Spatial Competition of Two-level Hierarchical Transportation Systems under Economies of Scale in Transportation Cost

NISHIDA, Kiheiji

Hyogo University of Health Sciences

2018

Spatial Competition of Two-level Hierarchical Transportation Systems under Economies of Scale in Transportation Cost

Kiheiji NISHIDA

Lecturer, General Education Center, Hyogo University of Health Sciences.

1-3-6, Minatojima, Chuo-ku, Kobe, Hyogo 650-8530, JAPAN.

E-mail: kiheiji.nishida@gmail.com

TEL: +81-78-304-3044

Key Words: Hierarchy system, Economy of scale in volume, Economy of scale in travelling distance, Competitive location, Planning location.

ABSTRACT

In this short paper, we consider how the hierarchical structure in transportation, in which goods are transported from demand points to nearby terminal stations in the first level of hierarchy and from station to station in the second, emerges in competition between transportation companies having economies of scale in volume and distance. We conmpare the competitive and planning locations of terminal stations of the hierarchical system using a Hotelling-style spatial competition model (1929). We find that a competitive location can be generated in an area where the planning location can never be generated. We also find no difference between the two when point-to-point transportation is replaced by transportation in bulk.

1 Introduction

One possible mode of transporting goods is a point-to-point transportation system, which transports goods directly from their origin to the final destination without any transshipments. Another mode is a hierarchical transportation system, in which goods are once collected in a nearby terminal station and each terminal station directly transports goods in bulk from station to station and delivers goods transported from other stations to their final destination points. The hierarchical transportation system faces two types of transportation costs: first from the origin/destination to its nearby station and the second from station to station. In the presense of economy of scale of volume in transportation, collecting many goods in a terminal station can result in high transport density and reduce the average costs of station-to-station transportation. Consequently, the hierarchical transportation system can outweigh point-to-point transportation in total transportation cost although even its path takes a detour in moving distance. Many researchers have tackled the problem of the emergence of hierarchical structure in transportation in relation to point-to-point transportation.

Suzuki and Kawaguchi (1998) supposed that a company plans to locate multiple transshipment stations in a linear city and numerically studied their optimal location in the presence of economy of scale in volume to be transported. In their model, customers are uniformly distributed in the linear city and each customer has a constant unit of demand to be transported to the final destination point fixed in the city center, experiencing one transshipment at the station in their nearest neighbor. The company chooses the location of multiple stations to minimize the total transportation cost, consisting of one from each customer to the nearby station (point-to-point) and one from the station to the final destination (station-to-station). Matching the optimal location with the city center indicates diseconomy in transshipment is diseconomy. In an optimal location, the stations tend to concentrate in the city center if the economy of scale and the demand for transportation are small.

Watanabe and Suzuki (2000) and Suzuki and Watanabe (2009) studied the nature of hierarchical transportation systems using the total number of transshipments. In other words, levels of the hierarchical transportation system, in which a trip goes from its origin to final destination and the number of stations in a level, both of which are optimized to minimize total transportation cost. They assume that the transportation cost is subject to economy of scale for not only to volume but also traveling distance. The essence of their studies is that the logarithm of the optimal number of stations in a level is written by the first-order decreasing function with respect to the number of relevant levels from the lowest, with the intercept term being the logarithm of total transportation demand. In Beckmann(1958), we can observe similarity of rank size rule in city sizes, the relationship between the number of cities in a level and the number of the level from the highest level of hierarchy of cities.

We observe economy of scale play an important role in determining the structure of a hierarchical transportation system. However, we realize that strategic behaviors between competitive transportation companies matter as well as economies of scale in determining the system. In this study, we employ spatially competitive location model in Hotelling (1929) and analyze the emergence of a hierarchical transportation system under competition in the presence of economies of scale.

2 Model

We consider a Hotelling style linear city expressed by a line segment $[0, L]$. In the city, customers are uniformly distributed and each customer has a constant demand for transporting goods of $1/L$ in volume from their locational point $t \in [0, L]$ to the final destination point fixed at $t_D = L/2$. Transportation companies denoted as i, $i = 1, 2, ...n$, are planning to enter the city, which can choose their location of single terminal station denoted as $x_i \in [0, L]$, and all of them are obliged to transport the goods they collect at their terminal station to the market center $t_D = L/2$. If $x_i = t_D = L/2$, transshipment does not happen in company i's system. Each customer utilizes the terminal station at their nearest neighbor. Figure 1 illustrates the assumption of the model.

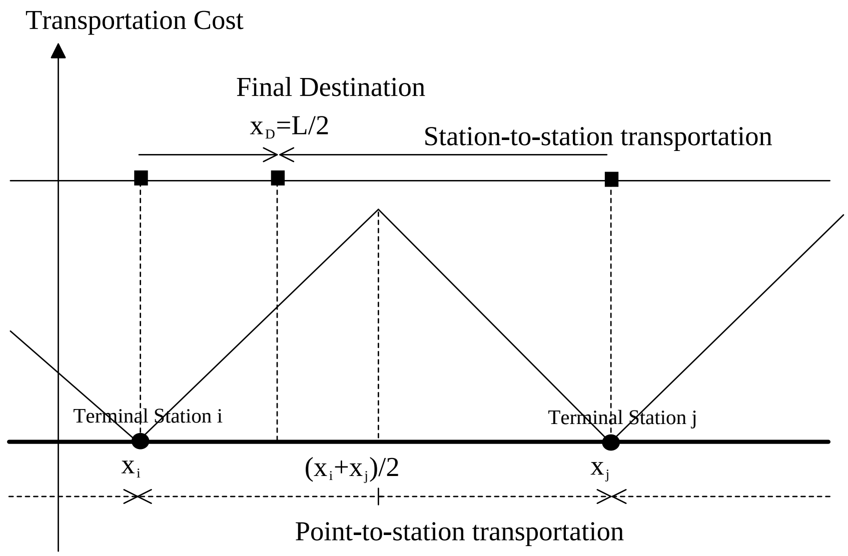

Figure 1: Assumption of our model.

All the companies face a common transportation cost function, which is an increasing function with respect to traveling distance d and volume to be transported v written as

$$c_i(v,t) = \text{Const.} \times v^{\alpha} d^{\beta},\ 0 < \alpha,\ 0 < \beta. \tag{1}$$

If $0 < \alpha < 1$ $(0 < \beta < 1)$, economy of scale with respect to volue (distance) exists.

Competitive location of terminal stations

Suppose that there are two transporting companies i and j in the city whose terminal station locations are $x_i < x_j$ without loss of generality. Then, the companies i and j face the following transportation cost functions

$$C_i(x_i, x_j) \ = \ \int_0^{\frac{x_i+x_j}{2}} \left[\frac{1}{L}\right]^{\alpha} |t - x_i|^{\beta}\, dt + \left[\int_0^{\frac{x_i+x_j}{2}} \frac{1}{L} dt\right]^{\alpha} \left|\frac{L}{2} - x_i\right|^{\beta}, \tag{2}$$

and

$$C_j(x_i, x_j) \;=\; \int_{\frac{x_i+x_j}{2}}^{L} \left[\frac{1}{L}\right]^{\alpha} |t - x_j|^{\beta}\, dt + \left[\int_{\frac{x_i+x_j}{2}}^{L} \frac{1}{L} dt\right]^{\alpha} \left|x_j - \frac{L}{2}\right|^{\beta}, \tag{3}$$

respectively. The first terms in (2) and (3) represent the sum of transportation cost from each customer to its nearby station and the second terms represent the sum of transportation costs from the station to the final destination in bulk transportation. Each company tries to minimize average cost defined by $AC_i(x_i, x_j) = C_i(x_i, x_j)/V_i(x_i, x_j)$, where $V_i(x_i, x_j) = \frac{x_i+y_j}{2}$ and $V_j(x_i, x_j) \;=\; L - \frac{x_i+y_j}{2}$. Then, companies choose their locations to satisfy the Nash equilibrium $AC_i(x_i^*, x_j^*) \le AC_i(x_i, x_j^*)$ and $AC_i(x_i^*, x_j^*) \le AC_i(x_i^*, x_j)$, where x_i^* and x_j^* are equilibrium locations of terminal stations for i and j, respectively.

Planning location of terminal stations

For comparative reasons, we consider the optimal location of terminal stations x_i and x_j. The planner chooses the set of locations x_i and x_j to minimize the average transportation cost function,

$$\min_{x_i, x_j} \left[\frac{C_i(x_i, x_j) + C_j(x_i, x_j)}{L}\right]. \tag{4}$$

3 Numerical Study

We conducted numerical calculations with respect to parameters α, β, and L because it is impossible to derive the solutions analytically in (2), (3), and (4). In Figure 2, we present the plots of the equilibrium and the planning locations with respect to α, β, and L in typical cases. We observe the following features in the results.

1. When $L = 1$, $\beta = 1.0$, and $\alpha \ge 1.5$ in the upper left panel of Figure 2, no equilibrium locations exist.

2. In the absence of economy of scale in distance, which is illustrated in the upper three panels, we observe the dispersed locations emerge in both the compatitive and planning situations, as the demand for transportation becomes larger (L gets larger) and

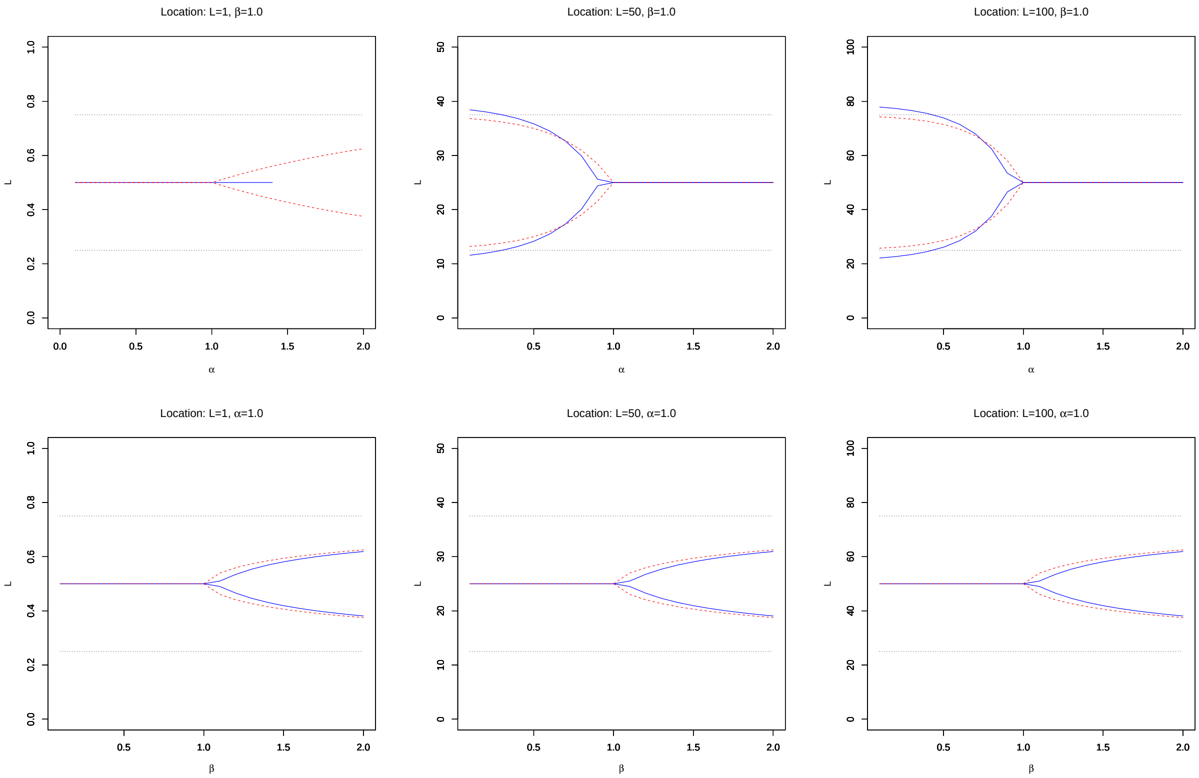

Figure 2: Blue solid lines: Competitive locations. Red dashed line: Planning locations. Black dotted lines: The first and the third quartile locational points.

economy of scale in volume becomes prominent (α gets smaller). This is because station-to-station transportation can save more total cost than point-to-point transportation when demand is large and economy of scale in volume works.

3. In the absence of economy of scale in volume, which is illustrated in the lower three panels, we observe the dispersed locations emerge in both compatitive and planning situations, as economy of scale in distance declines (β becomes larger). This is because companies have the incentive to reduce transportation cost from the demand points to the nearby terminal station, when economies of scale in distance and volume do not work.

4. When economy of scale in volume is prominent and the demand for transportation is

large, in the cases in the upper three panels of Figure 2, we observe the competitive locations in the area $x \leq L/4$, $3L/4 \leq x^*$, whereas we do not observe the planning locations in the area. This is proved because the first derivative of the total cost function (4) with respect to x_i under symmetry of the model $x_j = L - x_i$, is written as

$$\begin{aligned} & \frac{d}{dx_i}\left[\frac{C_i(x_i, L-x_i) + C_j(x_i, L-x_i)}{L}\right] \\ & = \frac{2}{L^{\alpha+1}}\left[x_i^\beta - \left(\frac{L}{2} - x_i\right)^\beta\right] - \frac{2^{1-\alpha}\beta}{L}\left(\frac{L}{2} - x_i\right)^{\beta-1} < 0 \end{aligned} \tag{5}$$

in the area $0 \leq x_i \leq \frac{L}{4}$.

5. Point-to-point transportation is replaced by transportation in bulk at $\alpha = 1$ or $\beta = 1$ both in planning and competition.

4 Conclusion and Discussion

In this short paper, we consider the problem of how hierarchical structure in transportation emerges in competition between companies having scale of economies in volume and distance in transportation. We consider the differences between the competitive and planning locations of the terminal stations of a two-level hierarchical transportation system. The major difference between the two is that the competitive location can be generated in an area where the planning location can never be generated. This difference in location is caused by the strategic behavior of companies. We also find no difference between the two in the values of α or β at which the point-to-point transportation is replaced by transportation in bulk.

For further study, we consider the competition between multiple transportation companies and the competition in two-dimensional space. We would also like to consider how the number of terminal stations in a level and the total number of levels in a hierarchy are competitively determined.

Acknowledgments

This study is financially supported by Japan Society for the Promotion of Science under Grant-in-Aid for Young Scientists (B) JP16K17142 and Grant-in-Aid for Scientific Research (C) 26520110.

References

Beckmann, M.J.(1958). “City Hierarchies and the distribution of City Size”, *Economic Development and Cultural Changes*, **6**, pp.243-248.

Hotelling, H.(1929). “Stability in Competition”, *The Economic Journal*, **39**, pp.41-57.

Suzuki, T. and Watanabe, D. (2009) “Optimal Hierarchical Transportation System with Economies of Scale, New Frontiers in Urban Analysis: In Honor of Atsuyuki Okabe”(Edited by Asami, Y., Sadahiro, Y. and Ishikawa, T.), pp.29-51, CRC Press.

Suzuki, T. and Kawaguchi, A. (1998). “Economy of Scale in Transportation and Optimal Partitioning in a one-Dimensional City”, Abstracts of the 1998 Fall National Conference of Operations Research Society of Japan, pp.24-25 (in Japanese).

Watanabe, D. and Suzuki, T. (2000). “A Study on the Hierarchical Collection/Distribution System with Economies of Scale”, *Journal of the City Planning Institute of Japan*, **35**, pp.1027-32 (in Japanese).