g_t^g}{g_t^g} \right)^{1-a_t^g-a_t^g} \]  
(F.33.)

\[ P_{t,g_{i,t}} = P_{t} \left( N^i c_t^i + N^{nr} c_t^{nr} \right) \]

\[ + P_{t} \frac{N^{nr}}{N} c_t^{nr} + P_t g_{i,t} + P_t g_{i,t}^o + P_t g_{i,t}^u \]  
(F.34.)
\[ y_t^H = y_t^{H,d} + x_t \]  
(F.36.)

\[ \frac{N_i}{N} l_t^{H,i} = \frac{N_f}{N} l_t^{H,r} + \frac{N_{nr}}{N} l_t^{H,nt} \]  
(F.37.)

\[ \frac{N_i}{N} l_t^{NT,i} = \frac{N_f}{N} l_t^{NT,r} + \frac{N_{nr}}{N} l_t^{NT,nt} \]  
(F.38.)

\[ l_t^q = \frac{N_r}{N} l_t^{p,r} + \frac{N_{nr}}{N} l_t^{p,nt} \]  
(F.39.)

\[ (1 - \lambda^q) P_t d_t - \frac{\partial P_t}{\partial t} \bigg| \frac{\partial P_t}{\partial t} f_t^r = Q_{t-1} \frac{\partial}{\partial t} + (1 - \lambda^q) P_{t-1} d_{t-1} - Q_{t-1} \frac{\partial}{\partial t} + \frac{P^F}{P_t} y_t^F + \frac{\partial N_{NT}}{\partial t} \bigg| \frac{\partial P_t}{\partial t} f_t^r \]  
(F.40.)

\[ k_t^{H,r} = (1 - \delta^H) k_{t-1}^{H,r} + x_t^{H,r} - \Phi^H \left( k_t^{H,r}, k_{t-1}^{H,r} \right) \]  
(F.41.)

\[ k_t^{NT,r} = (1 - \delta^{NT}) k_{t-1}^{NT,r} + x_t^{NT,r} - \Phi^{NT} \left( k_t^{NT,r}, k_{t-1}^{NT,r} \right) \]  
(F.42.)

\[ P_t y_t^{gd,p} = P_t^H y_t^H + P_t^{NT} y_t^{NT} + P_t g_t^w \]  
(F.43.)

\[ Q_t = Q_t^* + \psi^d \left( \frac{P_t^{sd}}{P_t} - \frac{P_t^{sd}}{P_t} - 1 \right) + e^{\gamma^d} - 1 \]  
(F.44.)

\[ t_{tot} = \frac{P_t^H}{P_t} \]  
(F.45.)

\[ x_t = \rho^x x_{t-1} + (1 - \rho^x) \left( t_{tot} \right)^{-x} \]  
(F.46.)

\[ P_t g_t^w = N_r P_t w_t^P l_t^{P,r} + N_{nr} P_t w_t^P l_t^{P,nt} \]  
(F.47.)

\[ s_t^{g,c} = \frac{g_t^{g,c}}{y_t^{gd,p}} \]  
(F.48.)

\[ s_t^{g,i} = \frac{g_t^{g,i}}{y_t^{gd,p}} \]  
(F.49.)

\[ s_t^{i} = \frac{g_t^{i}}{y_t^{gd,p}} \]  
(F.50.)

\[ s_t^{l} = \frac{g_t^{l}}{y_t^{gd,p}} \]  
(F.51.)
\[ s_t^{g,c} - s_{t-1}^{g,c} = \rho^{g,c} \left( s_{t-1}^{g,c} - s_{t-1}^{g,c} \right) - \gamma^{g,c} \left( \frac{d_t}{y_t^{dp}} - \frac{d}{y_t^{dp}} \right) + \varepsilon_t^{g,c} \] (F.52.)

\[ s_t^{g,i} - s_{t-1}^{g,i} = \rho^{g,i} \left( s_{t-1}^{g,i} - s_{t-1}^{g,i} \right) - \gamma^{g,i} \left( \frac{d_t}{y_t^{dp}} - \frac{d}{y_t^{dp}} \right) + \varepsilon_t^{g,i} \] (F.53.)

\[ s_t^{w} - s_{t-1}^{w} = \rho^{w} \left( s_{t-1}^{w} - s_{t-1}^{w} \right) - \gamma^{w} \left( \frac{d_t}{y_t^{dp}} - \frac{d}{y_t^{dp}} \right) + \varepsilon_t^{w} \] (F.54.)

\[ s_t^{l} - s_{t-1}^{l} = \rho^{l} \left( s_{t-1}^{l} - s_{t-1}^{l} \right) - \gamma^{l} \left( \frac{d_t}{y_t^{dp}} - \frac{d}{y_t^{dp}} \right) + \varepsilon_t^{l} \] (F.55.)

\[ \tau_t^{c} - \tau^{c} = \rho^{c} \left( \tau_{t-1}^{c} - \tau^{c} \right) + \gamma^{c} \left( \frac{d_t}{y_t^{dp}} - \frac{d}{y_t^{dp}} \right) + \varepsilon_t^{c} \] (F.56.)

\[ \tau_t^{k} - \tau^{k} = \rho^{k} \left( \tau_{t-1}^{k} - \tau^{k} \right) + \gamma^{k} \left( \frac{d_t}{y_t^{dp}} - \frac{d}{y_t^{dp}} \right) + \varepsilon_t^{k} \] (F.57.)

\[ \tau_t^{n} - \tau^{n} = \rho^{n} \left( \tau_{t-1}^{n} - \tau^{n} \right) + \gamma^{n} \left( \frac{d_t}{y_t^{dp}} - \frac{d}{y_t^{dp}} \right) + \varepsilon_t^{n} \] (F.58.)

\section*{G Functional forms}

We assume the following utility function for both types of households \( r, nr \):

\[ U^r \left( \epsilon_t, l_t^H, l_t^{NT}, y_t \right) = \frac{(\epsilon_t + \gamma \theta y_t)^{1-\sigma}}{1-\sigma} - \chi^H \left( l_t^H \right)^{1+\eta^H} - \chi^{NT} \left( l_t^{NT} \right)^{1+\eta^{NT}} - \chi^P \left( l_t^P \right)^{1+\eta^P} \] (98)

\[ \frac{\partial U_t}{\partial \epsilon} = (\epsilon_t + \gamma \theta y_t)^{-\sigma} \] (99)

\[ \frac{\partial U_t}{\partial l_t} = \chi^j \left( l_t^j \right)^{\eta^j} \] (100)

where \( j = H, NT, P \). Adjustment costs take the form:

\[ \Phi^j \left( k_t^j, k_{t-1}^j \right) = \frac{\phi^j}{2} \left( \frac{k_t^j}{k_{t-1}^j} - 1 \right)^2 k_{t-1}^j \] (101)

\[ \frac{\partial \Phi^j \left( k_t^j, k_{t-1}^j \right)}{\partial k_t^j} = \phi^j \left( \frac{k_t^j}{k_{t-1}^j} - 1 \right) \] (102)
\[
\frac{\partial \Phi^j}{\partial k_i^j} \left( k_{i+1}^j, k_i^j \right) = \phi^j \left( \frac{k_{i+1}^j}{k_i^j} - 1 \right)^2 - \phi^j \left( \frac{k_{i+1}^j}{k_i^j} - 1 \right) \left( \frac{k_{i+1}^j}{k_i^j} \right)^2
\]  

(103)

where \( j = H, NT \), and

\[
\Phi^* \left( f_t^* \right) = \phi^* \left( \frac{p_t^F f_t^* - p_t^F f_t^r}{p_t^F f_t^r} \right)^2
\]

(104)

\[
\frac{\partial \Phi^*}{\partial f_t^r} = \phi^* \left( \frac{p_t^F f_t^* - p_t^F f_t^r}{p_t^F f_t^r} \right)
\]

(105)

Below we simplify the DE system, in doing so we substitute out the following equilibrium conditions:

\[
\frac{N^i}{N} k_{i,t}^H,i = \frac{N^r}{N} k_{i,t}^H,r
\]

\[
\frac{N^j}{N} k_{i,NT,j} = \frac{N^r}{N} k_{i,NT,r}
\]

\[
y_{i,NT} = \frac{y_{i,NT}}{N} = \frac{N^i y_{i,NT,i}}{N}
\]

\[
y_{i,H} = \frac{y_{i,H}}{N} = \frac{N^j y_{i,H,j}}{N}
\]

Finally, we express the vector of prices \( \{P_t, P_t^T, P_t^F, P_t^H, P_t^{NT} \} \) in terms of final good price, \( P_t \), i.e., \( p_t^T \equiv \frac{P_t^T}{P_t} \), \( p_t^F \equiv \frac{P_t^F}{P_t} \equiv \frac{S_t P_t^T}{P_t} \), \( p_t^H \equiv \frac{P_t^H}{P_t} \), and inflation rate is defined as \( \Pi_t \equiv \frac{P_t}{P_t^{-1}} \). Also, we define \( \nu^r \equiv \frac{N^r}{N} \nu^{NT} \equiv \frac{N^r}{N} \), while, without loss of generality we assume \( N^i = N \).

### H Decentralized Equilibrium (in real terms)

\[
(c_t^c + \phi^{H} \frac{\beta}{g_t^c})^{-\sigma} = \Lambda_t^c \left( 1 + \tau_t^c \right)
\]

(1.H.)

\[
\chi^H \left( l_t^{H,r} \right)^{\eta^H} = \Lambda_t^c \left( 1 - \tau_t^c \right) w_t^{H}
\]

(2.H.)

\[
\chi^{NT} \left( l_t^{NT,r} \right)^{\eta^{NT}} = \Lambda_t^c \left( 1 - \tau_t^c \right) w_t^{NT}
\]

(3.H.)

\[
\chi^F \left( l_t^{F,r} \right)^{\eta^F} = \Lambda_t^c \left( 1 - \tau_t^c \right) w_t^{F}
\]

(4.H.)

\[
\Lambda_t^c \left( 1 + \phi^H \left( \frac{k_{t+1}^{H,r}}{k_{i}^{H,r}} - 1 \right) \right)
\]

\[
E_0 \beta \Lambda_t^c \left( 1 - \delta^H + (1 - \tau_t^H) \delta_{t+1}^H \right) \cdot \frac{\phi^H \left( \frac{k_{t+1}^{H,r}}{k_{i}^{H,r}} - 1 \right)^2 + \phi \left( \frac{k_{t+1}^{H,r}}{k_{i}^{H,r}} - 1 \right)}{2}
\]

(5.H.)
\[
\Lambda_t^r \left( 1 + \phi^H \left( \frac{k_t^{H,r}}{k_t^{H,r-1}} - 1 \right) \right) = E_0 \beta \Lambda_{t+1}^r \left( 1 - \delta^{NT} + (1 - r_{t+1}^{k_t}) r_{t+1}^{NT,k} - \frac{2}{\pi} \left( \frac{k_t^{NT,r}}{k_t^{NT,r-1}} - 1 \right)^2 + \phi \left( \frac{k_t^{NT,r}}{k_t^{NT,r-1}} - 1 \right) \left( \frac{k_t^{NT,r}}{k_t^{NT,r-1}} - 1 \right) \right)
\] (H.6)

\[
\Lambda_t^v = E_0 \beta \Lambda_{t+1}^v \frac{1}{\Pi_{t+1}}
\] (H.7)

\[
\left( p_t^F + \phi^* \left( p_t^F f_{t^v}^r - p_t^F f_{t^e}^r \right) \right) = E_0 \beta \Lambda_{t+1}^v Q_t^v \frac{1}{\Pi_{t+1}}
\] (H.8)

\[
(\sigma_t^{nr} + \theta_t^{\beta} \sigma_t^H)^{-\sigma} = \Lambda_t^{nr} (1 + \tau_t^v) 
\] (H.9)

\[
\chi^H \left( i_t^{H, nr} \right) \eta_t^H = \Lambda_t^{nr} (1 - \tau_t^v) w_t^H 
\] (H.10)

\[
\chi^{NT} \left( i_t^{NT, nr} \right) \eta_t^{NT} = \Lambda_t^{nr} (1 - \tau_t^v) w_t^{NT} 
\] (H.11)

\[
\chi^P \left( i_t^{P, nr} \right) \eta_t^P = \Lambda_t^{nr} (1 - \tau_t^v) w_t^P 
\] (H.12)

\[
(1 + \tau_t^v) e_t^{nr} = (1 - \tau_t^v) \left( w_t^H i_t^{H, nr} + w_t^{NT} i_t^{NT, nr} + w_t^P i_t^{P, nr} \right) - \tau_t^{nr} 
\] (H.13)

\[
y_t = \left[ (v)^{\frac{1}{\sigma}} \left( y_t^H \right)^{\frac{1}{\sigma^H}} + (1 - v) \right]^{\frac{1}{\sigma}} 
\] (H.14)

\[
y_t^{NT} = \frac{v}{1 - v} \left( \frac{p_t^F}{p_t^{NT}} \right)^{-\zeta} y_t^{NT} 
\] (H.15)

\[
1 = \left[ v \left( p_t^{f_{t^v}} \right)^{1-\zeta} + (1 - v) \left( p_t^{f_{t^e}} \right)^{1-\zeta} \right]^{\frac{1}{1-\zeta}} 
\] (H.16)

\[
y_t^H = \left[ (v^{H})^{\frac{1}{\sigma^H}} \left( y_t^{H, d} \right)^{\frac{1}{\sigma^H}} + (1 - v^H) \left( y_t^{F} \right)^{\frac{1}{\sigma^H}} \right]^{\frac{1}{\sigma^H}} 
\] (H.17)

\[
y_t^{H, d} = \frac{v^H}{1 - v^H} \left( \frac{p_t^{H}}{p_t^F} \right)^{-\zeta^H} y_t^F 
\] (H.18)
\[ p_t^F = \left[ u^H (p_t^H)^{1-\zeta^H} + (1 - u^H) (p_t^F)^{1-\zeta^F} \right]^{1-\zeta^F} \]  \hspace{1cm} (H.19.)

\[ \frac{p_t^F}{p_{t-1}^F} = \frac{S_t - \Pi_t}{\Pi_t} \] \hspace{1cm} (H.20.)

\[ y_t^{NT} = A_t^{NT} \left\{ y_t^{NT} \right\}^{\kappa_t^{NT}} \left\{ (\nu^r k_{t-1})^{NT} t_{t-1}^{NT,i} \right\}^{1-a^NT} \] \hspace{1cm} (H.21.)

\[ r_t^k = m c_t^{NT} a^NT \] \hspace{1cm} (H.22.)

\[ w_t^{NT} = m c_t^{NT} (1 - a^NT) \kappa_t^{NT} \frac{y_{t1}^{NT}}{\nu^r k_{t-1}} \] \hspace{1cm} (H.23.)

\[ \nu^r \omega_t^{NT,i} = p_t^{NT} y_t^{NT} - r_t^k \nu^r k_{t-1}^{NT,i} - w_t^{NT} t_{t}^{NT,i} \] \hspace{1cm} (H.24.)

\[ -\phi^{NT} \left( \frac{y_{i,1}^{NT}}{w_{i,1}^{NT}} - 1 \right) p_t^{NT} y_t^{NT} p_{t+1}^{NT} \Pi_t + \beta^{NT} \left\{ \phi^{NT} \left( \frac{p_t^{NT} y_{i,1}^{NT}}{w_{i,1}^{NT}} - 1 \right) P_{t+1}^{NT} y_{i,1}^{NT} p_{t+1}^{NT} \Pi_{t+1} \right\} = 0 \] \hspace{1cm} (H.25.)

\[ y_t^{H} = A_t^{H} \left\{ y_t^{H} \right\}^{\kappa_t^{H}} \left\{ (\nu^H k_{t-1})^{H} (h_{t,j})^{1-a^H} \right\}^{\kappa_t^{H}} \] \hspace{1cm} (H.26.)

\[ r_t^{H,k} = m c_t^{H} a^H \omega_t^{H,k} \frac{y_{t1}^{H}}{\nu^H k_{t-1}} \] \hspace{1cm} (H.27.)

\[ w_t^{H} = m c_t^{H} (1 - a^H) \kappa_t^{H} \frac{y_{t1}^{H}}{h_{t,j}} \] \hspace{1cm} (H.28.)

\[ \nu^r \omega_t^{H,i} = p_t^{H} y_t^{H} - r_t^{H,k} \nu^H k_{t-1}^{H,i} - w_t^{H} t_{t}^{H,i} \] \hspace{1cm} (H.29.)

\[ -\phi^{H} \left( \frac{y_{i,1}^{H}}{w_{i,1}^{H}} - 1 \right) p_t^{H} y_t^{H} p_{t+1}^{H} \Pi_t + \beta^{H} \left\{ \phi^{H} \left( \frac{p_t^{H} y_{i,1}^{H}}{w_{i,1}^{H}} - 1 \right) P_{t+1}^{H} y_{i,1}^{H} p_{t+1}^{H} \Pi_{t+1} \right\} = 0 \] \hspace{1cm} (H.30.)

\[ d_t = R_t + \frac{1}{\Pi_t} \lambda_t^2 d_{t-1} + Q_t \frac{S_t}{\Pi_t} \frac{1}{\Pi_t} (1 - \lambda_t^2) d_{t-1} + g_t^e + g_t^i + g_t^w - \tau_t - \tau_{t} \] \hspace{1cm} (H.31.)

\[ \tau_{t} \equiv \tau_{t}^e (\nu^e c_t^e + \nu^{nr} c_t^{nr} +) + \tau_{t}^r \nu^r \left( w_t^{H} t_{t}^{H,i} + w_t^{NT} t_{t}^{NT,i} + w_t^{T} P_{t}^{T} \right) \] \hspace{1cm} (H.32.)
\[ k_t^g = (1 - \delta^g) k_{t-1}^g + g_t^g \]  
\[ y_t^g = A_t^g (k_{t-1}^g)^{a_t^g} (h_t^g)^{a_t^g} (g_t^i)^{1-a_t^g-a_t^g} \]  
\[ y_t = \nu^r \left( c_t^r + x_t^{H,r} + x_t^{NT,r} \right) + \nu^{nr} c_t^{nr} + g_t^c + g_t^i \]  
\[ y_t^H = y_t^{H,d} + x_t \]  
\[ l_t^{H,i} = \nu^r l_t^{H,r} + \nu^{nr} l_t^{H,nt} \]  
\[ l_t^{NT,i} = \nu^r l_t^{NT,r} + \nu^{nr} l_t^{NT,nt} \]  
\[ l_t^g = \nu^r l_t^{P,r} + \nu^{nr} l_t^{P,nt} \]  
\[ (1 - \lambda^g) d_t - p_t^{F, NT, f^{tr}} = Q_{t-1} \frac{S_t}{S_{t-1}} \frac{1}{H_t} \left( 1 - \lambda^g \right) d_{t-1} - Q_{t-1} \frac{S_t}{S_{t-1}} p_t^{F^r} m_t^r l_t^{f^{tr}} \]  
\[ + \frac{p_t^H}{H_t} y_t^H - \frac{p_t^{nr}}{nr_t} x_t + \phi^{NT} \left( p_t^{NT} y_t^{NT} - 1 \right)^2 p_t^{NT} y_t^{NT} \]  
\[ + \phi^H \left( \frac{p_t^H}{H_t} - 1 \right)^2 p_t^H y_t^H + \nu^r \phi^r \left( p_t^{F^r} f_t^{tr} - p_t^{P^r} f_t^{tr} \right)^2 \]  
\[ k_t^{H,r} = \left( 1 - \delta^H \right) k_{t-1}^{H,r} + x_t^{H,r} - \frac{\phi^H}{2} \left( \frac{k_t^{H,r}}{k_{t-1}^{H,r}} - 1 \right)^2 k_t^{H,r} \]  
\[ k_t^{NT,r} = \left( 1 - \delta^{NT} \right) k_{t-1}^{NT,r} + x_t^{NT,r} - \frac{\phi^{NT}}{2} \left( \frac{k_t^{NT,r}}{k_{t-1}^{NT,r}} - 1 \right)^2 k_t^{NT,r} \]  
\[ g_t^{dp} = p_t^H y_t^H + p_t^{NT} y_t^{NT} + g_t^w \]  
\[ Q_t = Q_t^r + \phi^d \left( \frac{d_t}{v_f^{r, f^{tr}}} - D \right) + e_t^{r, 1} - 1 \]  
\[ tot_t = \frac{p_t^H}{p_t^r} \]  
\[ x_t = \rho^x x_{t-1} + (1 - \rho^x) \left( \frac{tot_t}{tot} \right)^{-r} \]  
\[ g_t^w = w_t^P \left( \nu^r l_t^{P,r} + \nu^{nr} l_t^{P,nr} \right) \]
s^g.c_t = \frac{y^g.c_t}{y^d_t} \quad \text{(H.48.)}

s^g.i_t = \frac{y^g.i_t}{y^d_t} \quad \text{(H.49.)}

s^w_t = \frac{y^w_t}{y^d_t} \quad \text{(H.50.)}

s^l_t = \frac{x^l_t}{y^d_t} \quad \text{(H.51.)}

s^g.c_t - s^g.c_t = \rho^g.c_t \left( s^g.c_{t-1} - s^g.c_t \right) - \gamma^g.c_t \left( \frac{d_t}{y^d_t} - \frac{d}{y^d_t} \right) + \epsilon_t^g.c \quad \text{(H.52.)}

s^g.i_t - s^g.i_t = \rho^g.i_t \left( s^g.i_{t-1} - s^g.i_t \right) - \gamma^g.i_t \left( \frac{d_t}{y^d_t} - \frac{d}{y^d_t} \right) + \epsilon_t^g.i \quad \text{(H.53.)}

s^w_t - s^w_t = \rho^w_t \left( s^w_{t-1} - s^w_t \right) - \gamma^w_t \left( \frac{d_t}{y^d_t} - \frac{d}{y^d_t} \right) + \epsilon_t^w \quad \text{(H.54.)}

s^l_t - s^l_t = \rho^l_t \left( s^l_{t-1} - s^l_t \right) - \gamma^l_t \left( \frac{d_t}{y^d_t} - \frac{d}{y^d_t} \right) + \epsilon_t^l \quad \text{(H.55.)}

\tau^c_t - \tau^c_t = \rho^c \left( \tau^c_{t-1} - \tau^c_t \right) + \gamma^c \left( \frac{d_t}{y^d_t} - \frac{d}{y^d_t} \right) + \epsilon_t^c \quad \text{(H.56.)}

\tau^k_t - \tau^k_t = \rho^k \left( \tau^k_{t-1} - \tau^k_t \right) + \gamma^k \left( \frac{d_t}{y^d_t} - \frac{d}{y^d_t} \right) + \epsilon_t^k \quad \text{(H.57.)}

\tau^n_t - \tau^n_t = \rho^n \left( \tau^n_{t-1} - \tau^n_t \right) + \gamma^n \left( \frac{d_t}{y^d_t} - \frac{d}{y^d_t} \right) + \epsilon_t^n \quad \text{(H.58.)}

**I Construction of capital stock series**

Following Conesa et al. (2007) and Gogos et al. (2014), we construct time series for the sectoral capital stocks, i.e. tradable and non-tradable, and public capital stock. Let us start with the construction of time series for capital stock in the tradable sector. To do this, we employ the law of motion of tradable physical capital:

\[ k^H_t = \left( 1 - \delta^H \right) k^H_{t-1} + x^H_t \quad \text{(106)} \]
Having Irish data on real investment, \(x_t^H\), (i.e. gross fixed capital formation in the tradable sector) we construct time series for physical capital stock, \(k_t^H\), by solving for a constant value of the depreciation rate, \(\delta^H\), and an initial value for the physical capital stock, \(k_0^H\) in the tradable sector. The value of \(\delta^H\) is chosen to be consistent with the average consumption of fixed capital in the tradable sector to GDP ratio observed in Irish data over 1995-2014. We have data only for total consumption of fixed capital which averages to 0.11207 as a share of GDP. Thus, to approximate the share of consumption of fixed capital in the tradable sector to total consumption of fixed capital we use the share of gross fixed capital formation in the tradable sector to gross fixed capital in both sectors which is equal to 34% over 1995-2014. Thus the average of consumption of fixed capital in the tradable sector as a share of GDP for 1995-2014 is written:

\[
\frac{1}{19} \sum_{t=1995}^{2014} \frac{\delta^H k_t^H}{y_t^{gdp}} = 0.038
\]  

(107)

The initial capital stock, \(k_0^H\), is chosen so that the capital-to-output ratio matches the average ratio over 1995-2014, which yields the following equation:

\[
\frac{k^H_{1995}}{y_{1995}} = \frac{1}{19} \sum_{t=1995}^{2014} \frac{k_t^H}{y_t^{gdp}}
\]  

(108)

Thus we end up with 21 unknowns, \(\delta^H\), \(k^H_{1995}\), \(k^H_{1996}\), ..., \(k^H_{2014}\) in 21 equations, i.e. equation (106) for \(t = 1996, \ldots, 2014\), and equations (107) and (108). The solution of this system results in a time series for tradable capital stock, \(k^H_{1995}, k^H_{1996}, \ldots, k^H_{2014}\) and a calibrated value for the depreciation rate, \(\delta^H = 0.071\).

Similarly we construct the time series for the capital stock in the non-tradable sector. We employ the law of motion of physical capital:

\[
k_t^{NT} = \left(1 - \delta^{NT}\right) k_{t-1}^{NT} + x_t^{NT}
\]  

(109)

As before we use data for real investment in the non-tradable sector, \(x_t^{NT}\), i.e. gross fixed capital formation in the non-tradable sector. We assume a constant depreciation rate, \(\delta^{NT}\), which is consistent with the average consumption of fixed capital in the non-tradable sector to GDP ratio over 1995-2014. As before to extract this ratio from total consumption of fixed capital we assume that the ratio related to non-tradable sector is equal to the average ratio of gross fixed capital in the non-tradable sector to gross fixed capital in both sectors.
which is equal to 66%. This gives us the following equation for the average of consumption of fixed capital in
the non-tradable sector as a share of GDP for 1995-2014:

\[
\frac{1}{19} \sum_{t=1995}^{2014} \frac{\delta^NT k^NT_t}{y^gdp_t} = 0.074 \tag{110}
\]

The initial capital stock, \(k^NT_0\), is chosen so that the capital-to-output ratio matches the average ratio over
1996-2014, which yields the following equation:

\[
\frac{k^NT_{1995}}{y^gdp_{1995}} = \frac{1}{19} \sum_{t=1995}^{2014} \frac{k^NT_t}{y^gdp_t} \tag{111}
\]

Thus we end up with 21 unknowns, \(\delta^NT\), \(k^NT_{1995}\), \(k^NT_{1996}\), ..., \(k^NT_{2014}\) in 21 equations, i.e. equation (109) for \(t = 1996, ..., 2014\) and equations (110) and (111). The solution of this system results in a time series for non-tradable
capital stock, \(k^NT_{1995}\), \(k^NT_{1996}\), ..., \(k^NT_{2014}\) and a calibrated value for the depreciation rate, \(\delta^NT = 0.051\).

Finally, we construct a time series for public capital stock following the same methodology (as in Papa-
georgiou (2014)). Using the law of motion of public capital, data for government consumption of fixed capital
and gross government fixed capital formation we obtain the following equations:

\[
k^g_t = (1 - \delta^g) k^g_{t-1} + g^t \tag{112}
\]

\[
\frac{1}{19} \sum_{t=1995}^{2014} \frac{\delta^g k^g_t}{y^gdp_t} = 0.01767 \tag{113}
\]

\[
\frac{k^g_{1995}}{y^gdp_{1995}} = \frac{1}{19} \sum_{t=1995}^{2014} \frac{k^g_t}{y^gdp_t} \tag{114}
\]

Similarly, we end up with 21 unknowns, \(\delta^g\), \(k^g_{1995}\), \(k^g_{1996}\), ..., \(k^g_{2014}\) in 21 equations, i.e. equation (112) for
\(t = 1996, ..., 2014\) and equations (113) and (114). The solution of this system results in a time series for public
capital stock, \(k^g_{1995}\), \(k^g_{1996}\), ..., \(k^g_{2014}\) and a calibrated value for the depreciation rate, \(\delta^g = 0.0741\).
### Data and sources

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