Affirmative Action Subcontracting Regulations in Dynamic Procurement Auctions

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Abstract

I study affirmative action subcontracting regulations in a model where governments use auctions to repeatedly procure goods and services at the lowest possible price. Through using disadvantaged subcontractors, prime contractors build relationships over time, resulting in lower subcontracting costs in future periods. I find that regulation in the form of a minimum subcontracting requirement expands bidder asymmetries, favoring prime contractors with stronger relationships over those with weaker ones. Simulations show that the manner in which relationships evolve determines not only the utilization of disadvantaged subcontractors but also the procurement costs attained under affirmative action.

1 Introduction

Public procurement is a substantial part of government spending. In 2015, government procurement accounted for 29.1 percent of all government spending and 11.9 percent of GDP in OECD countries.[1] Embedded within many of these procurement markets are affirmative action regulations, which are implemented to facilitate the participation of disadvantaged[2] firms in government contracting. Although affirmative action can take on many different forms, a common brand of policies in procurement are mandatory subcontracting goals. Under these policies, a prime contractor (or prime) must set aside a percentage share of a contract for subcontractors (or subs) designated as disadvantaged.

A key factor in a prime's disadvantaged subcontractor selection and associated subcontracting cost is their relationship with their pool of disadvantaged firms. In a needs assessment report by the Minnesota

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[1]See Government at a Glance 2017, which is available at `http://dx.doi.org/10.1787/gov_glance-2017-en`.

[2]In U.S. procurement, firms that benefit from these policies tend to be small firms owned and controlled by ethnic minorities or women, but veteran-owned small businesses and small businesses in general can also benefit from affirmative action policies.

Department of Transportation,[3] surveys revealed that prime contractors rely on relationships to identify and hire disadvantaged subcontractors and that primes prefer to hire disadvantaged firms with whom they have existing relationships. The report concluded that relationships and credibility were integral to primes meeting their affirmative action requirements. The economics literature indicates that relationships can serve as a mechanism to lower coordination costs, promote learning-by-doing (Kellogg (2011)), and establish reputations (Banerjee and Duflo (2000)). Gil and Marion (2013) find that prior subcontracting interactions reduce bids on California highway procurement contracts, which is suggestive of lower subcontracting costs. Thus, the goal of expanding disadvantaged subcontractor utilization and the cost of affirmative action are both tied to the relationships primes build with their disadvantaged subs.

A shortcoming of the literature is that it does not directly address this relationship dynamic. Instead, the literature often relies on a static framework or uses proxies for future demand to approximate continuation values.[4] In this paper, I seek to fill that gap in the literature by investigating how dynamic relationship formation impacts procurement auctions in a model with affirmative action. To do so, I numerically solve for the Markov-perfect equilibrium of a repeated auction game with relationship dynamics and contrast the equilibria obtained with and without affirmative action subcontracting quotas. Primes in my model stochastically improve relationships through the continued utilization of disadvantaged subs, leading to an expectation of lower disadvantaged subcontracting costs in future periods. This relationship-building dynamic endogenously creates asymmetries between bidders, where primes with stronger relationships have a cost advantage over primes with weaker ones. By requiring primes to use disadvantaged firms, affirmative action expands this asymmetry and increases the marginal value from attaining a better state of relationships in the future. As a result, farsighted primes have more of an incentive to subcontract with disadvantaged firms relative to myopic primes, and this incentive is amplified in markets with affirmative action.

The dynamic framework in this paper can answer several questions that a static framework cannot. Given that an objective of these programs is to remove barriers to the participation of disadvantaged firms in contracting,[5] and in the subcontracting case, one of those barriers is a lack of established relationships – a dynamic analysis can explore how relationships evolve and how much affirmative action contributes to that evolution. Equally relevant is the long-run impact of removing affirmative action programs, which has been implemented through laws such as California's Proposition 209.[6] In this case, a dynamic structure can

[3] The full name of the report is the MnDOT DBE and OJT Program Needs Assessment.

[4] Rosa (2018) and De Silva et al. (2012) use a static framework and Marion (2009) proxies for future demand using upcoming opportunities for disadvantaged firms.

[5] For U.S. Department of Transportation contracting, see 64 FR 5126, Feb. 2, 1999, as amended at 79 FR 59592, Oct. 2, 2014.

[6] See Marion (2009) for an empirical analysis of Proposition 209's effect on bidding and subcontracting and Holzer and

provide insights on how relationships – and therefore disadvantaged subcontracting – will adjust when the quotas are removed.

I explore these questions through a model simulated under a range of different parameter and starting values. I find that the manner in which relationships transition between periods has implications for how affirmative action affects a given market. When relationships are long-lasting, affirmative action has negligible effects on bids yet leads to marked increases in disadvantaged subcontracting. When relationships deteriorate, affirmative action still improves subcontracting but at the cost of higher bids. These simulations highlight the importance of accounting for relationships in evaluating affirmative action programs.

My paper is closely related to the literature on mandatory subcontracting goals in government procurement contracts. Rosa (2018) studies Disadvantaged Business Enterprise (DBE) subcontracting goals in New Mexico using an estimated model of bidding and subcontracting that is similar to my paper. He finds that subcontracting goals may not lead to significant changes in bids because primes have to use a common pool of disadvantaged subs, leading to lower markups in equilibrium. I extend his model by including relationship dynamics and allowing for asymmetries in the cost of using disadvantaged firms. Other empirical papers on DBE subcontracting goals include Marion (2009) who finds that subcontracting goals significantly increased subcontracting and the winning bids and De Silva et al. (2012) who use a structural model to compare costs across contracts with and without subcontracting goals in Texas, finding negligible differences in project costs. Similarly, Marion (2017) studies how a change in a goal exemption policy in Iowa impacted DBE utilization.

My model borrows methods from the dynamic auction literature. In that literature, the model that is closest to mine is Jeziorski and Krasnokutskaya (2016). They use a dynamic model of subcontracting and bidding to explore how subcontracting affects capacity-constrained bidders, finding that subcontracting reduces bidder asymmetries by allowing primes to modify current costs and control future costs via backlog accumulation. Although I borrow their subcontracting model, our papers differ in dynamics; their paper has dynamic capacity, while my paper has dynamic relationship formation. This distinction fundamentally changes the role of subcontracting. In my model, primes subcontract with disadvantaged firms to gain a cost advantage in future periods through relationship formation; therefore, subcontracting *increases* bidder asymmetry in future periods. My dynamics also differ from Saini (2012) who studies dynamic procurement with capacity-constrained firms, except without subcontracting.

The remainder of the paper has the following structure. Section 2 outlines the model, and section 3

Neumark (2000) for a general overview of affirmative action laws and literature.

characterizes the model's equilibrium. Section 4 contains solved examples of the model, which I use to show how affirmative action subcontracting regulations affect bidding and disadvantaged subcontracting and to study the long-run implications of subcontracting regulations. Section 5 concludes.

2 Model

In this section, I describe the dynamic procurement auction model with subcontracting regulation. Although I make references to a repeated construction project, the model applies to many repeated procurement settings with subcontracting possibilities.

Environment

In my environment, time is discrete, and the horizon is infinite. Each period, two infinitely lived prime contractors bid for the rights to complete a homogeneous construction project. Prime contractors can either complete the entire project in house or award part of it to subcontractors, some of which belong to the disadvantaged group of subcontractors. For simplicity, I group in-house costs and costs from non-disadvantaged subcontractors for prime contractor i in period t into one cost, $c_{i,t}$, which I refer to as a prime contractor's unregulated cost. Unregulated costs represent a prime contractor's cost of completing the entire project without using any disadvantaged subcontractors, and I assume that they are private information and are independently and identically distributed according to the CDF F_c with support $[\underline{c}, \overline{c}]$. In contrast, the disadvantaged subcontracting market determines the cost of using disadvantaged subcontractors.

Mirroring how the U.S. government awards its sealed-bid contracts, I assume that the government uses a first-price sealed-bid format to select the winning firm. In many procurement settings, government agencies have the right to reject any or all bids if they are irregularly high. Following the empirical literature,[7] I model that right as a secret reserve price with realization r_t that, for simplicity, is distributed independently across time according to the CDF F_R with support $[\underline{r}, \overline{r}]$. The winning prime contractor is then the one that submits the lowest bid, provided that it is below the reserve price. Observe that this bid-rejection power can come, in part, through smaller irregular (or fringe) bidders that bid myopically.

The government can also impose regulations on the amount of a project that primes must award disadvantaged subcontractors. Within the context of the model, the government requires that primes allocate a fraction of the project $\overline{s} \in [0, 1]$ to disadvantaged subs in each period; if $\overline{s} = 0$, then the market is unregulated. This feature of the model resembles the subcontracting regulations in the U.S. Department of

[7] See Jofre-Bonet and Pesendorfer (2003).

Transportation's (USDOT's) DBE Program.[8]

Disadvantaged Subcontracting Market

I assume that each prime has its own pool of disadvantaged subcontractors that myopically supply services according to their relationship with prime contractor i at time t. Suppose that this relationship can be summarized by a single state variable, $\omega_{i,t}$. Disadvantaged subcontractors then supply a share of total services, $s_{i,t} \in [0,1]$ according to inverse supply function, $P\left(s_{i,t}, \omega_{i,t}\right)$, which maps shares and relationships into a price of using disadvantaged firms.

In this setup, the inverse supply function is central to a prime contractor's disadvantaged subcontracting decision. In particular, it generates a trade-off between using disadvantaged subcontractors and their alternatives. This trade-off is affected by relationships in the sense that a prime with better relationships may face lower disadvantaged subcontracting costs for the same share of the project completed. In real-world procurement, part of the cost of using disadvantaged subs comes from monitoring and the increased risk of subcontractor insolvency. It is, therefore, plausible that primes with better relationships would have lower disadvantaged subcontracting costs since, through repeat usage, disadvantaged subs become more trustworthy. Primes may also have access to a more extensive network of disadvantaged subcontractors when their relationships are better, which can lead to lower quotes on subcontracting services.

Observe that my subcontracting model is stylized and rules out some potentially complex subcontracting issues. In particular, I do not allow the disadvantaged subcontractors to be capacity constrained, and I abstract away from task heterogeneity. My assumptions do, however, lead to a tractable model that will be shown to illustrate some of the primary facets of a prime contractor's disadvantaged subcontracting decisions. Namely, that more efficient primes are less inclined to use disadvantaged subcontractors and that subcontracting costs increase as disadvantaged subcontractors complete more of the project.

I make the following regularity assumptions on the inverse supply function:

Assumptions on P:

1. P is increasing and convex in its first argument and decreasing in its second argument.

2. $P\left(0, \omega_{i,t}\right) = 0$ for every $\omega_{i,t}$.

[8] Although it is currently illegal to use explicit quotas in the U.S., states will regularly set DBE goals on their procurement projects, which require prime contractors to award a pre-specified percentage share of a project to subcontractors that qualify as disadvantaged. Prime contractors typically meet this requirement; see Marion (2017) and Rosa (2018).

3. $\lim_{s_{i,t} \to 1} P(s_{i,t}, \omega_{i,t}) = +\infty$.

4. $P_{12}(s_{i,t}, \omega_{i,t}) < 0$.

Assumption 1 requires that disadvantaged subcontracting costs increase at an increasing rate as primes use more disadvantaged subcontractors and decrease with better relationships. This assumption helps generate a unique subcontracting solution. Assumption 2 ensures that prime contractors that use no disadvantaged subcontractors pay nothing to disadvantaged subcontractors, and assumption 3 makes subcontracting large portions of a project prohibitively costly. Governments typically have a maximum subcontracting threshold for firms to qualify as prime contractors. The USDOT, for example, has a subcontracting maximum that is around 60 percent for its highway procurement projects. Assumption 3 is meant to approximate that rule. My fourth and final assumption means that the marginal price increase decreases with better relationships. That is to say, the inverse supply function is flatter with better relationships.

Relationship Dynamics

I assume that the state variable that describes relationships evolves according to how often primes use disadvantaged subcontractors. In particular, I assume that the state variable transitions stochastically according to

$$\sigma_i(\omega_{i,t}, s_{i,t}) = \psi\omega_{i,t} + s_{i,t} + \epsilon_{i,t},$$

where ψ measures how easily past relationships carry over into future periods, $s_{i,t}$ is prime contractor i's disadvantaged subcontracting share, and $\epsilon_{i,t}$ is a time-invariant random shock that is distributed according to F_ϵ with $\mathbb{E}[\epsilon_{i,t}] = 0$. If $\psi = 1$, for example, then past relationships are expected to persist into future periods, whereas $\psi < 1$ means that past relationships tend to diminish over time. This parameter's value plays a major role in a prime contractor's dynamic incentive to use disadvantaged subs since more persistent relationships lead to longer stretches of time with lower disadvantaged subcontracting costs in the future. In real-life procurement, ψ might be low if there is high turnover in a prime's pool of disadvantaged subcontractors from firms exiting the market. The shock, $\epsilon_{i,t}$, captures any randomness that affects the stock of relationships next period.

To ensure that prime contractors will not face implausibly high or low disadvantaged subcontracting costs, I bound each state variable from above by a constant $\overline{M}$ and below by a constant $\underline{M}$ so that $\omega_{i,t} \in [\underline{M}, \overline{M}]$.

The state space is then given by $\Omega \in \left[\underline{M}, \overline{M}\right] \times \left[\underline{M}, \overline{M}\right]$, and from now on I will define $\boldsymbol{\omega}_t = (\omega_{i,t}, \omega_{-i,t})$ as the state vector for prime i at time t.

Timing and Equilibrium Description

At the beginning of each period, primes observe their unregulated cost and the public history of states, disadvantaged shares, and bids by them and their competitor. For tractability, I assume that primes use Markov strategies and focus attention on Markov-perfect equilibria, where strategies only depend on payoff-relevant information. In my model, that information is the state vector of prime relationships, as it summarizes the current period's disadvantaged cost while its transition gives disadvantaged costs in the future. Furthermore, payoffs do not depend on time given the state, so I can focus on stationary strategies.

Given my equilibrium assumptions, the relevant information that primes observe at the start of each period is their unregulated cost, $c_{i,t}$ and the state, $\boldsymbol{\omega}_t$. Primes use these two observations and their knowledge of the reserve price and unregulated cost distributions to choose their disadvantaged subcontracting shares, $s_{i,t} \in [\overline{s}, 1]$, and their bids on the project, $b_{i,t} \in \mathbb{R}_+$. A stationary Markov strategy is therefore a mapping between a prime's unregulated costs and the state, given the current regulatory regime, into disadvantaged subcontracting shares and bids. I define the disadvantaged subcontracting strategy for prime contractor i given regulation $\overline{s}$ as $S_i\left(\cdot, \boldsymbol{\omega}; \overline{s}\right)$ and the biding strategy by $B_i\left(\cdot, \boldsymbol{\omega}; \overline{s}\right)$, where stationarity allows me to drop the time subscript. The stationary strategy profile is then $\left\{B_i\left(\cdot, \boldsymbol{\omega}; \overline{s}\right), S_i\left(\cdot, \boldsymbol{\omega}; \overline{s}\right)\right\}_{i=1,2;\boldsymbol{\omega}\in\Omega}$.

Implicit in the timing is the assumption that prime contractors must commit to their disadvantaged subcontracting, which prevents primes from revising their subcontracting plans after letting. Many states have laws in place to avoid just that; in New Mexico, for example, prime contractors can be fined if their disadvantaged shares ex-post do not align with their planned shares. As a result, the proposed disadvantaged subcontracting shares rarely differ much from the final shares.

3 Equilibrium

Bellman Equation

I now construct a prime contractor's Bellman equation. Let $V_i\left(\boldsymbol{\omega}\right)$ be prime contractor i's value function given state $\boldsymbol{\omega}$ prior to observing its unregulated cost. For ease of notation, I drop the reliance of optimal disadvantaged shares and bids on the state and subcontracting regulation where it does not cause confusion. I then build the Bellman equation from the possible outcomes of each period's procurement.

In each period, a prime can either outbid their competitor or not and either outbid the reserve price or not. Therefore, there are four possible outcomes for prime contractor i:

1. Prime i bids below their opponent and reserve price.

2. Prime i bids below thier opponent but above the reserve price.

3. Prime i bids above their opponent and their opponent bids below the reserve price.

4. Prime i bids above their opponent, but their opponent bids above the reserve price.

Each of these cases yields different payoffs for the prime.

Case 1. If the prime outbids its competitor with an unregulated cost of c_i, a bid of b_i, and a disadvantaged subcontracting share of s_i while bidding below the reserve price, then its payoff is

$$b_i - (1 - s_i)\, c_i - P\,(s_i) + \delta \mathbb{E}_\epsilon V_i\,(\sigma_i\,(s_i)\,, \sigma_{-i}\,(0))\,.$$

Since the prime wins the contract in this case, it receives the static profit of $b_i - (1 - s_i)\, c_i - P\,(s_i)$, which consists of the bid less the fraction of the unregulated cost used on the project, $(1 - s_i)\, c_i$, and the disadvantaged subcontracting cost, $P\,(s_i)$. The prime contractor also receives a dynamic payoff of $\delta \mathbb{E}_\epsilon V_i\,(\sigma_i\,(s_i)\,, \sigma_{-i}\,(0))$, where $\delta \in [0, 1)$ is the common discount factor. This portion of the payoff accounts for the future value of using the disadvantaged subcontracting share s_i.

Case 2. In the event that the prime contractor outbids its opponent but bids above the reserve price, its payoff is

$$\delta \mathbb{E}_\epsilon V_i\,(\sigma_i\,(0)\,, \sigma_{-i}\,(0))\,.$$

Indeed, the prime contractor loses in this case, but since there is no winner, no primes use any disadvantaged subcontractors and the next period's payoff is the corresponding discounted value.

Case 3. When prime i is outbid by its competitor and the competitor beats the reserve price, prime i's expected payoff is

$$\delta \mathbb{E}_{c_{-i}}\left[\mathbb{E}_\epsilon V_i\,(\sigma_i\,(0)\,, \sigma_{-i}\,(S_{-i}\,(c_{-i})))\,\middle|\,B_{-i} < b_i\right].$$

In words, prime i receives the expected future value of its competitor using equilibrium share $S_{-i}\,(c_{-i})$. Since the prime must be outbid for this case to happen, the expectation is conditional on the competitor's equilibrium bid, B_{-i}, being less than their bid, b_i.

Case 4. The final case occurs when prime i bids above its competitor, but the competing firm bids above the reserve price. In this situation, prime i's expected payoff is

$$\delta \mathbb{E}_{c_{-i}} \left[\mathbb{E}_{\epsilon} V_i \left(\sigma_i \left(0\right), \sigma_{-i} \left(0\right)\right) \middle| B_{-i} < b_i \right].$$

The Value Function and Equilibrium. Combing all of the cases together and weighting them by their appropriate probabilities yields the value function. Let $W_i\left(b_i\right) = 1 - F_c\left(\left(B_{-i}\right)^{-1}\left(b_i\right)\right)$ be the probability of winning given a bid of b_i. The value function is then

$$\begin{aligned} V_i\left(\boldsymbol{\omega}\right) = \int\limits_{c_i} \max_{b_i, s_i} \Bigg\{ & W_i\left(b_i\right) \Bigg[\left(1 - F_R\left(b_i\right)\right) \big(b_i - \left(1 - s_i\right) c_i - P\left(s_i\right) \\ & + \delta \mathbb{E}_{\epsilon} V_i\left(\sigma_i\left(s_i\right), \sigma_{-i}\left(0\right)\right) \big) + F_R\left(b_i\right) \delta \mathbb{E}_{\epsilon} V_i\left(\sigma_i\left(0\right), \sigma_{-i}\left(0\right)\right) \Bigg] \\ & + \left(1 - W_i\left(b_i\right)\right) \delta \mathbb{E}_{c_{-i}} \Bigg[\left(1 - F_R\left(B_{-i}\left(c_{-i}\right)\right)\right) \mathbb{E}_{\epsilon} V_i\left(\sigma_i\left(0\right), \sigma_{-i}\left(S_{-i}\left(c_{-i}\right)\right)\right) \\ & + F_R\left(B_{-i}\left(c_{-i}\right)\right) \mathbb{E}_{\epsilon} V_i\left(\sigma_i\left(0\right), \sigma_{-i}\left(0\right)\right) \big| B_{-i} < b_i \Bigg] \Bigg\} dF_c\left(c_i\right). \end{aligned} \tag{1}$$

A stationary Markov-perfect equilibrium in this game consists of strategy profile $\left\{B_i\left(c_i, \boldsymbol{\omega}\right), S_i\left(c_i, \boldsymbol{\omega}\right)\right\}_{i=1,2;\boldsymbol{\omega}\in\Omega;c\in[\underline{c},\overline{c}]}$ and value functions $\left\{V_i\left(\boldsymbol{\omega}\right)\right\}_{i=1,2;\boldsymbol{\omega}\in\Omega}$ such that (i) given B_{-i} and S_{-i}, $V_i\left(\boldsymbol{\omega}\right)$ solves the bellman equation in (1) for every i and (ii) given B_{-i} and S_{-i}, B_i and S_i solve the optimization on the right-hand side of equation (1) for all $\boldsymbol{\omega}$ and all c_i given correct beliefs about the unregulated cost distributions and the reserve price distributions.

Subcontracting Strategies

I now turn to the prime contractor's optimal subcontracting strategy. Primes choose their disadvantaged subcontracting shares such that

$$S_i\left(c_i\right) \in \underset{s_i \in [\overline{s}, 1]}{\arg\max} \quad b_i - (1 - s_i)c_i - P(s_i) + \delta \mathbb{E}_{\epsilon} V_i\left(\sigma_i\left(s_i\right), \sigma_{-i}\left(0\right)\right).$$

Let $V_{i,1}\left(\cdot, \cdot\right)$ be the partial derivative of the value function with respect to its first argument and $V_{i,11}\left(\cdot, \cdot\right)$ be the second partial derivative again with respect to the first argument. To have a unique maximum, the prime's objective must be concave, or

$$-P''(s_i) + \delta\left[\mathbb{E}_\epsilon V_{i,11}(\sigma_i(s_i), \sigma_{-i}(0))\right] < 0.$$

Although this condition would be satisfied if V_i is concave in its second argument, proving the concavity of V_i is difficult in this setting. Instead, I follow the literature in verifying that this condition holds via simulation.

Assuming the second-order conditions hold, the prime contractor's optimal share is defined implicitly as the solution to

$$P'(s_i) = c_i + \delta\mathbb{E}_\epsilon V_{i,1}(\sigma_i(s_i), \sigma_{-i}(0)) \tag{2}$$

but takes on a corner value of $\overline{s}$ if

$$P'(\overline{s}) > c_i + \delta\mathbb{E}_\epsilon V_{i,1}(\sigma_i(\overline{s}), \sigma_{-i}(0)).$$

My third assumption on P prevents primes from subcontracting the entire project.

Intuitively, the left-hand side of equation (2) is a prime's marginal cost of increasing the share; the right-hand is the marginal benefit – which consists of the marginal savings on unregulated costs, c_i, and the discounted marginal change in future value, $\delta\mathbb{E}_\epsilon V_{i,1}(\sigma_i(s_i), \sigma_{-i}(0))$. With no regulations, prime contractors choose a share that equates their marginal benefit and marginal cost, meaning that they cost minimize in the event of a win. Prime contractors with higher unregulated costs have a higher marginal benefit from subcontracting and, therefore, are more inclined to subcontract. Since primes likely derive a higher value from having stronger relationships with disadvantaged firms, the dynamic component increases the marginal benefit – thus increasing the incentive to subcontract for farsighted primes.

Regulations also play an essential role. As the government requires higher disadvantaged shares, primes with strong relationships become more advantaged relative to primes with weak relationships since every firm must subcontract. Regulations then lead to more pronounced differences in costs between primes with strong and weak ties.

I summarize some properties of disadvantaged subcontracting below; the proofs are in the appendix.

Proposition 1. *Disadvantaged subcontracting shares are weakly increasing in unregulated costs,* c_i*.*

Proposition 2. *If* P_{12} *is sufficiently negative, then disadvantaged shares are weakly increasing in own relationship,* $\omega_{i,t}$*.*

These properties follow from the implicit function theorem applied to the first-order conditions on subcontracting and mean that more efficient primes and primes with weaker relationships will subcontract less with disadvantaged firms than less efficient primes and primes with stronger relationships.

Bidding Strategies

Next, I derive the bidding strategies. Throughout this section, I follow the literature in assuming properties on how the value function changes in state. In particular, I assume that the value function is weakly increasing in ω_i, $V_{i,1}(\omega_i, \omega_{-i}) \geq 0$, and weakly decreasing in ω_{-i}, $V_{i,2}(\omega_i, \omega_{-i}) \leq 0$. The intuition behind these assumptions is that primes with better relationships receive lower disadvantaged subcontracting prices, which lowers their costs and increases their profits. Similarly, when a prime's opponent has a better state, the prime must bid more aggressively to compete with its lower-cost competitor, leading to reduced profits. I verify these properties in my simulations. For ease of exposition, I also follow the literature in assuming that a prime's disadvantaged subcontracting function, $S(c_i)$, is smooth so that its derivative exists everywhere. This assumption is not too restrictive since a kink can only occur at the highest c_i such that $S(c_i) = \overline{s}$, which means that the derivative exists almost everywhere. Moreover, a kinked function can be approximated well with a smooth function.

In this environment, a prime's bid must account for both static costs and dynamic changes in state that come from winning. To this end, the lowest possible value from losing for prime i in state $\boldsymbol{\omega}$ is

$$\underline{V}_i(\boldsymbol{\omega}) = \mathbb{E}_\epsilon V_i(\sigma_i(0), \sigma_{-i}(S(\overline{c}_{-i}))),$$

which follows from my assumptions on V and the properties of S. Dropping the reliance of $\underline{V}_i$ on $\boldsymbol{\omega}$, I define the effective cost as

$$\phi(c_i; \boldsymbol{\omega}) = \underbrace{(1 - S(c_i)) c_i + P(S(c_i))}_{\text{static}} - \underbrace{\delta\left[\mathbb{E}_\epsilon V_i(\sigma_i(S(c_i)), \sigma_{-i}(0)) - \underline{V}_i\right]}_{\text{dynamic}}, \tag{3}$$

which is the cost relevant for bidding. Notice that the effective cost has two components: the static cost of completing the project and the dynamic opportunity cost of winning against an opponent with the highest unregulated cost. In contrast to Jeziorski and Krasnokutskaya (2016), the dynamic part is positive, so a forward-looking bidder has a lower effective cost than a myopic bidder.

Observe that

$$\phi'\left(c_i;\boldsymbol{\omega}\right)=\left(1-S\left(c_i\right)\right)+\left\{P'\left(S\left(c_i\right)\right)-\delta\mathbb{E}_\epsilon V_{i,1}\left(\sigma_i\left(S\left(c_i\right)\right),\sigma_{-i}\left(0\right)\right)-c_i\right\}S'\left(c_i\right)>0.$$

This property holds because the term inside of the brackets is zero by the first-order conditions, and $S'\left(c_i\right)$ is zero at the corner and positive otherwise. Effective costs are, therefore, monotone in c_i and can be inverted.

A prime's bid can now be expressed as a function that maps effective costs into bids given the state, $B_i\left(\cdot,\boldsymbol{\omega}\right):\left[\underline{\phi}_i,\overline{\phi}_i\right]\rightarrow\left[\underline{B}\left(\boldsymbol{\omega}\right),\overline{B}\left(\boldsymbol{\omega}\right)\right]$, instead of unregulated costs into bids. The inverse bid function, ξ_i, then maps bids into effective costs, $\xi_i\left(\cdot,\boldsymbol{\omega}\right):\left[\underline{B}\left(\boldsymbol{\omega}\right),\overline{B}\left(\boldsymbol{\omega}\right)\right]\rightarrow\left[\underline{\phi}_i\left(\boldsymbol{\omega}\right),\overline{\phi}_i\left(\boldsymbol{\omega}\right)\right]$. A prime chooses bid b_i to maximize expected profits:

$$\begin{aligned}\max_{b_i}&\left[\left(b_i-\phi_i\right)\left(1-F_R\left(b_i\right)\right)+\delta\left(\mathbb{E}_\epsilon V_i\left(\sigma_i\left(0\right),\sigma_{-i}\left(0\right)\right)-\underline{V}_i\right)F_R\left(b_i\right)\right]\left(1-F_{\phi,-i}\left(\xi_{-i}\left(b_i\right)\right)\right)\\&+\delta\int_{\underline{\phi}_{-i}}^{\xi_{-i}\left(b_i\right)}\left[\left(1-F_R\left(B_{-i}\left(\phi\right)\right)\right)\left(\mathbb{E}_\epsilon V_i\left(\sigma_i\left(0\right),\sigma_{-i}\left(S_{-i}\left(\phi\right)\right)\right)\right)\right.\\&\left.+F_R\left(B_{-i}\left(\phi\right)\right)\mathbb{E}_\epsilon V_i\left(\sigma_i\left(0\right),\sigma_{-i}\left(0\right)\right)-\underline{V}_i\right]f_{\phi,-i}\left(\phi\right)d\phi.\end{aligned}$$

Taking the first-order conditions leads to the system of differential equations:

$$\begin{aligned}&\left(1-F_{\phi,-i}\left(\xi_{-i}\left(b_i\right)\right)\right)\left[\left(1-F_R\left(b_i\right)\right)-\left(b_i-\phi_i\right)f_R\left(b_i\right)+\delta\left(\mathbb{E}_\epsilon V_i\left(\sigma_i\left(0\right),\sigma_{-i}\left(0\right)\right)-\underline{V}_i\right)f_R\left(b_i\right)\right]\\&-f_{\phi,-i}\left(\xi_{-i}\left(b_i\right)\right)\xi'_{-i}\left(b_i\right)\left(1-F_R\left(b_i\right)\right)\left[b_i-\phi_i-\delta\left(\mathbb{E}_\epsilon V_i\left(\sigma_i\left(0\right),\sigma_{-i}\left(S_{-i}\left(\xi_{-i}\left(b_i\right)\right)\right)\right)-\underline{V}_i\right)\right]=0.\end{aligned}\tag{4}$$

Notice that the support of a prime's effective cost distribution can change with the state. Indeed, a prime with the highest unregulated cost can use disadvantaged subcontractors to lower its effective cost and can achieve even lower effective costs with better relationships. Therefore, the auction is asymmetric as in Maskin and Riley (2000) but with different supports like in Kaplan and Zamir (2012). In these environments, a bidder with no probability of winning – which occurs either because their effective cost is above the highest equilibrium bid or the reserve price – is indifferent between any bid at or above its reserve price. I follow Kaplan and Zamir (2012) in assuming that primes that find themselves in this situation bid their effective cost.

Let $\left[\underline{b},\overline{b}\right]$ be the interval of equilibrium bids where a prime has a positive probability of winning. Assuming, without loss of generality, that $\overline{\phi}_1 \leq \overline{\phi}_2$, the boundary conditions are

$$\begin{aligned}
\xi_1\left(\overline{b}, \boldsymbol{\omega}\right) &= \overline{\phi}_1 \\
\xi_2\left(\overline{b}, \boldsymbol{\omega}\right) &= \overline{b} \\
\xi_1\left(\underline{b}, \boldsymbol{\omega}\right) &= \underline{\phi}_1 \\
\xi_2\left(\underline{b}, \boldsymbol{\omega}\right) &= \underline{\phi}_2,
\end{aligned} \tag{5}$$

where $\overline{b} = \min\left\{b_0, \overline{\phi}_2, \overline{r}\right\}$ and, following Jeziorski and Krasnokutskaya (2016), b_0 is defined implicitly by the equation

$$\begin{aligned}
&(1 - F_{\phi,2}(b_0))\left[(1 - F_R(b_0)) - \left(b_0 - \overline{\phi}_1\right) f_R(b_0) + \delta\left(\mathbb{E}_\epsilon V_1(\sigma_1(0), \sigma_2(0)) - \underline{V}_1\right) f_R(b_0)\right] \\
&\qquad - f_{\phi,2}(b_0)(1 - F_R(b_0))\left[b_0 - \overline{\phi}_1 - \delta\left(\mathbb{E}_\epsilon V_1(\sigma_1(0), \sigma_2(S_2(b_0))) - \underline{V}_1\right)\right] = 0.
\end{aligned} \tag{6}$$

The intuition behind the boundary conditions is that both primes must submit the same low bid in equilibrium or else the low prime could increase its bid without changing its probability of winning. The high bid is slightly more involved. When prime 1 has the lowest effective cost, it can use its first-order conditions to find its high bid, which is equivalent to finding b_0. In equilibrium, the least efficient prime cannot make a profit, so $\overline{b}$ must be the minimum of b_0 and $\overline{\phi}_2$. Moreover, $\overline{b}$ must be at or below the upper bound of the reserve price distribution since bids above the reserve price are rejected outright.

3.1 Discussion of Modeling Choices and Extensions

My analysis contains several abstractions used to produce a model with tractable results. In real procurement markets, projects can be more complex and can require primes to make more involved bidding and subcontracting decisions. In what follows, I discuss my modeling choices and consider additional factors that may influence how primes use disadvantaged subcontractors.

Task Heterogeneity. Procurement projects are rarely homogeneous and can consist of many different tasks. If tasks are sufficiently distinct, then primes may have to make subcontracting decisions in different markets, which will affect their disadvantaged subcontracting. Likewise, primes may have specialties in specific tasks, which again will perturb their disadvantaged subcontracting decisions. Since accounting for heterogeneous tasks in my model is infeasible because it would require separate states for each task and each prime, I abstract away from this feature of the disadvantaged subcontracting market.

Entry. For simplicity, my model assumes that the two competing primes invariably enter each period's auction, but in reality, primes can choose whether to participate. If entry is based on the expected profitability of a project, then subcontracting regulations can potentially deter entrants, resulting in higher costs of procurement than my model would predict. Less likely to be impacted, though, is a prime's disadvantaged subcontracting decisions since they are based on cost minimization rather than the number of entrants. In fact, Jeziorski and Krasnokutskaya (2016) show that subcontracting shares do not depend on the number of entrants in a closely related model, and empirically, studies such as Rosa (2018) find that disadvantaged subcontracting is uncorrelated with the number of participating firms. As a result, my findings on relationships formed through disadvantaged subcontracting are unlikely to change significantly with entry unless there is a substantial deterrence effect.

Capacity Constraints. The dynamic auction literature typically has a firm's capacity as the dynamic variable.[9] In these environments, a firm that wins a project now will have higher costs in the next period because they are operating closer to their capacity. In reality, disadvantaged firms are usually a small part of total subcontracting and are less likely to be impacted by a prime's capacity constraints. Moreover, some studies show that capacity constraints are not a major factor in a prime's overall subcontracting decisions; in the case of construction, see González et al. (2000). As such, my analysis focuses on dynamic relationship formation, which is one of the primary arguments for using affirmative action regulations in subcontracting.

Related to capacity issues for prime contractors is the possibility that the disadvantaged firms are capacity constrained. Incorporating subcontractor capacity constraints into the model would require additional state variables for each disadvantaged subcontractor, leading to a high-dimensional state space that quickly becomes unmanageable. Additionally, any capacity-related subcontracting concerns are likely short-lived because the work subcontracted to each firm tends to be small. For these reasons, I abstract away from capacity constraints in the disadvantaged subcontracting market.

The Reserve Price Distribution. I assume a time and state invariant reserve price distribution, but governments may wish to condition their reserve price on the state to allow for more leniency in the range of accepted bids when relationships are low. In practice, bids tend to be rejected for exceeding estimated costs, and cost estimates rarely factor in subcontracting regulations.[10] To keep the model simple, I abstract away

[9] See Saini (2012), Jeziorski and Krasnokutskaya (2016), Jofre-Bonet and Pesendorfer (2003) and Groeger (2014).

[10] In the state of Iowa, cost estimates and subcontracting regulations are handled by two different agencies.

from state-dependent reserve price distributions.

A somewhat related issue is that contracts rarely have only two bidders. There may be other less-frequent bidders that often bid, which the literature refers to as fringe bidders. Although the reserve price partially accounts for fringe bidders in my framework, an immediate extension would be to have the reserve price distribution depend on the number of fringe bidders. This extension complicates the model by adding another state variable, so I do not include it in this paper.

4 Simulation

To illustrate the impact of subcontracting regulations, I simulate bids, subcontracting, and industry outcomes in two situations: one in which there is no regulation and one in which there is regulation. This section contains those simulations and a brief description of my simulation methodology.

4.1 Simulation Method

To simulate the Markov-perfect equilibrium of the model, I must approximate the strategy profile $\{B_i(c_i, \boldsymbol{\omega}),$ $S_i(c_i, \boldsymbol{\omega})\}_{i=1,2;\boldsymbol{\omega}\in\Omega;c\in[\underline{c},\overline{c}]}$ and value functions $\{V_i(\boldsymbol{\omega})\}_{i=1,2;\boldsymbol{\omega}\in\Omega}$. Implicit in calculating the bids, I must also approximate the PDF and CDF of the effective cost distribution. For the bids, value function, and effective cost distribution, I use Chebyshev polynomial approximations; for subcontracting, I use Hermite splines, which perform well in approximating the flat area that can occur at the corner.

My algorithm consists of an inner loop that calculates the strategy profiles on a select grid of states and an outer loop that iterates on the value function. In the inner loop, I solve equation (2) to get the shares and use the shares to approximate the effective costs. In approximating the bids, I use projection methods with a mathematical programming with equilibrium constraints (or MPEC) optimization routine proposed by Hubbard and Paarsch (2009). These equilibrium objects allow me to find the following period's value function, and I iterate until the difference between the current and next value functions is small. A concern with this approach is that there can potentially be multiple equilibria. To ease that concern, I use the robust equilibrium selection rule proposed in Chen et al. (2009). To extend my strategy profile approximations to other states, I use linear interpolations. A detailed description of my numerical routine is contained in appendix B.

4.2 Simulation Parameters

Simulating outcomes from the theoretical model requires parametric assumptions on the inverse supply function, the unregulated cost distribution, the secret reserve price distribution, and the states. I begin by assuming that the inverse supply function takes the from

$$P\left(s_i, \omega_i\right) = \frac{s_i}{\omega_i\left(1 - s_i\right)}.$$

Observe that this specification adheres to my theoretical inverse supply function assumptions for $s_i \in [0, 1]$.

For simplicity, I assume that unregulated costs are distributed uniformly on an interval bounded below by 0 and above by 1.5. In real procurement environments, the government often provides a range of estimated project cost values to primes prior to bidding. I envision a realization of 1 equivalent to the midpoint of any cost range; a value of 1.5 would then be the upper bound, while a value of 0.5 would be the lower bound. Consequently, I assume that the reserve price distribution is also uniform, but bounded below by the lower cost estimate of 0.5 and above by the upper cost estimate of 1.5. In other words, bids can only be rejected if they are within or above the established cost range.

To simulate plausible disadvantaged shares, I set the lower bound for the states to 0.5 and the upper bound to 2. Given my previous assumptions, a prime that draws the average unregulated cost will not use any disadvantaged subcontractors in the lowest state absent regulations, while an equivalent draw in the highest state leads to a disadvantaged share of about 18 percent.

I investigate two different sets of values for the quota level, $\overline{s}$, and the relationship deterioration parameter, ψ. Subcontracting regulations typically range from 1 to 15 percent. In Iowa, for example, the DOT can recommend a DBE goal anywhere from 1 to 10 percent, and in states such as Texas, DBE goals can reach as high as 15 percent.[11] To highlight the distinction between projects with and without subcontracting regulations, I contrast the more extreme $\overline{s} = 0.15$ case with the case where $\overline{s} = 0$. I consider deterioration parameter values of $\psi \in \{0.95, 1\}$, which covers cases where relationships persist and where there is some deterioration. Finally, I hold the common discount factor fixed at $\delta = 0.95$, corresponding to a yearly interest rate of about 5 percent.

[11] See the 2018 Iowa DBE Program Plan for Iowa and De Silva et al. (2012) for Texas.

4.3 Simulation Results: The Role of Subcontracting Regulations

My simulation analysis begins by analyzing the role of subcontracting regulations. To isolate this effect, I fix ψ at 1 so that relationships persist over time in expectation, and I vary the quota from 0 to 15 percent. I first study bidding, disadvantaged subcontracting, and effective costs at select states and then move on to the value function.

Figure 1 illustrates how primes select their disadvantaged shares at different states and under various regulations. The two left panels depict optimal subcontracting when the market is unregulated and the right two when the market is regulated. In the poor state ($\boldsymbol{\omega} = (0.5, 0.5)$ in the top two panels) primes use no disadvantaged subcontractors at every unregulated cost when they are unregulated. As a result, primes in this state would never build any relationships with disadvantaged subcontractors without relationship shocks, even if those relationships would perfectly persist over time. Once the government imposes regulations, all primes in the poor state subcontract exactly at the quota.

Introducing asymmetries between prime contractors ($\boldsymbol{\omega} = (1.25, 0.5)$ in the bottom two panels) lead to different disadvantaged subcontracting behavior. With a stronger relationship than its competitor, prime 1 will subcontract at higher unregulated costs absent regulations, while prime 2 will still not subcontract at any unregulated cost. When the government introduces regulations, primes follow a similar pattern, except the lower bound on subcontracting increases to the quota level. These actions trace back to the optimal disadvantaged subcontracting conditions in equation (2): primes with better relationships have a lower marginal cost of subcontracting and will, therefore, subcontract more often.

Next, I turn to the implied effective costs – which I display in figure 2 in the same format as the disadvantaged shares. Because primes are symmetric in their disadvantaged subcontracting, their effective costs also overlap in the poor state. Effective costs become more distinct when prime 1 builds stronger relationships. In the unregulated case, effective costs are similar at lower unregulated costs but diverge at higher unregulated costs. This result arises because prime 1 can use disadvantaged subcontractors to achieve lower effective costs at higher unregulated costs. Effective costs are not exactly the same at lower unregulated cost levels because the option value from losing against an opponent is different.[12] The disparity between effective costs is magnified when the government adds on regulations, which indicates that regulations make bidders more asymmetric.

Figure 3 illustrates the equilibrium bids that have a positive probability of winning. As is implied by the

[12]In particular, the option value for prime 1 is zero since prime 2 will not subcontract even with the least efficient draw. The option value for prime 2 lowers effective costs since its competitor will have a higher state if it draws the highest unregulated cost.

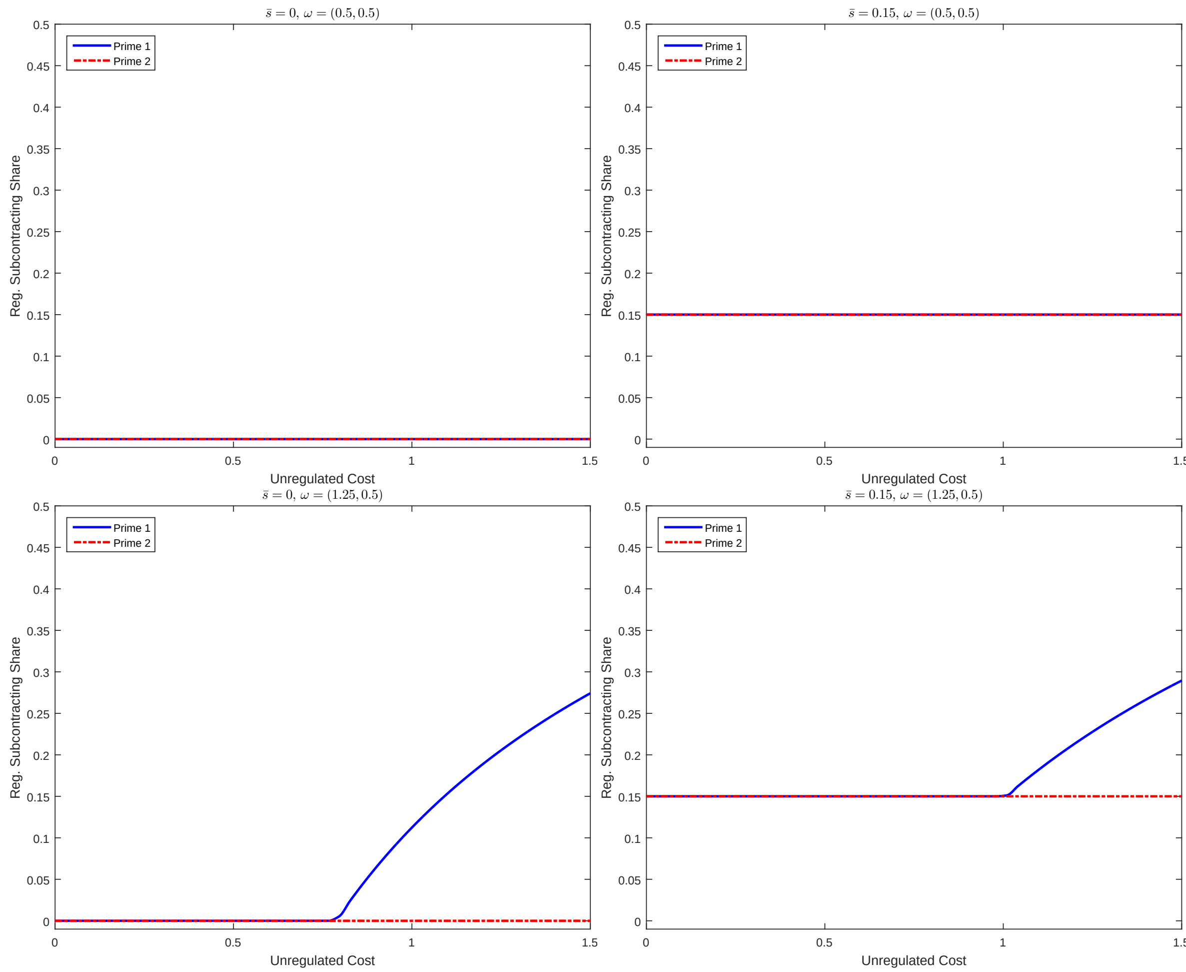

Figure 1: Disadvantaged Subcontracting

effective costs, symmetric bidders in the poor state bid the same irrespective of the regulation. Once bidders become asymmetric, the bid functions become more distinct in ways that mirror static asymmetric auctions. In particular, the bids have a "weakness-leads-to-aggression" property,[13] whereby primes with higher effective costs (or weak bidders) bid more aggressively (or closer to their effective costs) than strong bidders. As a result, asymmetry softens competition between firms, leading to a reduced impact of relationship-related cost savings on procurement costs.

In the no-regulation case, the weakness-leads-to-aggression property causes the bid functions to cross because the identity of the weak and strong bidder changes at different effective costs. When there are regulations, there is a pronounced separation in effective costs at every unregulated cost, leading to a similar separation in bids. Because the state generates asymmetries and arises as a result of each prime's past

[13]See Maskin and Riley (2000) and Krishna (2002) for the static case and Saini (2012) for the dynamic case with capacity constraints.

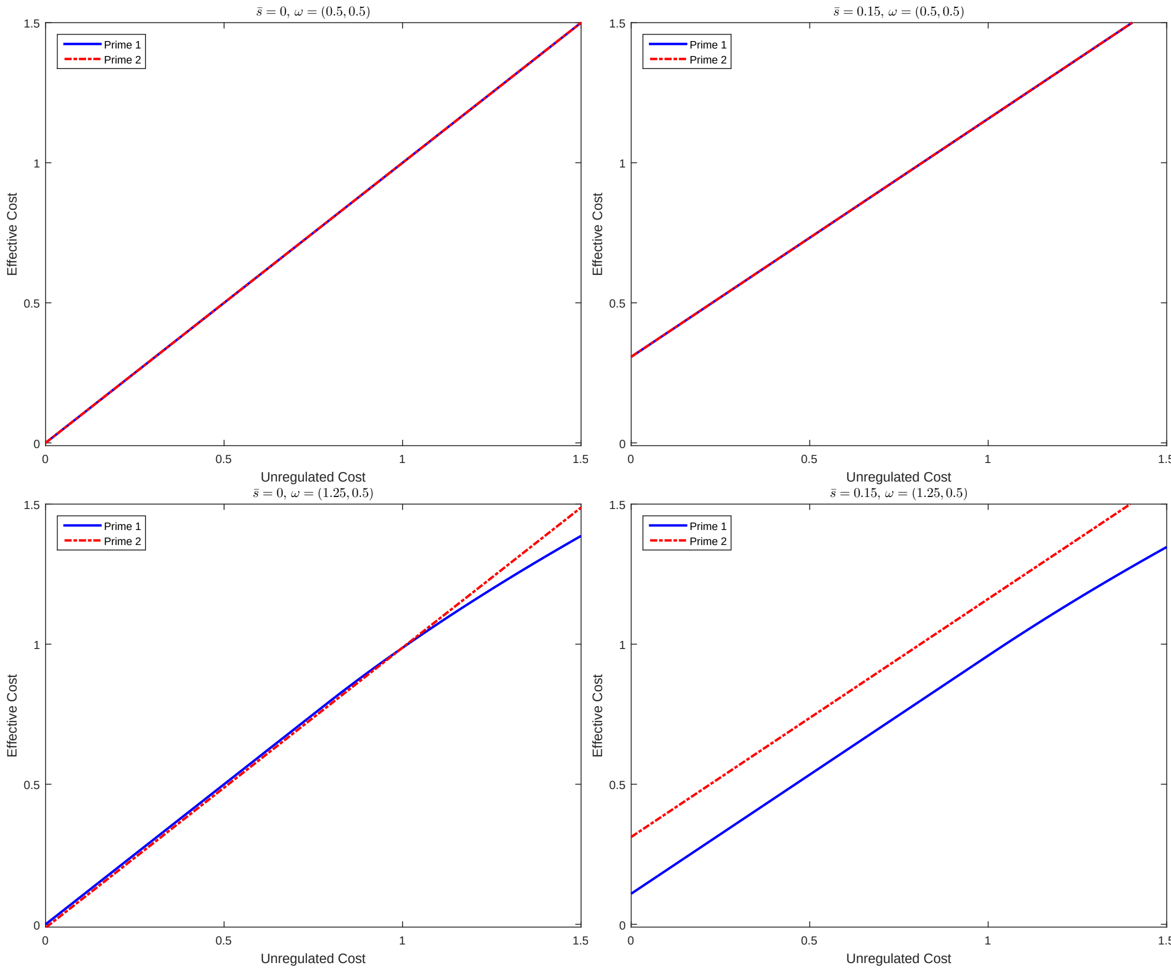

Figure 2: Effective Costs

actions, bidder asymmetry is endogenous. The key observation in this environment is that subcontracting regulations enhance that asymmetry.

The final equilibrium object is the value function, which I show in figures 4 and 5. My assumptions imply that the value function is symmetric for each prime. To illustrate how symmetry works, let $\omega_{own} \in \left[\underline{M}, \overline{M}\right]$ be an arbitrary state draw for a prime and $\omega_{rival} \in \left[\underline{M}, \overline{M}\right]$ be an arbitrary state draw for the prime's competitor. Symmetry implies that $V_1\left(\omega_{own}, \omega_{rival}\right) = V_2\left(\omega_{own}, \omega_{rival}\right) = V\left(\omega_{own}, \omega_{rival}\right),$ which obviates the need for computing multiple value functions for a given parameter configuration.

The value functions, which differ in whether there is any regulation, have all of my conjectured properties: the value function increases in own state and decreases in the competitor's state. Regulations change the steepness of the value function. Indeed, the gain in marginal value from attaining a higher state is only realized when unregulated costs are high in an unregulated market. On the other hand, the gain in marginal value

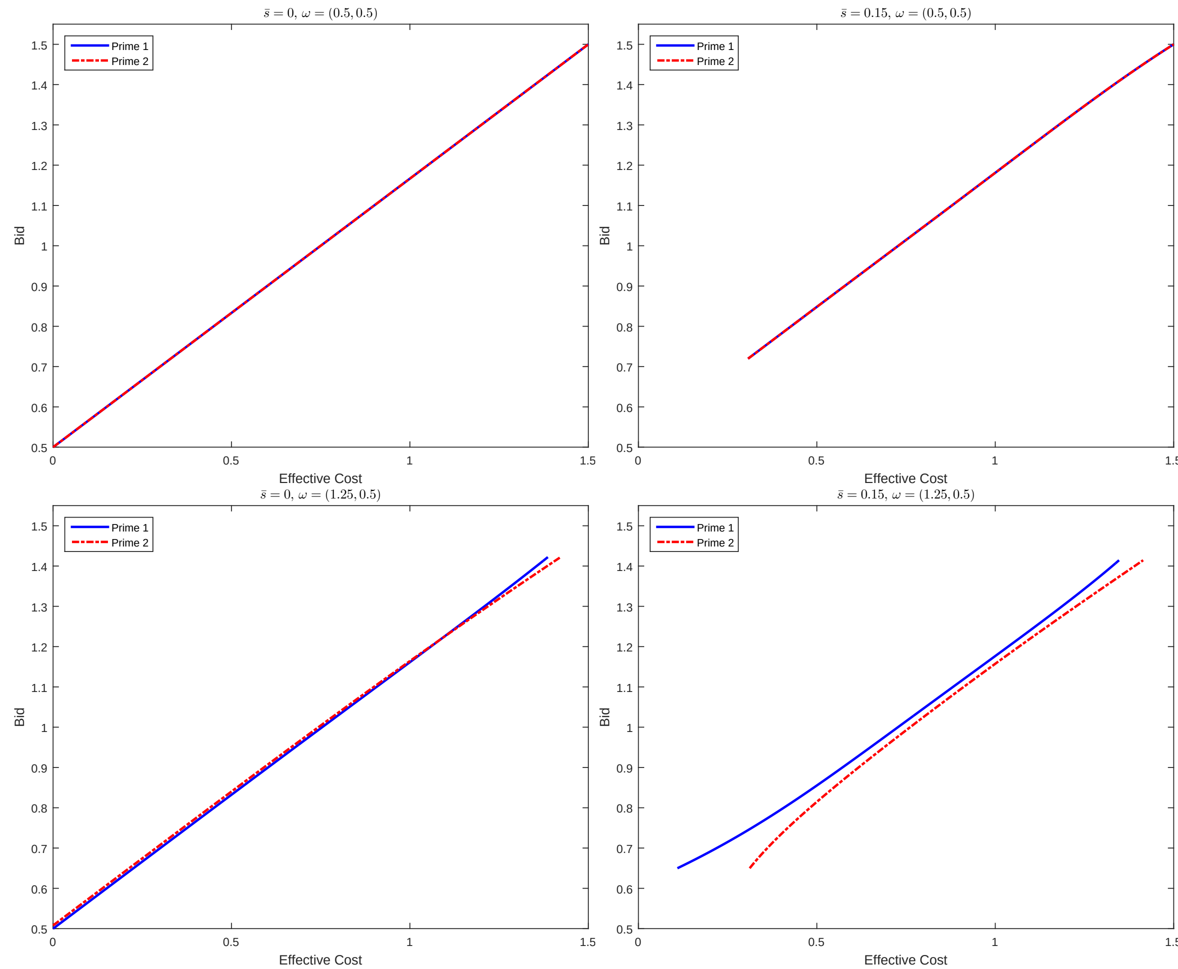

Figure 3: Bidding

is realized for every unregulated cost when there are regulations because primes are required to subcontract. As a result, regulations lead to a steeper value function.

4.4 Simulation Results: Dynamic Equilibrium

I now analyze the evolution of the relationship state variable over time. Central to this analysis is the initial state, $\boldsymbol{\omega}_0$, and the relationship deterioration parameter, ψ; the values that I consider for these variables are $\boldsymbol{\omega}_0 \in \{(0.5, 0.5), (2, 2)\}$ and $\psi \in \{1, 0.95\}$. My initial state values account for two extreme cases: one where the market begins at the lowest possible state and the other where the market starts at the highest possible state. My ψ values cover the cases of when relationships persist and when there is some deterioration. Since markets with different relationship deterioration will have different equilibria, I resolve the model every time there is a change in ψ.

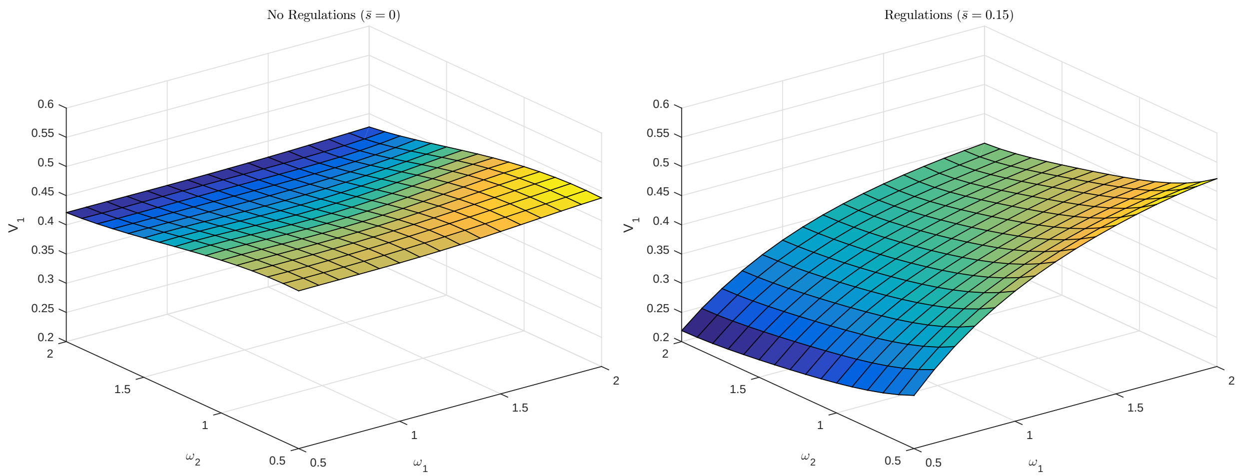

Figure 4: Three-Dimensional Value Functions

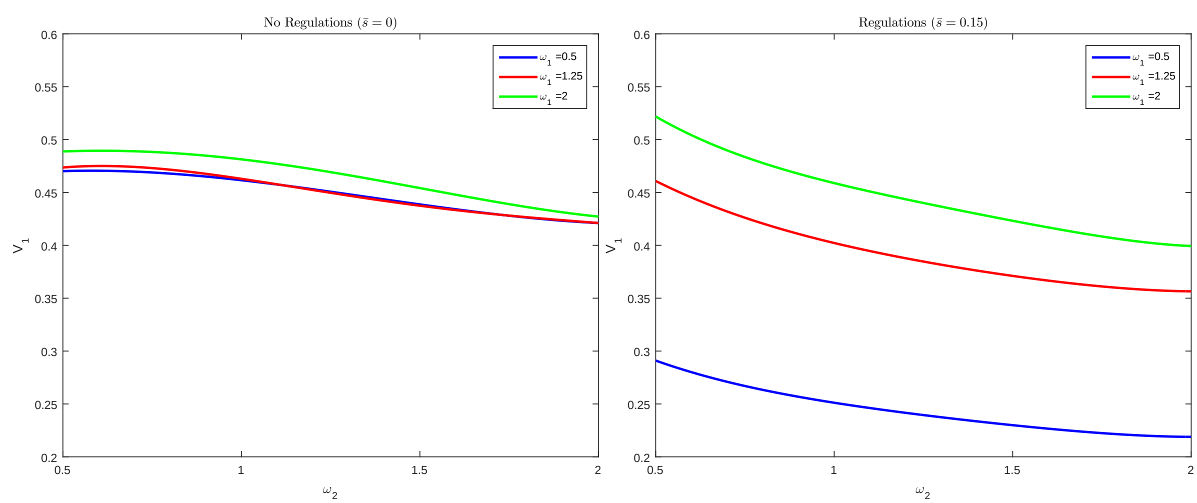

Figure 5: Value Function Level Sets

Figure 6 illustrates the evolution of different markets. In creating this figure, I simulated 100 procurement auctions for the first 100 periods, starting at various initial states. Figure 6 shows the average state for prime 1 across simulations in each period; the plot for prime 2 is similar and is, therefore, omitted.

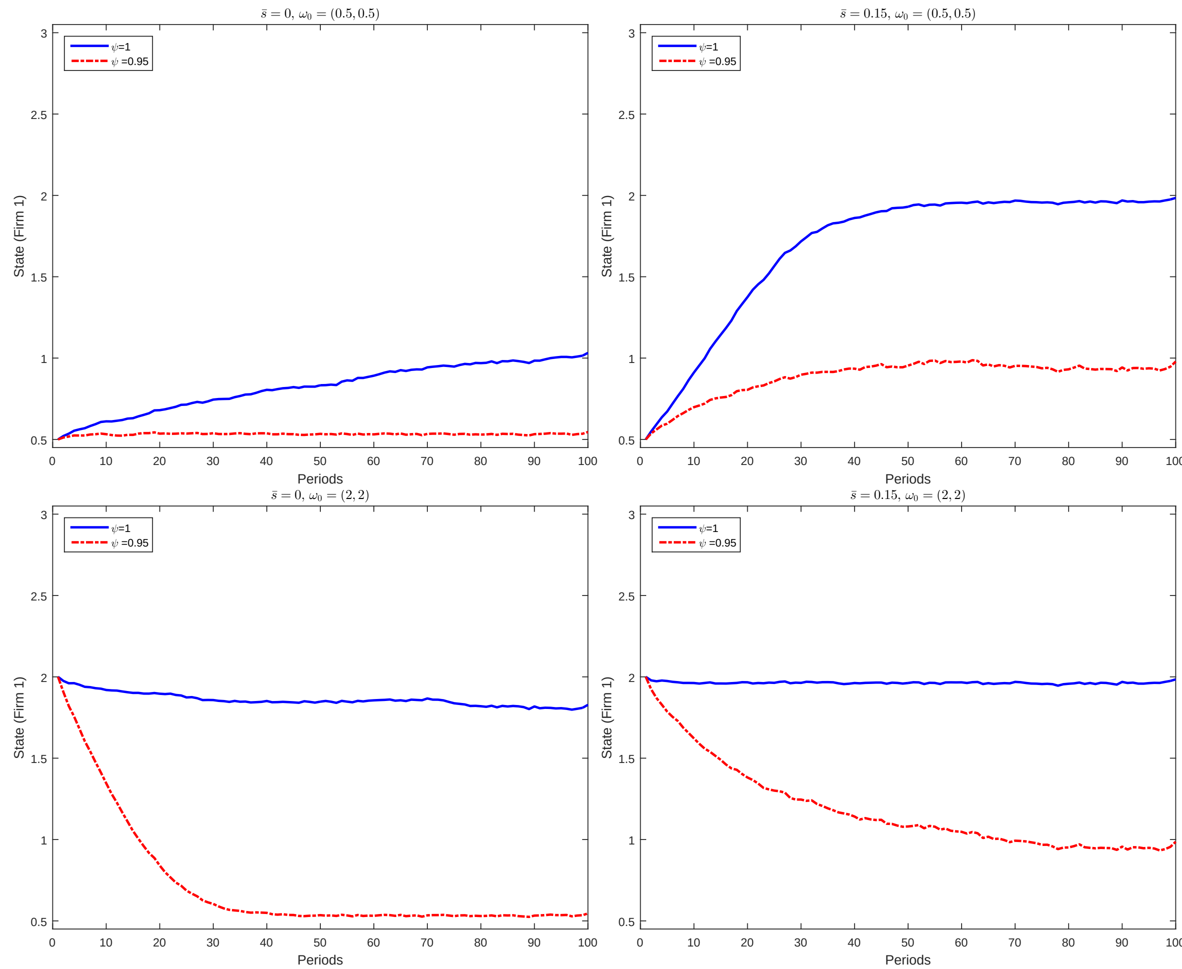

Figure 6: Time Paths

The top left panel depicts a market without regulations that starts in the lowest possible state. When relationships deteriorate, primes never develop any relationships because disadvantaged subcontractors are too expensive. This market is reminiscent of the lack of disadvantaged subcontractor utilization before affirmative action laws, as was the case in Atlanta in 1973 where one-tenth of one percent of the city's procurement business went to African American firms despite Atlanta's majority African American population.[14] When relationships persist, random forces cause primes to slowly build relationships to a point where they are willing to use disadvantaged subs. This scenario illustrates a case where affirmative action is not required for

[14]This example comes from Holzer and Neumark (2000).

long-term relationship formation.

When the government uses affirmative action quotas – which is displayed in the top right panel – relationships no longer remain stagnant at the lowest state when there is relationship deterioration between periods. The policy forces primes out of the lowest state through mandatory subcontracting, leading to a higher average state than would be obtained absent affirmative action. When relationships persist between periods, the average state increases more rapidly than without affirmative action and is not driven by random forces. These results justify the early use of affirmative action quotas in procurement: assuming primes develop long-lasting relationships, quotas can help markets reach states characterized by high disadvantaged subcontracting quicker; when relationships are more short-lived, the state still improves, but not as much as when relationships are long-lasting.

Now consider the market's evolution when it begins at the highest state, which is shown in the bottom two panels of figure 6. Even when relationships are long-lasting, random forces lead to a slight downward trend in the average state. That trend is more extreme when relationships deteriorate between periods because more factors lead to a reduction in the next period's state. The mandatory subcontracting required by affirmative action policies prevents the state from trending downward when relationships persist by correcting the random shocks that lower relationships between periods. With less persistent relationships, affirmative action is not strong enough to prevent a downward trend but does prevent primes from reaching the lowest state.

Figure 7 extends my analysis to the limiting (or ergodic) state distribution. I construct these figures by first simulating 1,000 periods and then treating the next 100 periods as draws from the ergodic distribution. I repeat this process 100 times, leading to a total of 10,000 draws. Since the limiting distributions look the same irrespective of the starting value, I only present the cases with differing levels of affirmative action and relationship deterioration.

The limit distributions are a natural consequence of extending the number of periods in figure 6. With long-lasting relationships, the limit distribution has more density at the higher state for both primes. This pattern emerges because past relationships tend to accumulate over time, culminating in a situation where primes eventually reach the higher states. With the mandatory subcontracting that affirmative action entails, the ergodic distribution becomes more concentrated in the higher states since winning primes are essentially required to keep adding to their stock of existing relationships.

The limit distributions look much different when relationships tend to deteriorate, which occurs in the two right-most graphs in figure 7. With deteriorating relationships and no affirmative action, the ergodic distribution is concentrated at the lower states, with variation driven by random forces. This distribution

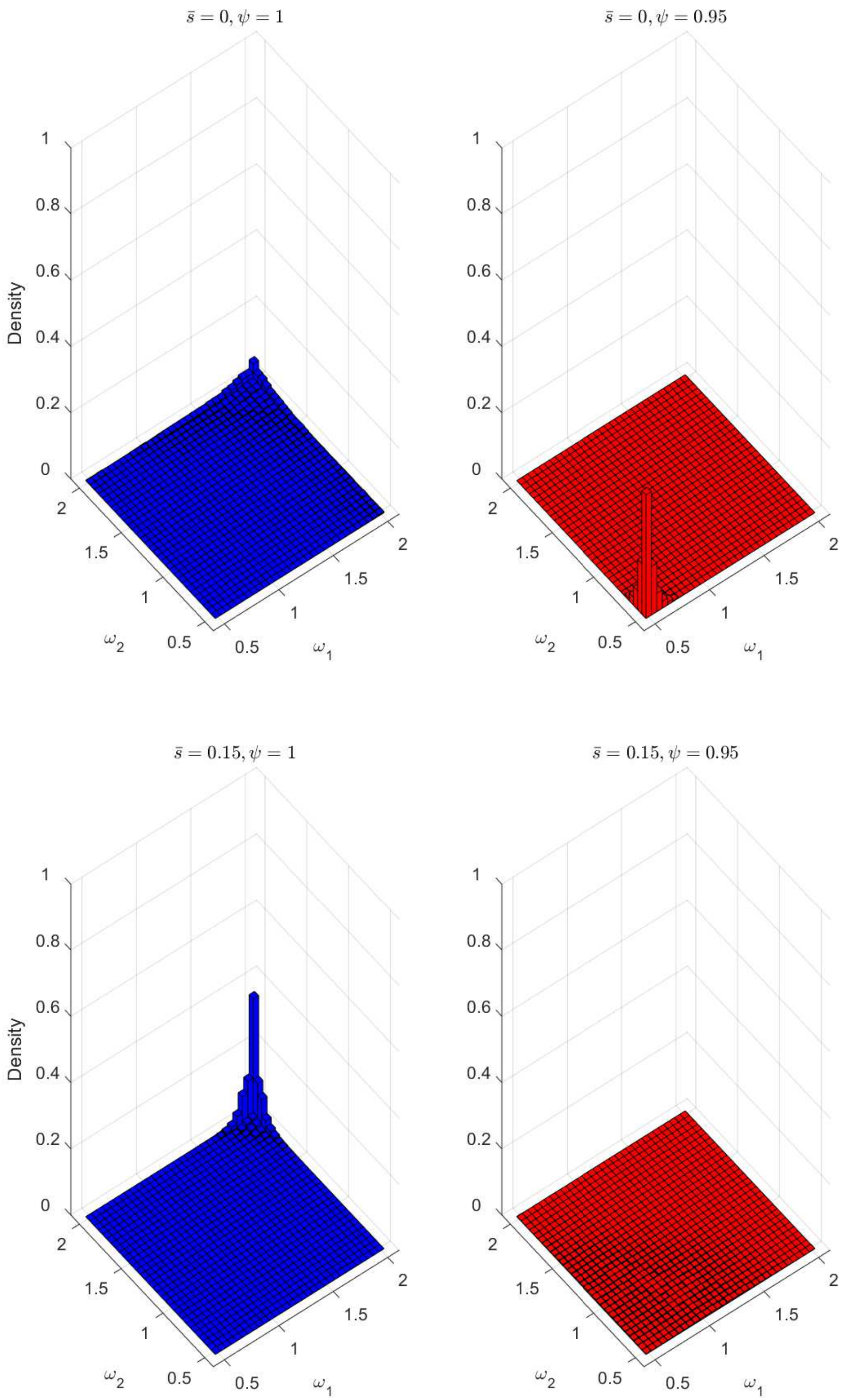

Figure 7: Ergodic State Distributions

arises as a consequence of the declining nature of relationships: a prime can only increase its stock conditional on winning, yet its relationships deteriorate regardless of a win. Consequently, relationships eventually hover around the lowest state, where primes avoid disadvantaged firms altogether. Affirmative action can work against deteriorating relationships by requiring a minimum level of subcontracting, leading to an ergodic distribution that is more disperse across states.

I conclude my simulation analysis by exploring industry statistics averaged over all simulations and time periods. I consider the following outcome variables. *Bid* is the average equilibrium bid, and *Subcon* is the average share of the project awarded to disadvantaged subcontractors. *Perc. Bind* is the percent of all projects where the quota binds, which I use as a measure of how constraining affirmative action regulations are. *Proc. Cost* is the average procurement cost conditional on the lowest bidder beating the reserve price, and *Perc. Allocated* is the percent of all projects awarded to one of the two primes. I remark here that a project awarded to neither prime does not necessarily go uncompleted; it is possible that the project is awarded to a fringe bidder, which is embedded in the reserve price distribution.

Table 1: Simulation Results

ψ	$\bar{s}$	$\boldsymbol{\omega}_0$	Bid	Subcon.	Perc. Bind	Proc. Cost	Perc. Allocated
1.00	0.00	(0.5,0.5)	0.95	0.12		0.75	69.22
1.00	0.00	(2,2)	0.93	0.15		0.74	69.99
0.95	0.00	(0.5,0.5)	1.00	0.00		0.75	66.76
0.95	0.00	(2,2)	1.00	0.00		0.75	66.79
1.00	0.15	(0.5,0.5)	0.93	0.24	43.55	0.76	69.41
1.00	0.15	(2,2)	0.92	0.24	42.81	0.76	69.65
0.95	0.15	(0.5,0.5)	1.03	0.16	74.52	0.83	60.70
0.95	0.15	(2,2)	1.03	0.17	73.22	0.83	61.09

Note: Table shows the simulated market statistics.

I summarize the industry statistics in table 1. I begin by analyzing the first four rows, which summarize markets without affirmative action regulations. A feature of these markets is that the average bid is lower and the average level of subcontracting is higher when relationships are long-lasting relative to when relationships deteriorate. These results are a consequence of disadvantaged subcontractors becoming a viable input in completing a project under the high stocks of relationships that accumulate with long-lasting relationships: primes willingly use disadvantaged subs to lower their project costs with sufficiently strong relationships, resulting in lower bids. Although the bids are lower, procurement costs remain relatively similar across the different relationship deterioration parameters because the winning prime is usually the more efficient one (in that it has a lower unregulated cost) and therefore is less likely to use disadvantaged subs. The two primes also win more projects when relationships last longer since disadvantaged subcontractors can work to reduce costs.

Next I turn to the markets with affirmative action quotas, which are contained in the bottom four rows of table 1. Remarkably, bids in these markets are comparable to the bids received without affirmative action, but procurement costs are generally higher. These results arise because subcontracting quotas are more likely to distort the winning prime's costs upwards, and the better states obtained through affirmative action allow less efficient primes to lower their costs – and therefore their bids – through subcontracting. Subcontracting under affirmative action is higher across the board, which is expected given that there is a quota. The values for the percent of times the quota binds, which is my measurement for how constraining affirmative action is, reveals that affirmative action is more binding when relationships are deteriorating because the lower stocks of relationships mean that primes would want to subcontract less relative when relationships are long-lasting.

Taken together, my industry statistics suggest that the impact of affirmative action programs depends crucially on how relationships between prime contractors and disadvantaged subcontractors evolve. With persistent relationships, affirmative action does not have much of an impact on the bids and procurement costs but leads to substantially higher subcontracting shares. This result is consistent with cross-project analyses of Rosa (2018) for highway construction in New Mexico and De Silva et al. (2012) for asphalt projects in Texas. On the other hand, deteriorating relationships lead to higher bids and procurement costs, although disadvantaged subcontracting is noticeably higher. These results are consistent with Marion (2009) for California highway procurement projects. From a policymaker's point of view, it is crucial to assess contractor-subcontractor relationships when setting or removing affirmative action regulations.

5 Conclusion

This paper explores the impact of affirmative action subcontracting regulations on a procurement environment with dynamic relationship formation. Although evidence from the literature and actual procurement agencies suggests that relationships are vital in a firm's cost of and ability to comply with affirmative action, much of the literature does not directly account for relationships. In this paper, I fill that gap by numerically solving for the Markov-perfect equilibria of an infinitely repeated procurement auction, where relationships evolve endogenously.

I first explore the role of affirmative action on farsighted bidders with relationship formation fixed. I find that affirmative action accentuates bidder asymmetries, favoring primes with stronger relationships over those with weaker ones. This leads to a value function that increases more rapidly in state when there are regulations, which translates into a stronger incentive for farsighted primes to subcontract with disadvantaged firms relative to myopic primes. Without affirmative action, the value function is much flatter, indicating less of a difference between myopic and farsighted bidders.

I then use the solved model to study how relationships evolve, allowing for the possibility that relationships can deteriorate. I find that the impact of affirmative action quotas on a procurement market depends crucially on the relationship formation process. When relationships are long-lasting, affirmative action rapidly improves the state of relationships, increases the average share of a project allocated to disadvantaged subs, and creates negligible differences in the cost of procurement. Conversely, a market characterized by higher relationship deterioration has more substantial differences in procurement costs with and without affirmative action, although affirmative action still markedly increases the utilization of disadvantaged firms.

My analysis highlights the importance of relationship formation in affirmative action subcontracting regulations. A potential avenue for future research would be to evaluate the long-run effectiveness of programs aimed at directly improving relationships and how they interact with markets with affirmative action. Examples of these types of programs include the Mentor-Protege Program sponsored by the USDOT, which seeks to match prime contractors with new disadvantaged firms. I leave the evaluation of such joint policies for future research.

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A Proofs

This section contains the proofs of my two propositions on optimal disadvantaged subcontracting. Because this section aims to recover comparative statics, I include the state in the optimal subcontracting function.

A.1 Proof of Proposition 1

Proposition. *Disadvantaged subcontracting shares are weakly increasing in c_i.*

Proof. There are two cases for $S(c_i, \boldsymbol{\omega})$: $S(c_i, \boldsymbol{\omega}) = \overline{s}$ or $S(c_i, \boldsymbol{\omega}) \in (\overline{s}, 1)$. If $S(c_i, \boldsymbol{\omega}) \in (\overline{s}, 1)$, then differentiating the first-order conditions with respect to c_i gives

$$S_1(c_i, \boldsymbol{\omega}) = -\frac{1}{\delta\left[\mathbb{E}_\epsilon V_{i,11}\left(\sigma_i\left(S\left(c_i, \boldsymbol{\omega}\right), \omega_i\right), \sigma_{-i}\left(0, \omega_{-i}\right)\right)\right] - P_{11}\left(S\left(c_i, \boldsymbol{\omega}\right), \omega_i\right)},$$

Since the denominator is negative by the second-order conditions, $S_1(c_i, \boldsymbol{\omega}) > 0$. If $S(c_i, \boldsymbol{\omega}) = \overline{s}$, then $S(c_i, \boldsymbol{\omega})$ does not change in c_i, implying that $S_1(c_i, \boldsymbol{\omega}) = 0$.[15] □

A.2 Proof of Proposition 2

Proposition. *If P_{12} is sufficiently negative, then disadvantaged shares are weakly increasing in own relationship, $\omega_{i,t}$.*

Proof. For interior shares, differentiating the first-order conditions with respect to ω_i gives

$$S_2(c_i, \boldsymbol{\omega}) = \frac{-P_{12}\left(S\left(c_i, \boldsymbol{\omega}\right), \omega_i\right) + \delta\left[\mathbb{E}_\epsilon V_{i,11}\left(\sigma_i\left(S\left(c_i, \boldsymbol{\omega}\right), \omega_i\right), \sigma_{-i}\left(0, \omega_{-i}\right)\right)\psi\right]}{P_{11}\left(S\left(c, \boldsymbol{\omega}\right), \omega_i\right) - \delta\left[\mathbb{E}_\epsilon V_{i,11}\left(\sigma_i\left(S\left(c_i, \boldsymbol{\omega}\right), \omega_i\right), \sigma_{-i}\left(0, \omega_{-i}\right)\right)\right]}.$$

[15] Observe that when there are unregulated costs that yield corner solutions, then there are unregulated cost draws where the optimal disadvantaged subcontracting function is non-differentiable. In particular, define $\underline{c}$ as the marginal unregulated cost such that $S(\underline{c}, \boldsymbol{\omega}) = \overline{s}$ and $S(\underline{c} + \tilde{\epsilon}, \boldsymbol{\omega}) > \overline{s}$ for an arbitrarily small $\tilde{\epsilon}$. Since $S(\underline{c} + \tilde{\epsilon}, \boldsymbol{\omega}) > S(\underline{c}, \boldsymbol{\omega})$, the increasing property is preserved when $S(\cdot, \cdot)$ is non-differentiable.

The denominator is positive by the second-order conditions, so if

$$-P_{12}\left(S\left(c_i,\boldsymbol{\omega}\right),\omega_i\right)+\delta\left[\mathbb{E}_\epsilon V_{i,11}\left(\sigma_i\left(S\left(c_i,\boldsymbol{\omega}\right),\omega_i\right),\sigma_{-1}\left(0,\omega_{-i}\right)\right)\psi\right]>0$$

(or if P_{12} is sufficiently negative), then $S_2\left(c_i,\boldsymbol{\omega}\right)>0$. For corner shares, take two arbitrary states ω' and ω'', where $\omega'<\omega''$and $S\left(c_i,\omega',\omega_{-i}\right)=\overline{s}$. Then, either $S\left(c_i,\omega',\omega_{-i}\right)=S\left(c_i,\omega'',\omega_{-i}\right)=\overline{s}$ or $S\left(c_i,\omega',\omega_{-i}\right)<S\left(c_i,\omega'',\omega_{-i}\right)$, so the share is weakly increasing in the state. □

B Numerical Details

My numerical algorithm is similar to the one used by Jeziorski and Krasnokutskaya (2016). Ultimately, I aim to approximate the solution to a dynamic auction game with subcontracting and asymmetric bidders. To this end, I need to approximate the value function for the prime contractors as well as bidding and subcontracting in each state. In this section, I will work backward, starting from bidding and subcontracting and then moving into the value function computation. Finally, I will outline my methods for simulation.

B.1 Bidding and Subcontracting

Given a state, ω, and the current value function guess, $\hat{V}(\cdot)$, each prime contractor uses their first-order conditions on subcontracting and bidding to determine their optimal disadvantaged subcontracting share and bids. I use Monte Carlo integration techniques to get an approximation of the expected future value, $\mathbb{E}_\epsilon\hat{V}$. To obtain the optimal subcontracting shares, I solve the first-order conditions in equation (2) on a grid of 50 uniformly spaced unregulated costs. I then interpolate that grid to a grid of $1,000$ unregulated costs using a Hermite polynomial, which is known to have fewer undulations than cubic splines. This property of Hermite polynomials allows me to capture the flat spot in subcontracting that occurs at the left corner solution.

Next, I calculate the effective costs on the fine grid of unregulated costs using equation (3). I then run a Chebyshev regression through the effective cost grid and CDF values of the unregulated costs to get a Chebyshev polynomial estimate of the effective cost CDF, $\hat{F}_{\phi,i}\left(\phi\right)$. I approximate the CDF with a Chebyshev polynomial of degree 50; the PDF approximation for effective costs is then the derivative of the Chebyshev approximation of the CDF.

The estimates of the effective cost distributions allows me to approximate the inverse bid functions. The first step in doing so is determining the equilibrium high bid, $\overline{b}$, which I find by solving equation (6) for b_0

and setting $\bar{b}$ to the minimum of b_0, the upper bound of the reserve price distribution, and the least efficient prime contractor's highest effective cost. When the states differ between prime contractors, the auction is asymmetric, so I then use projection methods in conjunction with the mathematical programming with equilibrium constraints (or MPEC) optimization strategy suggested by Hubbard and Paarsch (2009) in the case of asymmetric auctions. In particular, I assume that the inverse bid functions can be represented by an eighth-degree Chebyshev polynomial of the first kind:

$$\hat{\xi}_i\left(b_l, \boldsymbol{\omega}\right) = \sum_{m=0}^{M} \alpha_{i,m} \mathbb{T}_m \left[x\left(b_l\right)\right], \quad i = 1, 2;\ l = 1, \ldots, L,$$

where $\alpha_{i,m}$ is the m^{th} Chebyshev coefficient for prime contractor i, $x\left(\cdot\right)$ is in the interval $[-1, 1]$, and $\mathbb{T}_m$ is the m^{th}-degree Chebyshev polynomial. Here, the function $x\left(\cdot\right)$ maps the interval of equilibrium bids, $\left[\underline{b}, \bar{b}\right]$, into $[-1, 1]$ and is given by

$$x\left(b_l\right) = \frac{2\left(b_l - \underline{b}\right)}{\bar{b} - \underline{b}} - 1.$$

Following Judd (1998), I evaluate the inverse bid functions on the Chebyshev grid, which contains the zeros of the Chebyshev polynomials.

To then get approximations of the inverse bid functions, my objective is to choose the Chebyshev coefficients and an equilibrium low bid such that the first-order conditions in equation (4) are as close to zero as possible, subject to the boundary conditions in equation (5) as well as monotonicity (bid functions are increasing) and rationality (bidders do not bid below their effective costs) constraints. Let $\boldsymbol{\alpha}$ be a vector that collects all Chebyshev coefficients, $k = 1, \ldots, K$ be a grid to evaluate the constraint points, and $R_i\left(b \mid \boldsymbol{\alpha}\right)$ be the residual given by equation (4)– except using the Chebyshev approximations for the effective cost distributions and inverse bid functions. My objective function is then

$$\min_{\underline{b},\boldsymbol{\alpha}} \quad \sum_{l=1}^{L} R_1 \left(b_l \mid \boldsymbol{\alpha}\right)^2 + R_2 \left(b_l \mid \boldsymbol{\alpha}\right)^2$$

s.t.

$$\hat{\xi}_i \left(\underline{b}, \boldsymbol{\omega}\right) = \underline{\phi}_i,\ i = 1, 2$$

$$\hat{\xi}_i \left(\overline{b}, \boldsymbol{\omega}\right) = \min \left\{\overline{\phi}_i, \overline{b}, \overline{r}\right\},\ i = 1, 2$$

$$\underline{b} < \overline{b}$$

$$\hat{\xi}_i' \left(b_k, \boldsymbol{\omega}\right) > 0,\ i = 1, 2;\ k = 1, \ldots, K \quad \text{(Monotonicity)}$$

$$\hat{\xi}_i \left(b_k, \boldsymbol{\omega}\right) < b_k,\ i = 1, 2;\ k = 1, \ldots, K \quad \text{(Rationality).}$$

In practice, I use a grid of 10 uniformly spaced points on $\left[\underline{b}, \overline{b}\right]$ to evaluate the monotonicity and rationality constraints.

B.2 Value Function

Working backwards to the value function, I use the parametric value function procedure advocated by Judd (1998) to approximate the value function. In particular, I assume that a prime contractor's value function can be represented by a tensor product between two fourth-degree Chebyshev polynomials:

$$V\left(\omega_1, \omega_2\right) \approx \hat{V}\left(\omega_1, \omega_2 \mid \boldsymbol{\zeta}\right) = \sum_{m_1}^{M_1} \sum_{m_2}^{M_2} \zeta_{m_1, m_2} \mathbb{T}_{m_1} \left(x_1 \left(\omega_{1, l_1}\right)\right) \mathbb{T}_{m_2} \left(x_2 \left(\omega_{2, l_2}\right)\right),$$

where

$$x_i \left(\omega_{i, l_i}\right) = \frac{2\left(\omega_{i, l_i} - \underline{\omega}\right)}{\overline{\omega} - \underline{\omega}} - 1.$$

Let the current iteration's value function be denoted by $V_i^{(n)}\left(\boldsymbol{\omega}\right)$ and the next iteration's value function be denoted by $V_i^{(n+1)}(\boldsymbol{\omega})$. To approximate the next iteration's value function I solve

$$
\begin{aligned}
V_i^{(n+1)}(\boldsymbol{\omega}) = \int_{c_i} \max_{b,s} \Bigg\{ & W_i(b) \Bigg[(1 - F_R(b)) \big(b - (1-s) c_i - P(s) \\
& + \delta \mathbb{E}_\epsilon V_i^{(n)} (\sigma_i(s), \sigma_{-i}(0)) \big) + F_R(b) \delta \mathbb{E}_\epsilon V_i^{(n)} (\sigma_i(0), \sigma_{-i}(0)) \Bigg] \\
& + (1 - W_i(b)) \delta \int_{\underline{c}_{-i}}^{\xi_{-i,c}(b,\omega)} (1 - F_R(B_{-i}(c_{-i}))) \mathbb{E}_\epsilon V_i^{(n)} (\sigma_i(0), \sigma_{-i}(S_{-i}(c_{-i}))) \\
& + F_R(B_{-i}(c_{-i})) \mathbb{E}_\epsilon V_i^{(n)} (\sigma_i(0), \sigma_{-i}(0)) \, dF_c(c_{-i}) \Bigg\} dF_c(c_i)
\end{aligned}
$$

given my inverse bid and subcontracting approximations on a two-dimensional grid of state values, where $\xi_{-i,c}(\cdot)$ is the inverse bid function of the competing prime contractor in terms of unregulated costs. In practice, I use a Chebyshev grid with the boundary points in each state. To deal with the potential of multiple equilibria, I use the robust equilibrium selection rule described in Chen et al. (2009), which involves backwards inducting on the value function with a starting value of 0 to find the limit of a finite game. I continue to solve the value function and re-approximate the bids and subcontracting until $\|V^{(n)}(\boldsymbol{\omega}) - V^{(n+1)}(\boldsymbol{\omega})\|_\infty < \epsilon$, where $\|x\|_\infty = \sup |x|$.

B.3 Simulation

Given that the state space is continuous, I need a method to interpolate the bids and subcontracting for all states in Ω for the simulations. I do so by linearly interpolating the approximated bids and subcontracting functions across the missing states. Specifically, I fix a grid of 100 unregulated cost draws and the two-dimensional Chebyshev grid of states from the value function, yielding a three-dimensional grid. Observe that I have estimates of the inverse bid functions in terms of effective costs and the fine grid of equilibrium subcontracting shares for each state combination in the state grid at this point. Hence, I use the fine subcontracting grid to interpolate the subcontracting shares corresponding to the 100 unregulated costs in each state combination to get the shares in each state. I then linearly interpolate the states outside of the state grid. For bids, I find the effective costs corresponding to each unregulated cost for a given state combination and then use the inverse bid function to ascertain the bids. Similar to the subcontracting shares, I linearly interpolate bids outside of the grid.

To simulate an industry, I draw 1,100 unregulated costs for each prime contractor and 1,100 reserve price values. Given the current state, I use the three-dimensional grid to interpolate bids and subcontracting shares and then update the state using the state transition equation. I repeat these simulations 100 times and average them to get the time paths and industry statistics. For the stationary distribution, I assume that the industry reaches the limiting distribution after $1,000$ periods, which is generous given the time paths, and assume that the remaining draws are from the stationary distribution.

C Inverse Bid Function Error

This section evaluates the error associated with approximating the inverse bid functions. In an exact solution, the first-order conditions would be zero at every bid. Therefore, to evaluate the approximation error, I measure how close the first-order conditions are to zero given the approximated inverse bids. Figure 8 plots the residual function, $R_i\left(b \mid \hat{\boldsymbol{\alpha}}\right)$, for each prime contractor given the inverse bid function approximations used to generate figure 3. In general, I find that the first-order conditions are close to zero except at the left boundary. Even the errors for the least accurate approximation, which occurs when $\overline{s} = 0.15$ and $\omega = (1.25, 0.5)$, are comparable to the ones from the third example in Hubbard and Paarsch (2014), which suggests that my approximations are reasonable.

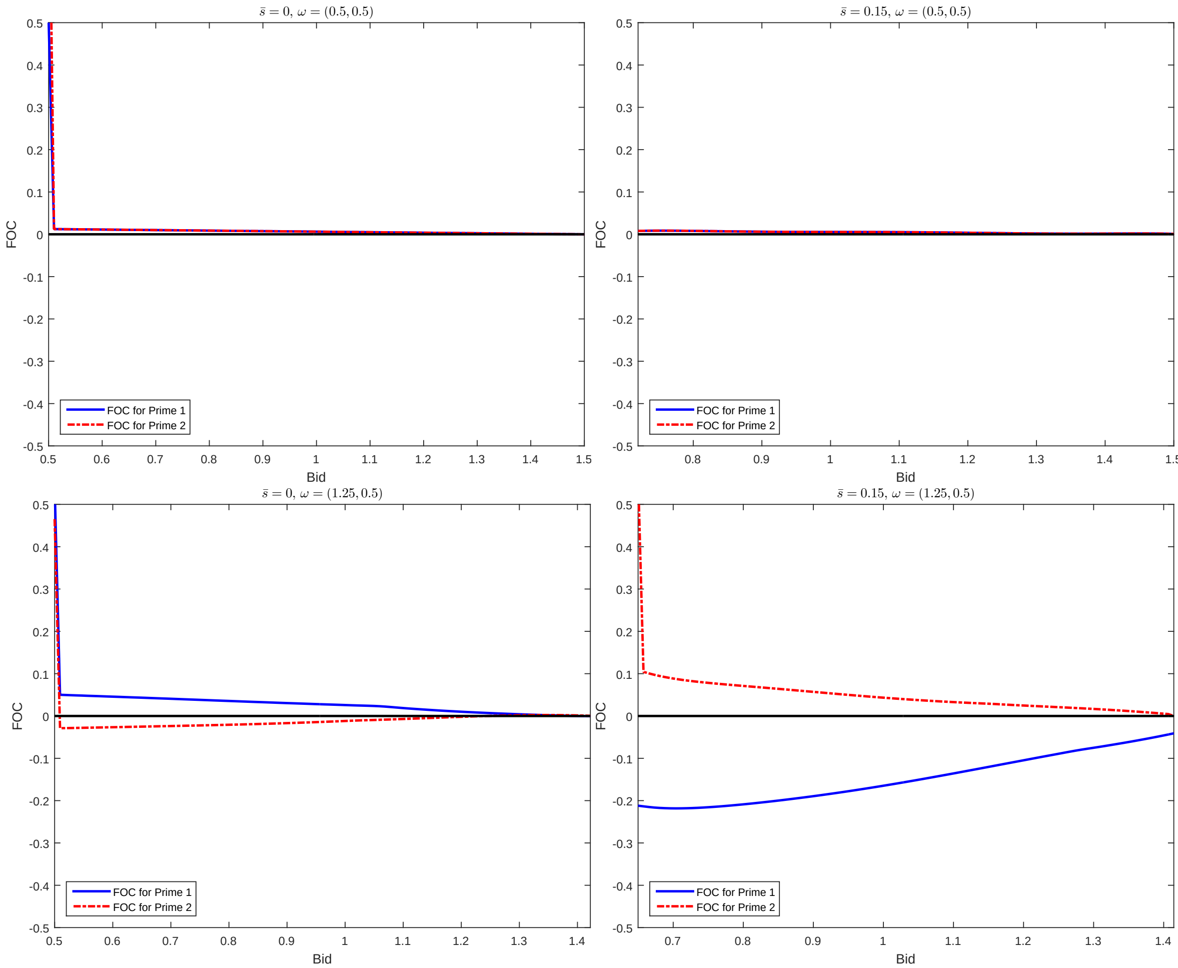

Figure 8: Bid Function Error