Terms-of-Trade Effects of Productivity Shocks in Developing Economies

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Abstract

This paper studies the terms of trade effects from unexpected economy-specific productivity increases in both developing and advanced economies using a panel vector autoregression model with interactive fixed effects and the “max-share” approach developed by Francis et al. (2014). First, we find that the terms of trade of developing economies do not deteriorate after unexpected productivity increases and display similar dynamics to those of advanced economies. Second, studying these shocks in a more detailed classification of developing economies shows that the terms of trade worsen following an unexpected productivity increase in the least developed economies, implying that economic underdevelopment can result in unexpected productivity increases causing a deterioration in the terms of trade. However, this adverse effect of productivity increases disappears in the developing economies with some success in moving up the ladder of economic development, as implied by our finding that the terms of trade of these economies improve after an unexpected productivity increase.

Keywords: Productivity shocks; The terms of trade; Developing economies; Advanced economies.

JEL Classification Numbers: O19, O47, O57
1 Introduction

The Prebisch-Singer hypothesis predicts that developing countries suffer from a secular downward tendency in their terms of trade. This emphasis on country factors differs from the one on commodity factors in the original form of the hypothesis. Indeed, in its original form proposed independently by Singer (1950) and Prebisch (1950), the hypothesis proclaimed that there would be a secular downward tendency for the relative price of primary commodities vis-à-vis manufactured goods. This stress on commodity factors in the 1950s can be attributed to the then trade patterns of developing economies, characterized by the export of primary commodities and the import of manufactures. However, following the success of many developing economies to diversify their exports into manufactures in the subsequent decades, the discussion on the Prebisch-Singer hypothesis shifted away from commodity factors to country factors, as noted by Singer (1987).

Singer (1987) notes four economic arguments explaining the Prebisch-Singer hypothesis. First, primary commodities have a small price elasticity of demand and supply, as compared to manufactured products. Since developing economies have a larger share of primary exports than advanced economies, Singer (1987) points out this argument as an underlying explanation for the declining terms of trade of developing economies. Second, the demand for primary commodities expand less than that for manufactured products following an increase in the world income. This is partly caused by the lower income elasticity of primary commodities and partly by energy-saving innovations in advanced economies, which reduce the demand for primary commodities as an input in production. Baffes and Etienne (2016) provide supporting evidence for this argument. Third, developing economies are characterized by competitive markets and disorganized labor. This results in technical progress and productivity increases causing a proportional fall in prices in developing economies. Consequently, the economic rents brought about by technical progress and increased productivity in devel-
oping economies accrue mostly to home and overseas consumers in the form of lower prices. In contrast, in advanced economies, these rents accrue mostly to producers and labor in the form of higher factor incomes due to the presence of monopolistic firms, strong producers’ organizations, and organized labor in trade union; see Singer (1950) and Sarkar (1997). Fourth, Singer (1987) claims that the technological superiority of advanced economies enables multinational firms headquartered in advanced economies to obtain a Schumpeterian rent due to their monopoly power.

This paper aims to shed some light on the validity of the third argument explaining the Prebisch-Singer hypothesis. In particular, we identify unexpected productivity improvements and study the effect of these improvements on the terms of trade of the economies. Our approach to identify such improvements can be briefly summarized as follows: first, a productivity measure is needed to identify the improvements considered. We use real GDP per employed person as a measure of productivity based on data availability. However, the movements in this productivity measure can be caused not only by the shocks specific to the economy considered but also by global shocks affecting all economies with varying degree. The Prebisch-Singer hypothesis is unequivocal about the effect of productivity improvements specific to the country and predicts a larger decline in the terms of trade of developing economies than that of advanced economies after such improvements due to structural differences between developing and advanced economies, as discussed above. However, the same hypothesis should be silent regarding the effects on the terms of trade from global productivity shocks since these shocks simultaneously occur in all economies and whether the terms of trade of an economy improve or deteriorate depends on the degree with which the economy and its trading partners are affected by such shocks.\(^1\) Consequently, to test the prediction that productivity shocks have a more adverse effect on the terms of trade of developing economies than the terms of trade of advanced economies, it is crucial to separate economy-specific productivity

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\(^1\)To make the abstract concept of global productivity shocks concrete, digital revolution since 1950s and the oil price spike in 2008 can be given as examples.
shocks from the ones simultaneously occurring across the economies.

To this end, we develop a panel vector autoregression (VAR) model with interactive fixed effects, which has the desirable feature of including common factors. These factors, affecting each economy differently, represent unobservable global shocks in the model and serve to isolate idiosyncratic shocks from global shocks. Using idiosyncratic shocks, we identify economy-specific structural productivity shocks with the “max-share” approach developed by Francis et al. (2014). We argue that these structural shocks are particularly useful for testing the hypothesis that a given productivity improvement leads to a larger terms of trade deterioration in developing economies than advanced economies.

After identifying economy-specific structural productivity shocks this way, we study three questions in this paper. First, does an unexpected productivity improvement cause a deterioration in the terms of trade of developing economies? Second, does the effect of the improvement on the terms of trade of developing and advanced economies differ? Third, how does the improvement affect export and import prices in developing and advanced economies? Apart from providing a telling clue on the quantitative weighting of the third argument in the Prebisch-Singer hypothesis, studying these questions is important to assess the gains resulting from the productivity improvement since the improvement benefits an economy less if it leads to a terms of trade deterioration. Indeed, in the extreme case of ‘immiserizing growth’ considered by Bhagwati (1958), the productivity improvement can even harm the economy by reducing national welfare when the loss due to deteriorating terms of trade outweighs the gain due to increased production.

The organization of the paper is as follows: Section 2 discusses our data and develops an econometric model to study the effect of a surprise productivity improvement on the terms of trade of developing and advanced countries. Section 3 presents our findings. Section 4 discusses the implications of our findings and concludes.
2 Data and Empirical Model

This section presents the data and the empirical model used for analyzing structural productivity shocks. We first discuss our data.

2.1 Data

The focus of this study is to understand whether the level of economic development plays an important role in the effect of productivity shocks on the terms of trade. To provide an answer to this, our approach is to divide the world into some major groups and to study productivity shocks in these groups separately. To this end, we use the country classification of the IMF’s World Economic Outlook, which classifies countries into two major groups: advanced economies and emerging and developing economies.\(^2\) Table A.1 in Appendix A reports countries included in each group in our sample. The group of advanced economies includes 36 economies and the group of emerging and developing economies includes 141 economies.

Let \(Y_{i,t}\) denote the vector of variables contained in our analysis, containing the log of GDP per person employed in constant 2011 purchasing power parity dollars (denoted by \(gdp_{i,t}\)), the log of export unit value index (denoted by \(p_{x,i,t}\)), and the log of net barter terms of trade index (denoted by \(tot_{i,t}\)):\(^3\)

\[
Y_{i,t} = \left[ gdp_{i,t}, \ p_{x,i,t}, \ tot_{i,t} \right]^T
\]

Our source of yearly data on \(Y_{i,t}\) is the World Bank’s World Development

\(^2\) This classification is based on three criteria: (1) an average per capita income level over a number of years, (2) export diversification, and (3) degree of integration into the global financial system. Taking an average in the first criterion is intended to eliminate excessive volatility in per capita income observed in some commodity-dependent countries due to marked year-to-year effects of commodity prices. Export diversification is useful not to include some largely commodity-dependent countries with high per-capita income in the group of advanced economies. It is remarkable that to classify countries according to their level of economic development, the World Economic Outlook’s country classification is more suitable than country classification of the World Bank since while the latter is based only on per capita income, the former also takes into account export diversification.

\(^3\) Net barter terms of trade index is defined as the ratio of export unit value index to import unit value index of exports and imports. Export unit value index expresses the US-dollar price of a unit of exports relative to that in the base year of 2000 and is defined as the ratio of export value index to export volume index.
2.2 A Panel VAR Model in the Presence of Common Shocks

We consider a panel VAR model with interactive effects which allows economy-specific productivity shocks to be separated from common productivity shocks, as we discuss below. The model is of the form:

$$
\Delta Y_{i,t} = \begin{bmatrix}
\alpha_0 \Delta gdp \\
\alpha_0 \Delta px \\
\alpha_0 \Delta tot
\end{bmatrix} + \sum_{k=1}^{\bar{r}} \begin{bmatrix}
\alpha_k \Delta gdp' \\
\alpha_k \Delta px' \\
\alpha_k \Delta tot'
\end{bmatrix} \Delta Y_{i,t-k} + \begin{bmatrix}
\lambda_i \Delta gdp' \\
\lambda_i \Delta px' \\
\lambda_i \Delta tot'
\end{bmatrix} f_t + \begin{bmatrix}
\Delta gdp \\
\Delta px \\
\Delta tot
\end{bmatrix} u_{i,t} (2.2)
$$

with \( \Delta Y_{i,t} = [\Delta gdp_{i,t} \ \Delta px_{i,t} \ \Delta tot_{i,t}]' \). \( f_t \) stands for \( \bar{r} \times 1 \) common factors, representing global shocks in the model, where \( \bar{r} \) denotes the number of common factors. \( \lambda_i \Delta gdp, \lambda_i \Delta px, \) and \( \lambda_i \Delta tot \) represent \( \bar{r} \times 1 \) factor loadings for the rate of changes in GDP per capita, export unit value index, and net barter terms of trade index, respectively. Idiosyncratic errors in the rate of change in GDP per capita, export unit value index, and net barter terms of trade index are denoted, respectively, by \( u_{i,t} \Delta gdp \), \( u_{i,t} \Delta px \), and \( u_{i,t} \Delta tot \). The 3×1 vector autoregression coefficients on \( \Delta gdp_{i,t}, \Delta px_{i,t}, \) and \( \Delta tot_{i,t} \) are given, respectively, by \( \alpha_k \Delta gdp \), \( \alpha_k \Delta px \), and \( \alpha_k \Delta tot \). The model can be written more compactly as:

$$
\Delta Y_{i,t} = \alpha_0 + \sum_{k=1}^{\bar{r}} \alpha_k \Delta Y_{i,t-k} + \lambda_i f_t + u_{i,t} (2.3)
$$

with \( \lambda_i = [\lambda_i \Delta gdp \ \lambda_i \Delta px \ \lambda_i \Delta tot]' \), \( u_{i,t} = [u_{i,t} \Delta gdp \ u_{i,t} \Delta px \ u_{i,t} \Delta tot]' \), and \( \alpha_k = [\alpha_k \Delta gdp \ \alpha_k \Delta px \ \alpha_k \Delta tot]' \) for \( k = 0, 1, \ldots, \bar{k} \). Idiosyncratic errors are assumed
to be uncorrelated across the economies and periods:

$$E(u_{i,t}u_{j,\tau}^\prime) = \begin{cases} 0 & \text{if } i \neq j \text{ or } t \neq \tau \\ \Sigma & \text{if } i = j \text{ and } t = \tau \end{cases} \quad (2.4)$$

We use the idiosyncratic errors $u_{i,t}$ to identify economy-specific structural shocks denoted by $\epsilon_{i,t}$:

$$u_{i,t} = A\epsilon_{i,t} \quad (2.5)$$

where $A$ is some $3 \times 3$ invertible matrix. Consequently, we assume idiosyncratic errors are given by some linear combination of economy-specific structural shocks, as is common in structural VAR analysis; e.g., see Christiano, Eichenbaum, and Evans (1999) and Arias, Rubio-Ramírez, and Waggoner (2018). We denote economy-specific productivity shocks as $\epsilon_{i,t}^{\Delta gdp}$. Before describing our strategy for recuperating $\epsilon_{i,t}^{\Delta gdp}$ in detail, we discuss our panel VAR model’s features.

First, $f_t$, which can be regarded as representing common shocks, plays a crucial role in recuperating $\epsilon_{i,t}^{\Delta gdp}$. To explain this, let $\eta_{i,t}$ denote the composite error terms in (2.2) given by the sum of interactive fixed effects terms $\lambda_i^t f_t$ and idiosyncratic errors $u_{i,t}$:

$$\eta_{i,t} = \lambda_i^t f_t + u_{i,t} \quad (2.6)$$

It is arguable that any shock which can be referred to as an economy-specific shock must be recuperated from idiosyncratic errors. However, were $f_t$ absent from the model, productivity shocks identified using the composite errors would not be economy specific since they would contain global shocks common to all economies. The presence of $f_t$ in the model serves to the purpose of obtaining idiosyncratic errors by purging the composite errors of global shocks.

Second, since $\lambda_i$ varies across the economies in the model, common shocks can have a different effect on the economies. In addition, the model has the
desirable feature that global shocks affect each variable of the same economy differently since their factor loadings are not the same (e.g., $\lambda_i^{\Delta gdp} \neq \lambda_i^{\Delta px}$).

The role played by global shocks in the model can be illustrated by the global financial crisis in 2008. The economies have been differently affected by the crisis. For example, while the net barter terms of trade of Australia improved greatly by 5.91% in 2008, Japan suffered from a large decline of 4.49% in its terms of trade in the same year. The presence of common factors together with economy-specific factor loadings in the model can account for such shocks affecting the economies differently and with varying degree. It is also notable that the only assumption we make about common factors and factor loadings is that their fourth moments are finite. Consequently, common factors representing global shocks in the model can have lasting effects since they can be serially correlated.

Third, the model is specified in first-differences based on our findings from appendix B that the endogenous variables are of integrated order one and no linear combination of them is stationary, resulting in that the model represented by finite-order vector autoregression in the differenced data is not misspecified; see Hamilton (1994, p. 574-575). By specifying the model this way, we aim to improve the small sample performance of the estimates from (2.2) and eliminate the non-standard distributions of the estimates occurring when the model is specified in levels, as noted in Hamilton (1994).

Fourth, we assume common slopes in the model for all the countries in the same group. To put it more clearly, $\alpha_k$ is assumed to be common across all economies in the same group. Also, the idiosyncratic shocks are assumed to have the same variance $\Sigma$ for all the countries in the same group. We assume common slopes and the same idiosyncratic errors' variance in the same group based on the general finding that heterogeneous panels have worse forecast performance than homogeneous models; e.g., see Baltagi (2005, chapter 10) who reviews the literature on heterogeneous and homogeneous panels and concludes

\footnote{However, we allow the slopes to differ between developing and advanced economies.}
that compared to the more parameter consuming heterogeneous panels, homogeneous panels yield better forecast performance due to their simplicity and parsimony in model estimation. Also, he notes forecast performance is particularly better in homogeneous panels of international studies whose variables have large variation.

Five, let $\mu_i$ denote additive economy-fixed effects. It is notable that the panel VAR model with additive effects is nested in our panel VAR model as the special case that $f_t^r$ is constant across all periods where $f_t^r$ denotes the $r^{th}$ common factor. Consequently, the least squares estimates from the panel model with only interactive fixed effects are still consistent even when $\mu_i$ is present but not imposed in the model since additive economy-fixed effects can be largely accounted for by an estimated common factor with little variation across periods in the model.

2.2.1 The Problem with Pooling All Available Data of Advanced and Developing Economies Together

Consider the following panel VAR model where all available data of advanced and developing economies is pooled together:

$$
\Delta Y_{i,t} = \alpha^g_0 + \sum_{k=1}^k \alpha^g_k \Delta Y_{i,t-k} + \lambda_t^r f_t + u_{i,t}; \quad g = A \text{ or } D \quad (2.7)
$$

where $g$ is the group in which economy $i$ is included and is given by either $A$ if economy $i$ is an advanced economy, or by $D$ if economy $i$ is a developing economy.

It is notable that common factors in Model (2.7) can be estimated with a larger cross section units than those in Model (2.3), where the two panels of advanced and developing economies are studied separately. Does this result in Model (2.7) being more desirable than Model (2.3)? In our opinion, the answer is no for basically three reasons.
First, the efficiency gain in the estimates of parameters in Model (2.7) from pooling is not likely to be larger than it is in Model (2.3). Indeed, in both Model (2.7) and (2.3), the rates of convergence of the least squares estimator of the coefficients and common factors are given by \( \frac{1}{\sqrt{TN_g}} \) and \( \frac{1}{\sqrt{T}} \), respectively, with \( N_g \) denoting the number of economies included in group \( g \); see, Bai (2009), Moon and Weidner (2017), and Tugan (2018).

Second, Model (2.7) is questionable since it imposes the same common factors affecting developing and advanced economies. However, common factors affecting advanced and developing economies can be different in each period. For example, the food-price boom in the 2000s lasting about a decade can be regarded as a common factor for developing economies, given its sizable effect on the terms of trade of many developing economies with a large share of food in trade. However, the boom is unlikely to be termed as a common factor affecting the terms of trade of advanced economies, given their small share of food in trade. Unlike Model (2.7), Model (2.3) allows common factors affecting advanced economies to be different from those affecting developing economies.

Third, in our subsequent analysis, we discuss the results from an extended sample of 1991-2016, for which we have an unbalanced panel due to unavailability of data between 1991 and 1999 for many economies. Model (2.7) is problematic with this unbalanced panel due to the sample-selection bias. Indeed, only a small fraction of advanced economies has data between 1991 and 1999, as compared to that of developing economies, as indicated in appendix A.1. This would result in missing observations in Model (2.7) being non-random, inducing a sample-selection problem in Model (2.7).

### 2.3 Empirical Strategy for Identifying Productivity Shocks

Next, we discuss our strategy for identifying economy-specific productivity shocks which is based on Francis et al. (2014). We recuperate productivity shocks as structural shocks best explaining the idiosyncratic variation in GDP
per employed person over some long but finite horizon. As discussed in Beaudry, Nam, and Wang (2011), this method has some favorable features, as compared to its alternatives. For example, Gali (1999) identifies productivity shocks as the only shock which have a long-run effect on labor productivity measured as GDP per capita. However, this identification strategy requires estimating infinite-order lag polynomials which are very uncertain and notoriously difficult to estimate with a reasonable sample size; see Christiano, Eichenbaum, and Vigfusson (2007). Such uncertainty in the long-run identification scheme also results in tests related to impulse responses being inconsistent; see Faust and Leeper (1997). Barsky and Sims (2011) suggest another method for recovering surprise technology shocks. In this method, one has to assume fluctuations in total factor productivity are caused either by surprise technology shocks or news shocks in all periods over some finite horizon. Other shocks which may have an effect on total factor productivity over this finite horizon are disregarded; see Beaudry, Nam, and Wang (2011). The max share approach of Francis et al. (2014), which we use in this paper to study productivity shocks, can be regarded as free of these critics. Indeed, it does not require estimating infinite-order lag polynomials. Moreover, while productivity shocks are assumed to play a profound role in labor productivity over some long but finite horizon, other shocks may also cause labor productivity to fluctuate over this horizon.

Now, we present our identification strategy in detail. Under some general conditions, $\Delta \gamma_{i,t}$ has a moving-average representation of the form:

$$\Delta \gamma_{i,t} = \Delta \gamma_{i} + \sum_{h=0}^{\infty} B(h) \lambda_{i,t} + \sum_{h=0}^{\infty} B(h) u_{i,t}$$ \hspace{1cm} (2.8)

where $\Delta \gamma_{i}$ denotes the population mean of $\Delta \gamma_{i,t}$ and the moving-average

\footnote{Indeed, while the former shocks are identified as innovations to total factor productivity having an immediate effect, the latter shocks are identified as innovations having an effect on total factor productivity with some delay}
coefficients given by $B(h)$ can be obtained from (2.2) as:

$$B(h) = \begin{cases} \sum_{k=1}^{h} \alpha_k B(h-k) & \text{for } h > 0 \\ I_3 & \text{for } h = 0 \\ 0 & \text{for } h < 0 \end{cases} \quad (2.9)$$

To study the effects of economy-specific productivity shocks on the terms of trade, the variables should be purged of their variation caused by $f_t$ and the forecast error in the variables caused only by idiosyncratic errors should be considered. We refer to the latter as the idiosyncratic variation in the variables. We define the $H$-period ahead idiosyncratic variation in the variables as:

$$\sum_{h=0}^{H} B(h)u_{i,t+H-h} \quad (2.10)$$

Since $\Delta gdp_{i,t+H}$ is the first element of $\Delta Y_{i,t+H}$, the idiosyncratic variation in $gdp_{i,t+H}$ can be obtained as that of the cumulative sum of (2.10):

$$I_3^{1'} \sum_{h=0}^{H} \sum_{j=0}^{h} B(j)u_{i,t+H-h} \quad (2.11)$$

where $I_3^1$ denotes the first column of $I_3$. Let $C(h)$ denote the $h$-period ahead moving average coefficients for the level of the variables which can be written as:

$$C(h) = \sum_{j=0}^{h} B(j) \quad (2.12)$$

Using (2.11) and (2.12), the $H$-period ahead idiosyncratic forecast error variance of $gdp_i$ can be written as:

$$I_3^{1'} \left( \sum_{h=0}^{H} C(h)\Sigma C(h)' \right) I_3^1 \quad (2.13)$$

Next, we discuss how economy-specific productivity shocks are identified in
Our analysis. We begin with rewriting (2.5) as

\[ u_{i,t} = \tilde{A}Q \epsilon_{i,t} \]  

(2.14)

where \( Q \) is some orthogonal matrix and \( \tilde{A} = AQ' \). As is common, we assume that idiosyncratic structural shocks are uncorrelated and have unit variance, \( E(\epsilon_{i,t}\epsilon'_{i,t}) = I_3 \). Consequently, \( \tilde{A} \) must satisfy the following:

\[ \tilde{A}\tilde{A}' = \Sigma \]  

(2.15)

For example, \( \tilde{A} \) can be given by the Cholesky decomposition of \( \Sigma \). Let productivity shocks be given by the \( j^{th} \) element of \( \epsilon_{i,t} \). Then, the \( H - step \) ahead forecast error variance of \( gdp_{i,t} \) due to productivity shocks can be written from (2.11) and (2.12) as

\[ I_3' \left( \sum_{h=0}^H C(h)\tilde{A}q_jq_j' \tilde{A}'C(h)' \right) I_3 \]  

(2.16)

where \( q_j \) is the \( j^{th} \) column of \( Q \). Since \( I_3' C(h)\tilde{A}q_j \) is scalar, (2.16) can be rewritten as:

\[ q_j' \left( \sum_{h=0}^H \tilde{A}'C(h)'I_3 I_3' C(h)\tilde{A} \right) q_j \]  

(2.17)

Let \( G(H) \) be given by

\[ G(H) = \frac{\sum_{h=0}^H \tilde{A}'C(h)'I_3 I_3' C(h)\tilde{A}}{I_3' \sum_{h=0}^H C_h\Sigma C_h' I_3} \]  

(2.18)

Using the “max-share” approach developed by Francis et al. (2014), we identify economy-specific productivity shocks by maximizing the share of \( H - step \) ahead forecast error variance of \( gdp_i \) due to economy-specific productivity shocks in \( H - step \) ahead total forecast error variance of \( gdp_i \). This involves choosing \( q_j \) as the vector ensuring the role played by productivity shocks in the \( H - step \) ahead forecast error variance.
ahead idiosyncratic variation in $gdp_i$ is greatest:

$$\max_{q^j} q^j G(H)q^j \quad \text{subject to } q^j'q^j = 1. \quad (2.19)$$

From this constrained maximization, it is easy to show $q^j$ can be obtained as
the eigenvector of $G(H)$ corresponding to its largest eigenvalue.

## 3 Results

In this section, we present the main results of the paper. We begin with reporting the results from the benchmark case. Next, we investigate whether the results from the benchmark case are robust to changes in the model specification and the sample period. Lastly, developing economies are more detailedly classified and grouped into the least developed countries and the remaining advanced economies and analyze the effects from a productivity improvement in the former and the latter separately to see whether the improvement considered affects the economies in the former differently from those in the latter.

### 3.1 The Benchmark Case

In this section, we discuss the results from a positive productivity shock. Without loss of generality, we normalize the impact effect of the shock on output per employed person ($gdp_{i,t}$) to unity. In the benchmark model, we choose the lag length in the model as one ($\bar{k} = 1$) and the horizon at which idiosyncratic forecast-error variance share of productivity in $gdp_i$ is maximized as 10 years for our annual data ($H = 10$). The lag length of one in our model with our annual data is consistent with four lags chosen by Christiano, Eichenbaum, and Evans (2005) for their quarterly data and approximately corresponds to 13 lags chosen by Bernanke, Boivin, and Eliasz (2005) and Boivin, Giannoni, and Mihov (2009) for their monthly data. Also, the choice of 10 years as the horizon in the max-share approach is the same as the choice of 40 quarters in Francis et al. (2014).
Figure 1 display the impulse responses of the terms of trade, the export unit value index, the import unit value index, and output per employed person to a positive productivity shock, resulting in an unexpected one percent increase in output per employed person in the impact period.\(^6\) In Figure 1, \(p^m\) denotes the import unit value index whose impulse responses are calculated by subtracting the impulse responses of terms of trade (\(tot\)) from those of export unit value index (\(p^x\)). As is evident from Figure 1, we find that a positive productivity shock

- results in an improvement in the terms of trade of both advanced and developing economies, which is initially distinguishable from zero but insignificant thereafter;

- gives rise to an insignificant fall and rise in the export unit value index of advanced and developing economies, respectively; and

- causes an insignificant increase and almost no change in the import unit value index of advanced and developing economies, respectively.

- induces a permanent increase in output per employed person in advanced and developing economies, which is significant at all horizons that we compute the impulse responses;

Consequently, our findings largely disagree with the argument put forward to explain the Prebisch-Singer hypothesis that productivity improvements in developing economies lead to more unfavorable terms of trade dynamics than in advanced economies. Our findings do not provide supporting evidence that this explanation holds in reality. In contrast, our results indicate that while the terms of trade of developing and advanced economies initially improve, they ultimately follow insignificant dynamics after a positive productivity shock. Moreover, the

\(^6\)An impulse response of a variable shows the change in the variable caused by the productivity shock over some horizon. A positive (negative) impulse response indicates that the variable would attain (fall to) a high (low) level in the presence of the shock, as compared to that in the undistorted path.
Panel A: Advanced Economies

Panel B: Developing Economies

Note: Our calculations are based on the World Bank’s World Development Indicators. The solid lines indicate the median impulse responses. The area between the dashed lines shows the 68% confidence interval estimated using the Monte Carlo method presented in appendix C.

Figure 1: Impulse Responses to a Positive Productivity Shock
(The Benchmark Specification: $\bar{k} = 1$)

magnitudes of the change in the median responses of the terms of trade in developing economies are similar to those in advanced economies.

The dynamics of the export unit value indexes in developing economies after an unexpected productivity improvement is also different from what the Prebisch-Singer hypothesis predicts. Indeed, while the Prebisch-Singer hypothesis predicts a certain fall in the export unit value index following a productivity improvement, we find no evidence of this. In fact, the export unit value index can initially show increasing dynamics after the productivity increase in developing economies, as is evident from Figure 1.

3.2 Robustness Checks

In this section, we consider two robustness checks. In the first robustness check, we use the specification that $H = 10$ and $\bar{k} = 2$. Consequently, in this alternative specification, while the horizon at which the forecast-error variance share of
Panel A: Advanced Economies

Panel B: Developing Economies

Note: Our calculations are based on the World Bank’s World Development Indicators. The solid lines indicate the median impulse responses. The area between the dashed lines shows the 68% confidence interval estimated using the Monte Carlo method presented in appendix C.

Figure 2: Impulse Responses to a Positive Productivity Shock
Robustness Check: \( \bar{k} = 2 \)

Productivity in output per employed person is maximized is the same as in the benchmark specification, we allow for richer dynamics by choosing the lag length in Model 2.3 as two instead of one as in the benchmark specification.

Figure 2 displays the responses from the specification allowing richer dynamics. It is discernible that the fall in the export unit value index of advanced economies is more pronounced under this specification than under the benchmark specification; see Figure 1 and Figure 2. Apart from this, the results differ little between the two specifications.

As a second robustness check, the specification that \( H = 20 \) and \( \bar{k} = 1 \) is considered. In this alternative specification, while the lag length in Model 2.3 is the same as in the benchmark specification, the anticipation horizon is longer than that in the benchmark specification. Since the results from this alternative
specification are almost identical to those from the benchmark specification, they are not reported for the reasons of brevity.

3.3 Results from a Longer Sample Period

In this section, we extend our sample period back to 1991 for developing and advanced economies and discuss the results under both the benchmark specification and the specification allowing richer dynamics discussed above. It is notable that the responses to a productivity shock can be more precisely estimated when the sample period is extended back to 1991. However, doing so results in an unbalanced panel since data is available only for a small fraction of advanced economies and about half of developing economies between 1991-1999. To eliminate the sample selection bias, the additional assumption that selection is unrelated to the idiosyncratic errors $u_{i,t}$ in Model (2.3) must be made for the longer unbalanced panel; see Wooldridge (2002, chapter 17.7). Our decision to study the sample period of 2000-2016 in our main analysis stems from the fact that our panel for 2000-2016 is balanced and by construction, free of sample-selection bias, which may plague the results from the longer unbalanced panel if the assumption that selection is unrelated to the idiosyncratic errors is violated.

It is also notable that in the unbalanced panel, estimating common factors in Model (2.3) requires imputing some missing values. We impute these values using the expectation-maximization algorithm suggested by Stock and Watson (2002) and Bai (2009). As simulation studies done by Bai, Liao, and Yang (2015) show, this algorithm yields consistent estimates, converging rapidly to their true values for both smooth and stochastic factors.

Before discussing the results from the longer sample, a caveat must be mentioned. Four out of 36 advanced economies in our sample have data between 1991-1999, resulting in the unbalanced panel of advanced economies having only 36 more observations than the balanced panel of advanced economies. Consequently, a small gain in precision from extending the sample period back to 1991 may not be worth of the possibility of sample-selection bias in the estimates from
Panel A: Advanced Economies

The Benchmark Identification ($\bar{k} = 1$, $H = 10$)  
Robustness Check ($\bar{k} = 2$, $H = 10$)

<table>
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<th>Year 1</th>
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Note: Our calculations are based on the World Bank’s World Development Indicators. The solid lines indicate the median impulse responses. The area between the dashed lines shows the 68% confidence interval estimated using the Monte Carlo method presented in appendix C.

Figure 3: Impulse Responses to a Positive Productivity Shock (Extended Sample Period)

Panel B: Developing Economies

The Benchmark Identification ($\bar{k} = 1$, $H = 10$)  
Robustness Check ($\bar{k} = 2$, $H = 10$)

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the unbalanced panel, causing them to be inconsistent.

Panel A of Figure 3 illustrates the impulse responses to a positive productivity shock in advanced economies from the unbalanced panel estimated using the benchmark specification and the specification allowing richer dynamics ($\bar{k} = 2$). There are two findings which are robust to different choices of $\bar{k}$ and are largely similar to those from the shorter balanced panel. First, the terms of trade of advanced economies initially improve and show no significant dynamics thereafter. Second, the import unit value index of advanced economies initially falls
and follows insignificant dynamics thereafter.

Panel B of Figure 3 displays the impulse responses to an unexpected productivity improvement in developing economies from the unbalanced panel estimated using the aforementioned specifications. As is evident from the panel, there is no supporting evidence for the prediction of the Prebisch-Singer hypothesis that the terms of trade and the export unit value index fall in developing economies following a productivity improvement. As a matter of fact, they can even show increasing dynamics, as is evident from the responses under the specification with $\tilde{k} = 2$. These findings are largely in conformity with those from the balanced panel.

3.4 A More Detailed Classification of Developing Economies

In this section, we study the effects from an economy-specific productivity shock in a more detailed classification of developing economies by dividing developing economies into two groups: the group of economies included in the list of the least developed countries of the United Nations and the group of the remaining developing economies not characterized as a least developed country by the United Nations. The numbers of the economies in the former and the latter are 42 and 99, respectively. Table A.1 in the appendix presents the economies on the list of the least developed countries.

Figure 4 displays the impulse responses of the variables to a positive productivity shock in the least developed countries and the remaining developing economies from these specifications. Consistent with the previous analysis, we consider the benchmark specification given by $(\tilde{k} = 1, H = 10)$ and the specification with richer dynamics given by $(\tilde{k} = 2, H = 10)$.

Our findings are striking. Regarding the least developed countries, we find that an unexpected productivity increase leads to a deterioration in the terms

\footnote{For the sake of brevity, the impulse responses of output per employed person, which stay positive and are distinguishable from zero across all the periods that we compute impulse responses are not shown in Figure 4.}
Panel A: The Least Developed Countries

The Benchmark Identification ($\hat{k} = 1$, $H = 10$)  
Robustness Check ($\hat{k} = 2$, $H = 10$)

![Graphs showing impulse responses of productivity shock in Least Developed Countries](image)

Panel B: Remaining Developing Economies

The Benchmark Identification ($\hat{k} = 1$, $H = 10$)  
Robustness Check ($\hat{k} = 2$, $H = 10$)

![Graphs showing impulse responses of productivity shock in Remaining Developing Economies](image)

Note: Our calculations are based on the World Bank’s World Development Indicators. The solid lines indicate the median impulse responses. The area between the dashed lines shows the 68% confidence interval estimated using the Monte Carlo method presented in appendix C.

Figure 4: Impulse Responses to a Positive Productivity Shock (More Detailecly Classified Developing Economies)

of trade, which is caused by a decrease in $p^e$ and insignificant dynamics in $p^m$. This finding is robust to the specification allowing richer dynamics ($\hat{k} = 2$) and consistent with the Prebisch-Singer hypothesis. However, the effects of an unexpected productivity increase in the remaining developing economies stand in stark contrast to the Prebisch-Singer hypothesis. Indeed, let alone a deterioration, the productivity increase results in an improvement in the terms of trade of the remaining developing economies with the responses being initially distinguishable from zero and barely significant thereafter. The improvement in the terms of trade results from the increase in the export unit value index and
Panel A: The Least Developed Countries

The Benchmark Identification ($k = 1, H = 10$)  
Robustness Check ($k = 2, H = 10$)

(a) $tot$  
(b) $p^s$  
(c) $p^m$

(d) $tot$  
(e) $p^s$  
(f) $p^m$

Panel B: Remaining Developing Economies

The Benchmark Identification ($k = 1, H = 10$)  
Robustness Check ($k = 2, H = 10$)

(a) $tot$  
(b) $p^s$  
(c) $p^m$

(d) $tot$  
(e) $p^s$  
(f) $p^m$

Note: Our calculations are based on the World Bank’s *World Development Indicators*. The solid lines indicate the median impulse responses. The area between the dashed lines shows the 68% confidence interval estimated using the Monte Carlo method presented in appendix C.

**Figure 5:** Impulse Responses to a Positive Productivity Shock  
(More Detailedly Classified Developing Economies and Extended Sample Period)

no sizable change in their import unit value index over the period of 2000-2016.

It is notable that the group of the remaining developing economies has substantial heterogeneity in the structure of their exports, as compared with the group of the least developed countries. This substantial heterogeneity brings about the question of whether unexpected productivity improvements cause notable heterogeneous dynamics in the group of the remaining developing economies. We study this question by further dividing this group into two subgroups: the economies with the *industrial export quality index* greater than 0.5 in 2005 and those with the index lower than 0.5; see Table A.1 in appendix A for
the list of economies included in these subgroups. The industrial export quality index is a composite indicator of UNIDO’s *Competitive Industrial Performance Index* database, calculated as the simple mean of the share of manufactured exports in total exports and the share of medium- and high-tech manufactured exports in total manufactured exports; see UNIDO (2017). While not shown for the reasons of brevity, an unexpected productivity improvement causes dynamics similar to those in Panel B of Figure 4 in both of the two subgroups of the remaining developing economies considered. Consequently, there is little evidence that the dynamics across the remaining developing economies differs largely due to the substantial heterogeneity in the structure of exports in the group.

Next, we consider the longer sample period of 1991-2016. Since data is unavailable for a large number of developing economies between 1991-1999, the panel is unbalanced. Consequently, the consistency of the results requires the additional assumption that selection is unrelated to idiosyncratic errors. The impulse responses to an unexpected productivity improvement from the unbalanced panel estimated using both of the aforementioned specifications in the least developed countries and the remaining developing economies are shown, respectively, by Panel A and Panel B of Figure 5. As is evident from the figure, our main finding from this section that the terms of trade deteriorate in the least developed countries and improve in the remaining developing economies after the shock is robust to extending the sample period. Another robust finding is that the export unit value index increases and the import unit value index shows insignificant dynamics after the shock in the remaining developing economies. However, our finding from the balanced panel of a fall in the export unit value index and no significant change in the import unit value index after the shock in the least developed countries is not robust to extending the sample period since these dynamics differ largely from the ones reported in Panel B of Figure 5.
4 Discussion and Conclusions

Are surprise productivity improvements associated with a large terms of trade deterioration in developing economies, as compared to that of advanced economies? Our findings indicate that they are not. Indeed, in both developing and advanced economies, the terms of trade show no significant dynamics after a productivity improvement. In this regard, our findings disagree with the Prebisch-Singer hypothesis. Concerned about the great heterogeneity of developing economies in our sample, we also study productivity improvements in a more detailed classification of developing economies. Specifically, developing economies are divided into two groups: the least developed countries and the remaining developing economies. Our findings are striking. While the terms of trade in the former deteriorate after a surprise productivity increase, it improves in the latter. This finding is robust to different model specifications and the sample period that surprise productivity shocks are analyzed. Consequently, we provide some evidence that economic underdevelopment can cause unfavorable terms of trade movements following an unanticipated increase in productivity, as predicted by the Prebisch-Singer hypothesis. However, the remaining developing economies, having some success in moving up the ladder of economic development, do not suffer from a decline in their terms of trade after a productivity increase. On the contrary, their terms of trade improve following unanticipated productivity increases. This sharply contrasts with the pessimist prediction of the Prebisch-Singer hypothesis for developing economies regarding the terms of trade effect of productivity increases.

What are the reasons behind the failure of the Prebisch-Singer hypothesis to account for our finding that the terms of trade in advance and developing economies have similar dynamics following unexpected productivity increases? In our opinion, this mainly results from the two defining characteristics of advanced and developing economies being radically different today from those in the 1950s, when the Prebisch-Singer hypothesis was first proposed. First,
the Prebisch-Singer hypothesis characterizes labor as organized in advanced economies and as disorganized in developing economies, implying that trade unions in advanced economies have more power to prevent labor shares from declining by securing the sufficient rents from productivity gains accruing to labor. Contrasting with this prediction, labor shares have considerably fallen over the last two decades in advanced economies. This largely result from real median compensation in advanced economies not keeping up with labor productivity, causing a decoupling of wages from productivity growth; see Schwellnus, Kappeler, and Pionnier (2017). As a matter of fact, as compared to developing economies, advanced economies have had a small labor share since 2000 when self-employment income is taken into account; see Guerriero (2012). Consequently, there is little evidence that labor in advanced economies benefits more from a given productivity improvement due to being more organized, as argued by the Prebisch-Singer hypothesis.

Second, many developing economies can no longer be characterized as producers of primary commodities since they have succeeded in diversifying their exports into differentiated products. This is evident from the fact that developing economies increased the share of differentiated and science-based products in their manufacture exports from barely three percent to more than a quarter between 1965 and 1990; see ul Haque et al. (1995). Similarly, Artopoulos, Friel, and Hallak (2013) report the considerable increase in the share of differentiated goods exported by developing economies to OECD countries since 1980. Since product differentiation can give some degree of price-setting power to exporters in developing economies, it can stand as a factor in accounting for productivity increases not having as an unfavorable effect on the terms of trade of developing economies as predicted by the Prebisch-Singer hypothesis.

Apart from this, product differentiation has two subtle favorable effects on the terms of trade of developing economies. First, differentiated goods have lower price volatility than non-differentiated goods. Gopinath, Itskhoki, and Neiman (2012) provides some supporting evidence on this by documenting that
while the prices of differentiated manufactures showed marked stability during the trade collapse of 2008-2009, those of non-differentiated goods sharply declined. Second, differentiated exports have a larger price elasticity of demand and supply than primary commodities. This implies that a productivity gain in developing economies with differentiated exports can lead to an increase in export earnings by causing an increase in export volumes larger than a fall in export prices in the traditional sectors that these economies trade.

Next, we discuss our finding that productivity improvements cause the terms of trade to worsen in the least developed countries. This can be traced to the fact that these economies are characterized as exporters of goods with a low price elasticity of demand. With such a structure of exports, productivity increases are likely to have an unfavorable effect on the terms of trade, as predicted by the Prebisch-Singer hypothesis. This effect works through the channel that a fall in unit labor costs brought about by productivity increases causes export prices to fall.

However, productivity increases can also have a favorable effect on the terms of trade of the developing economies with some success to export differentiated goods, which the Prebisch-Singer hypothesis overlooks. This effect operates through a possible increase in export earnings with product differentiation, which can relieve the balance of payment constraints on the imports of required capital goods and help developing economies upgrade their exports. Since upgrading exports can enable developing economies to escape from the fallacy of composition problem in their traditional export sectors, which depressed the prices of the primary and labor-intensive goods, it can have a favorable effect on their terms of trade; see UNCTAD (2002). Our finding of an improvement in the terms of trade of the remaining developing economies following unexpected productivity gains can result from this favorable effect of productivity gains prevailing over the unfavorable effect discussed above.

8Specifically, commodities accounted for more than two thirds of merchandise exports in the overwhelming majority of the least developed countries between 2013-2015, as noted in UNCTAD (2016).
A Economies in Our Sample

Panel A. Developing Economies included in Our Sample

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<th>Nigeria*</th>
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<td>Sao Tome and Principe*</td>
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<td>NIGER</td>
<td>Zimbabwe*</td>
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</tbody>
</table>

Panel B: Advanced Economies Included in Our Sample

| Australia | Hong Kong SAR, China* | Netherlands |
| Austria   | Iceland | New Zealand |
| Belgium   | Ireland | Norway |
| Canada    | Israel | Portugal |
| Cyprus    | Italy | Singapore* |
| Czech Republic | Japan | Slovak Republic |
| Denmark   | Korea, Rep.* | Slovenia |
| Estonia   | Latvia | Spain |
| Finland   | Lithuania | Sweden |
| France    | Luxembourg | Switzerland |
| Germany   | Macao SAR, China | United Kingdom |
| Greece    | Malta | United States* |

Note:

- The sample period of the economies with an asterisk extends back from up to 1991 until 2016. The economies without an asterisk have data between 2000-2016.
- The economies emphasized are characterized by the United Nations as a least developed country.
- The economies written in capital letters are the developing economies with UNIDO’s *Industrial Export Quality Index* greater than 0.5 in 2005.

Table A.1: Economies in Our Sample
B Specifications Issues

While estimating a vector autoregression often provides a convenient way of describing dynamics of variables, it involves making a number of critical decisions which may have an influential impact on inference. For example, when some variables included in the VAR contain a unit root, should they be included in levels or in differences? Hamilton (1994, p. 651-653) discusses this issue in detail. He notes if the true process is VAR in differences, differencing should improve the small-sample performance of all of the estimates and can eliminate non-standard distributions associated with certain hypothesis testing. However, differencing all variables in a VAR can result in a misspecified regression in case that some variables are already stationary or there is a stationary linear relationship between $I(1)$ variables included in the VAR.\footnote{In fact, as shown by Hamilton (1994, p. 574-575), a cointegrated system cannot be represented by a VAR in differences.}

We represent our system with a panel VAR with interactive fixed effects in differences based on two findings. First, using a panel unit root test, we show all of the variables included in our panel VAR model are $I(1)$. Second, we show there is no stationary linear combination of the variables.

B.1 Panel Unit Root Tests in the Presence of Common Shocks

We investigate whether series contain a unit root or not using the modified Sargan–Bhargava test (the MSB test) proposed by Stock (1999) and discussed extensively by Bai and Carrion-I-Silvestre (2009) and Bai and Ng (2010) in the context of a panel data with cross-sectional dependence. The model on which the MSB test is based is given by

\[
y_{i,t} = \mu_i + \nu_i t + f'_t \lambda_i + u_{i,t}
\]

\[i = 1, 2, \ldots, N; \quad t = 1, 2, \ldots, T
\]

where $y_{i,t}$ denotes one of the three variables contained in $Y_{i,t}$. $\mu_i$ and $\nu_i$ stand for economy-specific intercept and trend terms, respectively. $\lambda_i$, $f_t$, and $u_{i,t}$ stand for factor loadings, common factors, and idiosyncratic errors, respectively.

The error structure in the MSB test is useful for two reasons. First, it is notable
that since $\lambda_i$ is an economy-specific parameter, the error structure allows common shocks to have a different effect on individual economies. This assumption is useful to make in our analysis since global shocks can differently affect individual economies. For example, the unprecedented increase in the IMF’s crude oil price index from its trough of 19.54 to its peak of 249.66 between 1998:12 and 2008:7 can be regarded as a global oil shock. It can be argued that while this shock favorably affected the terms of trade of oil-exporting economies, it had an unfavorable effect on the terms of trade of oil-importing economies. Second, since global shocks are likely to cause high cross-section dependence, assuming the error terms are uncorrelated across countries as in Choi (2001) would result in large size distortion, as shown by Pesaran (2007).

One can write the differenced form of (B.1) in matrix notation as

$$\Delta y_i = \iota_T \nu_i + \Delta f \lambda_i + \Delta u_i \quad (B.2)$$

where $\iota_T$ is a $T \times 1$ vector of ones. Let $M_x = I_T - \frac{1}{T} \iota_T \iota_T'$. Multiplying (B.2) with $M_x$ eliminates the constant from this equation:

$$\Delta y_{i,t}^* = \Delta f \lambda_i + \Delta u_{i,t}^* \quad (B.3)$$

where $\Delta y_{i,t}^* = M_x \Delta y_{i,t}$, $\Delta f \lambda_i = M_x \Delta f t$, and $\Delta u_{i,t}^* = M_x \Delta u_{i,t}$. Let $\hat{u}_{i,t}^*$ be the least squares estimates of $\sum_{s=2}^{T} \Delta u_{i,s}^*$. The $MSB_{u^*}(i)$ test statistics is defined by

$$MSB_{u^*}(i) = \frac{(T-2)^2 \sum_{t=3}^{T} \hat{u}_{i,t-1}^2}{\hat{\sigma}_{u_i^*}^2} \quad (B.4)$$

where $\hat{\sigma}_{u_i^*}^2$ denotes an estimator of the long-run variance of $u_{i,t}^*$. As suggested by Bai and Carrion-i Silvestre (2013), $\hat{\sigma}_{u_i^*}^2$ can be estimated as

$$\hat{\sigma}_{u_i^*}^2 = \frac{\hat{\sigma}_{\hat{u}_{i,t}}^2}{1 - \hat{\phi}_{i,1}} \quad (B.5)$$

with $\hat{\sigma}_{\hat{u}_{i,t}}^2 = (T-3)^{-1} \sum_{t=4}^{T} \hat{\nu}_{i,t}$, where $\hat{\phi}_{i,1}$ and $\hat{\nu}_{i,t}$ are the least squares estimates from the following equation:

$$\Delta \hat{u}_{i,t}^* = \phi_{i,0} \hat{u}_{i,t-1}^* + \phi_{i,1} \Delta \hat{u}_{i,t-1}^* + \nu_{i,t} \quad (B.6)$$

$$i = 1, 2, \ldots, N \quad ; \quad t = 2, 3, \ldots, T$$
Panel A: Advanced Economies

<table>
<thead>
<tr>
<th></th>
<th>gdp_{i,t}</th>
<th>p_{i,t}^x</th>
<th>tot_{i,t}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00</td>
<td>-0.62</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(0.27)</td>
<td>(0.64)</td>
</tr>
</tbody>
</table>

Panel B: Developing Economies

<table>
<thead>
<tr>
<th></th>
<th>gdp_{i,t}</th>
<th>p_{i,t}^x</th>
<th>tot_{i,t}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.10</td>
<td>-1.08</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td>(0.14)</td>
<td>(0.55)</td>
</tr>
</tbody>
</table>

Note: Panel A and Panel B report the MSB test statistics of the log-level of the variables for advanced and developing economies, respectively. The numbers in parenthesis refer to the p-values of the MSB test statistics.

Table B.1: Panel Unit Root Test Results

The MSB test statistics discussed by Bai and Carrion-I-Silvestre (2009), which asymptotically has standard normal distribution, is given by

\[ MSB_{u} = \sqrt{N} \frac{MSB_{u} - \frac{1}{N}}{\sqrt{\frac{N}{T}}} \]  

with \( MSB_{u} = N^{-1} \sum_{i=1}^{N} MSB_{u}(i) \).

Panel A and Panel B of Table B.1 report the MSB test statistics of \( gdp_{i,t} \), \( p_{i,t}^x \), and \( tot_{i,t} \) for advanced and developing economies, respectively. It is notable that when obtaining the MSB\( u \) test statistics, the number of common factors in (B.3) is treated as unknown and is estimated using the eigenvalue ratio estimator suggested by Ahn and Horenstein (2013) allowing up to five common factors. The number of factors chosen based on the eigenvalue ratio estimator is one for \( gdp_{i,t} \), \( p_{i,t}^x \), and \( tot_{i,t} \) in our sample of both advanced and developing economies. From the p-values reported in Panel A, it is evident that the null hypothesis of a unit root in neither \( gdp_{i,t} \), nor \( p_{i,t}^x \), and nor \( tot_{i,t} \) across all advanced economies cannot be rejected. Similarly, we fail to reject the null in the sample of developing economies for \( gdp_{i,t} \), \( p_{i,t}^x \), and \( tot_{i,t} \) since the p-values of the MSB\( u \) test statistics reported in Panel B are larger than .05, the significance level we choose in our analysis.
B.2 A Panel Cointegration Test in the Presence of Common Shocks

Can there be a linear combination of $gdp_{i,t}$, $p_{i,t}$, and $tot_{i,t}$ which is stationary and suggests a long-run equilibrium relationship between $gdp_{i,t}$, $p_{i,t}$, and $tot_{i,t}$ despite the presence of a unit root in the series? Put differently, are $gdp_{i,t}$, $p_{i,t}$, and $tot_{i,t}$ cointegrated? The answer to this question is instrumental in specifying the panel VAR model discussed in section 2.2. Indeed, when a linear combination of the series is stationary, the level specification must be preferred since a panel VAR in differences is not consistent with a cointegrated system, as shown by Hamilton (1994). Before explaining how we test for cointegration, it is useful to review two well-known economic models related to the terms of trade determination: the Balassa-Samuelson model and the Prebisch-Singer model. In the Balassa-Samuelson model, the terms of trade between any two countries are mainly determined by productivity differences in traded sectors and productivity increases are associated with a certain fall in export prices. The Prebisch-Singer model argues that apart from productivity differences, the good and labor market structures also play a key role in the determination of the terms of trade. Indeed, the effect that productivity increases have on export prices is more unfavorable in economies with more competitive good and labor markets. Can $gdp_{i,t}$, $p_{i,t}$, and $tot_{i,t}$ have a cointegration relation based on either the Balassa-Samuelson model or the Prebisch-Singer model? When common factors are omitted from the analysis, in our opinion, the answer is no since the series in our analysis include neither a measure of productivity in the foreign traded sector nor a measure of competition in the good and labor markets. However, in the presence of common factors, the answer is not so simple to give. Indeed, when common factors are added to the analysis, a measure of productivity changes in the foreign traded sector can be reflected in a linear combination of common factors, possibly yielding that a long-run relationship between $gdp_{i,t}$, $p_{i,t}$, and $tot_{i,t}$ can exist up to some common global trends represented by these factors.

Next, we discuss the issue from the statistical point of view. We test whether the series in our analysis are cointegrated or not with the panel cointegration test developed by Bai and Carrion-i Silvestre (2013), which allows cross-sectional dependence with
common factors. The model we consider can be written as

\[ gdp_{i,t} = \mu_i + X_{i,t}'\beta + f_t'\lambda_i + u_{i,t} \]

where \( X_{i,t} = \begin{bmatrix} \text{tot}_{i,t} & p_{i,t} \end{bmatrix}' \). This equation has the desirable feature that a cointegration relationship is tested in the presence of dynamic common factors, \( f_t \). Such factors in the model can substitute for some important missing variables not included in our analysis due to the lack of data such as unobservable global productivity changes affecting many countries. In addition, we test for cointegration under the assumption that idiosyncratic errors, \( u_{i,t} \), are independent across the economies and allow cross-section correlation in the error terms by means of common factors included in the error terms. Were such factors omitted from the errors, there would be no cross-section correlation in the error terms. This can be viewed as implausible given that global shocks frequently happening are likely to result in a large correlation in the error terms across economies.

The cointegration equation in (B.8) has the normalization that the coefficient on \( gdp_{i,t} \) in a possible long-run equilibrium relation is one since \( gdp_{i,t} \) is specified as the dependent variable in (B.8). As suggested by Hamilton (1994), it is important to test for cointegration with alternative normalizations. In our study, these alternative normalizations are the cointegration equations given by (B.8) with \( \text{tot}_{i,t} \) as the dependent variable and \( X_{i,t} = \begin{bmatrix} gdp_{i,t} & p_{i,t} \end{bmatrix}' \) or \( p_{i,t} \) as the dependent variable and \( X_{i,t} = \begin{bmatrix} gdp_{i,t} & \text{tot}_{i,t} \end{bmatrix}' \).

It is notable that the presence of a cointegration relationship between the series is studied with (B.8) under the assumption of homogeneous slopes. Indeed, \( \beta \) is assumed to be common across all economies of the same group, implying that a homogeneous cointegration relationship between the series is assumed. The assumption of homogeneous slopes in the cointegration equations is made to be consistent with the assumption of homogeneous slopes in the panel VAR model which we use in forecasting the effects of a productivity improvement on the terms of trade of advanced and developing economies, as discussed in the next section. Assuming homogeneous slopes in our panel VAR model can be defended by the fact that the models with homogeneous slopes are shown to have better forecast performance thanks to their parsimonious
representation and does not suffer from parameter estimate instability which occurs
in heterogeneous slope models due to estimating several parameters with short time
series; see Baltagi (2005, chapter 10).

Now, we can discuss the panel cointegration test statistics in our analysis, which
is based on the following equation:

\[
\Delta gdp_{i,t} = \Delta X_{i,t} \beta + \sum_{j=-1}^{1} \Delta^2 X_{i,t+j} A_j + \Delta f'_t \lambda_i + \Delta \xi_{i,t}
\]

(B.9)

\[i = 1, 2, \ldots, N; \ t = 3, 4, \ldots, T - 1\]

This equation can be regarded as the differenced form of the model given by (B.8)
with the additional term \(\sum_{j=-1}^{1} \Delta^2 X_{i,t+j} A_j\), which is added to the equation since we augment the model using
the dynamic least squares method, as in Bai and Carrion-i Silvestre (2013). (B.9) can
be rewritten more compactly as

\[
\Delta gdp_{i,t} = x'_{i,t} \delta + \Delta f'_t \lambda_i + \Delta \xi_{i,t}
\]

(B.10)

\[i = 1, 2, \ldots, N; \ t = 3, 4, \ldots, T - 1\]

where \(x_{i,t} = \begin{bmatrix} \Delta X_{i,t} & \Delta^2 X_{i,t+1} & \Delta^2 X_{i,t} & \Delta^2 X_{i,t-1} \end{bmatrix}\) and \(\delta = \begin{bmatrix} \beta & A_{-1} & A_0 & A_1 \end{bmatrix}'\). Let \(\Hat{\xi}_{i,t}\) be the least squares estimates of \(\sum_{s=3}^{t} \Delta \xi_{i,s}\)
from (B.9). As noted in Bai and Carrion-i Silvestre (2013), the least squares estimates
of \(\sum_{s=3}^{t} \Delta \xi_{i,s}\) can be obtained with an iterative procedure. Indeed, \(\Delta f_t\) can
be estimated for given \(\delta\). With this estimate of \(\Delta f_t\), \(\delta\) can be estimated and used in
the next iteration. The same steps in iteration are performed until convergence.

It is notable that in this iterative estimation, the number of common factors must
first be chosen. To decide on the number of common factors in the context of such
iterative estimation, Bai (2009) suggests two criteria, referred to as the IC and PC
criteria. We allow up to five common factors in estimation, as in Byrne, Fazio, and
Fiess (2013), and choose the number of common factors as the simple average of the
numbers of common factors chosen by the aforementioned two criteria.
Next, using computed $\hat{\xi}_{i,t}$, we define the $MSB_{\xi}(i)$ statistics as

$$MSB_{\xi}(i) = \frac{(T - 4)^2 \sum_{t=4}^{T-1} \hat{\xi}_{i,t-1}^2}{\hat{\sigma}_i^2}$$

(B.11)

where $\hat{\sigma}_i^2$ is the estimate of the long-run variance of $\xi_{i,t}$, which is obtained using the method discussed in (B.5).

Next, we define the $MSB_{\xi}$ statistics which is obtained by pooling $MSB_{\xi}(i)$ from (B.11) across the economies under the assumption of no cross-section correlation in the idiosyncratic errors:

$$MSB_{\xi} = \sqrt{N \frac{MSB_{\xi}(i) - \frac{1}{2}}{\sqrt{3}}}$$

(B.12)

with $MSB_{\xi}(i) = N^{-1} \sum_{i=1}^{N} MSB_{\xi}(i)$. To test for cointegration between the series in our analysis, we use the $MSB_{\xi}$ statistics, which has the standard normal distribution, as shown by Bai and Carrion-I-Silvestre (2009).

Now, we can discuss the results on cointegration. Panel A and Panel B in Table B.2 present the results for advanced and developing economies. The variables in the rows indicate the normalized variable in the cointegration equation, as discussed above. For each normalization, test for cointegration is performed with and without common factors. In the case that common factors are contained in the analysis, we report the number of common factors, estimated as pointed out above. Also, we compute the number stochastic trends in common factors with the $MQ_c$ test proposed by Bai and Ng (2004).

When common factors are contained in the analysis and $gdp_{i,t}$ is chosen as the normalized variable in (B.10), the null hypothesis that $\xi_{i,t}$ is of integrated order one can be rejected for advanced and developing economies at the 5% significance level (the p-values of the $MSB_{\xi}$ statistics are .004 and .000, respectively). However, with this normalization, we estimate there are four common factors having two stochastic trends for advanced economies and three common factors having three stochastic trends for developing economies. Consequently, we conclude that the variables are cointegrated only up to some global stochastic trends with this normalization.
Panel A: Advanced Economies

<table>
<thead>
<tr>
<th>Normalized Variable</th>
<th>With factors</th>
<th>Without factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$MSB_\xi$</td>
<td>$#$ of factors</td>
</tr>
<tr>
<td>$\text{gdp}_{i,t}$</td>
<td>-2.669</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>(.004)</td>
<td></td>
</tr>
<tr>
<td>$p_{i,t}'$</td>
<td>1.091</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(.862)</td>
<td></td>
</tr>
<tr>
<td>$\text{tot}_{i,t}$</td>
<td>2.509</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(.994)</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Developing Economies

<table>
<thead>
<tr>
<th>Normalized Variable</th>
<th>With factors</th>
<th>Without factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$MSB_\xi$</td>
<td>$#$ of factors</td>
</tr>
<tr>
<td>$\text{gdp}_{i,t}$</td>
<td>-7.968</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(.000)</td>
<td></td>
</tr>
<tr>
<td>$p_{i,t}'$</td>
<td>3.472</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(1.000)</td>
<td></td>
</tr>
<tr>
<td>$\text{tot}_{i,t}$</td>
<td>5.590</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(1.000)</td>
<td></td>
</tr>
</tbody>
</table>

Note: The numbers in parenthesis are the p-values of the $MSB_\xi$ statistics.

Table B.2: Panel Cointegration Test Results

Can there be a stationary linear combination of $\text{gdp}_{i,t}$, $p_{i,t}'$, and $\text{tot}_{i,t}$ when they are cointegrated up to some global stochastic trends? To answer this question, we first write from (B.9) that

$$ \text{gdp}_{i,t} - \mu_i + X_{i,t}\beta - \sum_{j=-1}^{1} \Delta X_{i,t+j} A_j - f_i'\lambda_i = \xi_{i,t} \quad (B.13) $$

While we find that $\xi_{i,t}$ in (B.13) is stationary when $\text{gdp}_{i,t}$ is the normalized variable, $\text{gdp}_{i,t} - \mu_i + X_{i,t}\beta$ is not. This results from $f_i$ including stochastic trends and $\lambda_i$ differing across economies. Consequently, even when we find $\xi_{i,t}$ is stationary, there cannot be a linear combination of the variables which is stationary if common factors have stochastic trends.
For the alternative normalizations for cointegration with common factors, we find that the $MSB_{\xi}$ statistics have large p-values. For example, with $p_{i,t}^{x}$ as the normalized variable, the p-values of the estimated $MSB_{\xi}$ statistics are .862 and 1.000 for advanced and developing economies, respectively, implying that the null that $\xi_{i,t}$ contains a unit root is true with a large probability. Consequently, we find no cointegration in this case even when global stochastic trends in common factors are allowed in cointegration. Similar results hold when $tot_{i,t}$ is chosen as the normalized variable, as is evident from Table B.2.

Lastly, we discuss the $MSB_{\xi}$ statistics from the model omitting common factors in (B.9). The results from this model should be interpreted with caution since the $MSB_{\xi}$ statistics are obtained by pooling individual $MSB_{\xi}$ ($i$) statistics from (B.11) under the implausible assumption of no cross-section correlation in the error terms. In contrast, the model with common factors has $\Delta f'_{t} \lambda_{i}$ in the error terms resulting in them being correlated across economies. With the caution in mind, the results from the model without common factors indicate that regardless of the normalized variable in the cointegration equation, the $MSB_{\xi}$ statistics have a large p-value. Consequently, we find that the hypothesis of no cointegration between the series is quite likely to hold in this model.
C Calculating Standard Errors for Impulse Responses

In this section, we describe the Monte Carlo method using which the standard errors for impulse responses to a positive idiosyncratic shock are calculated.

First, let $\xi_{i,t}$, $z_{i,t}$, $\beta$, and $F_t$ are defined by

\[
\begin{align*}
    z_{i,t} &= \begin{bmatrix} I_3 & I_3 \otimes \xi_{i,t}' \end{bmatrix}; \\
    \beta &= \begin{bmatrix} a_0' & \alpha_1 \Delta gdp' & \cdots & \alpha_k \Delta gdp' & \alpha_1 \Delta p_x' & \cdots & \alpha_k \Delta p_x' & \alpha_1 \Delta tot' & \cdots & \alpha_k \Delta tot' \end{bmatrix}'; \\
    \xi_{i,t} &= \begin{bmatrix} \Delta Y_{i,1} & \cdots & \Delta Y_{i,T} \end{bmatrix}' ; \\
    F_t &= \begin{bmatrix} I_3 \otimes f_t' \end{bmatrix}
\end{align*}
\]

Our panel vector autoregression model with interactive fixed effects in (2.3) can be written more compactly as

\[
\Delta \gamma_{i,t} = z_{i,t} \beta + F_t \lambda_i + u_{i,t} \quad \text{(C.1)}
\]

Stacking economy $i$’s observations in (C.1) yields

\[
\Delta \gamma_i = Z_i \beta + F \lambda_i + u_i \quad \text{(C.2)}
\]

where

\[
\begin{align*}
    \Delta \gamma_i &= \begin{bmatrix} \Delta \gamma_{i,1} & \cdots & \Delta \gamma_{i,T} \end{bmatrix}' ; \\
    Z_i &= \begin{bmatrix} z_{i,1}' & \cdots & z_{i,T}' \end{bmatrix}' ; \\
    \lambda_i &= \begin{bmatrix} \lambda_{i,1} & \cdots & \lambda_{i,T} \end{bmatrix}' ; \\
    F &= \begin{bmatrix} F_{i,1}' & \cdots & F_{i,T}' \end{bmatrix}' ; \\
    u_i &= \begin{bmatrix} u_{i,1}' & \cdots & u_{i,T}' \end{bmatrix}'
\end{align*}
\]

The least squares estimates $\hat{\beta}$ and $\hat{f_t}$ can iteratively be computed, as discussed in Tugan (2018). Indeed, for given $F$, $\hat{\beta}$ can be written as

\[
\hat{\beta} = \left( \sum_{i=1}^{C} Z_i' M_F Z_i \right)^{-1} \sum_{i=1}^{C} Z_i' M_F Y_i \quad \text{(C.4)}
\]
with $M_F = I_T - F \left( F'F \right)^{-1} F'$. Next, using $\widehat{\beta}$, $\widehat{f}_t$ can be obtained as the eigenvectors corresponding to the largest $r$ eigenvectors of $1/T \left( \sum_{i=1}^C \widehat{v}_i \widehat{v}_i' / 3C \right)$:

$$
\begin{bmatrix}
\frac{1}{T} \left( \sum_{i=1}^C \widehat{v}_i \widehat{v}_i' / 3C \right)
\end{bmatrix}
\widehat{f} = \widehat{f} \widehat{\mu}_{vv}' 
$$

(C.5)

where $\widehat{\mu}_{vv}'$ is a diagonal matrix whose diagonal elements are given by the eigenvalues of $1/T \left( \sum_{i=1}^C \widehat{v}_i \widehat{v}_i' / 3C \right)$ with $\widehat{v}_i$ and $\widehat{f}$ defined by

$$
\begin{bmatrix}
\Delta Y_{i,1} - z_{i,1} \widehat{\beta} \\
\Delta Y_{i,T} - z_{i,T} \widehat{\beta}
\end{bmatrix} \\
\begin{bmatrix}
\widehat{f}_1 \\
\widehat{f}_T
\end{bmatrix}'
$$

(C.6)

Beginning with $F = 0$, the steps above are repeated many times until the estimates converge. However, in this iterative estimation, the number of factors denoted by $r$ must first be chosen. Bai (2009) suggest the IC and PC criteria for choosing the number of common factors in the context of such an iterative procedure. We choose $r$ as the simple average of the numbers of common factors chosen by the aforementioned two criteria allowing up to five common factors. For known $r$, Tugan (2018) shows that under some minimal assumptions

$$
\sqrt{T C} \left( \widehat{\beta} - \beta_0 \right) = N \left( \widehat{B}_T + \widehat{B}_B, \mathcal{D}(\widehat{F}, \widehat{\lambda})^{-1} \mathcal{D}(\widehat{F}, \widehat{\lambda})^{-1}' \right) + o_p(1) 
$$

(C.7)

where $\mathcal{D}(\widehat{F}, \widehat{\lambda})$ is defined by

$$
\begin{align*}
\mathcal{D}(\widehat{F}, \widehat{\lambda}) &= \left( \frac{1}{3TC} \sum_{i=1}^C Z_i' M_F Z_i - \frac{1}{9T^2C^2} \sum_{i,c=1}^C Z_i' M_F \left[ I_T \otimes \widehat{\lambda}' \left( \frac{\widehat{\lambda} \widehat{\lambda}}{3C} \right)^{-1} \right] Z_c \right) \\
\end{align*}
$$

(C.8)

with

$$
\begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3 & \cdots & \lambda_C
\end{bmatrix}
$$

(C.9)

and

$$
\widehat{B}_T \text{ is defined by}
$$

$$
\begin{align*}
\widehat{B}_T &= -\mathcal{D}(\widehat{F}, \widehat{\lambda})^{-1} \left( \frac{1}{\sqrt{T}} \sum_{i=1}^C \sum_{t=1}^{T-1} \sum_{r=1}^{3C} \left( z_{i,t} \widehat{u}_{i,t} \right) \left( \frac{\widehat{f} \widehat{f}'}{T} \right)^{-1} \widehat{f}_t \right)
\end{align*}
$$

(C.10)
\[ \hat{u}_{i,t} = \Delta y_{i,t} - z_{i,t} \tilde{\beta} - (I_3 \otimes \tilde{f}) \tilde{\lambda}_i \]  
(C.11)

\[ \hat{B}_\Phi = \mathcal{D}(\tilde{F}, \tilde{\lambda})^{-1} (\hat{\Psi}^* - \hat{\Psi}^{**}) \]  
(C.12)

\[ \hat{\Psi}^* = -\sqrt{\frac{1}{3}} \frac{1}{T C} \sum_{j=1}^{C} \left( Z_j' \hat{F} \right) \left( I_N \otimes \left( \frac{\tilde{\lambda}}{3C} \right)^{-1} \right) \left( \frac{1}{T C} \sum_{j=1}^{C} \sum_{t=1}^{T} \hat{u}_{i,t} \otimes \hat{u}_{i,t} \right) \]  
(C.13)

\[ \hat{\Psi}^{**} = -\sqrt{\frac{1}{3}} \frac{1}{T C} \sum_{j=1}^{C} \sum_{k=1}^{C} Z_j' \hat{F} \left( \tilde{\lambda}_k' \left( \frac{\tilde{\lambda}}{3C} \right) \right)^{-1} \left( \frac{1}{T C} \sum_{j=1}^{C} \sum_{t=1}^{T} \hat{u}_{i,t} \otimes \hat{u}_{i,t} \right) \]  
(C.14)

Lastly, \( \hat{\Omega} \) is defined by

\[ \hat{\Omega} = \frac{1}{T C} \sum_{j=1}^{C} \sum_{t=1}^{T} \tilde{z}_{j,t} \hat{u}_{i,t} \otimes \hat{u}_{i,t} \]  
(C.15)

with

\[ \Gamma_{j,t} = \frac{1}{3} \left( Z_j' \tilde{M} \hat{F} - \frac{1}{3C} \sum_{k=1}^{C} Z_k' \tilde{M} \hat{F} \left( I_T \otimes \hat{\lambda}_k' \left( \frac{\tilde{\lambda}}{3C} \right)^{-1} \right) \right) \left( I_T \otimes I_3 \right) \]  
(C.16)

where \( I_T^l \) is the \( l \)th column of \( I_T \).

To compute the 68% confidence bands, we use a Monte Carlo method: let \( \beta_n \) be a random draw from

\[ N \left( \hat{\beta} - \frac{\tilde{B}_\Phi + \hat{B}_\Phi}{\sqrt{TC}}, \frac{1}{T C} \mathcal{D}(\tilde{F}, \tilde{\lambda})^{-1} \hat{\Omega} \mathcal{D}(\tilde{F}, \tilde{\lambda})^{-1} \right) \]  
(C.17)

Next, \( F^n \) and \( \Sigma^n \) corresponding to \( \beta^n \) are obtained using (C.5) and (C.11). Then, using \( \beta^n \) and \( \Sigma^n \), the impulse response functions to a positive productivity shock are obtained using the max-share approach over the 10-year horizon, as described in section 2.3. The number
of random draws we use in estimating the confidence bands is 500. We sort the impulse responses from these 500 impulse responses over all horizons we compute impulse responses. The lower and upper confidence bands for each horizon displayed in the figures corresponds to the $16 \times 500^{th}$ and $84 \times 500^{th}$ of the sorted impulse responses, respectively.
References


