A proposed method to estimate dynamic panel models when either N or T or both are not large

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Abstract

Traditionally the bias of an estimator has been reduced asymptotically to zero by enlarging data panel dimensions N or T or both. This research proposes a novel econometric modelling method to separate and measure the bias of an estimator without altering data panel dimensions. This is done by recursively decomposing its bias in systematic and nonsystematic parts. This novel method addresses the bias of an estimator as a type of asymptotic serial correlation problem. Once this method disentangles bias components it could provide consistent estimators and adequate statistic inference. This recursive bias approach is missed from the current bias literature. This novel method results do not cast doubt about the asymptotic bias approach conclusions, but made them incomplete. Monte Carlo simulations find consistent sample estimators asymptotic convergence with population estimators by enlarging the sample size. In these simulations the population estimator value is provided beforehand the simulation begins. The mean advantage of the alternative recursive estimator bias approach is that the sample estimator recursively converges with population estimators without enlarging sample size. Importantly this novel method avoids researcher bias criteria, which consist on an arbitrary a priori population estimator value selection.

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Introduction

Panel data involves two dimensions. The first dimension is N, which represents the number of individuals. The second dimension is T, which stands for the number of time periods. In the literature there are abundant papers related with panel data dynamic estimators, and their asymptotic properties, i.e., MacKinnon and Smith (1998); Hsiao, et al., (2002); Hsiao and Tahmiscioglu (2008); Abadie and Imbens (2011); Hsiao and Zhang (2015). These studies are concerned with measuring panel data estimators sensitivities when either N or T, or both are large, by means of Monte Carlo simulations and Bootstrap experiments. The idea behind the sample size enlargement of N or T or both is to obtain a diminishing estimator asymptotic bias distribution converging with a population estimator. When convergence is achieved between the asymptotic estimator with the population estimator, it is said that the asymptotic estimator is consistent.

Here, for the sake of simplicity the above literature is identified with an "asymptotic bias approach literature." This approach is concerned with different panel data dynamic asymptotic estimators properties and values. These estimators properties and values depend on several factors, i.e., assumptions; initial values; functional forms; sample size; endogeneity treatments; econometric techniques, i.e., MLE (maximum likelihood) or GMM (general method of moments); recursive AR(1) (autoregressive vector of order one); just to mention a few. Some estimators properties and values from the asymptotic bias approach literature are reported next: Arellano and Bond (1991) find a GMM panel consistent estimator asymptotically unbiased if T is fixed, and N goes to infinite. For its part, Hahn and Kuersteiner (2002) show that β_{cv} (covariance estimator) is asymptotically biased of order $\sqrt{\frac{N}{T}}$, when N and T go to infinite and $\frac{T}{N}$ goes to a constant different to zero and less than infinite and T grows faster than N. In a similar manner Alvarez and Arellano (2003) report a GMM panel estimator that is asymptotically biased of order $\sqrt{c^*}$, when $\lim_{N\to\infty}\frac{T}{N} = c^*$ when T grows faster or at the same rate than N, where $0 < c^* < \infty$; and the star signals an optimum. As a specific variation of this last result, Hsiao (2003) finds that if y_{i0} is fixed and α_i measures the individual specific effects, then the maximum likelihood estimator converges to the covariance estimator β_{cv}, where β_{cv} is asymptotically normally distributed with mean 0 if N is fixed and T is large, implying that the only panel dimension being enlarged is T.

The asymptotic bias approach properties are studied in the literature because consistent estimators have an important function in statistics and other sciences where optimization plays an important role. Under the light of this approach a consistent estimator means efficiency, and decreasing bias under sample size enlargement. According with Carbajal (2017) a consistent estimator implies information convergence and diminishing scaling variance. Thus efficiency could be interpreted as information convergence and decreasing bias could be interpreted as diminishing scaling variance, where the application of such concepts should be mediated by the operational context. The central limit theorem postulates that as the sample size enlarges, the sample estimator will converge with the population estimator. The asymptotic bias approach assumes that the central limit theorem holds, and if bias goes to zero as N or T or both go to infinite, then the estimators become consistent. Consistent estimators are important because they become the benchmark to set up confidence intervals; t-test power; significance levels; just to mention a few. Statistic inference refutes or validated sample estimator against consistent estimators benchmark.

This is why a lot of statistic tests includes p-values obtained from this asymptotic approach, i.e., Mackinnon p-values to accept or reject the unit root null hypothesis.

This paper proposes a novel method to obtain consistent estimators, without need to increase panel data dimensions. Here this novel method is called an "alternative recursive estimator bias approach." This method treats bias as a type of serial correlation problem. Also, this method provides an analytical solution to solve this type of serial correlation problem. This solution separates consistent estimators and their biases in a linear fashion.

This paper is organized as follows: Section 2 presents the model, and the proposed novel method. Section 3 discusses the findings and concludes.

Section 2. Model and proposed novel method

Suppose that a dynamic panel data model has the following form:

(1) $y_{it} = \alpha_i + \beta y_{i,t-1} + u_{it}$

where β is assumed stationary, meaning that its roots are inside the unit circle; $i = 1, \cdots, N$; $t = 1, \cdots, T$. Also suppose that:

Assumption 1. u_{it} the error term is a random variable. It has a standard normal distribution N(0, 1). Its moments are $E[u_{it}] = 0$; $var[u_{it}] = \sigma_u^2 = 1$; third and fourth moments are finite;

Assumption 2. $E[\hat{\beta}|\alpha_i]$ the expected value of β estimator conditional to α_i; its bias, and its movil average terms can be decomposed on systematic, and nonsystematic parts. The systematic part is represented by its mean. The nonsystematic part is represented as a type of serial correlation problem.

The alternative recursive estimator bias approach provides an analytical solution to separate consistent estimators from their bias. In equation (1) α_i computation introduces a serial correlation problem. This is because the specific individual-effects are presented in both estimators α_i and β. The estimator β takes into account time effects, and also individual-effects, because $y_{i,t-1}$ has two panel data dimensions: i, and $t-1$. This leads to a measurement error given a specific individual-effects double accounting on α_i and β. If this measurement error were closed to zero, then bias would almost disappear. Thus, bias is a result of an estimation error when individual effects double accounting holds. The celebrated omitted variable formula is used to represent the expected value of β estimator, and its bias for equation (1) as follows:

(2) $E[\hat{\beta}|\alpha_i] = \beta + \frac{cov[\alpha_i, y_{i,t-1}]}{var[\alpha_i]} u_{it}$

where $E[\hat{\beta}|\alpha_i]$ expresses the expected value of β estimator conditional to α_i; β is a consistent estimator, and $\frac{cov[\alpha_i, y_{i,t-1}]}{var[\alpha_i]} u_{it}$ represents its bias. Assumption 2 applied to equation (2) points out that the systematic part is β and the nonsystematic part is $\frac{cov[\alpha_i, y_{i,t-1}]}{var[\alpha_i]} u_{it}$. Here, bias is a result of individual-effects double accounting and its interactions with the error term demonstrate a serial correlation problem.

Equation (2) is highly nonlinear in the bias component. For simplicity sake bias is treated with a Beverage-Nelson decomposition. This decomposition linearizes bias into two parts: mean and error terms. Thus, assume that bias is equal to ξ_{it} and has the following linear representation:

(3) $\xi_{it} = \delta_i + u_{it}$

where $\xi_{it} = \frac{cov[\alpha_i, y_{i,t-1}]}{var[\alpha_1]} u_{it}$; δ_i represents the mean and u_{it} represents error terms. Assumption 2, applied to equation (3), points out that the systematic part is δ_i, and the nonsystematic part is u_{it}. Also, consider the following functional movil average form for u_{it}:

(4) $u_{it} = \psi(L)\xi_{it}$

expanding the infinite lag polynomial $\psi(L)$ yields

(5) $u_{it} = \psi(1)\xi_{it} + \psi(2)\xi_{i,t-2} + \cdots + \psi(T)\xi_{i,0}$

where $\psi(1)$ represents a movil average of order one; $\psi(2)$ represents a movil average of order two; $\cdots$; $\psi(T)$ represents a movil average of order T. Plugging equation (5) into equation (3) gives

(6) $\xi_{it} = \delta_i + \psi(1)\xi_{i,t-1} + \psi(2)\xi_{i,t-2} + \cdots + \psi(T)\xi_{i,0}$

then, equation (2) can be rewritten as:

(7) $E[\hat{\beta}|\alpha_i] = \beta + \delta_i + \psi(1)\xi_{i,t-1} + \psi(2)\xi_{i,t-2} + \cdots + \psi(T)\xi_{i,0}$

where β represents a consistent estimator, and bias is $\delta_i + \psi(1)\xi_{it} + \psi(2)\xi_{i,t-1} + \cdots + \psi(T)\xi_{i,0}$. As the infinite lag polynomial expands up to the term T, bias components are disentangled from a nonlinear representation on equation (2) to a linear fashion as demonstrated on Equation (7).

The asymptotic bias approach applied to the bias linear decomposition would expect that as T goes to infinite the sum of its terms converges to zero:

(8) $\lim_{T\to\infty} \delta_i + \psi(1)\xi_{it} + \psi(2)\xi_{i,t-1} + \cdots + \psi(T)\xi_{i,0} = 0$

If equation (8) holds, then equation (7) reduces to $E[\hat{\beta}|\alpha_i] = \beta$, where β is a consistent estimator. Without loss of generality, it is fair to say that equation (8) represents the result that the asymptotic bias approach looks at when N or T or both are large.

Theorem 1. A consistent estimator in presence of specific individual-effects correlation is obtained by estimating its bias components.

Proof

Plugging equation (7) into equation (1) provides:

(9) $y_{it} = \alpha_i + [\beta + \delta_i + \psi(1)\xi_{i,t-1} + \psi(2)\xi_{i,t-2} + \cdots + \psi(T)\xi_{i,0}]y_{i,t-1} + u_{it}$

distributing the $y_{i,t-1}$ term gives:

(10) $y_{it} = \alpha_i + \beta y_{i,t-1} + \delta_i y_{i,t-1} + \psi(1)\xi_{i,t-1}y_{i,t-1} + \psi(2)\xi_{i,t-2}y_{i,t-2} + \cdots + \psi(T)\xi_{i,0}y_{i,0} + u_{it}$

collecting the individual effects estimators in only one term, i.e., $\eta_i = \alpha_i + \delta_i y_{i,t-1}$ yields:[2]

(11) $y_{it} = \eta_i + \beta y_{i,t-1} + \psi(1)\xi_{i,t-1}y_{i,t-1} + \psi(2)\xi_{i,t-2}y_{i,t-2} + \cdots + \psi(T)\xi_{i,0}y_{i,0} + u_{it}$

Equation (11) represents the first recursive iteration of the proposed novel method to separate and quantify bias components.

For the moment, consider only the term $\psi(1)\xi_{i,t-1}y_{i,t-1}$. Its estimator could be decomposed in systematic and nonsystematic parts.

(12) $E[\hat{\psi}(1)|\eta_i] = \psi(1) + \frac{cov[\eta_i,\xi_{i,t-1}y_{i,t-1}]}{var[\eta_i]}u_{it}$

Assumption 2 applied to equation (12) points out that the systematic part is $\psi(1)$ and the nonsystematic part is $\frac{cov[\eta_i,\xi_{i,t-1}y_{i,t-1}]}{var[\eta_i]}u_{it}$.

Next, the analogs of equations (3)-(8) are presented for $\psi(1)\xi_{i,t-1}y_{i,t-1}$ estimator in equations (13)-(18). To use a notational that facilitates comparisons between equations (3)-(8) and equations (13)-(18), let the $\psi(1)\xi_{i,t-1}y_{i,t-1}$ components be represented with an underbar, i.e., $\underline{\xi}_{\text{it}} = \frac{cov[\eta_i,\xi_{i,t-1}y_{i,t-1}]}{var[\eta_i]}u_{it}$.

(13) $\underline{\xi}_{it} = \underline{\delta}_i + u_{it}$

(14) $u_{it} = \underline{\psi}(L)\underline{\xi}_{it}$

(15) $u_{it} = \underline{\psi}(1)\underline{\xi}_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}$

(16) $\underline{\xi}_{it} = \underline{\delta}_i + \underline{\psi}(1)\underline{\xi}_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}$

(17) $E[\hat{\psi}(1)|\eta_i] = \psi(1) + \underline{\delta}_i + \underline{\psi}(1)\underline{\xi}_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}$

(18) $\lim_{T\to\infty} \underline{\delta}_i + \underline{\psi}(1)\underline{\xi}_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0} = 0$

If equation (18) holds, then equation (17) reduces to $E[\hat{\psi}(1)|\eta_i] = \psi(1)$, where $\psi(1)$ is a consistent estimator. By symmetry, equation (17) can be generalized for the following estimators $\psi(2), \cdots, \psi(T)$.

2 Although $y_{i,t-1}$ contains both data panel dimensions, δ_i takes into account only mean specific individual-effects.

(19) $E[\hat{\psi}(2)|\eta_i] = \psi(2) + \underline{\delta_i}_2 + \underline{\psi}(2)\underline{\xi}_{i,t-2} + \underline{\psi}(3)\underline{\xi}_{i,t-3} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0}$

$$\vdots$$

(20) $E[\hat{\psi}(T)|\eta_i] = \psi(T) + \underline{\delta_i}_T + \underline{\psi}(T)\underline{\xi}_{i,0}$

plugging equations (17); (19), and (20) into equation (11) yields:

(21) $y_{it} = \eta_i + \beta y_{i,t-1} +$
$\psi(1) + \underline{\delta_i} y_{i,t-1} + \underline{\psi}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0} y_{i,0} +$
$\psi(2) + \underline{\delta_i}_2 y_{i,t-2} + \underline{\psi}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \underline{\psi}(3)\underline{\xi}_{i,t-3} y_{i,t-3} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0} y_{i,0} +$
$\cdots +$
$\psi(T) + \underline{\delta_i}_T y_{i,0} + \underline{\psi}(T)\underline{\xi}_{i,0} y_{i,0} + u_{it}$

collecting again the individual effects terms in only one term, i.e., $\underline{\eta_i} = \eta_i + \underline{\delta_i} y_{i,t-1} + \underline{\delta_i}_2 y_{i,t-2} + \cdots + \underline{\delta_i}_T y_{i,0}$ provides:

(22) $y_{it} = \underline{\eta_i} + \beta y_{i,t-1} +$
$\psi(1) + \underline{\psi}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + \underline{\psi}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0} y_{i,0} +$
$\psi(2) + \underline{\psi}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \underline{\psi}(3)\underline{\xi}_{i,t-3} y_{i,t-3} + \cdots + \underline{\psi}(T)\underline{\xi}_{i,0} y_{i,0} +$
$\cdots +$
$\psi(T) + \underline{\psi}(T)\underline{\xi}_{i,0} y_{i,0} + u_{it}$

Now collecting similar terms yields:

(23) $y_{it} = \underline{\eta_i} + \beta y_{i,t-1} + \psi(1) + \psi(2) + \cdots + \psi(T) +$
$\underline{\psi}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + 2\underline{\psi}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + T\underline{\psi}(T)\underline{\xi}_{i,0} y_{i,0} + u_{it}$

Consider that the estimated variables for $\psi(1)$; $\psi(2); \cdots; \psi(T)$ are a vector of ones in each case. In consequence, they are the movil average means of y_{it} at each lag value. If panel data is stacked by individuals, then movil average means also gauge specific individual-effects. For the sake of simplicity, these movil average means could be collected together with the specific individual-effects means. Thus, these terms can be compiled together with $\underline{\eta_i}$ in only one term representing all individual effects in equations (23), i.e., $\underline{\eta_i}_2 = \underline{\eta_i} + \psi(1) + \psi(2) + \cdots + \psi(T)$. Thus equation (23) can be rewritten as:

(24) $y_{it} = \underline{\eta_i}_2 + \beta y_{i,t-1} + \underline{\psi}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + 2\underline{\psi}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + T\underline{\psi}(T)\underline{\xi}_{i,0} y_{i,0} + u_{it}$

Equation (24) represents the second iteration of the proposed novel method. With this recursive method, and after T interactions, the $\hat{\beta}$ estimator bias components are linearly separated in systematic and nonsystematic parts. Thus, the estimator bias nonsystematic part is fully determined. □

Theorem 2. Consistent estimator could be computed at any panel data dimension size.

Proof
Theorem 1 provides the alternative recursive estimator bias approach to decompose β estimator bias in systematic and nonsystematic parts. This method iterates recursively bias

systematic and nonsystematic parts until the systematic part converges to a consistent estimator. Thus, the following bias equality follows:

$$(25)\ \frac{cov[\alpha_i, y_{i,t-1}]}{var[\alpha_1]} u_{it} = \left[\left(\hat{\eta}_{\underline{i}_2} - \hat{\alpha}_i\right) + \left(\underline{\hat{\psi}}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + 2\underline{\hat{\psi}}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + T\underline{\hat{\psi}}(T)\underline{\xi}_{i,0} y_{i,0}\right)\right]$$

where hat denotes estimates. The left hand side in equation (25) is theoretical bias. The right hand side displays empirical bias decomposition. This decomposition provides bias systematic $\left(\hat{\eta}_{\underline{i}_2} - \hat{\alpha}_i\right)$ and nonsystematic $\left(\underline{\hat{\psi}}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + 2\underline{\hat{\psi}}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + T\underline{\hat{\psi}}(T)\underline{\xi}_{i,0} y_{i,0}\right)$ components.

After estimating equation (24) right hand side, it is subtracted from equation (2) as follows:

$$(26)\ E[\hat{\beta}|\alpha_i] = \beta + \frac{cov[\alpha_i, y_{i,t-1}]}{var[\alpha_1]} u_{it} - \left[\left(\hat{\eta}_{\underline{i}_2} - \hat{\alpha}_i\right) + \left(\underline{\hat{\psi}}(1)\underline{\xi}_{i,t-1} y_{i,t-1} + 2\underline{\hat{\psi}}(2)\underline{\xi}_{i,t-2} y_{i,t-2} + \cdots + T\underline{\hat{\psi}}(T)\underline{\xi}_{i,0} y_{i,0}\right)\right]$$

The right hand side two last terms on equation (26) conform equation (25). Because equation (25) equality, theoretical and empirical bias cancels out on equation (26). Therefore:

$$E[\hat{\beta}|\alpha_i] = \beta$$

where β is a consistent estimator. This is the result that the recursive bias approach looks at when N or T or both are not large. □

Section 3. Discussion and Conclusions

The asymptotic bias approach uses Monte Carlo simulations and Bootstraps experiments to enlarge N or T or both. The asymptotic bias approach look at reducing estimator asymptotic bias distribution to zero. This is with the end of achieving convergence between sample enlarge size estimator with population estimator. However, Monte Carlo simulations introduce researcher bias criteria, which consist on an arbitrary a priori population estimator value selection.

The theoretic literature addressing this alternative recursive estimator bias approach is not known by the author. On one hand, this approach is missing on the theoretical asymptotic bias approach, i.e., Hsiao and Zhang (2015) consider that bias is the result of using instruments to purge correlations between estimators and equation errors. For Makowski, et al., (2006) omision bias is the result of a measurement error between independet variables. Thus, the bias of an estimator is not econometrically modelled in the symptotic bias approach. In constrast, the alternative recursive estimator bias approach relies on bias econometric modelling. On the other hand, the alternative recursive estimator bias approach is also neglected by the empirical asymptotic bias approach, i.e., Arellando and Bond (1991); Alvarez and Arellano (2003); Anderson and Hsiao (1981); Hsiao et al., (2002), and Carbajal (2014).

Perhaps the alternative recursive estimator bias approach is not present in the empirical asymptotic bias approach, because this approach is not theoretically implemented in a first place. As a result, consistent estimators study derived from the application of this alternative recursive estimator bias approach is ommitted from the current econometric modelling literature. This omission does not cast doubt about the conclusions of the asymptotic bias approach, but made them incomplete.

This alternative recursive estimator bias approach is a novel method that separates consistent estimators from their bias components. It seems that this novel method is filling a bias theoretic and empirical literature gap. This novel method relies on two assumptions. Assumption 1 sets a finite error process. In particular, the fourth moment finiteness is associated with a stationary solution in a strict-sense (Douc et al., 2014 and Spanos, 1999); meanwhile, Arellano and Bond (1991) consider the fourth-order moment as indicating a lower or quicker convergence to normality. Assumption 2 sets the conditional estimator, its bias, and its movil averages as decomposable in systematic, and nonsystematic parts. This method characterizes the nonsystematic part as a type of serial correlation problem. These assumptions are the base upon which the novel method econometric modelling relies on.

In the alternative recursive estimator bias approach, bias is the result of individual-effects double accounting. This approach considers bias as an independent random process that could be linearly decomposed and quantifiable after a series of recursive steps. The alternative recursive and asymptotic bias approaches provide the same consistent estimator β.

Importantly, the alternative recursive estimator bias approach does not need N or T or both being large to compute consistent estimators. Thus, panel dimensions N or T or both are not large. In contrast, the asymptotic bias approach obtains asymptotic convergence between sample and population estimators, when N or T or both are large.

The alternative recursive estimator bias approach has two mean advantages with respect to the asymptotic bias approach.

1. It provides the means by which a consistent estimator value does not have to be predetermined. This avoids researcher bias criteria, which consist on an arbitrary a priori population estimator value selection, which is considered by definition a consistent estimator;
2. Asymptotic bias properties are not needed to compute a consistent estimator.

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