Interactions between Credit and Market Risk, Diversification vs Compounding effects

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Abstract

The relations between credit and market risk have deep roots in financial and economic theory. After a brief theory review, we select five variables and calculate their historical shortfalls. This shortfall is taken as a proxy for market risk quantification. Relating this shortfall to non performing loans as a proxy for credit risk allows us to study the nature of the relation between credit and market risk. The nonlinearity of the relation is discussed in view of diversification and compounding effects.

Keywords: Credit risk, Market risk, Aggregation, Diversification, Compounding effect.

JEL Classification: D81, E44, G21

1 Introduction

A correct evaluation of the interactions between market and credit risk is an important requirement for an adequate measure of total risk. Risk quantification is a necessary condition for calculating economic capital and acts as a cornerstone of management decisions. Furthermore, risk measurement constitute a fundamental part of shareholders information and is an essential tool for economic regulators.

Banking income has different sources. First, the spread between the interest rates that it receives on investments and those what it pays for resources. Second, the revaluation of on and off-balance sheet positions. Therefore, banks income is affected by credit risk (related to counterpart default) and market risk (influencing on and off-balance sheet positions prices).

Traditionally credit risk was associated with the banking book and market risk with the trading book (Breuer, Jandačka, Rheinberger, & Summer, 2010),

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which was the origin for the separate treatment of both risks[1]. An important debate related to the interaction between the two types of risk is that of diversification vs compounding effects. If risks are linearly correlated, diversification leads to a total risk measure that is less than the sum of both. Compounding effects act in the other direction. If the relation between risks is nonlinear, the sum of risks underestimate total risk. Diversification benefits are highlighted by Rosenberg & Schuermann (2006). They quantify those in about 30% to 40% percent, comparing the copula approach with the additive approximation. Drehmann et al.(2010) show how measuring credit risk without taking in account the impact of net interest income leads to overestimate the overall negative impact. Lucas & Verhoef (2012) find that model specification involve diversification benefits for aggregated market and credit risk. They quantify this specification effect to Value-at-Risk reductions in the range of 3% to 47%. Estimates of the impact of diversification on aggregate economic capital differ significantly: between 10% - 30% for banks (Brockmann & Kalkbrener, 2010). An interesting attempt to model the relation with Gaussian copulas is proposed by Broker & Hillebrand (2009). The linear correlation framework that this work uses leads naturally to question what happen in nonlinear cases.

Jarrow & Turnball (2000) point out that market and credit risks are not separable, incorporating a convenience yield as determinant of credit spread. The Basel Committee on Banking Supervision (2009) argues that, apart from diversification benefits; the two types of risks may reinforce each other[2]. Compounding effects may appear due to nonlinear interactions[3].

Not taking in account diversification leads to overestimate capital buffers and to dampen economic growth due to less availability of loanable funds. On the other side, not considering compounding effects induce smaller capital buffers and ultimately leads to higher probability of crisis due to the possible failure of the financial system. As capital buffers are most relevant in crisis time, when the diversification benefits are most needed; this paper also tries to contribute to stress testing analysis. Some limitations of this work may provide a stricter delimitation of the scope. We are not studying the effect of one risk over the other, rather we are interested in the joint effect over total risk. We do not use holding periods and consequently do not analyse market liquidity. We also don't see the proportion in which the different risk classes combines. Hence, direct consequences of the relations for capital calculation, risk measurement and stress testing forms are out of the range of this work.

[1] *"The risk types have been measured separately, managed separately, and economic capital against each risk type has been assessed separately"* (Basel Committee on Banking Supervision, 2009).

[2] With adjustable rate loans market risk (interest rate risk) is passed on to borrowers when default risk is fixed. Under linearity, each risk is estimated separately, maintaining fixed the other. Total risk is measured only as the sum of both risks. If default risk increases with interest rate both risks raise and total risk may be understated.

[3] Nowadays some aspects are calculated. For market risk, within the Sensitivities-based method, nonlinear risks are captured with the curvature risk factor. For a comprehensive overview see the work of the Basel Committee on Banking Supervision (2014, 2016) and its related documents.

2 Economic dynamics

We take a brief review of economic theory to provide a broad knowledge of agents incentives and possible dynamic sequences; giving guidance for identifying potential economic causes and helping to interpret econometric results. Understanding if there is an intrinsic non separable nature of credit-market risk relations may help to better control those risk for all stakeholders[4]. The interactions of credit and market risk are elaborated since long in economic theory. In a nutshell, when Gross Domestic Product (GDP) does not grow above some limit value, generally; a) unemployment grows and credit risk increases, and; b) at the same time, asset values fall and market risk grows. Interest rate hikes produce higher exigencies over credit payments, higher credit risk; lower asset prices and therefore higher market risk. These interrelations tent to move together with other variables trough the economic cycle. The connections of the relation between macroeconomic variables and credit and market risk are so significant that they are worked out with detailed procedures, using satellite models (Henry et al., 2013).

The economic theory of this section provide explanation and the logic of these relations and their impulse-response mechanisms. In particular, it may clarify what are the arguments for nonlinear effects to appear[5] and those that act against. The relation between lower asset value and payment capacity (disposition of means of payment) has been analysed since the beginning of modern economic thought.

Adam Smith[6] put risk at the centre of his theory, as one of the primary explanation of profit. The seminal author connects the risk of low asset value of foreign trade companies to lower payment capacity[7]. In book two[8] Smith links high interest rate (and therefore low asset value) to the adverse selection problem and consequently to lower repayments (Smith, 1976). Thornton maintained that in time of low confidence money would be hoarded (a proto keynesian liquidity theory), therefore other assets would lose value and this hoarding may have the potential to produce difficulties in the punctuality of payments (Thornton, 1802)[9]. Keynes point out that: *"A collapse in the price of equities, which has had disastrous reactions on the marginal efficiency of capital, may have been due to the weakening either of speculative confidence or of the state of credit"*[10]. The combination of doubts of prospective yield, the collapse of the marginal efficiency of capital, increasing liquidity preference and consequently higher interest rate

[4]Jarrow state that *"Economic theory tells us that market and credit risks are intrinsically related to each other and not separable"* (Jarrow & Turnbull, 2000).

[5]Nonlinear effects are the basis of the compounding effect. For nonlinearity a necessary condition must hold; the impossibility to maintain fixed one risk while quantifying the other (see section 1 for an example).

[6] *"Part of that profit naturally belongs to the borrower, who runs the risk"* Book 1, chapter 6, paragraph 18.

[7]Book 4, chapter 1, paragraph 16.

[8]Book 2, chapter 4, paragraph 15.

[9]Chapter III, pages 97-98.

[10]Chapter 12, point 4.

are causes of lower production which leads to the deterioration of the state of credit and payment capacity[11] (Keynes, 2008). The credit channel of monetary policy is another branch of this kind of work; falling asset prices leading to lower collateral value hindering the capacity to refinancing (Bernanke, Gertler, & Gilchrist, 1994).

It is worth to quote some theories where causality is flowing from credit risk as cause to market risk effects. This kind of dynamics may have complementary or substitutive character with those that have market risk variables as independent variables. If complementary, the response of the market risk side to movements in the credit risk side may be neutral, act as limit or induce amplifications of the processes created when the market risk side is considered as explicative. In particular, Hayek's theory is substitutive; contrary to Smith and Keynes, it postulates that low interest rates cause default as credit creation generates inefficient capital allocation (and low value of these assets), leading to the default of these projects (Von Hayek, 1933). Which of the two effects of the interest rate prevails is a matter of the particular circumstances that each situation under analysis possess. Fisher (1933) claims that (higher) default risk affects (lowers) the market price of assets; giving rise to the debt deflation theory of crisis. To a certain point this theory is complementary, for instance, with that of the credit channel. The theoretical links are confirmed empirically by Borio et al.; they show asset price booms leading to real fluctuations exploring data of the 1980 decade onwards (Borio, Kennedy, & Prowse, 1994).

To this point, the theories cited in this section generate dynamics that may give rise to compounding effects trough different incentives and path sequences.

Adam Smith already highlights the benefit of diversification[12]. In the 20th century financial economics becomes rigorous bases. The structure of Irving Fisher's (1930) two-period model allows to compare assets in an intertemporal framework as an agent choice problem. The Arrow-Debreu (1954) formulation of General Equilibrium with the complete set of future state-contingent markets gives the foundation for the complete pricing of the intertemporal economy and introduce the Arrow security (1953) that allows to buy goods given a future state of nature. Fisher's agent choice problem (allowing the incorporation of assets) and Arrow-Debreu contingents markets (giving the possibility of risk incorporation) translate into financial economics.

Markowitz (1952) setting the modern portfolio theory introduce diversification in a context of maximizing expected return of portfolio given risk preferences; the capital asset pricing model (CAPM) (Treynor, 1961) measuring asset risk respect to a free risk alternative and the overall market risk and Tobin's (1958) paper with the separation theorem, shape the modern form of diversification theory. Later evolutions of probabilistic measures of risk such as Value at Risk (Guldimann, 2000) or Expected shortfall (ES) (Acerbi & Tasche, 2002)

[11] Chapter 22, point 2.

[12] *"When a great company, or even a great merchant, has twenty or thirty ships at sea, they may, as it were, insure one another. The premium saved upon them all, may more than compensate such losses as they are likely to meet with in the common course of chances."* Book 1, chapter 10, paragraph 28.

take in account only tail losses of the distribution (not profits), evolving from a measure based on variance (volatility) to one founded in metrics on extreme losses. The important assumption for this research is that they all need separable assets to construct their models. The results (capital gains or losses and positive or negative returns) need to be attributed to the individual position and the relation with other assets requires to have a linear characterization. In particular, Markowitz identifies linear correlation between assets to get his measure of expected return; also CAPM individualizes specific asset risk in a context of linear correlation and Tobin individual choice is based likewise in linear correlation of assets. VaR methodologies (Guldimann, 2000) use linear apparatus (correlation coefficients) as well and ES (Acerbi & Tasche, 2002) may be seen as a transformation of VaR by a linear operator (if the average exist), an average of VaR's beyond some tail probability of the distribution.

3 Variables

For market risk classes we employ[13] the Dow Jones *dowjones* (as an example of stocks), the price of wheat *wheat* (as an example of commodity price). The ten year treasury bond *bondyield* allows us to capture interest rate risk, whereas the difference between prices of government and corporate bonds represents spread, *spread*. The exchange rate fx lets us follow exchange rate risk. We work with data from 1988 to 2017. The procedure for all variables of the market risk side is to select from daily data the three biggest losses (this makes near 5 percent) of a given year and take the average of them. Thus we have the empirical shortfall for each variable for each period[14].

We use only one variable for the credit risk side of the empirical study; nonperforming loans (NPL)[15] *nplx*. They are only four data points for each year, to approximate shortfall we use the higher percentage of each year. This variable allows to see simultaneously the effect of market risk variables and factors on probability of default (PD), the exposure at default (EAD) and the loss given default (LGD); addressing the channels postulated by Breuer et.al. (Breuer, Jandacka, Rheinberger, & Summer, 2008). Nonperforming loans is a direct measure of default whereas credit default swaps (CDS) works with

[13]Percentage of price changes in the case of all variables. All variables are expressed as positive values to facilitate interpretation.

[14]Data sources for market risk side are prices of a ten-year treasury bond (Board of Governors of the Federal Reserve System (US), 1988-2017, (accessed October 5, 2018)), prices of a ten-year corporate bond minus to the price of the ten-year treasury bond (Federal Reserve Bank of St. Louis, 1988-2017, (accessed October 5, 2018)), the Dow Jones index (Yahoo Finance, 1988-2017, (accessed July 11, 2018)), prices of wheat ("www.macrotrends.net", 1988-2017, (accessed July 11, 2018)) and the exchange rate between the US dollar vs Euro (previous 1999 the ECU) (Board of Governors of the Federal Reserve System (US), 1988-1998, (accessed July 11, 2018)) and (Board of Governors of the Federal Reserve System (US), 1999-2017, (accessed October 5, 2018)).

[15](Federal Financial Institutions Examination Council (US), 1988-2017, (accessed October 5, 2018)).

expectation over defaults[16]. Nonetheless, CDS is a usual choice as variable for credit risk proxy as we may see in section 4. In our case, to highlight the points of possible nonlinearity suffices to decompose one risk only. In order to make the argument as simple as possible we break down the market risk side. It is necessary to remember that feedback effects (mutual influence) and simple explanatory power may also come from the credit risk side as we have seen in the economic theory in section 2.

4 Statistical Analysis

The aim of this section is to explore if both types of risk are related in a nonlinear way.

The benefits of diversification inducing lower risk quantifications depend critically on the assumption of linearity and the possibility to use the following equation[17],

$$R_{C_t+M_t} = \sqrt{R_{M_t}^2 + R_{C_t}^2 + 2\rho R_{C_t} R_{M_t}} \tag{4.1}$$

which has deep roots in the Cauchy-Schwarz inequality and is justified by economic theory exposed in section 2 assuming separability. If the relation between market and credit risk is nonlinear then it is not possible to use equation 4.1.

The election of statistical procedures depends usually on specific characteristics of the problem, and frequently there is more than one choice available[18].

The characteristics of the problem create the need to work with direct observable data[19] taking the simplest method that is consistent with the exposed theory[20] and goal of the paper. We observe the empirical distributions of the

[16]The measure option of CDS as credit risk proxy has some of his dynamic attributed to characteristics singular to that market as reported by Boehmer *"The fact that the informational efficiency of equity prices is reduced suggests that the information environment changes after CDS introduction. It is possible that there is more speculation in CDS markets when market conditions are poor[,] CDS markets are opaque and highly decentralized"* (Boehmer, Chava, & Tookes, 2015); evolution of CDS may be (erroneously) viewed as caused or correlated by/with market risk variables.

[17]Where R states for Risk, C_t for credit, M_t for market and ρ for the correlation coefficient.

[18]For instance, principal component analysis (PCA) is a powerful technique that allows to concentrate analysis on the group factors that are responsible for most of the variation of the system. The methodology is still evolving and a very active field of research (Kelly, Pruitt, & Su, 2018) but have some shortcomings that are particularly relevant in the case studied in this paper. For instance in a very recent publication Kelly states that *"PCA is ill-suited for estimating conditional versions it can only accommodate static loadings. Furthermore, PCA lacks the flexibility for a researcher to incorporate other data beyond returns to help identify a successful asset pricing"*. Moreover, the parameters of PCA are linear (at least in the canonical version) and difficult to analyse as they are mathematical constructions without direct interpretation in terms of the original set of variables. Reconstruction of the original variables is not possible; only approximations are achievable. This is critical in our case because of separability (needed for diversification); if we can't separate linear components of market risk, diversification cannot be achieved.

[19]We can check if expectation of default corresponds to default rates or not, whereas CDS pricing may be linked to special characteristics of the market.

[20]Necessity to know which is the positions source (separability) of price or return variability to make possible to benefit from diversification. This feature does not allow to use PCA.

shortfalls and use ordinary last square (OLS) for regression analysis.

The division of the components of credit and market risks sides in dependent and independent variables has been done using both possibilities; credit risk proxy's determining market risk proxy's and the other way around.

Reviewing some results may help to establish the complementary or substitutive character of the results that have they theoretical counterpart in section 2. As stated there, the responses of the market risk side to movements in the credit risk side are neutral, act as limits or enhancements of the movements when the market risk side is considered as independent.

Gilchrist and Zakrajšek[21](2012) stated that shocks to the price of a default risk proxy that are uncorrelated to the current state of the economy, cause significant declines in aggregate demand components and output as well as in equity prices, providing evidence of the statement of Keynes who relates the "collapse in the price of equities" to the "state of credit" in one of the possible dynamics.

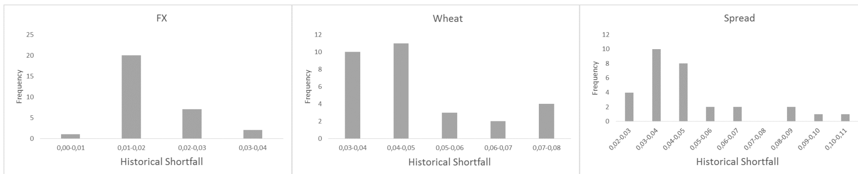
As Hayek and Fisher in the theoretical analysis, Friewald et al. on the base of empirical data (Friewald, Wagner, & Zechner, 2014) encounter that the term structure of CDS spreads contains risk premium information that is relevant for stock valuation and is not captured by conventional risk measures. Also in the framework of Hayek and Fisher; Norden and Weber (2009) owing to econometric evidence note that at firm level CDS spread changes Granger cause bond spread changes for more individual firms than the other way around, but equity prices lead credit default swaps more in the theoretical posture of the credit channel, Thornton and Keynes as stated in section 2.

Market risk variables has been used as independent variables explaining credit default swap spreads with equity volatility (Zhang, Zhou, & Zhu, 2009) or by banks that use internal ratings-based (IRB) approaches assigning credit risk in the credit portfolio depending on their market risk exposure in the trading book (Abbassi & Schmidt, 2018).

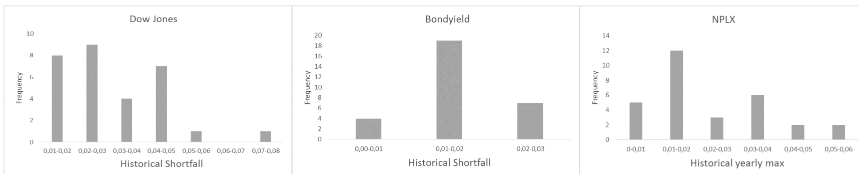
4.1 Distributions

As ES depends on the shape of the profit-loss distribution tail, we construct distributions of the shortfalls based on empirical data. Actual shapes for the distributions of average ES have no theoretical limitations. High volatility of possible values is a condition that allows nonlinearity because large changes have nonnegligible probabilities. Consequently, of all the features of the distribution (taking the conservative assumption of unimodality); for nonlinearity analysis is relevant that the shape can varies from normal or exponential to those that have non negligible probabilities of extreme events (e.g. crisis such as the of stock market of 1987 or the general of 2008) such as power law distributions.

[21] Gilchrist and Zakrajšek divides a "*credit spread index into a predictable component that captures the available firm-specific information on expected defaults and a residual component, the excess bond premium, which we argue reflects the price of default risk*".



(a) fig 1 (b) fig 2 (c) fig 3

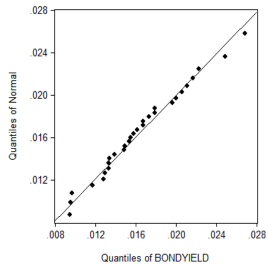


(d) fig 4 (e) fig 5 (f) fig 6

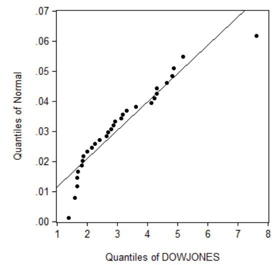
Figure 1: Empirical Shortfall Distributions

If the central limit theorem works for the samples of averages each of the distribution should approach a normal. Only *bondyield* has an approximately normal shape. As for the others, a visual inspection shows that the distributions does not converge to the normal distribution for the thirty samples of averages that we have. Except for *bondyield*, none of the distributions are monotonic decreasing and they maximum value is situated near the lowest losses. After the maximum, *dowjones* and *nplx* have a positive first difference (derivative) leading to a local maximum. The next step is to take a more rigorous test to confirm the visual analysis. We plot the distributions in a quantile-quantile graph for each variable in figure 2.

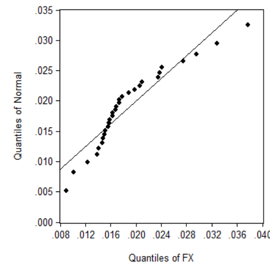
The Q-Q plots of figure 2 show a good fit of *bondyield* to a normal distribution. All other variables does not display a reasonable correspondence with the normal distribution, in particular the plots are consistent with right skewness (curved pattern with slope decreasing from left to right) and (or) heavy right tails (points below the line on the right side). Given that the fitting lines are flatter than a 45 degree line the empirical distributions appear to have data more dispersed than the normal, which is consistent with the heavy tail hypothesis. Given the rather reduced sample size (30), although the shortfall does not converge to a normal; this is not sufficient evidence to discard it, convergence may be slower due to the skewness of the parent distribution. The test of figure 2 is not indisputable evidence of heavy tails, nevertheless it is consistent with its existence and should be considered in further investigations. Therefore in subsection 4.2 we complement the distribution test with tests of the relations between variables using regressions.



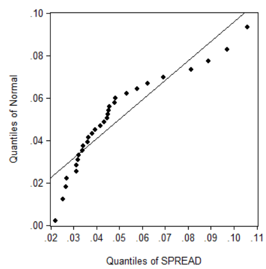
(a) fig 1



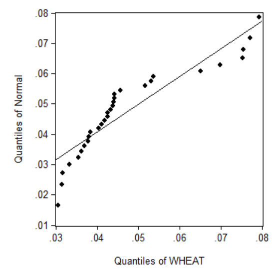
(b) fig 2



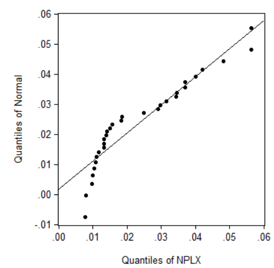
(c) fig 3



(d) fig 4



(e) fig 5



(f) fig 6

Figure 2: Q-Q Plots

4.2 Regressions

From section 3 dependence relations[22] are stated in the following form:

$$nplx = f(bonyield, dowjones, fx, spread, wheat) \tag{4.2}$$

All variables are stationary except *spread*[23]; in this case the other variables would be explained by the stationary part and the nonstationary part (trend)[24]. All regressions have a constant term.

For the quadratic model we may safely assume that the extrema occur at independent risk 0 and dismiss linear terms. We may suppose that we are studying only the nondecreasing part of the curve, $\dfrac{dR_{C_t}}{dR_{M_t}} \geqslant 0$[25]. This is consistent

[22] Credit risk is not only explained by economy wide market risk related variables. Idiosyncratic variables also play a huge role, such as dependence of a particular market, the risk aversion of the management of each institution or exposure to fraud.

[23] All test are done at 5 percent significance when not indicated otherwise. In the text, standard deviations are given in parenthesis.

[24] Not detrending spread's decreasing trend does not modify the conclusions of the analysis. From an economic point of view the trend showed by the econometric analysis may be temporal, otherwise it could lead to nonconsistent results such as very negative spread rates.

[25] In other words we are assuming that a higher independent risk cannot trigger lower dependent risk. Dependent risk is a monotonic nondecreasing function of independent risk in the relevant section of the curve.

with the theory exposed in section 2, in particular with Adam Smith respect to the adverse selection problem and the credit channel formulation.

We begin first to quantify the relations between variables that measures market risk in table 1. This will allow us to see the network of dependence relations and to elaborate the form of information flow from the market risk variables to credit risk that come out from the statistical relations.

Variable *bondyield* explains *spread* and *dowjones*, but have not direct explanation power over *nplx*. Through the influence in *spread* which determines *wheat* and *dowjones* which explains *fx* it determines *nplx*.

Variable *spread* determines *wheat* with one lag, but have not direct explanation power over *nplx*.

The variable *dowjones* is determined by *bondyield*, *fx* and *wheat* (squared). It determines *fx* and *wheat* (squared); explains *nplx* individually but has not significant contribution together with *fx* and *wheat* and any of them individually over *nplx*.

Variables *fx* and *wheat* determines *nplx* alone and also combined.

Table 1: Significance of pairwise relations between market risk variables

Market Risk variables	Indep.	Bondyield		Spread				Dow Jones				Wheat	FX	
Dep.		linear	square	linear	lag	square	lag squared	linear	lag	square	lag squared	square	linear	square
Bondyield	coefficient			0,1046		0,7101		0,1349		1,5571				
	st dev			0,0347		0,2895		0,0546		0,6844				
	p-value			0,0054		0,0206		0,0198		0,0308				
	Adj-R^2			0,2178		0,1475		0,1498		0,1259				
Spread	coefficient	2,3401	65,7418											
	st dev	0,7768	22,6379											
	p-value	0,0054	0,0071											
	Adj-R^2	0,2178	0,2040											
Dow Jones	coefficient	1,3273	34,3150									3,2523	0,9168	19,8671
	st dev	0,5370	15,8474									1,5350	0,3796	8,2298
	p-value	0,0198	0,0390									0,0431	0,0225	0,0226
	Adj-R^2	0,1498	0,1128									0,1074	0,1428	0,1427
Wheat	coefficient				-0,2969		-2,2571			4,7701				
	st dev				0,1203		0,9624			2,2294				
	p-value				0,0202		0,0266			0,0412				
	Adj-R^2				0,1538		0,1385			0,1098				
FX	coefficient							0,1880	0,1897	2,4114	2,2516			
	st dev							0,0779	0,0816	0,9550	1,0001			
	p-value							0,0225	0,0279	0,0175	0,0327			
	Adj-R^2							0,1428	0,1358	0,1564	0,1269			

The relations between variables shows different patterns.

Sensitivity: The linear response of *spread* to *bondyield* with a coefficient of near $2,3$ shows a high elasticity. *Bondyield* has in both cases (*spread* and *dowjones*) a significant linear coefficient greater than one, whereas *fx* influence over *dowjones* is near one, being one within the band of confidence.

Negative relation (Oscillations): The only relation that can produce negative relation (oscillatory) dynamics are that of *spread* explaining *wheat*. Oscillations arrives because *wheat* is a proportion α of *spread* (from an accounting point of view expressed both shortfalls as percentage) in any period $wheat = \alpha * spread$.

Giving that the coefficient is lower than 0 and greater than -1, approx. $-3/10$, if the proportion α is lower than approx. $3/10$ the oscillation would be explosive whereas if it is greater there would be damped[26].

Squared explicative variables: Only when squared have *wheat* explanation power over *dowjones*. Apart from *spread* explaining *wheat* all relation are of an increasing from.

Model significance: The only variables that have relevant influence over the other in all models is *dowjones* over fx.

The adjusted R^2 is rather low in all cases, but this is to expect given that al those variables have other influences; from macroeconomic level, other risk variables and sector level variables. Once we have established the relations of table 1 we may see the quantitative nature of influences of the variables that directly explains *nplx*.

Table 2: Significant direct relations of market risk variables with non performing loans

Market risk variables	Dependent nplx	alone				combined			
		coefficient	st dev	p-value	Adj-R^2	coefficient	st dev	p-value	Adj-R^2
Dow Jones	lag	0,4717	0,1814	0,0149	0,1707				
	lag squared	5,8963	2,1962	0,0123	0,1815				
Wheat	linear	0,5019	0,1647	0,0050	0,2223	0,4655	0,1520	0,0049	0,3437
	lag	0,5436	0,1613	0,0023	0,2702	0,4989	0,1305	0,0007	0,5254
	square	4,6182	1,4718	0,0040	0,2337	4,1691	1,4104	0,0064	0,3132
	lag squared	5,1313	1,4231	0,0012	0,3000	4,4800	1,1751	0,0008	0,5329
FX	linear	0,9603	0,3919	0,0208	0,1472	0,8584	0,3454	0,0194	0,3437
	lag	1,2775	0,3654	0,0016	0,2862	1,1781	0,2990	0,0005	0,5254
	square	19,4149	8,6122	0,0322	0,1234	15,8883	7,7158	0,0492	0,3132
	lag squared	28,1578	7,8311	0,0013	0,2987	24,5678	6,4603	0,0008	0,5329

The econometric analysis of table 2 shows that market risk proxy's are leading indicators of credit risk proxy's. We will work with this result in section 5.

The response to lagged variables of *nplx* is a signal of slow reaction to (adverse) movements in the asset market expressed via the shortfall function and an indication that readjustment take some time. This lagged response of *nplx* may also be related to an adjustment process to non expected changes. If changes would be expected; adaptation would be instantaneous because the reaction would be anticipated, avoiding adjustment processes. Thornton's state of confidence, Keynes' speculative confidence and state of credit are especially important to induce that lagged behaviour. Only when expectation for the future have changed then the process of adaptation may develop.

More in detail, fx_{t-1} as explicative variable indicates that devaluations in the specific shortfall framework may be manifestations of a relative lower interest rate in the home market (capital is allocated outside) or a current account with importations bigger than exportations; leading to higher default rates and

[26] α is not necessary a constant because ES proportions between classes may change.

giving some indications of a possible recession (later or at the moment) in the home market (lower activity and therefore lower output). Deterioration in the exchange rate may not conduct to a better economic performance if the Marshall-Lerner condition (Marshall, 1923) is not satisfied. Even if fulfilled, the depreciation may affect some sectors that cannot compensate the devaluation.

On the side of $wheat_{t-1}$ lower prices of commodities are anticipating higher default rates. Fragile conditions in the commodity market translate to higher default rates, as this weaker circumstances are indications of recession (later or at the moment). It produce a sector crisis (change in the exchange terms of some sector) that have the potential to expand to the entire economy via lower spending of the affected sector (not compensated by other public or private spending) that account for the augmenting default rates (Prebisch, 1962).

Together, theory and econometric evidence supports that shocks in the capital market represented by the shortfall function (large falls in one day) have repercussions in the good market (lagged response) with less income ($wheat$) or higher debt (fx), which finally elevate the default rate.

As we work with expected shortfall we are looking at the worst results in some Δt period. It is not necessary that this translates to the results of the overall period; big losses may be compensated by big gains. In our case big losses of market risk classes are interrelated to growing default outcomes. Events occurring in a small proportion of the period considered have statistical significance over default rates; representing shocks that work out his influence in the next period.

To sum up, we present figure 3; continuous lines denote mutual significant coefficients. The dashed line from DJ to $nplx$ stand for significant coefficient only in pairwise relation. Arrows indicate direction of significant coefficient. When no arrows are depicted, coefficients both ways are significant (feedback)[27].

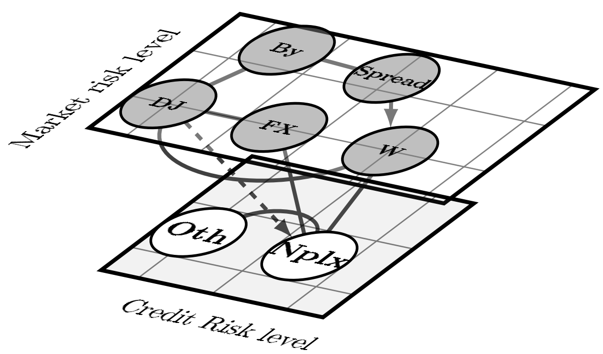


Figure 3: Network of variable relations

From figure 3 we can see that although not all market risk variables have direct explanation power of $nplx$, all have influence over the credit risk proxy.

[27]Where By stand for $Bondyield$, DJ for Dow Jones, W for $wheat$ and FX for fx, $spread$ has the usual interpretation. Oth represents other variable that influence $nplx$.

Variable *nplx* has a significant coefficient 0.183893 (0.075042) and p-value of 0.0208 respect to fx. Also, *nplx* hold a significant coefficient 0.496410 (0.162859) with p-value of 0.0050 respect to *wheat*. This is a statistical confirmation that credit risk variables may influence market risk variables as stated in section 2 from a theoretical point of view and at the beginning of section 4 with practical examples. Complementary theories like the credit channel view and the debt deflation theory may reinforce each other and give rise to feedbacks and nonliniearities. So, holding market risk fixed while quantifying credit risk would not be possible in this case.

5 Tests of nonlinearity: functional forms and parameter stability in regressions

We use two regressions, one with *wheat* and fx with one lag each and the other with those lagged variables squared[28].

$$S = c_2 + \beta_{sw} wheat_{t-1}^2 + \beta_{sf} fx_{t-1}^2 = nplx_t \tag{5.1}$$

$$L = c_1 + \beta_{lw} wheat_{t-1} + \beta_{lf} fx_{t-1} = nplx_t \tag{5.2}$$

We employ two sample sizes, one of 15 and the other of 20 (except for the first regression due to the lag of the independent). This allow us to have 16 and 11 regressions respectively.

The source of the nonlinearity may be nonlinearity in the coefficient or the independent variable itself can be nonlinear[29].

The analysis is performed using a rolling-window regression, based on a modified bootstrap estimation with a fixed window size for various sample sizes for the two models (Swanson, 1998). This allow us to compare the two specifications, on the basis of their performance in the evolution of measures such as Durbin-Watson, F probability and adjusted R^2. The next step is to examine the path of the coefficients of the independent variables.

In subsection 5.1 we study the case of squared vs linear variables; in this case the regression framework itself is still linear since the parameters of the variables remains linear. In subsection 5.2 we study if the parameters are linear. If they are not, the regression framework is not linear.

5.1 Lagged response: Linear and Squared Variables

The results of lagged vs lagged squared regressions are striking similar. Besides minor divergences, the results are analogous.

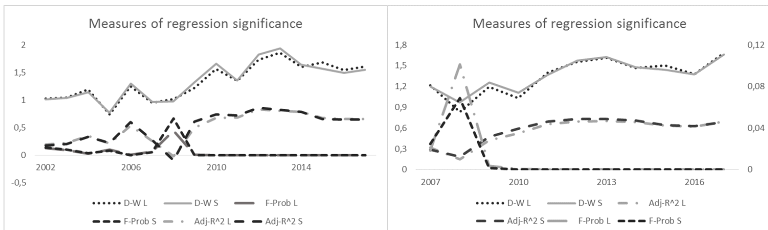
The sum of the result of both independent variables estimated contribution[30] give equal curvature for equations 5.2 and 5.1; different constant for both cases

[28] In this section S stand for squared and L for linear using upper and lower case indistinctly.

[29] For example, when squared, the derivative changes with value of the variable. The variable changes are not proportional, they change when the value of the variable changes.

[30] Coefficient times value of variable.

adjust to produce very similar approximations. The linear equation need to have a negative constant term -0,022654 to reach the best approximation. The constant term of the squared equation is 0,002509, very near to 0. Differing in the constant term and having the same curvature they must have the same derivative. Therefore, as we can see in equation 5.6, there is a precise analytical relation between equations 5.2 and 5.1[31].

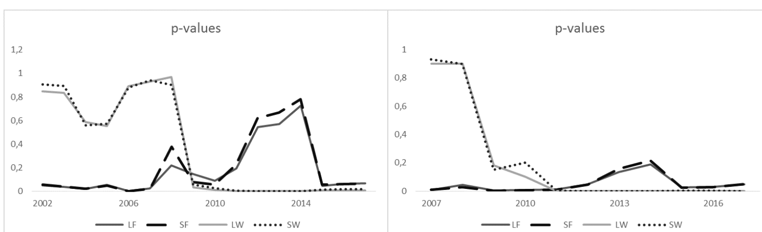


(a) Series 15 data points

(b) Series 20 data points. Secondary axis for F probability

Figure 4: Variations of measures of regression significance

The Durbin-Watson (D-W), F-probability and adjusted R^2 ($Adj - R^2$) all follow very similar patters with almost no divergence.



(a) Series 15 data points

(b) Series 20 data points

Figure 5: Variations of measures of p-value

The same reasoning for the other measures of significance applies for the value of the sequence of p-values.

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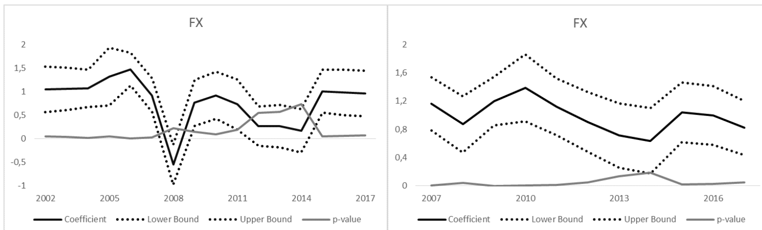
$$\frac{dS}{dt} = 2\beta_{sw} wheat_{t-1} + 2\beta_{sf} fx_{t-1} = \frac{dnplx_t}{dt} \quad (5.3)$$

$$\frac{dL}{dt} = \beta_{lw} + \beta_{lf} = \frac{dnplx_t}{dt} \quad (5.4)$$

$$2\beta_{sw} wheat_{t-1} + 2\beta_{sf} fx_{t-1} = \beta_{lw} + \beta_{lf} \quad (5.5)$$

$$2\beta_{sw} wheat_{t-1} - \beta_{lw} = \beta_{lf} - 2\beta_{sf} fx_{t-1} \quad (5.6)$$

5.2 Lagged response: Parameter change



(a) Series 15 data points (b) Series 20 data points

Figure 6: Coefficient variation FX

In the case of fx_{t-1}:

1) Not all the coefficient values are statistically significant. For the 15 sample size series this is due to that the standard deviation remains in the range of 0,4-0,5 (except for 2005 and 2011) whereas the coefficient value changes in the range 1,0-1,5 for significant coefficients. For the case of 20 sample size series the variation of the standard deviation is even lower, between 0,37 and 0,47 and the coefficient varies between 0,8 and 1,2.

2) For the 15 sample size series the samples that end in the year 2006, 1,472622 (0,345568) is the maximum with the minimum at 2015 reaching 1,005671 (0,453055). The 20 sample size case had a maximum 1,387125 (0,472888) in 2010 and minimum 0,819761 (0,382871) in 2017.

3) For the 15 sample size series the sample that end in the year 2006 1,472622 (0,345568) is approx. 3/2 of that of 2003 1,056686 (0,447204) and that of 2015. In the 20 sample size case the maximum value of 2010 is approx. 7/4 of that of 2017.

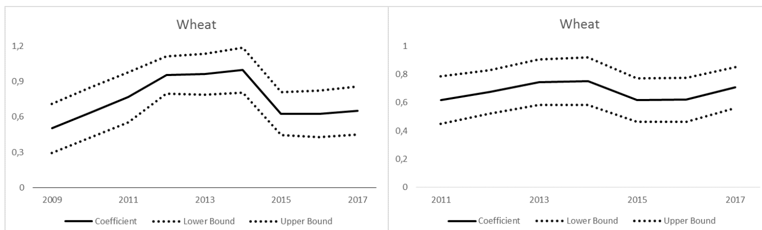
4) For the 15 sample size series the change between years 2003-06 are very fast. And so for the 20 sample size case between 2008-10. It changes from 0,871311 (0,396553) in 2008 to 1,200404 (0,341352) in 2009 with maximum in 1,387125 (0,472888) in 2010. This is not limited to the crisis year, from 2011 to 2012 it change from 1,119881 (0,400607) to 0,903654 (0,422161) also for the 20 sample size case.

5) For the 15 sample size series the samples that end in the year 2006 1,472622 (0,345568) is approx. 3/2 of that of the 2003 1,056686 (0,447204) although both values are contained in the bands of confidence of the other. For 20 sample size series the coefficient of 2008 0,871311 (0,396553) is not in the lower side of the band of confidence of the 2010 coefficient 1,387125 (0,472888).

6) For the 15 sample size series there is no higher lower bound of confidence than any upper bound, but the band of confidence of 2006 does not cover the parameter value of 2003; only the upper side of the band of confidence of 2003 is contained. And the upper side of the band of confidence of the 2004 coefficient does not cover the value of the 2006 parameter. The minimum of 2015 with

1,005671 (0,453055) upper side of the band of confidence does not contain the maximum of 2006. For 20 sample size series there is no higher lower bound of confidence than any upper bound but the band of confidence of 2009-10 does not cover the value of 2017. In all commented cases the coefficients are statistically significant.

7) Coefficient changes over and bellow one imply (regime) changes in the relation of the variables from very-sensitive (coefficient>1) to less sensitive (coefficient<1). Those changes are present in the fx_{t-1} case.



(a) Series 15 data points (b) Series 20 data points

Figure 7: Coefficient variation Wheat

In the case of $wheat_{t-1}$:

1) Only the variation of statistically significant coefficients of the $wheat_{t-1}$ variable are showed. For the 15 sample size series the standard deviation remains fairly stable near 0,2. For the 20 sample size case, it is also relative stable, approx. between 0,45 and 0,6.

2) For the 15 sample size series the minimum is 0,503563 (0,208042) in 2009 whereas the maximum of 2014 lies at 0,997029 (0,189984). For the 20 sample size case, the maximum changes are 0,617204 (0,167755) in 2011 to 0,750769 (0,167846) in 2014.

3) For the 15 sample size series the relation of coefficients between 2014 and 2009 is almost 2/1. For the 20 sample size case the relation 2014-2011 is about 6/5.

4) For the 15 sample size series, the coefficient drops a proportion of 2/5 from 2014 to 2015. For the 20 sample size case the biggest increase is of approximate 1/6 between 2016 and 2017.

5) For the 20 sample size case, although been the changes in the coefficient relevant; it may be argued that they are restricted and preserve a linear relation, the maximum changes are 0,617204 (0,167755) in 2011 to 0,750769 (0,167846) in 2014, where both coefficients are in the error margin of the other.

6) For the 15 sample size series the lower side of the band of confidence of the 2012-14 coefficients does not cover the upper bound of confidence of the 2009 coefficient. For the 20 sample size, no upper bound of confidence is smaller than any lower bound of confidence.

7) Only for the 15 sample size series the upper bound of confidence is greater than one for the period 2012-14.

6 Concluding remarks

After briefly reviewing the economic theory behind credit-market risk relations, we use five time series to represent market risk via its price changes. Next we take the three worst results of a year for each of them and take their average, identifying the result as the historical shortfall. With these values, using those as independent variables, we were able to perform a regression with the worst yearly data of nonperforming loans as dependent one.

The test of the empirical distributions of shortfalls of the selected variables shows that they don't follow a normal distribution; except for *bondyield*. This may be attributed to the slow convergence due to the skewness of the original distribution or to the power law nature of the distribution of the shortfall. Therefore the presence of extreme values in a greater proportion than in a normal distribution cannot be discarded.

After reviewing briefly the network of variables we compare (both lagged) linear and squared regressions.

The lagged response of credit risk has theoretical justification in changing expectations (Thornton and Keynes) and dynamics running via the good market (Marshall and Prebisch). Lagged linear and lagged squared models are very similar respect to quantitative statistical measures of significance and does not allow to extract further conclusions.

At the final stage we study (for the lagged linear regression) if the parameters of variables are stable linear. The change in the parameter value is the most robust indication that nonlinearity may be present. The parameter variation in the case of fx gives significant evidence of nonlinearity in terms of relative variation and speed of change. In contrast, in the 20 sample case; for *wheat* the proportional variation is not so large in terms of relative variation and speed of change[32]. It is precisely the case of foreign currency loans one of the examples of the Basel Committee of Banking Supervision (2009), where compounding effects may appear[33]. This is also in supported by the theories of Smith, Thornton, Keynes and the credit channel theory as exposed in section 2.

A separate analysis of credit risk and exchange risk class of market risk would lead to incorrect measurements of total risks. Respect to the commodity risk class of market risk the quantitative change of the parameter may be more in accordance with the linear hypothesis.

A changing parameter is in term of calculus the analogue of a changing derivative $\frac{dR_{C_t}}{dR_{M_t}}$. Hence, a constantly changing association, implies that it would be impossible to maintain one risk fixed while quantifying the other. In other words, if the derivative is changing the change of credit risk is not always (proportionally) the same when market risk changes. Changes in market risk may trigger bigger or smaller changes (second derivative of credit risk respect to market risk is not zero) in credit risk which changes the total risk sum. From the

[32]Although the 15 sample case give evidence of nonlinearity, the 20 sample case is a very clear case for linearity.

[33]*"The ability of a domestic borrower to repay a loan in foreign currency depends in a nonlinear way on fluctuations in the exchange rate."*

empirical data we see that the pattern of the parameter for fx follows oscillations, which correspond to what can be expected from theory and observation; relations are changing and are not monotonic. Therefore, the econometric evidence suggest that credit risk is positively related to the fx part of market risk with different intensities over time.

There are still good arguments for diversification. A careful selection of positions; all subject to market risk or credit risk, that have linear statistical relationship between them in term of risks, may be chosen. The rather surprising consequence is that they are limits to diversification; for being realistically beneficial it must exclude those assets and positions that are subject to nonlinear relations respect to risks. This implies that the portfolio that includes all possible positions in some proportion (Mutual fund theorem, (Tobin, 1958)) is also constrained by the condition to not include positions that have nonlinear risk connexions. In very basic terms this remit to the statement of the seminal models that assets need to be separable. If the interactions are so constituted that the relations cannot be obtained by a linear model, the assets under consideration are not separable. Furthermore, nonlinearities may appear only after a threshold; default rates only increasing if the interest rate is bigger than some value, therefore if expected shortfall is smaller than this value diversification may still work.

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