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New Essentials of Economic Theory I.
Assumptions, Economic Space and Variables

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Abstract

This paper develops economic theory framework free from assumptions on market equilibrium, utility functions, rational expectations and etc. We describe macroeconomics as a system of economic agents under action of n risks. Economic and financial variables of agents, their expectations and transactions between agents define macroeconomic variables. Agents variables depend on transactions between agents and transactions are performed under agents expectations. Agents expectations are formed by economic variables, transactions, expectations of other agents, by all factors that impact macroeconomic evolution. We describe evolution of macroeconomic variables, transactions and expectations by systems of economic partial differential equations. We develop asset pricing model as a result of equations on transactions and expectations and derive equations that describe price dynamics. To do this we use risk ratings of economic agents as their coordinates on economic space. We approximate description of economic and financial variables, transactions and expectations of numerous separate agents by description of variables, transactions and expectations as density functions on economic space. We take into account flows of economic variables, transactions and expectations induced by motion of separate agents on economic space due to change of agents risk ratings and describe macroeconomic impact of these economic flows. We apply our model to description of business cycles, describe models of wave propagation for disturbances of economic variables and transactions, model asset price fluctuations and argue hidden complexities of classical Black-Scholes-Merton option pricing.

Keywords: Economic Theory, Risk Ratings, Economic Space, Economic Flows, Density Functions

JEL: C00, C02, C10, E00

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1. Introduction

Economic policy and regulation rely heavily on economic theory. Currently general equilibrium theory (GE) (Arrow and Debreu, 1954; Tobin, 1969; Arrow, 1974; Smale, 1976; Kydland and Prescott, 1990; Starr, 2011) and DSGE (Fernández-Villaverde, 2010; Komunjer and Ng, 2011; Negro, et.al., 2013; Farmer, 2017) are ground for building and implementation of macroeconomic and financial management and policy making. Existing flaws and weaknesses of general equilibrium and GSGE may bring economic authorities to unjustified decisions and add excess perturbations and shocks into unsteady global economic and financial processes. Numerous papers study for pro and contra of general equilibrium theory (Hazlitt, 1959; Morgenstern, 1972; Ackerman, 1999; Stiglitz, 2017). A special issue of Oxford Review of Economic Policy on “Rebuilding macroeconomic theory” (Vines and Wills, Eds., 2018a) presents 14 papers of 18 authors those discuss issues of macroeconomic theory: “What new ideas are needed? What needs to be thrown away? What might a new benchmark model look like? Will there be a ‘paradigm shift’?” (Vines and Wills, 2018b).

In this paper we present economic model that entirely differs from mainstream GE approach. There is not much sense to argue pro and contra of our approach to compare with mainstream before we introduce main economic assumptions and formal frame of the model. Thus we avoid any general discussions and comparisons with GE and move forward to our model.

The sketch of our approach is based on common and well-known economic statements. We treat macroeconomics as system of numerous economic agents. Agents have different economic and financial variables and are engaged into various economic and financial transactions with other agents. Agents perform transactions under different expectations. Agents form expectations on base of their forecasts of macroeconomic variables, transactions, expectations of other agents, policy, technology or regulatory changes, climate forecasts and so on. In this paper we develop macroeconomic model that describe relations between three core economic notions - variables, transactions and expectations.

Our paper has three Parts. In Part I Sec.2 we argue main economic assumptions and explain key concepts of our model. Further we introduce formal notions, definitions and economic equations that describe macroeconomic dynamics. In Part I Sec.3 we argue economic agents as simple units on macroeconomic processes and introduce economic space notion as ground of our theory. In Part I Sec.4 we discuss meaning of economic and financial variables and introduce notions of flows of economic variables on economic space and describe them by certain intermediate approximation. We derive equations that describe dynamics of economic

and financial variables and their flows on economic space and argue their economic meaning. In Part II of our paper we study definitions and description of transactions and expectations on economic space and develop asset pricing model as result of equations on transactions and expectations. In Part III we present applications of our model to description of several particular economic problems. We describe approximations of business cycles, describe models of wave propagation for disturbances of different economic variables and transactions, describe asset pricing model and price fluctuations and argue hidden complexities of classical Black-Scholes-Merton option pricing model.

We number equations independently in each Part of the paper and refer (II.4) as equation (4) in Part II. We use bold letters to denote vectors and roman letters – scalars.

2. Main assumptions and economic model

It is obvious that single theory that can explain and describe all possible economic and financial phenomena don't exist. Any theory may give approximations of economic reality only. Thus any economic theory should respond several simple questions: What economic approximations and assumptions are used in the model? What economic variables are described? How these variables are measured? What economic processes are described? Due to common understanding macroeconomics is as set of economic and financial variables that depend on transactions between economic agents and expectations of agents. Thus economic theory should give definitions and describe general frame for interactions between economic variables, transactions and expectations. Let's do it for our model.

We use standard look on macroeconomics as a system of numerous economic agents that interact each other by performing economic and financial transactions made under different expectations of agents. Agent-based models (Tesfatsion and Judd, 2005; Gaffard and Napoletano, 2012) are widely used but our approach has almost nothing common with them. Transactions from sellers with definite trading volume of commodities, assets, service and etc., are supported by trading value as payments from buyers. The ratio of trading value to trading volume determines the price of particular transaction between agents. Transactions are performed under expectations of agents - sellers and buyers. Agents form their expectations as their forecasts of economic variables as return of the deal, inflation, price index, currency rate and etc. Expectations can reflect forecasts of economic growth rate, decline or growth of demand, different expectations of other agents, assumptions of possible impact of policy, regulatory or technology changes and etc. Some macroeconomic variables are determined as sum (without doubling) of corresponding variables of economic agents. For

example, macroeconomic demand, supply, investment, credits are determined as sum of demand, supply, investment and credits of economic agents. Let's call such variables as additive. Other macroeconomic variables are determined as ratio of two additive variables and are non-additive. For example prices are determined as ratio of trading value and trading volume. Inflation, indexes are determined as ratio of prices in different moments of time and are non-additive also. We present these obvious considerations to make simple statement: agents additive variables that define additive macro variables describe all macroeconomic and financial variables.

Now let's argue agents variables those involved into transactions between agents. Any transaction imply that seller transfer certain volume of commodities, assets, service, investment and etc., to buyer. Let's call agents variables involved into transactions between agents as additive variables of type 1. All other variables are determined as functions of additive variables of type 1. For example sum of agents value-added define macroeconomic additive variable – GDP (Fox, et.al., 2014). As well agents value-added variables are not subject of any transaction and are determined as difference between agents aggregate sales and expenditures. Sales and expenditures are result of transactions between agents and their linear function define agents value-added. These easy examples result second simple statement: all agents variables are determined by additive variables of type 1 those involved into transactions between agents. Hence description of transactions between agents permit model all agents variables and hence model all macroeconomic variables. This statement is well-known at least since Leontief's models (Leontief, 1941; 1955; Horowitz and Planting, 2006). Now let's present three issues that distinguish our approach from common economic treatment (Olkhov, 2016a-b; 2017a-d; 2018; 2019a-b):

- I. *We use risk ratings of economic agents as their coordinates on economic space.*
- II. *We approximate description of economic and financial variables, transactions and expectations of numerous separate agents by description of variables, transactions and expectations as density functions on economic space.*
- III. *We take into account flows of economic variables, transactions and expectations induced by motion of separate agents on economic space due to change of agents risk ratings and describe macroeconomic impact of such economic flows.*

Now let's discuss these issues in details.

- I. *We use risk ratings of economic agents as their coordinates on economic space.*

ABM macroeconomic models (Tsfatsion and Judd, 2005; Gaffard and Napoletano, 2012) are well known. We develop macroeconomic model using relations between economic agents

in a way different from ABM. Our main issue concern assessments of agents risk ratings. International rating agencies as S&P, Moody's, Fitch (Metz and Cantor, 2007; S&P, 2014; Fitch, 2018) for decades provide risk assessments for major banks, corporations, securities and etc., and deliver distributions of biggest banks by their risk ratings (Moody's, 2018; South and Gurwitz, 2018). These assessments are basis for investment expectations of biggest hedge funds, investors, traders etc. According to current risk assessment methodologies (Altman, 2010; Moody's, 2010; S&P, 2016; Fitch, 2018) risk ratings take values of risk grades like AAA, AA, BB, C etc. Different rating agencies use different risk assessment methodologies and risk grades notions differs slightly.

Let's outline that risk grades AAA, AA, BB, C can be treated as points x_1, \dots, x_N of space that we call further as economic space. Risk assessment methodology use available economic statistics and determine number N of risk points. Let's propose that economic statistics and econometrics can provide sufficient data to assess risk ratings for all economic agents and for all risks that may hit macroeconomic evolution and growth. Thus we assume that rating agencies will be able to estimate risk ratings as to large corporations and banks as to small companies, firms and even households. Let's assume that risk assessment can be provided to all agents of entire economics. Now let's assume that risk assessment methodologies can be developed so that define continuous spectrum of risk grades on space R . Risk methodology always can take continuous risk grades as $[0,1]$ with point 0 as most secure and 1 as most risky grades. A lot of different risks can disturb macroeconomic processes (McNeil, Frey and Embrechts, 2005;). Assessments of single risk, like credit risk, distributes agents over range $[0,1]$ of 1-dimensional economic space R . Assessments of two or three risks, like credit, exchange rate and liquidity for example, distribute economic agents over unit square or cube. For given configuration of n macroeconomic risks, assessments of agents risk rating distribute agents by their risk coordinates $\mathbf{x}=(x_1, \dots, x_n)$ over economic domain

$$0 \leq x_i \leq 1, i = 1, \dots, n \quad (1.1)$$

of n -dimensional economic space R^n . Distribution of economic agents by their risk coordinates $\mathbf{x}=(x_1, \dots, x_n)$ over economic domain (1.1) mean that all economic and financial variables of agents are also distributed on economic domain (1.1). Aggregation of similar variables for agents with coordinates near point $\mathbf{x}=(x_1, \dots, x_n)$ of (1.1) define economic variables as functions of \mathbf{x} . Aggregations of similar transactions between agents with coordinates \mathbf{x} and \mathbf{y} determine transactions as functions of \mathbf{x} and \mathbf{y} on economic space. As we show below this helps describe dynamics of macroeconomic variables, transactions and expectations by partial differential equations on economic space.

Let's repeat our main assumptions:

1. We assume that economic statistics can provide sufficient data for risk assessment of almost all economic agents for wide range of macroeconomic risks. That assumption permits distribute economic agents by their risk ratings as coordinates on economic space.
 2. We propose that risk assessment methodologies can be developed so that will define continuous risk grades $[0,1]$ on R for all macroeconomic risks. Ratings of n risks define risk coordinates $\mathbf{x}=(x_1, \dots, x_n)$ on economic domain (1.1) of n -dimensional economic space R^n .
- II. *We approximate description of economic and financial variables, transactions and expectations of numerous separate agents by description of economic and financial variables, transactions and expectations as density functions on economic space.*

Transition from description of economic properties, like variables, transactions and expectations, of separate agents to same economic properties as density functions on economic space has clear economic meaning. Risk assessment distributes agents by their ratings as coordinates on economic domain (1.1). Description of variables and transactions of numerous separate agents requires a lot of econometric data. We propose approximation that gives more rough description but requires significantly less economic data. To establish such approximation let's aggregate variables, transactions and expectations of agents with risk coordinates inside small volume and than average them. To do that let's chose economic space scale d and time scale Δ . For n -dimensional economic space R^n let's take unit volume $dV=d^n$ near point \mathbf{x} of (1.1) and assume that space scales $d < 1$ are small to compare with scales of economic domain (1.1) but a lot of economic agents have risk coordinates inside this unit volume dV near point \mathbf{x} . The similar requirements concern time scale: Δ should be small to compare with time scale of the problem under consideration but a lot transactions should be performed during Δ . For example, the number of economic agents in economics with population around 10^8 - 10^9 can be estimated as 10^8 - 10^9 . Thus space scale $d \sim 10^{-2}$ on 2-dimensional economic space defines unit volume $dV \sim 10^{-4}$ with estimate 10^4 - 10^5 agents inside it. Description of macroeconomics during one quarter or year with 1 transaction between agents per second mean that there are around $6 \cdot 10^5$ transactions per $\Delta = 1$ week. As example let's consider Credits provided by agents with coordinates inside dV near point \mathbf{x} and average it during $\Delta = 1$ week. Let's take that $C(t, \mathbf{x})$ equals sum of credits over volume dV and averaged during time Δ . Function $C(t, \mathbf{x})$ has meaning of density of credits provided by agents from point \mathbf{x} at moment t . Indeed, integral of $C(t, \mathbf{x})$ by $d\mathbf{x}$ over economic domain

equals total credits provided by all economic agents in economics at moment t . Averaging over time Δ reduce high frequency fluctuations of the sum of credits and makes this variable smooth. Introduction of space scale d and time scale Δ reduce accuracy of the model approximation. If one chose space scale $d=1$ then volume dV will be equal economic domain and aggregation of credits provided by agents inside economic domain equals all credits provided in macroeconomics. Thus introduction of scales $d \ll 1$ establishes economic approximation that is intermediate between precise description of variables of numerous separate economic agents and rough macroeconomic approximation based on aggregation of variables of all agents in economy. Below we define economic space density functions for economic and financial variables, transactions and expectations. Nevertheless expectations are not additive variables, we show in Part II how apply aggregation procedure to obtain correct form for density functions of expectations. Description of density functions of economic variables, transactions and expectations require significantly less economic data then same description with accuracy of each agent and hence simplifies the models. The same time descriptions of mutual relations between density functions of economic variables, transactions and expectations are much more informative then modeling relations between macroeconomic variables as functions of time only. The most important factor that impact evolution of density functions of variables, transactions and expectations is determined by aggregative flows of variables, transactions and expectations induced by motion of agents on economic space. Such economic flows are results of motion of agents on economic space due to change of their risk rating.

III. We take into account flows of economic variables, transactions and expectations induced by motion of separate agents on economic space due to change of agents risk ratings. We describe macroeconomic impact of such economic flows.

Change of agents risk ratings due to their economic activity, variation of economic environment, action of different risk factors and other reasons cause change of agents risk coordinates on economic space. Such change means that agents move on economic space with certain speed v . Motion of agent with speed v indicates that agents carries its economic and financial variables, expectations and transactions. For example if agent provides credits C and moves with speed v then its carries flow P_C of credits as $P_C=Cv$. Flows of variables, expectations and transactions carried by agents due to change of their risk ratings have important impact on macroeconomic evolution. To describe action of these flows on macroeconomics let's develop approximation similar to one we use to describe densities functions of variables, expectations and transactions. As we show below, aggregations of

flows of separate agents define densities of economic flows of variables, transactions and expectations. Motion of different flows of variables, expectations and transactions have certain parallels to flows of liquids but all properties of economic flows are completely different from hydrodynamics. Numerous flows of economic and financial variables, expectations and transactions induce on economic domain (1.1) a great variety of mutual interactions and economic effects.

Now let's argue derivation of equations that should govern density functions of variables, transactions and expectations and their flows. These equations have similar form and we explain their derivation for credit density function $C(t, \mathbf{x})$ as example. Credit density function $C(t, \mathbf{x})$ aggregates credits of agents with coordinates inside small volume dV at point \mathbf{x} . Each agent moves on economic space with some velocity \mathbf{v} due to change of its risk ratings. This motion of agents induces aggregate credit flows $\mathbf{P}_C(t, \mathbf{x}) = C(t, \mathbf{x})\mathbf{v}(t, \mathbf{x})$. Function $\mathbf{v}(t, \mathbf{x})$ describes velocity of flow of credit density $C(t, \mathbf{x})$. To describe change of credit density function $C(t, \mathbf{x})$ during time dt in a small volume dV on economic space let's take into account two factors of such change. The first factor describes change of $C(t, \mathbf{x})$ due to change of agents credits in time dt in a small volume dV . That can be presented as

$$\int dV \frac{\partial}{\partial t} C(t, \mathbf{x})$$

The second factor that impact change of credit density $C(t, \mathbf{x})$ is determined by credit flows $\mathbf{P}_C = C\mathbf{v}$ of agents that during time dt may flow in or flow out of small volume dV . Agents that flow in volume dV with credit flow $\mathbf{P}_C = C\mathbf{v}$ increase credit density function $C(t, \mathbf{x})$ and agents that flow out volume dV with credit flow $\mathbf{P}_C = C\mathbf{v}$ decrease credit density function $C(t, \mathbf{x})$. Balance of aggregated $\mathbf{P}_C(t, \mathbf{x}) = C(t, \mathbf{x})\mathbf{v}(t, \mathbf{x})$ credit flows in and credit flows out takes form of integral of credit flows $\mathbf{P}_C(t, \mathbf{x}) = C(t, \mathbf{x})\mathbf{v}(t, \mathbf{x})$ over the surface of small volume dV :

$$\oint ds \mathbf{P}_C(t, \mathbf{x}) = \oint ds C(t, \mathbf{x}) \mathbf{v}(t, \mathbf{x})$$

Due to well-known divergence theorem (Gauss' Theorem) (Strauss 2008, p.179), surface integral of the flows equals volume integral of the flows divergence. Thus balance of credit flows equals integral of the divergence of flow over small volume dV :

$$\oint ds C(t, \mathbf{x}) \mathbf{v}(t, \mathbf{x}) = \int dV \nabla \cdot (C(t, \mathbf{x}) \mathbf{v}(t, \mathbf{x})) \quad (1.2)$$

Hence total change of credit density function during time dt in a small volume dV equals:

$$\int dV \left[\frac{\partial}{\partial t} C(t, \mathbf{x}) + \nabla \cdot (C(t, \mathbf{x}) \mathbf{v}(t, \mathbf{x})) \right]$$

As small volume dV is arbitrary one can take equations on density functions as:

$$\frac{\partial}{\partial t} C(t, \mathbf{x}) + \nabla \cdot (C(t, \mathbf{x}) \mathbf{v}(t, \mathbf{x})) = F_C(t, \mathbf{x}) \quad (1.3)$$

Function $F_C(t, \mathbf{x})$ in the right side (1.3) describes action of any factors like variables, transactions and expectations and their flows on credit density function $C(t, \mathbf{x})$. Equation (1.3) depends on flow $\mathbf{P}_C(t, \mathbf{x}) = C(t, \mathbf{x}) \mathbf{v}(t, \mathbf{x})$ and hence one should derive equation on this flow. Completely same considerations as we argue above cause equations on flows $\mathbf{P}_C(t, \mathbf{x}) = C(t, \mathbf{x}) \mathbf{v}(t, \mathbf{x})$ as:

$$\frac{\partial}{\partial t} \mathbf{P}_C(t, \mathbf{x}) + \nabla \cdot (\mathbf{P}_C(t, \mathbf{x}) \mathbf{v}(t, \mathbf{x})) = \mathbf{G}_C(t, \mathbf{x}) \quad (1.4)$$

Function $\mathbf{G}_C(t, \mathbf{x})$ describes action of any factors like variables, transactions and expectations and their flows on credit flows $\mathbf{P}_C(t, \mathbf{x})$. Let's underline that equations (1.3; 1.4) define "simple" relations for macroeconomic variables as functions of time only. Indeed, integral by $d\mathbf{x}$ of credit density $C(t, \mathbf{x})$ over economic domain (1.1) equals macroeconomic credits $C(t)$ issued by all agents:

$$C(t) = \int d\mathbf{x} C(t, \mathbf{x}) \quad (1.5)$$

Integral by $d\mathbf{x}$ for equations (1.3) over economic domain (1.1) equals

$$\frac{d}{dt} C(t) + \int d\mathbf{x} \nabla \cdot (C(t, \mathbf{x}) \mathbf{v}(t, \mathbf{x})) = \int d\mathbf{x} F_C(t, \mathbf{x}) = F_C(t) \quad (1.6)$$

Due to (1.2) integral in left side (1.6) equals zero as no in- or out- flows exist outside surface of economic domain (1.1) and no economic agents exist outside economic domain (1.1). Thus (1.6) takes simple form of ordinary differential equation:

$$\frac{d}{dt} C(t) = F_C(t) \quad (1.7)$$

The problems of (1.7) are hidden by function $F_C(t)$ determined by integral in the right side of (1.6). Function $F_C(t, \mathbf{x})$ may depend on other variables, transactions, expectations and their flows and integral in (1.6) may define $F_C(t)$ as very complicated function. Thus time evolution of macroeconomic variables like macro credits $C(t)$ may depend on properties of hidden dynamics of variables, transactions and expectations and their flows on economic space. Integral by $d\mathbf{x}$ for equations (1.4) over economic domain (1.1) define ordinary differential equation on new macroeconomic variables $\mathbf{P}_C(t)$:

$$\mathbf{P}_C(t) = \int d\mathbf{x} \mathbf{P}_C(t, \mathbf{x}) = C(t) \mathbf{v}(t) \quad (1.8)$$

$$\frac{d}{dt} \mathbf{P}_C(t) = \int d\mathbf{x} \mathbf{G}_C(t, \mathbf{x}) = \mathbf{G}_C(t) \quad (1.9)$$

Integral (1.8) define macroeconomic flows $\mathbf{P}_C(t)$ of credits $C(t)$ (1.5) with velocity $\mathbf{v}(t)$ and equation (1.9) describes evolution of macroeconomic credit flows $\mathbf{P}_C(t)$ determined by function $\mathbf{G}_C(t)$ in the right side of (1.9). Similar equations are valid to macroeconomic flows of other additive variables as demand and supply, investment and GDP and etc. Economic

meaning of equations (1.9) is following. Velocity $v(t)$ of macroeconomic flow $P_C(t)$ of credits $C(t)$ describes motion on economic domain (1.1) that is bounded along each risk axes by most secure and most risky grades $[0,1]$. Thus for each axis motion from secure to risky direction with velocity $v(t)$ should change by opposite motion from risky to secure area of (1.1). Thus velocity $v(t)$ and macroeconomic flow $P_C(t)$ of credits $C(t)$ should fluctuate in time and such fluctuations describe credit cycles of macroeconomics. Similar fluctuations of flows model business cycles of GDP, investment and etc. Description of correlations between cycles of different macro variables and particular models that define forms of functions $F_C(t)$ and $G_C(t)$ should be studied for each economic case. In Part III we present one simple model of business cycles caused by interactions between transactions.

In Part II we show that equations on transactions have form similar to (1.3; 1.4) taking into account that transactions density functions depend on two coordinates x and y . In Part II we argue that expectations of agents can't be treated as additive variables and derivation of equations on aggregated expectations requires further considerations. We propose that economic value or economic importance of agents expectations should be taken proportional to value of transactions approved by this particular expectation. In Part II we introduce additive factors that we call – expected transactions – that are proportional to product of transactions and expectations. Our approach permits define density functions of expected transactions and flows of expected transactions. Further we derive equations on expected transactions and their flows that have form similar to (1.3; 1.4). That permits derive definitions and equations for density functions of expectations and their flows. Further in Part II we show that considerations similar to those we use for description of expectations can be applied for description of prices as densities functions on economic space and we derive definitions and equations for price density functions and their flows. That allows model dynamics of asset pricing determined by corresponding transactions. Well-known that asset pricing is one of the most important problems of economics and finance and papers by (Cochrane and Hansen, 1992; Cochrane and Culp, 2003; Hansen, 2013; Campbell, 2014; Fama, 2014; Cochrane, 2017) refer only few but important studies on asset pricing. These studies argue models that determine “correct” price of assets. In our paper we don't argue “correct” price and don't study why asset price should take certain value. We describe prices as results of transactions performed by agents in economy. In Part II we study different definitions of prices caused by different aggregations of transactions and show how economic equations on transactions, expectations and their flows determine equations on prices caused by transactions.

Let's argue some consequence of our macroeconomic approximations. As we mention above equations similar to (1.3; 1.4) describe density functions and flows of numerous economic and financial variables, transactions and expectations. Thus equations (1.3; 1.4) define macroeconomic approximations for each selected set of variables, transactions and expectations. Let's take a model that describes macroeconomics by set of k different transactions. As such transactions one can study for example credit transactions, investment, buy-sell transactions and etc. Each type k of transactions defines change of variables of sellers and buyers. For example credit transaction change value of credits provided by Creditor (seller) and amount of loans received by Borrowers (buyers). Hence each type of transactions can change only two additive variables of type 1 – one for seller and one for buyer and their prices. Thus k types of transactions can change $2k$ additive variables of type 1 and their prices. Transactions of each type can be performed under different expectations. Let's assume that k types of selected transactions are performed under W expectations. To develop self-consistent macroeconomic model that describe $2k$ additive variables of type 1 determined by k types of selected transactions one should assume that all W expectations are formed by endogenous $2k$ additive variables, k selected transactions and their flows. If part of W expectations can depend on exogenous variables, transactions, expectations and their flows or exogenous shocks and etc., then one describes macroeconomic model in presence of exogenous factors. Expectations approve transactions and thus impact change of economic and financial variables. Hence only expectations transfer impact of exogenous variables, transactions or shocks on macroeconomic evolution, transactions and variables.

Importance of expectations is not reduced by their role as transfer of exogenous shocks on macroeconomic dynamics. As we argue above expectations can depend on economic flows of variables, transactions and other expectations. Dependence of expectations on economic flows makes them key factors that determine impact of economic flows on macroeconomics. Dynamics of economic flows like credit flows $P_C(t, \mathbf{x}) = C(t, \mathbf{x})\mathbf{v}(t, \mathbf{x})$, flows of variables, transactions and expectations and their mutual interactions on economic domain (1.1) establish very complex picture. For example economic flows on economic domain (1.1) generate business cycles that describe slow oscillations of macroeconomic variables. On the other hand perturbations of economic flows cause wave propagation of disturbances and shocks of economic variables, transactions and expectations those induce fast oscillations of economic parameters. Consistent macroeconomic model on base of economic equations (1.3; 1.4) that describe dynamics of $2k$ variables that depend on k transactions under action of W expectations establish a really tough problem. Reductions of complete system of equations

permit study various approximations of macroeconomic evolution. In Part III we study approximations of equations (1.3; 1.4) that describe “simplified” model interactions between two variables, between two transactions, between transactions and expectations. Such approximations help describe model examples of business cycles and different examples of wave propagation of disturbances of economic variables and transactions inside economic domain and on surface of economic domain (1.1). Similar approximations permit develop model of price fluctuations induced by interactions between transactions and numerous expectations.

In Sec. 3 and 4 of current Part I we present formal treatment of economic space, definitions of economic variable density functions and their flows and derive of equations on density functions and their flows. In Part II we define transactions density functions and their flows and derive equations. Further we argue description of expectations and introduce notion of expected transactions density functions and their flows. We derive equations on expected transactions and show how they cause equations on expectations. Further in Part II we explain how equations on transactions generate equations on price density functions and derive these equations. In Part III we describe economic systems under different approximations of our general scheme. That permits in a unified approach describe wide range of economic processes: starting with description of business cycles and economic wave propagation till modeling asset price fluctuations and description of hidden problems for classical treatment of option pricing via Black-Scholes-Merton model.

3. Economic space and economic agents

Notion of economic agents is a basic economic term (Giovannini, 2008): “One of the fundamental characteristics of activities defined as economic processes is that they involve relations between various agents. The definition of economic agent is therefore absolutely fundamental in determining the nature of the economic processes: economic agent refers to a person or legal entity that plays an active role in an economic process”. There are a lot of studies of agent-based economic and financial models (Tsfatsion and Judd, 2005; Gaffard and Napoletano, 2012). Our approach has nearly nothing with them. We regard agent as economic unit that has a lot of economic or financial variables like asset and debts, investment and credits, supply and demand and etc. Economic and financial variables can be additive or non-additive. Additive variables are investment, credits, volume and cost of commodities and etc. Non-additive variables are prices, bank rates, inflation, indexes and etc. Non-additive variables can be presented as ratio of two additive variables or ratio of non-

additive variables. For example price of commodity equals ratio of cost and volume of commodities purchased by particular transaction. Inflation index during time term $[0, T]$ equals ratio of prices at moment T and at moment 0 . All additive macroeconomic or financial variables like GDP, investment, credits, supply and demand and etc., are composed as sum of agents variables. For example macroeconomic investment equals sum of investment (without doubling) of all agents of the entire economics. Non-additive macroeconomic variables like inflation, economic growth are determined as ratios of macroeconomic additive variables. Thus description of agents additive economic and financial variables determine evolution of all macroeconomic and financial variables. Let's introduce economic space notion and explain how macroeconomic additive variables can be described by additive variables of economic agents.

To define economic space let's use well-known economic tool – risk ratings. Risk management and risk assessment (Horcher, 2005; Skoglund and Chen, 2015) during at least 50 years establish well-developed sector of economics. Risk assessment is a core tool for banking and corporate management and is necessary issue for any investment and financial markets operations. Top international rating agencies provide risk assessments for major banks, financial securities and etc. Risk ratings of particular agent like bank or corporation or ratings of their securities impact on decisions of financial markets traders. There are many risks that affect macroeconomics and finance like credit, inflation, market risks and etc. We don't argue particular risks but treat any risks as factors that may affect economic and financial properties of agents and hence entire economics.

Let's treat assessments of risk ratings of agents as coordinates of agents alike to coordinates of physical particles. Let's note space that imbed agents by their risk coordinates as economic space (Olkhov, 2016a-b; 2017a-d). Current risk methodologies measure risk ratings by risk grades (Wilier, 1901; McNeil, Frey and Embrechts, 2005; Metz and Cantor, 2007; SEC, 2012; S&P, 2014) that have notations as *AAA*, *AA*, *BB*, *C* etc. Let's take current risk grades as points x_1, \dots, x_n of economic space. Such economic space imbed economic agents by their risk ratings \mathbf{x} . Risk grades of single risk establish 1-dimensional economic space. Grades of two or three risks establish 2 or 3 dimensional economic space. Number of risk grades depends on risk assessment methodology. Let's assume that one can extend risk methodology so that it adopts continuous risk grades. Then n -dimensional economic space that describe action of n risks can be treated as R^n . Let's propose that economic statistics provide sufficient data for risk assessments of all economic agents of the macroeconomics. Let's state that risk ratings take continuous values between most secure grade equals 0 and most risky grade equals 1 .

Partition of agents by their risk ratings for n risks define economic domain (1.1) on economic space R^n . All agents have their risk coordinates inside economic domain (1.1). Partition of all agents on economic domain (1.1) establishes distribution of agents economic and financial variables over economic domain. Distribution of agents on economic domain (1.1) defines distribution of agents economic and financial variables. Change of agents risk ratings due to their economic activity, market dynamics, other endogenous or exogenous shocks induce evolution of agents variables and thus change macroeconomic variables. In the next section we show how usage of risk ratings as coordinates of economic agents describes evolution of macroeconomic and financial variables.

4. Economic variables on economic space

In this Section we describe economic and financial variables on economic space. Description of economic dynamics of numerous separate agents and their economic and financial variables is too complex problem. Uncertainty of agents risk coordinates and of their economic and financial variables makes such description too ambiguity. To simplify macroeconomic model and develop more sustainable and reasonable model let's rougher our description. The main idea is simple: let's rougher description of separate agents and their variables and describe same variables as aggregates of variables of agents with coordinates at point \mathbf{x} of economic space.

Let's regard macroeconomics as system of numerous agents on n -dimensional economic domain (1.1). Let's state that agents at moment t have risk ratings coordinates $\mathbf{x}=(x_1, \dots, x_n)$ and velocities $\mathbf{v}=(v_1, \dots, v_n)$. Velocities $\mathbf{v}=(v_1, \dots, v_n)$ describe change of agents risk coordinates. Let's assume that a unit volume $dV(\mathbf{x})$ at point \mathbf{x} on economic space contains many agents but scales d_i (2) of a unit volume $dV(\mathbf{x})$ are small to compare with scales of domain (1.1)

$$d_i \ll 1, i = 1, \dots, n ; \quad dV = \prod_{i=1, \dots, n} d_i \quad (2)$$

Let's regard only additive variables of agents and assume that economic statistics able select "independent" agents. Let's call agents as "independent" if sum of their additive variables equals same variable of the entire group. For example sum of Credits of k agents equals Credits of the group of these k agents. Additive variables are Credits, Investment, Asset and etc. There are a lot of non-additive variables as bank rates, inflation, prices and etc. Non-additive variables are defined as ratio of additive variables or ratio of non-additive variables. For example non-additive variable - price of transaction equals the ratio of cost and volume of this deal. Hence all economic variables are determined by additive variables only. Let's show how description of additive variables models evolution of macroeconomic variables.

Let's define additive economic variable $A(t, \mathbf{x})$ at point \mathbf{x} as sum of variables $A_i(t, \mathbf{x})$ of agents i with coordinates in a unit volume $dV(\mathbf{x})$ (2) and then average it during term Δ as:

$$A(t, \mathbf{x}) = \sum_{i \in dV(\mathbf{x}); \Delta} A_i(t, \mathbf{x}) \quad (3)$$

$$\sum_{i \in dV(\mathbf{x}); \Delta} A_i(t, \mathbf{x}) = \frac{1}{\Delta} \int_t^{t+\Delta} d\tau \sum_{i \in dV(\mathbf{x})} A_i(\tau, \mathbf{x}) \quad (4)$$

We use $i \in dV(\mathbf{x})$ to denote that risk coordinates \mathbf{x} of agent i belong to unit volume $dV(\mathbf{x})$. For brevity we use left hand sum (4) to denote averaging during time Δ in a unit volume $dV(\mathbf{x})$. We repeat meaning of space scales d_i and time scale Δ given in Sec.2. Scales $d_i \ll 1$ of volume $dV(\mathbf{x})$ are small to compare with scales of economic domain (1.1) but volume $dV(\mathbf{x})$ contains a lot of economic agents. Scale Δ is small to compare with time scales of the problem under consideration but a lot of economic and financial transactions between agents are performed during time Δ . Time averaging smooth changes of variables under numerous transactions during time Δ . We aggregate values of variables of numerous agents with risk coordinates inside volume $dV(\mathbf{x})$, smooth their changes during time Δ and denote result as density function of variable at point \mathbf{x} . Density function $A(t, \mathbf{x})$ describes economic variable at point \mathbf{x} on economic domain (1.1). For example let's take $A_i(t, \mathbf{x})$ as Credits of agent i . Then density of Credits $A(t, \mathbf{x})$ describe sum of Credits issued by all agents with coordinates \mathbf{x} inside unit volume $dV(\mathbf{x})$ and averaged during time Δ . Total value of macroeconomic variable $A(t)$ is determined by integral (5) over economic domain (1.1):

$$A(t) = \int d\mathbf{x} A(t, \mathbf{x}) \quad (5)$$

Function $A(t)$ equals sum (without doubling) of variables $A_i(t, \mathbf{x})$ of all agents i of entire economics averaged during time Δ . For example Credits $A(t)$ issued in macroeconomics equal integral of Credits $A(t, \mathbf{x})$ by $d\mathbf{x}$ over economic domain (1.1). Thus function $A(t, \mathbf{x})$ (3) can be treated as economic density of variable $A(t)$ (5) on economic space. Now let's describe evolution of aggregated economic and financial densities $A(t, \mathbf{x})$ defined by relations (3). Economic density $A(t, \mathbf{x})$ (3) is composed by variables $A_i(t, \mathbf{x})$ of agents i . Risk ratings of each agent can change during time Δ . Such time Δ can be equal a day, a week, a quarter etc. Let's describe change of agent's i risk coordinates on economic space during time Δ by velocity $\mathbf{v}_i = (v_1, \dots, v_n)$. Thus each agent i with economic variable $A_i(t, \mathbf{x})$ carries flow of this economic variable with velocity $\mathbf{v}_i = (v_1, \dots, v_n)$. Flow $\mathbf{p}_{iA}(t, \mathbf{x})$ of economic variable $A_i(t, \mathbf{x})$ of agent i with velocity $\mathbf{v}_i = (v_1, \dots, v_n)$ equals:

$$\mathbf{p}_{iA}(t, \mathbf{x}) = A_i(t, \mathbf{x}) \mathbf{v}_i(t, \mathbf{x}) \quad (6)$$

Different agents induce different flows of economic variable A in different directions with different velocities. Let's aggregate flows of variable $A_i(t, \mathbf{x})$ in the direction of velocity \mathbf{v}_i of

agents i with coordinates in a unit volume $dV(\mathbf{x})$ (2) and then average this flow during time Δ similar to relations (3, 4). Let's define aggregated flow $\mathbf{P}_A(t, \mathbf{x})$ of variable $A(t, \mathbf{x})$ as:

$$\mathbf{P}_A(t, \mathbf{x}) = \sum_{i \in dV(\mathbf{x}); \Delta} A_i(t, \mathbf{x}) \mathbf{v}_i(t, \mathbf{x}) \quad (7)$$

Similar to (5) integral of (7) by $d\mathbf{x}$ over economic domain (1.1) define macro flow $\mathbf{P}_A(t)$ of variable $A(t)$ as:

$$\mathbf{P}_A(t) = \int d\mathbf{x} \mathbf{P}_A(t, \mathbf{x}) \quad (8)$$

Flow $\mathbf{P}_A(t, \mathbf{x})$ (7) of variable $A(t, \mathbf{x})$ (3) defines aggregated velocity $\mathbf{v}_A(t, \mathbf{x})$ of economic variable $A(t, \mathbf{x})$ that adjust the flow (7) as:

$$\mathbf{P}_A(t, \mathbf{x}) = A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x}) \quad (9)$$

Thus (9) describes flow $\mathbf{P}_A(t, \mathbf{x})$ of economic variable $A(t, \mathbf{x})$ with velocity $\mathbf{v}_A(t, \mathbf{x})$. Relations (5) and (8) define macro velocity $\mathbf{v}_A(t)$ on domain (1.1) of macro variable $A(t)$ as:

$$\mathbf{P}_A(t) = A(t) \mathbf{v}_A(t) \quad (10)$$

Let's mention that due to (3; 5; 7-9 and 10) velocity $\mathbf{v}_A(t)$ is not equal to integral of velocity $\mathbf{v}_A(t, \mathbf{x})$ over economic domain (1.1). Aggregation of agents economic variables defines corresponding economic densities and velocities. Due to relations (3-10) different economic variables A define different economic flows $\mathbf{P}_A(t, \mathbf{x})$ and different velocities $\mathbf{v}_A(t, \mathbf{x})$. In other words – motion of different economic variables $A(t, \mathbf{x})$ on economic space has different velocities $\mathbf{v}_A(t, \mathbf{x})$. For example flow $\mathbf{P}_C(t, \mathbf{x})$ of Credits $C(t, \mathbf{x})$ has velocity $\mathbf{v}_C(t, \mathbf{x})$ different from velocity $\mathbf{v}_L(t, \mathbf{x})$ that describe flow $\mathbf{P}_L(t, \mathbf{x})$ of Loans $L(t, \mathbf{x})$ or velocity $\mathbf{v}_I(t, \mathbf{x})$ that describe flow $\mathbf{P}_I(t, \mathbf{x})$ of Investment $I(t, \mathbf{x})$ on economic space. Macroeconomic models should describe dynamics and mutual interactions between numerous economic and financial variables and their flows. Properties of economic flows are completely different from properties of any physical flows.

To model dynamics of economic variables $A(t, \mathbf{x})$ and flows $\mathbf{P}_A(t, \mathbf{x})$ let's describe their change in small unit volume dV . There are two factors that change $A(t, \mathbf{x})$ in a unit volume dV . The first factor describes change of $A(t, \mathbf{x})$ on a unit volume dV in time and can be presented by time derivative as:

$$\int d\mathbf{x} \frac{\partial}{\partial t} A(t, \mathbf{x}) \quad (11)$$

The second factor describe change of variable $A(t, \mathbf{x})$ due to flows $\mathbf{P}_A(t, \mathbf{x})$. Indeed, amount of economic density $A(t, \mathbf{x})$ in a unit volume dV during time dt can grow up or decrease due to in- or out- flows $\mathbf{P}_A(t, \mathbf{x})$. If there are more in-flows $\mathbf{P}_A(t, \mathbf{x})$ than out-flows then amount of $A(t, \mathbf{x})$ will increase in a volume dV . To calculate balance of in- and out-flows let's take integral of flow $\mathbf{P}_A(t, \mathbf{x})$ over the surface of a unit volume dV :

$$\oint ds \mathbf{P}_A(t, \mathbf{x}) = \oint ds A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x}) \quad (12)$$

Due to divergence theorem (Strauss 2008, p.179) surface integral of flux $A(t, \mathbf{x})\mathbf{v}_A(t, \mathbf{x})$ through surface equals volume integral of divergence of flow $A(t, \mathbf{x})\mathbf{v}_A(t, \mathbf{x})$

$$\oint ds A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x}) = \int d\mathbf{x} \nabla \cdot (A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) \quad (13)$$

Relations (11,13) give total change of variable $A(t, \mathbf{x})$ in a unit volume dV :

$$\int d\mathbf{x} \left[\frac{\partial}{\partial t} A(t, \mathbf{x}) + \nabla \cdot (A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) \right]$$

As unit volume dV is arbitrary one can take equations on economic density $A(t, \mathbf{x})$ as

$$D_A A(t, \mathbf{x}) = \frac{\partial}{\partial t} A(t, \mathbf{x}) + \nabla \cdot (A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) = F_A(t, \mathbf{x}) \quad (14)$$

Function $F_A(t, \mathbf{x})$ in right side (14) describe factors that impact change of economic density $A(t, \mathbf{x})$ as: other variables, transactions, expectations and etc. Equations like (14) are reproduced in any treatise on physics of fluids (Batchelor, 1967; Resibois and De Leener, 1977; Landau and Lifshitz, 1987) and are valid for any additive economic or financial variables defined as aggregates of agents variables on economic space similar to (3). Due to (13) integral of divergence of flow $\nabla \cdot (A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x}))$ over economic domain (1.1) equals integral over surface of economic domain (1.1) and hence equals zero as no economic or financial flows exist outside of (1.1):

$$\int d\mathbf{x} \nabla \cdot (A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) = \oint ds A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x}) = 0$$

Hence integral over economic domain (1.1) for (14) due to (5) equals:

$$\int d\mathbf{x} \left[\frac{\partial}{\partial t} A(t, \mathbf{x}) + \nabla \cdot (A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) \right] = \frac{d}{dt} \int d\mathbf{x} A(t, \mathbf{x}) = \frac{d}{dt} A(t) \quad (15)$$

Thus operator D_A (14) on economic space for economic or financial variable $A(t, \mathbf{x})$ (3) plays the same role as usual ordinary derivation by time d/dt for macro variable $A(t)$ (5). Let's underline that different variables $A(t, \mathbf{x})$ and $B(t, \mathbf{x})$ follow different operators (14) due to different velocities $\mathbf{v}_A(t, \mathbf{x})$ and $\mathbf{v}_B(t, \mathbf{x})$. So, economic variable $B(t, \mathbf{x})$ follows equations:

$$D_B B(t, \mathbf{x}) = \frac{\partial}{\partial t} B(t, \mathbf{x}) + \nabla \cdot (B(t, \mathbf{x}) \mathbf{v}_B(t, \mathbf{x})) = F_B(t, \mathbf{x}) \quad (16)$$

Equations (14; 16) are valid for additive variables. Flow $\mathbf{P}_A(t, \mathbf{x})$ follows the same operator D_A (14) as variable $A(t, \mathbf{x})$ and

$$D_A \mathbf{P}_A(t, \mathbf{x}) \equiv \frac{\partial}{\partial t} \mathbf{P}_A(t, \mathbf{x}) + \nabla \cdot (\mathbf{P}_A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) = \mathbf{G}_A(t, \mathbf{x}) \quad (17)$$

$$\nabla \cdot (\mathbf{P}_A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) = \sum_{j=1, \dots, n} \frac{\partial}{\partial x_j} (\mathbf{P}_A(t, \mathbf{x}) v_{Aj}(t, \mathbf{x}))$$

Function $\mathbf{G}_A(t, \mathbf{x})$ in right side (17) describe factors that impact change of economic flow $\mathbf{P}_A(t, \mathbf{x})$ as: other variables, transactions, expectations and etc.

Equations (14, 17) describe evolution of $A(t, \mathbf{x})$ (3) and $\mathbf{P}_A(t, \mathbf{x})$ (9) under action of factors $F_A(t, \mathbf{x})$ and $\mathbf{G}_A(t, \mathbf{x})$. Integrals of (14; 17) by $d\mathbf{x}$ over domain (1.1) give ordinary differential equations as no economic or financial flows exist outside of (1.1) (Strauss 2008, p.179):

$$\int d\mathbf{x} \left[\frac{\partial}{\partial t} A(t, \mathbf{x}) + \nabla \cdot (A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) \right] = \frac{d}{dt} A(t) = F_A(t) \quad (18.1)$$

$$\int d\mathbf{x} \left[\frac{\partial}{\partial t} \mathbf{P}_A(t, \mathbf{x}) + \nabla \cdot (\mathbf{P}_A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) \right] = \frac{d}{dt} \mathbf{P}_A(t) = \mathbf{G}_A(t) \quad (18.2)$$

Ordinary differential equations (18.1, 18.2) describe time evolution of macroeconomic and financial variables of entire economics. It is clear that all complexity of economic dynamics is described by form of right hand side factors $F_A(t, \mathbf{x})$ and $\mathbf{G}_A(t, \mathbf{x})$ in (14, 17). Equations (14, 17) permit model self-consistent interactions between two macro variables. The simplest case of mutual dependence between two macro variables can be presented as

$$\frac{\partial}{\partial t} A(t, \mathbf{x}) + \nabla \cdot (A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) = F_A(t, \mathbf{x}) \quad (19.1)$$

$$\frac{\partial}{\partial t} B(t, \mathbf{x}) + \nabla \cdot (B(t, \mathbf{x}) \mathbf{u}_B(t, \mathbf{x})) = F_B(t, \mathbf{x}) \quad (19.2)$$

$$\frac{\partial}{\partial t} \mathbf{P}_A(t, \mathbf{x}) + \nabla \cdot (\mathbf{P}_A(t, \mathbf{x}) \mathbf{v}_A(t, \mathbf{x})) = \mathbf{G}_A(t, \mathbf{x}) \quad (19.3)$$

$$\frac{\partial}{\partial t} \mathbf{P}_B(t, \mathbf{x}) + \nabla \cdot (\mathbf{P}_B(t, \mathbf{x}) \mathbf{u}_B(t, \mathbf{x})) = \mathbf{G}_B(t, \mathbf{x}) \quad (19.4)$$

$$F_A(t, \mathbf{x}) = F_A(t, \mathbf{x}; B, \mathbf{P}_B) \quad ; \quad F_B(t, \mathbf{x}) = F_B(t, \mathbf{x}; A, \mathbf{P}_A) \quad (19.5)$$

$$\mathbf{G}_A(t, \mathbf{x}) = \mathbf{G}_A(t, \mathbf{x}; B, \mathbf{P}_B) \quad ; \quad \mathbf{G}_B(t, \mathbf{x}) = \mathbf{G}_B(t, \mathbf{x}; A, \mathbf{P}_A) \quad (19.6)$$

Relations (19.5, 19.6) may describe dependence of $F_A(t, \mathbf{x})$ and $\mathbf{G}_A(t, \mathbf{x})$ on variable $B(t, \mathbf{x})$ and flow $\mathbf{P}_B(t, \mathbf{x})$ and $F_B(t, \mathbf{x})$ and $\mathbf{G}_B(t, \mathbf{x})$ on variable $A(t, \mathbf{x})$ and flow $\mathbf{P}_A(t, \mathbf{x})$. $F_A(t, \mathbf{x})$ and $\mathbf{G}_A(t, \mathbf{x})$ may depend on operators like divergence, gradient, rotor and etc. on functions $B(t, \mathbf{x})$ and $\mathbf{P}_B(t, \mathbf{x})$. It is obvious that in real economics macro variables depend on numerous economic and financial factors but (19.1-19.4) permit study simple approximations of mutual relations between two – three or four macroeconomic variables and their flows.

In Part II we describe economic transactions and expectations as density functions on economic space. We derive equations on transactions, expectations and their flows. We show how our approach helps describe asset pricing on economic space and derive equations on price evolution. In Part III of our paper we apply our model equations to description of particular economic problems.

5. Conclusions

The first part of our paper gives general economic treatment of economic model. We introduce notions of economic space, economic and financial variables and density functions flows. We derive economic equations on density functions and flows of economic variables

and show that simple approximation permit study self-consistent relations between economic variables and their flows.

Our economic model uses no assumptions on market equilibrium, utility functions, rational expectations and etc., those ground general equilibrium (Arrow and Debreu, 1954; Tobin, 1969; Arrow, 1974; Smale, 1976; Kydland and Prescott, 1990; Starr, 2011). These assumptions are not necessary for economic modeling and economic theory can be based on economic statistics as source for agents risk assessments, alike to measuring the coordinates in physics. Hence excessive and unnecessary assumptions can be put aside of economic modeling or may be applied for description of some specific, particular cases of economic problems only.

Our approach uncovers a lot of economic problems that should be studied further to clarify elements of the economic model. Let's argue some those concern economic space. For example dimension of economic space is determined by choice of n risks those impact macroeconomic evolution. To develop reasonable economic model one should reduce number of risks and chose major two-three risks to define economic space of 2 or 3 dimensions. Hence one should develop methods to compare and forecast impact of risks on macroeconomic dynamics and procedure for selection most important risks. Choice of definite risks defines distribution of agents, form of density functions and economic dynamics on selected economic space. Different sets of risks cause different economic dynamics. Random nature of economic risks means that impact of some current risks may decline in time and influence of some new risk may unexpectedly grow up. Such collision underlines internal random properties of macroeconomic evolution and modeling. We state that economic development can occurs only under action of risks and different risks may set different directions for economic dynamics. Thus change of major risks results in change of dynamics determined by economic equations on density functions and flows of variables, transactions and expectations. In this paper we study economic evolution in the assumption that major risks and economic space don't change. The problems of random change of major risks should be studied further.

Risk assessments play central role for our model. It is clear that exact risks assessments of all agents in the entire economics are impossible. This is similar inability to measure coordinates of all physical particles of macro system. We propose the roughening procedure that transfers description of numerous separate particles to description of density functions on economic space. Such roughening procedure has parallels to transition from description of separate physical particles in kinetic approximation to description of continuous media or physics of

fluids in hydrodynamic approximation. Such transition in physics significantly reduce amount of data required for model description. We seek the same effect in economic modeling. Roughening of risk ratings of separate agents and transition to description of density functions and flows of economic variables, transactions and expectations reduce amount of econometric statistics required for such approximation. Our approximation becomes intermediate between extra precise description based on modeling macroeconomics as system of numerous separate agents and description based on modeling macroeconomics as aggregated functions of time only. We propose that achievements of econometrics (Fox, et.al, 2014) and efforts in developing risks assessments methodologies will solve that complex problem for sure.

We assume that our approach to economic modeling may help improve description and forecasting of macroeconomic processes and impact development of economic policy making for sustainable economic growth.

References

- Ackerman, F., (1999). Still Dead After All These Years: Interpreting the Failure of General Equilibrium Theory. Global Development and Environment Institute, Tufts University.
- Altman, E., (Ed), (2010). The Z-Metrics™ Methodology For Estimating Company Credit Ratings And Default Risk Probabilities. RiskMetrics Group, Inc.
- Arrow, K.J., Debreu, G., (1954). Existence of an Equilibrium for a Competitive Economy. *Econometrica*, 22, (3), 265-290.
- Arrow, K.J., (1974). General Economic Equilibrium: Purpose, Analytic Techniques, Collective Choice. *The American Economic Review*, 64, (3), 253-272
- Batchelor, G.K., (1967). An introduction to fluid dynamics. Cambridge Univ. Press, UK.
- Campbell, J.Y., (2014). Empirical Asset Pricing: Eugene Fama, Lars Peter Hansen, and Robert Shiller. *Scandinavian J. of Economics*. 116, (3), 593-634
- Cochrane, J.H., Hansen, L.P., (1992). Asset Pricing Explorations for Macroeconomics. Ed. Blanchard, O.J. and Fischer, S., NBER Macroeconomics Annual, 7, 115 - 182
- Cochrane, J.H., Culp, C.L., (2003). Equilibrium Asset Pricing and Discount Factors: Overview and Implications for Derivatives Valuation and Risk Management. In *Modern Risk Management. A History*, Ed. S.Jenkins, 57-92
- Cochrane, J.H., (2017). Macro-Finance. *Review of Finance*, EFA, 21(3), 945-985
- Dominitz, J., Manski, C.F., 2005. Measuring And Interpreting Expectations Of Equity Returns. NBER, WP 11313, Cambridge, MA.
- Fama, E.F., (2014). Two Pillars of Asset Pricing. *American Economic Review*, 104, (6), 1467–1485
- Farmer, R.E.A., (2017). Post Keynesian Dynamic Stochastic General Equilibrium Theory. WP 23109, NBER, Chicago, Cambridge, MA
- Fernández-Villaverde, J., (2010). The econometrics of DSGE models. *Jour. Spanish Economic Association*, 1, 3–49
- Fox, D.R., et.al., (2014). Concepts and Methods of the U.S. National Income and Product Accounts. BEA DoC. 1-373.
- Fitch, (2018). Procedures and Methodologies for Determining Credit Ratings. Fitch Ratings, Inc., Form 25-101F1
- Gaffard, J.L., and Napoletano, M., (Ed)., (2012). Agent-based models and economic policy, OFCE, Paris Cedex.
- Giovannini, E., (2008). Understanding Economic Statistics. An OECD Perspective. OECD.

Hansen, L.P., (2013). Uncertainty Outside and Inside Economic Models. Nobel Prize Lecture, University of Chicago, Chicago, IL, USA.

Hazlitt, H., (1959). The Failure Of The "New Economics". An Analysis Of The Keynesian Fallacies. D. Van Nostrand Company, Inc

Horcher, K.A., (2005). Essentials of Financial Risk Management. J.Wiley & Sons, NJ

Horowitz, K.J. and Planting, M.F., (2006). Concepts and Methods of the Input-Output Accounts. BEA, US DoC.

Komunjer, I., NG, S., (2011). Dynamic Identification Of Dynamic Stochastic General Equilibrium Models. *Econometrica*, 79, (6), 1995–2032

Kydland, F.E., Prescott, E.C., (1990). The Econometrics Of The General Equilibrium Approach To Business Cycles. FRB Minneapolis, Staff Report 130.

Landau, L.D., E.M. Lifshitz (1987). *Fluid Mechanics* , Pergamon Press Ltd., NY.

Leontief, W. (1941). *The Structure of American Economy 1919–1939* , New York: Oxford University Press.

Leontief, W., (1955). Some Basic Problems of Empirical Input-Output Analysis. 9-52. in Goldsmith, R.W., (Ed.). (1955). *Input-Output Analysis: An Appraisal*. NBER, Princeton Univ.Press.

McNeil, A.J., Frey, R., Embrechts, P., (2005). *Quantitative Risk Management. Concepts, Techniques and Tools*. Princeton Univ. Press, Princeton.

Metz, A. and Cantor, R., (2007). Introducing Moody's Credit Transition Model, Moody's Investor Service, 1-26

Moody's, (2010). *Rating Symbols and Definitions*. Moody's Investors Service.

Moody's, (2018). *Procedures and Methodologies Used to Determine Credit Ratings*. Moody's Investors Service.

Morgenstern, O., (1972). Thirteen Critical Points in Contemporary Economic Theory: An Interpretation. *Journal of Economic Literature*, 10, (4), 1163-1189

Negro, M.D., et.al. (2013). *The FRBNY DSGE Model*. Fed. Reserve Bank of NY Staff Report No. 647

Olkhov, V., (2016a). On Economic space Notion, *Int. Rev. Financial Analysis*, 47, 372-381

Olkhov, V., (2016b). Finance, Risk and Economic space, *ACRN Oxford J. of Finance and Risk Perspectives*, Special Issue of Finance Risk and Accounting Perspectives, 5, 209-221

Olkhov, V., (2017a). Quantitative Wave Model of Macro-Finance. *Int. Rev. Financial Analysis*, 50, 143-150

Olkhov, V., (2017b). Quantitative Description of Financial Transactions and Risks” ACRN Oxford Journal of Finance and Risk Perspectives 6, (2), 41-54.

Olkhov, V., (2017c). Credit-Loans Non-Local Transactions and Surface-Like Waves. <https://ssrn.com/abstract=2971417>

Olkhov, V., (2017d). Econophysics of Business Cycles: Aggregate Economic Fluctuations, Mean Risks and Mean Square Risks. arxiv.org/abs/1709.00282, q-fin.EC

Olkhov, V., (2018). Expectations, Price Fluctuations and Lorenz Attractor. MPRA WP89105, <https://mpra.ub.uni-muenchen.de/89105/>

Olkhov, V., (2019a). Economic Transactions Govern Business Cycles. ACRN Oxford J. of Finance and Risk Perspectives, 7 (1/2), 102-122.

Olkhov, V., (2019b). Econophysics of Asset Price, Return and Multiple Expectations. <https://papers.ssrn.com/abstract=3319018>

Resibois, P., De Leener, M. (1977). Classical Kinetic Theory of Fluids. John Wiley, NY

Skoglund, J., Chen, W., (2015). Financial Risk Management: Applications in Market, Credit, Asset and Liability Management, and Firmwide Risk. John Wiley&Sons Inc.

SEC, (2012). Credit Rating Standardization Study. Report to Congress.

Smale, S., (1976). Dynamics in General Equilibrium Theory. Am.Economic Review, 66, (2), 288-294

South, A.H., Gurwitz, Z.R., (2018). 2017 Annual Global Structured Finance Default Study And Rating Transitions. S&P Global Ratings.

S&P, (2014). Guide To Credit Rating Essentials. What are credit ratings and how do they work? McGraw Hill Financial

S&P&, (2016). S&P Global Ratings Definitions. S&P Global Ratings.

Starr, R.M., 2011. General Equilibrium Theory, An Introduction, Cam. Univ. NY

Stiglitz, J.E., (2017). Where Modern Macroeconomics Went Wrong. NBER, WP 23795

Strauss, W.A., (2008). Partial Differential Equations. An Introduction. John Wiley&Sons, NJ, US. p.179.

Tesfatsion, L., and K. L. Judd (editors), (2006). Handbook of Computational Economics, Volume 2: Agent-Based Computational Economics, Handbooks in Economics Series, North-Holland, Amsterdam.

Tobin, J., (1969). A General Equilibrium Approach To Monetary Theory. Journal of Money, Credit and Banking, 1, (1), 15-29

Vines, D., and Wills, S., (Eds.), (2018a). Rebuilding macroeconomic theory. Oxford Review of Economic Policy, 34, (1–2), 2-353

Vines, D. and Wills, S., (2018b). The rebuilding macroeconomic theory project: an analytical assessment, *Oxford Review of Economic Policy*, **34**, (1–2), 1–42

Wilier, A.H., (1901). *The Economic Theory Of Risk And Insurance*. Introduction by Glenn Meyers. Reprinted 1951. Richard D. Irwin, Inc., Homewood, Illinois