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3 April 2019

Online at <https://mpra.ub.uni-muenchen.de/93353/>

MPRA Paper No. 93353, posted 25 April 2019 17:50 UTC

Bitcoin's return behaviour: What do We know so far?

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April 16, 2019

Abstract

In this paper we study the daily return behavior of Bitcoin digital currency. We propose the use of generalized hyperbolic distributions (GH) to model Bitcoin's return. Our results show that GH is a very good candidate to model this return.

Keywords: Bitcoin, Cryptocurrency, Jumps, Generalized Hyperbolic distributions.

JEL codes: G1, C22

1 Introduction

Since Nakamoto (2008), the interest for cryptocurrencies has increased a lot, as today we have more than 2000 cryptocurrencies and many platforms trading them, but only 25 respond for 90% of market cap. In the top of the list we have Bitcoin who was the first and was online in 2009, since them has received a lot of attention mainly because its transparency.

Recently, Uquhart (2016) and Wei (2018) have explored the Bitcoin predictability, finding evidence of inefficiency. Also, Balcilar et al. (2017), argue that transaction volume can sometimes help predict returns. On the other hand, researchers have focused their attention on the Bitcoin's returns behaviour, Katsiampa (2017) use GARCH models to fit Bitcoin volatility. Earlier, Gronwald (2014) applies an autoregressive jump-intensity GARCH model. Scaillet et al. (2017) using high-frequency data find evidence that jumps are frequent in time. More recently, Chaim and Laurini (2018) finds evidence of discontinuous return jumps. Finally, Tiwari et al. (2018) and Bariviera (2017) finds some evidence of long memory in volatility.

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In this paper, we explore another kind of jumps activity in the dynamic of Bitcoins daily returns, the infinite-activity jump class. It is well known in the finance literature that infinite-activity jumps models are more suitable to model asset returns, basically because these jumps can capture both small and frequent jumps as well as large and infrequent ones. For these reason we propose a family of distributions called generalized hyperbolic distributions,¹ to model Bitcoin daily returns.

These GH distributions allow us to model excess of kurtosis and skewness, as for example Fajardo and Farias (2004) and Eberlein and Prause (2002) showed for financial asset returns. Our results show a very good fit of GH with the empirical data, we use five measures of fit and two subsamples, one starting in early 2013 and the other in 2017, this latter with more turbulence. In both cases the GH fit was excellent.

The paper is organized as follows Section 2 presents our model. Section 3 presents our sample. In Section 4 we present the results with GH and some of its subclasses. Section 5 concludes.

2 Generalized Hyperbolic Distributions

For any $x \in R$ the generalized hyperbolic distribution are is defines as

$$gh(x; \alpha, \beta, \delta, \mu, \lambda) = a(\lambda, \alpha, \beta, \delta)(\delta^2 + (x - \mu)^2)^{\frac{(\lambda - \frac{1}{2})}{2}} K(\lambda, \alpha, \delta\mu, \beta) \\ \times K_{\lambda - \frac{1}{2}}(\alpha\sqrt{\delta^2 + (x - \mu)^2}) \exp(\beta(x - \mu))$$

where,

$$a(\lambda, \alpha, \beta, \delta) = \frac{(\alpha^2 - \beta^2)^{\frac{\lambda}{2}}}{\sqrt{2\pi}\alpha^{(\lambda - \frac{1}{2})}\delta^\lambda K_\lambda(\delta\sqrt{\alpha^2 - \beta^2})}$$

and

$$K_\lambda(x) = \frac{1}{2} \int_0^\infty y^{\lambda-1} \exp\left(-\frac{1}{2}x(y + y^{-1})\right) dy, x > 0$$

is the modified Bessel function of the third kind with index λ . $\alpha, \beta, \lambda, \delta$ and μ are the historical parameters that satisfy the conditions $0 \leq |\beta| < \alpha, \mu, \lambda \in R$, and $\delta > 0$.

In this family we find subclasses of interest as normal inverse Gaussian distribution ($\lambda = -0.5$), Hyberbolic ($\lambda = 1$), Variance-Gamma distribution, Cauchy, Student-t, among others.

¹Introduced by Ole E. Barndor-Nielsen (1977).

3 Data

We use data from <https://coinmarketcap.com>. The data consists of daily closing prices for Bitcoin in USD from April 28th, 2013 to March 27th, 2019. Fig. 1 shows Bitcoin returns over the whole period. Bitcoin prices are relatively stable before this period 2013. Also, we use a subsample starting January 1st, 2017. In Table 1 we have the descriptive statistics and also we add SP500 just for comparison, we can see that Bitcoin is more fat tailed than SP500 and positively skewed whereas SP500 is negatively skewed.

Table 1: Bitcoin descriptive statistics

Sample	Min	Max	Mean	Std.	Skewness	Kurtosis
All sample	-0.3575	0.2662	-0.0016	0.0432	0.1881	8.0163
Since 2017	-0.2251	0.2075	-0.0017	0.0446	0.1068	3.3427
SP500	-0.0402	0.0383	0.0004	0.0082	-0.4616	3,6840
SP500 (since 2017)	-0.0418	0.0484	0.0004	0.0082	-0.3647	2.2508

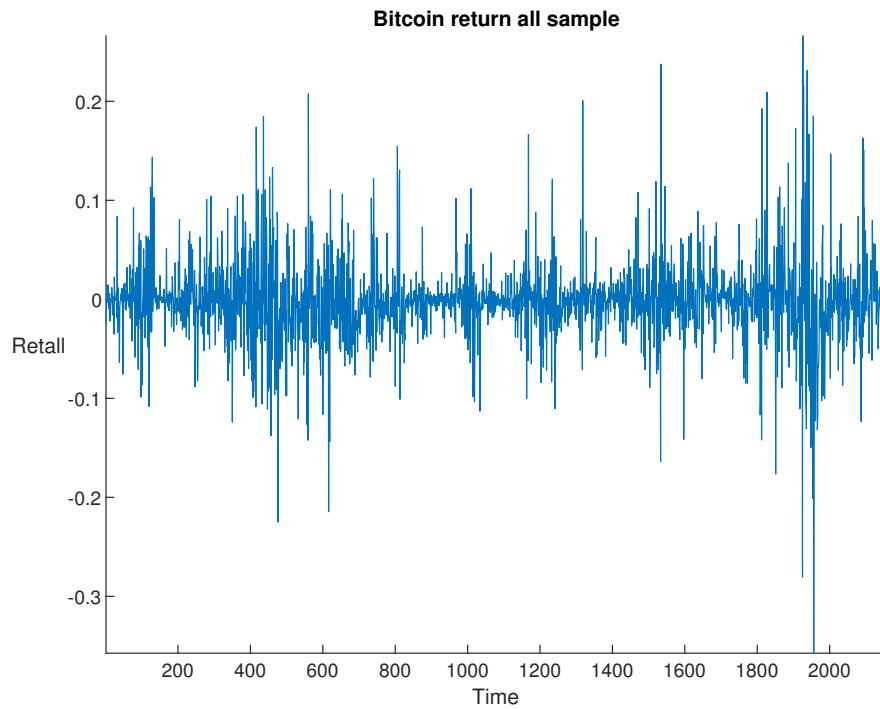


Figure 1: Bitcoin return

4 Results

First, we estimate the GH, NYG and Hyp parameters using maximum log-likelihood using both samples. The results are presented in Table 2 below.

Table 2: Estimated parameters of each distribution using the two samples

	GH		NIG		Hyp	
	All sample	Since 2017	All sample	Since 2017	All sample	Since 2017
α	19.4674	23.5019	9.1377	11.4462	37.0056	32.2970
β	-0.4301	-0.1917	0.0008	0.0006	-0.0060	0.1684
δ	0.0060	0.0013	0.0188	0.0265	0.0012	0.0011
μ	-0.0014	-0.0013	-0.0021	-0.0017	-0.0012	-0.0011
λ	0.2290	0.5961	-0.5	-0.5	1	1

With these parameters we obtain the respective GH, NIG and Hyp densities and can compare with the empirical ones. The figure 2, allow us to compare graphically the distributions.

[Fig. 2 around here]

Moreover, if we change the scale to the log-scale we can observe the good fit around the tails. In figure 3, we have the comparison for both samples.

[Fig. 3 around here]

Now we use statistical tests and distances to see this goodness-of-fit. As we can see in Table 3 below, the fit of Bitcoin's return with GH model is very good. Both Kolmogorov and Kuiper tests perform very well. Additionally, the other two distances Anderson-Darling and FOF². distances provides evidence of such goodness-of-fit. In all cases GH shows to be better than its subclasses. In the χ^2 test GH perform very well in the sample starting in January 2017.

[Table 3 around here]

5 Conclusions

We conclude that GH distributions fits very well Bitcoin's returns. Two interesting applications of our results are the use of GH distributions to price option in Bitcoins and Value at Risk calculations.

²See Fajardo, Ornelas and Farias (2008).

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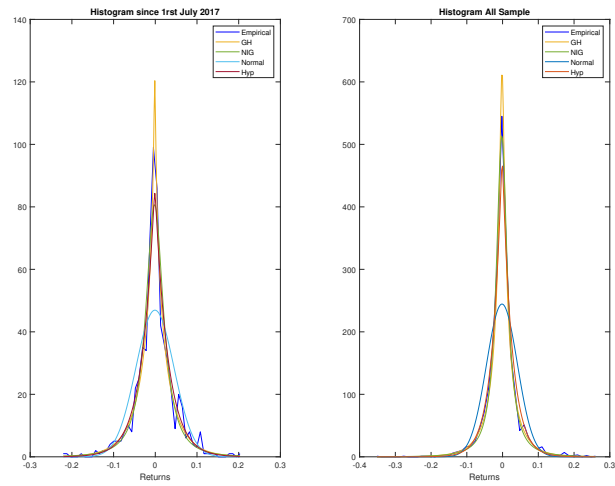


Figure 2: Densities

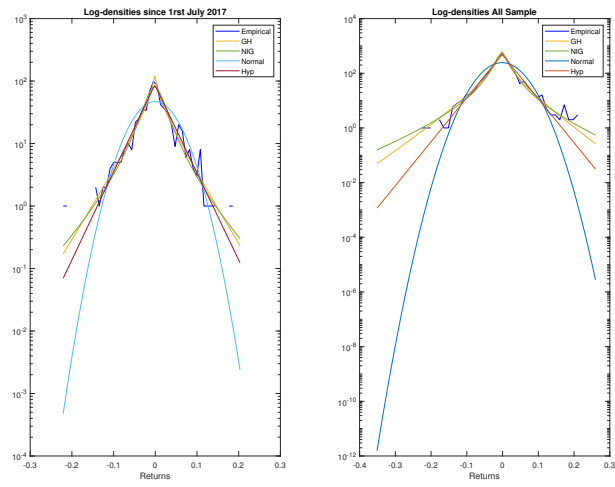


Figure 3: Log-Densities

Table 3: Tests and Measures of Fit

	GH		NIG		Hyp	
	All sample	Since 2017	All sample	Since 2017	All sample	Since 2017
KDist	0.0156 (0.6639)	0.0231 (0.883)	0.0243 (0.1556)	0.0326 (0.5028)	0.0598 (0.000)	0.0403 (0.249)
ADDist	0.0650	0.0783	0.0667	0.1152	1.1096	0.1205
KPDist	0.0261 (0.5092)	0.0344 (0.9434)	0.0420 (0.0134)	0.0612 (0.1411)	0.0824 (0.000)	0.7070 (0.0383)
FOFDist	0.1300	0.1566	0.1333	0.2304	2.2191	0.2409
$\chi^2(44)$	74.9438 (0.0017)	52.6961 (0.2025)	71.4507 (0.0044)	70.3057 (0.0059)	1152.8 (0.000)	83.3533 (0.0001)

Values in parenthesis are p - values.